GRAVITATIONAL WAVES FROM FIRST ORDER PHASE TRANSITIONS

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OUTLINE

- Overview of GWs from cosmological sources in the early universe
- First order phase transitions as sources of GWs
- Identify the relevant parameters that enter in the GW spectrum in connection with physics of the source
- Sketch the construction of an analytical model of the GW source to determine the shape of the spectrum
- GW signal for two known PT cases
- Detection prospects

GWs: unique probe of the very early universe

- because of the weakness of the gravitational interaction, the universe becomes transparent to GWs very early

- GWs propagate freely after generation in the early universe, carrying direct information on the process itself

photon background (CMB):	$T_{\rm dec} = 0.3\,{\rm eV}$	
neutrino background:	$T_{\rm dec} = 1\,{\rm MeV}$	$\frac{\Gamma(T)}{H(T)} < 1$
"primordial GW background" :	$T_{\rm dec} = 10^{19}\,{\rm GeV}$	

- Limits or detection of the GW background could provide a big step forward in the knowledge of the very early universe, falsifying models

GWs from cosmological sources

tensor perturbations of FRW metric: $\quad (h_i{}^i = h_i{}^j{}_{|j} = 0)$

$$ds^2 = a^2(t)[-dt^2 + (\delta_{ij} + 2h_{ij})dx^i dx^j]$$

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu} \qquad \ddot{h}_{ij} + 2\frac{\dot{a}}{a}\dot{h}_{ij} + k^2 h_{ij} = 8\pi G\, a^2\, \Pi_{ij}$$

- source: Π_{ij} tensor anisotropic stress
  - fluid : $\Pi_{ij} = \gamma^2(\rho + p)\, v_i v_j$
  - electromagnetic field : $\Pi_{ij} = \frac{(E^2 + B^2)}{3} - E^i E^j - B^i B^j$
  - scalar field : $\Pi_{ij} = \partial_i \phi\, \partial_j \phi$

GWs from cosmological sources

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- source: Π_{ij} tensor anisotropic stress
- stochastic background of GW, statistically homogenous, isotropic and Gaussian

$$\langle \dot{h}_{ij}(\mathbf{k})\dot{h}^*_{ij}(\mathbf{q})\rangle = (2\pi)^3\delta(\mathbf{k}-\mathbf{q})|\dot{h}(k)|^2$$

GWs from cosmological sources

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$$\langle \dot{h}_{ij}(\mathbf{k})\dot{h}^*_{ij}(\mathbf{q})\rangle = (2\pi)^3\delta(\mathbf{k}-\mathbf{q})|\dot{h}(k)|^2$$

- GW energy density:

$$\Omega_{\rm GW} = \frac{\langle \dot{h}_{ij}\dot{h}_{ij}\rangle}{8\pi G\, a^2\, \rho_c} = \int \frac{{\rm d}k}{k}\frac{{\rm d}\Omega_{\rm GW}}{{\rm d}\ln k} \qquad \frac{{\rm d}\Omega_{\rm GW}}{{\rm d}\ln k} = \frac{k^3|\dot{h}(k)|^2}{2(2\pi)^3\, G\, a^2\, \rho_c}$$

Characteristic frequency

causal source of GW cannot operate beyond the cosmological horizon:

$$k_* = \frac{H_*}{\epsilon_*} \qquad\qquad \epsilon_* \leq 1$$

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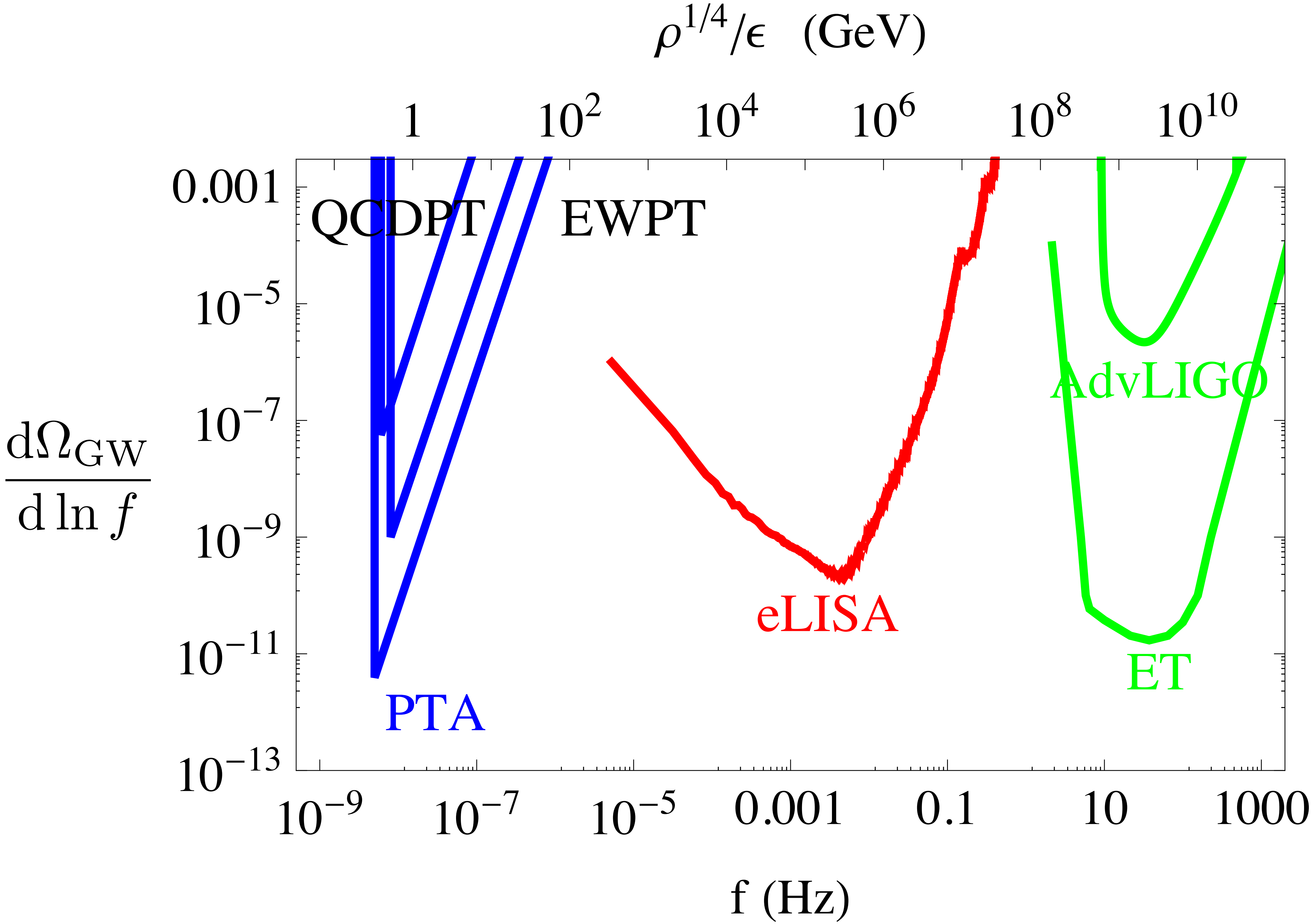
$$k_* = \frac{H_*}{\epsilon_*} \qquad\qquad \epsilon_* \leq 1$$

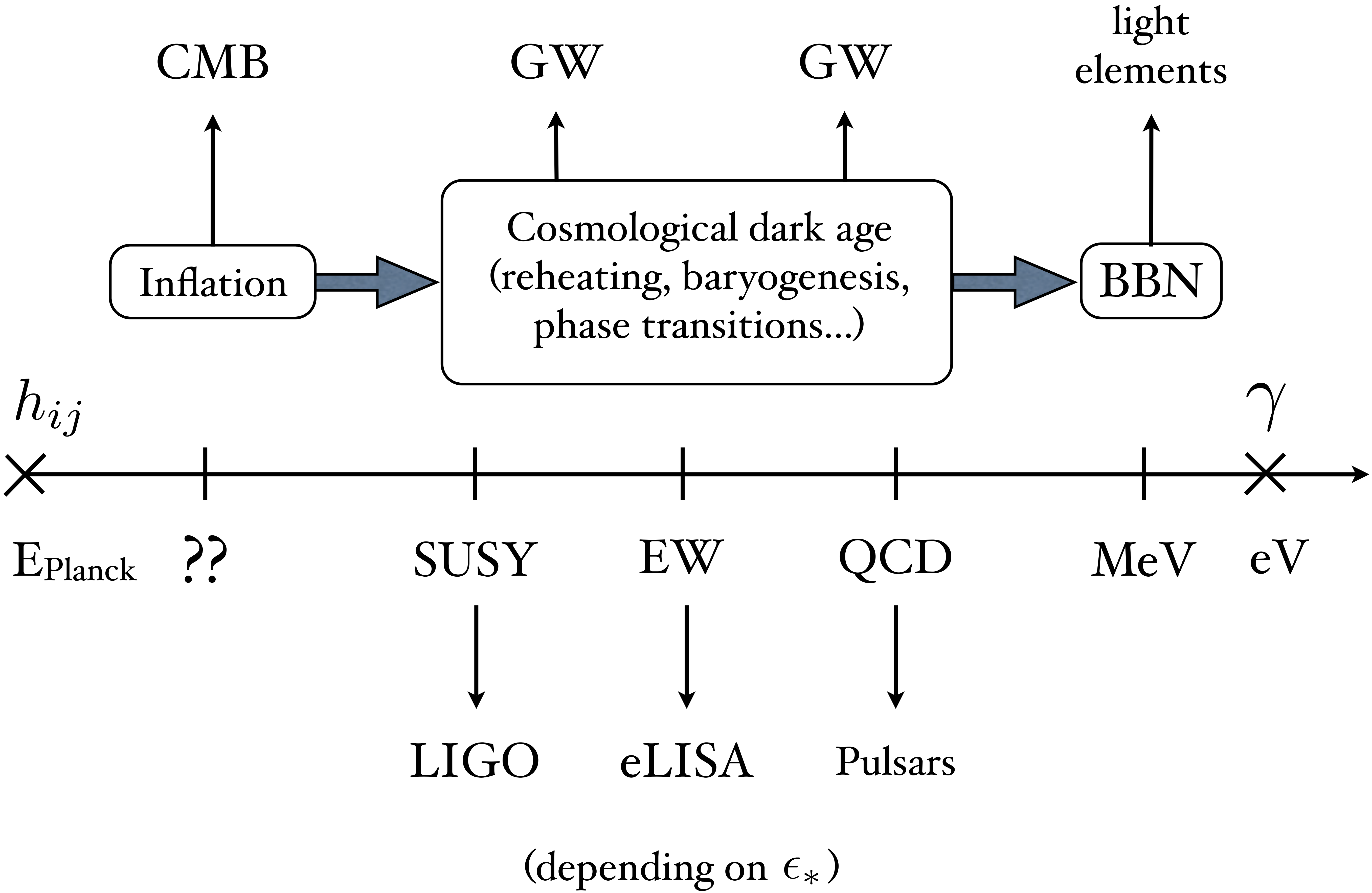
$$f_c = \frac{k_*}{2\pi}\frac{a_*}{a_0} = \frac{1.6 \cdot 10^{-4}\,\text{Hz}}{\epsilon_*}\left(\frac{T_*}{1\,\text{TeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$

- f_c → characteristic frequency today
- ϵ_* → dynamics of the source
- T_* → temperature (energy density) of the universe at the source time

