Holographic thermalization patterns

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Motivation

Quark gluon plasma

- Produced in heavy collisions at RHIC and LHC
- Behaves as a strongly coupled liquid
- Thermalization process not well understood: $\tau < 1 fm/c$

Goals

- Gain insight into the thermalization process
- Modification of production rates of photons
- Modification of energy momentum tensor correlators
- Which modes thermalize first: top-down or bottom-up ?
- Dependence on coupling strength

Strategy

- SYM where strong and weak coupling regimes are accessible
- AdS/CFT: strongly coupled N=4 SYM is dual to classical SUGRA in AdS space

Outline

Photons in N=4 SYM plasma

- Motivation
- Equilibrium properties at infinite and finite coupling
- Out of equilibrium at $\lambda = \infty$
- Out of equilibrium at $\lambda \neq \infty$

} thermalization scenarios

Plasma constituents

- Properties of energy momentum tensor correlators

Thermalization scenarios

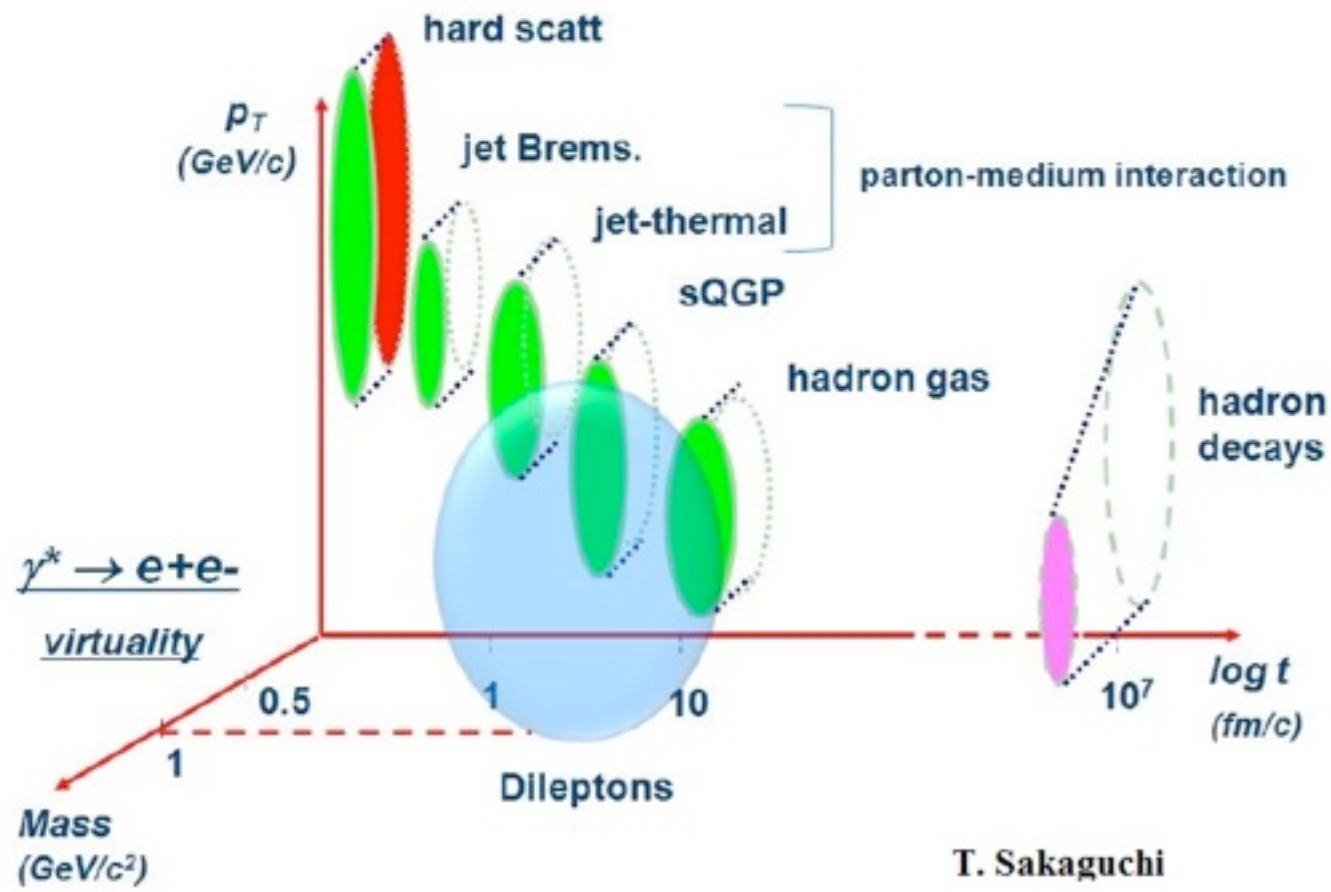
Bottom up scenario

- At weak coupling
- Scattering processes
  - In the early stages many soft gluons are emitted which then thermalize the system (*Baier et al (2001)*)
  - Supported by classical Yang-Mills simulations *(Berges et al (2013))*
- Driven by instabilities
  - Instabilities isotropize the momentum distributions more rapidly than scattering processes (*Kurkela, Moore (2011)*)

Top down scenario

- At strong coupling
- UV modes thermalize first
- In AdS calculations, follows naturally from causality

Photon emission in heavy ion collisions



Photons are emitted at all stages of the collision

- Initial hard scattering processes: quark anti-quark annihilation:
  - on-shell photon or virtual photon → dilepton pair
- Strongly coupled out of equilibrium phase: no quasiparticle picture
- Additional (uninteresting) emissions from charged hadron decays

Probing the plasma

Probing the plasma

- Once produced photons stream through the plasma almost unaltered
- Provide observational window in the thermalization process of the plasma

Quantity of interest

- Spectral density : $\chi^{\mu}_{\mu} = -2\mathrm{Im}(\Pi^{\mathrm{ret}})^{\mu}_{\mu}(k_0)$
- Number of emitted photons

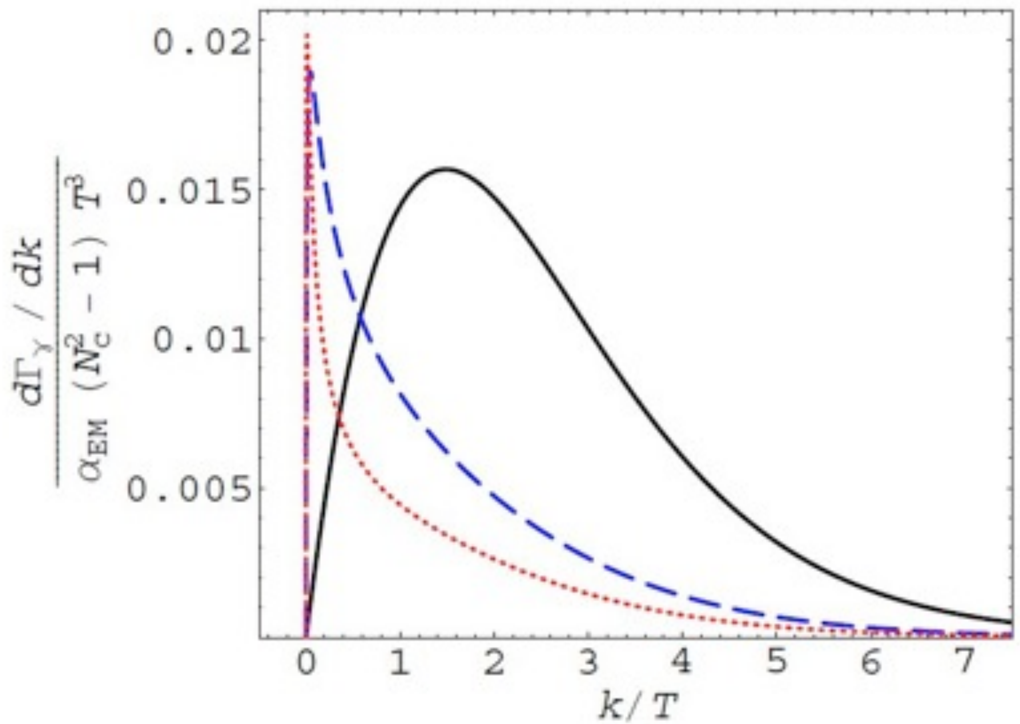
Fluctuation dissipation theorem

$$\eta^{\mu\nu}\Pi^{<}_{\mu\nu}(\omega) = -2n_B(\omega)\mathrm{Im}(\Pi^{ret})^{\mu}_{\mu}(\omega) = n_B(\omega)\chi(\omega)$$

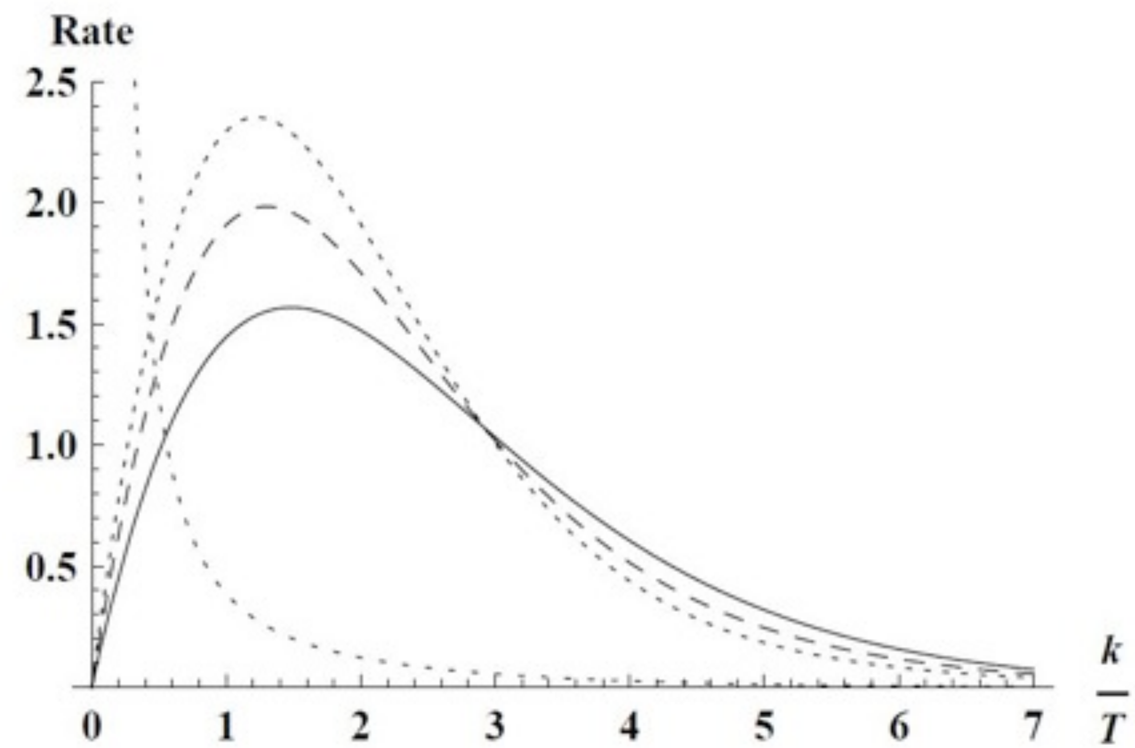
Production rate

$$k^0\frac{d\Gamma_\gamma}{d^3k} = \frac{\alpha}{4\pi^2}\eta^{\mu\nu}\Pi^{<}_{\mu\nu}(\omega = k^0)$$

