

Recent developments in jet quenching theory

Liliana Apolinário

*Universidade de Santiago de Compostela
CENTRA-IST*

Special thanks to : Nestor Armesto, Guilherme Milhano and Carlos Salgado

July 19th, 2013

FPS-HEP, Stockholm, Sweden

*For recent developments in lattice, see Panero's talk
(Saturday, Non-perturbative QCD session)*

(Some) Recent developments in jet quenching theory

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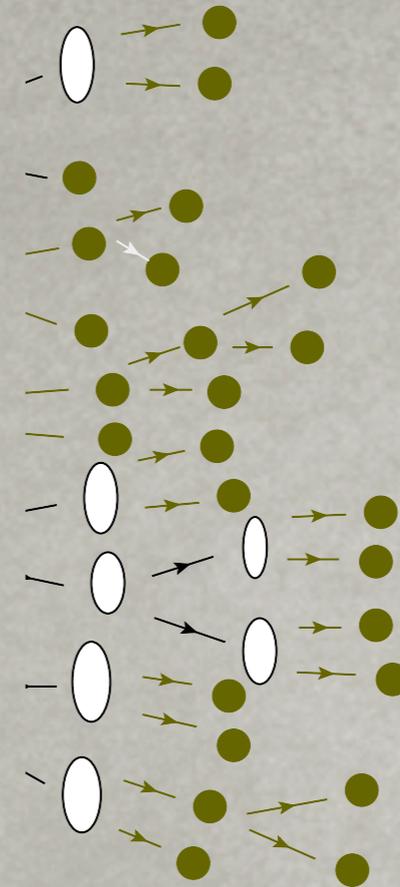
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Introduction

- Parton Branching:

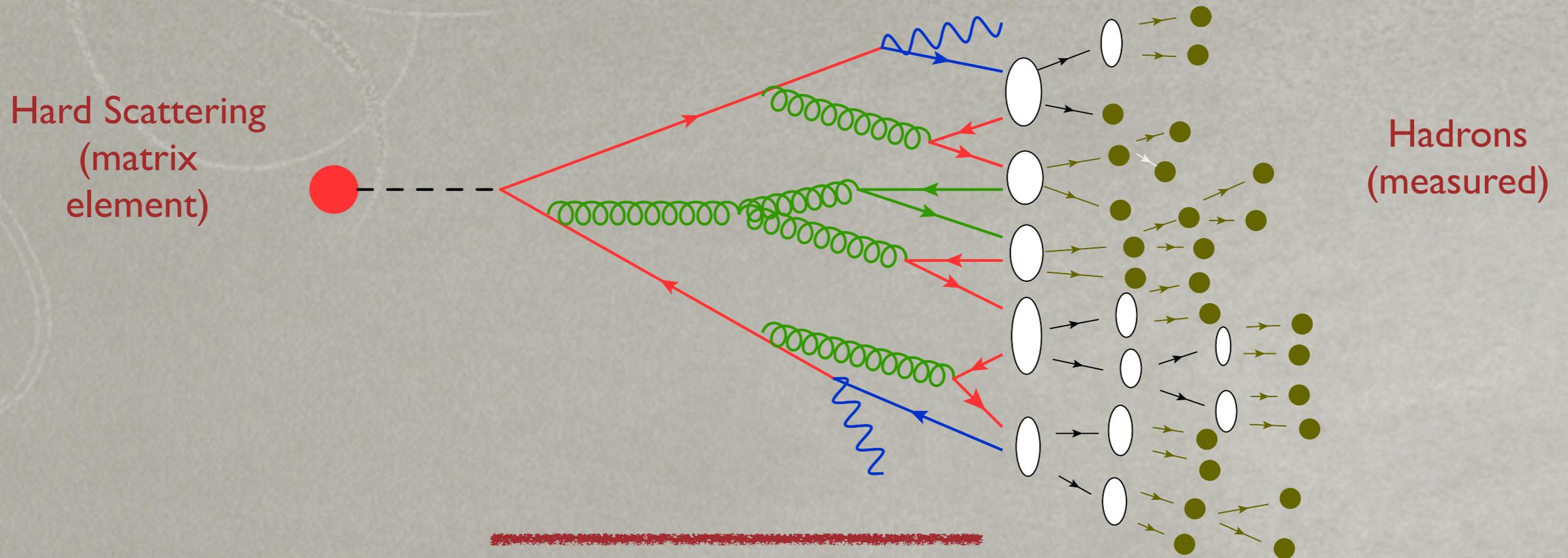
Hard Scattering
(matrix
element)



Hadrons
(measured)

Introduction

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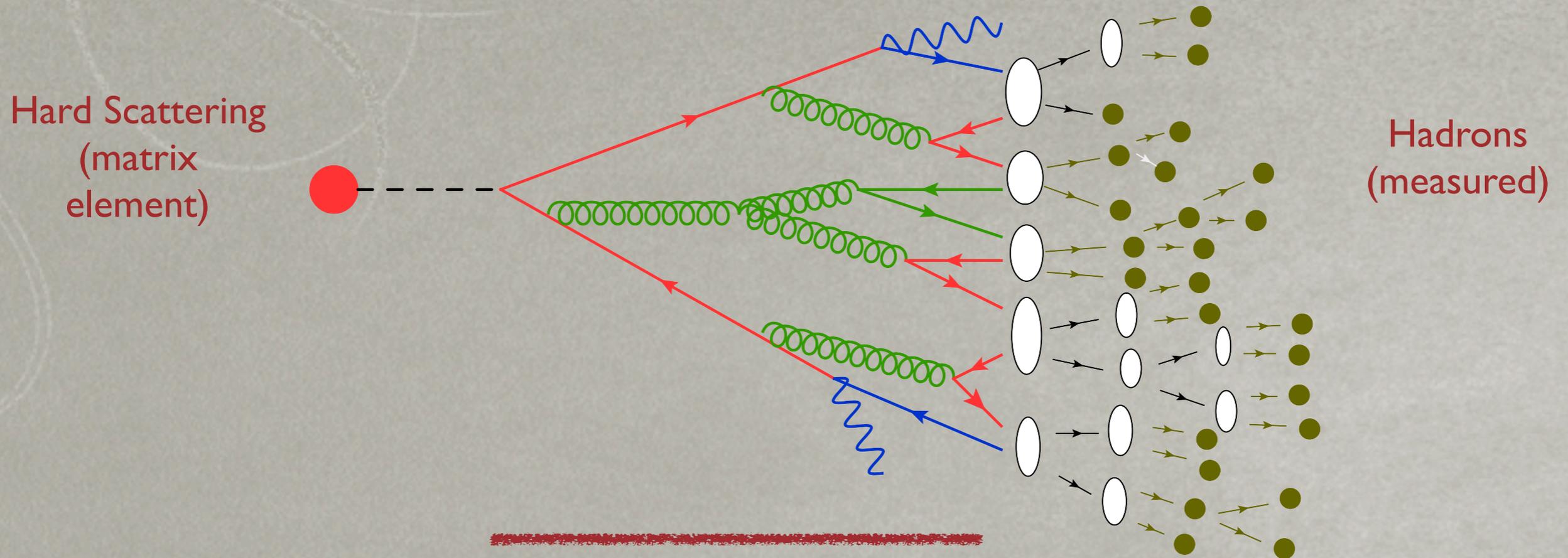


Parton showering:

- Calculable through pQCD
- Known in vacuum

Introduction

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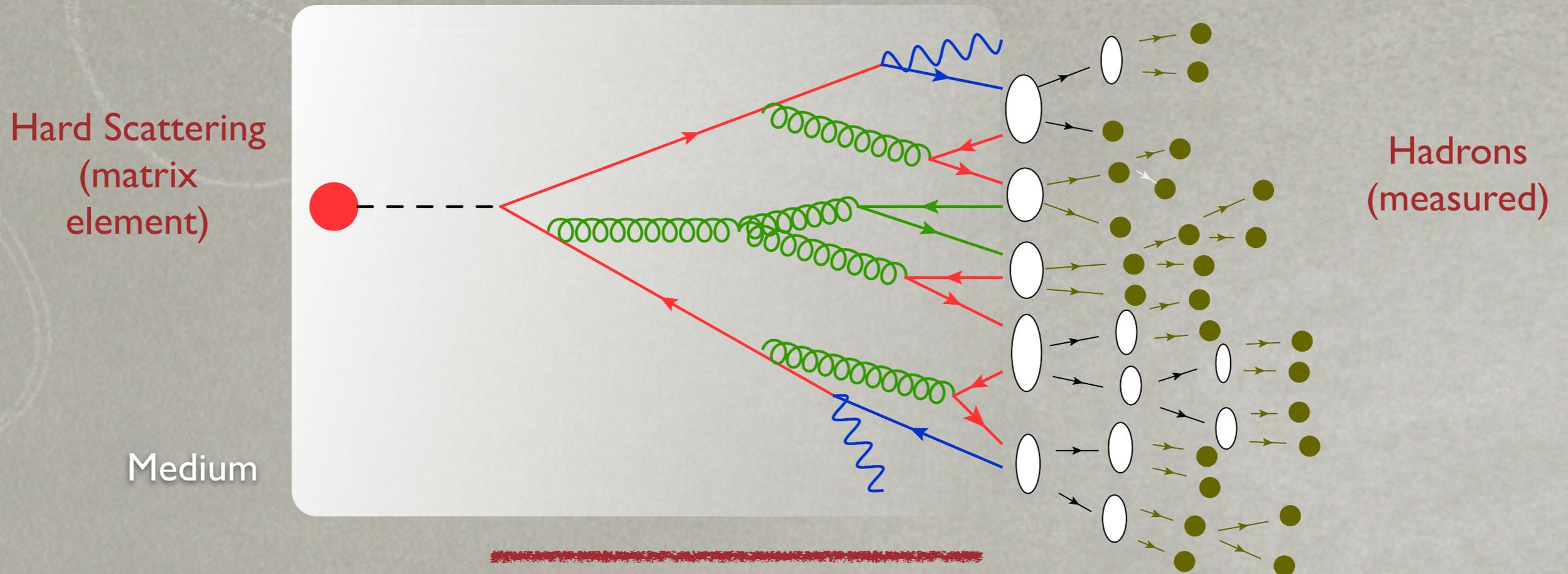
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Hadronization

- Universal/process independent
- Not described by pQCD

Introduction

- Parton Branching:



What are the main mechanisms of in-medium energy loss?