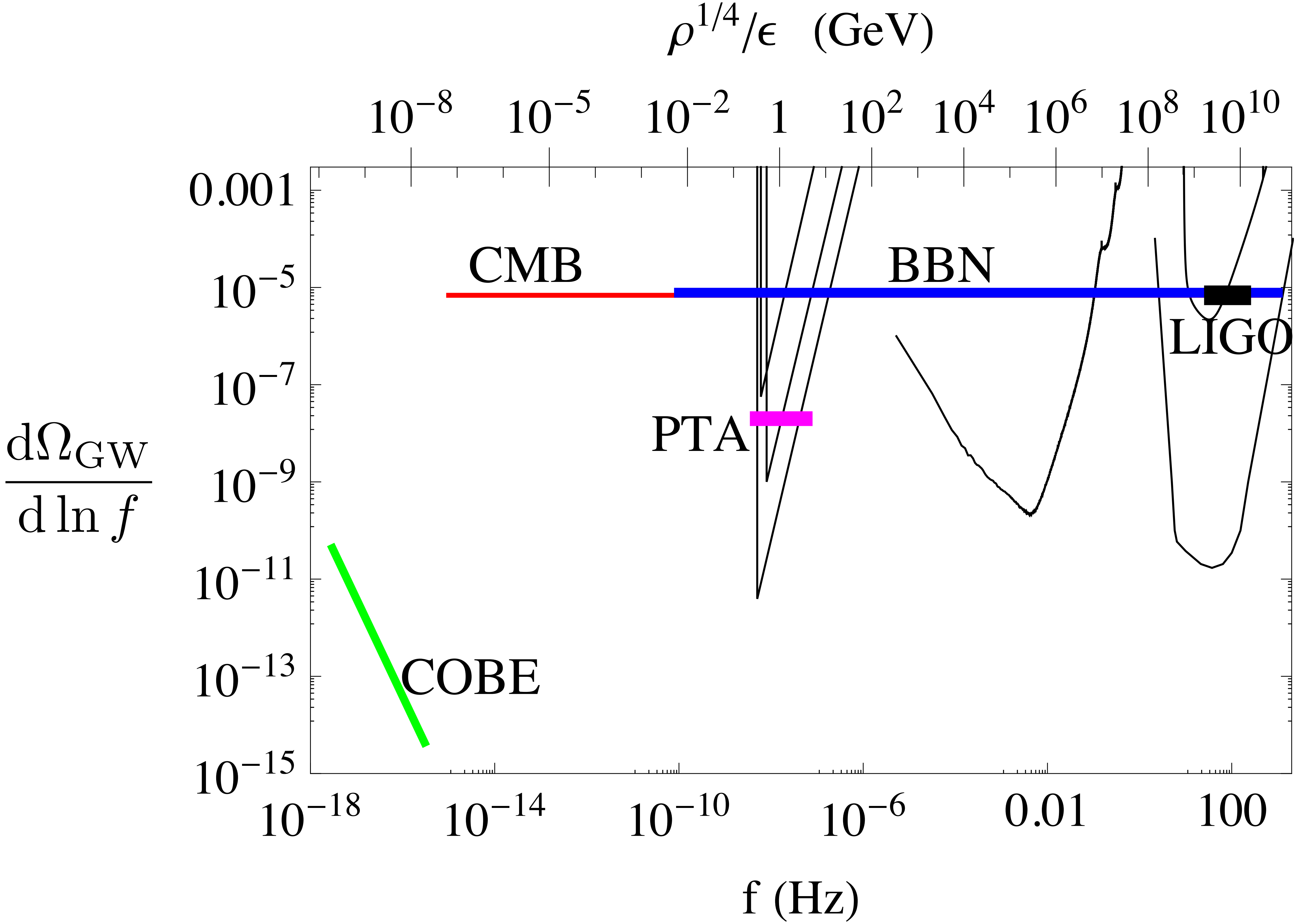
standard thermal history $\quad H = \frac{\dot{a}}{a} \qquad a = \left(\frac{g_0}{g}\right)^{\frac{1}{3}}\frac{T_0}{T}$

Characteristic frequency




CMB
GW
GW
light elements
Inflation
Cosmological dark age (reheating, baryogenesis, phase transitions...)
BBN
h_{ij}
γ
E_{Planck}
??
SUSY
EW
QCD
MeV
eV
LIGO
eLISA
Pulsars
(depending on ϵ_*)

Current limits on a stochastic background



GWs from first order phase transitions

- universe expands and temperature decreases : PT
- nature of PT depends on the particle theory model
- if it is first order it can lead to the production of GW

(Witten '84, Hogan '86...)

(Turner et al '92, Kosowsky et al '92, Kosowsky and Turner '93, Kamionkowski et al '94, Kosowsky et al '02, Dolgov et al '02, Gogoberidze et al '08, Kahniashvili et al '08...)

- EWPT : beyond the standard model (baryogenesis)

(Apreda et al '01, Nicolis '04, Grojean et al '05, Huber and Konstandin '08, Kahniashvili et al '09, Kehayias and Profumo '09, Chung and Long '10...)

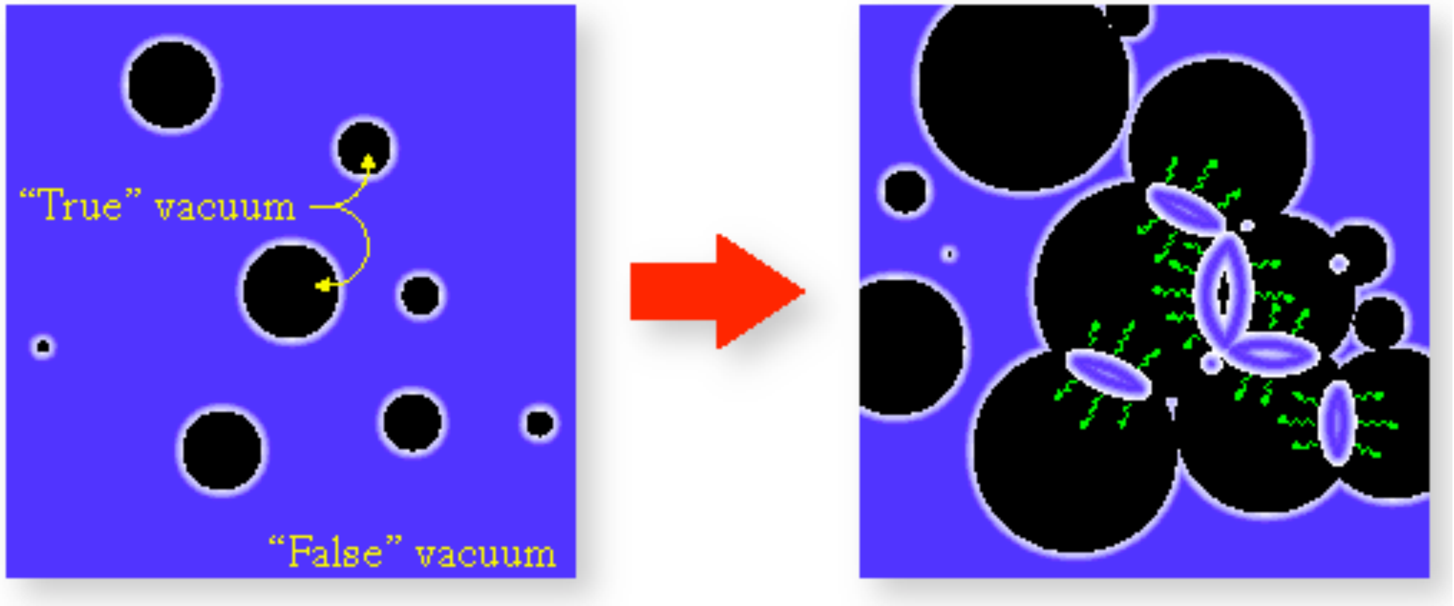
- QCDPT : if lepton asymmetry is large (Schwarz and Stuke '09)

GWs from first order phase transitions

$$\ddot{h}_{ij} + 2\frac{\dot{a}}{a}\dot{h}_{ij} + k^2 h_{ij} = 8\pi G\, a^2\, \Pi_{ij}$$

source: Π_{ij}

tensor anisotropic stress



- collisions of bubble walls
- turbulence in the primordial fluid
- magnetic fields (MHD turbulence)

GWs from first order phase transitions

$$\ddot{h}_{ij} + 2\frac{\dot{a}}{a}\dot{h}_{ij} + k^2 h_{ij} = 8\pi G\, a^2\, \Pi_{ij}$$

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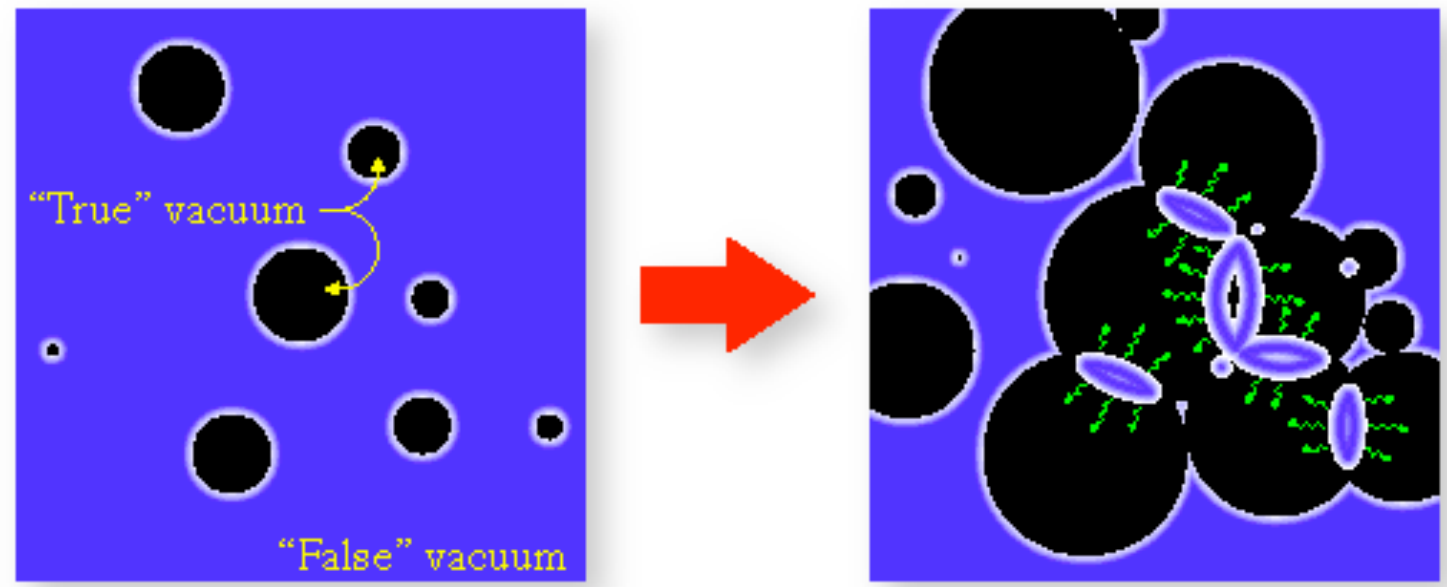


