Planck and LHC results for the new physics

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Outline

Current experimental and theoretical state of affairs

- Planck results and ΛCDM
- LHC results and SM

Model I – Light φ^4 non-minimally coupled inflaton

- Small non-minimal coupling and tensor modes
- From cosmology to particle physics

Model II – Higgs Inflation

- Large non-minimal coupling
- Cosmology with HI
- Radiative corrections and Higgs boson mass

Model I|I – R^2 inflation

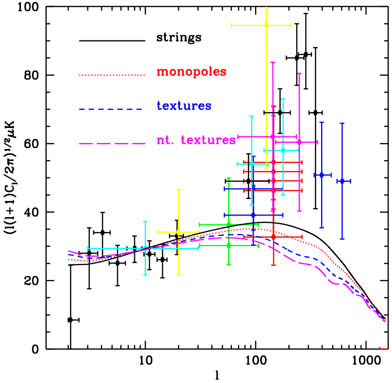
- Inflation and reheating
- Any Higgs mass is ok?

Conclusions

Prehistorical era

Before ∼97

Still analysis at primordial perturbations generated from defects

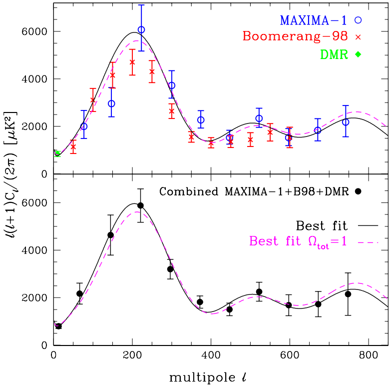


Pen, Seljak, Turok'97

Prehistorical era

Beginning of millenium

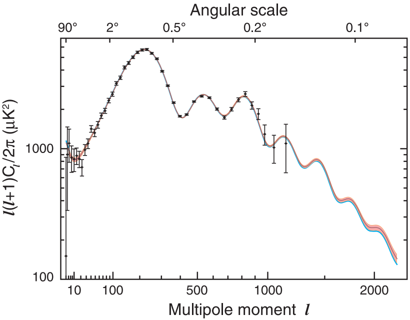
Inflation winning



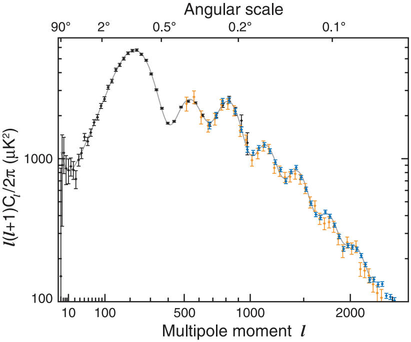
Jaffe et al.’01

Recent past

Just at the beginning of the year



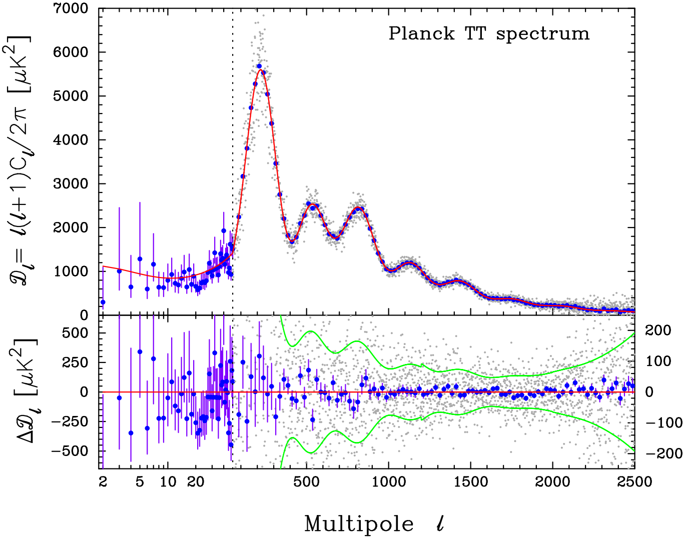
WMAP



WMAP+SPT+ACT

WMAP 9year

Glorious present



Planck’13

Inflation predicts nearly scale invariant spectra of scalar and tensor perturbations

