5d SYM and the correspondence to AdS_7/CFT_6

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based on
J.Källen, J.A.Minahan, A.N., M.Zabzine; hep-th/1207.3763
and
J.A.Minahan, A.N., M.Zabzine; hep-th/1304.1016

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Content

- Motivation: 6d (2,0) SCFT
- 5d SYM
- Localization
- Matrix Model Results
- AdS_7/CFT_6 Results

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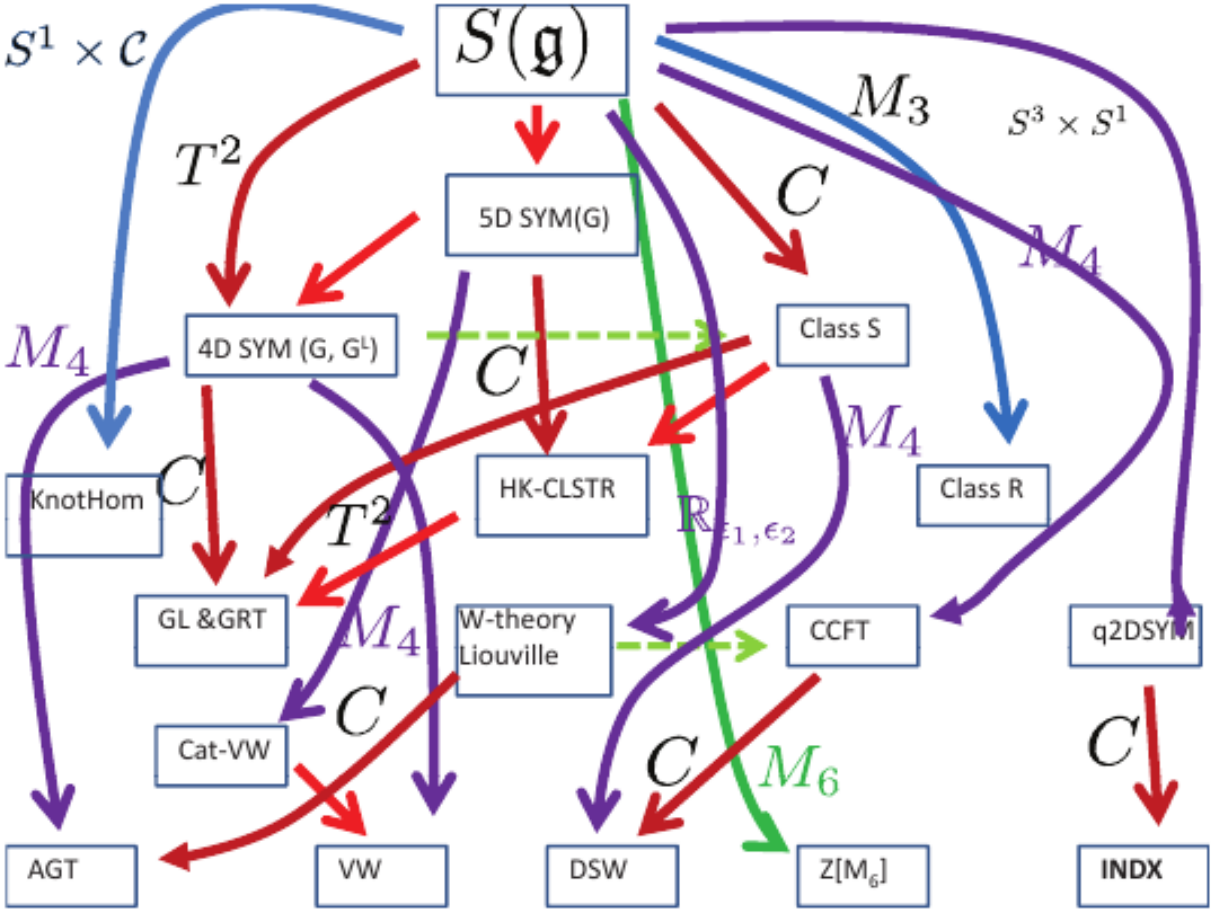
- Has a lot of useful compactifications