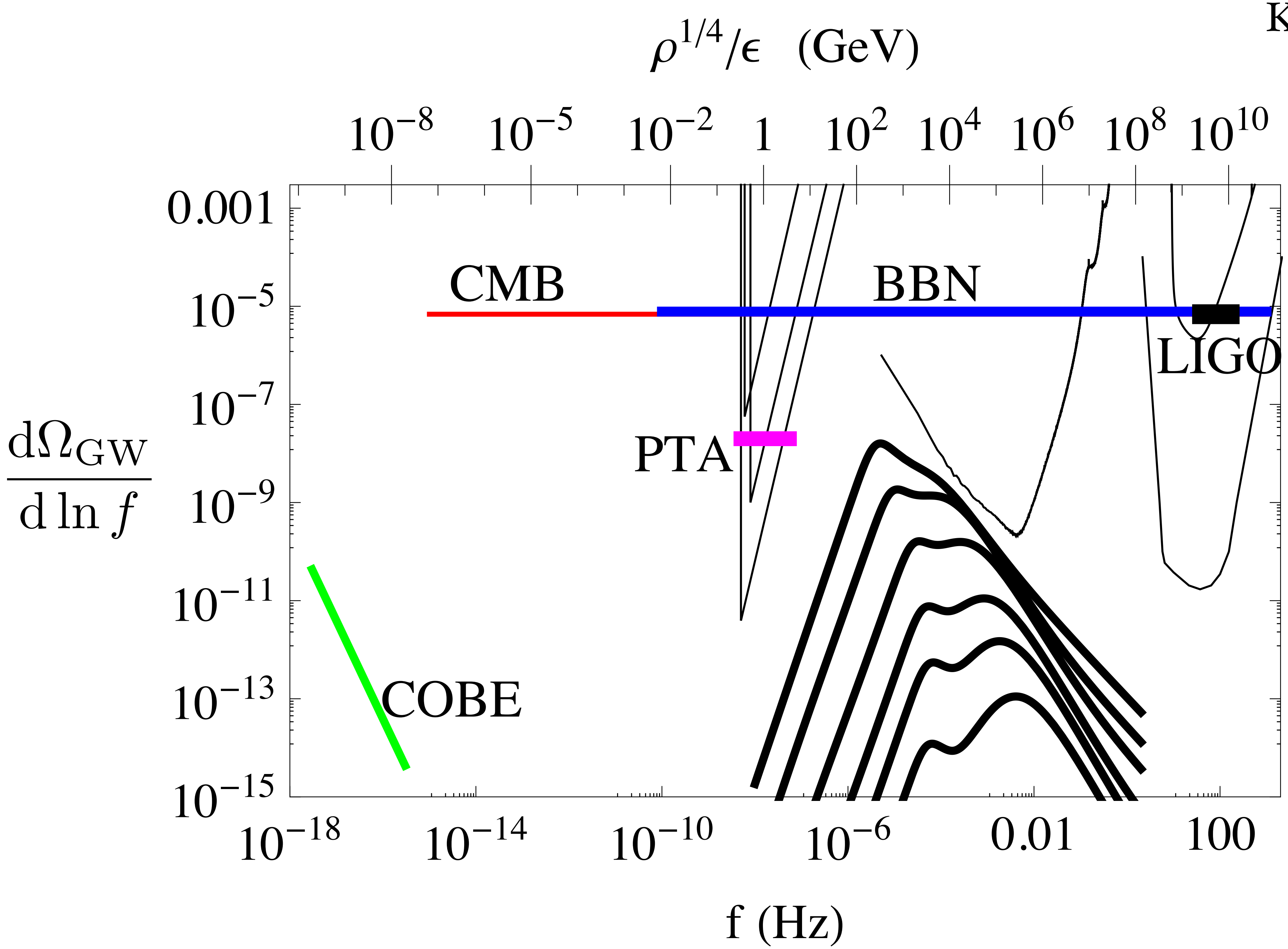
$$\Pi_{ij} = \partial_i\phi\,\partial_j\phi$$

$$\Pi_{ij} = \gamma^2(\rho + p)\,v_i v_j$$

$$\Pi_{ij} = \frac{(E^2 + B^2)}{3} - E^i E^j - B^i B^j$$

Example of GW spectrum from EWPT

(Huber and Konstandin 2008)



GWs from PT : peak frequency

GW generation processes related to size of the bubbles
towards the end of the PT

$$k_* = 1.6 \cdot 10^{-7} \epsilon^{-1} \left(\frac{T_*}{1\,\mathrm{GeV}} \right) \left(\frac{g_*}{100} \right)^{\frac{1}{6}} \mathrm{Hz}$$

$$\epsilon \simeq \frac{H_*}{\beta}, \; H_* R_*$$

β^{-1} duration of the PT

$R_* = v_b \, \beta^{-1}$ size of bubbles at collision

$v_b \leq 1$ speed of bubble walls

corresponding to the source characteristic time or scale depending on the source properties: space and time correlations

GWs from PT : scaling of the peak amplitude

energy density of GWs: $\rho_G \sim \frac{\dot{h}^2}{8\pi G}$

$\delta G_{ij} = 8\pi G\, T_{ij}$ $\beta^2 h \sim 8\pi G\, T$ $\dot{h} \sim \frac{8\pi G\, T}{\beta}$

β → characteristic time of evolution

h → tensor perturbation

T → energy momentum tensor

$$\Omega_{\rm GW} \sim \Omega_{\rm rad} \left(\frac{H_*}{\beta}\right)^2 \left(\frac{\rho_{\rm s}^*}{\rho_{\rm tot}^*}\right)^2$$

$\Omega_{\rm rad}$ → radiation parameter

H_*/β → DURATION of the source with respect to Hubble time

$\rho_{\rm s}^*/\rho_{\rm tot}^*$ → RELATIVE ENERGY DENSITY available in the source for the GW generation: T

GWs from PT : scaling of the peak amplitude

energy density of GWs: $$\rho_G \sim \frac{\dot{h}^2}{8\pi G}$$

$$\delta G_{ij} = 8\pi G\, T_{ij}$$

$$\beta^2 h \sim 8\pi G\, T$$

β: characteristic time of evolution; h: tensor perturbation; T: energy momentum tensor

$$\dot{h} \sim \frac{8\pi G\, T}{\beta}$$

$$\Omega_{\rm GW} \sim \Omega_{\rm rad} \left(\frac{H_*}{\beta}\right)^2 \left(\frac{\rho_{\rm s}^*}{\rho_{\rm tot}^*}\right)^2$$

$$\Omega_{\rm GW} \gtrsim 10^{-10} \quad \Rightarrow \quad \left(\frac{H_*}{\beta}\right)\left(\frac{\rho_{\rm s}^*}{\rho_{\rm tot}^*}\right) \gtrsim 3 \cdot 10^{-3}$$

Parameters entering the GW spectrum

$$k_* = 1.6 \cdot 10^{-7} \, \epsilon^{-1} \left(\frac{T_*}{1\,\mathrm{GeV}} \right) \left(\frac{g_*}{100} \right)^{\frac{1}{6}} \mathrm{Hz}$$

↓

$$\epsilon \simeq \frac{H_*}{\beta} \, , \;\; H_* \, R_*$$

⇨

$$\frac{\beta}{H_*} \, , \quad v_b \, , \quad T_*$$

$$\Omega_{\mathrm{GW}} \sim \Omega_{\mathrm{rad}} \left(\frac{H_*}{\beta} \right)^2 \left(\frac{\rho_{\mathrm{s}}^*}{\rho_{\mathrm{tot}}^*} \right)^2$$

⇨

$$\alpha = \frac{\rho_{\mathrm{vac}}}{\rho_{\mathrm{rad}}^*}$$

$$\kappa = \frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{vac}}}$$

$$\frac{\rho_{\mathrm{s}}^*}{\rho_{\mathrm{tot}}^*} = \frac{\kappa \, \alpha}{1 + \alpha}$$

Parameters entering the GW spectrum

$$\frac{\beta}{H_*}\,, \quad \alpha\,, \quad T_*$$

are determined from the PT dynamics :

(Espinosa et al 2008)

T_* temperature at which the PT completes, when the fraction of space covered by bubbles is one:

$$\Gamma(T) \qquad\qquad f(T_*) = \frac{4\pi}{3}\int_{T_*}^{T_c} \frac{d\bar{T}}{\bar{T}}\frac{\Gamma(\bar{T})}{H} R^3(T,\bar{T}) = 1$$

$\frac{\beta}{H_*}$ duration of the PT:

$$S(t) \simeq S(t_*) - \beta(t - t_*) + \mathcal{O}(t - t_*)^2$$

$$\frac{\beta}{H_*} = T_* \, \frac{\mathrm{d}}{\mathrm{d}T}\frac{S_3(T)}{T}\bigg|_{T_*}$$

α determined from the latent heat $\quad L = \Delta V(\varphi) - T\Delta S$

Parameters entering the GW spectrum

$$\frac{\beta}{H_*}\,, \quad \alpha\,, \quad T_*$$

are determined from the PT dynamics :

(Espinosa et al 2008)

$\frac{\beta}{H_*}$ and α are not independent parameters : stronger PT lasts longer

(Huber and Konstandin 2008)
(Grojean and Servant 2007)

$$\Omega_{\rm GW} \sim \Omega_{\rm rad} \left(\frac{H_*}{\beta}\right)^2 \left(\frac{\kappa\,\alpha}{1+\alpha}\right)^2$$

for a stronger PT the GW amplitude is higher

$$\epsilon \simeq \frac{H_*}{\beta}\,, \;\; H_*\,R_*$$

but the GW spectrum peaks at lower frequency: bad for detection

Parameters entering the GW spectrum

$$k_* = 1.6 \cdot 10^{-7}\, \epsilon^{-1} \left(\frac{T_*}{1\,\mathrm{GeV}}\right) \left(\frac{g_*}{100}\right)^{\frac{1}{6}} \mathrm{Hz}$$

$$\downarrow$$

$$\epsilon \simeq \frac{H_*}{\beta}\,,\quad H_*\, R_*$$

$$\Rightarrow \quad \boxed{\frac{\beta}{H_*}\,, \quad v_b\,, \quad T_*}$$

$$\Omega_{\mathrm{GW}} \sim \Omega_{\mathrm{rad}} \left(\frac{H_*}{\beta}\right)^2 \left(\frac{\rho_{\mathrm{s}}^*}{\rho_{\mathrm{tot}}^*}\right)^2$$

$$\Rightarrow \quad \alpha = \frac{\rho_{\mathrm{vac}}}{\rho_{\mathrm{rad}}^*} \qquad \kappa = \frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{vac}}}$$

$$\frac{\rho_{\mathrm{s}}^*}{\rho_{\mathrm{tot}}^*} = \frac{\kappa\, \alpha}{1+\alpha}$$

Parameters entering the GW spectrum

$\kappa \, , \quad v_b$ are determined from the dynamics of bubble expansion :

(Espinosa et al 2010)

$v_b(\alpha, \eta)$ wall velocity : balance among
driving force (pressure difference - α) and the
friction force (interaction of the wall with surrounding plasma -
phenomenological description of several particle theory models
introducing a new parameter, the friction η)

(Steinhardt '92,
Kamionkowski et al '94,
Ignatius et al '94...)