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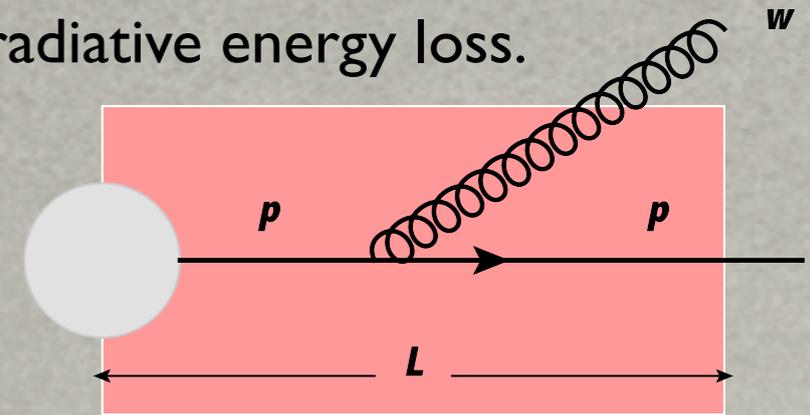
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- Medium?

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Jet Quenching "before LHC"

- Standard jet quenching models: description in terms of radiative energy loss.
 - Four classes of formalisms based on pQCD:
 - Baier-Dokshitzer-Mueller-Peigné-Schiff-Zakharov and Armesto-Salgado-Wiedemann (BDMPS-Z/ASW)
 - Gyulassy-Levai-Vitev (GLV)
 - Arnold-Moore-Yaffe (AMY)
 - Higher-Twist (HT)



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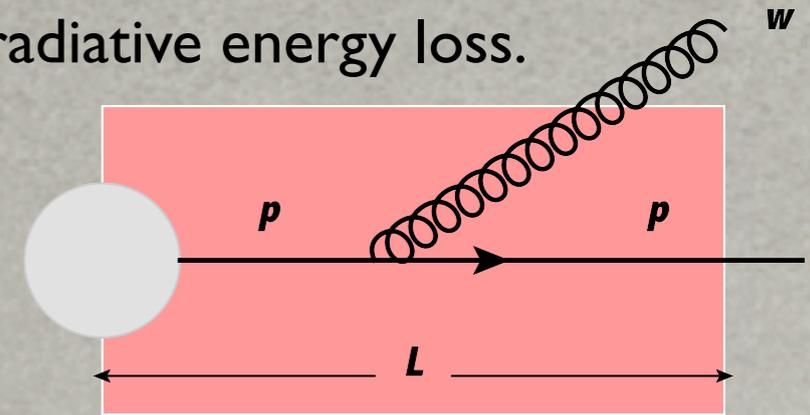
- Higher-Twist (HT)

- Usual characteristics:

- Neglect recoil (elastic energy loss not included);

- Work in the limit of soft ($x \rightarrow 0$) (except AMY) and collinear ($k_T \ll \omega$) gluon emissions;

- Assume multiple gluon emissions as a factorization of single gluon emissions.



Jet Quenching "before LHC"

- Problems/limitations of jet quenching models:
 - Phenomenological extension into large x and large angle domain:
 - Extensions beyond the formalism domain lead to large discrepancies [1106.1106].
⇒ Calculation of single emission process in a more general kinematic range
Several efforts within the path-integral formalism and SCET

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Several efforts within the path-integral formalism and SCET
 - Shower evolution based on multi-gluon radiation:
 - Problem of color coherence between different emitters.
⇒ Propagation of color correlated sources inside a colored medium
Several approaches address coherence effects within the antenna setup

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- Hadronization unmodified by the presence of a medium;

- Color (de)coherence can induce modifications independently of where hadronization takes place.

⇒ Extension of medium-induced gluon radiation to the study of color-differential case

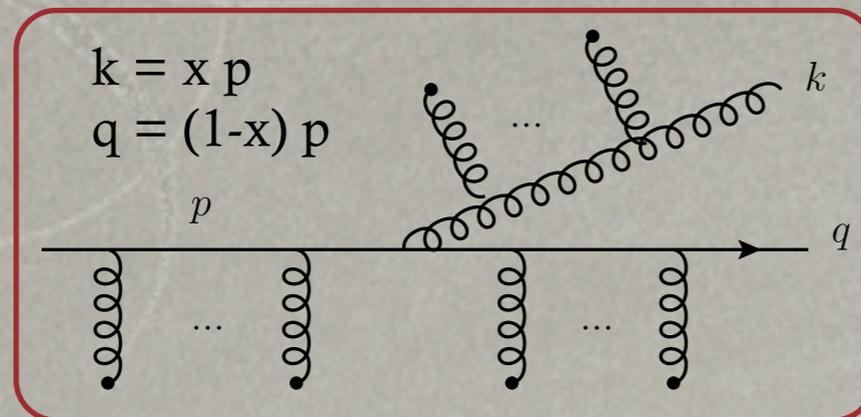
Study of medium-modified color connections

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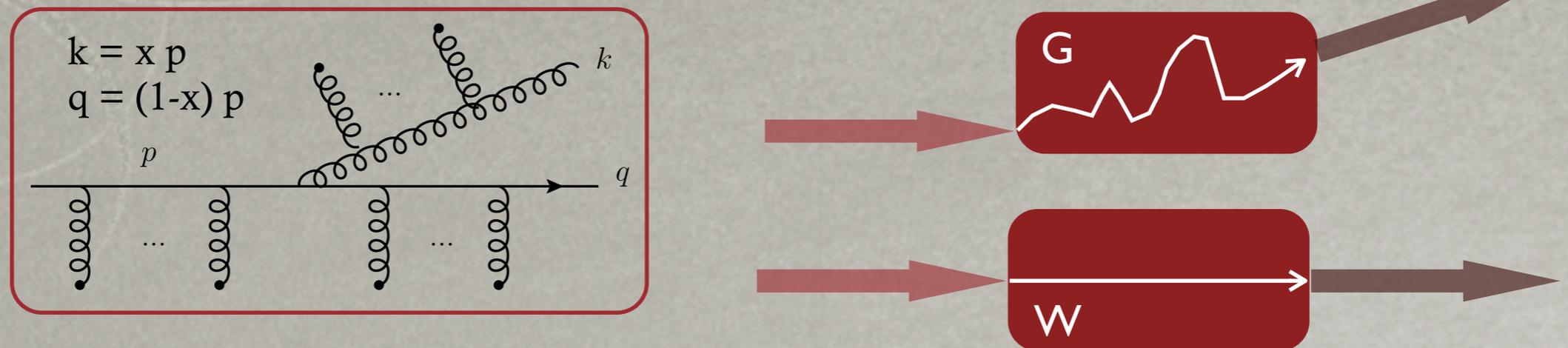
Medium-induced radiation

- Radiative energy loss assumed to be the dominant process:
 - Particle propagation inside a medium:



Medium-induced radiation

- Radiative energy loss assumed to be the dominant process:
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- Highly energetic partons described by a Wilson Line:

$$W(x_{0+}, L_+; \mathbf{x}_\perp) = \mathcal{P} \exp \left\{ ig \int_{x_{0+}}^{L_+} dx_+ A_-(x_+, \mathbf{x}_\perp) \right\}$$

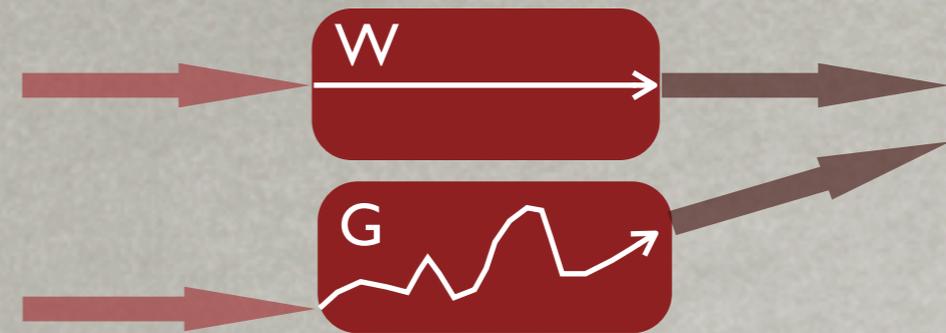
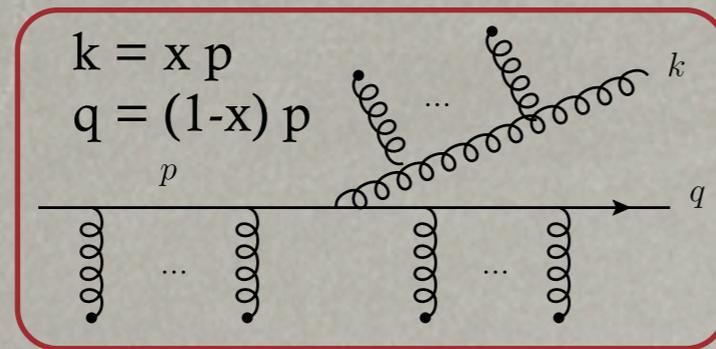
- Less energetic particles described by a Green's Function:

$$G(x_{0+}, \mathbf{x}_{0\perp}; L_+, \mathbf{x}_\perp | p_+) = \int_{\mathbf{r}_\perp(x_{0+})=\mathbf{x}_{0\perp}}^{\mathbf{r}_\perp(L_+)=\mathbf{x}_\perp} \mathcal{D}\mathbf{r}_\perp(\xi) \exp \left\{ \frac{ip_+}{2} \int_{x_{0+}}^{L_+} d\xi \left(\frac{d\mathbf{r}_\perp}{d\xi} \right)^2 \right\} \times W(x_{0+}, L_+; \mathbf{r}_\perp(\xi)),$$

Medium-induced radiation

LA, Armesto and Salgado
[1204.2929]

- Radiative energy loss assumed to be the dominant process:
 - Particle propagation inside a medium:
 - Contribution in the opposite limit allows to find an interpolation function between the two limits:

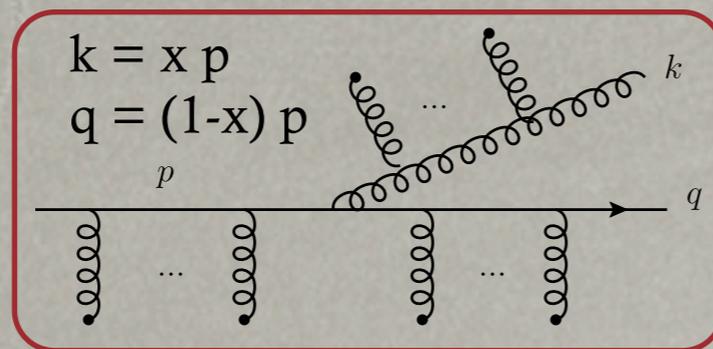


- Numerical results:

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Dimensionless variables

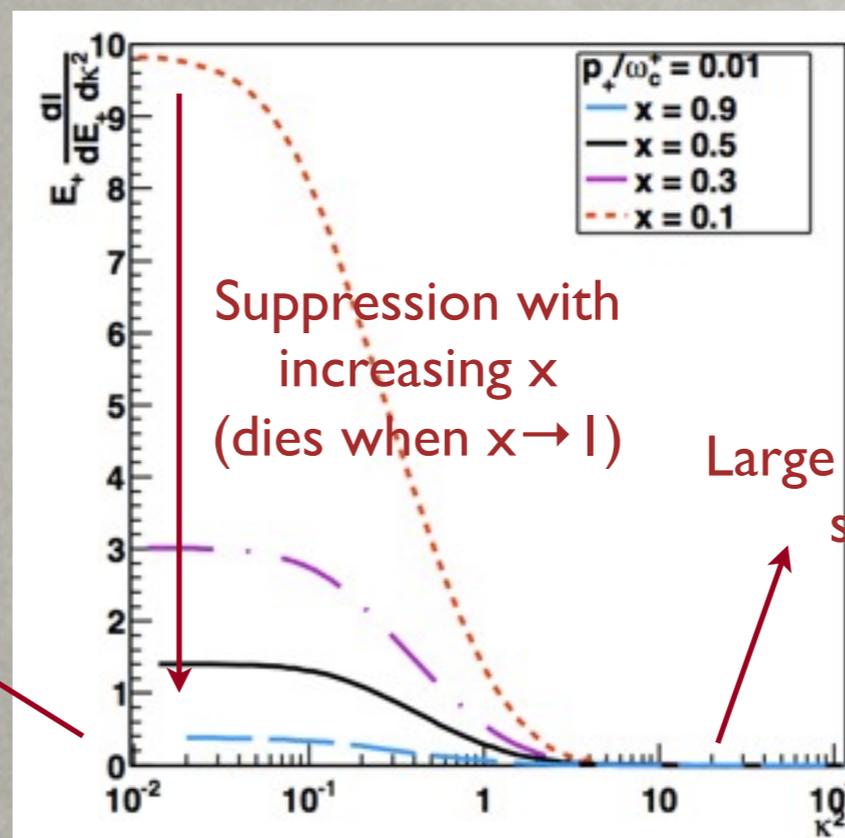
Energy:

$$\frac{p_+}{\omega_c^+} = \frac{2p_+}{\hat{q}_A L_+^2}$$

Transverse momentum:

$$\kappa^2 = \frac{q_\perp^2}{\hat{q}_A L_+}$$

No collinear divergency

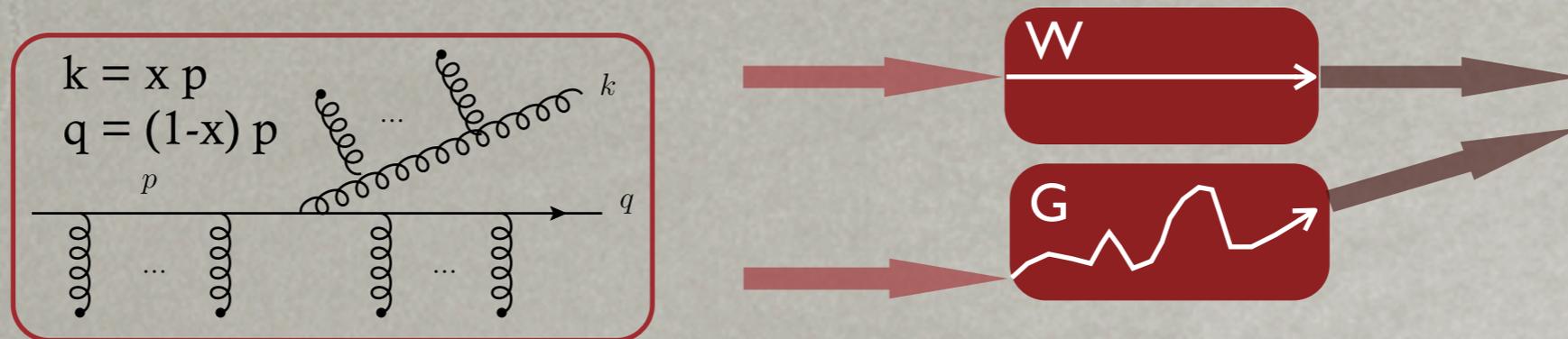


Medium information encoded in:
 L_+ : Medium length
 \hat{q}_A : Transport coefficient

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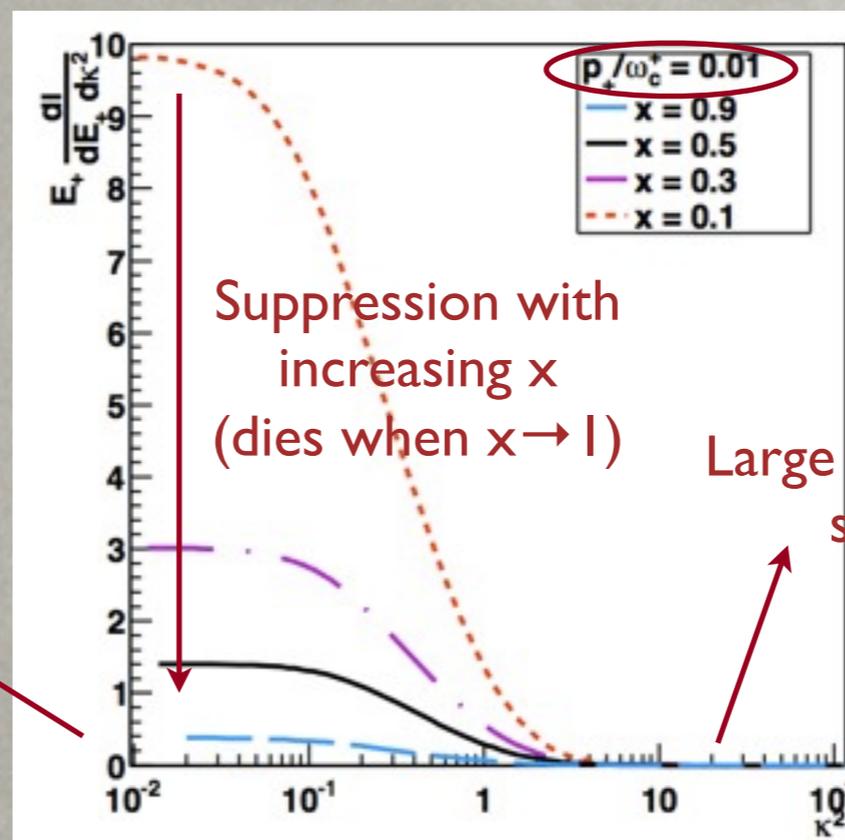
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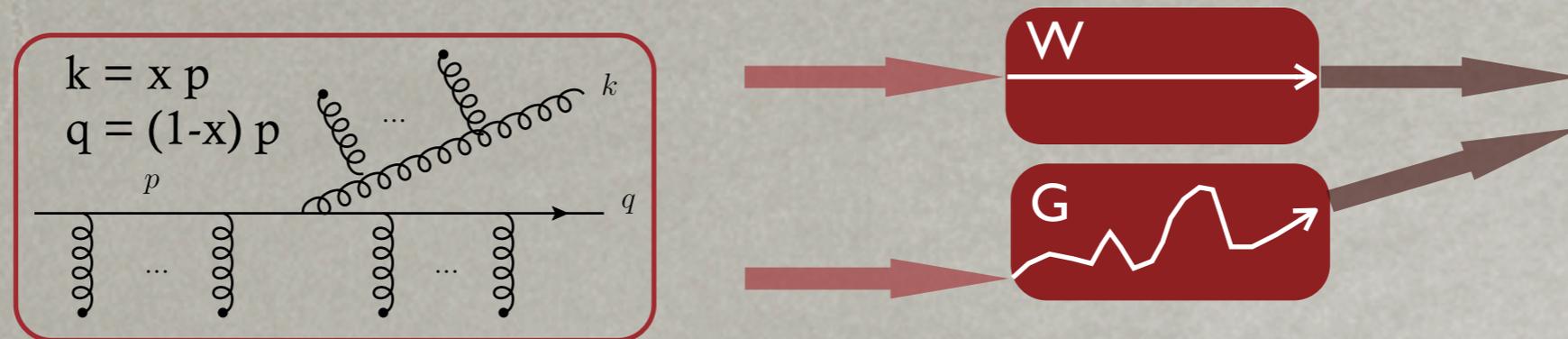


