Karlsruhe Institute of Technology

SFB TR9

β-function for the Higgs self-interaction in the Standard Model at three-loop level

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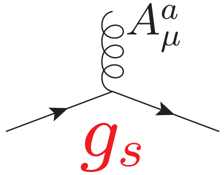
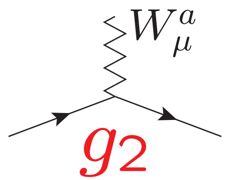
in collaboration with K. G. Chetyrkin

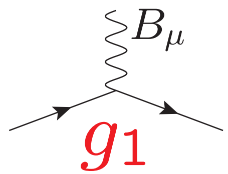
EPS HEP, Stockholm, 19th July 2013

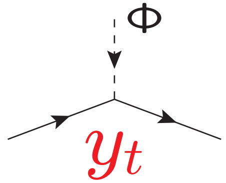
Motivation: Vacuum Stability in the SM

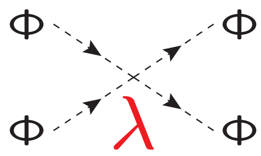
SM interactions:

- gauge couplings: QCD: 

Electroweak: 



- Yukawa couplings and Higgs self-interaction: 

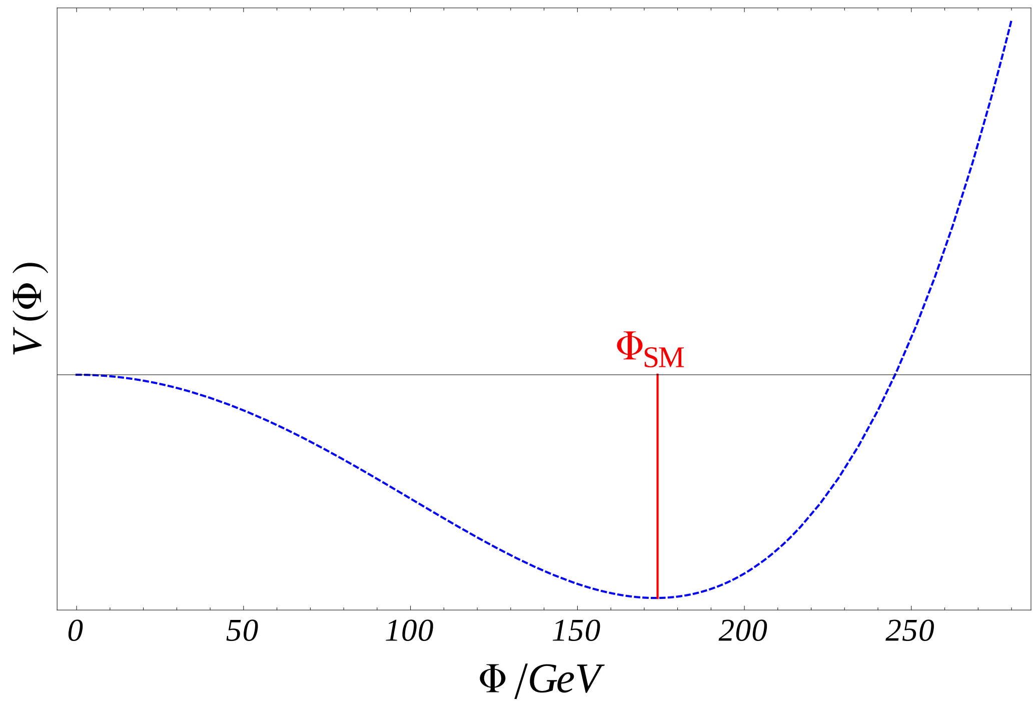


Classical Higgs potential:

$$V(\Phi) = \left(m^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2 \right)$$

$$\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \xrightarrow{\text{SSB}} \begin{pmatrix} \Phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\chi) \end{pmatrix}$$

