Hamiltonian approach to QCD: The Polyakov loop potential

H. Reinhardt

EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

H. R. & J. Heffner
Phys. Lett.B718(2012)672 and arXiv:1304.2980

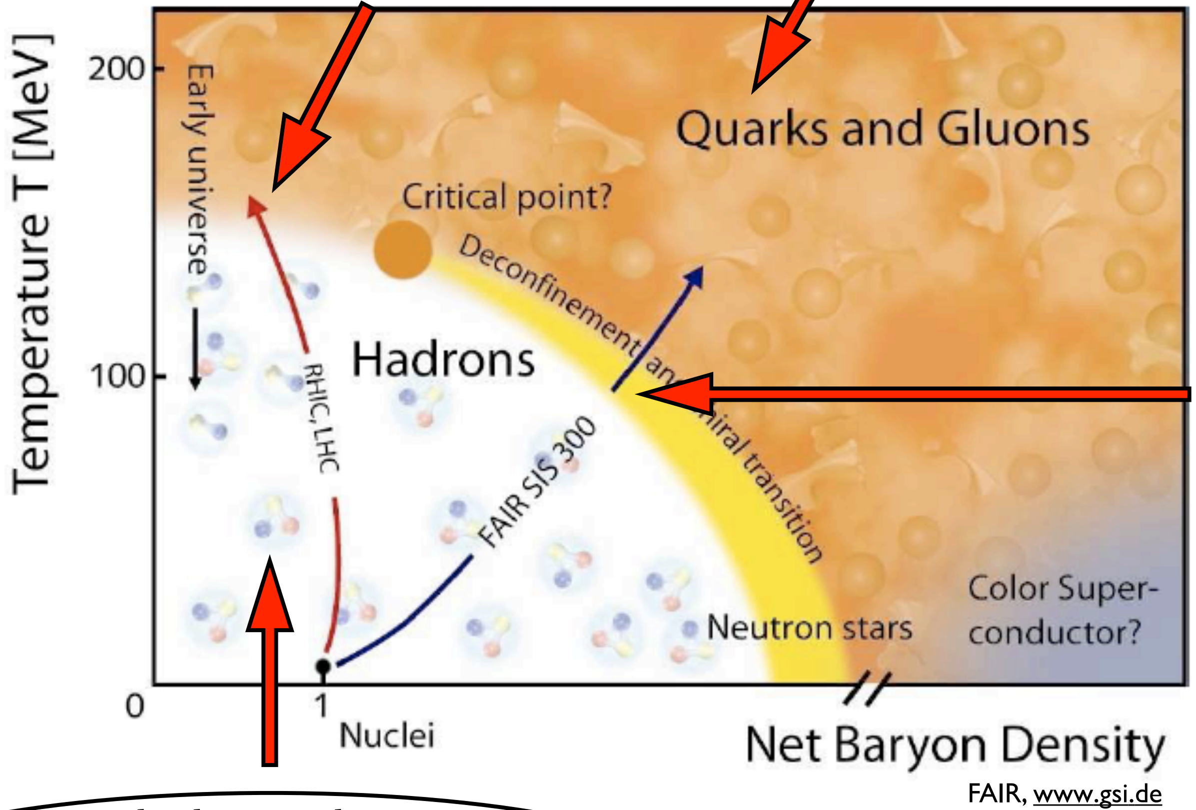
Phase diagram of QCD

Strongly correlated quark-gluon-plasma

'RHIC serves the perfect fluid'

massless quarks (chiral symmetry)

deconfinement



quarkyonic:

confinement & chiral symmetry?

hadronic phase

confinement & chiral symmetry breaking

Outline

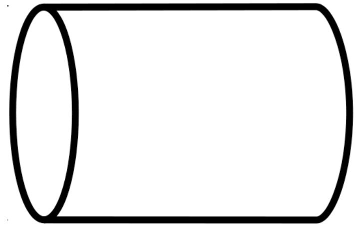
- introduction
  - order parameter for confinement:
  - Polyakov loop & alternatives
- Hamiltonian approach to YMT in background gauge
- effective potential of the Polyakov loop
- deconfinement phase transition
- conclusions

Polyakov loop

- YMT at finite temperature T :compact Euclidean time

$$P[A_0](\vec{x}) = \tfrac{1}{d_r} tr P \exp\left[i\int_0^L dx_0 A_0(x_0,\vec{x}) \right]$$

$$T^{-1} = L$$



- order parameter for confinement: $\langle P[A_0](\vec{x})\rangle \sim \exp\left[-F_\infty(\vec{x})L\right]$
  - conf. phase: center symmetry $\langle P[A_0](\vec{x})\rangle = 0$
  - deconf. phase: center symmetry-broken $\langle P[A_0](\vec{x})\rangle \neq 0$

- Polyakov gauge $\partial_0 A_0 = 0,\quad A_0 = diagonal$ $\quad P[A_0](\vec{x}) = \cos(\frac{A_0(\vec{x})L}{2})$
  - fundamental modular region $0 < A_0 L/2 < \pi$ $\quad P[A_0] - unique\ function\ of\ A_0$
  - alternative order parameters: $\langle P[A_0](\vec{x})\rangle \qquad P[\langle A_0(\vec{x})\rangle] \qquad \langle A_0(\vec{x})\rangle$

- *F.Marhauser and J. M. Pawlowski, arXiv:0812.11144*
- *J. Braun, H. Gies, J. M. Pawlowski, Phys. Lett. B684(2010)262*

Effective potential of the order parameter for confinement

- background field calculation $a_0 = \langle A_0(\vec{x}) \rangle -$ const, diagonal (Polyakov gauge)
- effective potential $e[a_0] \rightarrow \min \qquad \Rightarrow a_0 = \bar{a}_0$
- order parameter $\langle P[A_0] \rangle \approx P[\bar{a}_0]$

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- 1-loop perturbation theory

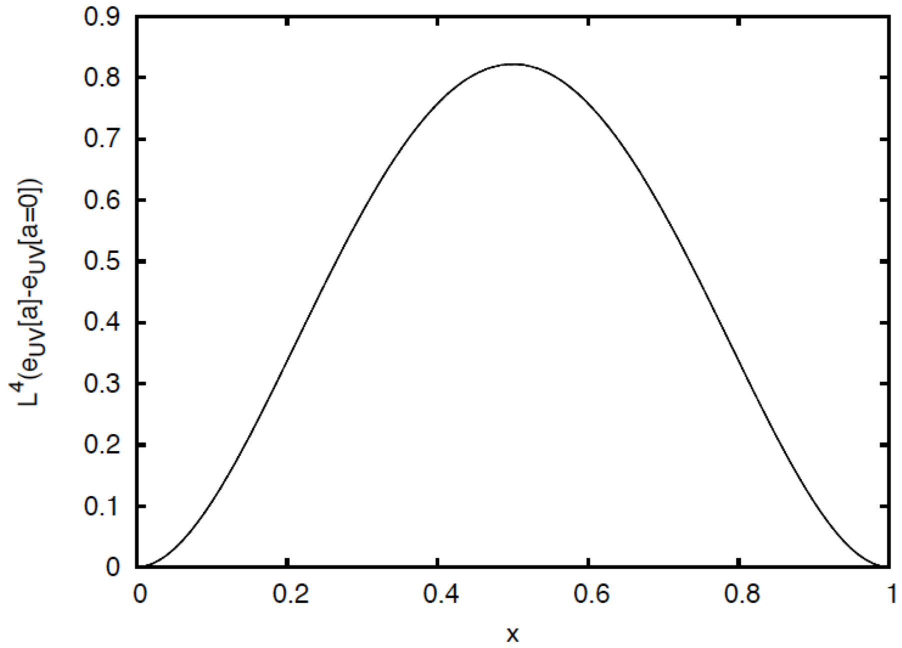
$$e_{PT}[a_0 = x 2\pi / L]$$

Gross, Pisarski, Yaffe, Rev.Mod.Pys.53(1981)

N.Weiss, Phys.Rev.D24(1981)

$$P[\bar{a}_0 = 0] = 1$$

deconfined phase



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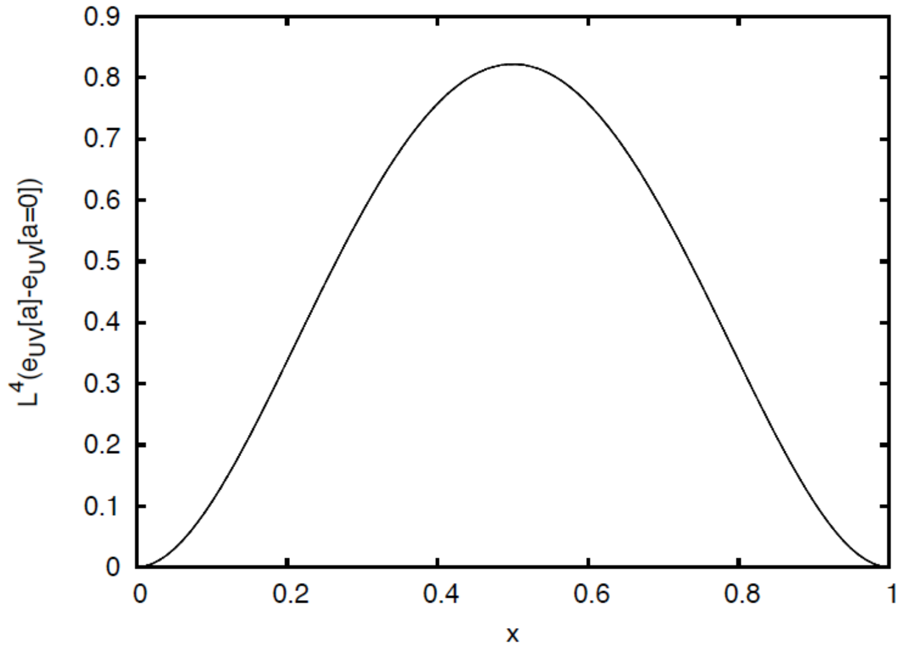

aim of this talk: non-perturbative evaluation of $e[a_0]$ in the Hamiltonian approach