Photon emission in equilibrium SYM plasma



Huot et al (2006)



Hassanain, Schvellinger (2012)

Perturbative result

- Increasing the coupling: slope at k=0 decreases, hydro peak broadens and moves right

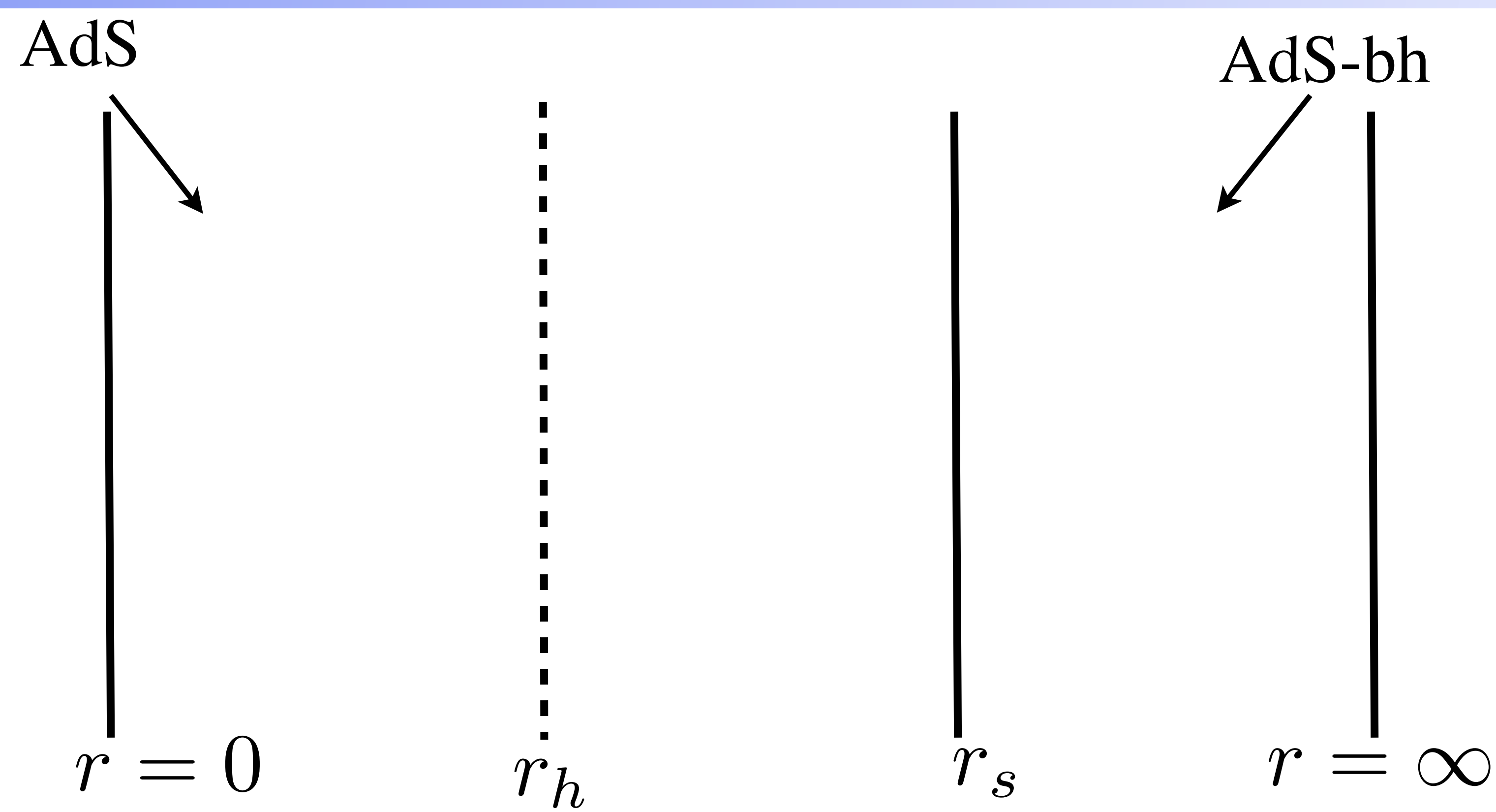
Strong coupling result

- Decreasing coupling from $\lambda = \infty$: peak sharpens and moves left

Equilibrium summary

- Equilibrium picture in SYM fairly complete
- How does photon/dilepton production get modified out of equilibrium
- Can one access thermalization at finite coupling?

The falling shell setup



Danielsson, Keski-Vakkuri, Kruczenski (1999)

Outside and inside spacetime

- metric: $$ds^2 = \frac{(\pi T L)^2}{u}\left(f(u)dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{L^2}{4u^2 f(u)}du^2 \qquad u = \frac{r_h^2}{r^2}$$

$$f(u) = \begin{cases} f_+(u) = 1 - u^2\,, & \text{for } u > 1 \\ f_-(u) = 1\,, & \text{for } u < 1 \end{cases} ,$$

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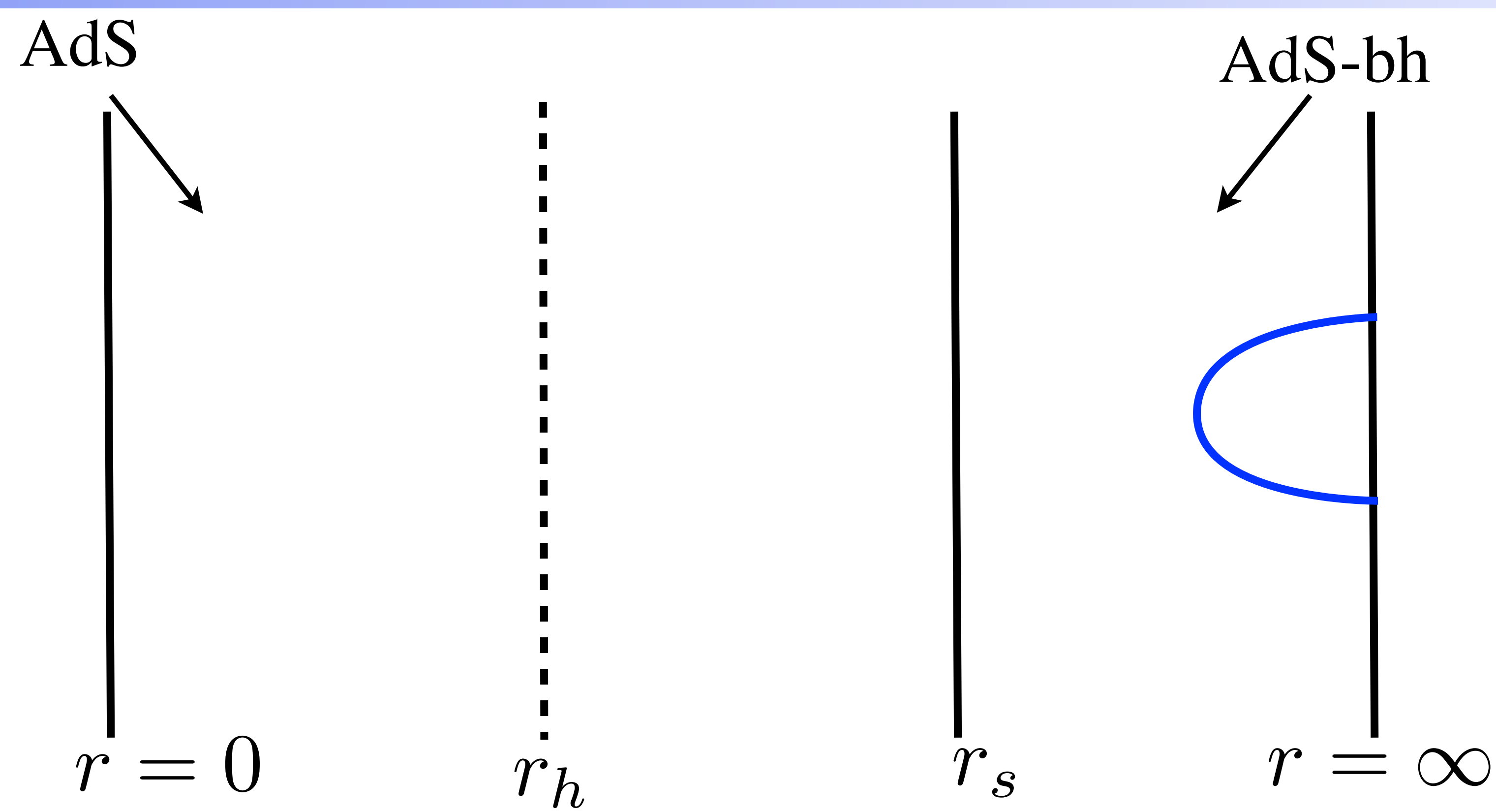

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Thermalization from geometric probes:

- Entanglement entropy and Wilson loop: always top down thermalization

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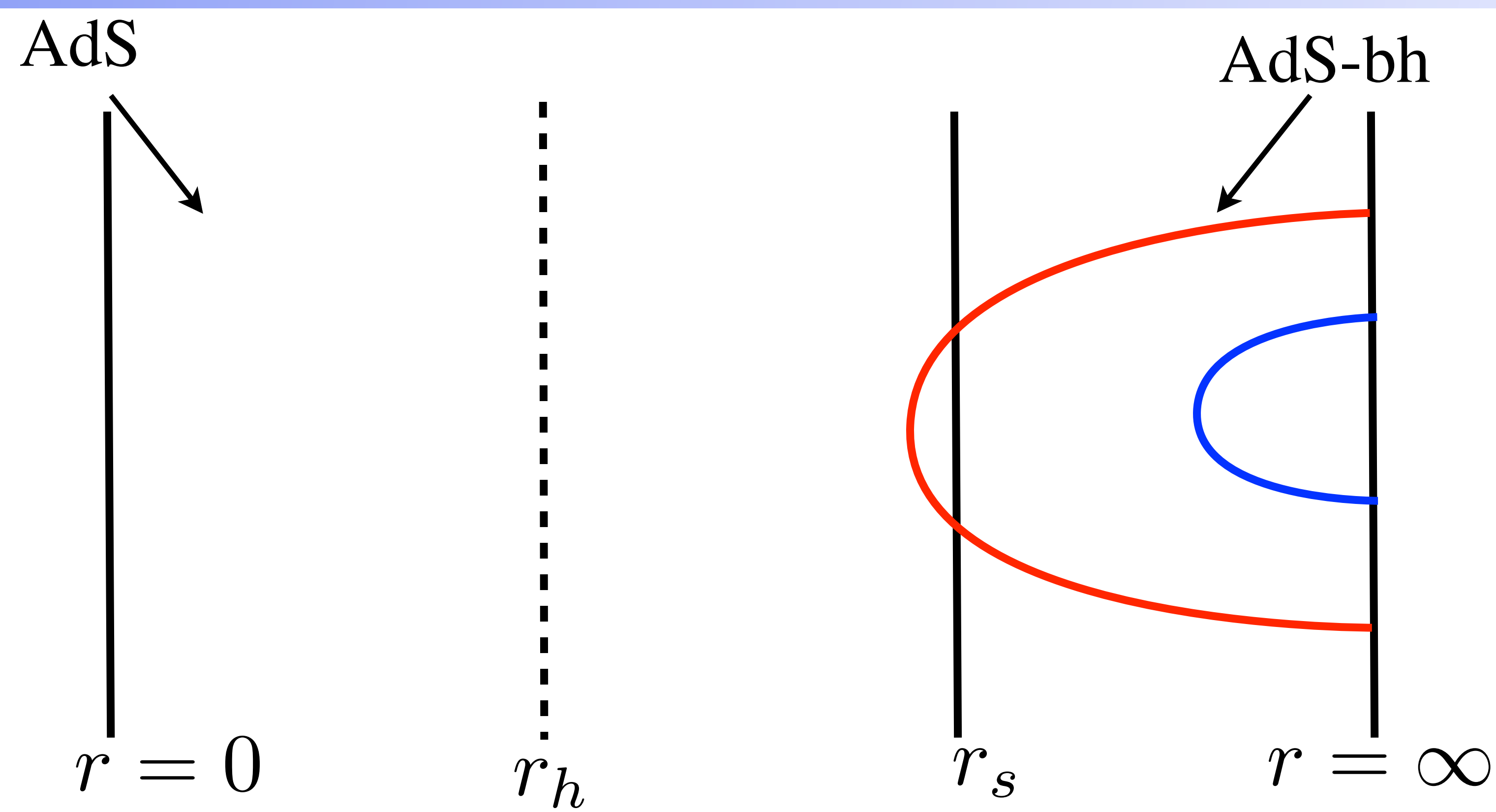

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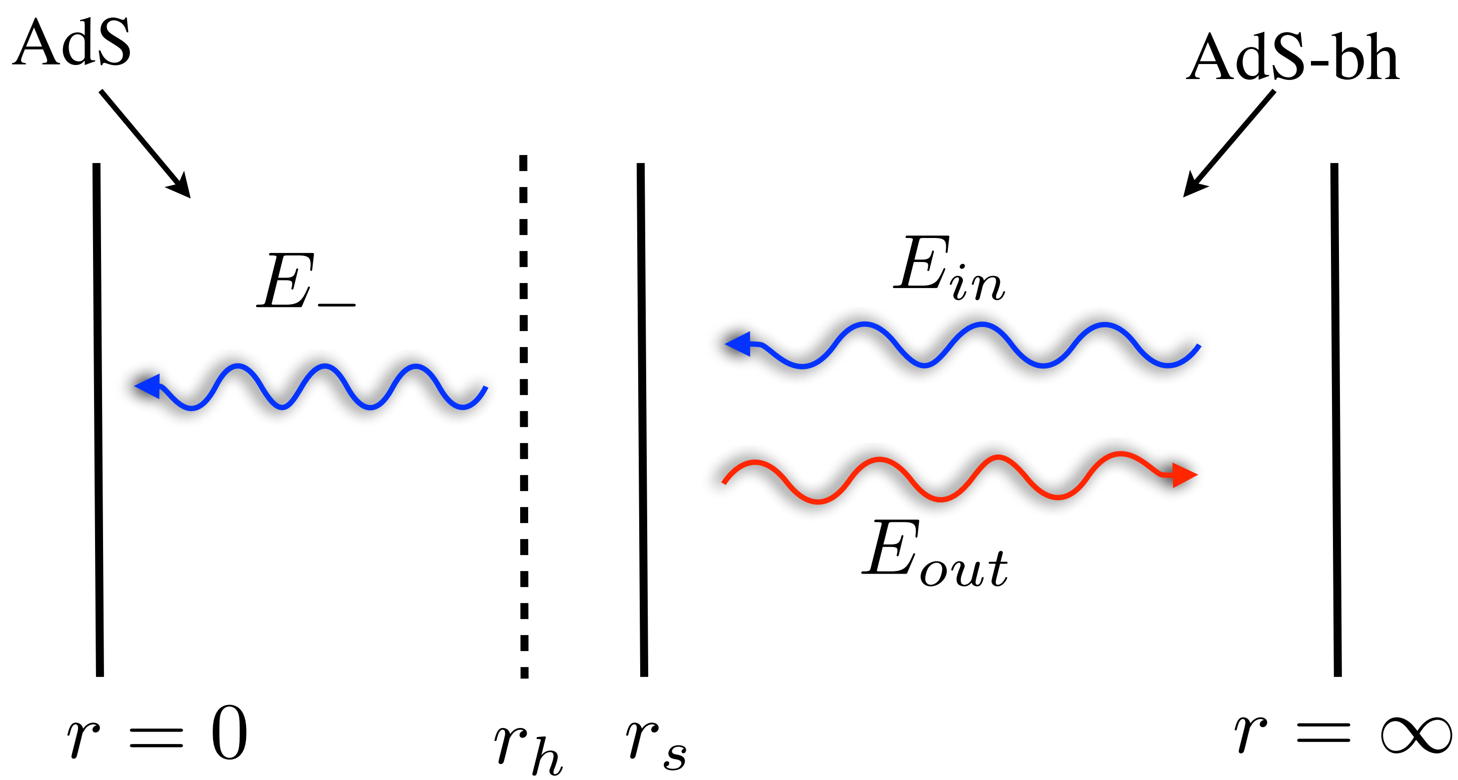

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$$f(u) = \begin{cases} f_+(u) = 1 - u^2\,, & \text{for } u > 1 \\ f_-(u) = 1\,, & \text{for } u < 1 \end{cases},$$

Outside solution

$$E_+ = c_+ E_{in} + c_- E_{out}$$

Holographic Green's functions

Off-equilibrium correlators offer a useful window to thermalization:

- Probe how different energy scales approach equilibrium
- Related to measurable quantites, e.g. production rates

Some computational details

- Solve classical EoM for bulk electric field E inside and outside the shell
- Matching conditions: $$\begin{aligned} E_-(\omega_-)|_{u_s} &= \sqrt{f_m} E_+(\omega_+)|_{u_s}, \\ E'_-(\omega_-)|_{u_s} &= f_m E'_+(\omega_+)|_{u_s}. \end{aligned} \quad \Longrightarrow \quad \left.\frac{c_-}{c_+}\right|_{u_s}$$
- Use conventional methods to obtain retarded correlator

$$\Pi(\omega, \mathbf{q}) = -\frac{N_c^2 T^2}{8} \lim_{u \to 0} \frac{E'(u,Q)}{E(u,Q)} = -\frac{N_c^2 T^2}{8} \Pi_{therm} \frac{1 + \frac{c_-}{c_+} \frac{E'_{out}}{E'_{in}}}{1 + \frac{c_-}{c_+} \frac{E_{out}}{E_{in}}}$$

- Behaviour of c_-/c_+ crucial for out of equilibrium dynamics

Quasistatic approximation:

- Energy scale of interest >> characteristic time scale of shell's motion

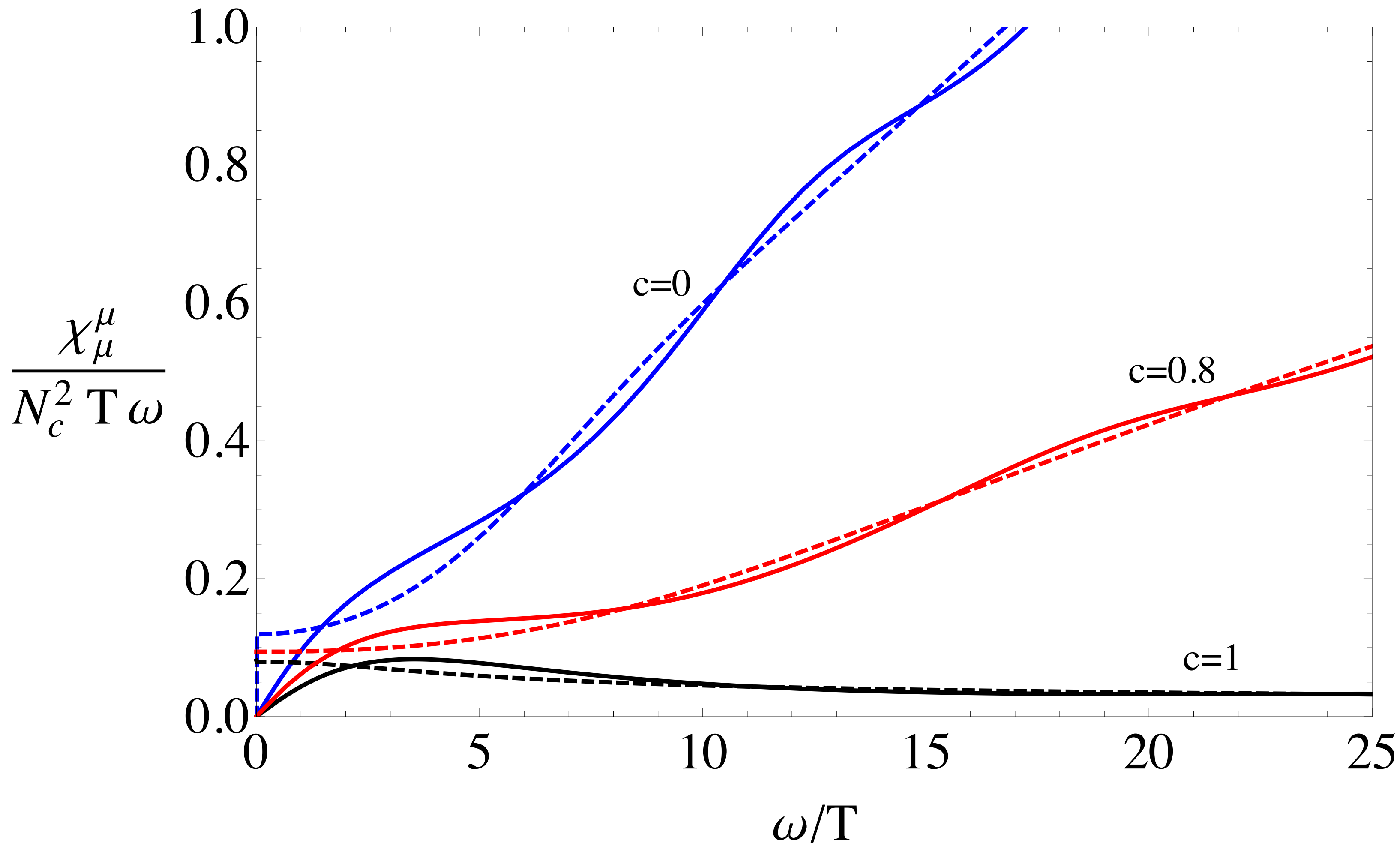
Photon spectral density

- virtuality

$$v = \frac{\hat{\omega}^2 - \hat{q}^2}{\hat{\omega}^2}$$

- parametrize

$$q = c\,\hat{\omega}$$



spectral density for $r_s/r_h = 1.1$ for different virtualities

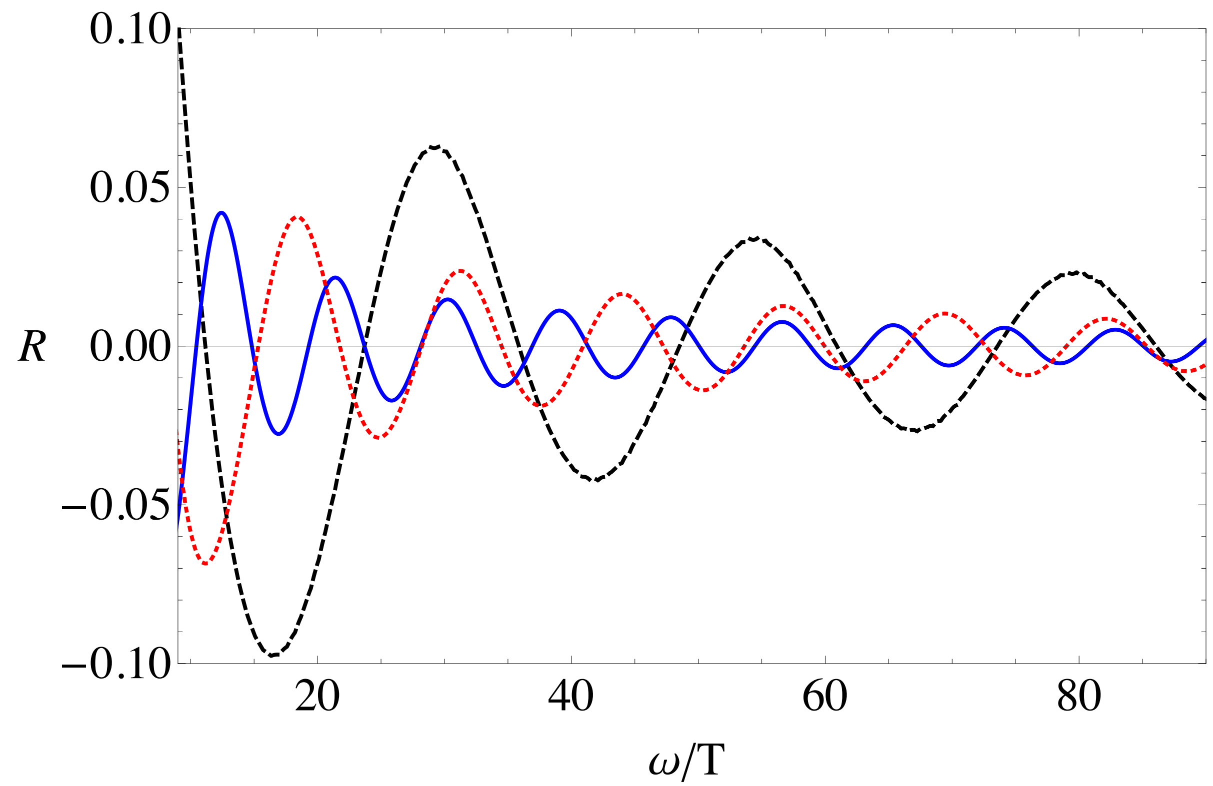
- Out of equilibrium effect: oscillations around thermal value

- As the shell approaches the horizon equilibrium is reached

Relative deviation of spectral density

- Relative deviation from thermal equilibrium

$$R(\hat{\omega}) = \frac{\chi(\hat{\omega}) - \chi_{th}(\hat{\omega})}{\chi_{th}(\hat{\omega})}$$

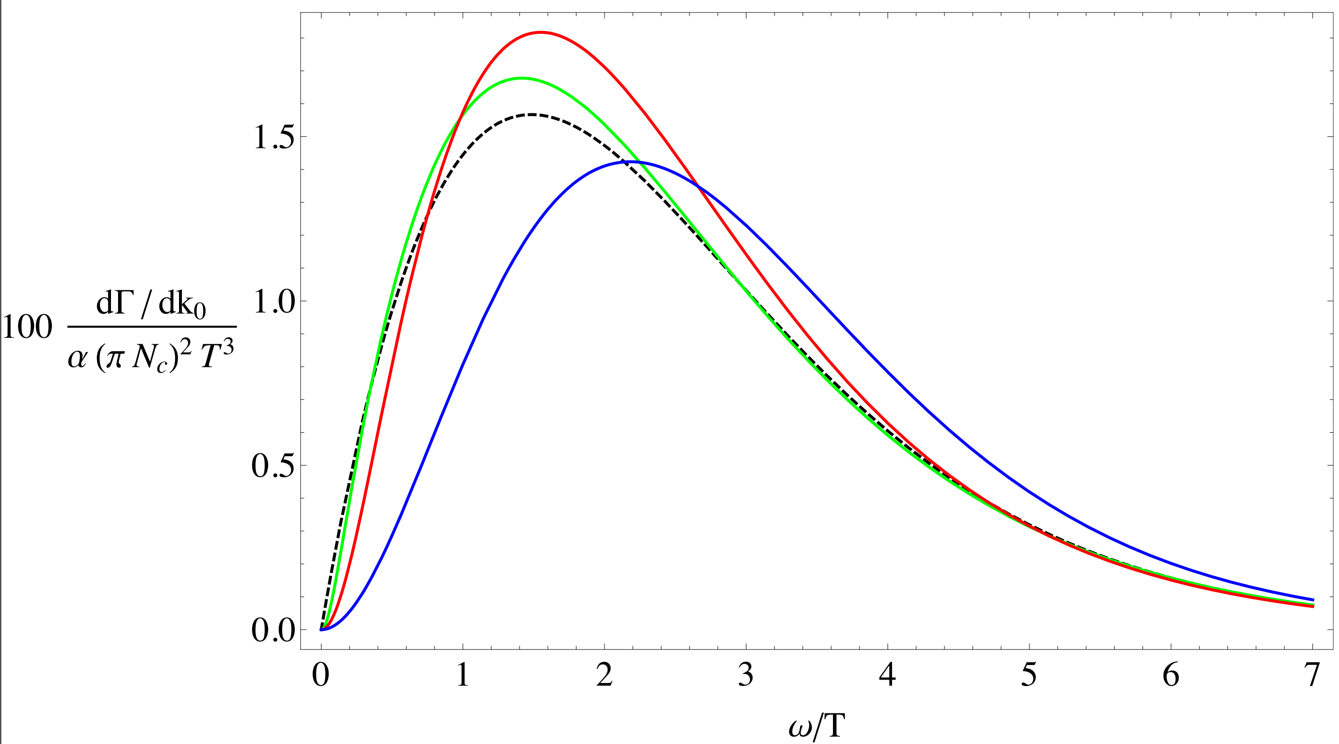


relative deviation R for r_s=1.1 and c=1, 0.8, 0

- Top down thermalization: highly energetic modes are closer to equ. value
- Highly virtual field modes thermalize first

$$\chi(\hat{\omega}) \approx \hat{\omega}^{\frac{2}{3}} \left(1 + \frac{f_1(u_s)}{\hat{\omega}}\right), \qquad R \approx \frac{1}{\hat{\omega}}$$

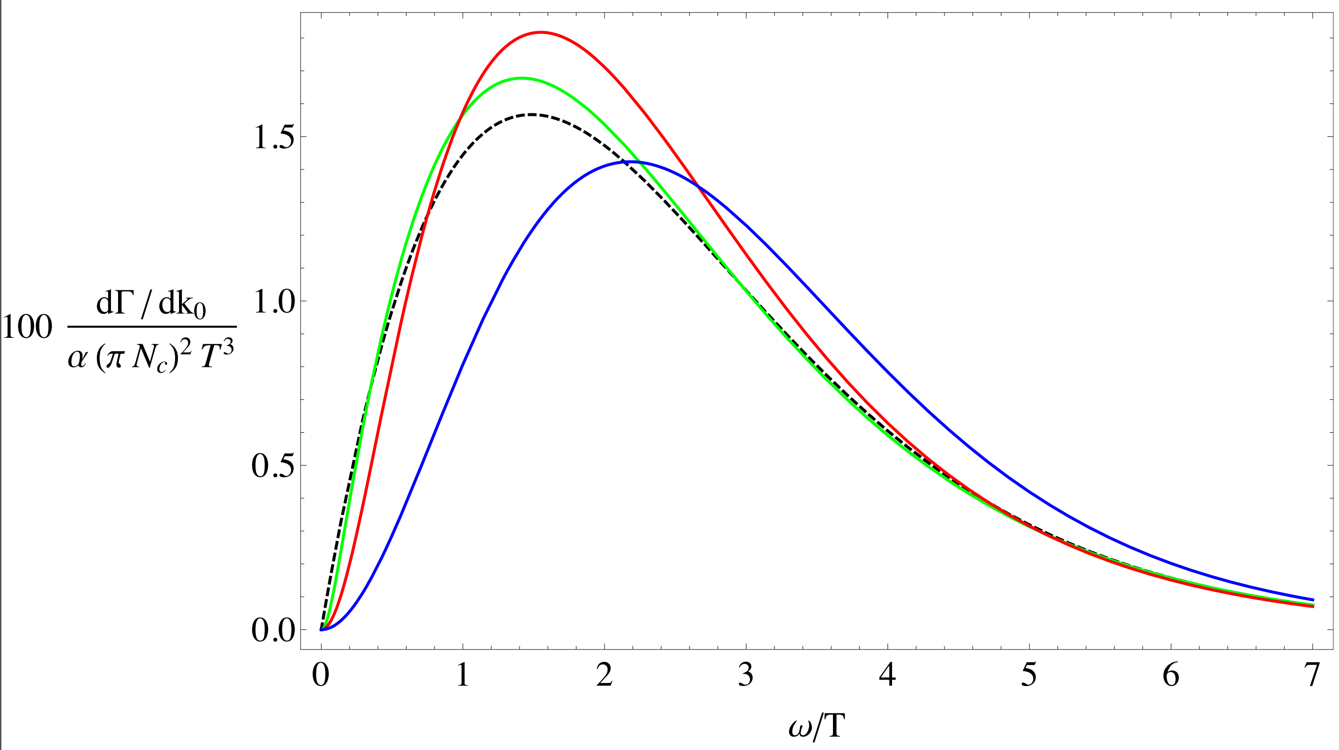
Photon production rate at infinite coupling