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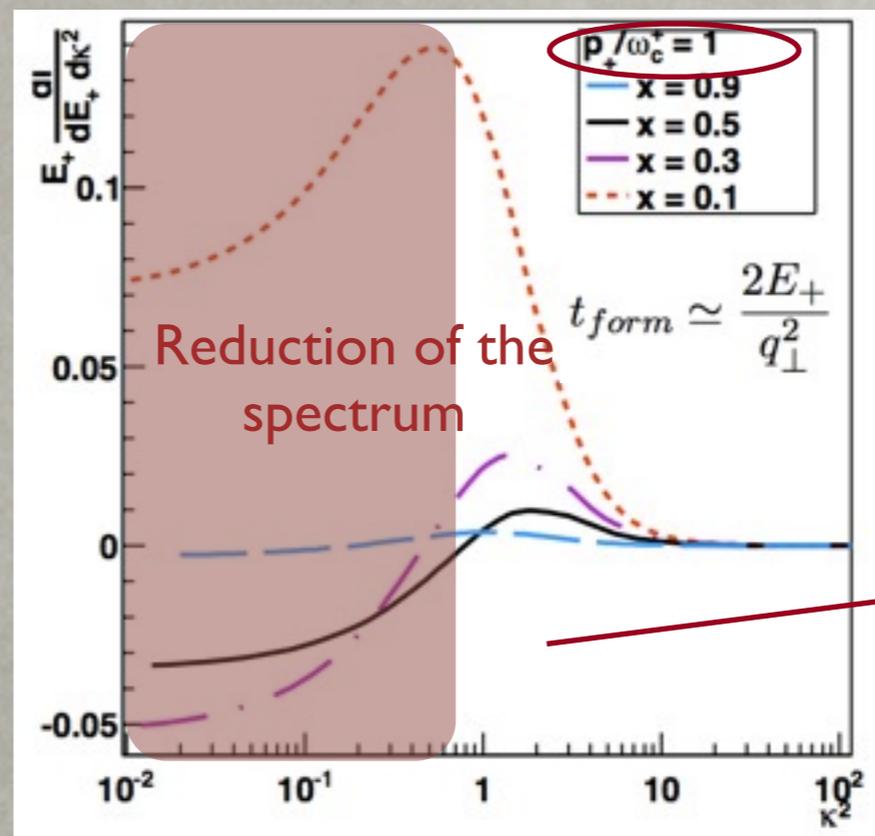
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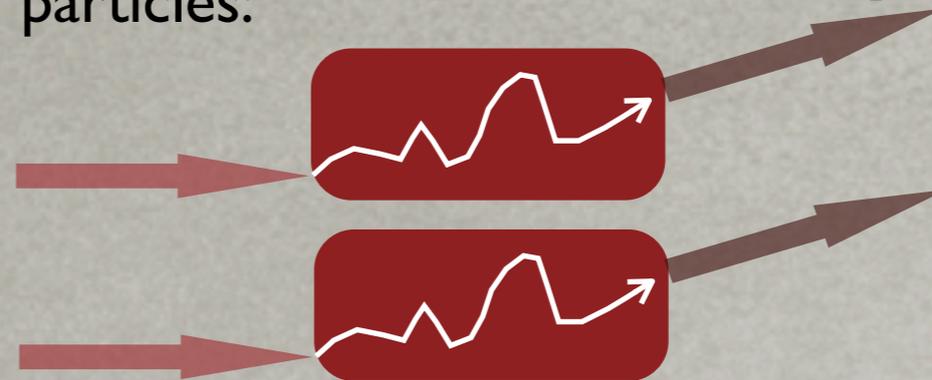
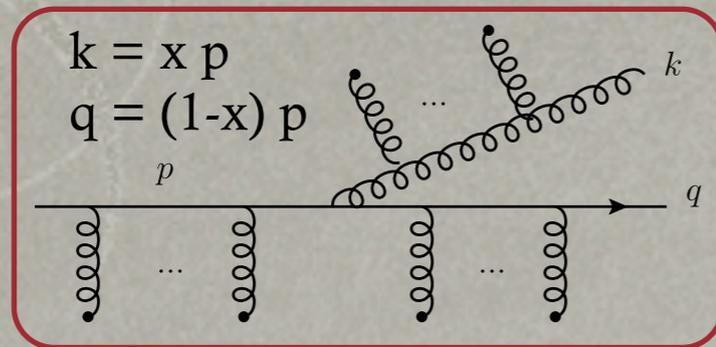
If $t_{\text{form}} \gg L$, whole medium act as a single scattering center (LPM effect in QCD)

Medium-induced radiation

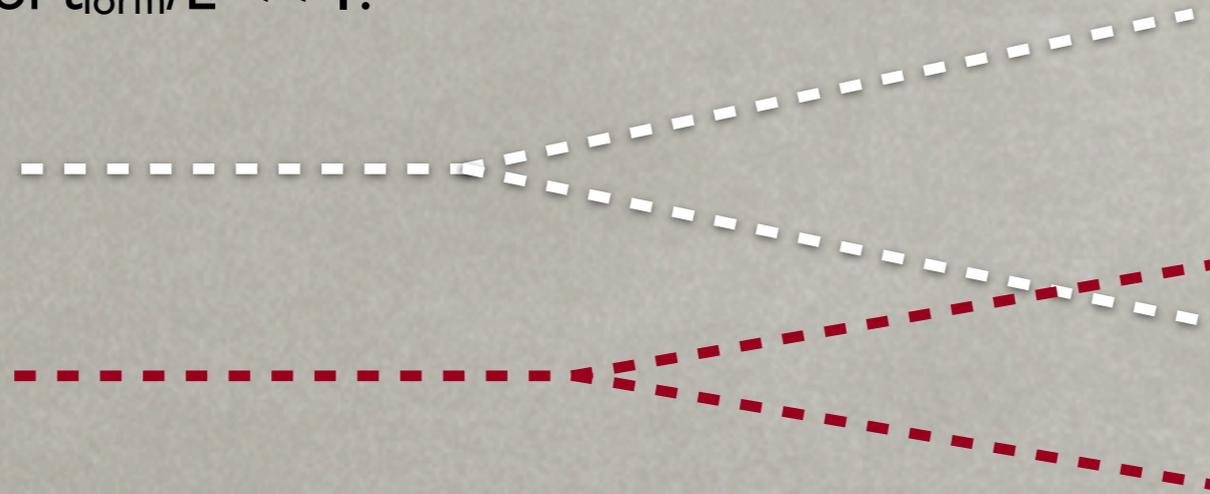
Blaizot, Dominguez, Iancu, Mehtar-Tani
[1209.4585]

Extension beyond small t_{form} :
LA, Armesto, Milhano and Salgado
[in prep]

- Gluon branching process:
 - Free the transverse motion of all particles:



- Limit of $t_{\text{form}}/L \ll 1$:

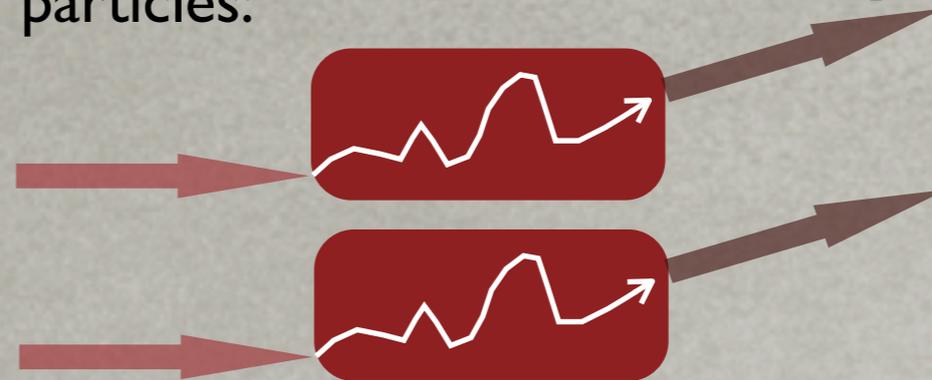
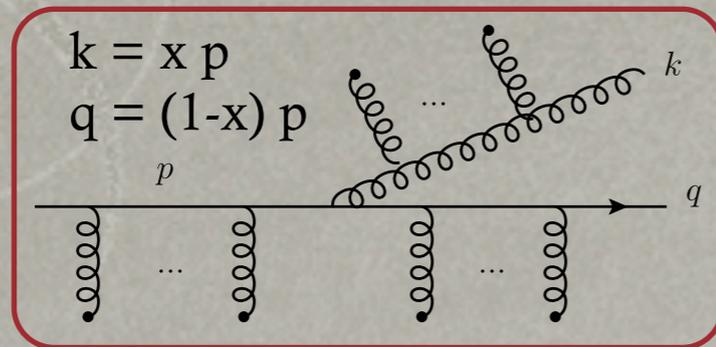


