Normalized Multijet Cross Sections using Regularized Unfolding and Extraction of $\alpha_s(M_Z)$ in Deep-inelastic Scattering at high $Q^2$ at HERA

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Jet production in ep scattering

Deep-inelastic ep scattering

Jet measurements are performed in ‘Breit frame’

Breit frame \( 2x_{Bj}p + k = 0 \)

Jet production in leading-order pQCD

Photon virtuality \( Q^2 \)

\[ Q^2 = -q^2 = -(k - k')^2 \]

Inelasticity

\[ y = \frac{p \cdot q}{p \cdot k} \]

Bjorken variable

\[ x_{Bj} = \frac{Q^2}{2p \cdot q} \]
**H1 detector at HERA**

**HERA ep collider**

**HERA collider in Hamburg, Germany**
- $e^\pm p$ collider
- $\sqrt{s} = 319$ GeV
  - $E_e = 27.6$ GeV
  - $E_p = 920$ GeV

**HERA-2 period**
Years 2003 – 2007
Electron and positron runs
$\mathcal{L} \simeq 356$ pb$^{-1}$

**Integrated H1 Luminosity**

HERA-1
HERA-2

Days of running
H1 Integrated Luminosity / pb$^{-1}$

- Electrons
- Positrons
- Low $E$
Normalized multijet measurement at H1

Four measurements are performed

**Neutral current phase space**

\[ 150 < Q^2 < 15000 \text{ GeV}^2 \]

\[ 0.2 < y < 0.7 \]

**Jet acceptance**

\[ -1.0 < \eta_{\text{lab}} < 2.5 \]

**Inclusive Jet**

\[ 7 < p_T^{\text{jet}} < 50 \text{ GeV} \]

**Dijet** \((n_{\text{jet}} \geq 2)\)

\[ 5 < p_T^{\text{jet}} < 50 \text{ GeV} \]

\[ M_{12} > 16 \text{ GeV} \]

\[ 7 < \langle p_T \rangle_2 < 50 \text{ GeV} \]

**Trijet** \((n_{\text{jet}} \geq 3)\)

\[ \langle p_T \rangle_2 = \frac{p_T^{\text{jet}1} + p_T^{\text{jet}2}}{2} \]
Correction of detector effects using regularized unfolding

Detector effects
- Acceptance and efficiency
- Migrations due to limited resolution

Aim
- Cross section on hadron level
- Direct matrix inversion of $A$ often not possible

Detector response
- Measured vector $y$
- Hadron level vector $x$
- Detector response matrix $A$
- Covariance matrix $V_y$

Regularized unfolding using Tunfold ([JINST 7 (2012) T10003])
- Find hadron level $x$ by analytic minimization of $\chi^2$

$$\chi^2(x, \tau) = (y - Ax)^T V_y^{-1} (y - Ax) + \tau^2 (x - x_0)^T (L^T L) (x - x_0)$$

Matrix inversion: $\chi^2_A$

Regularization: $\chi^2_L$

- Find stationary point ($\partial \chi^2 / \partial x = 0$) by solving analytically as function of $x$
- ‘True’ hadron level can be determined directly

$$x = (A^T V_y^{-1} A + \tau^2 L^2)^{-1} A^T V_y^{-1} y =: By$$

- $\tau$ (and $L$) are free parameters
Schematic definition of migration matrix

Simultaneous unfolding
NC DIS, inclusive jet, dijet and trijet

Covariance matrix $V_y$
takes statistical correlations of observables into account

Individual unfolding schemes
- $E, J_1, J_2, J_3$ studied in detail
- Are optimized separately using MC

Matrices $B_i$
Constrain reconstructed but not generated contributions

Two MC generators
Django and Rapgap

Phase space is enlarged
in all variables where migrations are relevant

Migration Matrix

3-dimensional unfolding in $p_T, Q^2, y$

Four measurements are unfolded simultaneously: stat. correlations are considered
Correlation matrix of all data points

Covariance matrix
Obtained through linear error propagation of statistical uncertainties

Correlations
• Resulting from unfolding
• Physical correlations
  • Between measurements
  • Within inclusive jet