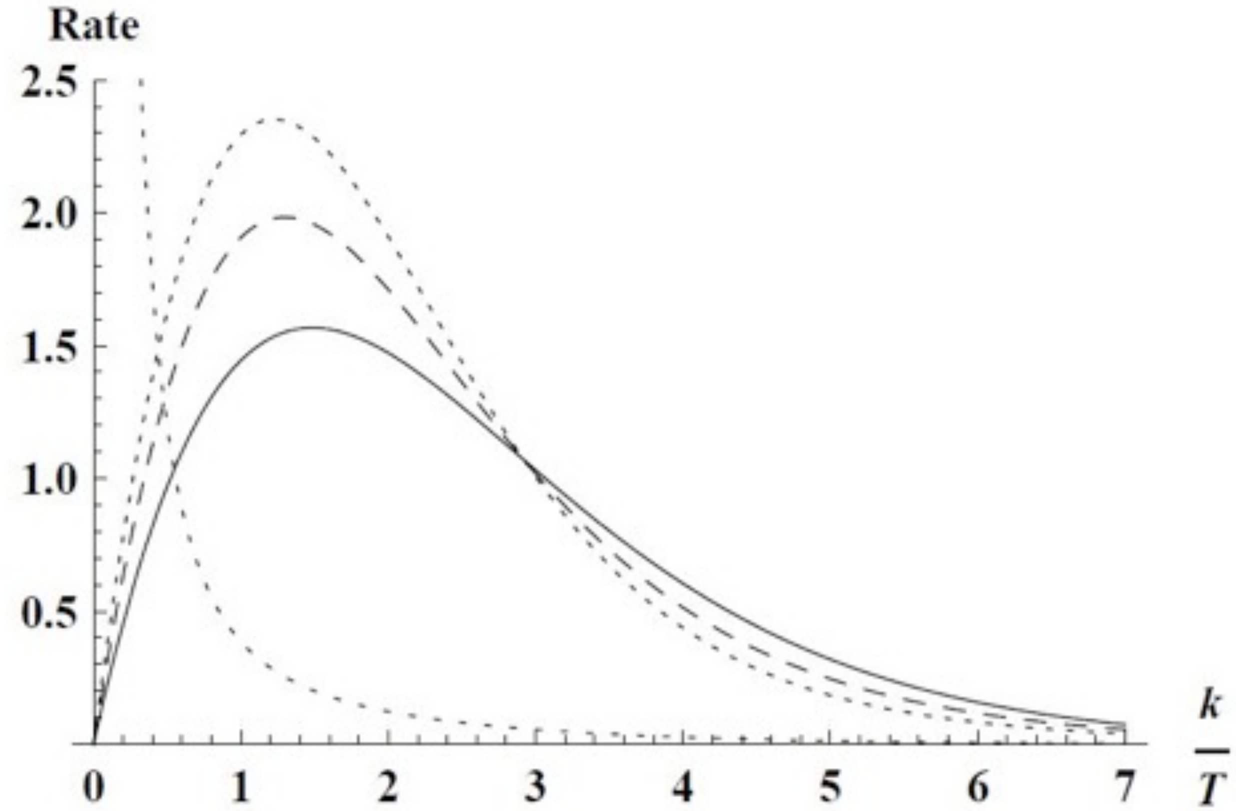


photon production rate for r_s/r_h=1.1, 1.01, 1.001

- Enhancement of production rate
- Hydro peak broadens and moves right
- Apparently no dramatic observable signature in off-equilibrium photon production

Photon production rate at infinite coupling





photon production rate for r_s/r_h=1.1, 1.01, 1.001

- Enhancement of production rate
- Hydro peak broadens and moves right
- Apparently no dramatic observable signature in off-equilibrium photon production
- Combining the two allows to study thermalization at finite coupling!

Finite coupling corrections

Key relation in AdS/CFT: $(L/l_s)^4 = L^4/\alpha'^2 = \lambda$

- Go beyond $\lambda = \infty$: add α' terms to SUGRA action, i.e. first non trivial terms in a small curvature expansion
- Leading order corrections: $\mathcal{O}(\alpha'^3) = \mathcal{O}(\lambda^{-3/2})$

Gubser et al; Pawelczyk, Theisen (1998)

Improved type IIB SUGRA action:

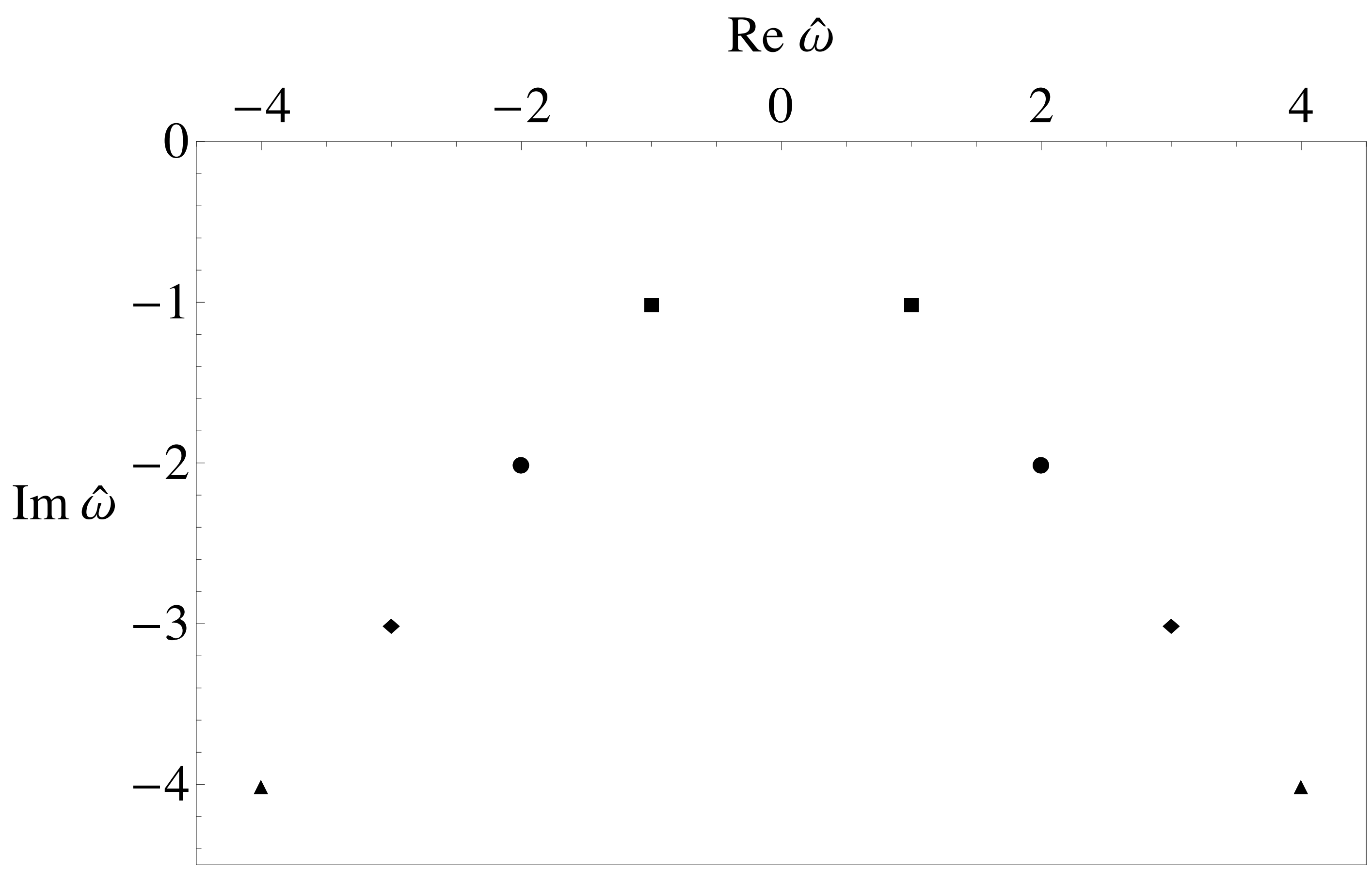
$$S^0_{IIB} = \frac{1}{2\kappa_{10}} \int d^{10}x\sqrt{-g}\left(R_{10} - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4.5!}(F_5)^2 + \gamma e^{\frac{-3}{2}\phi}(C+\mathcal{T})^4\right)$$

$$\mathcal{T}_{abcdef} = i\nabla_a F^+_{bcdef} + \frac{1}{16}\left(F^+_{abcmn}F^{+\ mn}_{def} - 3F^+_{abfmn}F^{+\ mn}_{dec}\right), \quad \gamma \equiv \frac{1}{8}\zeta(3)\lambda^{-\frac{3}{2}}$$

Paulos (2008)