6d SCFT compactifications



Gregory W. Moore, "Lecture Notes for Felix Klein Lectures" , 2012

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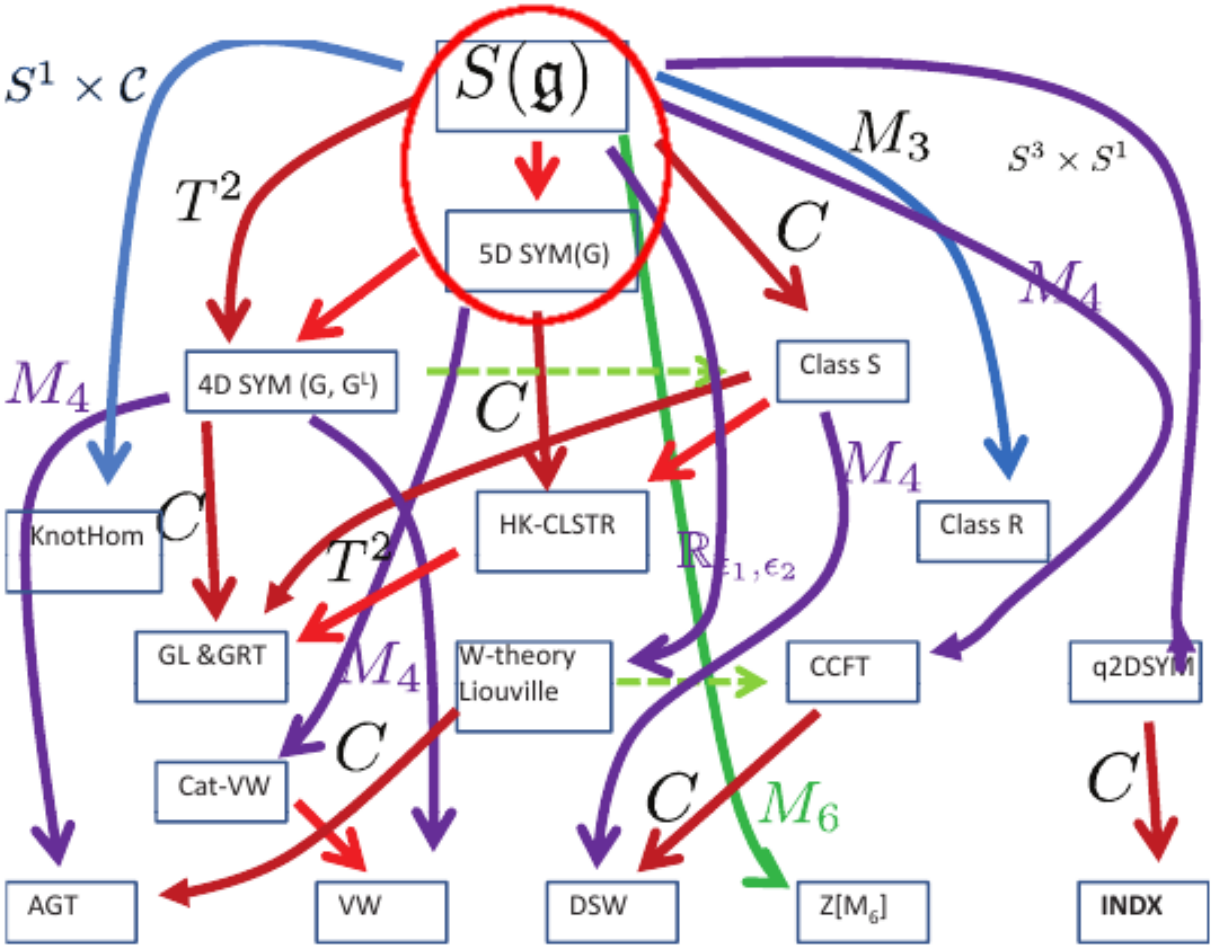

