Extraction of $\gamma$ from three-body $B$ decays

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Based on arXiv:1303.0846:
work done in collaboration with B. Bhattacharya and D. London.
Extracting weak phases

A procedure for extracting $\gamma$

Application to $B \rightarrow KK\bar{K}$ and $B \rightarrow K\pi\pi$

Caveats

Conclusions
Extracting weak phases

State of the art:

Direct measurements (world averages from CKMfitter):

- $\alpha = (88.7^{+4.6}_{-4.3})^\circ$
- $\beta = (21.38^{+0.79}_{-0.77})^\circ$
- $\gamma = (66 \pm 12)^\circ$  
  (Latest result from Belle (1301.2033) : $(68^{+15}_{-14})^\circ$)
Extracting weak phases

An ideal situation (E.g., $\beta$ from $B_d \rightarrow J/\psi K_s$):

- The final state is a CP eigenstate
- The amplitude contains a single contribution (E.g., pure penguin or pure tree)
- The indirect CP asymmetry measures the weak phase

Three-body $B$ decays

- In general, the final state is NOT a CP eigenstate (E.g., $K_s \pi^+ \pi^-$)
- The amplitude contains several different contributions
- Electroweak penguin (EWP) pollution
- The amplitude is momentum dependent (Dalitz plots)

How to overcome these problems?
Extracting weak phases

In general, the final state is NOT a CP eigenstate

Solution: Construction of a CP eigenstate by symmetrizing the amplitude

The amplitude contains several different contributions

Solution: Combination of several decays parametrized in the same framework

There is the same problem for some two-body $B$ decays. E.g., extracting $\alpha$ from $B_d \rightarrow \pi^0\pi^0$, $B_d \rightarrow \pi^+\pi^-$ and $B^\pm \rightarrow \pi^\pm\pi^0$

Electroweak penguin (EWP) pollution

Solution: SU(3) relations between trees and EWP’s (requires a fully symmetric final state for three-body decays)

The amplitude is momentum dependent

Solution: Extract $\gamma$ independently for all points of the Dalitz plot
A procedure for extracting $\gamma$

**Step 1 : Construction of the fully symmetric amplitude**

**Isobar model:**

For a decay $B \rightarrow M_1 M_2 M_3$:

The amplitude ($\mathcal{A}$) is expressed in terms of isobar parameters ($c_j$ and $\theta_j$):

$$\mathcal{A}(s_{12}, s_{13}) = \mathcal{N} \sum_j c_j e^{i\theta_j} F_j(s_{12}, s_{13})$$

- Momentum dependence (invariant masses $s_{ij} = (p_i + p_j)^2$)
- Isobar coefficients ($c_j e^{i\theta_j}$) are obtained by fitting the data of the Dalitz plot

**Fully symmetric amplitude:**

$$\mathcal{A}_{fs}(s_{12}, s_{13}) = \frac{1}{\sqrt{6}} \left( \mathcal{A}(s_{12}, s_{13}) + \mathcal{A}(s_{13}, s_{12}) + \mathcal{A}(s_{12}, s_{23}) + \mathcal{A}(s_{23}, s_{12}) + \mathcal{A}(s_{13}, s_{23}) + \mathcal{A}(s_{23}, s_{13}) \right)$$

Step 2 : Construction of the fully symmetric observables

The momentum dependent observables are constructed from the fully symmetric amplitude

- **CP averaged branching fraction**
  \[
  X(s_{12}, s_{13}) = |A_{fs}(s_{12}, s_{13})|^2 + |\bar{A}_{fs}(s_{12}, s_{13})|^2
  \]

- **Direct CP asymmetry**
  \[
  Y(s_{12}, s_{13}) = |A_{fs}(s_{12}, s_{13})|^2 - |\bar{A}_{fs}(s_{12}, s_{13})|^2
  \]

- **Indirect CP asymmetry**
  \[
  Z(s_{12}, s_{13}) = \text{Im} \left( A_{fs}^*(s_{12}, s_{13}) \bar{A}_{fs}(s_{12}, s_{13}) \right)
  \]
A procedure for extracting $\gamma$

**Step 3 : Parametrizing three-body $B$ decays**

Diagrammatic parametrization:
E.g.,

Same as in two-body $B$ decays, but:
- “pop” an extra quark pair
- Diagrams are momentum dependent
- All permutations of the final states must be considered (symmetrization)

Several topologies: $T'_1, T'_2, C'_1, C'_2, P'_{EW1}, P'_{EW2}$, etc.

A procedure for extracting $\gamma$

**Step 4 : Removing the EWP pollution**

Generalization of two-body $B$ decays relations between trees and EWP’s:

$$P'_{EWi} = \kappa T'_i$$

$$P'_{EWi}^C = \kappa C'_i$$

with

$$\kappa = -\frac{3}{2} \frac{|\lambda_t^{(s)}|}{|\lambda_u^{(s)}|} \frac{c_9 + c_{10}}{c_1 + c_2}$$

- The $c_i$’s are Wilson coefficients of the effective Hamiltonian
- Assume flavor-SU(3) symmetry
- Hold only for fully symmetric amplitudes

A procedure for extracting $\gamma$

Step 5 : Fit for $\gamma$

For a given value of $s_{12}$ and $s_{13}$:

\[
\text{# of observables} (X's, Y's and Z's) \geq \text{# of parameters} (|T_1'|, |T_2'|, \ldots, \text{relative strong phases and } \gamma)
\]

The weak phase $\gamma$ is extracted for any given value of $s_{12}$ and $s_{13}$

