$\mathcal{N} = 1$ Susy Field Theories on Four Manifolds

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arXiv:1105.0689 Seiberg, GF.

arXiv:1205.1115 Dumitrescu, Seiberg, GF.

TBA, Closset, Dumitrescu, Komargodski, Shamir, GF.

Why Susy in curved space

We can get a new handle on the dynamics of strongly coupled supersymmetric field theories by studying them in curved space.

Susy allows to compute exactly many interesting observables:

- Partition function Z on a compact manifold $\mathcal{M}$.
- Expectation value of supersymmetric operators.

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For instance:

- The partition function on $S^3 \times S^1$ of $\mathcal{N} = 1$ theories with a $U(1)_R$ symmetry. [Romelsberger;...]
- Wilson loops for $\mathcal{N} = 2$ theories on S^4. [Pestun;...]
- The partition function on S^3 of $\mathcal{N} = 2$ theories with a $U(1)_R$ symmetry.[Kapustin, Willett, Yaakov; Jafferis ...]

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Which Riemannian manifolds $\mathcal{M}$ allow for Susy?

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In this talk I will consider $\mathcal{N} = 1$ field theories with a $U(1)$ R-symmetry in 4D.

Susy on Curved Manifolds

There is no unique prescription to place a flat space theory on a manifold $\mathcal{M}$. In general this breaks all Susy.

In a systematic approach we couple a Susy theory to frozen Sugra background fields. The Sugra background fields include auxiliary fields. We do not impose their e.o.m.

Susy is preserved if, on the selected background, the gravitino variation $\delta_\zeta \psi_{\mu\alpha} = 0$.

New Minimal Sugra

An $\mathcal{N} = 1$ theory with a $U(1)_R$ symmetry has a 12+12 multiplet

$S_{\mu\alpha}$ $T_{\mu\nu}$ j^R_μ $C_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}\partial^\rho \mathcal{A}^\sigma$.

see [Komargodski, Seiberg]

It couples to the fields in New Minimal Sugra:

ψ^μ_α $g_{\mu\nu}$ A_μ V^μ.

In the Rigid Limit we set $\psi^\mu_\alpha = 0$ and freeze the metric and auxiliary fields to arbitrary background values.

$$\delta_\zeta \psi_{\mu\alpha} = 0 \quad \Rightarrow \quad (\nabla_\mu - iA_\mu)\zeta = -iV_\mu\zeta - iV^\nu \sigma_{\mu\nu}\zeta$$

If the theory has a global symmetry group G we can also couple the corresponding current multiplet to an background gauge multiplet.

Susy on Complex $\mathcal{M}$

A necessary and sufficient condition for one supercharge Q_ζ is that $\mathcal{M}$ is complex with an Hermitian metric.
see also [Klare, Tomasiello, Zaffaroni]

This supercharge satisfies $\{Q_\zeta, Q_\zeta\} = 0$.

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When the manifold is locally a torus T^2 fibered over a Riemann surface Σ there are two supercharges (of opposite R-charge).

As an example $S^3 \times S^1$ is an Hopf fibration of T^2 over S^2. If the S^3 is round we can preserve 4 supercharges.

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What data does the partition function $Z_{\mathcal{M}}$ of an $\mathcal{N} = 1$ theory with a $U(1)$ R-symmetry on a compact complex manifold $\mathcal{M}$ depend on?

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- $Z_{\mathcal{M}}$ is holomorphic in the moduli parametrizing the choice of holomorphic G bundle.
- For manifolds which are locally fibrations of T^2 over Σ there is no dependence on changes in the c.s. over Σ.

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A change of Hermitian metric corresponds to turning on $\delta g^{i\bar{j}}$.

The other background fields A_μ and V_μ also have to vary to preserve susy.

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All these variations couple to Q_ζ exact operators. Hence they don't change Z.

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A change of complex structure gives $\delta J^i{}_{\bar{j}}$ with $\partial_{\bar{i}}\delta J^i{}_{\bar{j}} - \partial_{\bar{j}}\delta J^i{}_{\bar{i}} = 0$. It also implies a change in the metric δg^{ij} (and c.c) and V_μ, A_μ

The variations corresponding to $\delta J^{\bar{i}}{}_j$ are Q_ζ exact but not the ones for $\delta J^i{}_{\bar{j}}$

Example

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The moduli space of complex structures on $S^3 \times S^1$ is parametrized by two complex variables s, t. [Kodaira,Spencer]

The parameters p, q are simply related to (s, t).

The $U(1)$ fugacities correspond to the moduli of holomorphic $U(1)$ bundles over $S^3 \times S^1$.

Conclusions

For $\mathcal{N} = 1$ theories with a U(1) R-symmetry in 4D we can preserve at least one supercharge on any Hermitian manifold.

The partition function $Z_{\mathcal{M}}$ computes invariants of the complex structure and of the holomorphic G bundles.

Similarly for $\mathcal{N} = 2$ theories (with a U(1) R-symmetry) in 3D, the partition function depends on the choice of a certain integrable almost contact structure.

Open Questions

Extension to different number of dimensions or $\mathcal{N} = 2$ in 4D.
What are the relevant geometrical structures?

Extend discussion to include more observables.

Exploit meromorphy to evaluate Z on any Hermitian $\mathcal{M}$.

When is the answer completely topological?

Thank You!