Useful for
• Cross section ratios
• Combined fits
• Normalized cross sections

Correlation matrix is employed for correct error propagation for norm. cross sections
Normalized multijet cross sections

\[ \frac{\sigma_{\text{Jet}}}{\sigma_{\text{NC}}} \times 10^i \]

- Normalised Inclusive Jet
- Normalised Dijet
- Normalised Trijet

**H1**

Preliminary

- NLO \( \otimes c_{\text{had}} \)
- NLOJet++ and fastNLO
- QCDNUM
- CT10, \( \alpha_s = 0.118 \)

- \( 150 < Q^2 < 200 \text{ GeV}^2 \) (\( i = 10 \))
- \( 200 < Q^2 < 270 \text{ GeV}^2 \) (\( i = 8 \))
- \( 270 < Q^2 < 400 \text{ GeV}^2 \) (\( i = 6 \))
- \( 400 < Q^2 < 700 \text{ GeV}^2 \) (\( i = 4 \))
- \( 700 < Q^2 < 5000 \text{ GeV}^2 \) (\( i = 2 \))
- \( 5000 < Q^2 < 15000 \text{ GeV}^2 \) (\( i = 0 \))

**H1prelim-12-031**
Data are employed for extraction of $\alpha_s(M_Z)$

**Experimental input** $m_i$
- Normalized incl. jet, dijet and trijet data

**Experimental uncertainties** $\delta_k m_i$
- Taken into account in fit
- Covariance matrix $V$ takes correlations into account
- Experimental uncertainties $k$ are respected with nuisance parameters

**Theoretical input** $t_i$
- NLO predictions from
  - NLOJET++ and fastNLO
  - QCDNUM
- Hadronization corrections
- PDF: CT10
- Scale choices
  - $\mu_r^2 = (Q^2 + E_T^2)/2$
  - $\mu_f^2 = Q^2$
- FastNLO provides fast repeated calculation of cross section predictions

**Iterative $\chi^2$ minimization using TMinuit with $\alpha_s(M_Z)$ and $\varepsilon_k$ are free parameters**

$$
\chi^2(\alpha_s(M_Z), \varepsilon_k) = pV^{-1}p + \sum_{k}^{N_{sys}} \varepsilon^2_k \\
\quad = d_i - t_i \left( 1 - \sum_{k}^{N_{sys}} \delta_{i,k} \varepsilon_k \right)
$$
\( \alpha_s \) fits to individual measurements

**Normalized inclusive jet**
\[ \alpha_s(M_Z) = 0.1197 \pm 0.0008 \, \text{(exp)} \pm 0.0014 \, \text{(PDF)} \pm 0.0012 \, \text{(had)} \pm 0.0054 \, \text{(theo)} \]
\[ \chi^2 / \text{ndf} = 28.7/23 = 1.25 \]

**Normalized dijet**
\[ \alpha_s(M_Z) = 0.1142 \pm 0.0010 \, \text{(exp)} \pm 0.0017 \, \text{(PDF)} \pm 0.0009 \, \text{(had)} \pm 0.0048 \, \text{(theo)} \]
\[ \chi^2 / \text{ndf} = 27.0/23 = 1.18 \]

**Normalized trijet**
\[ \alpha_s(M_Z) = 0.1185 \pm 0.0018 \, \text{(exp)} \pm 0.0013 \, \text{(PDF)} \pm 0.0016 \, \text{(had)} \pm 0.0043 \, \text{(theo)} \]
\[ \chi^2 / \text{ndf} = 12.0/16 = 0.75 \]

**Results**
- High experimental precision
- Reasonable \( \chi^2 / \text{ndf} \) for each fit

**Tension between inclusive jet and dijet**
Visible also in previous H1 and ZEUS analyses

**Theory uncertainties using offset method**
- PDF
  Obtained from PDF eigenvectors (90\%CL)
- Hadronization (had)
  Half-difference between ‘Lund string’ and ‘cluster’ fragmentation
- Missing higher orders (theo)
  Scale variations: \( \mu_r \) and \( \mu_f \) between 0.5 and 2
$\alpha_s$ fit to normalized multijet cross sections

Simultaneous fit to
- Normalized inclusive jet
- Normalized dijet
- Normalized trijet