Polyakov loop potential in the Hamiltonian approach

- Hamiltonian approach assumes Weyl gauge $A_0 = 0$

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- O(4)-invariance

> - *compactify (instead of time) one spatial*
> - (x_3-)*axis to a circle of circumference* L *and interprete* L^{-1} *as temperature*

- YMT at finite length L in a constant, color diagonal background field a_3

- calculate the effective potential $\boxed{e[a_3]}$

The effective potential in the Hamiltonian approach

- effective potential $e(\vec{a})$ of a spatial background field $\vec{a}$

$$\langle H \rangle_{\vec{a}} = \min \langle H \rangle \qquad \langle \vec{A} \rangle = \vec{a}$$

$$\langle H \rangle_{\vec{a}} = (spatial\ volume) \times e(\vec{a})$$

$$e(\vec{a}) - effective\ potential$$

- variational calculation of $e(\vec{a})$

Hamiltonian approach to Yang-Mills theory

Weyl gauge: $A_0^a(x) = 0$ cartesian coordinates $A_i^a(x)$

momenta $\Pi_i^a(x) = \delta S / \delta \dot{A}_i^a(x) = E_i^a(x)$

$$H = \tfrac{1}{2}\int d^3x(\Pi^2(x) + B^2(x))$$

$$\Pi_k^a(x) = \delta / i\delta A_k^a(x)$$

YM Schrödinger equation $H\Psi[A] = E\Psi[A]$

Gauss law $D\Pi\Psi = 0$ *gauge invariant wave functionals:* $\Psi[A]$

more convenient: gauge fixing
explicit resolution of Gauss´law

$$\left[\vec{\partial} + \vec{a}, \vec{A}\right] \equiv \left[\vec{d}, \vec{A}\right] = 0$$

$\vec{a}$ – *background field*

for $\vec{a} = 0$

$\Rightarrow \partial A = 0$

Hamiltonian approach to YMT in background gauge [d,A]=0

$$\boxed{H = \tfrac{1}{2}\int (J^{-1}\Pi^{\perp} J \Pi^{\perp} + B^2) + H_C} \qquad \Pi^{\perp} = \delta / i\delta A^{\perp}$$

$$J(A^{\perp}) = Det(-Dd) \qquad D = \partial + A \qquad \mathrm{d} = \partial + a$$

$$\mathrm{H_C} = \tfrac{1}{2}\int \mathrm{J}^{-1}\rho \mathrm{J}(-\mathrm{Dd})^{-1}(-\mathrm{d}^2)(-\mathrm{Dd})^{-1}\rho$$ Coulomb term

color charge density: $\rho = -(\mathrm{A}^{\perp} - \mathrm{a})\Pi^{\perp}$

$$\langle \Phi | \ldots | \Psi \rangle = \int_{\Lambda} \mathrm{DAJ(A)}\Phi^{*}(\mathrm{A})\ldots\Psi(\mathrm{A})$$

Hamiltonian approach to YMT in background gauge [d,A]=0

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Coulomb term

color charge density: $\rho = -(\mathrm{A}^{\perp} - \mathrm{a})\Pi^{\perp}$

$$\langle \Phi | \ldots | \Psi \rangle = \int_{\Lambda} \mathrm{DAJ(A)}\Phi^{*}(\mathrm{A})\ldots\Psi(\mathrm{A})$$

$$\langle \Psi | H | \Psi \rangle \to \min \qquad \langle \Psi | \vec{A} | \Psi \rangle = \vec{a}$$

Variational approach

- trial ansatz

c.f. C.Feuchter & H. R. PRD70(2004)

$$\Psi_a(A) = \frac{1}{\sqrt{\mathrm{Det}(-\hat{D}\hat{d})}} \exp\left[-\frac{1}{2}\int dx\, dy (A(x)-a)\omega(x,y)(A(y)-a)\right]$$

gluon field $\langle A \rangle_a = a$

gluon propagator $\langle A(x), A(,y) \rangle_a = (2\omega(x,y))^{-1}$

variational kernel $\omega(x,x')$ determined from

$$\boxed{\langle \Psi | H | \Psi \rangle \to \min}$$

Propagators in the background field

- background field $a = a^k H_k =: a \cdot H$ in the Cartan algebra $[H_k, H_l] = 0$

$$H_k |\sigma\rangle = \sigma_k |\sigma\rangle$$
$$\sigma = (\sigma_1, \ldots, \sigma_r) - roots$$

$SU(2)$: $H_1 = T_3$ $\quad \sigma_1 = 0, \pm 1$

$SU(3)$: $H_1 = T_3$ $\quad H_2 = T_8$

$$\sigma = (1,0), \quad \left(\frac{1}{2}, \frac{1}{2}\sqrt{3}\right), \quad \left(\frac{1}{2}, -\frac{1}{2}\sqrt{3}\right)$$

- propagators in presence of the diagonal background field
- in background gauge $[\partial + a, A] = 0$

- exact relation:

$$G^{\sigma}_{a,\ background\ gauge}(p) = G_{a=0,\ \partial A=0}(p^{\sigma})$$

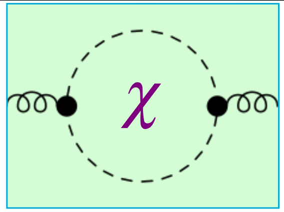
$$p^{\sigma} = p - \sigma \cdot a$$

- ordinary Coulomb gauge propagators $G_{a=0,\ \partial A=0}(p)$ C. Feuchter & H. Reinhardt, Phys. Rev.D71(2005)

- compactify 3-axis $\vec{a} = a\vec{e}_3$ $\quad \vec{p}^{\sigma} = \vec{p}_{\perp} + (p_n - \sigma \cdot a)\vec{e}_3, \quad p_n = 2\pi n / L$

The effective potential

- energy density $$e(a,L)=\sum_{\sigma}\frac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_{\perp}\left(\omega(p^{\sigma})-\chi(p^{\sigma})\right)$$



- background field $p^{\sigma}=p_{\perp}+(p_n-\sigma a)$ $\quad p_n=2\pi n/L$ $\quad \sigma=0\pm 1$

- periodicity $e(a,L)=e(a+\mu_k/L,L)$ $\quad \exp\left(i\mu_k\right)=z_k\in Z(N)$

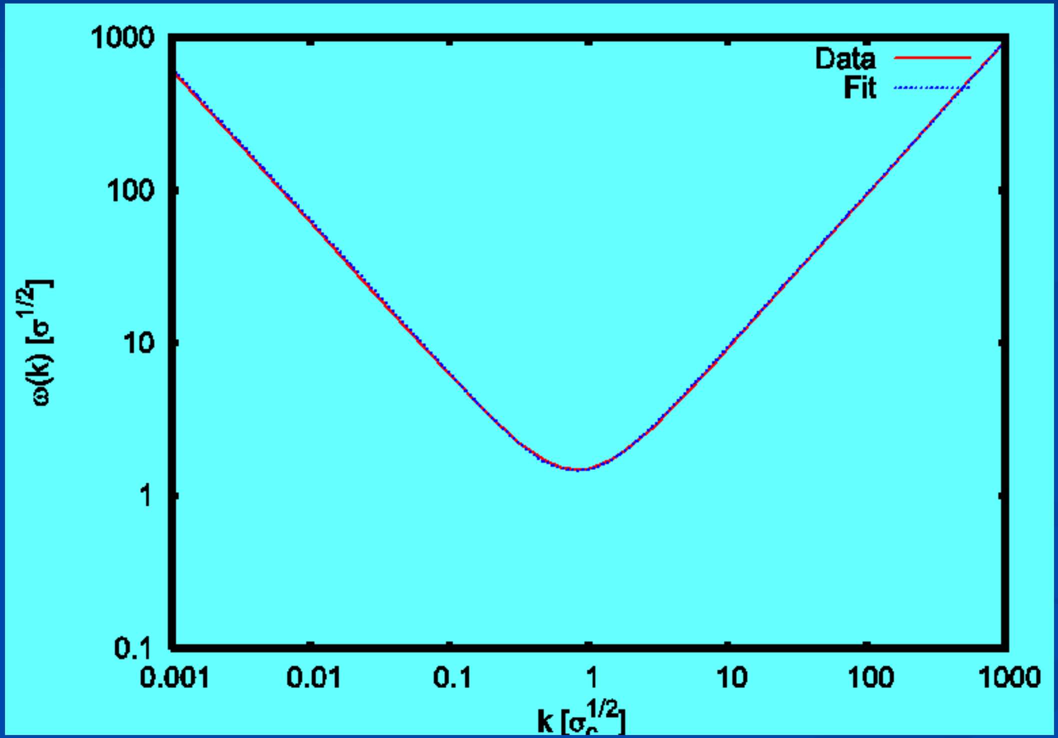
ghost loop χ arises from the FP determinant in the kinetic energy