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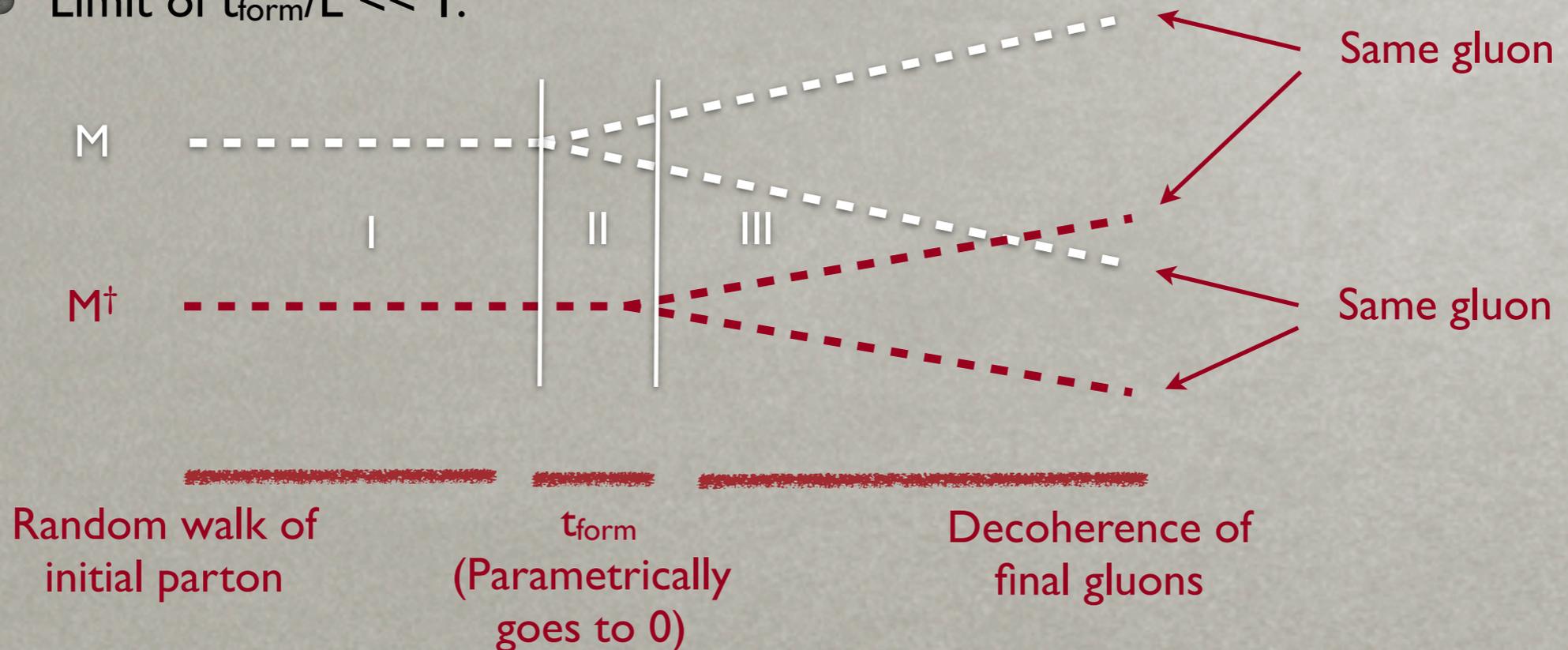
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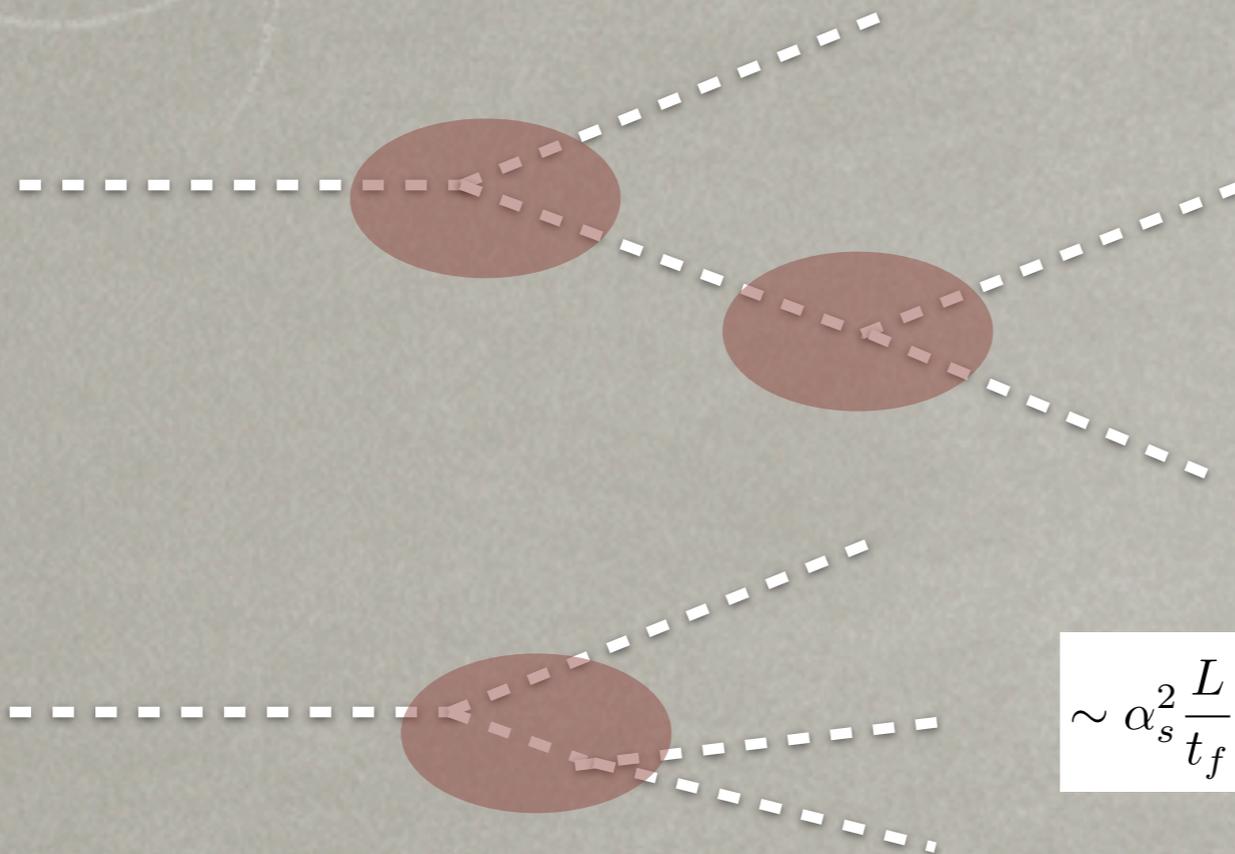


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Extension beyond small t_{form} :
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- Gluon branching process:
 - Limit of $t_{\text{form}}/L \ll 1$:
 - Decoherence of final gluons can provide a probabilistic picture



$$\sim \left(\alpha_s \frac{L}{t_f} \right)^2$$

Independent
emissions are
enhanced within
the considered limit

$$\sim \alpha_s^2 \frac{L}{t_f}$$

Showering can be understood as a factorization of single gluon emissions for $t_{\text{form}} \ll L$

Momentum broadening

Original works:
Idilbi, Majumder [0808.1087]

- Soft Collinear Effective Theory (SCET):
 - Other framework to describe dynamics of highly energetic quarks and gluons (radiative and collisional energy loss);

- Based on a scale separation: $Q \gg l_T \gg T \Rightarrow \lambda \equiv \frac{T}{Q} \ll 1$

Hard parton energy

Soft scale characteristic of the medium

Transverse momentum of radiated gluon

D'Eramo, Liu, Rajagopal
[1010.0890, 1006.1367]
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Transverse momentum of radiated gluon

- Setup:

- High energy parton with $p_0 = (0, Q, 0)$ that exchanges Glauber gluons $p_G \sim Q (\lambda^2, \lambda^2, \lambda)$ with the medium.

- After interaction, $p_0 \rightarrow p_c \sim Q (\lambda^2, l, \lambda)$

- Continuous kicks to hard parton result in significant transverse momentum broadening (Brownian motion)

- Lagrangian degrees of freedom: p_G, p_c and $p_s \sim (\lambda^2, \lambda^2, \lambda^2)$ (subset of Glauber gluons)

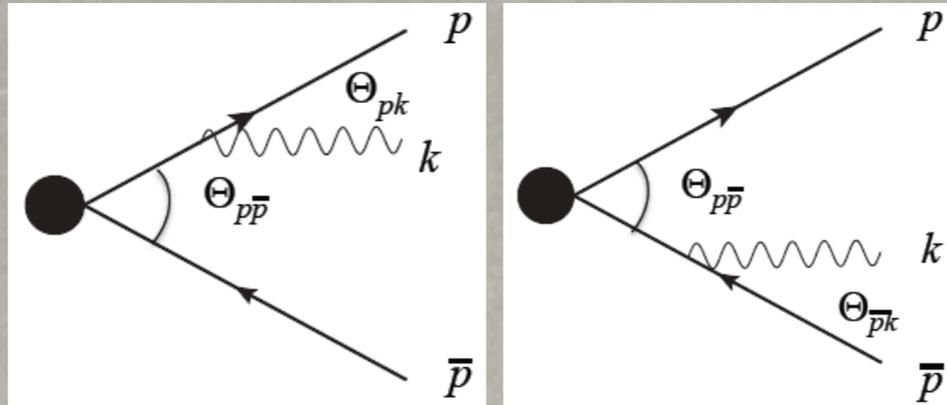
- Feynman rules \Rightarrow Medium-induced splitting kernels

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Color (de)coherence

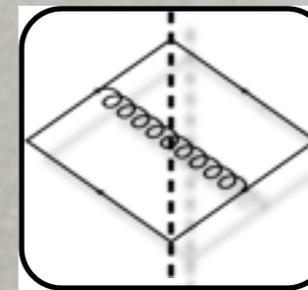
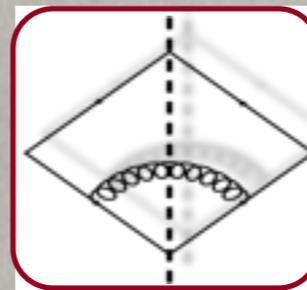
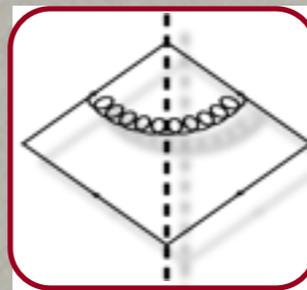
- Vacuum angular ordering property determined in the antenna setup:

- Diagrams:



- Radiation spectrum contributions:

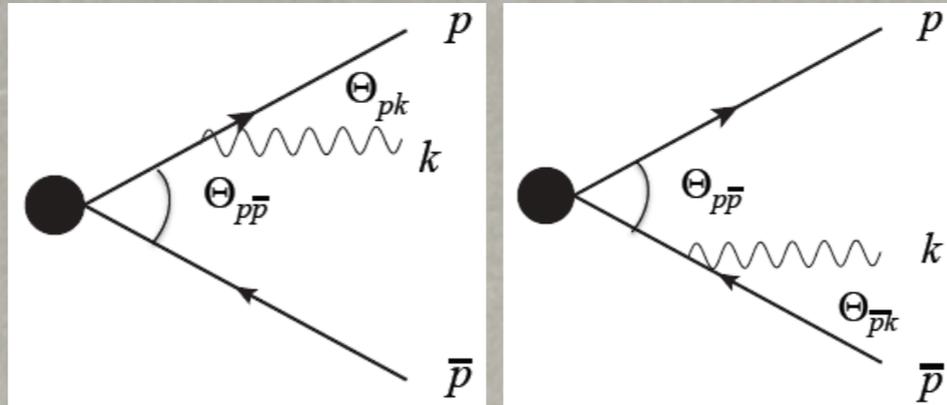
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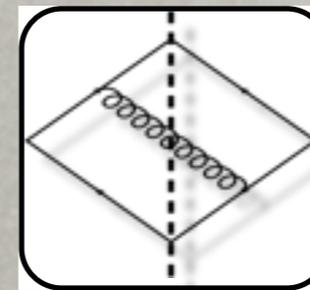
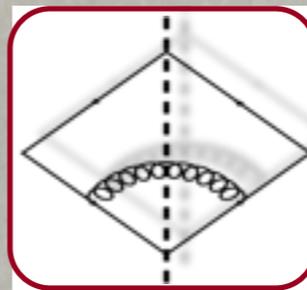
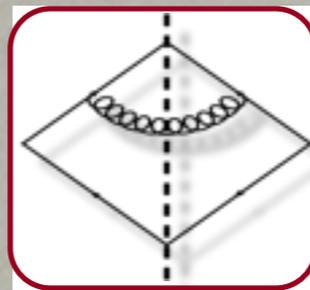
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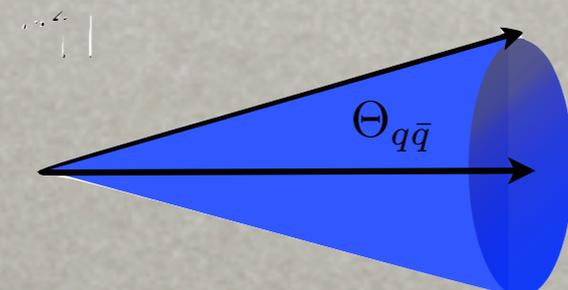
Destructive interferences suppress large angle emission