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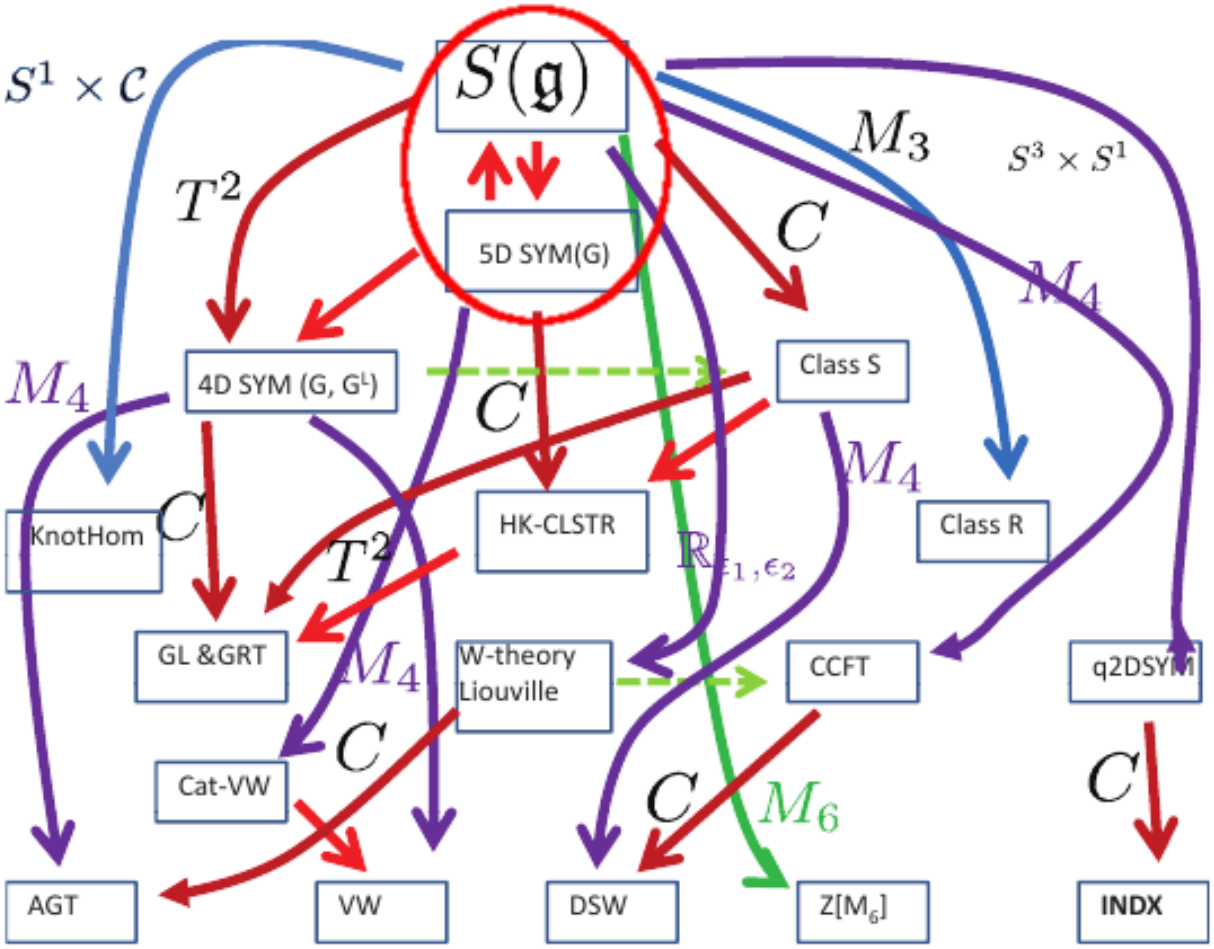

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6d SCFT $\leftrightarrow$ 5d MSYM correspondence

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Main arguments:

- Kaluza-Klein modes of compactified 6d SCFT can be identified with instanton particles in 5d MSYM, with the following relation between YM coupling and compactification radius arising
  5d instanton particles: $M_6 \sim \frac{1}{g_{YM}^2}$; *6d KK modes:* $M_6 \sim \frac{1}{R_6} \Rightarrow R_6 \sim g_{YM}^2$

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- Complete KK spectrum of 6d theory compactified on S_1 matches the instanton solitons of 5d MSYM, with the identification

$$R_6 = \frac{g^2_{YM}}{8\pi^2}.$$

  This was proven for the broken phase with $N = 2$ (i.e. 2 slightly separated M5-branes) *N.Lambert, C.Papageorgakis, M.Schmidt-Sommerfeld '11*

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- For the details of putting 5d SYM on the sphere see *K. Hosomichi, R. -K. Seong and S. Terashima '12*

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- *path integral (hard)* $\to$ *matrix integral (easier)*

Localization for 5D SYM on S^5

To localize 5D SYM one should

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- This in turn implies localization locus to be
  $F = 0; \quad \sigma = const; \quad q_I = 0$
  Here we have chosen only flat connection solution to the equations above, ommiting instantons contribution.

