Extraction of γ from three-body B decays

Maxime Imbeault

Cégep de Saint-Laurent

July 18, 2013

EPS 2013, Stockholm

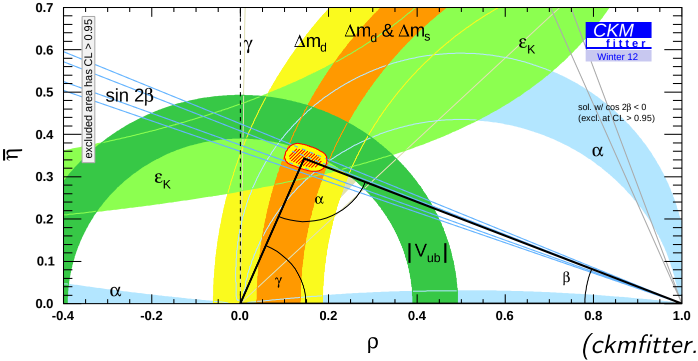
Based on arXiv:1303.0846:
work done in collaboration with B. Bhattacharya and D. London.

Outline

1. Extracting weak phases
2. A procedure for extracting γ
3. Application to $B \to K K \bar{K}$ and $B \to K\pi\pi$
4. Caveats
5. Conclusions

Extracting weak phases

State of the art:



(ckmfitter.in2p3.fr)

Direct measurements (world averages from CKMfitter):

- $\alpha = (88.7^{+4.6}_{-4.3})^\circ$
- $\beta = (21.38^{+0.79}_{-0.77})^\circ$
- $\gamma = (66 \pm 12)^\circ$ (Latest result from Belle (1301.2033) : $(68^{+15}_{-14})^\circ$)

Extracting weak phases

An ideal situation (E.g., β from $B_d \to J/\psi K_s$):

- The final state is a CP eigenstate
- The amplitude contains a single contribution (E.g., pure penguin or pure tree)
- The indirect CP asymmetry measures the weak phase

Three-body B decays

- In general, the final state is NOT a CP eigenstate (E.g., $K_s\pi^+\pi^-$)
- The amplitude contains several different contributions
- Electroweak penguin (EWP) pollution
- The amplitude is momentum dependent (Dalitz plots)

How to overcome these problems?

Extracting weak phases

In general, the final state is NOT a CP eigenstate

Solution: Construction of a CP eigenstate by symmetrizing the amplitude

The amplitude contains several different contributions

Solution: Combination of several decays parametrized in the same framework

There is the same problem for some two-body B decays. E.g., extracting α from $B_d \to \pi^0\pi^0$, $B_d \to \pi^+\pi^-$ and $B^\pm \to \pi^\pm\pi^0$

Electroweak penguin (EWP) pollution

Solution: SU(3) relations between trees and EWP's (requires a fully symmetric final state for three-body decays)

The amplitude is momentum dependent

Solution: Extract γ independently for all points of the Dalitz plot

A procedure for extracting γ

Step 1 : Construction of the fully symmetric amplitude

Isobar model:

For a decay $B \to M_1 M_2 M_3$:

The amplitude ($\mathcal{A}$) is expressed in terms of isobar parameters (c_j and θ_j):

$$\mathcal{A}(s_{12}, s_{13}) = \mathcal{N} \sum_j c_j e^{i\theta_j} F_j(s_{12}, s_{13})$$

- Momentum dependence (invariant masses $s_{ij} = (p_i + p_j)^2$)
- Isobar coefficients ($c_j e^{i\theta_j}$) are obtained by fitting the data of the Dalitz plot

Fully symmetric amplitude:

$$\begin{aligned} \mathcal{A}_{\rm fs}(s_{12}, s_{13}) &= \frac{1}{\sqrt{6}} \left(\mathcal{A}(s_{12}, s_{13}) + \mathcal{A}(s_{13}, s_{12}) + \mathcal{A}(s_{12}, s_{23}) \right. \\ & \left. + \mathcal{A}(s_{23}, s_{12}) + \mathcal{A}(s_{13}, s_{23}) + \mathcal{A}(s_{23}, s_{13}) \right) \end{aligned}$$

(N. Rey-Le Lorier and D. London, Phys. Rev. D**85**, 016010 (2012).)