- input:

$\omega(p),\chi(p)$ from the variational calculation in Coulomb gauge at T=0

Variational approach in Coulomb gauge
Numerical results for SU(2)

D. Epple, H. R., W.Schleifenbaum, PRD 75 (2007)



$IR: \quad \omega(k) \sim 1/k \qquad\qquad UV: \quad \omega(k) \sim k$

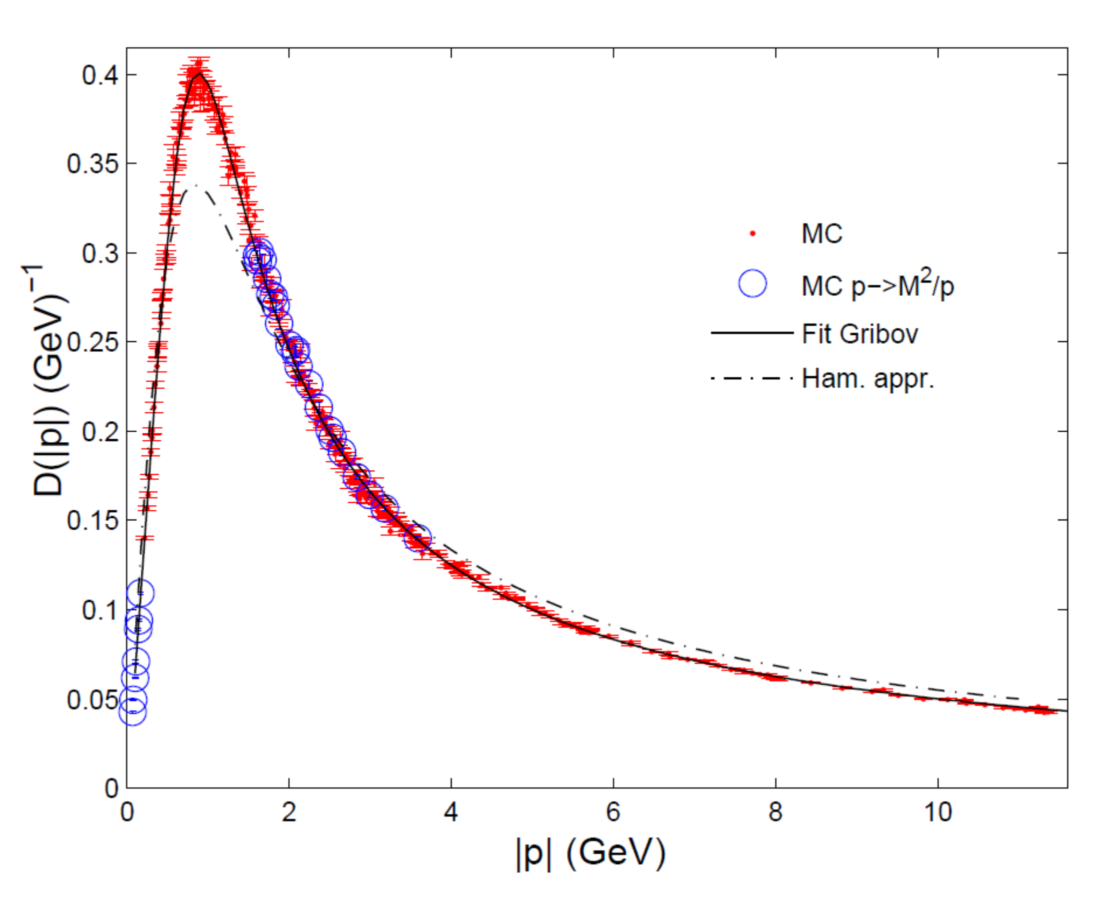
Static gluon propagator in D=3+1

$$D(k) = (2\omega(k))^{-1}$$

Gribov's formula

$$\omega(k) = \sqrt{k^2 + \frac{M^4}{k^2}}$$

$$M = 0.88 GeV$$



missing strength in
mid momentum regime:
missing gluon loop

G. Burgio, M.Quandt , H.R., **PRL102(2009)**

Variational approach to YMT with non-Gaussian wave functional

D. Campagnari & H.R, Phys.Rev.D82(2010)

wave functional

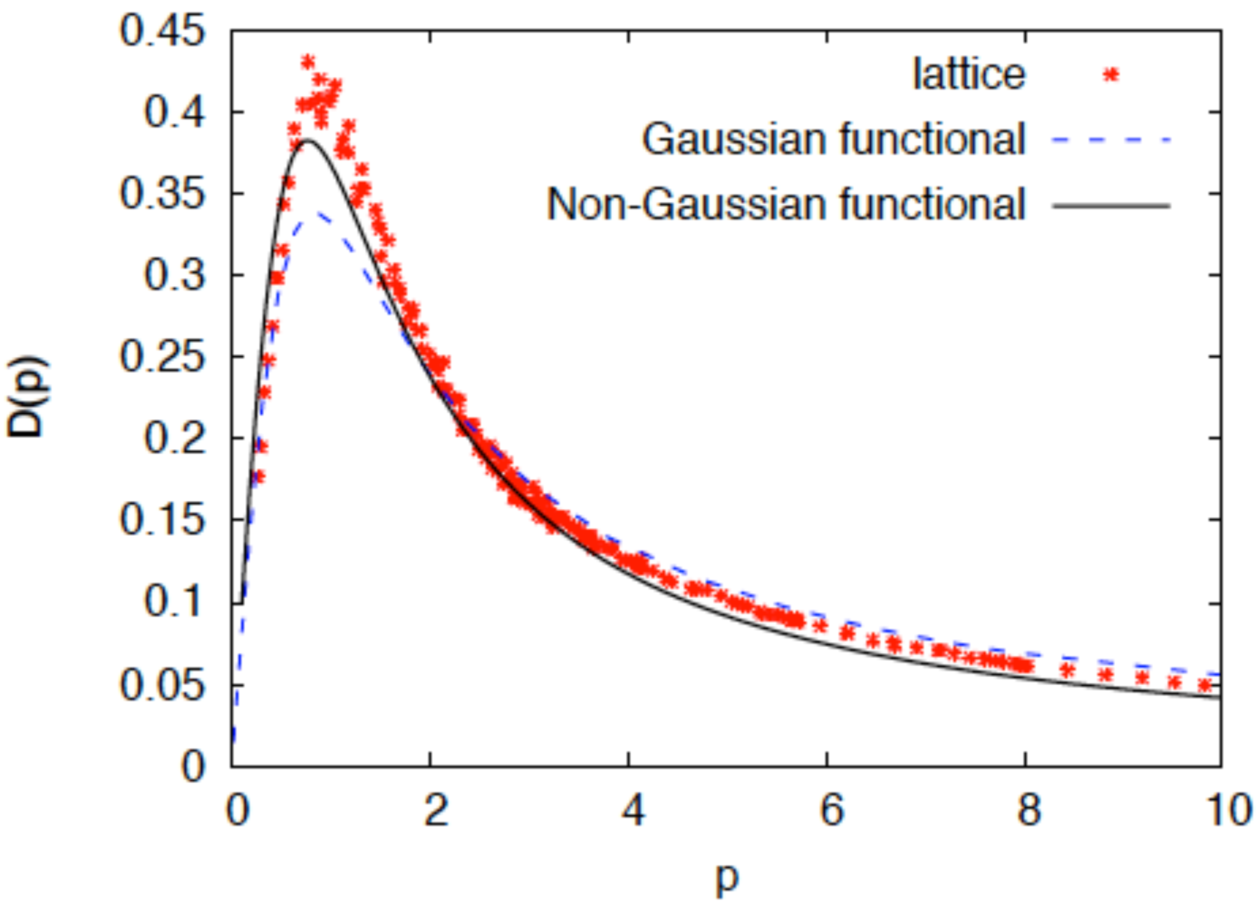
$$|\psi[A]|^2 = \exp(-S[A])$$

ansatz

$$S[A] = \int \omega A^2 + \tfrac{1}{3!}\int \gamma^{(3)} A^3 + \tfrac{1}{4!}\int \gamma^{(4)} A^4$$

exploit DSE

Corrections to the gluon propagator

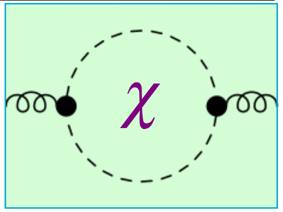


D. Campagnari & H.R, Phys.Rev.D82(2010)

The effective potential

- energy density

$$e(a,L)=\sum_{\sigma}\frac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_{\perp}(\omega(p^{\sigma})-\chi(p^{\sigma}))$$



- background field

$$p^{\sigma}=p_{\perp}+(p_n-\sigma a) \qquad p_n=2\pi n/L \qquad \sigma-roots$$

- periodicity

$$e(a,L)=e(a+\mu_k/L,L) \qquad \exp\left(i\mu_k\right)=z_k\in Z(N)$$

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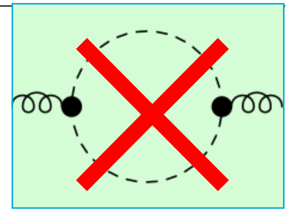