$$|\Phi_{SM}| = \frac{v}{\sqrt{2}}, \qquad v \approx 246 \text{ GeV}$$

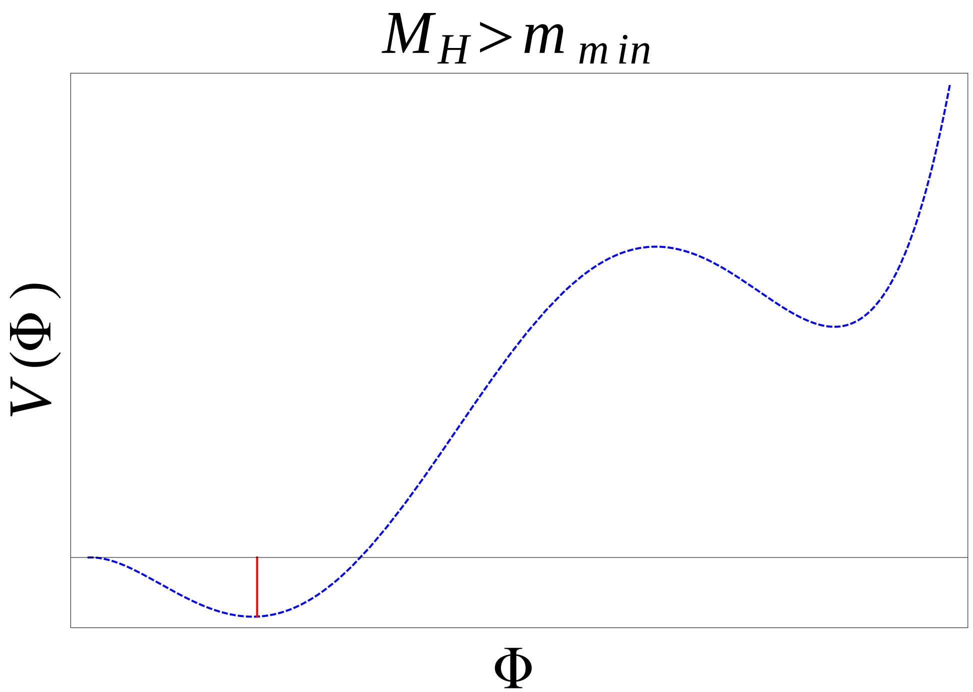


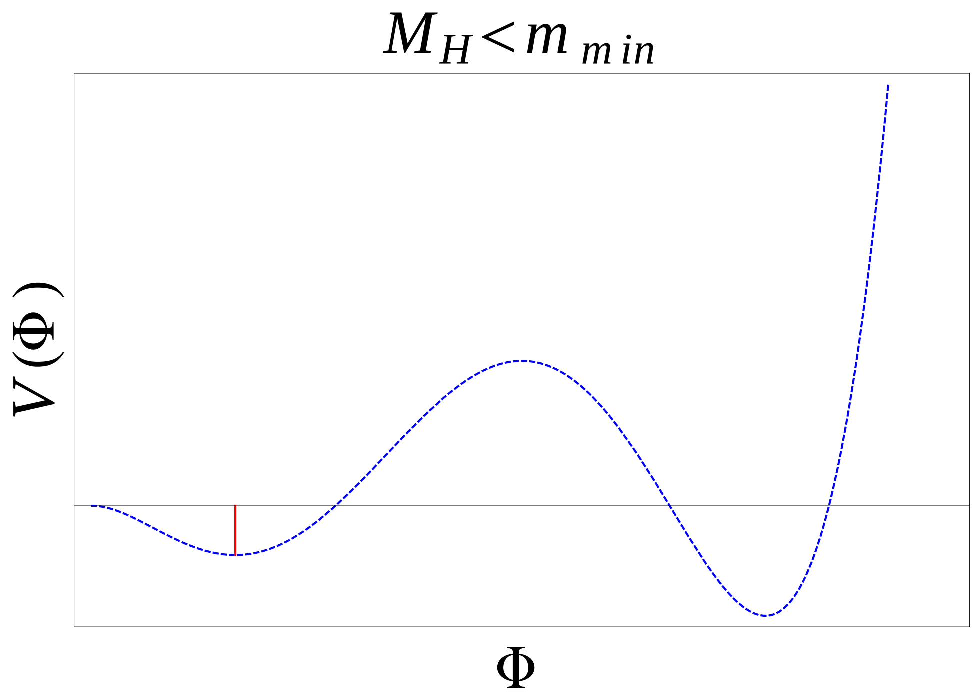
The effective Potential

QFT: Radiative corrections $\Rightarrow$ Evolution of couplings λ, g_1, g_2, g_s, y_t, ... and fields Φ, ...

Higgs potential $\rightarrow$ $V_{eff}\,(\lambda(\Lambda), g_i(\Lambda), y_t(\Lambda), \ldots)\,[\Phi(\Lambda)]$ [Coleman, Weinberg]

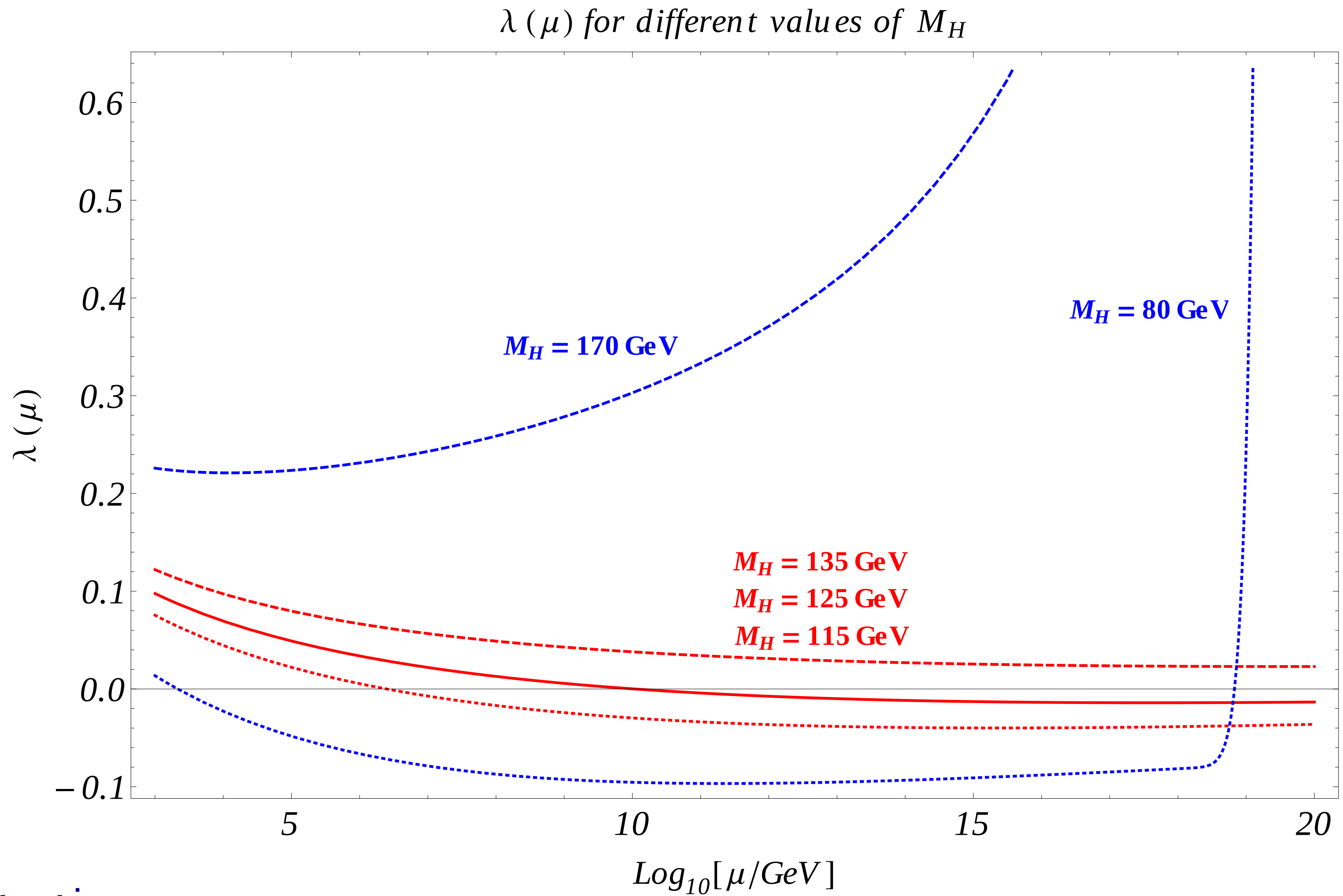
(Λ: scale up to which the SM is valid, starting scale for running e.g. $\mu_0 = M_t$)





For $\Phi \sim \Lambda \gg M_Z$: $V_{eff}[\Phi] \approx \lambda(\Lambda)\Phi(\Lambda)^4 + \mathcal{O}(\lambda^2, g_i^2)$ [Altarelli, Isidori; Ford, Jack, Jones]

