Discrete symmetries in quantum electrodynamics with electric and magnetic sources

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Abstract

QED with magnetic monopoles gives Maxwell's equations with dual symmetry and leads to the quantization of electric charge. However the transformation of parity P and time inversion T are no longer the symmetries of theory. Also the CP symmetry is broken. The symmetry is restored for PT and CPT transformations. These conclusions follow from the classical Maxwell's equations and the quantum field analysis of the 2-point Wightman functions in Zwanziger's model of QED. Also the general form of the Wightman function is given for an arbitrary gauge fixing condition.

Maxwell's equations with electric and magnetic sources

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho_e}{\epsilon_0},$$
$$\vec{\nabla}\cdot\vec{B} = \mu_0\rho_m,$$
$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} - \mu_0\vec{K},$$
$$\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} + \mu_0\vec{J},$$

$(\rho_e, \vec{J})$ 4-vector current of electric sources,

$(\rho_m, \vec{K})$ 4-vector current of magnetic sources

Parity transformation P - classical case

$$x = (\vec{r}, t) \xrightarrow{P} x^p = (-\vec{r}, t), \qquad a = \{e, m\}$$

$$\rho_a(x) \xmapsto{P} {}^p\rho_a(x) = \rho_a(x^p), \qquad \vec{J}_a(x) \xmapsto{P} {}^p\vec{J}_a(x) = -\vec{J}_a(x^p)$$

$$\vec{E}(x) \xmapsto{P} {}^p\vec{E}(x) = e_p\vec{E}(x^p), \qquad \vec{B}(x) \xmapsto{P} {}^p\vec{B}(x) = b_p\vec{B}(x^p)$$

$$\vec{\nabla}\times {}^p\vec{B}(x) = \mu_0\epsilon_0\frac{\partial}{\partial t}{}^p\vec{E}(x) + \mu_0{}^p\vec{J}_e(x) \quad \Longrightarrow b_p = -e_p = +1$$
$$\vec{\nabla}\times {}^p\vec{E}(x) = -\frac{\partial}{\partial t}{}^p\vec{B}(x) - \mu_0{}^p\vec{K}(x) \quad \Longrightarrow b_p = -e_p = -1$$

no parity symmetry

Time reversal transformation T - classical case

$$x = (\vec{r}, t) \xrightarrow{T} x^t = (\vec{r}, -t), \qquad a = \{e, m\}$$

$$\rho_a(x) \xmapsto{T} {}^t\rho_a(x) = \rho_a(x^t), \qquad \vec{J}_a(x) \xmapsto{T} {}^t\vec{J}_a(x) = -\vec{J}_a(x^t),$$

$$\vec{E}(x) \xmapsto{T} {}^t\vec{E}(x) = e_t\vec{E}(x^t), \qquad \vec{B}(x) \xmapsto{T} {}^t\vec{B}(x) = b_t\vec{B}(x^t)$$

$$\vec{\nabla}\times {}^t\vec{B}(x) = \mu_0\epsilon_0\frac{\partial}{\partial t}{}^t\vec{E}(x) + \mu_0{}^t\vec{J}_e(x) \Longrightarrow \qquad b_t = -e_t = -1$$
$$\vec{\nabla}\times {}^t\vec{E}(x) = -\frac{\partial}{\partial t}{}^t\vec{B}(x) - \mu_0{}^t\vec{K}(x) \Longrightarrow \qquad b_t = -e_t = +1$$

no time reversal symmetry

Charge conjugation transformation C - classical case

$$x = (\vec{r}, t) \xrightarrow{C} x^c = (\vec{r}, t), \qquad a = \{e, m\}$$

$$\rho_a(x) \xmapsto{C} {}^c\rho_a(x) = \rho_a(x^c), \qquad \vec{J}_a(x) \xmapsto{C} {}^c\vec{J}_a(x) = -\vec{J}_a(x^c),$$

$$\vec{E}(x) \xmapsto{C} {}^c\vec{E}(x) = e_c\vec{E}(x^c), \qquad \vec{B}(x) \xmapsto{C} {}^c\vec{B}(x) = b_c\vec{B}(x^c)$$