For simple single field slow-roll inflation

- scalar density perturbations

$$\Delta^2_{\mathcal{R}}(k) = \frac{H^2(t_k)}{8\pi^2\varepsilon(t_k)} \simeq \Delta^2_{\mathcal{R}} \left(\frac{k}{k_*}\right)^{n_s-1}$$

- spectral index

$$n_s - 1 = 2\eta - 6\varepsilon$$

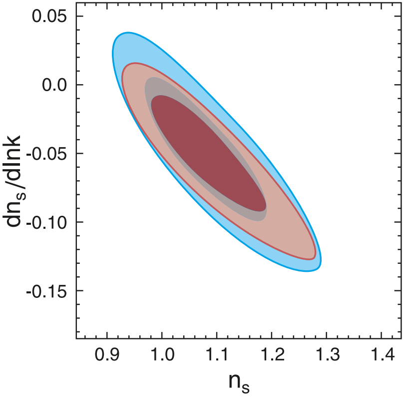
- differs from 1 by small slow-roll parameters

$$\varepsilon = -\frac{\dot{H}}{H^2} = \frac{M_P^2}{2}\left(\frac{U'}{U}\right)^2, \quad \eta = M_P^2 \frac{U''}{U}$$

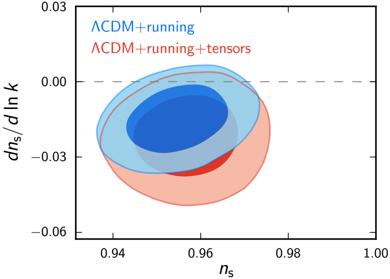
- Nearly (but not completely) scale invariant spectrum

Planck is very confident in nearly scale invariant spectrum $n_s < 1$

WMAP (5 year)



Planck



Inflation predicts nearly scale invariant spectra of scalar and tensor perturbations

For simple single field slow-roll inflation

- tensor modes (primordial gravity waves)

$$\Delta_h^2(k) = \frac{2H^2(t_k)}{\pi^2}$$

- tensor to scalar ratio

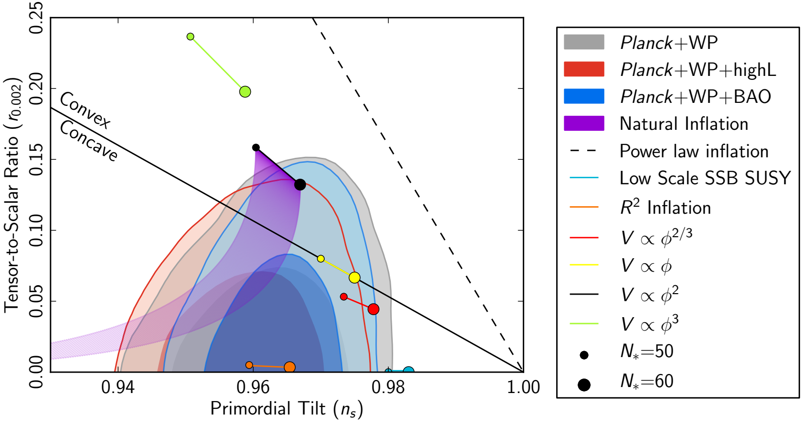
$$r = \frac{\Delta_h^2}{\Delta_{\mathcal{R}}^2} = 16\varepsilon$$

- determines the energy scale at inflation

$$U_{\text{inflation}}^{1/4} = 1.06 \times 10^{16}\,\text{GeV}\left(\frac{r}{0.01}\right)^{1/4}$$

- (measured only indirectly for the moment)