- background field $p^{\sigma}=p_{\perp}+(p_n-\sigma a)$ $\quad p_n=2\pi n/L$ $\quad \sigma-roots$
- periodicity $e(a,L)=e(a+\mu_k/L,L)$ $\quad \exp\left(i\mu_k\right)=z_k\in Z(N)$
- neglect ghost loop $\chi(p)=0$ $$e(a,L)=\sum_{\sigma}\tfrac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2 p_{\perp}\omega(p^{\sigma})$$
  - quasi-gluon gas

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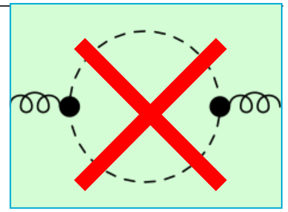

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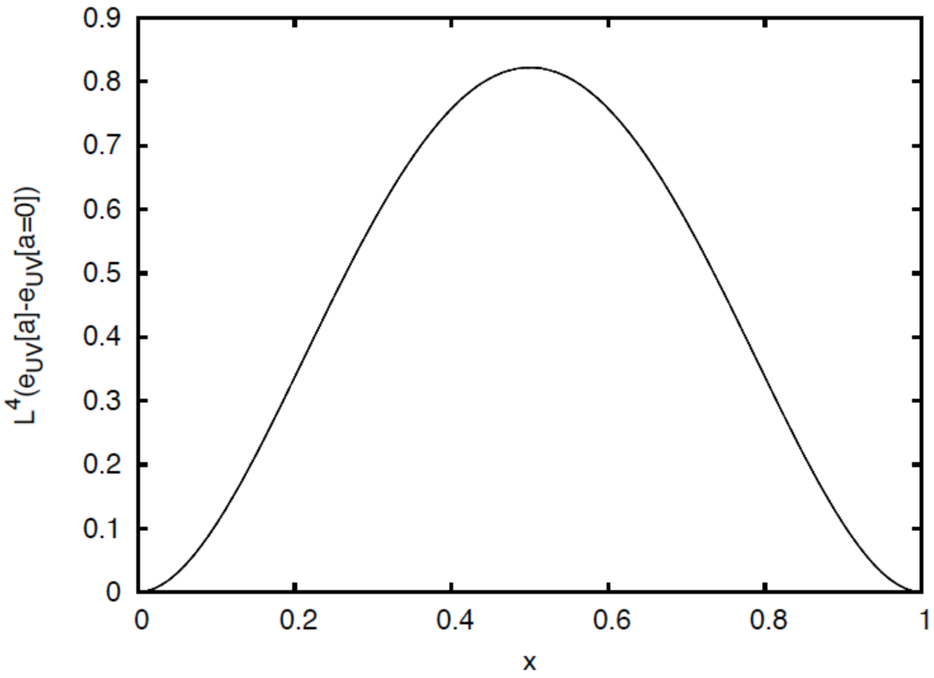
- limiting cases
  - UV: $\omega_{UV}(p)=p$
  - IR: $\omega_{IR}(p)=M^2/p$
  - Gribov: $\omega(p)=\sqrt{(p^2+M^4/p^2)}\approx\omega_{IR}(p)+\omega_{UV}(p)$

The UV-effective potential

$\chi(p) = 0 \qquad \omega(p) = p \qquad e(a,L) = \sum_{\sigma} \frac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_{\perp} (\omega(p^{\sigma}) - \cancel{\chi(p^{\sigma})})$

$$e(a,L) = \frac{8}{\pi^2 L^4} \sum_{n=1}^{\infty} \frac{\sin^2(naL/2)}{n^4}$$
$$= \frac{4\pi^2}{3L^4} \underbrace{\left(\frac{aL}{2\pi}\right)}_{x}^2 \left[\frac{aL}{2\pi} - 1\right]^2$$

N.Weiss 1-loop PT



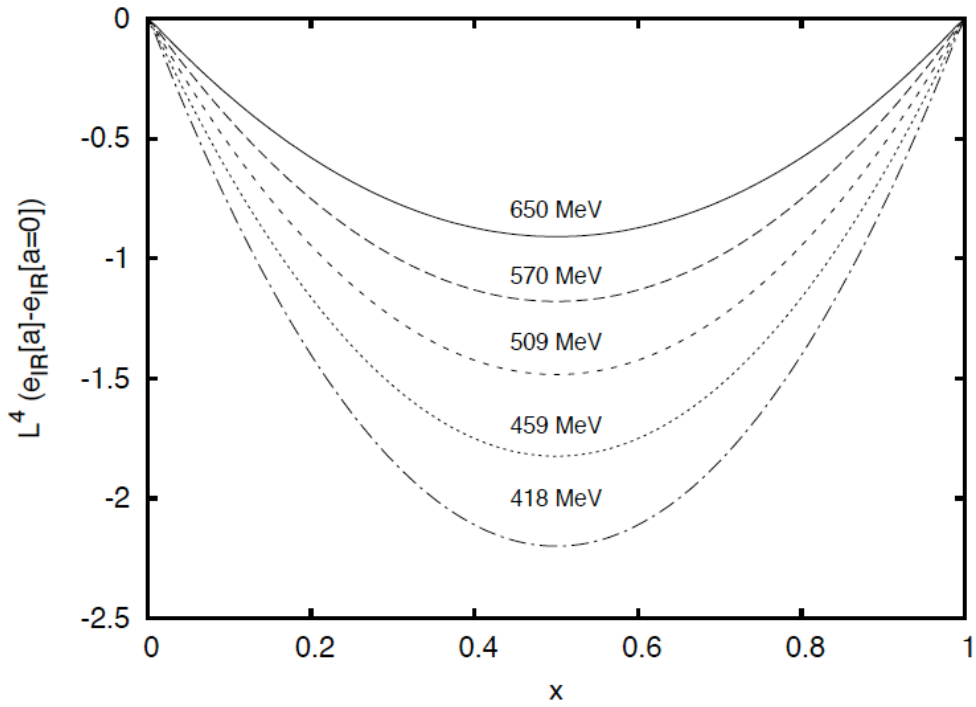
$Polyakov - loop \qquad \langle P \rangle \simeq P[a_{\min} = 0] = 1 \qquad deconfining \quad phase$

The IR-effective potential

$$\chi(p)=0 \qquad \omega(p)=M^2/p \qquad e(a,L)=\sum_\sigma \tfrac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_\perp(\omega(p^\sigma)-\cancel{\chi(p^\sigma)})$$

$$e_{IR}(a,L)=-\frac{4M^2}{\pi^2L^2}\sum_{n=1}^{\infty}\frac{\sin^2(naL/2)}{n^2}$$

$$=\frac{2M^2}{L^2}\underbrace{\left(\frac{aL}{2\pi}\right)}_{x}\left[\frac{aL}{2\pi}-1\right]$$



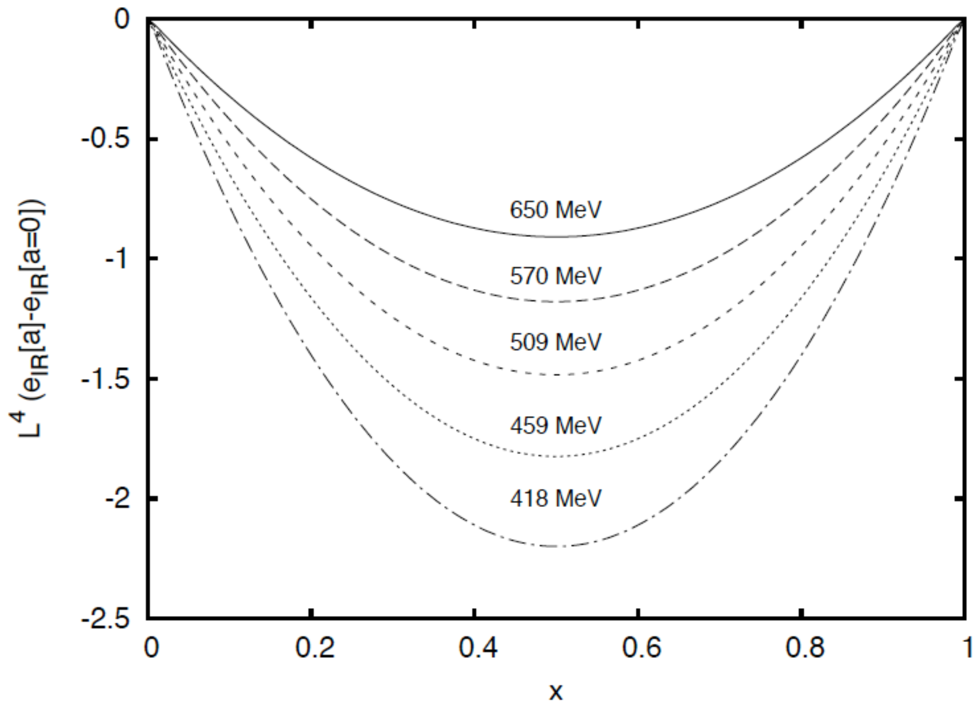
$$Polyakov-loop \qquad \langle P\rangle \simeq P[a_{\min}=\pi/L]=0 \qquad confining \quad phase$$