A procedure for extracting γ

Step 2 : Construction of the fully symmetric observables

The momentum dependent observables are constructed from the fully symmetric amplitude

- CP averaged branching fraction

$$X(s_{12}, s_{13}) = |\mathcal{A}_{\rm fs}(s_{12}, s_{13})|^2 + |\bar{\mathcal{A}}_{\rm fs}(s_{12}, s_{13})|^2$$

- Direct CP asymmetry

$$Y(s_{12}, s_{13}) = |\mathcal{A}_{\rm fs}(s_{12}, s_{13})|^2 - |\bar{\mathcal{A}}_{\rm fs}(s_{12}, s_{13})|^2$$

- Indirect CP asymmetry

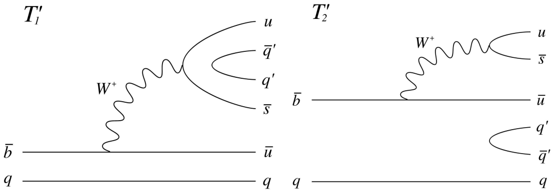
$$Z(s_{12}, s_{13}) = \mathrm{Im}\left(\mathcal{A}^*_{\rm fs}(s_{12}, s_{13})\bar{\mathcal{A}}_{\rm fs}(s_{12}, s_{13})\right)$$

A procedure for extracting γ

Step 3 : Parametrizing three-body B decays

Diagrammatic parametrization:

E.g.,



Same as in two-body B decays, but

- "pop" an extra quark pair
- Diagrams are momentum dependent
- All permutations of the final states must be considered (symmetrization)

Several topologies : T_1', T_2', C_1', C_2', P'_{EW1}, P'_{EW2}, etc.

(N. Rey-Le Lorier, M. Imbeault and D. London, Phys. Rev. D**84**, 034040 (2011).)

Step 4 : Removing the EWP pollution

Generalization of two-body B decays relations between trees and EWP's:

$$P'_{EWi} = \kappa T'_i$$
$$P'^{C}_{EWi} = \kappa C'_i$$

with

$$\kappa = -\frac{3}{2}\frac{|\lambda_t^{(s)}|}{|\lambda_u^{(s)}|}\frac{c_9 + c_{10}}{c_1 + c_2}$$

- The c_i's are Wilson coefficients of the effective hamiltonian
- Assume flavor-SU(3) symmetry
- Hold only for fully symmetric amplitudes

(M. Imbeault, N. Rey-Le Lorier and D. London, Phys. Rev. D**84**, 034041 (2011).)

Step 5 : Fit for γ

For a given value of s_{12} and s_{13}:

# of observables (X's, Y's and Z's)	$\geq$	**# of parameters** ($\|T_1'\|$, $\|T_2'\|$, ... , relative strong phases and γ)

The weak phase γ is extracted for any given value of s_{12} and s_{13}
$\Rightarrow$ **Several values of γ**

All extracted values of γ are averaged

Application to $B \to KK\bar{K}$ and $B \to K\pi\pi$

We have used results from *BaBar*: (B. Bhattacharya, M. Imbeault and D. London, arXiv:1303.0846.)

Mode	Observables	Reference	Note
$B^0 \to K^+\pi^0\pi^-$	X, Y	PRD **83**, 112010 (2011)	
$B^0 \to K^0\pi^+\pi^-$	X, Y, Z	PRD **80**, 112001 (2009)	
$B^+ \to K^+\pi^+\pi^-$	X, Y	PRD **78**, 012004 (2008)	To probe flavor-SU(3) breaking
$B^0 \to K^+K^0K^-$	X, Y, Z	PRD **85**, 112010 (2012)	
$B^0 \to K^0K^0\bar{K}^0$	X, ~~Y~~, ~~Z~~	PRD **80**, 054023 (2012)	$\mathcal{A} = \bar{\mathcal{A}}$ was assumed $(\lvert P'_{uc}\rvert = 0)$

Parametrization:

$$
\begin{aligned}
2\mathcal{A}(B^0 \to K^+\pi^0\pi^-)_{\rm fs} &= be^{i\gamma} - \kappa c \\
\sqrt{2}\mathcal{A}(B^0 \to K^0\pi^+\pi^-)_{\rm fs} &= -de^{i\gamma} - a + \kappa d \\
\sqrt{2}\mathcal{A}(B^+ \to K^+\pi^+\pi^-)_{\rm fs} &= -ce^{i\gamma} - a + \kappa b \\
\sqrt{2}\mathcal{A}(B^0 \to K^+K^0K^-)_{\rm fs} &= \alpha_{SU(3)}(-ce^{i\gamma} - a + \kappa b) \\
\mathcal{A}(B^0 \to K^0K^0\bar{K}^0)_{\rm fs} &= \alpha_{SU(3)}a
\end{aligned}
$$