- Leads to γ-corrected metric
- EoM for different fields

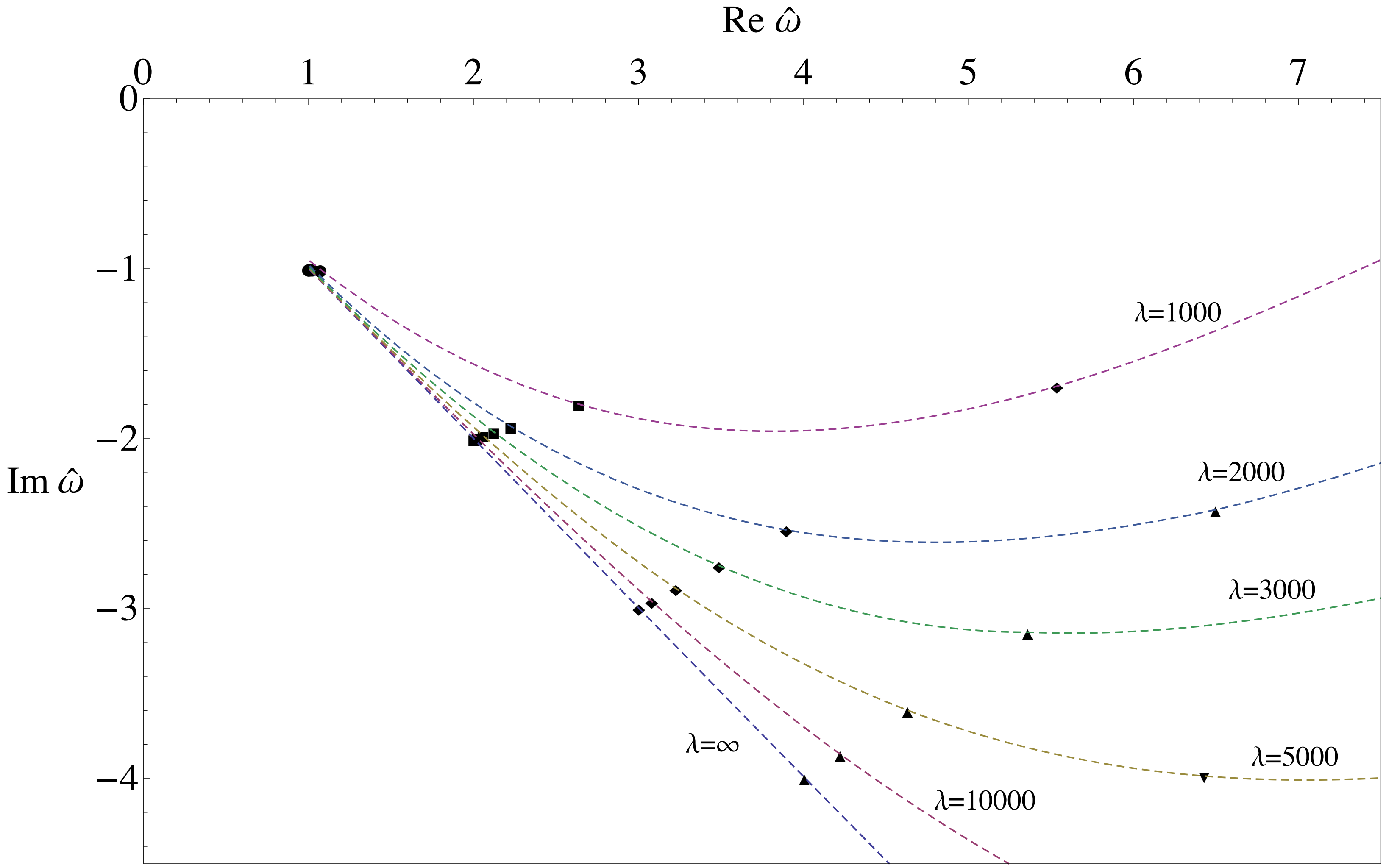
Quasinormal modes infinite coupling



$$\omega_n|_{q=0} = n(\pm 1 - i)$$

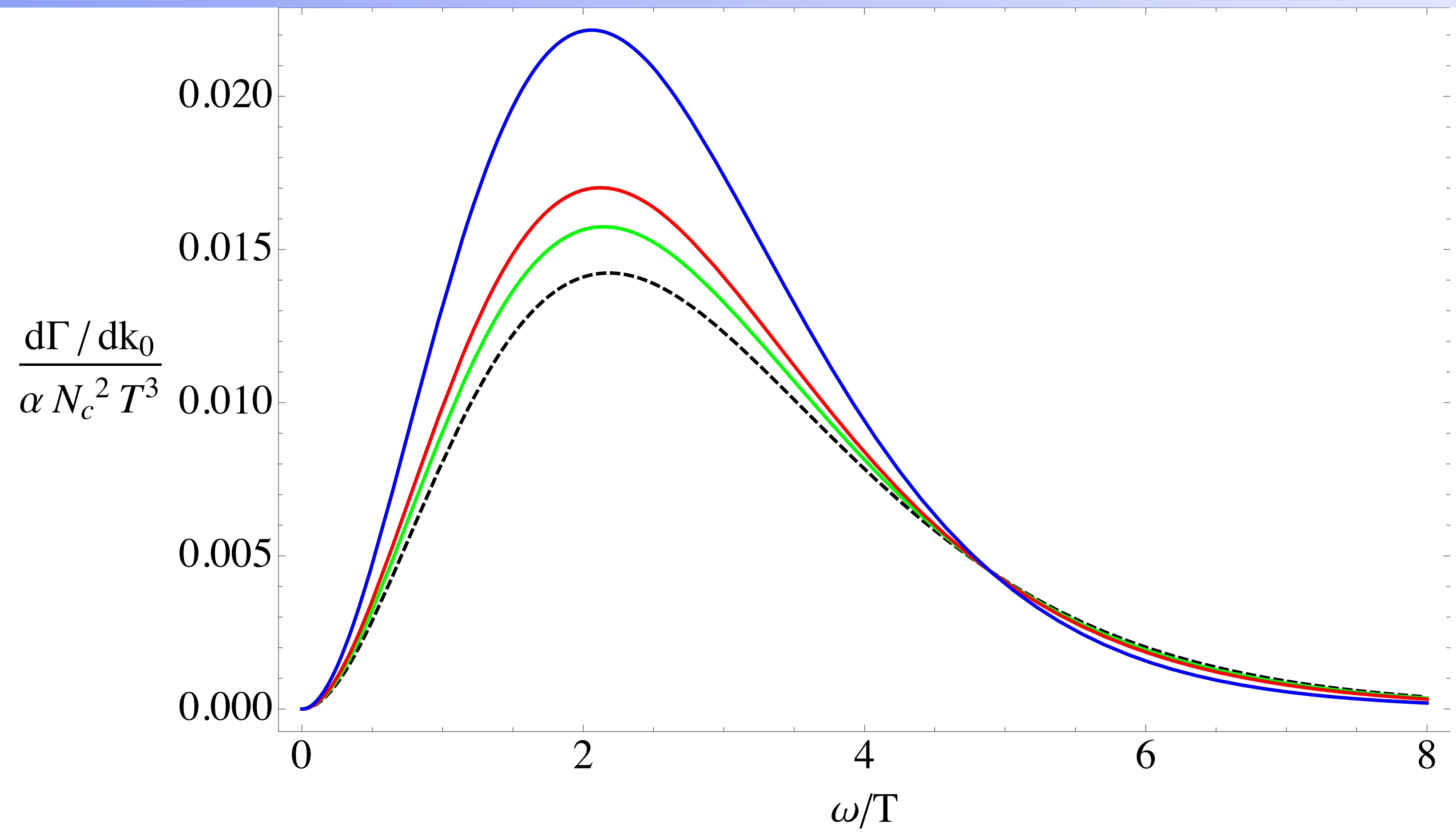
- Strong coupling equivalent to quasiparticle picture at weak coupling
- Characterize the response of the system to inf. perturbation
- Appear as poles in the retarded correlator: $\omega_n(q) = M_n(q) - i\Gamma_n(q)$,
- First indication of top down thermalization at strong coupling

Quasinormal modes finite coupling



- Effect of decreasing coupling: Imaginary part decreases
- Outside the $\lambda = \infty$ limit, response of the plasma appears to change, moving towards a quasiparticle picture
- Larger impact on higher energetic modes
- Convergence of strong coupling expansion not guaranteed when shift is of $\mathcal{O}(1)$
- What happens if we take system further away from equilibrium?