The IR-effective potential

$$\chi(p) = 0 \qquad \omega(p) = M^2 / p \qquad e(a,L) = \sum_\sigma \frac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_\perp (\omega(p^\sigma) - \cancel{\chi(p^\sigma)})$$

$$e_{IR}(a,L) = -\frac{4M^2}{\pi^2 L^2} \sum_{n=1}^{\infty} \frac{\sin^2(naL/2)}{n^2}$$

$$= \frac{2M^2}{L^2} \underbrace{\left(\frac{aL}{2\pi}\right)}_{x} \left[\frac{aL}{2\pi} - 1\right]$$



$$Polyakov - loop \qquad \langle P \rangle \simeq P[a_{\min} = \pi / L] = 0 \qquad confining \quad phase$$

deconfinement phase transition results from the interplay between the confining IR-potential and deconfining UV-potential

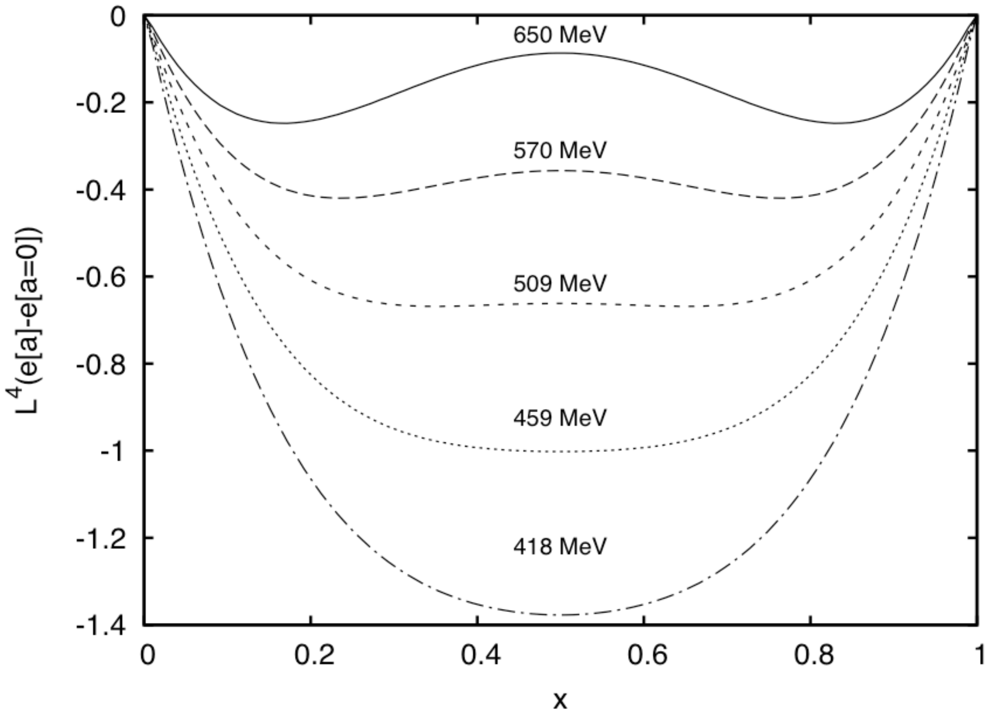
The IR+UV effective potential:

$\chi(p) = 0$ $\omega(p) = p + M^2 / p$ $e(a, L) = e_{UV}(a, L) + e_{IR}(a, L)$

phase transition

critical temperature:

$T_C = \sqrt{3} M / \pi$



$lattice : M \simeq 880\, MeV \quad \Rightarrow \quad T_C \simeq 485\, MeV$

The IR+UV effective potential:

$$\chi(p)=0 \qquad \omega(p)=p+M^2/p \qquad e(a,L)=e_{UV}(a,L)+e_{IR}(a,L)$$

phase transition

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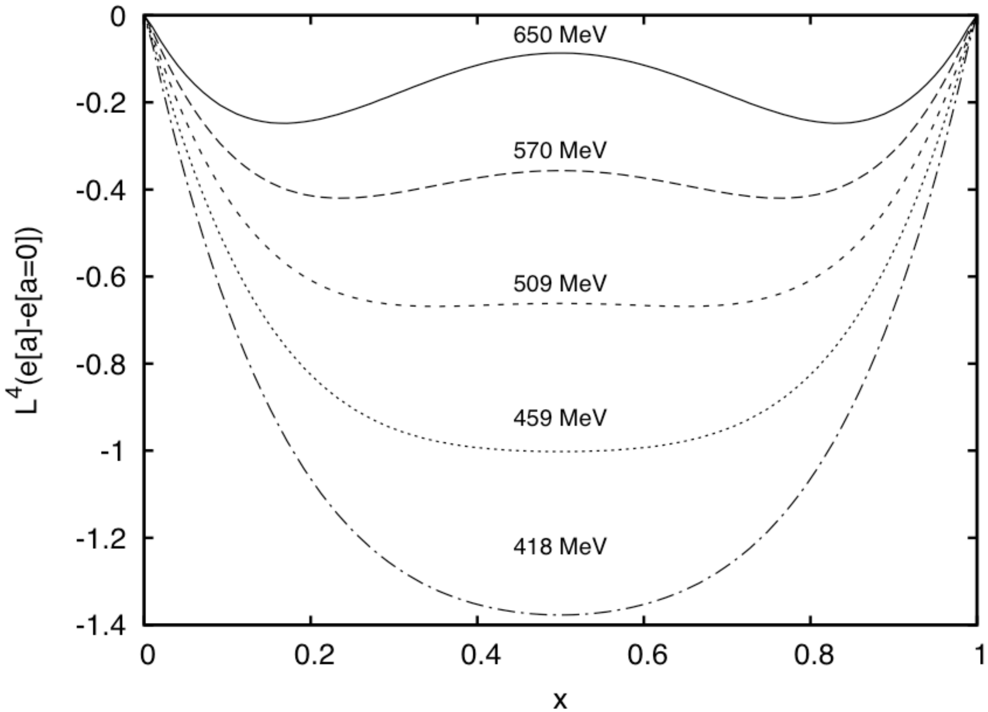
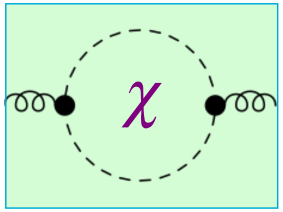


$$lattice: M\simeq 880\,MeV \quad \Rightarrow \quad T_C\simeq 485\,MeV$$

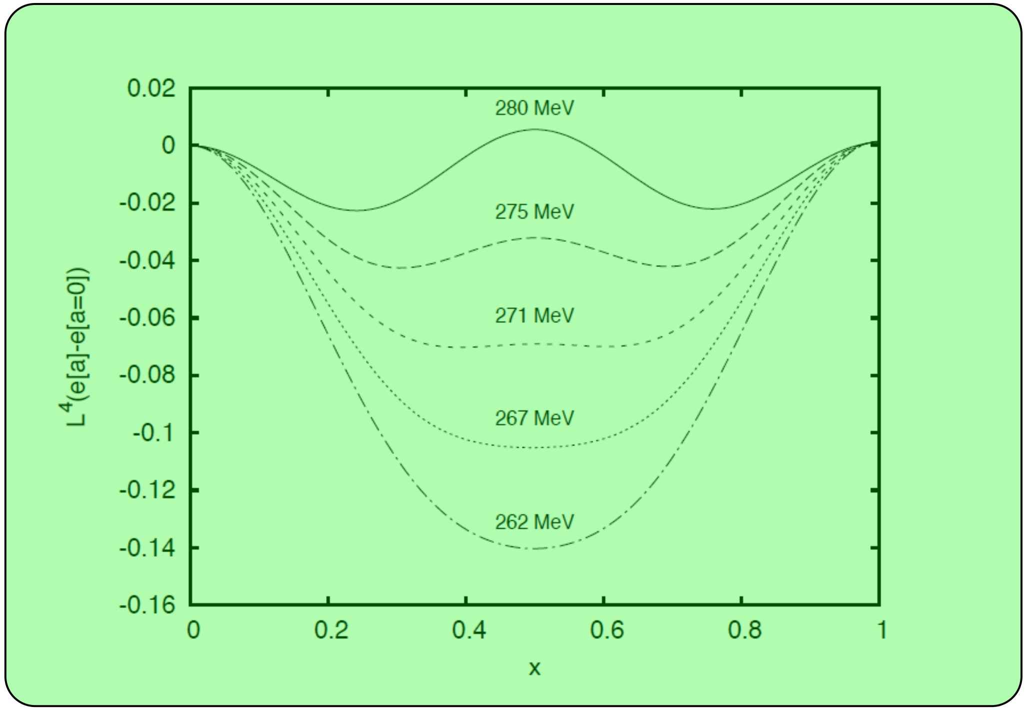
$$\chi(p)=0 \qquad \omega(p)=\sqrt{p^2+M^4/p^2} \qquad T_C\simeq 432\,MeV$$