Taking statistical correlations between observables into account
Demanding NLO corrections < 30%: $k<1.3$

Normalized Multijet ($k < 1.3$)

$$\alpha_s(M_Z) = 0.1163 \pm 0.0011\,(\text{exp}) \pm 0.0014\,(\text{PDF}) \pm 0.0008\,(\text{had}) \pm 0.0040\,(\text{theo})$$

$$\chi^2 / \text{ndf} = 53.3 / 41 = 1.30$$

$\alpha_s(M_Z)$ from H1 multijet cross section

Value consistent with world average (Phys. Rev. D 86 (2012) 010001)
- $\alpha_s(M_Z) = 0.1184 \pm 0.0007$

High experimental precision
Precision limited by theory predictions
Summary

Regularized unfolding
Simultaneous unfolding of four measurements
Unfolding of NC DIS, inclusive jet, dijet and trijet data
Migrations in up to 6 observables
Normalized cross sections can be obtained

Normalized Multijet Measurement at high $Q^2$
- Normalized Inclusive Jet
- Normalized Dijet
- Normalized Trijet

Extraction of $\alpha_s(M_Z)$
$\alpha_s$ fit using unfolded data to NLO QCD
Covariance matrix is considered
Also correlations between observables
Experimental error 1%
Theoretical errors dominate with 3.4%

$\frac{\sigma_{\text{had}}}{\sigma_{\text{NLO}}} = 0.118$

$\alpha_s^{CT10, QCDNUM NLOJet++ and fastNLO}$
Backup
Comparision of $\alpha_s$ values

Uncertainties: exp. ———— theo. ———————

- **H1 norm. multijets at high $Q^2$ (this talk)**  
  H1-prelim-12-031
- **H1 norm. inclusive jet at high $Q^2$ (this talk)**  
  H1-prelim-12-031
- **H1 norm. dijet at high $Q^2$ (this talk)**  
  H1-prelim-12-031
- **H1 norm. trijet at high $Q^2$ (this talk)**  
  H1-prelim-12-031
- **H1 norm. multijets at high $Q^2**  
- **H1 multijets at low $Q^2**  
- **ZEUS inclusive jet at high $Q^2**  
  ZEUS-prel-10-002
- **ZEUS dijet at high $Q^2**  
- **Aleph 3-jet rate, NNLO**  
- **World average**  
HERA and H1

H1 detector

- Multi-purpose detector
- Asymmetric design
- Trackers
  - Silicon tracker
  - Jet chambers
  - Proportional chambers
- Calorimeters
  - Liquid Argon sampling calorimeter
  - Scintillating fiber calorimeter
- Muon detectors
- Superconducting solenoid
  - 1.15T axial-symmetric magnetic field
Comparing unfolding with bin-by-bin method: Monte Carlo

Compare with Monte Carlo pseudo-data
- Bin-by-bin correction
  Bin-wise correction factors
- Regularized unfolding

Two Incl. DIS Models
- Rapgap (MEPS)
- Django (CDM)
Statistically independent samples

Checking
- Pseudo-data from ‘one’ model
- Unfolding matrix from 'other' model

Pull distributions
Corrected vs. true distribution
$$p_i = \frac{x_i^{\text{unfold}} - x_i^{\text{true}}}{\Delta x_i}$$

Unfolding is less biased by Model predictions
Comparing unfolding with bin-by-bin method: Data

Using H1 Data

Average of two MCs for migration matrix

\[ A = \frac{A_{Dj} + A_{Rg}}{2} \]

Compare pull values between

- Unfolded data points
- Bin-by-bin corrected data points

Bin-by-bin bias also in data

Statistical uncertainties of unfolding are larger

\[ \text{Mean } -0.87 \pm 0.17 \]

\[ \text{Mean } -0.86 \pm 0.11 \]

\[ \text{Mean } -0.74 \pm 0.10 \]

Unfolding features a full linear error propagation of (statistical) uncertainties

Unfolding has small, but visible effect on cross sections
Comparing unfolding with bin-by-bin method: Data

Normalised Inclusive Jet Cross Section

Normalised Dijet Cross Section

Normalised Trijet Cross Section