Photon production rate at intermediate coupling

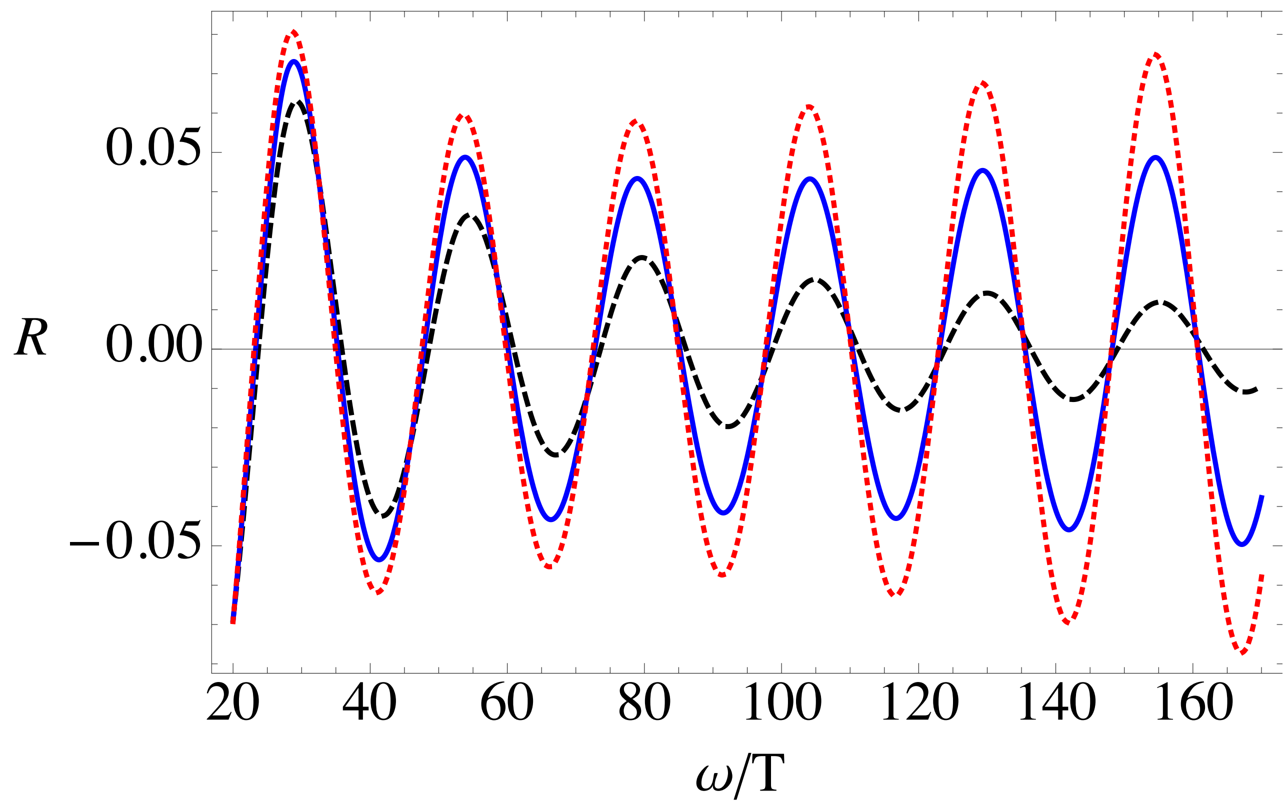


emission rate for photons r_s/r_h=1.01 and $\lambda = \infty, 120, 80, 40$

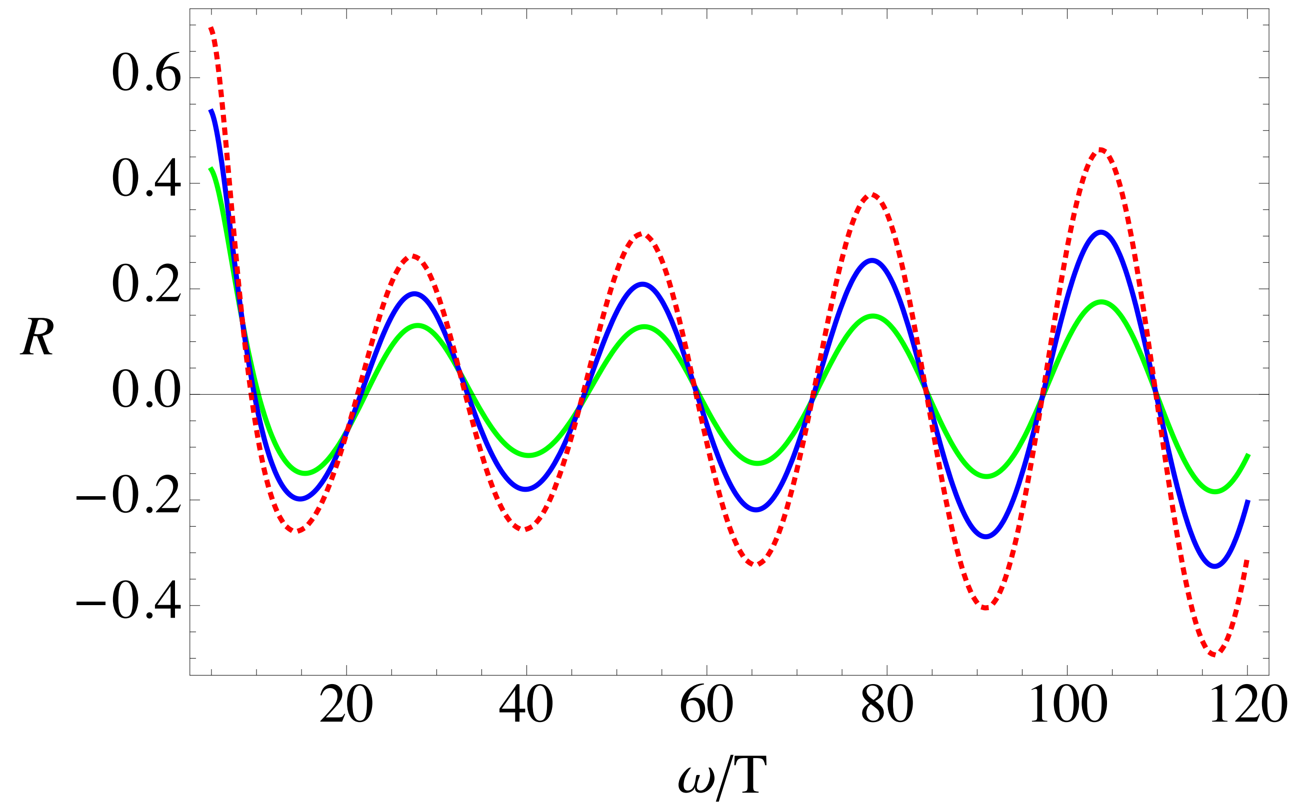
- Behaviour qualitatively similar to equilibrium case: in particular the result is much less sensitive to finite coupling corrections than QNM spectrum

Thermalization at finite coupling

Relative deviation from thermal limit for on shell photons



R for r_s/r_h=1.1 and $\lambda = \infty,\ 500,\ 300$

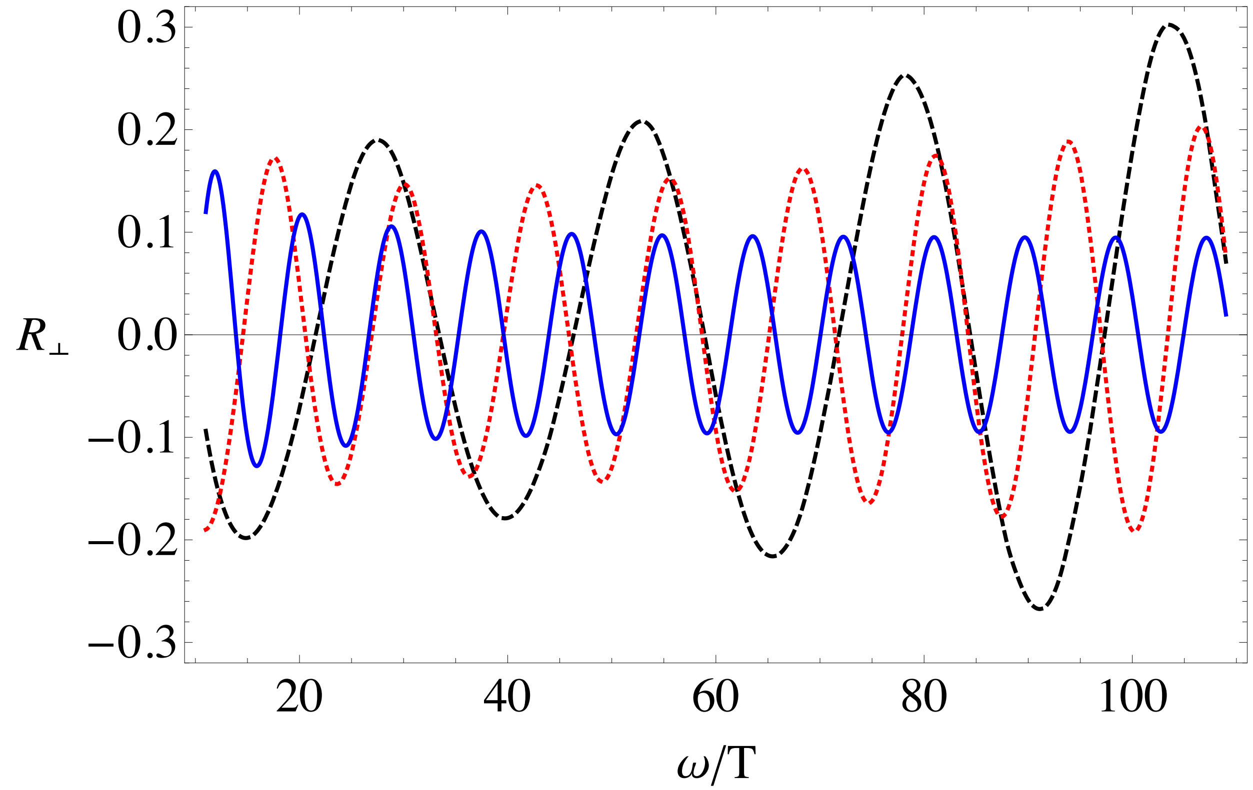


R for r_s/r_h=1.1 and $\lambda = 150,\ 100,\ 75$

- Behaviour of relative deviation changes at large frequency
- UV modes are no longer first to thermalize
- Decreasing the coupling: change happens at lower frequency

Thermalization at finite coupling

Virtuality dependence of the relative deviation



R for r_s/r_h=1.1 and c=1, 0.8, 0 for $\lambda = 100$

- For maximally virtual photons (c=0), R approaches a constant at $\omega \to \infty$
- For on-shell photons (c=1): amplitude of R rises linearly with ω
- Indication that thermalization pattern changes from top-down towards bottom-up

Interpretation

- What to make of all this? Evidence for the holographic plasma starting to behave like a system of weakly coupled quasiparticles

- Or is this simply a peculiarity of photon production but not a fundamental feature of the collective behaviour of the plasma constituents?

- Next: Repeat the same analysis for energy momentum tensor correlators

Energy momentum tensor correlators $\langle T_{\mu\nu} T_{\alpha\beta} \rangle$

Metric fluctuations

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$$

- $h_{\mu\nu}$ corresponds to the energy momentum tensor of the field theory

Construct gauge invariants

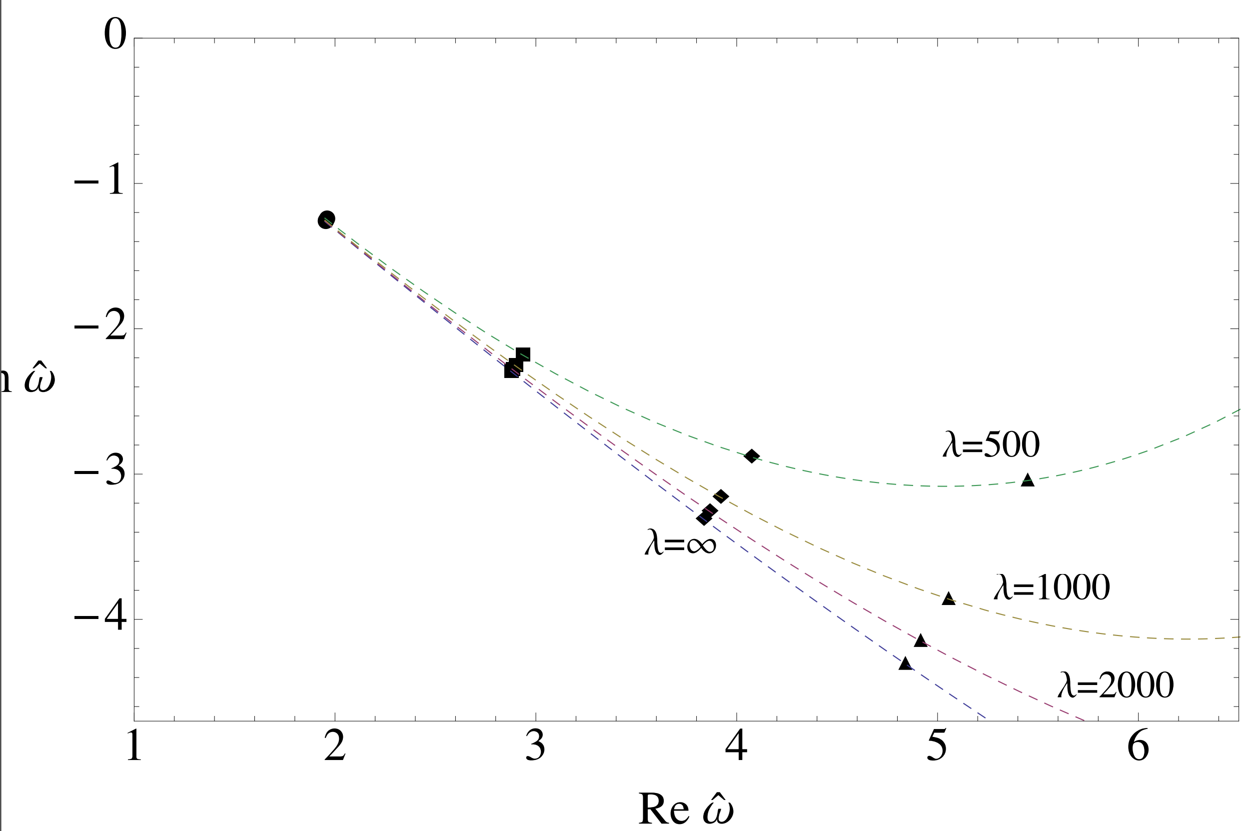
- Scalar channel: h_{xy}
- Shear channel: $h_{tx},\ h_{zx}$
- Sound channel: $h_{tt},\ h_{tz},\ h_{zz},\ h$