For detailed procedure of 5D SYM localizations see
K. Hosomichi, R. -K. Seong and S. Terashima '12;
J. Kallen and M. Zabzine '12;
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Matrix Model

- After localization is done path integral takes following form:

  $$Z \sim \int d\phi Z^{vect}_{1-loop}(\phi) Z^{hyper}_{1-loop}(\phi) e^{-\frac{8\pi^3 r}{g^2_{YM}} \mathrm{Tr}\left(\phi^2\right)} + \textit{instantons}$$

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  $$f(z) = \frac{i\pi z^3}{3} + z^2 \log\left(1 - e^{-2\pi i z}\right) + \frac{iz}{\pi}\mathrm{Li}_2\left(e^{-2\pi i z}\right) + \frac{1}{2\pi^2}\mathrm{Li}_3\left(e^{-2\pi i z}\right) - \frac{\zeta(3)}{2\pi^2}$$

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- Derivatives of this complicated functions are given by simple expressions

  $$\frac{df(z)}{dz} = \pi z^2\cot(\pi z)\,; \qquad \frac{dl(z)}{dz} = -\pi z\cot(\pi z)\,;$$

Case of Adjoint Hypermultiplet

- In the case of $SU(N)$ gauge group with adjoint hypermultiplet path integral turns into
  $Z \sim$
  $$\int \prod_{i=1}^{N} d\phi_i \exp\left(-\frac{8\pi^3 N}{\lambda} \sum_i \phi_i^2 + \sum_{j\neq i} \sum_i \left[\frac{1}{2} \log\left[\sinh(\pi(\phi_i - \phi_j))\right] + \frac{1}{4} f(i(\phi_i - \phi_j)) \right.\right.$$
  $$\left.\left. -\frac{1}{4} I\left(\frac{1}{2} - im + i(\phi_i - \phi_j)\right) - \frac{1}{4} f\left(\frac{1}{2} - im + i(\phi_i - \phi_j)\right) + (\phi_i \to -\phi_i) \right] \right)$$
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  where $\lambda = \frac{g_{YM}^2 N}{r}$, is t' Hooft coupling constant
- In large N limit partition function is dominated by the saddle point
  $$\frac{16\pi^3 N}{\lambda}\phi_i = \pi \sum_{j\neq i} \left[\left(2 - (\phi_i - \phi_j)^2\right) \coth(\pi(\phi_i - \phi_j)) \right.$$
  $$\left. + \frac{1}{2}\left(\frac{1}{4} + (\phi_i - \phi_j - m)^2\right) \tanh(\pi(\phi_i - \phi_j - m)) + (m \to -m) \right]$$

Strong Coupling Limit

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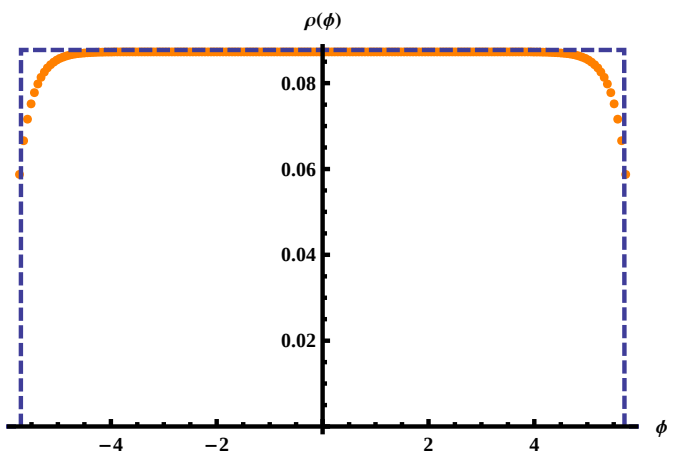
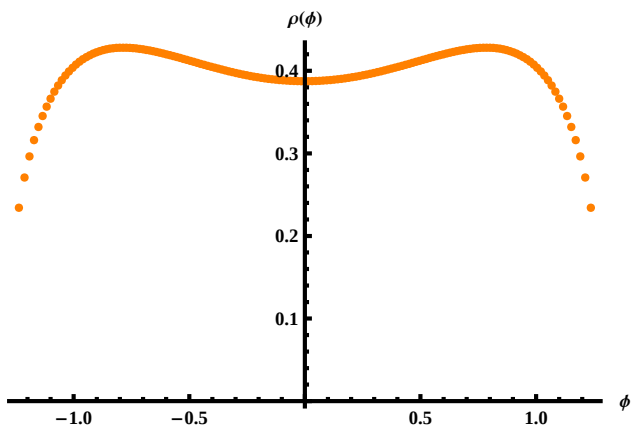
- Corresponding eigenvalue density is

$$\begin{aligned}\rho(\phi) &= \frac{32\pi^2}{\left(9+4m^2\right)\lambda} & |\phi| &\leq \phi_m\ , & \phi_m &= \frac{\left(9+4m^2\right)\lambda}{64\pi^2}\\ &= \quad 0 & |\phi| &> \phi_m\ .\end{aligned}$$