$\boxed{b_c = e_c = -1}$ for all Maxwell's equations

charge conjugation symmetry

PCT symmetry

Zwanziger Model of QED

$$F^{\mu\nu} = -n^\mu(\partial_3 A^\nu - \partial^\nu A_3) + n^\nu(\partial_3 A^\mu - \partial^\mu A_3)$$
$$-\epsilon^{\mu\nu 3\rho}(\partial_3 C_\rho - \partial_\rho C_3)$$

$$n^\mu = (0,0,0,1), \qquad \epsilon^{0123} = 1, \qquad g_{\mu\nu} = (+,-,-,-)$$

$$E_i = -\epsilon^{0ij3}(\partial_3 C_j - \partial_j C_3) \qquad E_3 = \partial_0 A_3 - \partial_3 A_0$$
$$B_i = -\epsilon^{0ij3}(\partial_3 A_j - \partial_j A_3) \qquad B_3 = \partial_0 C_3 - \partial_3 C_0$$

where $i, j = \{1, 2\}$

A_μ, C_μ are the independent gauge potentials - gauge symmetry $U(1)\times U(1)$ gauge transformations

$$A_\mu(x) \to A_\mu(x) + \partial_\mu\chi_e(x), \qquad C_\mu(x) \to C_\mu(x) + \partial_\mu\chi_g(x)$$

D.Zwanziger, *Phys.Rev.* **D 3** 880 (1971)

Discrete transformation - quantum case

$\mathcal{P}$ – unitary operator for parity

$\mathcal{T}$ – anti-unitary operator for time reversal

$\mathcal{C}$ – unitary operator for charge reversal

$$\mathcal{PCT}X_\mu(x)(\mathcal{PCT})^\dagger = -X_\mu(-x), \quad X_\mu = \{A_\mu, C_\mu\}$$
$$\mathcal{PCT}|0\rangle = \langle 0|$$

Duality transformation

$$A_\mu(x) \mapsto C_\mu(x) \mapsto -A_\mu(x)$$

Mixed Wightman functions

$$\langle 0\,|A_\mu(x)C_\nu(0)|\,0\rangle = \langle 0\,|\mathcal{PCT}A_\mu(x)C_\nu(0)(\mathcal{PCT})^\dagger|\,0\rangle =$$
$$= \langle 0\,|C_\nu(x)A_\mu(0)|\,0\rangle =$$
$$= -\langle 0\,|A_\nu(x)C_\mu(0)|\,0\rangle\,,$$

$$(n\cdot\partial)^2\epsilon^{\lambda\alpha\beta\rho}\partial_\lambda\langle 0|A_\alpha(x)\,C_\beta(0)|0\rangle = (n\cdot\partial)\langle 0|(n\cdot F)_\beta(x)\,F^{\rho\beta}(0)|0\rangle$$
$$+n^\rho\langle 0|(n\cdot F)_\beta(x)\,\partial_\lambda F^{\lambda\beta}(0)|0\rangle$$
$$\stackrel{\star}{=} 2(n\cdot\partial)^2\partial^\rho D_+(x).$$

* R. E. Peierls, *Proc. Roy. Soc.*, **A 124** 143 (1952)

If $n^2 < 0$ then we can integrate out

$$\frac{1}{2}\epsilon^{\mu\nu\lambda\rho}\partial_\lambda\langle 0|A_\mu(x)\,C_\nu(0)|0\rangle = \partial^\rho D_+(x)$$

There are no Lorentz covariant solutions**

** E.Dzimida-Chmielewska, J.A.Przeszowski, *Acta Phys. Polon.* **B 6** no.1 364 (2013)

Spherical symmetric solution

$$\langle 0|A_\mu(x)\,B_\nu(0)|0\rangle_0 = -\epsilon_{\mu\nu\alpha\beta}\bar{\partial}^\alpha\partial^\beta\Delta^{-1}\star D_+(x),$$

where

$$\bar{\partial}^\mu = \partial^\mu - t^\mu\partial_0, \qquad t^\mu = (1,0,0,0),$$
$$\Delta^{-1}(\vec{x}) = -\frac{1}{4\pi}\frac{1}{|\vec{x}|}$$

Mixed Wightman functions for arbitrary gauge

$$\langle 0|A_\mu(x)\,B_\nu(0)|0\rangle = \langle 0|A_\mu(x)\,B_\nu(0)|0\rangle_0 + \partial_\mu\phi_\nu - \partial_\nu\phi_\mu$$

ϕ_μ may depend on a gauge fixing condition and n_μ