Repeat analysis from photons

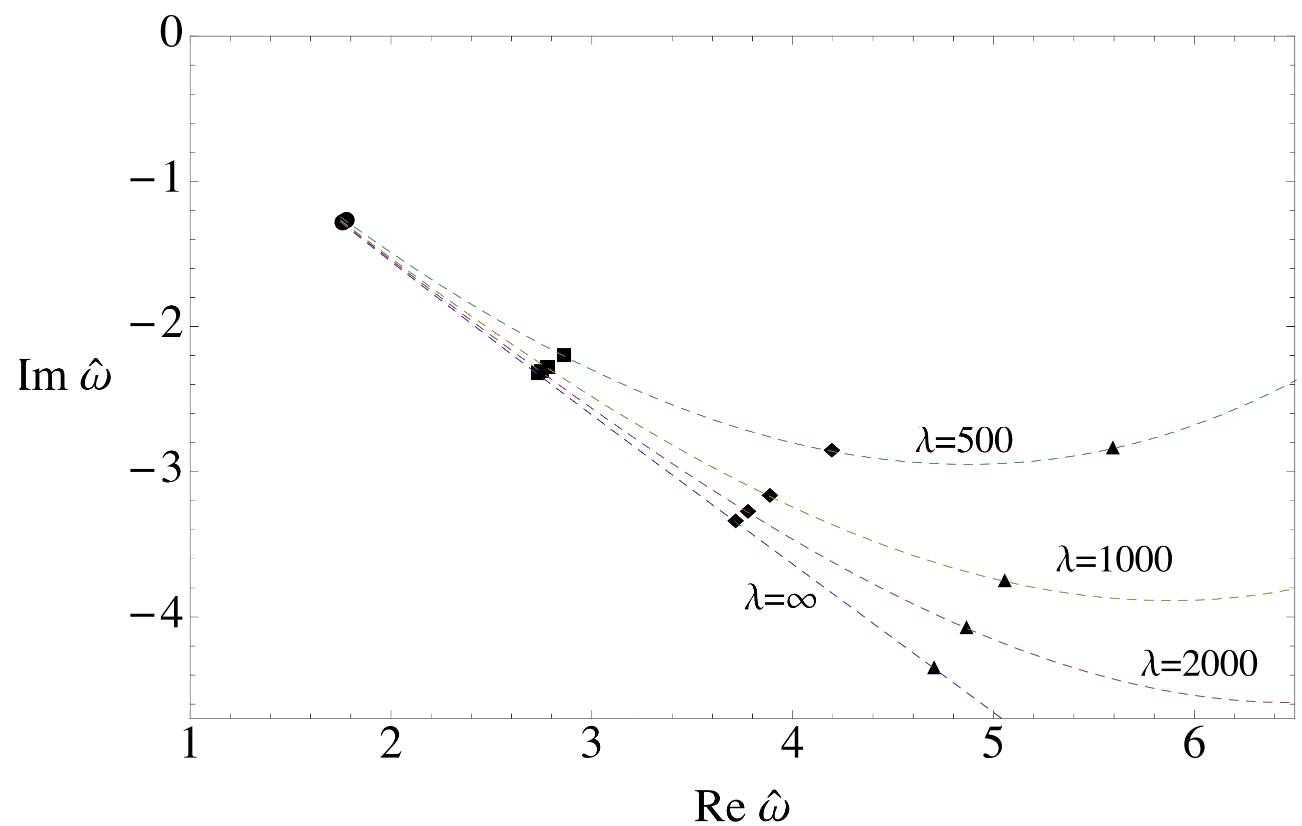
- Straight forward but more involved

Quasinormal modes at finite coupling

Scalar channel



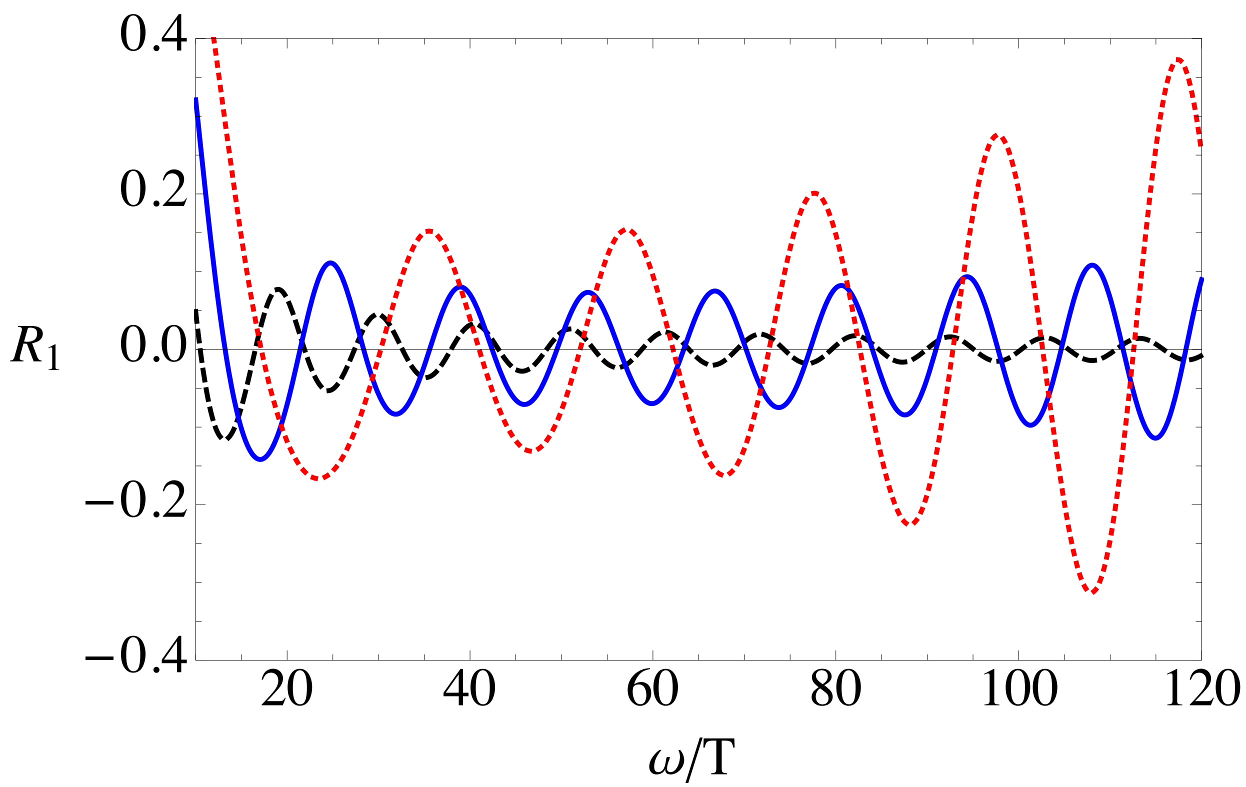
Shear channel



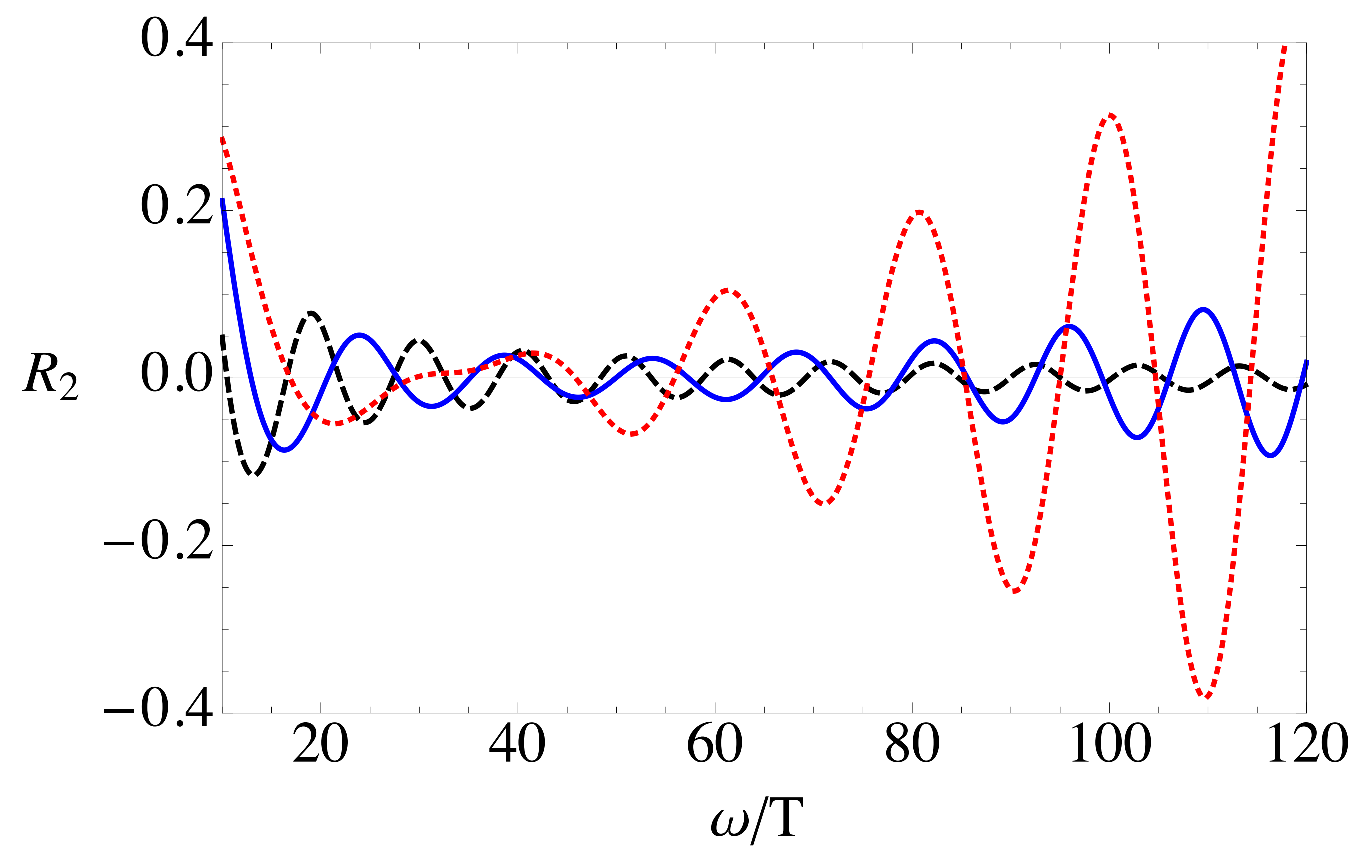
- QNM spectrum for the shear and shear channel for q=2πT
- All three channels show the same behaviour
- Qualitatively identical to virtual photons

Relative deviation at finite coupling

Scalar channel



Shear channel



- Relative deviation for the scalar/shear channel for r_s/r_h=1.1, c=0, 6/9, 8/9 and $\lambda = 100$
- All three channels show the same behaviour
- Again similar to photons with the same dependence on c
- shift from top-down towards bottom-up

Conclusions

- Holographic (thermalization) calculations at finite coupling are possible and potentially a very fruitful exercise
- All calculations suggest that the system starts to behave like a weakly coupled system in the realm of the strong coupling expansion
    - QNM modes: flow towards quasiparticle picture, independent of the thermalization model
    - Thermalization pattern shifts from top-down towards bottom-up
- Open questions
    - How universal is the shift from top-down towards bottom-up
    - Go beyond quasistatic approximation