MODEL INDEPENDENT $f_0(500)$ AND $f_0(980)$ MESON PARAMETERS BY PION SCALAR FORM FACTOR ANALYSIS

S. Dubnička[1], A. Z. Dubničková[2], A. Liptaj[1]

[1]Institute of Physics, Slovak Academy of Sciences, Bratislava, Slovakia
[2]Faculty of Mathematics, Physics and Informatics, Comenius University, Bratislava, Slovakia

Introduction

Scalar mesons $f_0(500)$ and $f_0(980)$ were a long time subject to a controversy, appearing, disappearing and reappearing in PDG tables under different names: σ, $f_0(400\text{-}1200)$, $f_0(600)$, ... Presently the existence of the two particles seems to be well experimentally established with, however, important uncertainties[1]. We propose a theoretical approach based on a model independent pion scalar form factor analysis which allows to determine the parameters of these two mesons.

Pion scalar form factor $\Gamma_\pi(t)$

$\Gamma_\pi(t)$: scalar function in the matrix element parametrization

$$<\pi^i(p_2)|\hat{m}(\bar{u}u+\bar{d}d)|\pi^j(p_1)>=\delta^{ij}\Gamma_\pi(t)$$

with

$$t=(p_2-p_1)^2, \quad \hat{m}=(m_u+m_d)/2.$$

It has following properties:

- Is analytic in the whole complex t-plane besides the cut on the positive real axis above $t=4m_\pi^2$.
- Obeys the so-called reality condition $\Gamma_\pi(t^*)=\Gamma_\pi^*(t)$.
- Normalization $\Gamma_\pi(0)=1$ is adopted.
- Asymptotic behavior is $\Gamma_\pi(t)_{|t|\to\infty}\sim 1/t$.
- In the elastic region $4m_\pi^2 \le t \le 16m_\pi^2$ form factor respects so-called elastic unitary condition $Im\,\Gamma_\pi(t)=M_0^0\,\Gamma_\pi^*(t)$.

In the last point M_0^0 denotes I=J=0 partial wave ππ scattering amplitude. In phase representation one obtains

$$M_0^0=e^{i\delta_0^0}\sin\delta_0^0 \implies Im\Gamma_\pi=e^{i\delta_0^0}\sin\delta_0^0\Gamma_\pi^* \implies \delta_0^0\equiv\delta_\pi$$

Our method

Dispersion relations with no and with one subtraction derived from Cauchy formula:

$$\Gamma_\pi(t)=\frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{Im\Gamma_\pi(t')}{t'-t}dt' \qquad \Gamma_\pi(t)=1+\frac{t}{\pi}\int_{4m_\pi^2}^{\infty}\frac{Im\Gamma_\pi(t')}{t'(t'-t)}dt'$$

Dispersion relations + elastic unitary condition ⇒ Omnes-Muskelishvili integral equation. Solution is known:

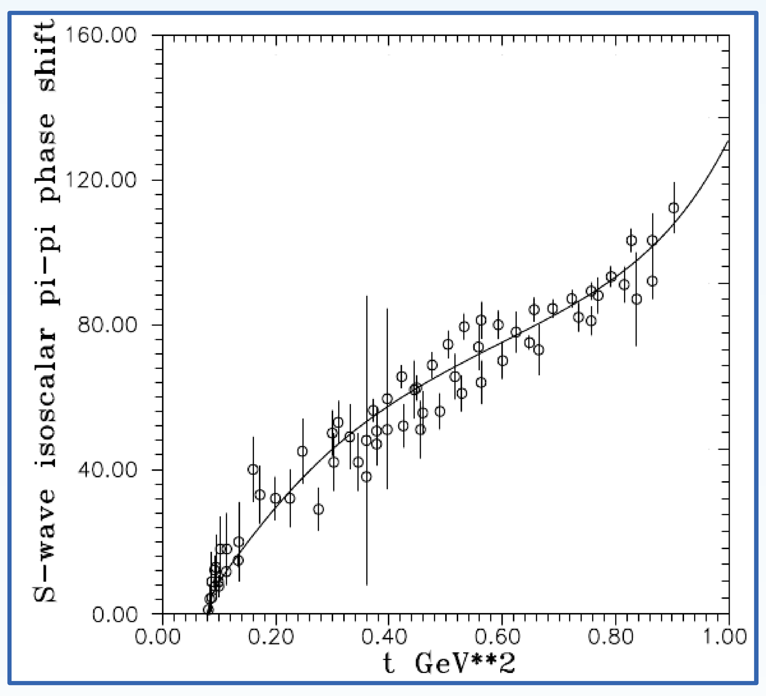
$$\Gamma_\pi(t)=P_n(t)\exp\left[\frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\delta_0^0(t')}{t'-t}dt'\right] \qquad \Gamma_\pi(t)=P_n(t)\exp\left[\frac{t}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\delta_0^0(t')}{t'(t'-t)}dt'\right]$$

with $P_n(t)$ an arbitrary but normalized (for $t=0$) polynomial.

If the phase shift δ_0^0 is known, one can find explicit form of the scalar pion form factor $\Gamma_\pi(t)$ and its poles $f_0(500)$ and $f_0(980)$.

Phase shift δ_0^0 determination

We describe the existing data points on δ_0^0



by performing a conformal mapping into q variable: $q=\sqrt{\frac{t-4}{4}} \qquad m_\pi=1$

Form factor $\Gamma_\pi(q)$ possesses in the q-plane poles and zeros only. Therefore and appropriate description can be achieved by a rational function (Padè type approximation):

$$\Gamma_\pi(t)=\frac{\sum_{n=0}^{M}a_nq^n}{\Pi_{i=1}^{N}(q-q_i)}$$

Poles q_i: 1) on imaginary axis
2) pair of poles symmetric according to imaginary axis

$\Gamma_\pi(t^*)$ - real analytic function ⇒ a_{2i} is real, a_{2i+1} is pure imaginary.