Allowed inflationary models



Planck non-gaussianities are compatible with simplest single field model

Bi-spectrum of the perturbations

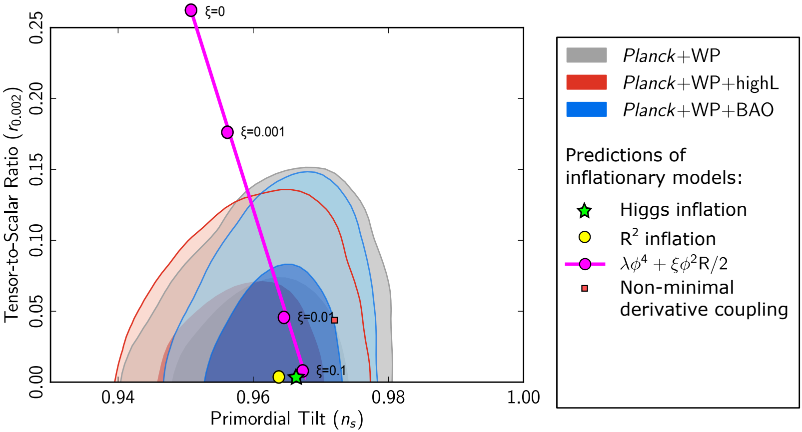
$$\langle \Phi(k_1)\Phi(k_2\Phi(k_3)\rangle = (2\pi)^3\delta(k_1+k_2+k_3)B_\Phi(k_1,k_2,k_3)$$

$$B_\Phi(k_1,k_2,k_3) = f_{NL}F((k_1,k_2,k_3)$$

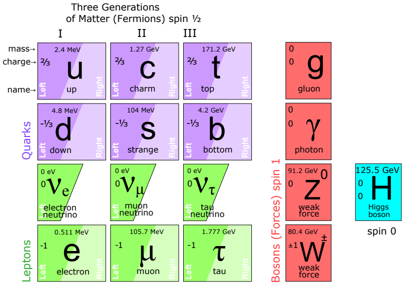
Different shapes correspond to different complicated models – multiple light fields during inflation, modified sound speed
All are compatible with zero (simplest one field model)

$$f_{\text{local}} = 2.7 \pm 5.8$$
$$f_{\text{equil}} = 42 \pm 75$$
$$f_{\text{ortho}} = 25 \pm 39$$

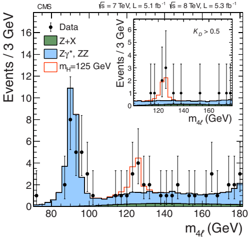
Allowed simple inflationary models

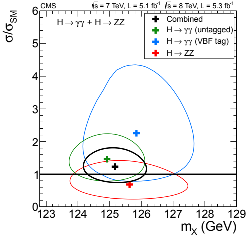


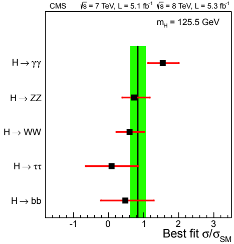
LHC is nicely compatible with the Standard Model



LHC – CMS “a Higgs boson” results





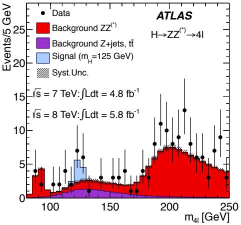


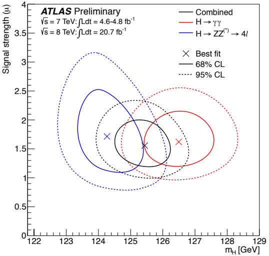
“New boson” mass

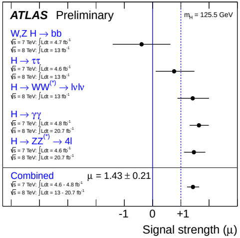
$$M_h = 125.3 \pm 0.4(\text{stat}) \pm 0.5(\text{syst})\,\text{GeV}$$