$\Rightarrow$ Several values of $\gamma$

All extracted values of $\gamma$ are averaged
Application to $B \rightarrow K K \bar{K}$ and $B \rightarrow K \pi \pi$

We have used results from BaBar: (B. Bhattacharya, M. Imbeault and D. London, arXiv:1303.0846.)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Observables</th>
<th>Reference</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0 \rightarrow K^+ \pi^0 \pi^-$</td>
<td>$X, Y$</td>
<td>PRD 83, 112010 (2011)</td>
<td></td>
</tr>
<tr>
<td>$B^0 \rightarrow K^0 \pi^+ \pi^-$</td>
<td>$X, Y, Z$</td>
<td>PRD 80, 112001 (2009)</td>
<td></td>
</tr>
<tr>
<td>$B^+ \rightarrow K^+ \pi^+ \pi^-$</td>
<td>$X, Y$</td>
<td>PRD 78, 012004 (2008)</td>
<td>To probe flavor-SU(3) breaking</td>
</tr>
<tr>
<td>$B^0 \rightarrow K^+ K^0 K^-$</td>
<td>$X, Y, Z$</td>
<td>PRD 85, 112010 (2012)</td>
<td></td>
</tr>
<tr>
<td>$B^0 \rightarrow K^0 K^0 \bar{K}^0$</td>
<td>$X, Y', Z'$</td>
<td>PRD 80, 054023 (2012)</td>
<td>$A = \bar{A}$ was assumed ($</td>
</tr>
</tbody>
</table>

Parametrization:

$$2A(B^0 \rightarrow K^+ \pi^0 \pi^-)_{fs} = be^{i\gamma} - \kappa c$$
$$\sqrt{2}A(B^0 \rightarrow K^0 \pi^+ \pi^-)_{fs} = -de^{i\gamma} - a + \kappa d$$
$$\sqrt{2}A(B^+ \rightarrow K^+ \pi^+ \pi^-)_{fs} = -ce^{i\gamma} - a + \kappa b$$
$$\sqrt{2}A(B^0 \rightarrow K^+ K^0 K^-)_{fs} = \alpha_{SU(3)}(-ce^{i\gamma} - a + \kappa b)$$
$$A(B^0 \rightarrow K^0 K^0 \bar{K}^0)_{fs} = \alpha_{SU(3)}a\quad (\alpha_{SU(3)} : SU(3)$$-breaking parameter)

Four effective diagrams:

$$a \equiv -\tilde{P}'_{tc} + \kappa \left(\frac{2}{3}T_1' + \frac{1}{3}C_1' + \frac{1}{3}C_2'\right)$$
$$b \equiv T_1' + C_2' \quad c \equiv T_2' + C_1' \quad d \equiv T_1' + C_1'$$
Application to $B \rightarrow K K \bar{K}$ and $B \rightarrow K \pi \pi$

- $\gamma$ is extracted for 50 points of the Dalitz plot
- Points are chosen in 1/6 of the Dalitz plot to avoid double counting due to symmetrization
- Three fits:
  1. $|\alpha_{SU(3)}| = 1$, $B^+ \rightarrow K^+ \pi^+ \pi^-$ is excluded (9 obs., 8 param.)
  2. $|\alpha_{SU(3)}|$ is fixed by comparing $B^+ \rightarrow K^+ \pi^+ \pi^-$ and $B^0 \rightarrow K^+ K^0 K^-$ (9 obs., 8 param.)
  3. $|\alpha_{SU(3)}|$ is a free parameter (11 obs., 9 param.)
Application to $B \to K K \bar{K}$ and $B \to K \pi \pi$

Combined $-2\Delta \log \mathcal{L}$ of all 50 points: [Updated]

![Graph showing combined $-2\Delta \log \mathcal{L}$](image)

- Four favored solutions: $(31^{+2}_{-1})^\circ$, $(77 \pm 2)^\circ$, $(261^{+2}_{-3})^\circ$, $(314 \pm 2)^\circ$ (statistical only)
- Only one solution, $(77 \pm 2)^\circ$, is consistent with established measurements
- Very small error bars
Caveats

- **Flavor-SU(3) breaking due to mismatched kinematical boundaries and resonances:**
  - $|\alpha_{SU(3)}|$ has little effect on fit results
  - Average values of $|\alpha_{SU(3)}|$ are very close to 1
    - Extracted from $|A|$’s: $0.97 \pm 0.04$ (stat.)
    - Extracted from $|\bar{A}|$’s: $0.99 \pm 0.04$ (stat.)
  - No perfect handle of flavor-SU(3) breaking, but all clues indicate a small effect

- **We do not work directly from data ⇒ some limitations:**
  - Systematic uncertainties are not quoted for isobar parameters
  - Correlations between observables at different points of the Dalitz plot can enlarge statistical error bars (lack of computing power)
  - The current analysis is model dependent (isobar model)
  - 50 points is ad hoc ⇒ optimal binning

- Discrete ambiguities cannot be resolved without an outside input
Conclusions

Summary:

- The weak phase $\gamma$ can be extracted from three-body $B$ decays
- Estimations of $\gamma$ obtained with the limitations of current available experiment values produce VERY small uncertainties
- The flavor-SU(3) breaking is cooperative
- Some caveats can be resolved with a more rigorous analysis performed directly on experimental data

Is it the beginning of the story?

- Can we also exploit the other S3 states (non-fully-symmetric)?
- Would it resolve discrete ambiguities?
- Are there other applications with multi-body $B$ decays?

Thank you!