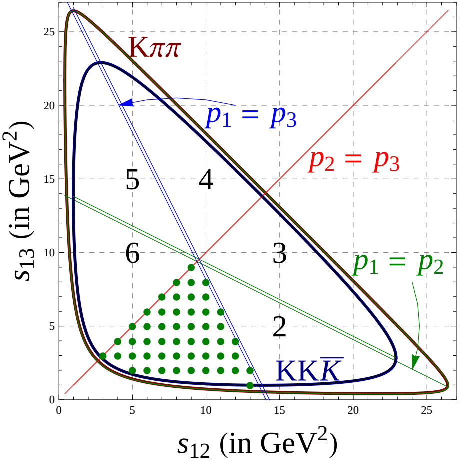
($\alpha_{SU(3)}$: SU(3)-breaking parameter)

Four effective diagrams:

$$a \equiv -\tilde{P}'_{tc} + \kappa\left(\frac{2}{3}T'_1 + \frac{1}{3}C'_1 + \frac{1}{3}C'_2\right)$$

$$b \equiv T'_1 + C'_2 \qquad c \equiv T'_2 + C'_1 \qquad d \equiv T'_1 + C'_1$$

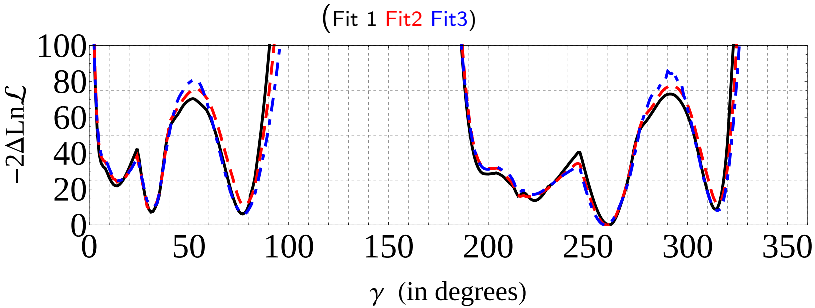
Application to $B \to KK\bar{K}$ and $B \to K\pi\pi$



- γ is extracted for 50 points of the Dalitz plot
- Points are chosen in 1/6 of the Dalitz plot to avoid double counting due to symmetrization
- Three fits:
  1. $|\alpha_{SU(3)}| = 1$, $B^+ \to K^+\pi^+\pi^-$ is excluded (9 obs., 8 param.)
  2. $|\alpha_{SU(3)}|$ is fixed by comparing $B^+ \to K^+\pi^+\pi^-$ and $B^0 \to K^+K^0K^-$ (9 obs., 8 param.)
  3. $|\alpha_{SU(3)}|$ is a free parameter (11 obs., 9 param.)

Application to $B \to KK\bar{K}$ and $B \to K\pi\pi$

Combined $-2\Delta\log\mathcal{L}$ of all 50 points: [Updated]



- Four favored solutions : $(31^{+2}_{-1})^\circ$, $(77 \pm 2)^\circ$, $(261^{+2}_{-3})^\circ$, $(314 \pm 2)^\circ$ (statistical only)
- Only one solution, $(77 \pm 2)^\circ$, is consistent with established measurements
- Very small error bars

Caveats

- Flavor-SU(3) breaking due to mismatched kinematical boundaries and resonnances:
    - $|\alpha_{SU(3)}|$ has little effect on fit results
    - Average values of $|\alpha_{SU(3)}|$ are very close to 1
        - Extracted from $|\mathcal{A}|$'s : 0.97 ± 0.04 (stat.)
        - Extracted from $|\bar{\mathcal{A}}|$'s : 0.99 ± 0.04 (stat.)
    - No perfect handle of flavor-SU(3) breaking, but all clues indicate a small effect
- We do not work directly from data $\Rightarrow$ some limitations:
    - Systematic uncertainties are not quoted for isobar parameters
    - Correlations between observables at different points of the Dalitz plot can enlarge statistical error bars (lack of computing power)
    - The currect analysis is model dependent (isobar model)
    - 50 points is *ad hoc* $\Rightarrow$ optimal binning
- Discrete ambiguities cannot be resolved without an outside input

Conclusions

Summary:

- The weak phase γ can be extracted from three-body B decays
- Estimations of γ obtained with the limitations of current available experiment values produce VERY small uncertainties
- The flavor-SU(3) breaking is cooperative
- Some caveats can be resolved with a more rigorous analysis performed directly on experimental data

Is it the beginning of the story?

- Can we also exploit the other S3 states (non-fully-symmetric)?
- Would it resolve discrete ambiguities?
- Are there other applications with multi-body B decays?

Thank you!