Stability of SM vacuum $\Leftrightarrow$ $\lambda(\Lambda) > 0$ [Cabibbo; Sher; Lindner; Ford]



for $\Lambda = M_{Planck}$:

- Upper bound $M_H < m_{max}$: no Landau pole (up to Λ)

  $m_{max} \approx 175$ GeV [Cabibbo, Maiani, Parisi, Petronzio, Lindner, Hambye, Riesselmann]

- Stability bound on the Higgs mass: $M_H > m_{min}$

  $m_{min} \approx 129 \pm 3$ GeV (2011) [Elias-Miro, Espinosa, Giudice, Isidori, Riotto, Strumia]

Evolution of couplings $X \in \{\lambda, g_1, g_2, g_s, y_t, \ldots\}$

β-functions: $\boxed{\mu^2 \frac{d}{d\mu^2} X(\mu^2) = \beta_X[\lambda(\mu^2), y_t(\mu^2), g_i(\mu^2), \ldots]}$

$\Rightarrow$ **Coupled system of differential equations with initial conditions:**

$$
\underbrace{\begin{aligned}
\mu^2 \frac{d}{d\mu^2}\lambda(\mu^2) &= \beta_\lambda[\lambda(\mu^2), y_t(\mu^2), g_i(\mu^2)], \\
\mu^2 \frac{d}{d\mu^2}y_t(\mu^2) &= \beta_{y_t}[\lambda(\mu^2), y_t(\mu^2), g_i(\mu^2)], \\
\mu^2 \frac{d}{d\mu^2}g_s(\mu^2) &= \beta_{g_s}[\lambda(\mu^2), y_t(\mu^2), g_i(\mu^2)], \\
\mu^2 \frac{d}{d\mu^2}g_2(\mu^2) &= \beta_{g_2}[\lambda(\mu^2), y_t(\mu^2), g_i(\mu^2)], \\
\mu^2 \frac{d}{d\mu^2}g_1(\mu^2) &= \beta_{g_1}[\lambda(\mu^2), y_t(\mu^2), g_i(\mu^2)],
\end{aligned}}
\qquad
\underbrace{\begin{aligned}
\lambda(\mu_0^2) &= \lambda_0, \\
y_t(\mu_0^2) &= y_{t\,0}, \\
g_s(\mu_0^2) &= g_{s\,0}, \\
g_2(\mu_0^2) &= g_{2\,0}, \\
g_1(\mu_0^2) &= g_{1\,0}
\end{aligned}}
$$

Calculated from theory,
power series in couplings
$\Rightarrow$ Theoretical uncertainty

Experimental data matched to theoretical $\overline{\text{MS}}$-scheme
$\Rightarrow$ Experimental and theoretical uncertainty

Starting values for SM couplings

- $g_s(M_t) \approx 1.16$

- $y_t(M_t) \approx 0.94$

- $g_2(M_t) \approx 0.65$

- $g_1(M_t) \approx 0.36$

- $\lambda(M_t) \approx 0.13$

- $y_b \approx 0.02,\ y_\tau \approx 0.01$

For

$M_t \approx 173.5$ GeV,
$M_H \approx 126$ GeV,
$\alpha_s \approx 0.1184$

one-loop + $\mathcal{O}(\alpha_s^2)$ + $\mathcal{O}(\alpha\alpha_s)$ matching for

pole masses $M_t, \ldots \leftrightarrow \overline{\text{MS}}$-parameters

[Sirlin, Zucchini; Hempfling, Kniehl; Jegerlehner, Kalmykov, Veretin; Bezrukov, Kalmykov, Kniehl, Shaposhnikov]

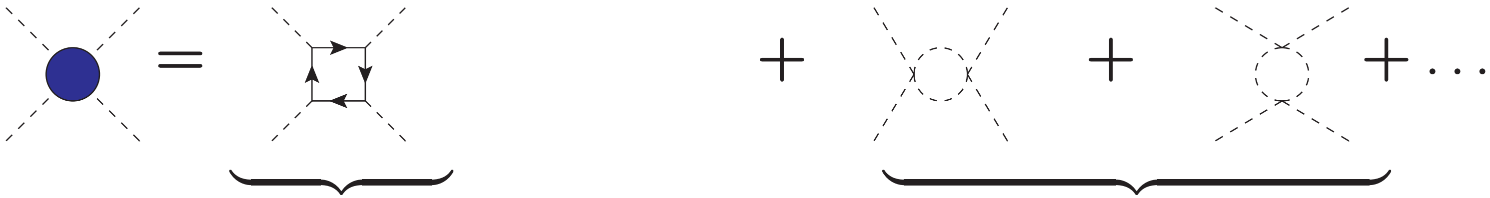
Status of β–functions in the SM

- for gauge couplings g_1, g_2, g_s:
  - 2 loop [M. Fischler, C. Hill (1981); D. Jones (1982); M. Machacek, M. Vaughn (1983); I. Jack, H. Osborn (1985)]
  - 3 loop [L. Mihaila, J. Salomon, M. Steinhauser (2012); A. Bednyakov, A. Pikelner, Velizhanin (2012)]

- for Yukawa couplings y_t, y_b, y_τ, etc.:
  - 2 loop [M. Fischler, J. Oliensis (1982); M. Machacek, M. Vaughn (1984); I. Jack, H. Osborn (1984); M. Luo, Y. Xiao (2003)]
  - 3 loop [K. Chetyrkin, M.Z. (2012); A. Bednyakov, A. Pikelner, Velizhanin (2013)]

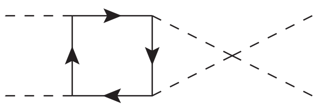
- for the Higgs self-coupling λ (and the mass parameter m^2):
  - 2 loop [M. Machacek, M. Vaughn (1985); C. Ford, I. Jack, D. Jones (1992); I. Jack, H. Osborn (1984); M. Luo, Y. Xiao (2003)]
  - 3 loop [K. Chetyrkin, M.Z. (2012 and 2013); A. Bednyakov, A. Pikelner, Velizhanin (2013)]

Calculation of $\beta_\lambda[\lambda, y_t, g_s, g_2, g_1, \ldots] = \mu^2 \frac{d}{d\mu^2}\lambda(\mu)$

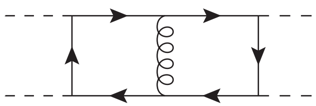


$\propto \frac{y_t^4}{(16\pi^2)}$ $\propto \frac{\lambda^2}{(16\pi^2)}$

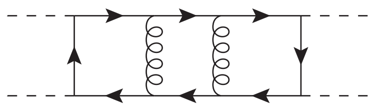
Higher orders:



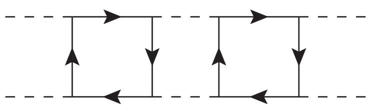
$\propto \frac{y_t^4 \lambda}{(16\pi^2)^2}$



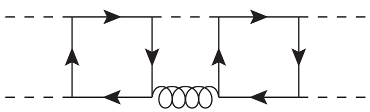
$\propto \frac{y_t^4 g_s^2}{(16\pi^2)^2}$



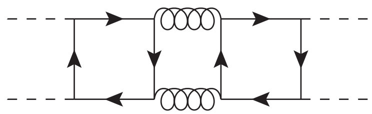
$\propto \frac{y_t^4 g_s^4}{(16\pi^2)^3}$



$\propto \frac{y_t^8}{(16\pi^2)^3}$



$\propto \frac{y_t^6 g_s^2}{(16\pi^2)^3}$



$\propto \frac{y_t^4 g_s^4}{(16\pi^2)^3}$

Challenges:

- Huge number of diagrams
  ($\sim 2.3 \times 10^6$ 1PI-diagrams with 4 ext. Φ at 3 loops)

- Treatment of $\boxed{\gamma_5 = \frac{i}{4!}\varepsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma}$ in $D = 4 - 2\varepsilon$ ['t Hooft, Veltman]

- IR divergencies
  $\Rightarrow$ auxiliary mass [Chetyrkin, Misiak, Münz]
  $\Rightarrow$ massive tadpole integrals $\Rightarrow$ MATAD [Steinhauser]

Results:

$$\mu^2 \frac{d}{d\mu^2}\lambda(\mu) = \beta_\lambda = \sum_{n=1}^{\infty} \frac{1}{(16\pi^2)^n} \beta_\lambda^{(n)}$$

(in the $\overline{\text{MS}}$-scheme)

$$\beta_\lambda^{(1)} = -y_t^4 3 + y_t^2 \lambda 6 - \lambda g_2^2 \frac{9}{2} + \lambda^2 12 + g_2^4 \frac{9}{16} - \lambda g_1^2 \frac{3}{2} + g_1^2 g_2^2 \frac{3}{8} + g_1^4 \frac{3}{16} + \lambda y_b^2 6 + \lambda y_\tau^2 2 - y_b^4 3 - y_\tau^4$$

$$\beta_\lambda^{(2)} = -g_s^2 y_t^4 16 + y_t^6 15 + g_s^2 y_t^2 \lambda 40 - y_t^2 \lambda^2 72 + y_t^2 \lambda g_2^2 \frac{45}{4} + +\lambda^2 g_2^2 54 - \lambda^3 156 + \dots$$

$$\begin{aligned}\beta_\lambda^{(3)} =& g_s^2 y_t^6 (-38 + 240\zeta_3) + y_t^8 \left(-\frac{1599}{8} - 36\zeta_3\right) + g_s^4 y_t^4 \left(-\frac{626}{3} + 32\zeta_3 + 40 N_g\right) \\ & + g_s^2 y_t^4 \lambda (895 - 1296\zeta_3) + g_s^4 y_t^2 \lambda \left(\frac{1820}{3} - 48\zeta_3 - 64 N_g\right) + y_t^4 \lambda^2 \left(\frac{1719}{2} + 756\zeta_3\right) \\ & + y_t^6 g_2^2 \left(\frac{3411}{32} - 27\zeta_3\right) + y_t^6 \lambda \left(\frac{117}{8} - 198\zeta_3\right) + \dots\end{aligned}$$

$$\beta_\lambda^{(1)}(\mu = M_t)/(16\pi^2) = -1.0 \times 10^{-2}$$

$$\beta_\lambda^{(2)}(\mu = M_t)/(16\pi^2)^2 = -2.3 \times 10^{-5}$$

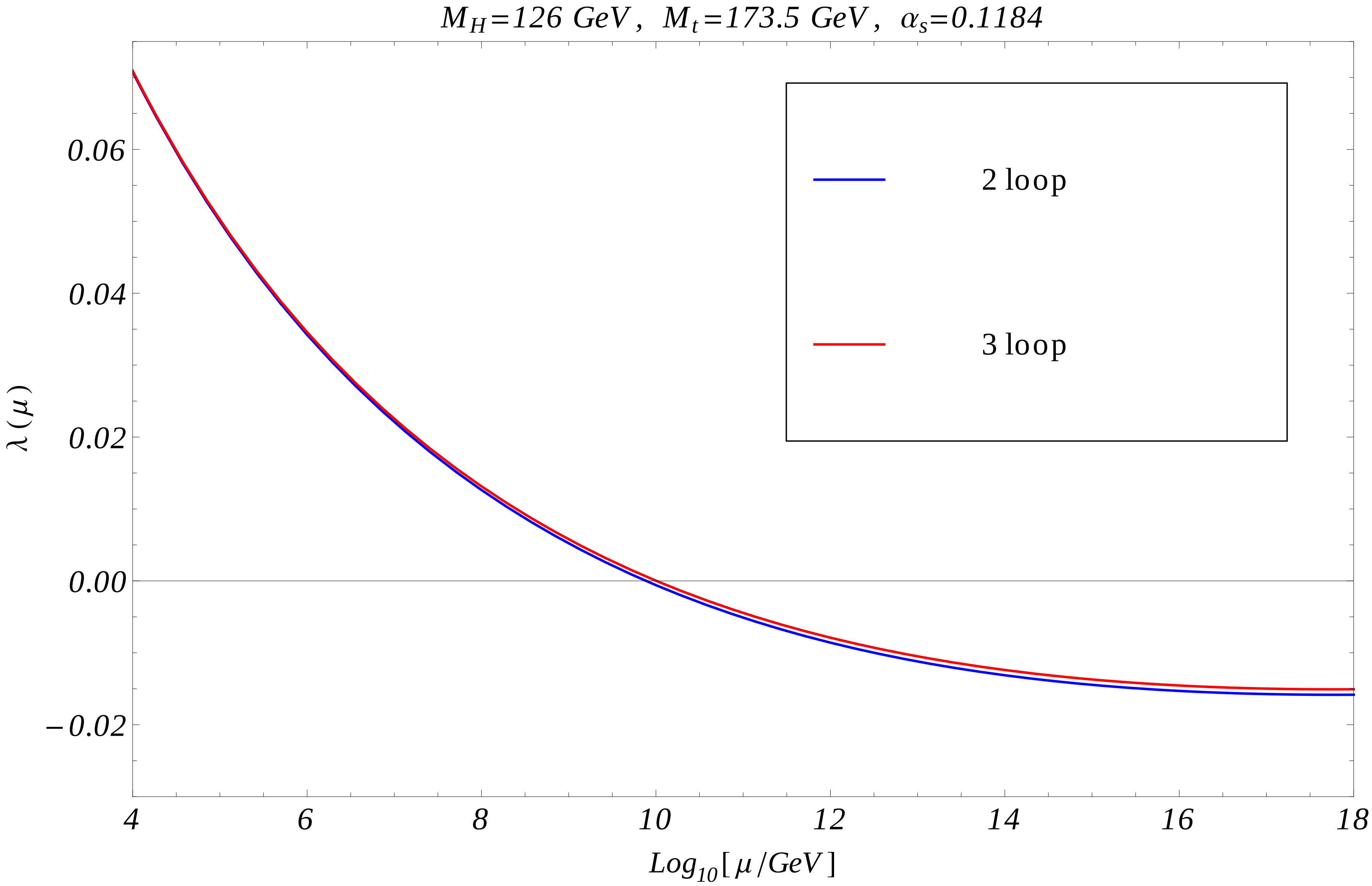
$$\beta_\lambda^{(3)}(\mu = M_t)/(16\pi^2)^3\Big|_{g_2, g_1 \to 0} = +2.1 \times 10^{-6}$$