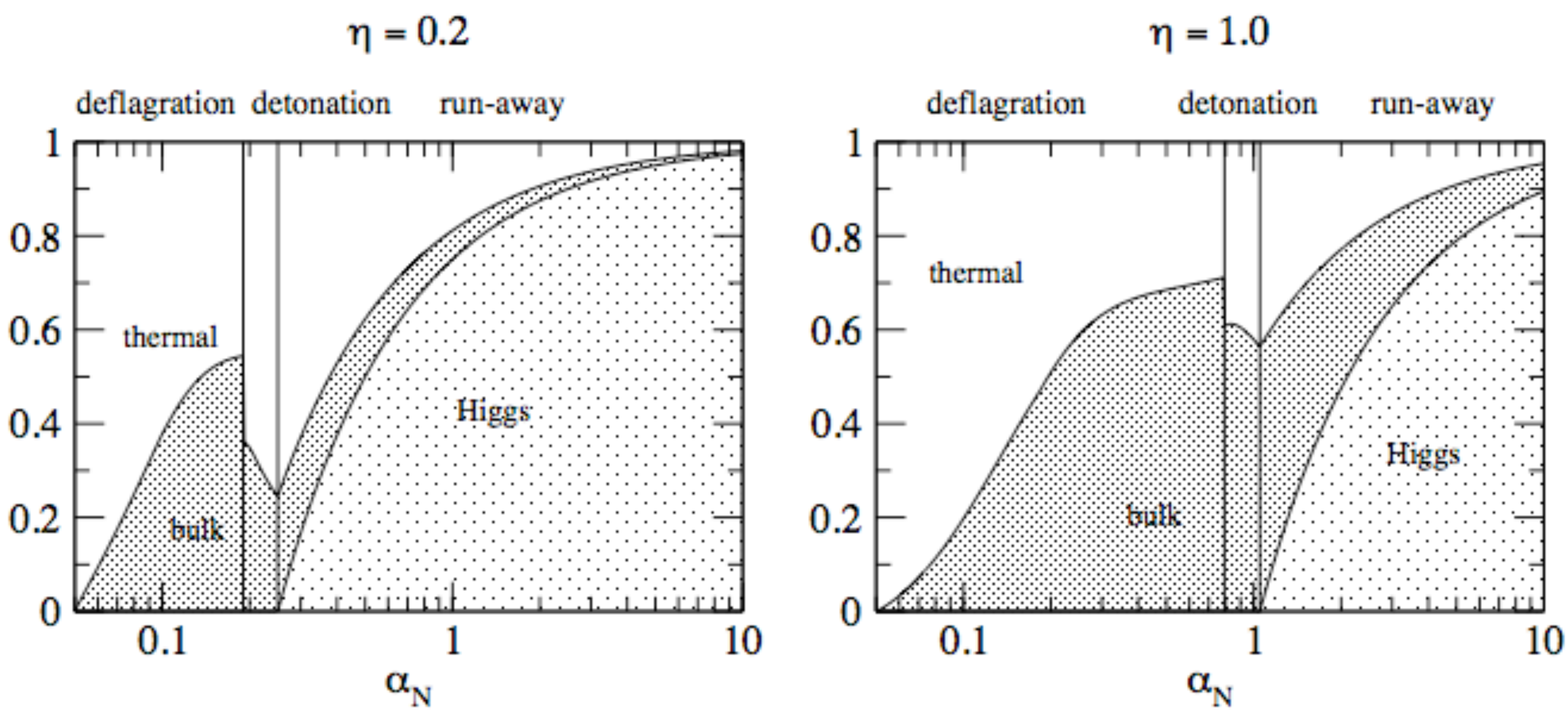
Parameters entering the GW spectrum

$\kappa\,, \quad v_b$ are determined from the dynamics of bubble expansion :

(Espinosa et al 2010)

$\kappa(\alpha, \eta)$ kinetic energy : hydrodynamics of bubble growth at late times

$\kappa(\alpha, \eta)$



Parameters entering the GW spectrum

$$\frac{\beta}{H_*}\,, \quad \alpha\,, \quad T_*\,, \quad \eta$$

- all known (at least in principle) for a given PT model
- not independent among each other
- correctly account for the contribution of bubble collisions (runaway) and MHD turbulence (deflagrations, hybrids and weak detonations)

what is the spectral shape of the GW spectrum
and the peak position?

Analytical evaluation of the GW spectrum

GW power spectrum: $$\Omega_{\rm GW} = \frac{\langle \dot{h}_{ij}\dot{h}_{ij}\rangle}{8\pi G\, a^2\, \rho_c} = \int \frac{{\rm d}k}{k}\frac{{\rm d}\Omega_{\rm GW}}{{\rm d}\ln k}$$

$$\frac{d\Omega_{\rm GW}}{d\ln k} \propto k^3 \int_{t_{\rm in}}^{t_{\rm fin}} \frac{dt_1}{t_1}\int_{t_{\rm in}}^{t_{\rm fin}} \frac{dt_2}{t_2} \cos[k(t_1-t_2)]\, \Pi(k,t_1,t_2)$$

source: anisotropic stress power spectrum at unequal time

$$\langle \Pi_{ij}(\mathbf{k},t_1)\Pi^*_{ij}(\mathbf{q},t_2)\rangle = \delta(\mathbf{k}-\mathbf{q})\Pi(k,t_1,t_2)$$

analytical model of the stochastic source for bubble collisions and MHD turbulence

1. space correlation structure (at equal times)
2. time correlation structure
3. overall time evolution

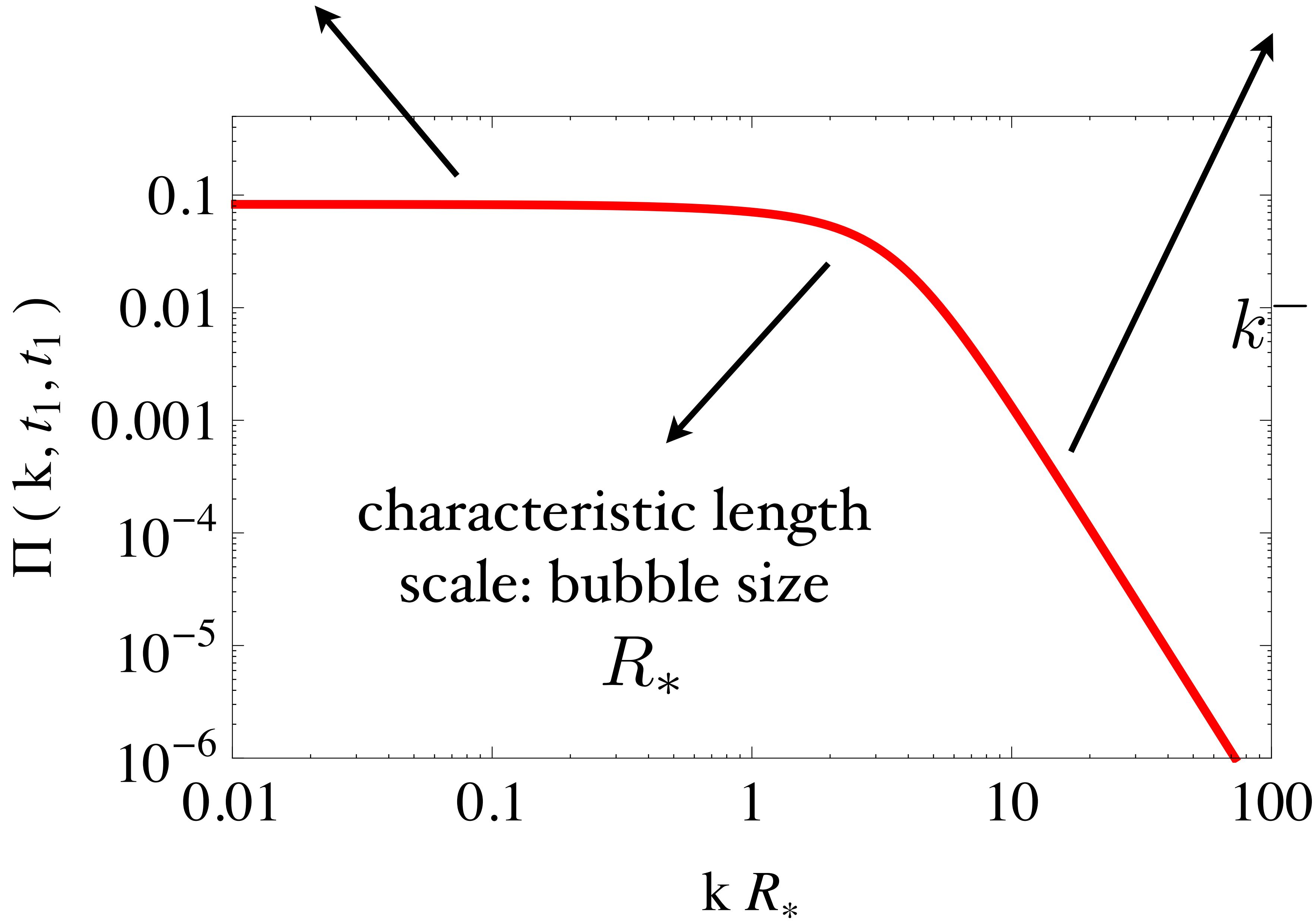
I. space correlation structure at equal times

bubbles and MHD causal processes with typical length scale: bubble size

flat: spatially uncorrelated, causality

k^3 slope in the GW power spectrum

slope depending on source power spectrum



k^{-4} bubbles ?

$k^{-11/3}$ Kolmogorov turbulence

2. time correlation structure

$$\epsilon \simeq \frac{H_*}{\beta}\,,\ H_* R_*$$

BUBBLES : completely coherent

peak at the characteristic time of the source $k_* \simeq \beta$

MHD TURBULENCE : decorrelating in time

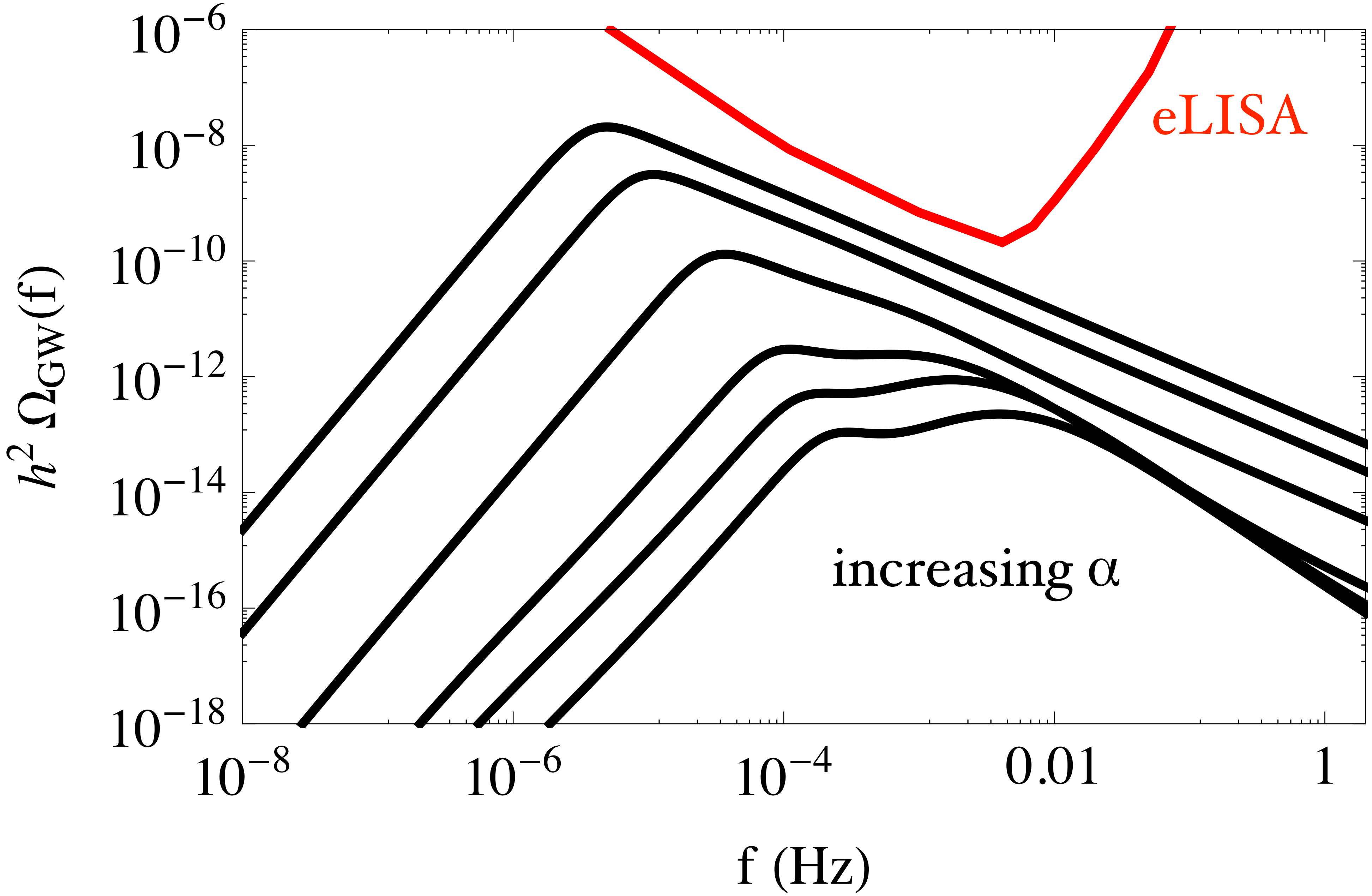
peak at the spatial correlation scale $k_* \simeq R_*^{-1}$