The full effective potential

$$e(a,L)=\sum_{\sigma}\frac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_\perp(\omega(p^\sigma)-\chi(p^\sigma))$$



variational calculation in Coulomb gauge



critical temperature:

$$T_C \simeq 270\,MeV$$

The color dielectric function of the QCD vacuum

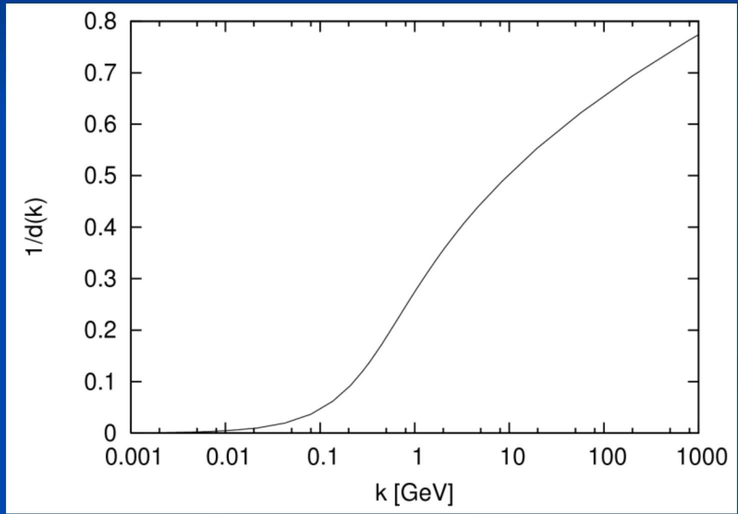
- ghost propagator

$$\left\langle (-D\partial)^{-1} \right\rangle = d / (-\Delta)$$

- dielectric „constant"

$$\varepsilon = d^{-1}$$

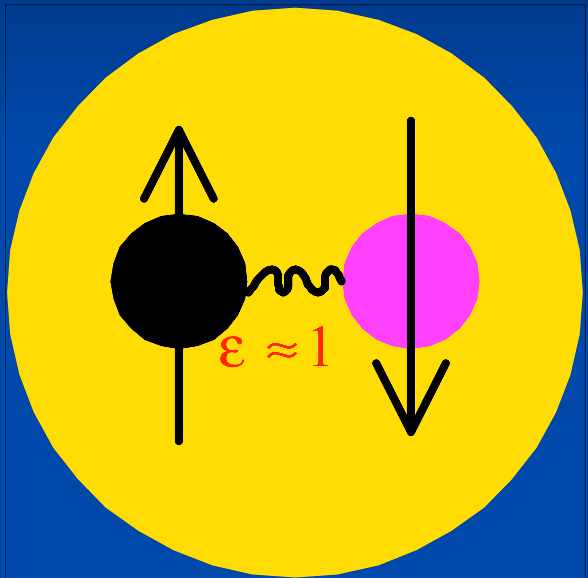
H.Reinhardt, PRL101 (2008)



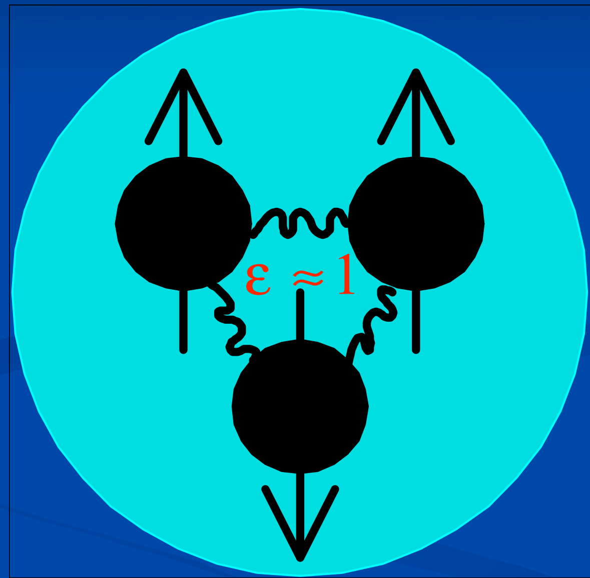
- horizon condition:
  - : $d^{-1}(k=0)=0 \qquad \varepsilon(k=0)=0$
- QCD vacuum: perfect color dia-electricum
  - dual superconductor:

$\varepsilon(k)<1$ anti-screening

$$D = \varepsilon E \qquad \partial D = \rho_{free}$$



$$\varepsilon = 0$$



no free color charges in the vacuum: confinement

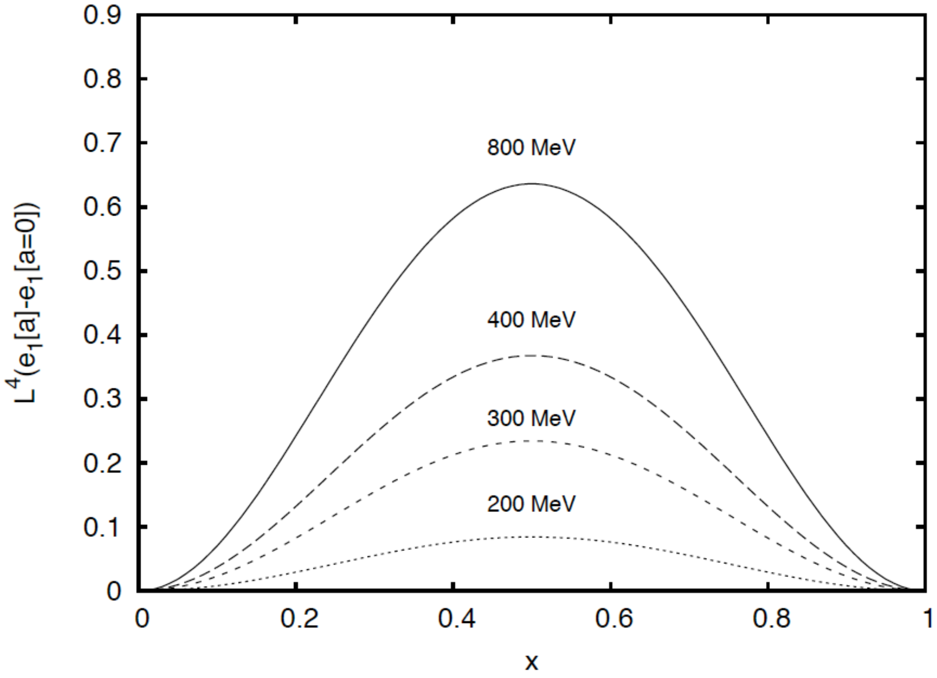
The effective potential for massive gluons

$$\chi(p)=0$$

$$e(a,L)=\sum_{\sigma}\frac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_\perp(\omega(p^\sigma)-\cancel{\chi(p^\sigma)})$$

$$\omega(p)=\sqrt{M^2+p^2}$$

$$x=\frac{aL}{2\pi}$$



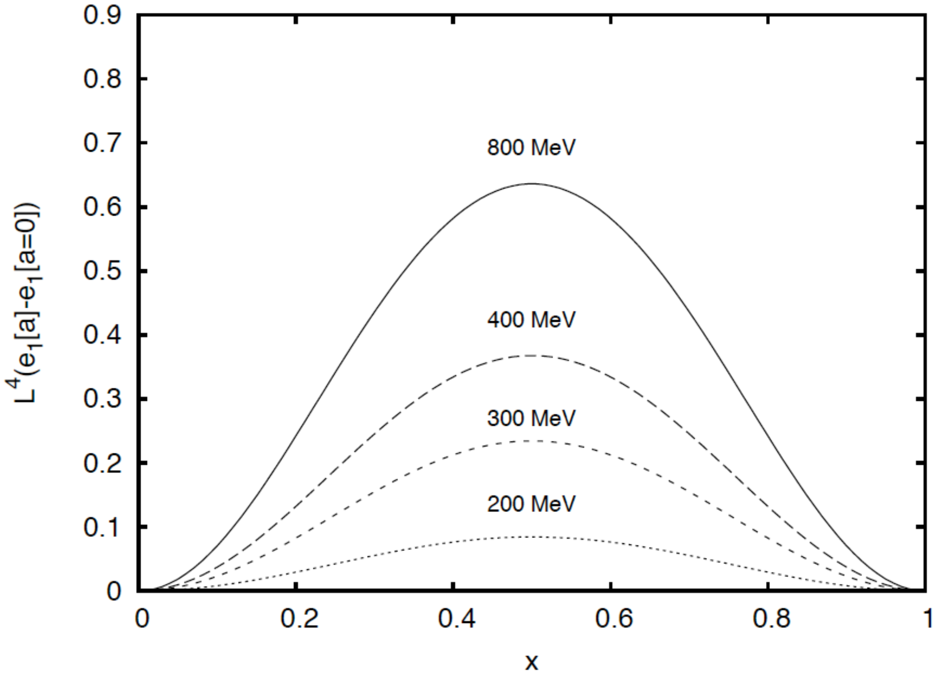
no phase transition

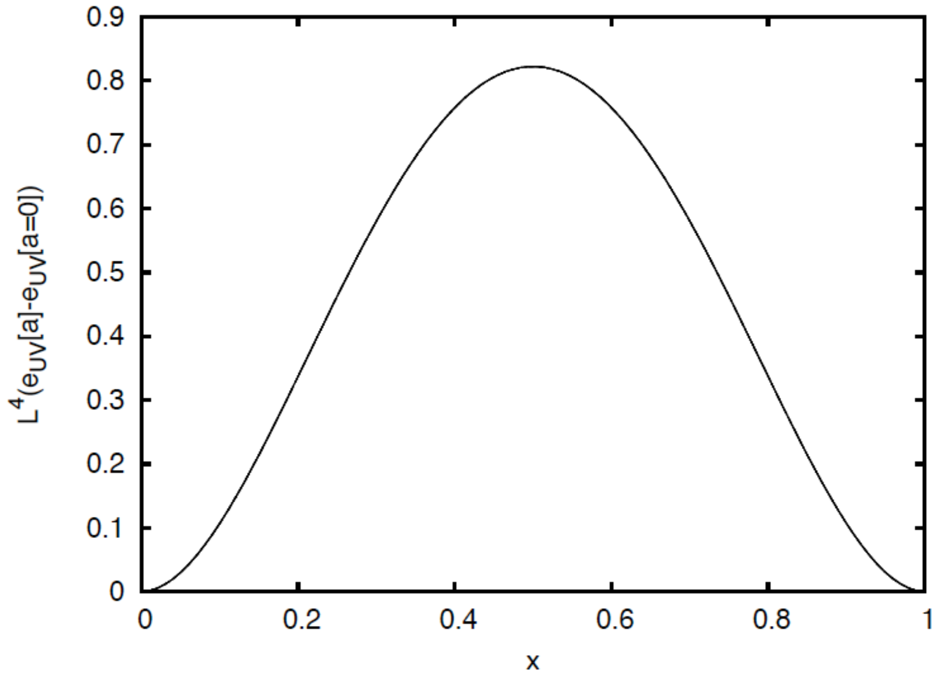
$$Polyakov-loop \quad \langle P\rangle \simeq P[a_{\min}=0]=1 \quad deconfining \quad phase$$