- Coherent radiation spectrum off a quark (same for anti-quark):

$$dN_q \propto \alpha_s \frac{d\omega}{\omega} \frac{d\theta_{pk}}{\theta_{pk}} \Theta(\theta_{p\bar{p}} - \theta_{pk})$$

IRC divergent

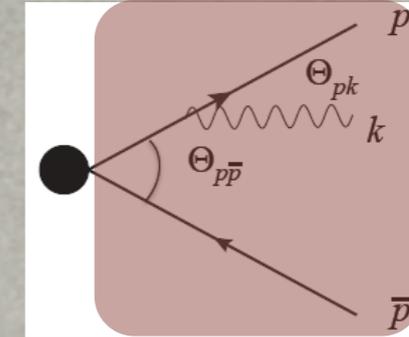
Angular Ordering



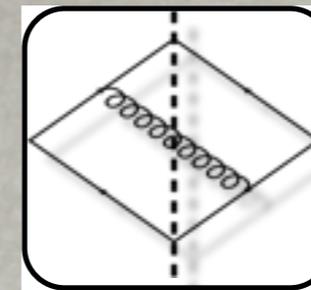
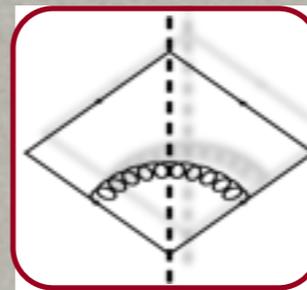
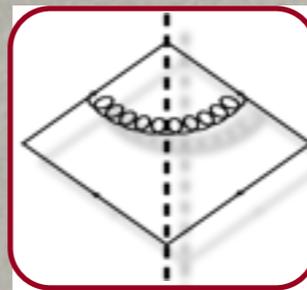
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Mehtar-Tani, Salgado and Tywoniuk
[1009.2965, 1102.4317, 1112.5031]

- Ordering properties within in-medium antenna setup:
 - Same contributions to radiation spectrum:



$$|\mathcal{M}^{tot}|^2 = |\mathcal{M}_q + \mathcal{M}_{\bar{q}}|^2 = |\mathcal{M}_q|^2 + |\mathcal{M}_{\bar{q}}|^2 + 2 \operatorname{Re}\{\mathcal{M}_q \mathcal{M}_{\bar{q}}^*\}$$



Casalderrey-Solana and Iancu
[1105.1760]

Massive antenna:
Armesto, Ma, Mehtar-Tani,
Salgado and Tywoniuk
[1110.4343]

BDMPS-Z/GLV independent spectrum:

- IRC safe;
- LPM interference;
- Broad angular structure;

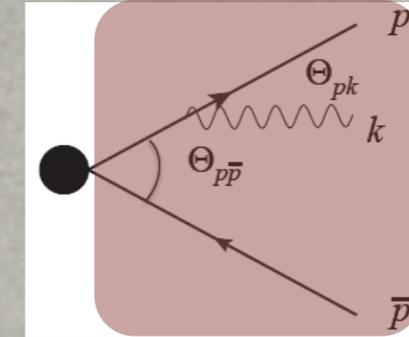
Interference spectrum:

- Vacuum-like radiation (IR divergent and collinear safe)
- In-cone radiation suppressed;

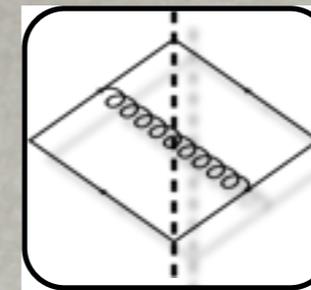
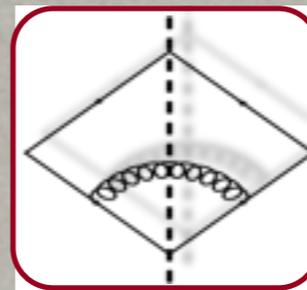
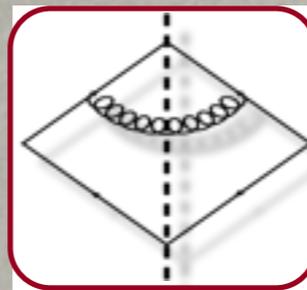
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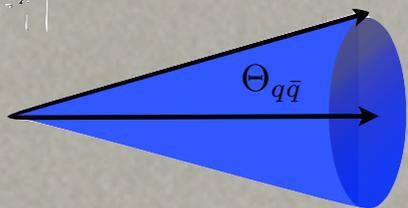
Interference spectrum:

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- Total gluon spectrum off an antenna in the soft limit (vacuum + medium):

$$dN_q^{tot} \Big|_{\omega \rightarrow 0} = \frac{\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{\sin \theta}{1 - \cos \theta} \frac{d\theta}{d\theta} \left[\Theta(\cos \theta - \cos \theta_{q\bar{q}}) + \Delta_{med} \Theta(\cos \theta_{q\bar{q}} - \cos \theta) \right]$$

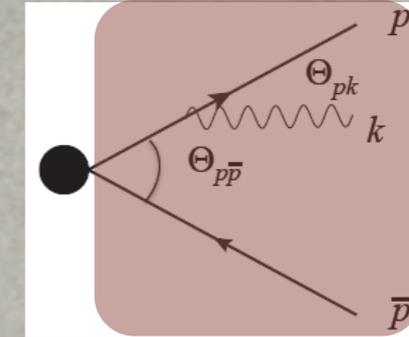
Angular ordering



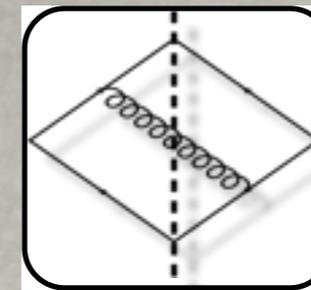
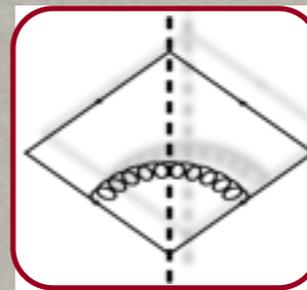
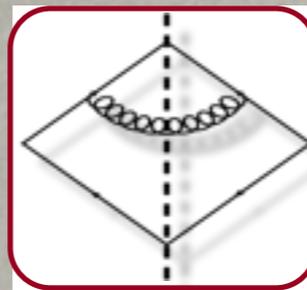
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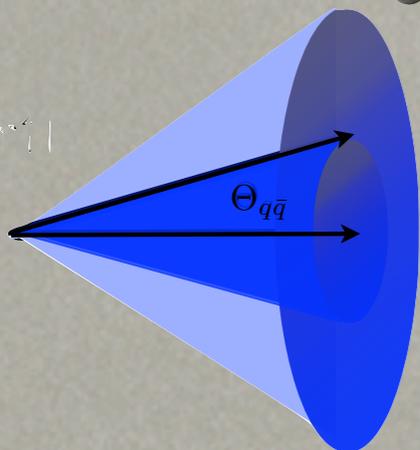
- Vacuum-like radiation (IR divergent and collinear safe)
- In-cone radiation suppressed;

- Total gluon spectrum off an antenna in the soft limit (vacuum + medium):

$$dN_q^{tot} \Big|_{\omega \rightarrow 0} = \frac{\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{\sin \theta}{1 - \cos \theta} \frac{d\theta}{d\theta} \left[\Theta(\cos \theta - \cos \theta_{q\bar{q}}) + \Delta_{med} \Theta(\cos \theta_{q\bar{q}} - \cos \theta) \right]$$

Angular ordering

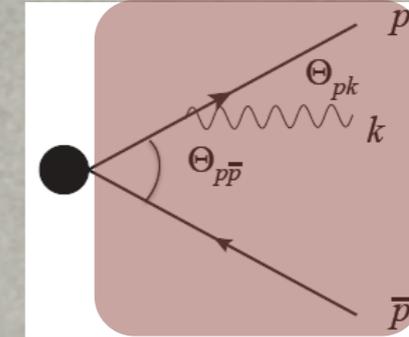
Antiangular ordering



Color (de)coherence

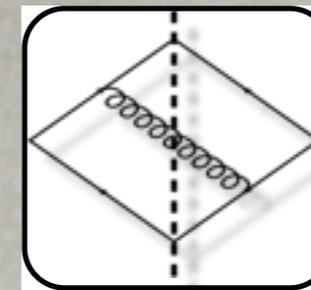
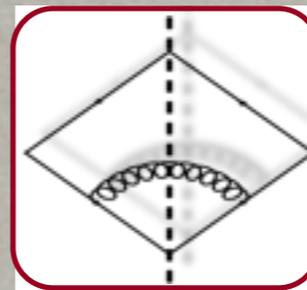
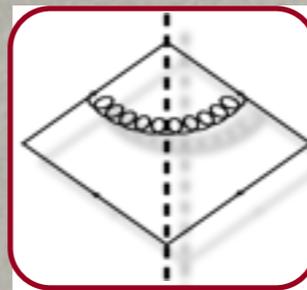
Mehtar-Tani, Salgado and Tywoniuk
[1009.2965, 1102.4317, 1112.5031]

- Ordering properties within in-medium antenna setup:
 - Same contributions to radiation spectrum:



$$|\mathcal{M}^{tot}|^2 = |\mathcal{M}_q + \mathcal{M}_{\bar{q}}|^2 = |\mathcal{M}_q|^2 + |\mathcal{M}_{\bar{q}}|^2 + 2 \operatorname{Re}\{\mathcal{M}_q \mathcal{M}_{\bar{q}}^*\}$$

$\Delta_{med} = 0$: No medium effects
 $\Delta_{med} = 1$: Complete decorrelation of emitters



Casalderrey-Solana and Iancu
[1105.1760]

Massive antenna:
Armesto, Ma, Mehtar-Tani,
Salgado and Tywoniuk
[1110.4343]

BDMPS-Z/GLV independent spectrum:

- IRC safe;
- LPM interference;
- Broad angular structure;

Interference spectrum:

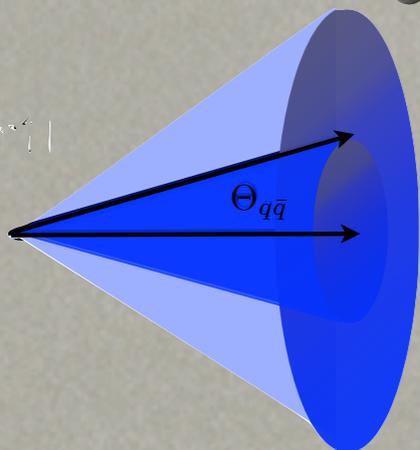
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Angular ordering

Antiangular ordering



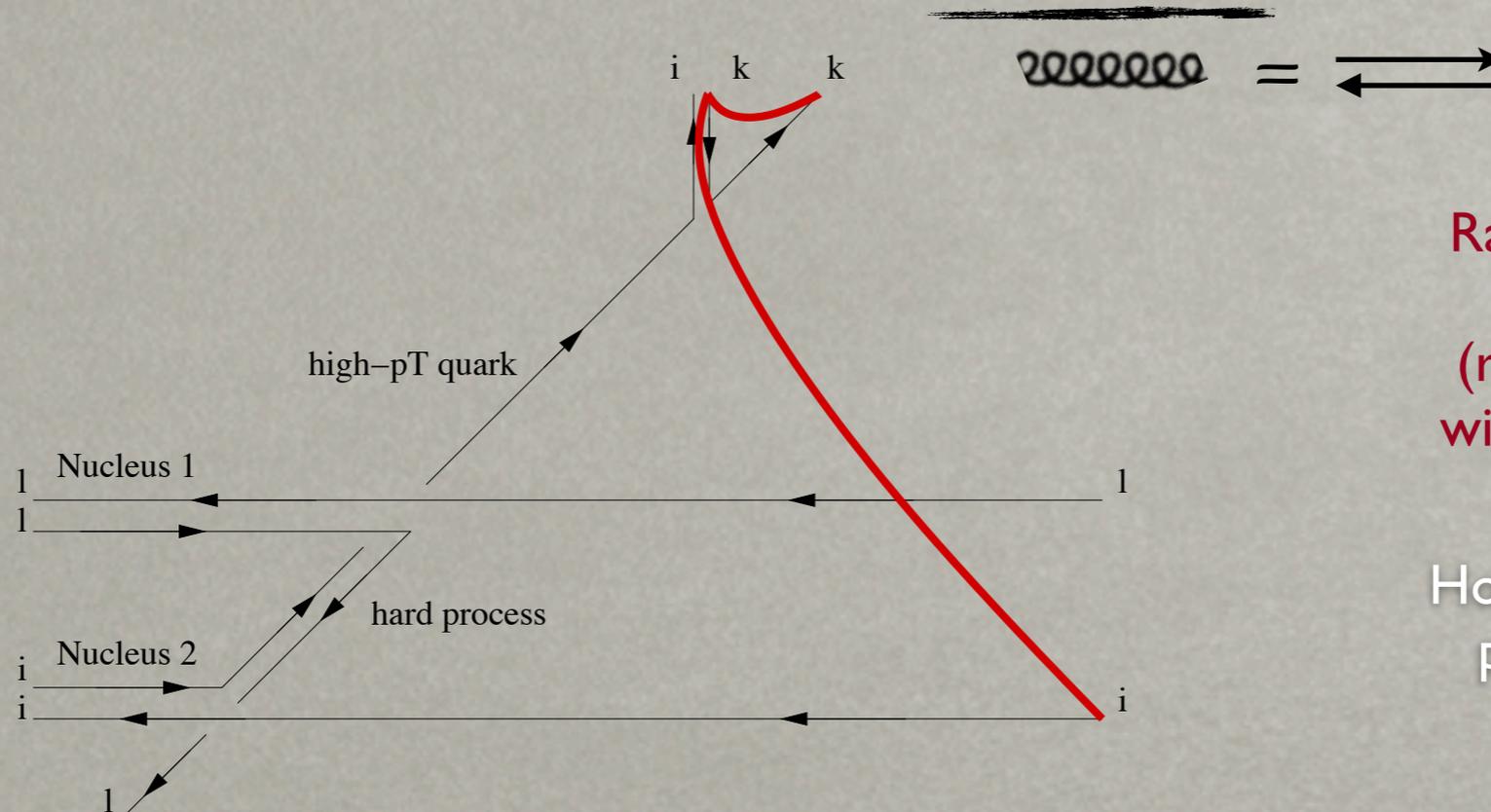
Interference effects open up radiation phase space

Color Flow

- Final state of a parton shower needs to be interfaced with an hadronization prescription:
 - Lund Model (implemented in PYTHIA): color connection through a string between a quark-antiquark pair, with gluons as kinks.
 - Singlet clusters (implemented in HERWIG): clusters color singlet objects, where gluons are split into quark-antiquark pairs.

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 - Singlet clusters (implemented in HERWIG): clusters color singlet objects, where gluons are split into quark-antiquark pairs.
- Vacuum color connections, in the large N_c limit, as:



Radiated gluons is part of the same string (remain color connected with the leading fragment)

How is this modified in the presence of a medium?

Color Flow

Beraudo, Milhano and Wiedemann
[1109.5025, 1204.4342]
Aurenche, Zakharov
[1109.6819]

- Color flows in the presence of a medium, within Lund string model (first order in opacity expansion):
 - Two possible situations:
 - No medium interaction after emission:
 - Medium interaction after emission:

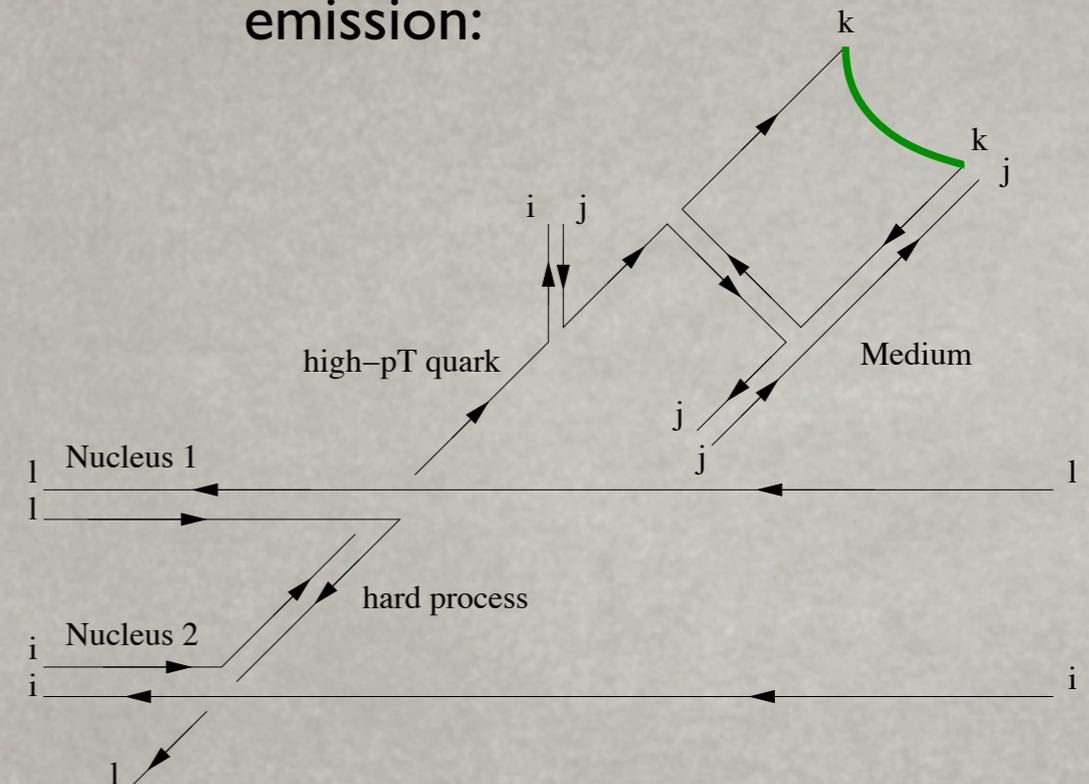
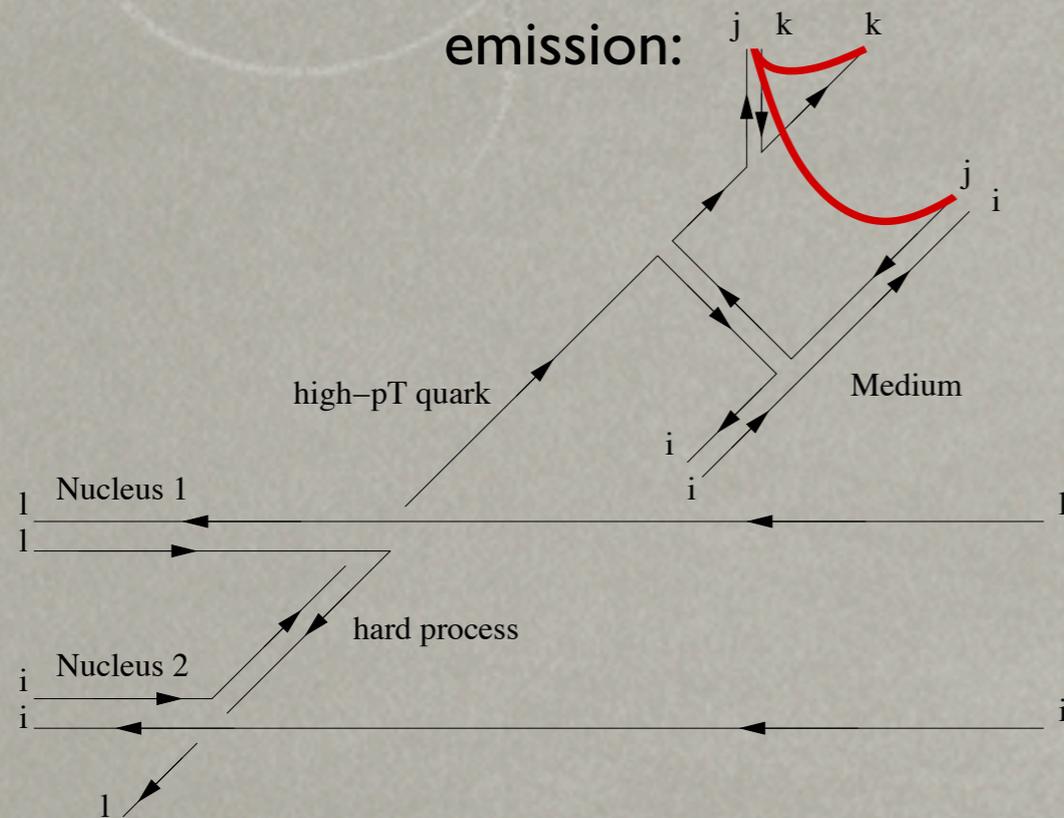
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Gluon belongs to the string that will form leading fragment (as in vacuum)