Examples of GW spectrum

$$V(H) = -\frac{\mu^2}{2}H^2 + \frac{\lambda}{4}H^4 + \frac{1}{8M^2}H^6$$

(Huber and Konstandin 2008)

SM + dimension six operator, η=0.2



Examples of GW spectrum

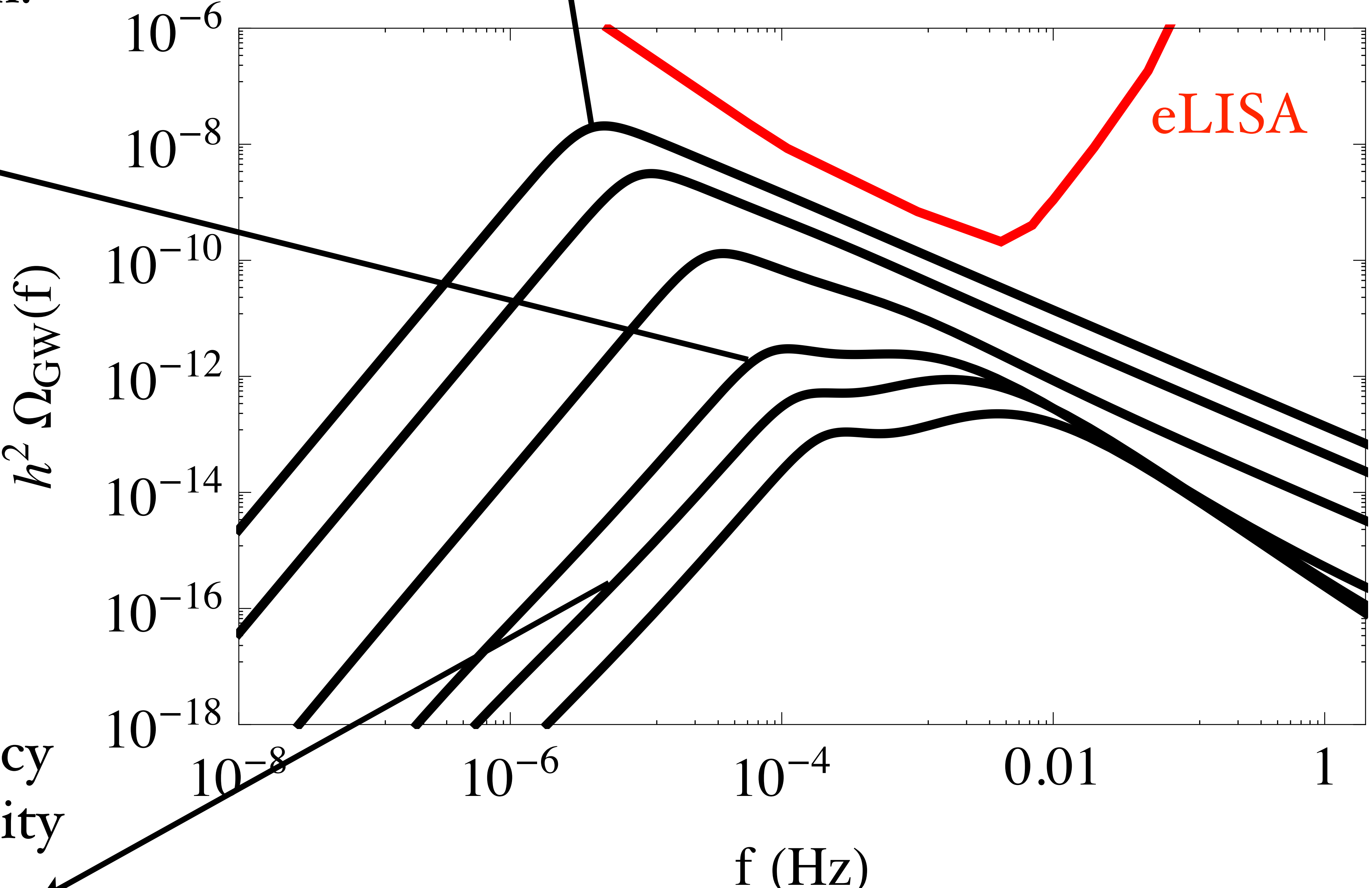
as PT gets stronger: 1. signal from bubble collision becomes dominant
gradient energy of the Higgs dominates bulk kinetic energy

peak position:

$k_* \sim R_*^{-1}$

$k_* \simeq \beta$

$R_* = v_b/\beta$



low frequency slope: causality

k^3

Examples of GW spectrum

as PT gets stronger: 2. the peak is at lower frequencies

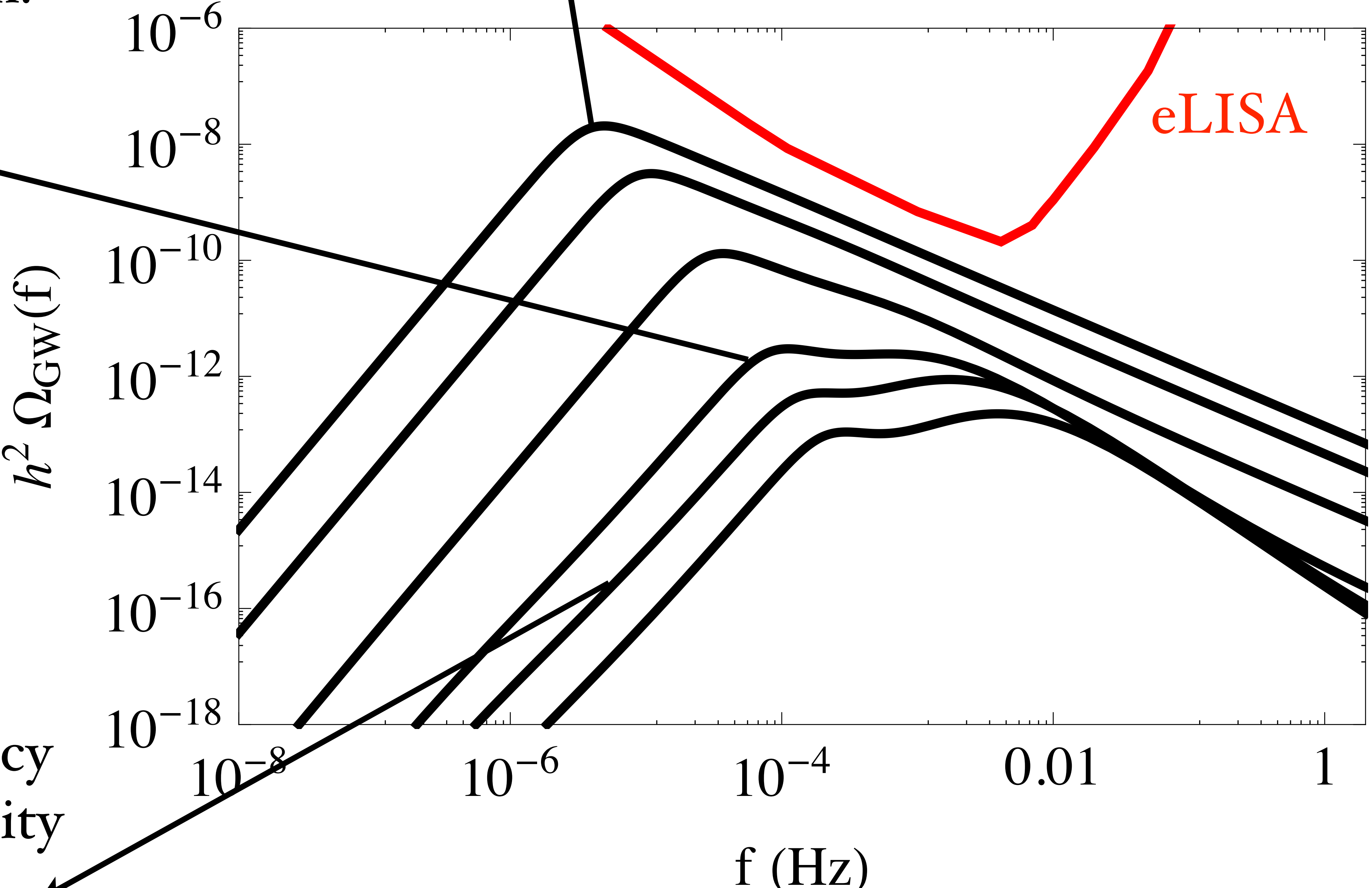
β/H_* and T_* decrease, bad for detection

peak position:

$k_* \sim R_*^{-1}$

$k_* \simeq \beta$

$R_* = v_b/\beta$



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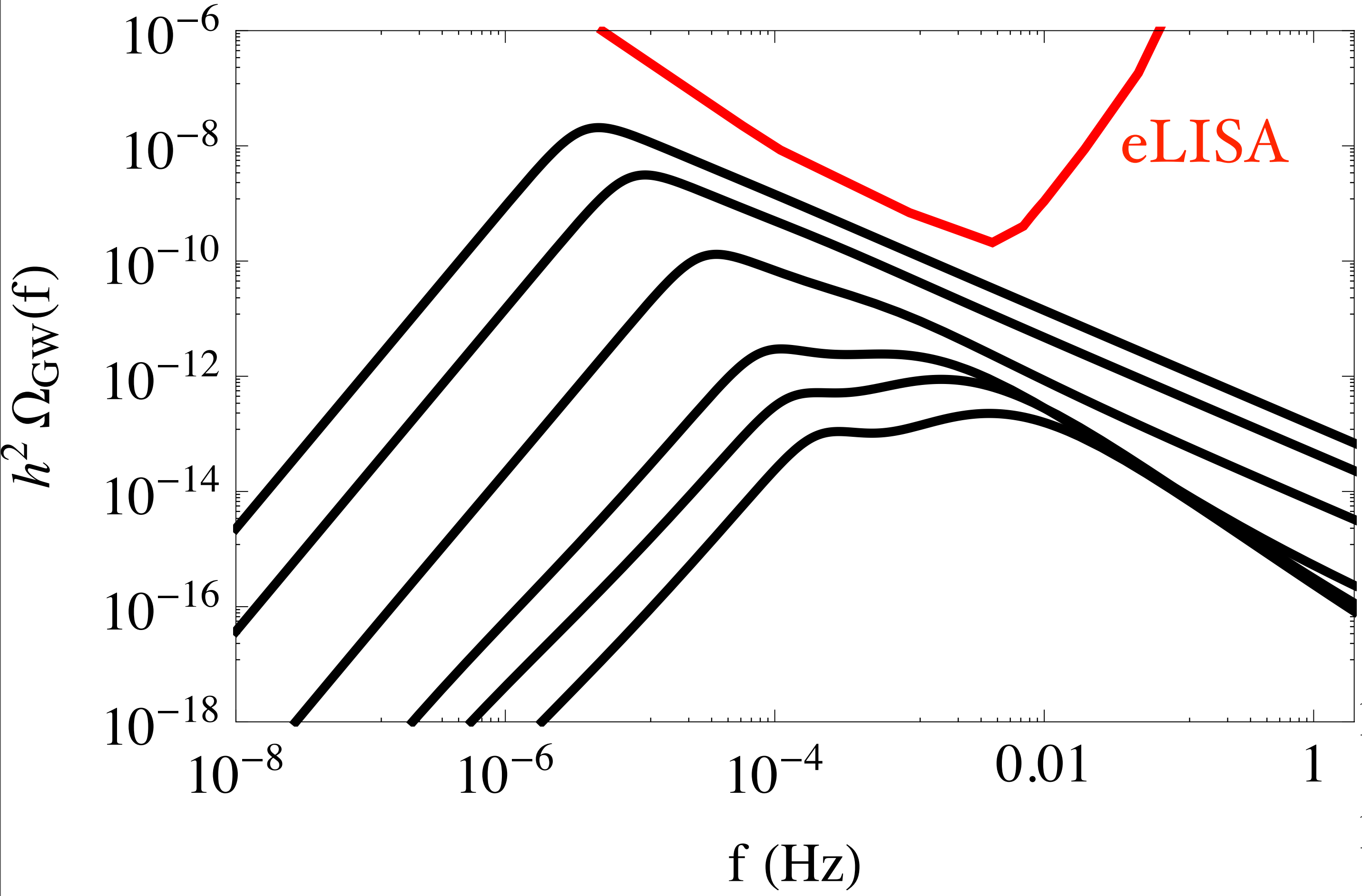
k^3