The effective potential for massive gluons

$$\chi(p)=0 \qquad e(a,L)=\sum_{\sigma}\frac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_\perp(\omega(p^\sigma)-\cancel{\chi(p^\sigma)})$$

$$\omega(p)=\sqrt{M^2+p^2} \qquad x=\frac{aL}{2\pi} \qquad M=0 \qquad \omega(p)=p$$





no phase transition **N.Weiss 1-loop PT**

$$Polyakov-loop \qquad \langle P\rangle \simeq P[a_{\min}=0]=1 \qquad deconfining\ \ phase$$

The effective potential for massive gluons

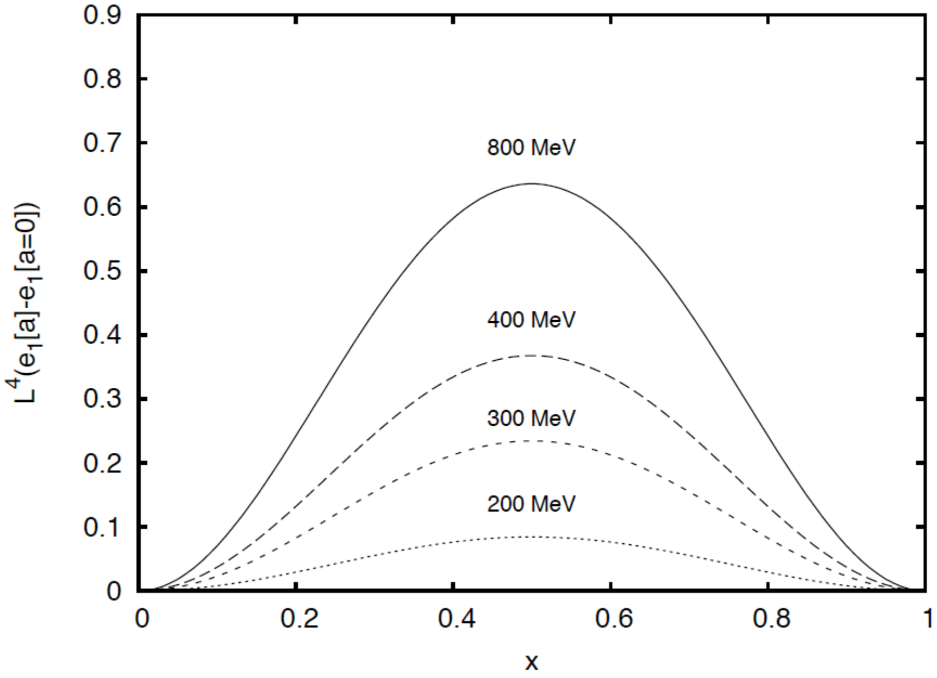
$$\chi(p) = 0$$

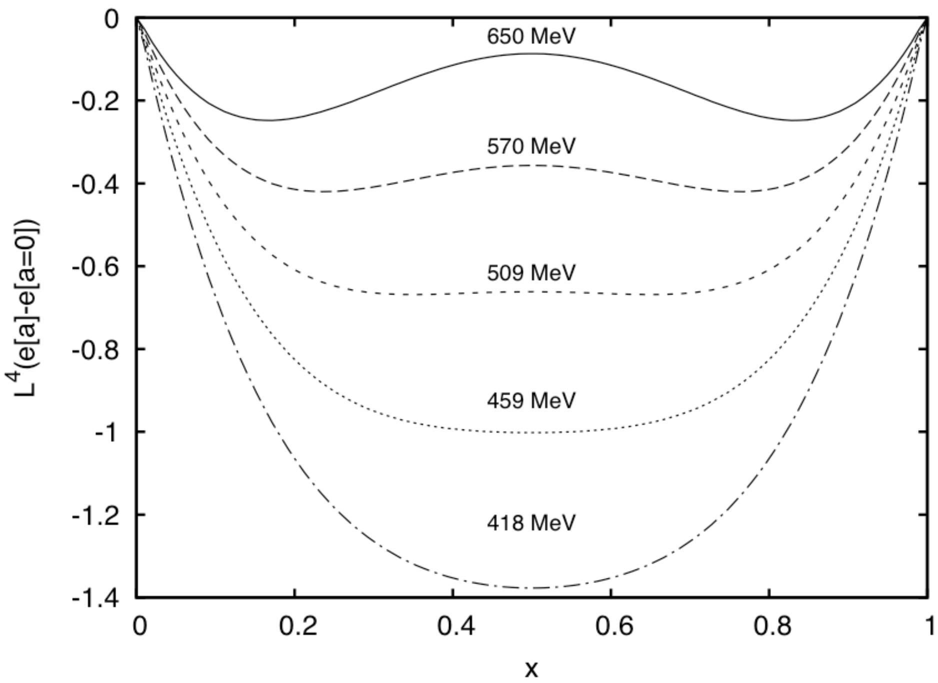
$$e(a,L) = \sum_{\sigma} \frac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_\perp (\omega(p^\sigma) - \cancel{\chi(p^\sigma)})$$

$$\omega(p) = \sqrt{M^2 + p^2}$$

$$x = \frac{aL}{2\pi}$$

$$\omega(p) = \sqrt{\frac{M^4}{p^2} + p^2}$$





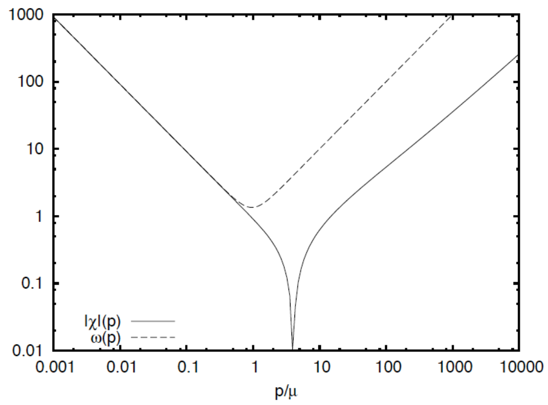
no phase transition

phase transition

The full effective potential for SU(3)

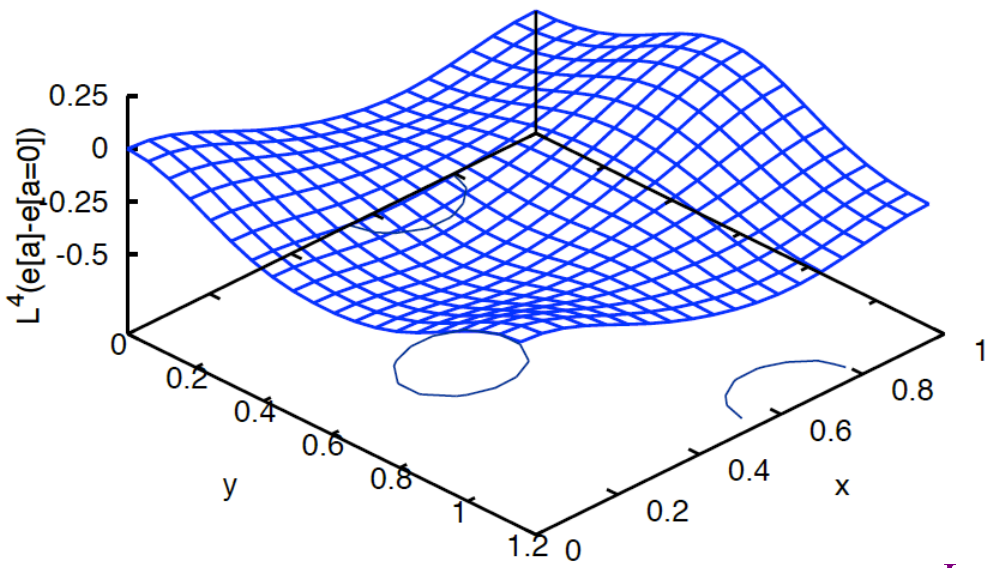
$$e(a,L) = \sum_{\sigma} \frac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_{\perp} (\omega(p^{\sigma}) - \chi(p^{\sigma}))$$