Taking into account:

- all previous statements
- threshold behavior of δ_0^0

It can be show that the phase shift takes the form:

$$\delta_0^0(t)=\arctan\frac{A_1q+A_3q^3+A_5q^5+A_7q^7+\ldots}{1+A_2q^2+A_4q^4+A_6q^6+\ldots}$$

where A_i are real coefficients (and A_1 is S-wave iso-scalar ππ scattering length a_0^0). Good fit (X^2/ndf = 1.41) achieved with 5-coefficient formula:

$A_1=0.2351\pm0.0107$; $A_2=0.2137\pm0.0283$; $A_3=0.2706\pm0.0162$;
$A_4=-0.0443\pm0.0048$; $A_5=-0.0248\pm0.0007$;

From $lim_{q\to\infty}\delta_0^0(t)=\frac{\pi}{2}$ ⇒ one-subtraction phase representation has to be used.

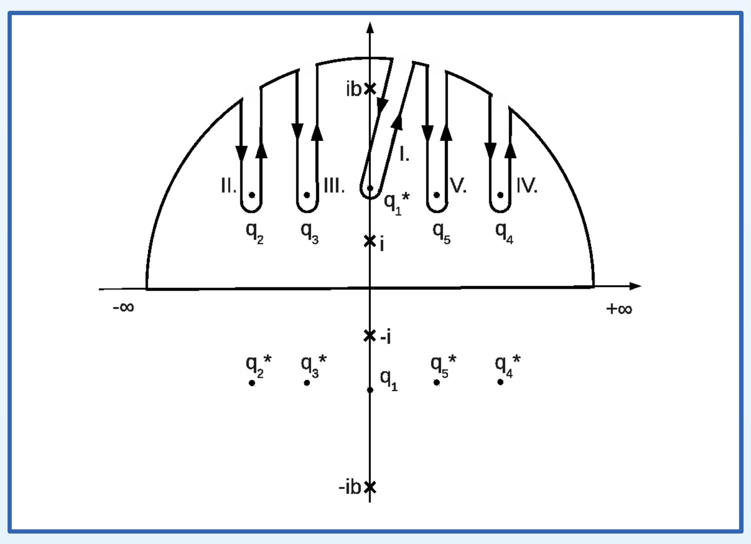
Determination of form factor $\Gamma_\pi(t)$ poles

Inserting δ_0^0 expression into dispersion relation, using the fact that integrand is a pair function and using complex identity $\arctan(z)=\frac{1}{2i}\ln\frac{1+iz}{1-iz}$ one gets

$$\Gamma_\pi(t)=P_n(t)\exp\left[\frac{q^2+1}{2\pi i}\frac{t}{\pi}\int_{-\infty}^{\infty}\frac{q'\ln\frac{(1+A_2q'^2+A_4q'^4)+i(A_1q'+A_3q'^3+A_5q'^5)}{(1+A_2q'^2+A_4q'^4)-i(A_1q'+A_3q'^3+A_5q'^5)}}{(q'^2+1)(q'^2-q^2)}dq'\right]$$

Now **theory of residues** is used. Logarithm generates branch points ⇒ roots of its denominator polynomial need to be found. Numerical analysis leads to:

$q_1=-1.863\,i$;
$q_2=-3.583+0.283\,i$;
$q_3=-1.333+1.280\,i$;
$q_4=3.583+0.283\,i$;
$q_5=1.333+1.280\,i$;



Integral (I) can be split and evaluated separately for the upper and lower half-planes: $I=I_1+I_2$

$$I_1=\int_{-\infty}^{\infty}\frac{q'\ln\frac{(q'-q_2)(q'-q_3)(q'-q_4)(q'-q_5)}{q'-q_1^*}}{(q'+i)(q'-i)(q'+ib)(q'-ib)}dq'$$

$$I_2=\int_{-\infty}^{\infty}\frac{q'\ln\frac{q'-q_1}{(q'-q_2^*)(q'-q_3^*)(q'-q_4^*)(q'-q_5^*)}}{(q'+i)(q'-i)(q'+ib)(q'-ib)}dq'$$

The contour related to the first integral is closed around upper half-plane, for the second integral it is closed around lower half-plane. To evaluate the integral we go around branch points q_i and calculate residuum in singular points (i, ib). So, in addition to the integral along the real axis, we get 5 integrals around cuts originating in branch points and two residua. The result than can be schematically written (analogically for I_2) :

$$I_1=2\pi i\sum_n Res_n-\left[-\int_{1^*}+\int_2+\int_3+\int_4+\int_5\right]$$

Results

As the last step, the integral previously evaluated is inserted into the expression of the form factor:

$$\Gamma_\pi(t)=P_n(t)\frac{q-q_1}{(q+q_2)(q+q_3)(q+q_4)(q+q_5)}\frac{(i+q_2)(i+q_3)(i+q_4)(i+q_5)}{i-q_1}$$

We get our final results by identifying the $(-q_3)$ and $(-q_2)$ poles of this expression as the scalar mesons $f_0(500)$ and $f_0(980)$:

$$\mathbf{m_{f_0(500)}=(360\pm33)\,MeV}, \qquad \mathbf{\Gamma_{f_0(500)}=(587\pm85)\,MeV}$$

$$\mathbf{m_{f_0(980)}=(957\pm72)\,MeV}, \qquad \mathbf{\Gamma_{f_0(980)}=(164\pm142)\,MeV}$$

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[1] J. Beringer *et al.* (PDG), Phys. Rev. D 86, 010001 (2012)