Free Energy

- Now substituting obtained solutions back to the free energy

$$F \equiv -\log Z \approx \frac{8\pi^3 N}{\lambda} \sum_i \phi_i^2 - \frac{\pi}{2}\left(\frac{9}{4} + m^2\right) \sum_{j \neq i} \sum_i |\phi_i - \phi_j|$$

and using

$$\sum_{i=1}^{N} (2i - N)^2 \approx \frac{1}{3} N^3 , \qquad \sum_{j \neq i} \sum_{i=1}^{N} |i - j| \approx \frac{1}{3} N^3 .$$

we can finally obtain expression for 5d SYM free energy

$$F \approx -\frac{g_{YM}^2 N^3}{96\pi r}\left(\frac{9}{4} + m^2\right)^2 \qquad N^3 \text{ -behavior !}$$

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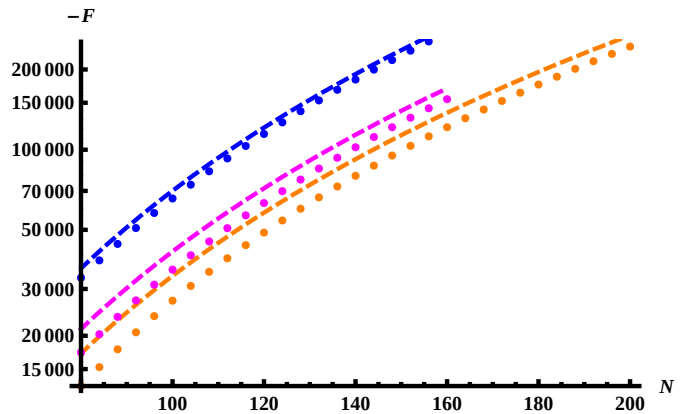
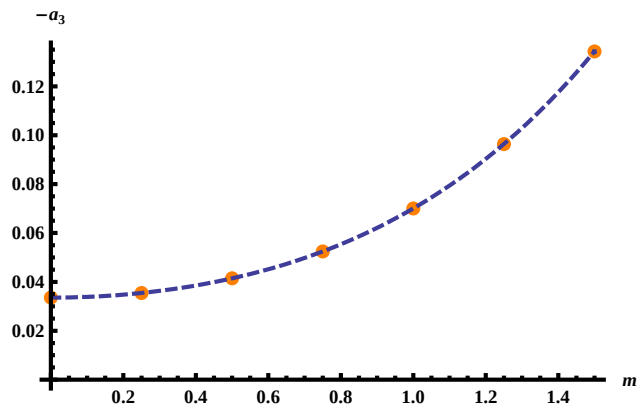
$$\sum_{i=1}^{N} (2i - N)^2 \approx \frac{1}{3} N^3 , \qquad \sum_{j \neq i} \sum_{i=1}^{N} |i - j| \approx \frac{1}{3} N^3 .$$

we can finally obtain expression for 5d SYM free energy

$$F \approx -\frac{g_{YM}^2 N^3}{96\pi r}\left(\frac{9}{4} + m^2\right)^2 \qquad N^3 \text{ -behavior !}$$

- Numerical results: $m = 0$ (*orange*); $m = \frac{1}{2}$ (*purple*); $m = 1$ (*blue*)





Free Energy

- Now substituting obtained solutions back to the free energy

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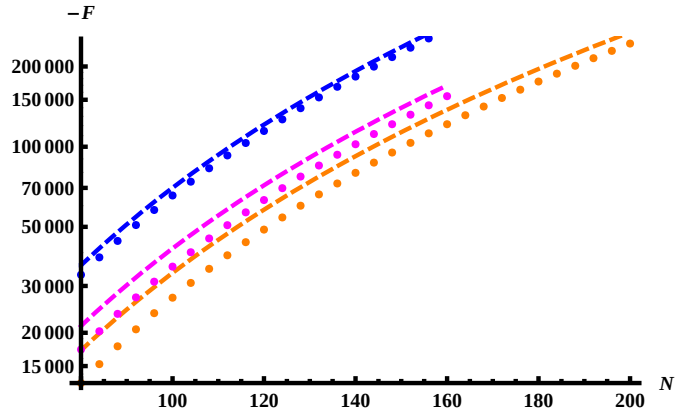