JHEP 1206 (2012) 033
arXiv:1205.2892

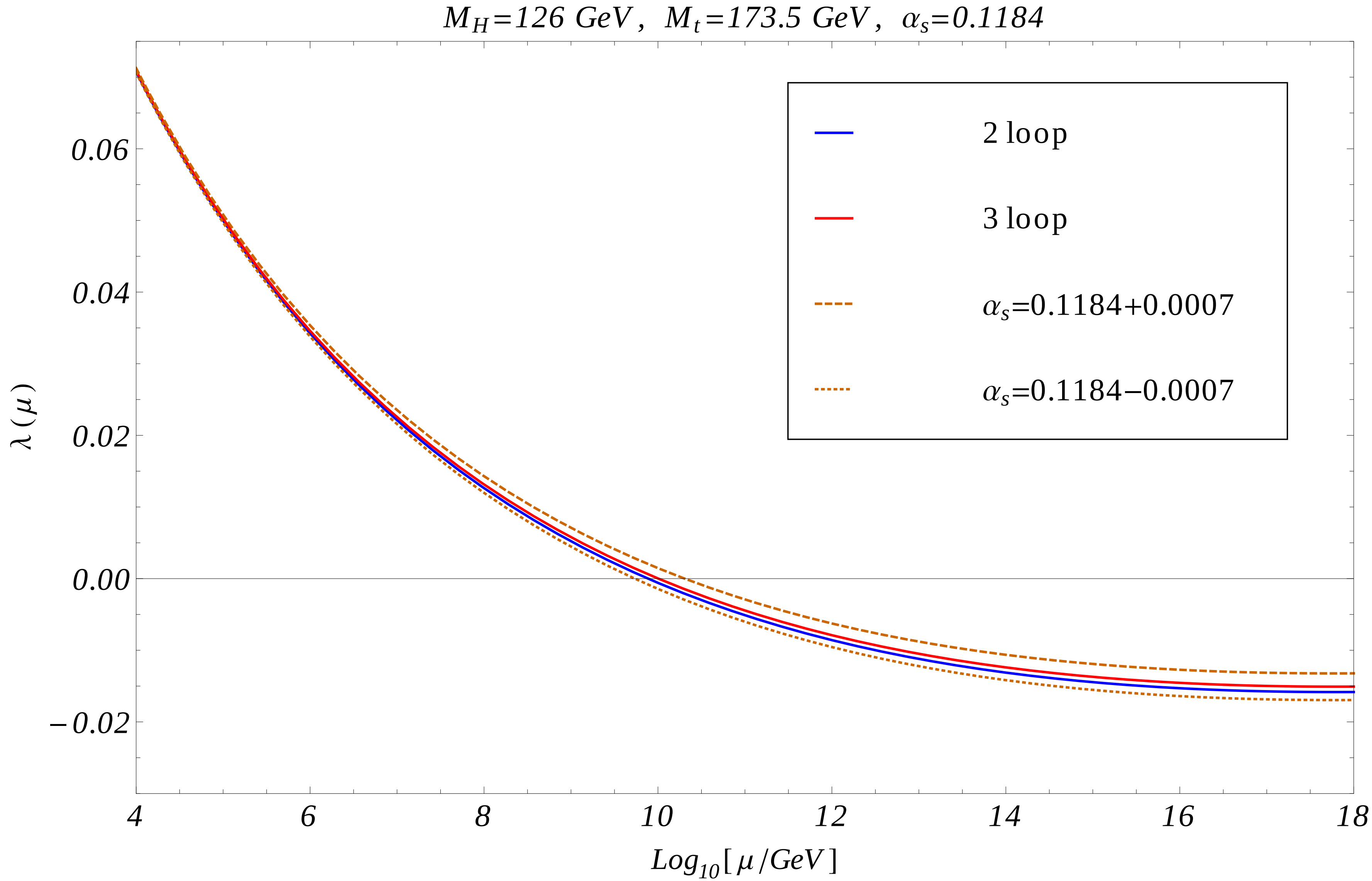
$$\beta_\lambda^{(3)}(\mu = M_t)/(16\pi^2)^3 = +1.1 \times 10^{-5}$$