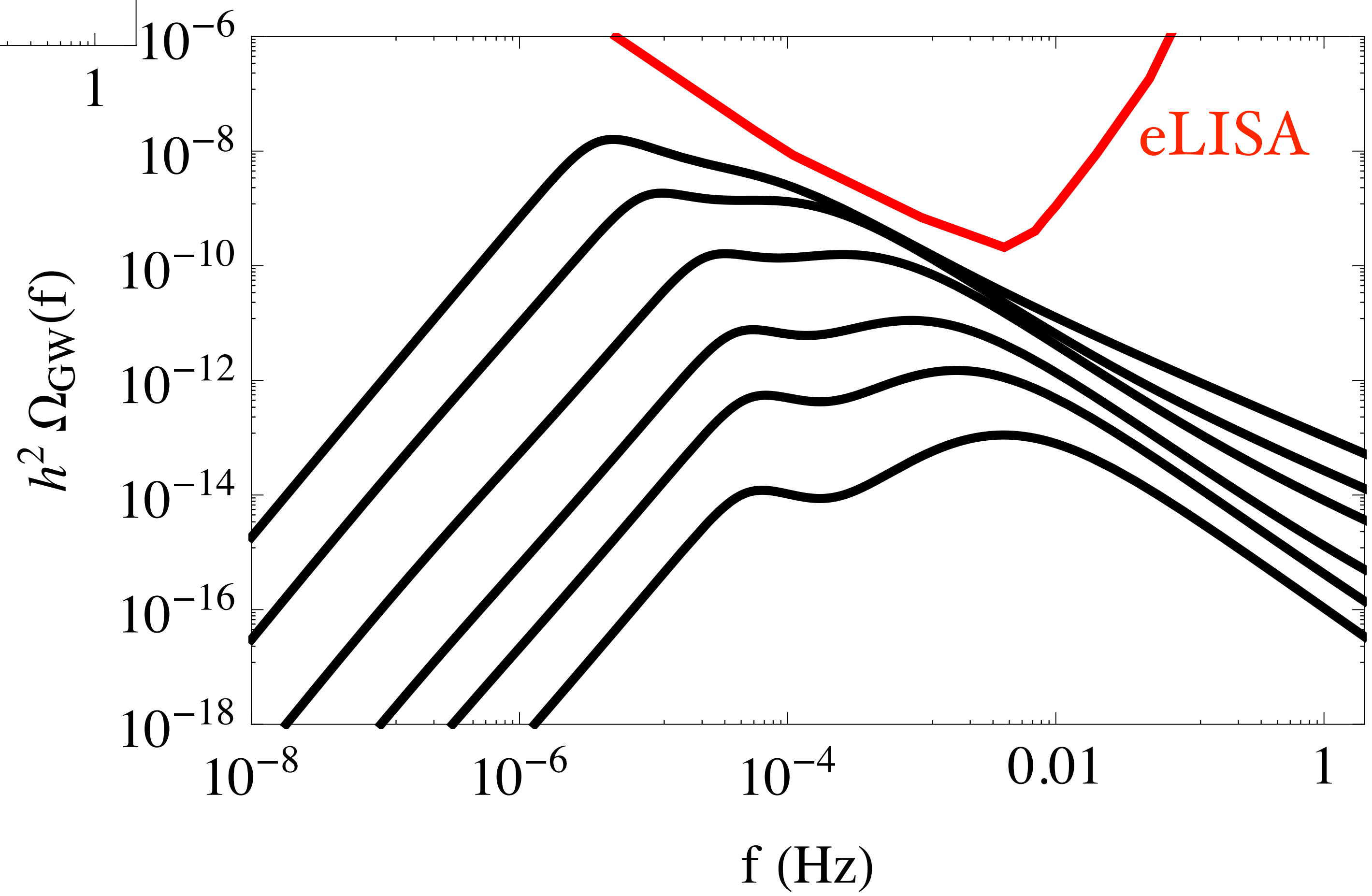
Examples of GW spectrum

SM + dimension six operator, η=0.2



increasing the friction the relative importance of the MHD signal increases

SM + dimension six operator, η=1

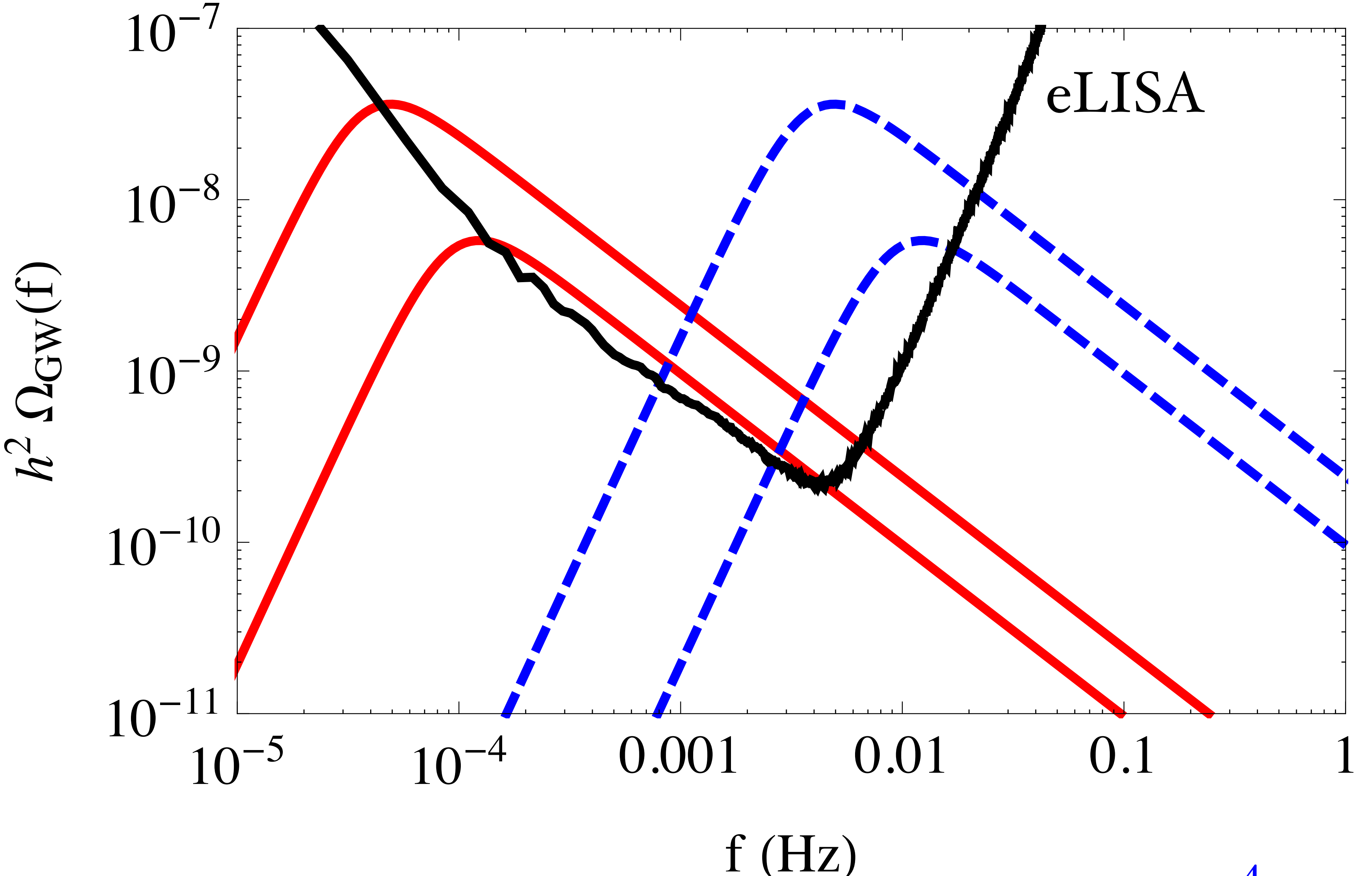


transfer of vacuum energy to bulk kinetic energy is more efficient

Examples of GW spectrum

(Randall and Servant 2007, Konstandin et al 2010)

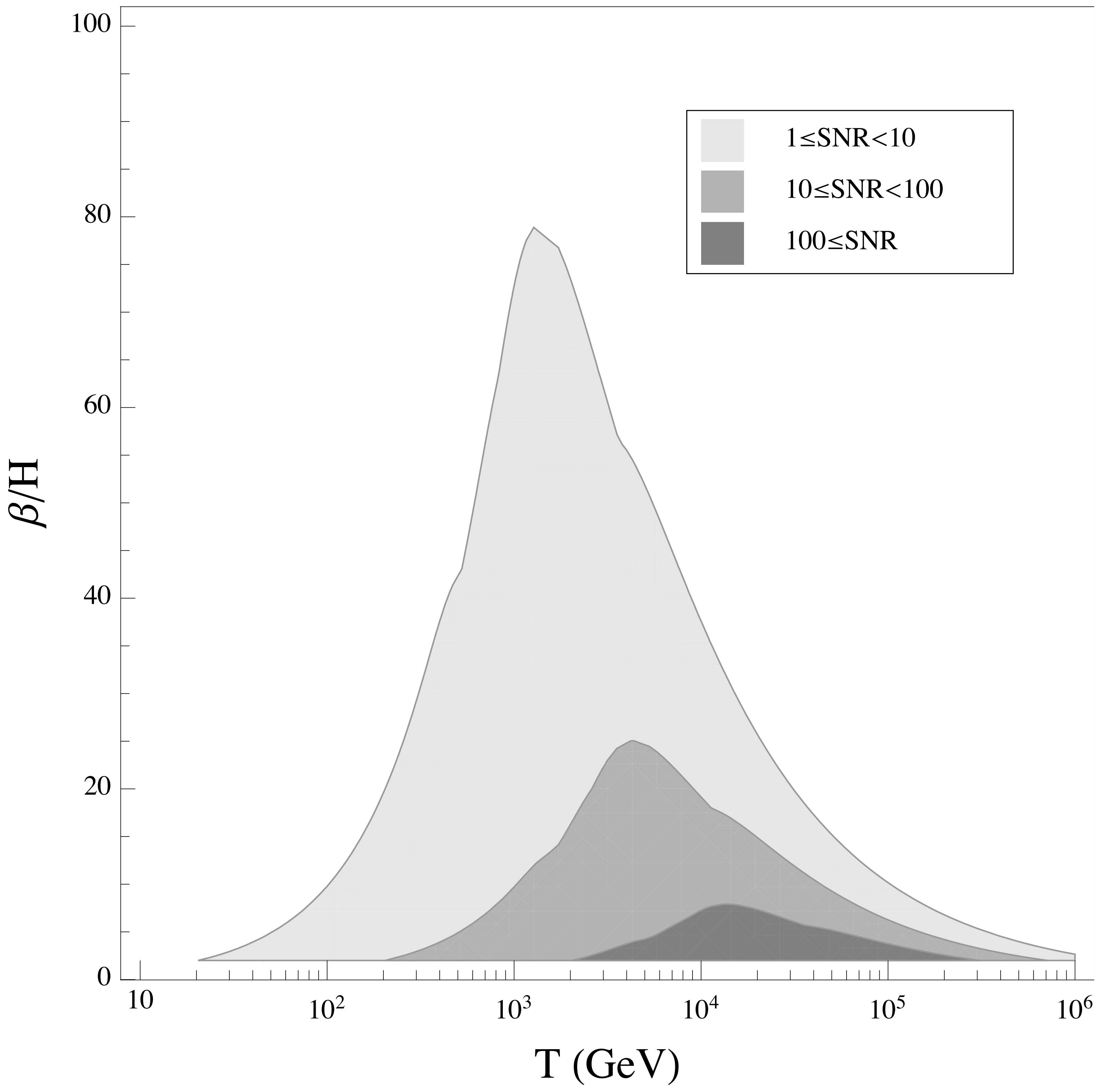
PT from the radion stabilisation in RS model