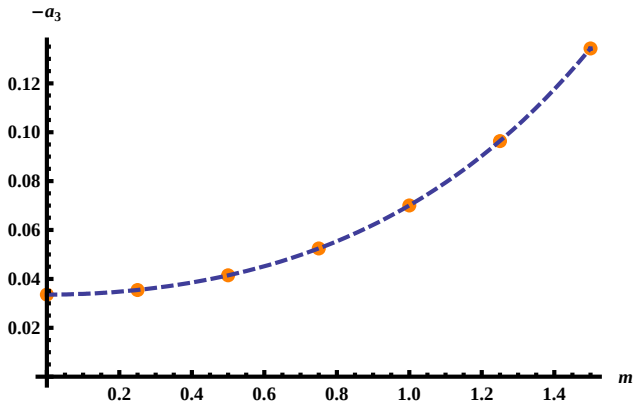

- Notice that essential element for N^3 behavior is hypermultiplet in adjoint representation !

Wilson Loop

- Wilson loop in 5D SYM:

$$W = \frac{1}{\dim R} \mathrm{Tr}_R P e^{\oint iA_M dx^m + \sigma ds}$$

As we are interested only in supersymmetric variables we further calculate loop wrapping equator

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- Saddle point position is the same as before, so we get:

$$\langle W \rangle = \int d\phi \rho(\phi) e^{2\pi\phi} \sim \exp\left(\frac{\lambda}{4\pi}\left(\frac{9}{4}+m^2\right)\right)$$

AdS_7/CFT_6

6d SCFT results can be obtained from $AdS_7 \times S^4$ supergravity calculations.

- boundary of AdS_7 is taken to be $S^1 \times S^5$
- radii of AdS_7 and S^4 are ℓ and $\ell/2$ correspondingly, $\ell = 2\ell_{pl}(\pi N)^{1/3}$
- AdS^7 metric is given by

$$ds^2 = \ell^2(\cosh^2\rho\, d\tau^2 + d\rho^2 + \sinh^2\rho\, d\Omega_5^2)$$

together with the identification $\tau = \tau + \frac{2\pi R_6}{r}$

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$G_N = 1/(16\pi^7\ell_{pl}^9)$ is Newton constant and e.o.m. give $R-2\Lambda = -\frac{12}{\ell^2}$
Integration gives

$$I_{\rm bulk} = \tfrac{4\pi R_6}{3\,r}N^3\left(\tfrac{1}{64}\epsilon^{-6} - \tfrac{3}{32}\epsilon^{-4} + \tfrac{15}{64}\epsilon^{-2} - \tfrac{5}{16} + \mathrm{O}(\epsilon^2)\right),$$

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- After minimal substruction we get free energy of 6d SCFT

$$I_{AdS} = -\frac{5\pi R_6}{12\,r}N^3$$

AdS_7/CFT_6: Wilson Loop

- In SUGRA Wilson loop is related to the world volume of the $M2$-brane:

$$\langle W \rangle \sim e^{-T^{(2)} \int dV}$$

where $T^{(2)} = \frac{1}{(2\pi)^2 \ell_{pl}^3}$ is the tension of the membrane.

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- After regularization we obtain:

$$\langle W \rangle \sim \exp \left(\frac{2\pi N R_6}{r} \right)$$

Comparison of results

- Summarizing results

	AdS_7/CFT_6	Matrix Model
Free Energy	$-\frac{5\pi R_6}{12\, r} N^3$	$-\frac{g_{YM}^2 N^3}{48\pi r}\left(\frac{9}{4}+m^2\right)^2$
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- For these two results to be consistent we need $m = \frac{1}{2}$, $R_6 = \frac{5g_{YM}^2}{16\pi^2}$
- $m = \frac{1}{2}$ is indeed very special point because with this value of mass enlargement of supersymmetry takes place (in difference with flat case where SUSY is maximal when hypers are massless).

Summary and Outlook

- From matrix model obtained with localization technique we found some observables in 5d SYM
- Large-N behavior coincides with AdS_7/CFT_6 results
- Coefficients for matrix model and AdS_7/CFT_6 calculations coincides for $\mathcal{N} = 1^*$ SYM with hyper masses $m = \frac{1}{2}$, when enlargement of SUSY takes place
- Relation between 5d SYM on S^5 and 6d SCFT is $R_6 = \frac{5g_{YM}^2}{16\pi^2}$ which differs from the flat case correspondence $R_6 = \frac{g_{YM}^2}{8\pi^2}$ obtained by Lambert et. al.

Thank you!