JHEP 1304 (2013) 091
arXiv:1303.2890

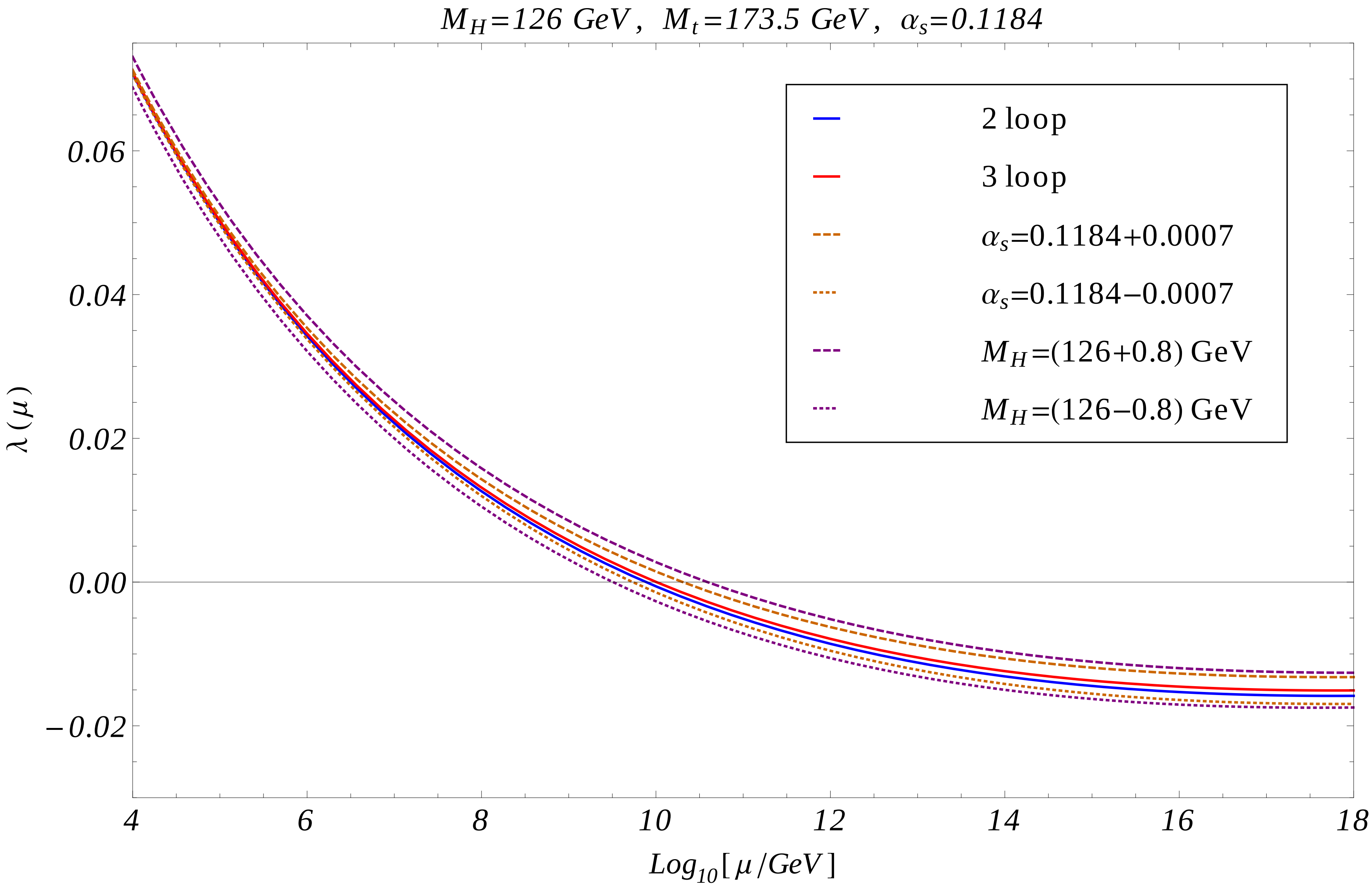
Evolution of $\lambda(\mu)$



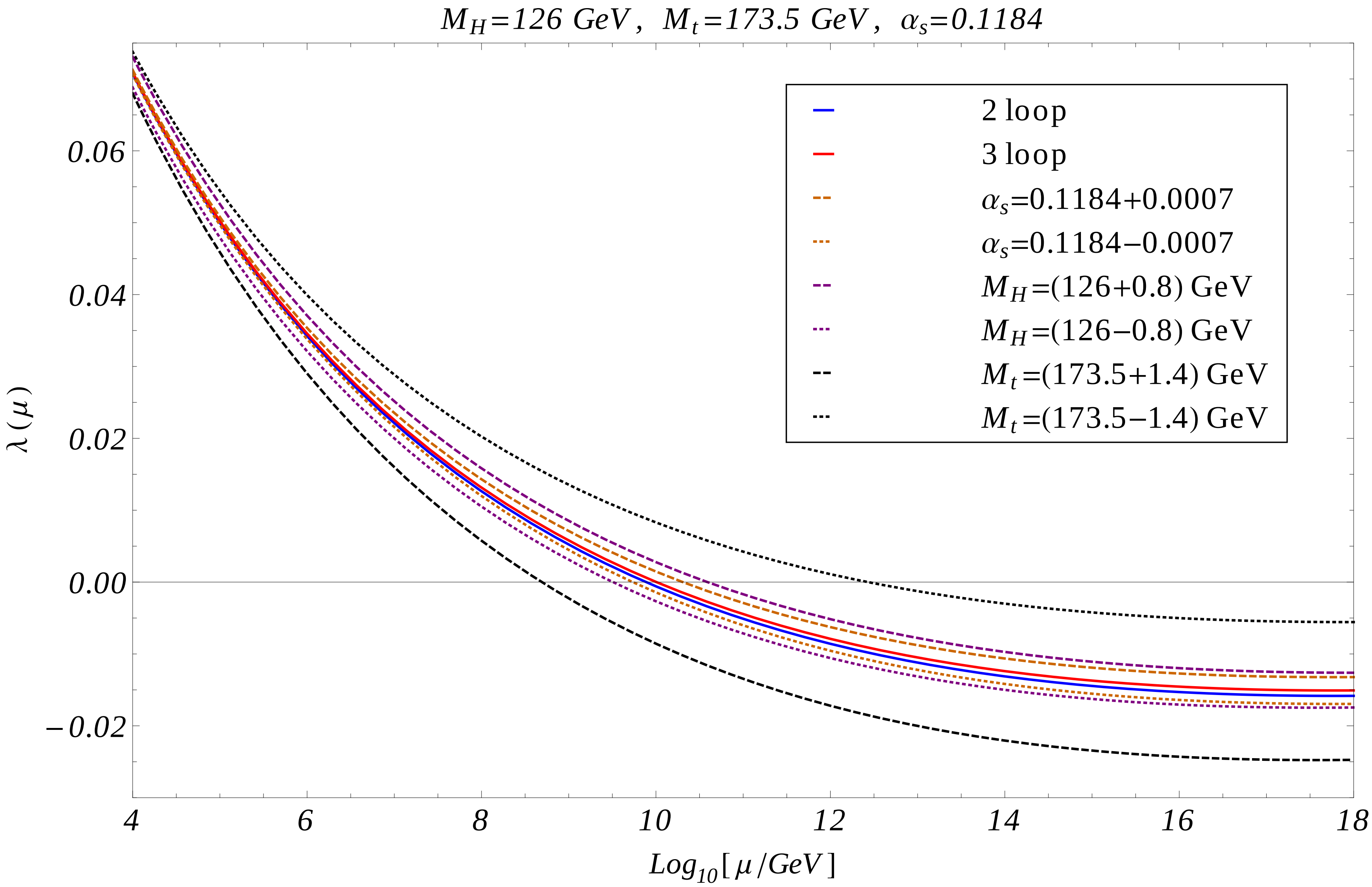
Evolution of $\lambda(\mu)$



Evolution of $\lambda(\mu)$