always in the runaway regime

$T_* \simeq 100\,\mathrm{GeV}$ $T_* \simeq 10^4\,\mathrm{GeV}$

Detection prospects for eLISA, $\alpha \gtrsim 10$



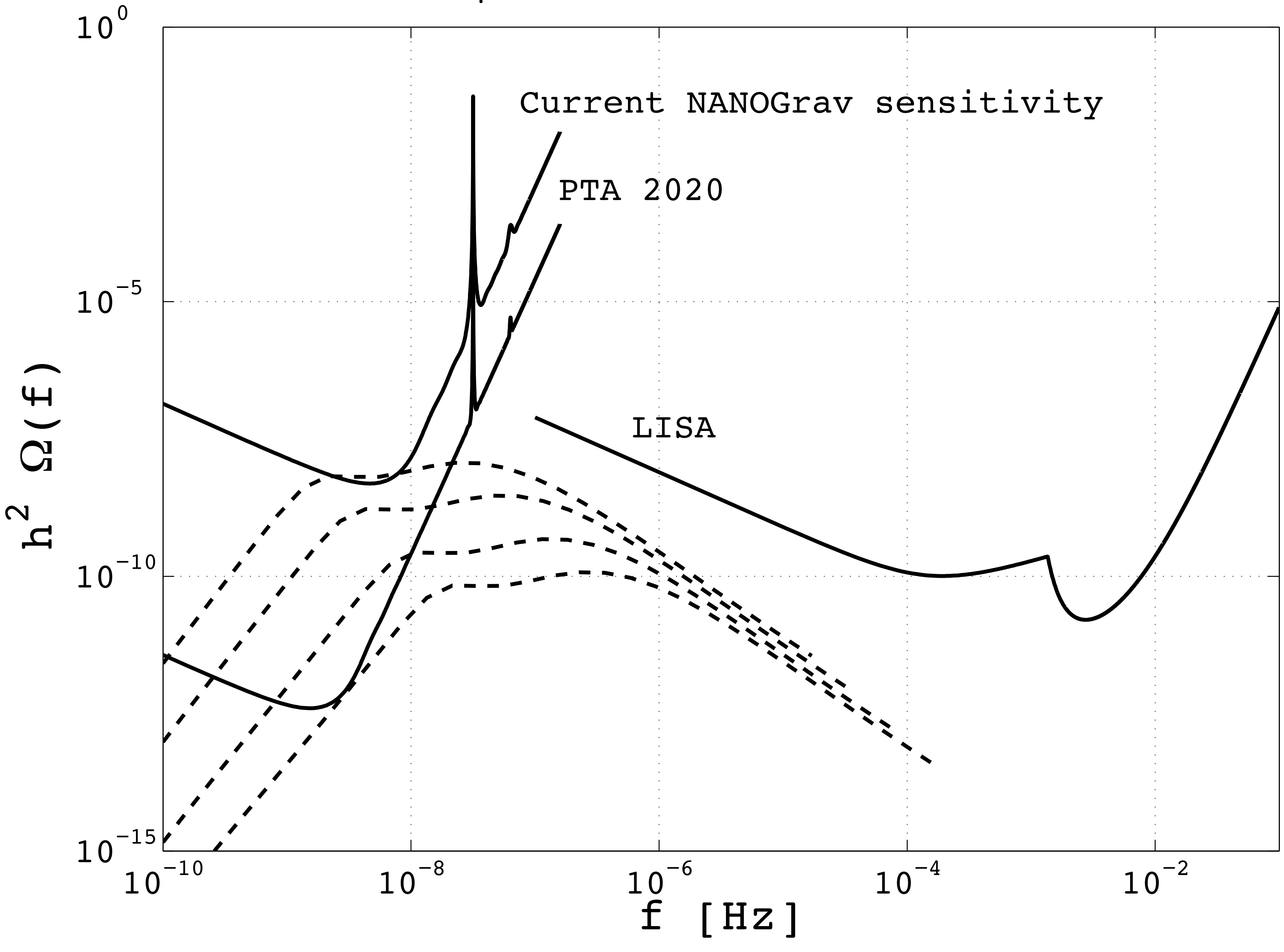
Most sensitive in the region around 10TeV

It can detect GWs from strong PTs, occurring slow

Example of GW spectrum for the QCDPT

$$T_* = 100\,\mathrm{MeV} \qquad \frac{\rho_s^*}{\rho_{tot}^*} = 0.1 \qquad v_b = 0.7$$

$$\frac{\beta}{H_*} = 1\,,\ 2\,,\ 5\,,\ 10$$



Conclusions

- we have little information about the physics and the processes operating in the very early universe
- it is in principle possible to generate a stochastic background of GWs detectable by the next generation interferometers or by PTA (ultimate sensitivity around 10^{-10} in energy density)
- eLISA is sensitive to very strong PTs occurring around 10 TeV and which last long
- a well defined prediction of the spectral shape is crucial for detection, to distinguish the signal from the noise
    - the large scale part of the spectrum raises as k^3 (causality)
    - the peak is given by β for bubble collisions, and by 1/R for MHD turbulence (analytical analysis)
    - the interplay between the strength and the duration of the PT badly affects detection prospects
- GWs are a powerful mean to learn about the early universe and high energy physics: detection is difficult but great payoff

Current limits on a stochastic background

- Nucleosynthesis and CMB: measure of the relativistic energy density in the universe

$$h^2\Omega_{\rm GW} \lesssim 7.8 \cdot 10^{-6} \qquad h^2\Omega_{\rm GW} < 6.9 \cdot 10^{-6}$$
$$f > 10^{-10}\,\text{Hz} \qquad f > 10^{-16}\,\text{Hz}$$

(Smith et al, astro-ph/0603144)

- LIGO science run 2005-2007 : cross-correlation of three interferometers

$$h^2\Omega_{\rm GW} \lesssim 6.9 \cdot 10^{-6} \qquad 41\,\text{Hz} < f < 169\,\text{Hz}$$

(Abbott et al, 0910.5772)

- combined data sets of observation of 7 pulsars for 8 years (Parkes pulsars timing array)

$$h^2\Omega_{\rm GW} < 2 \cdot 10^{-8} \qquad f \simeq 10^{-9}\,\text{Hz}$$

(Jenet et al, astro-ph/0609013)

- COBE and WMAP measurements of temperature fluctuations in CMB

$$h^2\Omega_{\rm GW} < 7 \cdot 10^{-11} \left(\frac{H_0}{f}\right)^2 \qquad 10^{-18}\,\text{Hz} < f < 10^{-16}\,\text{Hz}$$

I. space correlation structure - MHD turbulence

Reynolds number at energy injection scale

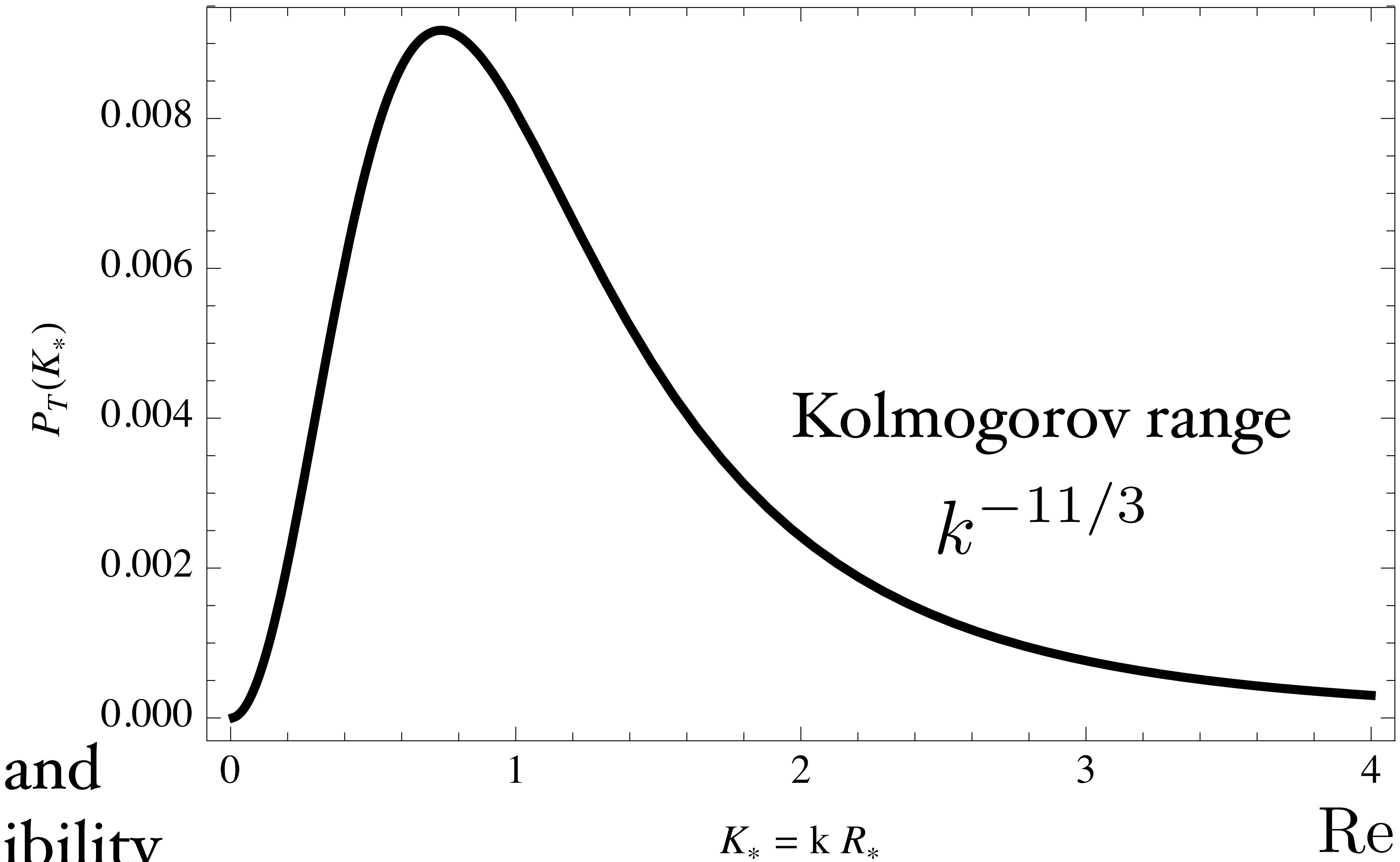
$$\mathrm{Re}(R_*) = \frac{v_R\, R_*}{\nu}$$