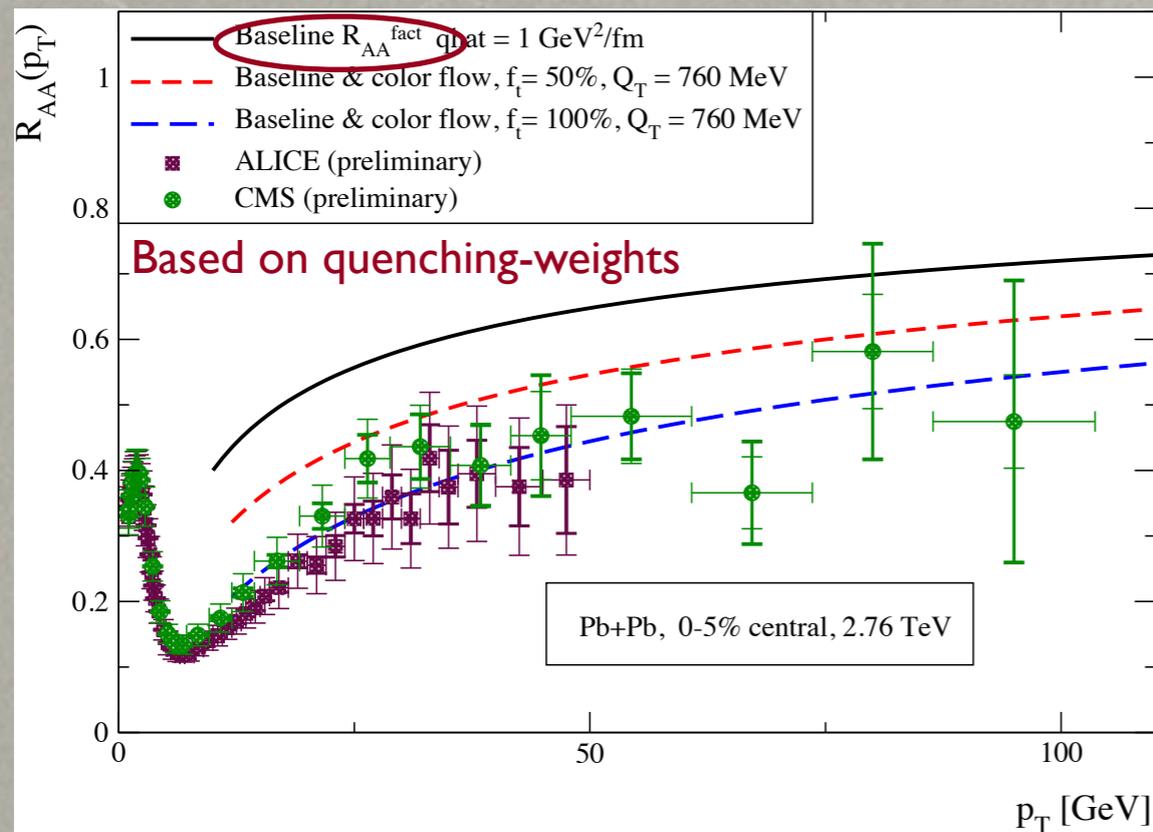
Gluon decoupled from the leading string
 ⇒ Energy is lost

Color Flow

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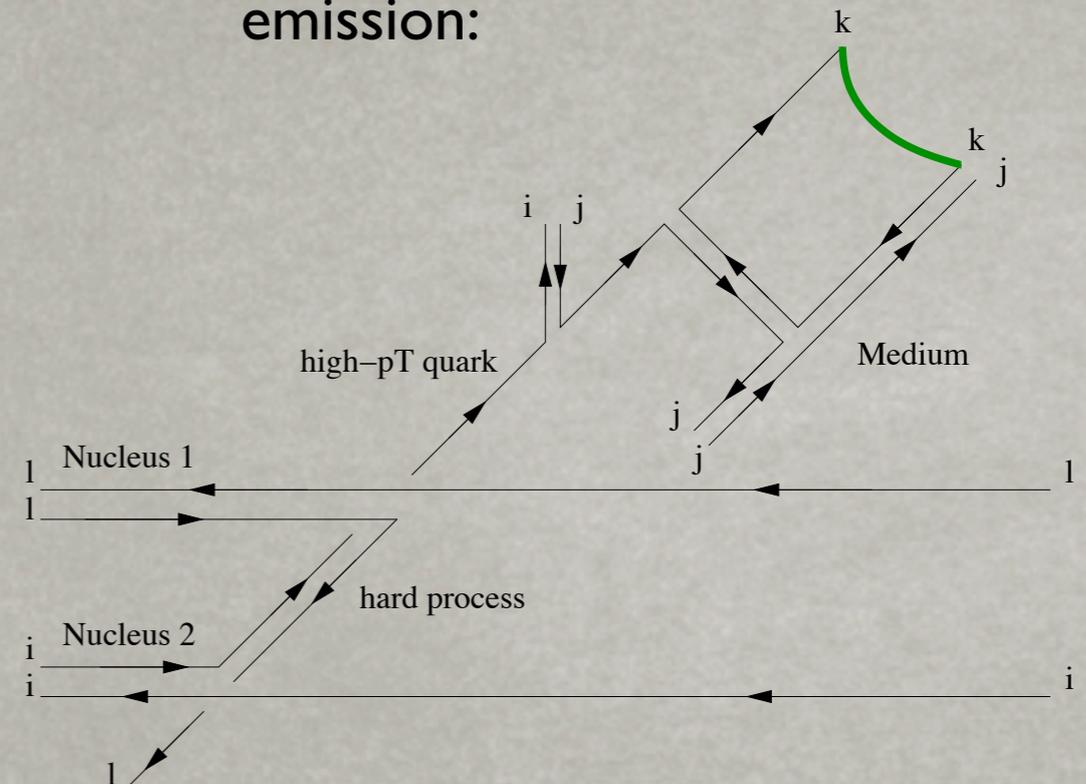
- Color flows in the presence of a medium, within Lund string model (first order in opacity expansion):
 - Two possible situations:

[1109.5025]



Induces an extra amount of energy loss
 \Rightarrow Same q_{hat} but a larger effect in the R_{AA}

- Medium interaction after emission:



**Gluon decohered from the leading string
 \Rightarrow Energy is lost**

Summary

- There has been several developments in the jet quenching theory, to build a complete picture of the shower branching in the medium.
 - Improvements in the kinematics:
 - Relaxation of approximations to account for finite energy loss, instead of soft and collinear radiation;
 - Several works within the path-integral and SCET formalisms.
 - Improvements in the parton shower evolution:
 - Identification of a probabilistic picture for $L \gg \tau_{\text{form}}$;
 - Developments in the angular structure of color correlated pairs, with a clear identification of the relevant scales that allow for a geometrical separation of the radiation;
 - Study of the modification of color connections inside a hot and dense medium;

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Relevant for particle and jet observables @ AA
Phenomenological use of these new theoretical ingredients is under development!

Thanks!

Backup slides

Color (de)coherence

Mehtar-Tani, Salgado and Tywoniuk
[1009.2965, 1102.4317, 1112.5031]

- Total gluon spectrum off an antenna in the soft limit:
 - Dominated by interference effects:

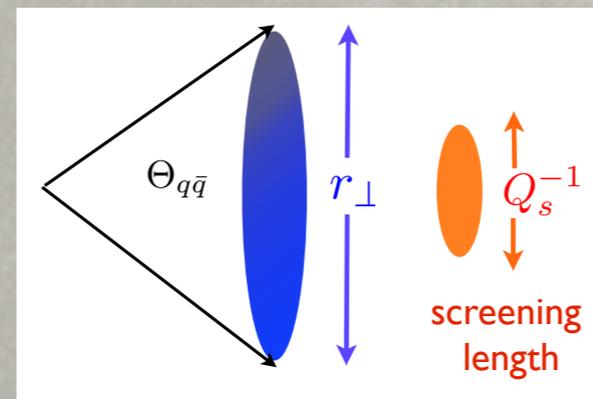
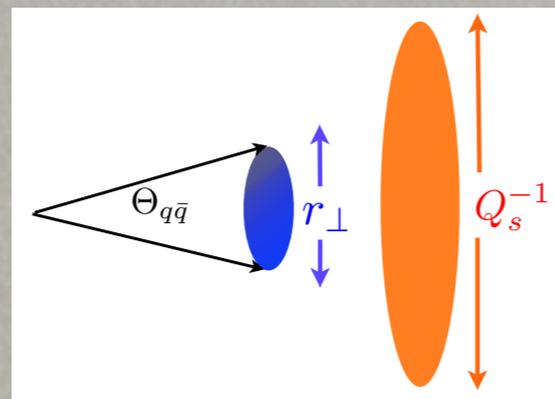
Casalderrey-Solana and Iancu
[1105.1760]

$$dN_q^{\text{tot}}|_{\omega \rightarrow 0} = \frac{\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{\sin \theta}{1 - \cos \theta} \frac{d\theta}{d\theta} \left[\Theta(\cos \theta - \cos \theta_{q\bar{q}}) + \Delta_{\text{med}} \Theta(\cos \theta_{q\bar{q}} - \cos \theta) \right]$$

$$\Delta_{\text{med}} \approx 1 - \exp\left[-\frac{1}{12} Q_s^2 r_{\perp}^2\right]$$

- Characteristic scale of the medium: $Q_s^2 = \hat{q} L$
- Transverse resolution of the antenna inside the medium: $r_{\perp} = \theta_{q\bar{q}} L$
- Two possible situations:

$\Delta_{\text{med}} \sim Q_s^2 r_{\perp}^2$
Medium probes the pair as a singlet. Gluon spectrum not much affected by the medium



$\Delta_{\text{med}} = 1$
Pair lose color coherence and radiation becomes independent (radiation out-of-cone)

- For the non-soft limit (large ω), spectrum dominated by BDMPS-Z

Massive antenna:
Armesto, Ma, Mehtar-Tani, Salgado and Tywoniuk
[1110.4343]