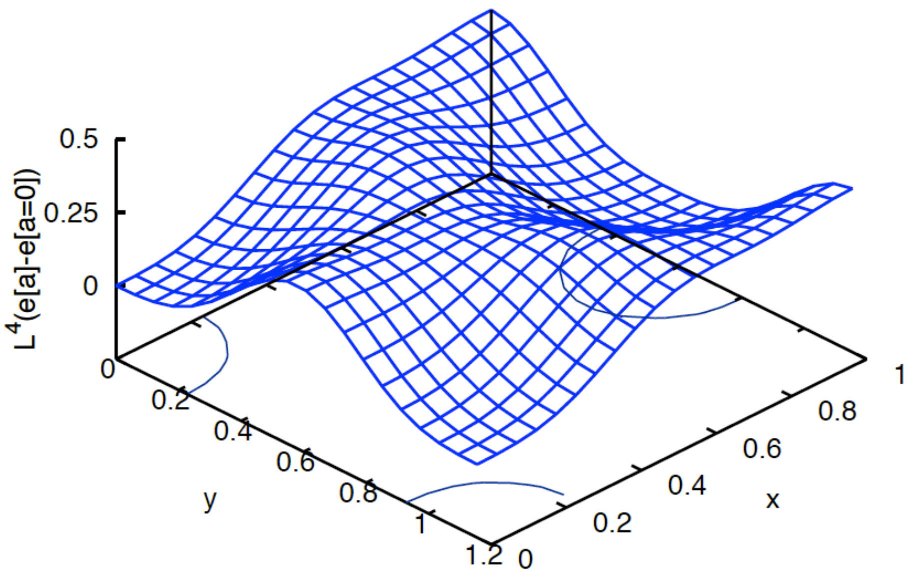
variational calculation in Coulomb gauge



$T < T_C$

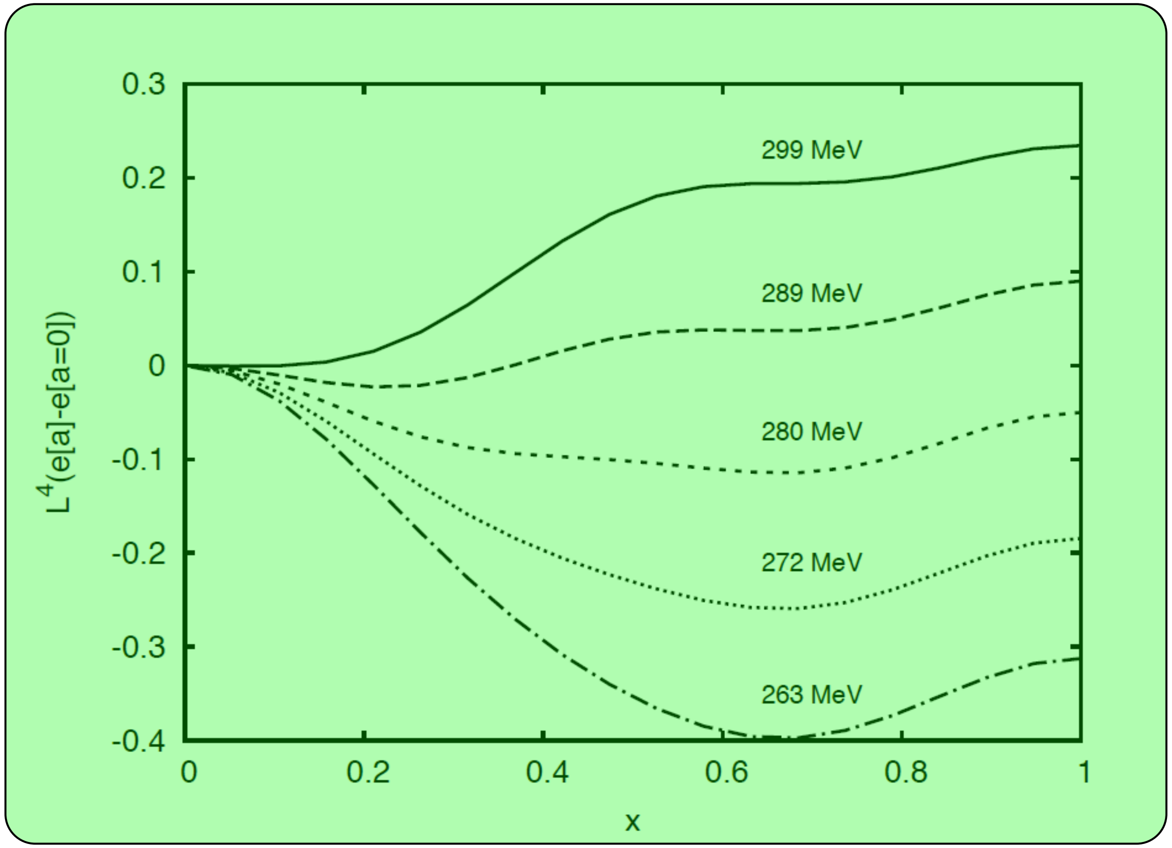
$T > T_C$





$$x = \frac{a_3 L}{2\pi}, \qquad y = \frac{a_8 L}{2\pi}$$

Polyakov loop potential for SU(3)



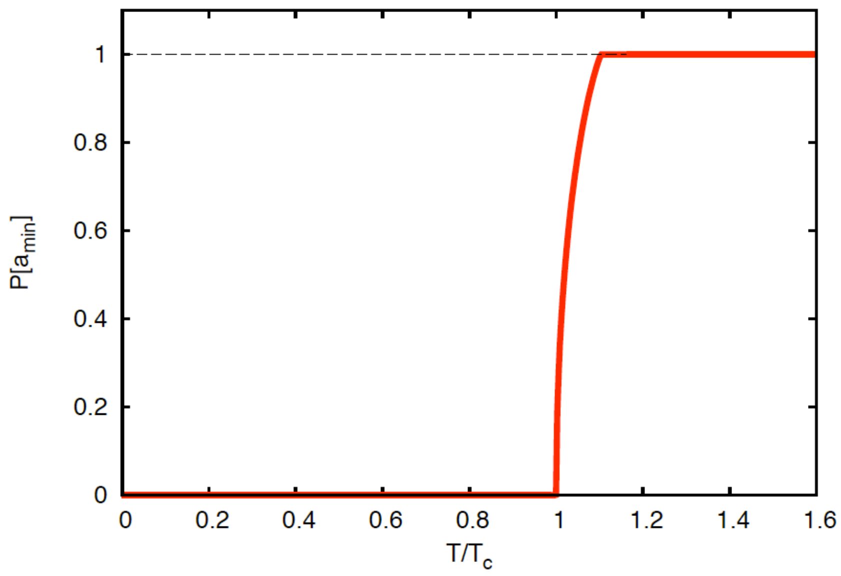
$$x = \frac{a_3 L}{2\pi}, \qquad y = \frac{a_8 L}{2\pi} = 0$$

$$input: \; SU(2) - data:$$

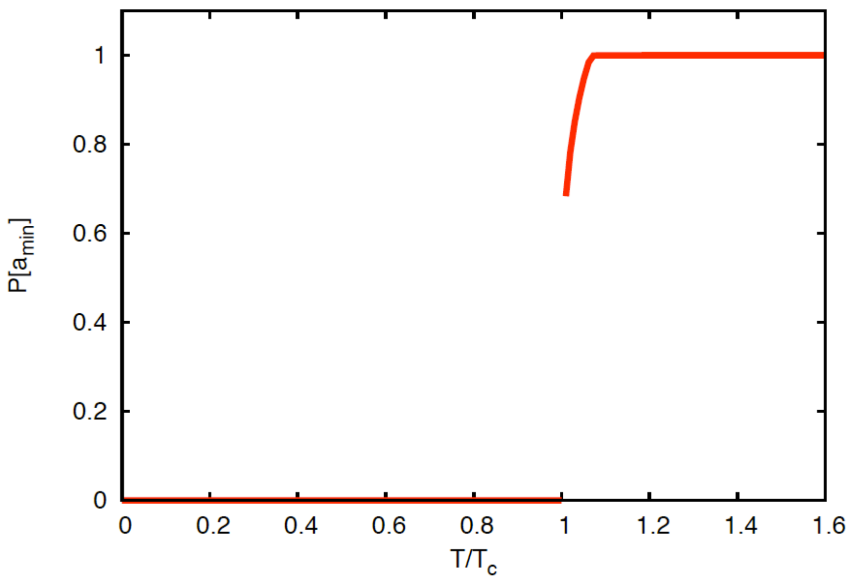
$$M = 880\,MeV$$

$$T_C = 283 MeV$$

The Polyakov loop



SU(2)



SU(3)

critical temperature

$lattice:$ $T_C^{SU(2)} = 295\,MeV$ $T_C^{SU(3)} = 270\,MeV$

$this\ work:$ $T_C^{SU(2)} = 267\,MeV$ $T_C^{SU(3)} = 277\,MeV$

$FRG(Fister\ \&\ Pawlowski):$ $T_C^{SU(2)} = 230\,MeV$ $T_C^{SU(3)} = 275\,MeV$

Conclusions

- effective potential of the Polyakov loop in the Hamiltonian approach

- input: vacuum propagators obtained in the variational calculation
- in Coulomb gauge

- neglect of ghost loop and use of the UV-gluon energy $\omega(p) = p$:
  ` Weiss potential

- full potential: deconfinement phase transition $T_C \simeq 270 MeV$
  - SU(2): 2.order
  - SU(3): 1.order

- deconfinement phase transition is encoded in the vacuum wave functional on $R^2 \times S^1$

Thanks for your attention