$\mathrm{Re(EW)} \simeq 10^{13}$

$\mathrm{Re(QCD)} \simeq 10^{4}$

energy injection scale: R_*

bubble size at collision



Kolmogorov range

$k^{-11/3}$

k^2

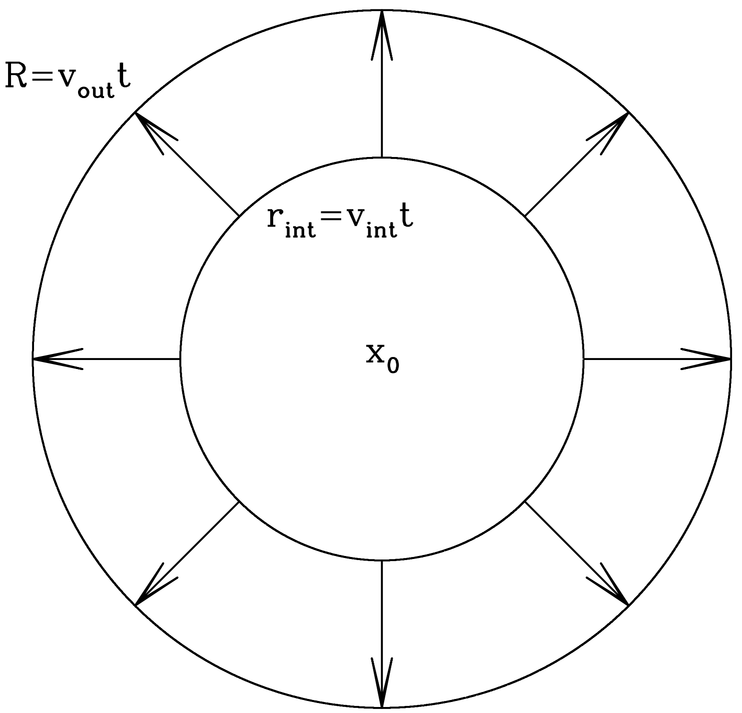
causality and incompressibility

dissipation scale λ

$$\mathrm{Re}(\lambda) = \frac{v_\lambda\, \lambda}{\nu} \simeq 1$$

1. space correlation structure - bubbles

method developed to be applied to thick bubbles



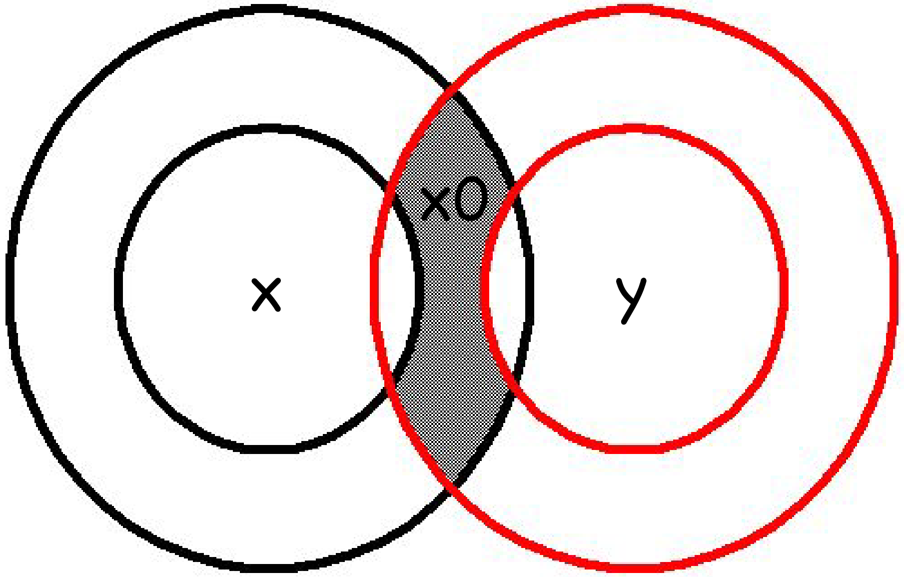
the power spectrum of the source due to bubble collisions is evaluated by Fourier transform of the two point space correlation function

$\langle v_i(\mathbf{x},t) v_j(\mathbf{y},t)\rangle$ non-zero if

1. $|\mathbf{x}-\mathbf{y}| \leq 2R(t)$

2. x and y are in the same bubble, and in the region of non-zero velocity

$$\langle v_i(\mathbf{x},t) v_j(\mathbf{y},t)\rangle = \frac{v_f^2}{R^2}\frac{p}{V}\int_V d^3x_0 (\mathbf{x}-\mathbf{x}_0)_i (\mathbf{y}-\mathbf{x}_0)_j$$



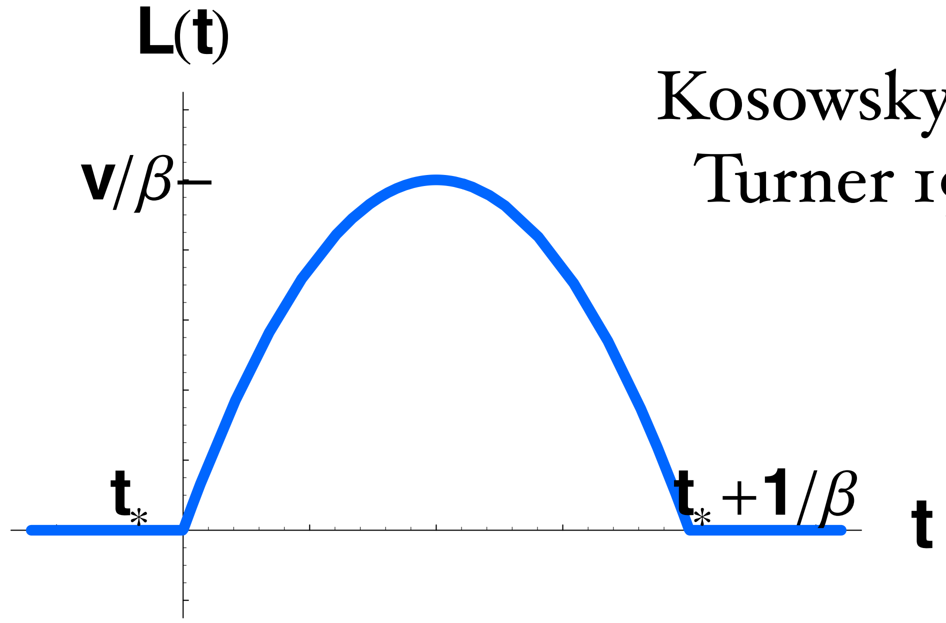
p: probability that there is a centre in the volume V

assume a linear velocity profile

3. overall time evolution

- bubbles: the source lasts for a short time compared to the Hubble time

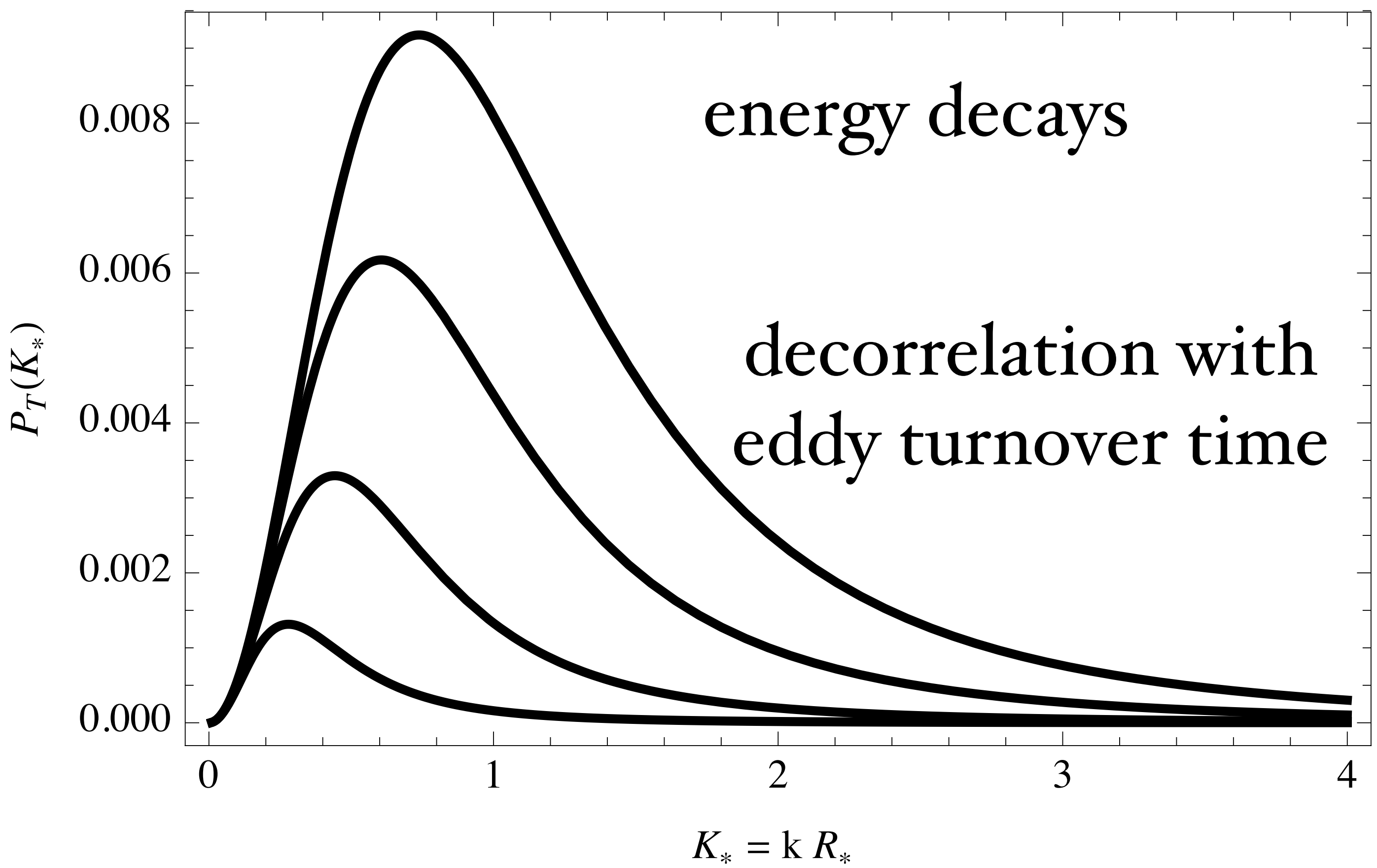
simulation: switches on with a kink enters in the time Fourier transform and strongly influences the spectral slope at high frequency



Kosowsky and Turner 1993

- turbulence: lasts for a long time compared to the Hubble time determined by the decay turbulence, slow because of low viscosity of the primordial fluid

ends when the entire Kolmogorov range is dissipated



2. time correlation structure

BUBBLES : completely coherent

- different collision events are uncorrelated in time

- single collision event is coherent : time evolution deterministic

$$\Pi(k,t_1,t_2) = \sqrt{\Pi(k,t_1,t_1)}\sqrt{\Pi(k,t_2,t_2)}$$

$$\frac{d\Omega_{\rm GW}}{d\ln k} \propto k^3 \int_{t_{\rm in}}^{t_{\rm fin}} \frac{dt_1}{t_1} \int_{t_{\rm in}}^{t_{\rm fin}} \frac{dt_2}{t_2} \cos[k(t_1-t_2)]\, \Pi(k,t_1,t_2)$$

- GW spectrum becomes the square of the time Fourier transform of the source : peak at the characteristic time of the source

$$k_* \simeq \beta \qquad\qquad (\beta < R_*^{-1})$$

2. time correlation structure

MHD TURBULENCE : decorrelating in time

- motions decorrelate with eddy turnover time $\tau_\ell \simeq \frac{\ell}{v_\ell}$

- decorrelation time depends on eddy size

correlated for $|t_1 - t_2| < \frac{1}{k}$

$$\Pi(k, t_1, t_2) = \{\Pi(k, t_1, t_1)\Theta[t_1 - t_2]\Theta[1 - k(t_1 - t_2)] + t_1 \leftrightarrow t_2\}$$

- no temporal Fourier transform: peak at the spatial correlation scale

$$k_* \simeq R_*^{-1}$$