Evolution of $\lambda(\mu)$



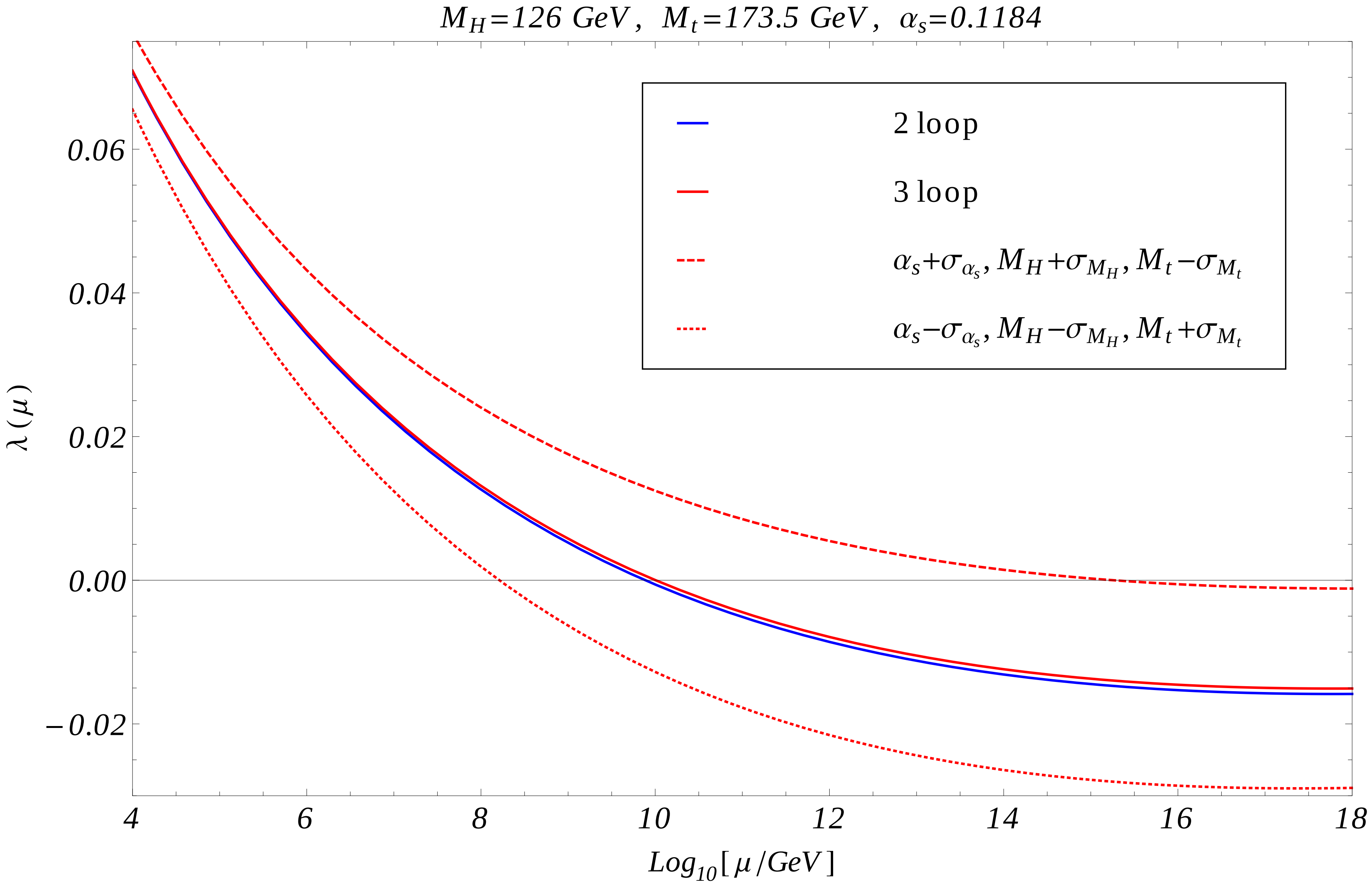
Summary

- Stability of SM vacuum $\leftrightarrow$ $\boxed{\lambda > 0}$
  3 loop result improves stability!

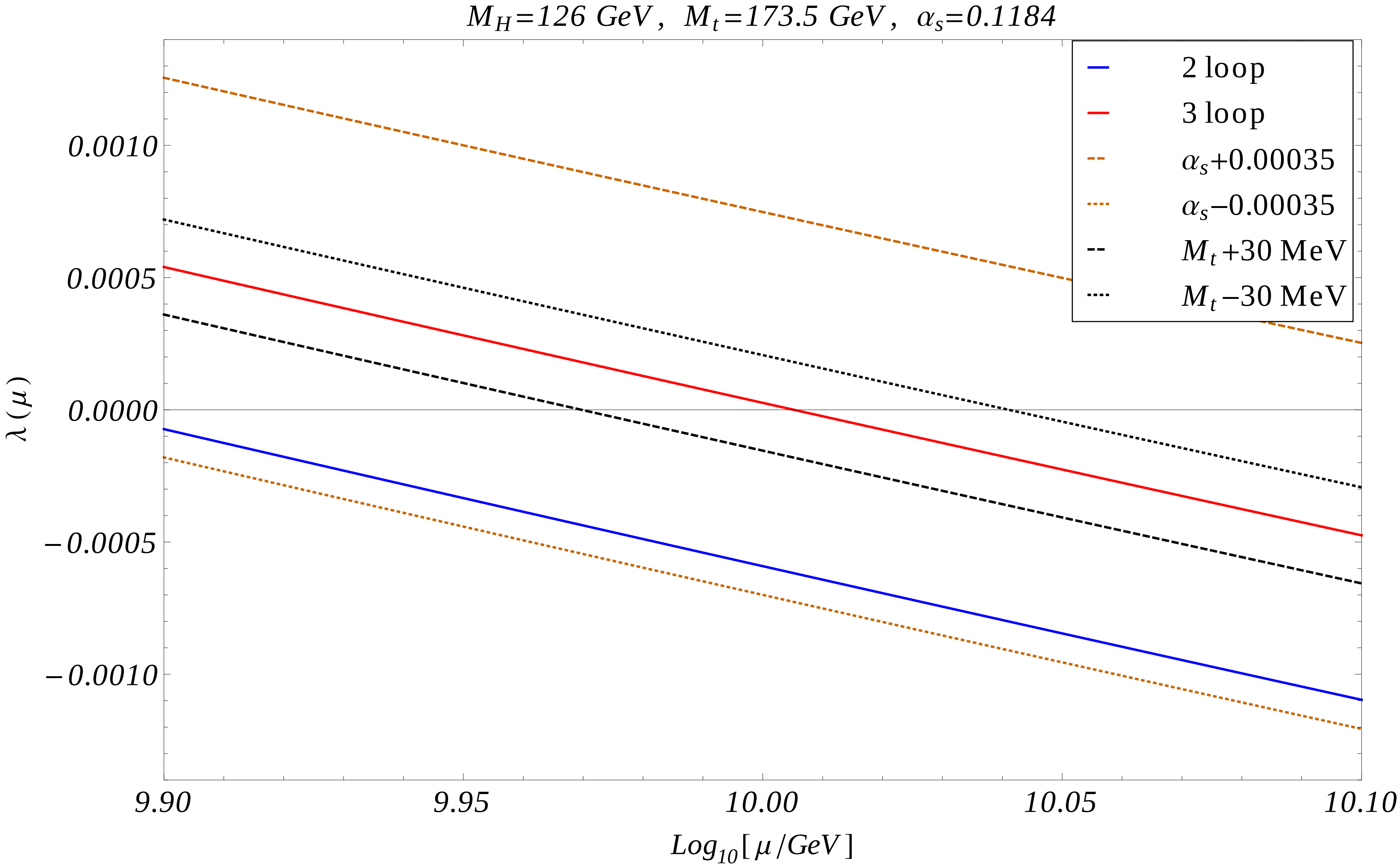
- Strong cancellation between individual contributions to β_λ at low scales
  $\Rightarrow$ β_λ well convergent for $M_H \approx 126$ GeV

- 3 loop β_λ effect smaller than experimental uncertainty
  for α_s, M_H and mainly M_t

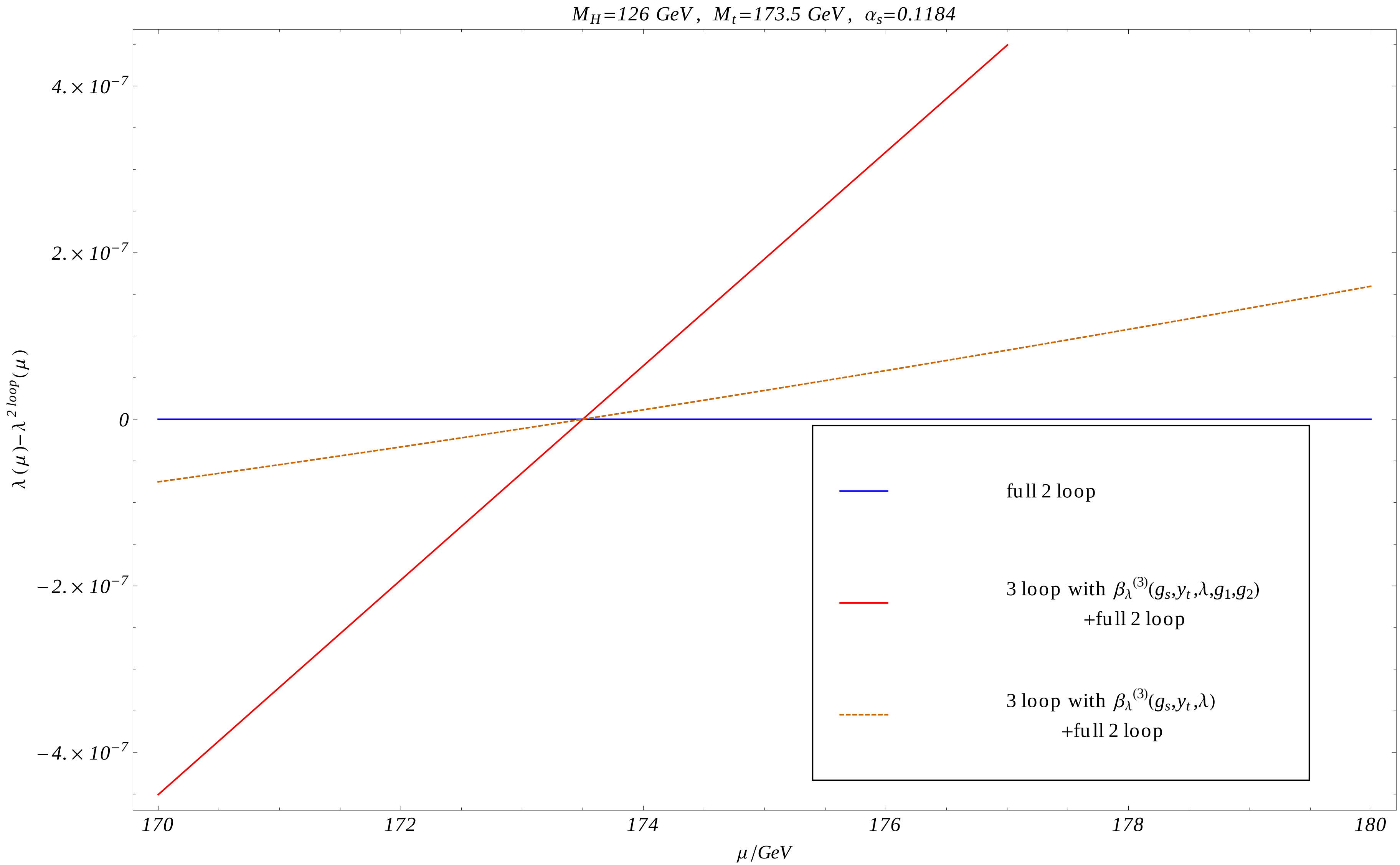
Backup I: Evolution of $\lambda(\mu)$: Combined error



Backup II: Evolution of $\lambda(\mu)$: future scenario



Backup III: Influence of EW corrections on the evolution of $\lambda(\mu)$



Backup IV: Influence of EW corrections on the evolution of $\lambda(\mu)$