CMS’13

LHC – ATLAS "a Higgs boson" results







"New particle" mass

$$M_h = 125.5 \pm 0.2(\text{stat}) + 0.6 - 0.6(\text{syst})\,\text{GeV}$$

ATLAS'13

Minimal extensions of the SM to account for everything

Should explain everything

- Neutrino oscillations
- Dark Matter
- Baryon asymmetry of the Universe

} *ν*MSM (Oleg's talk)

- Inflation

} this talk

in a minimal way

- Introduce minimal amount of new particle/parameters
  - Simple
  - Predictive
- No new scales up to gravity/inflation
  - With scale invariance – removes hierarchy problem
  - Allows to make relations between inflation and particle physics

Outline

“Standard” chaotic inflation

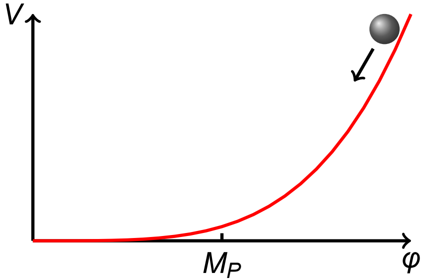
Scalar part of the action

$$S = \int d^4x\sqrt{-g}\left\{ -\frac{M_P^2}{2}R + \frac{\partial_\mu\varphi\partial^\mu\varphi}{2} - \frac{\beta}{4}\varphi^4 \right\}$$

Required to get $\delta T/T \sim 10^{-5}$

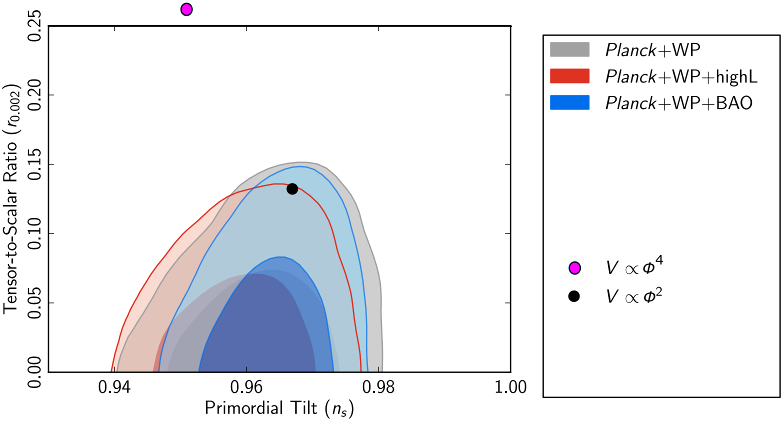
$$\beta \sim 10^{-13}$$

$$m \sim 10^{13}\,\mathrm{GeV}$$



Fields $\gtrsim M_P$, energy $\sim \beta^{1/4} M_P$.

Planck results disfavour plain φ^4 inflation



Non-minimal coupling to gravity leads to good inflation

Scalar action with non-minimal coupling

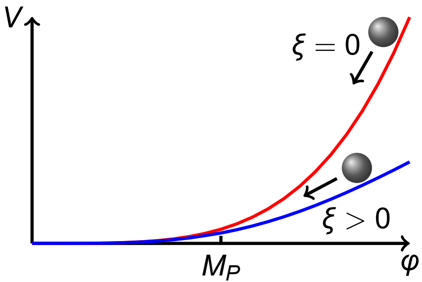
$$S = \int d^4x\sqrt{-g}\left\{ -\frac{M_P^2}{2}R - \frac{\xi}{2}\varphi^2 R + \frac{\partial_\mu\varphi\partial^\mu\varphi}{2} - \frac{\lambda}{4}\varphi^4 \right\}$$

Conformal transformation to the Einstein frame

$$\hat{g}_{\mu\nu} = \sqrt{1 + \frac{\xi\varphi^2}{M_P^2}}\, g_{\mu\nu},$$

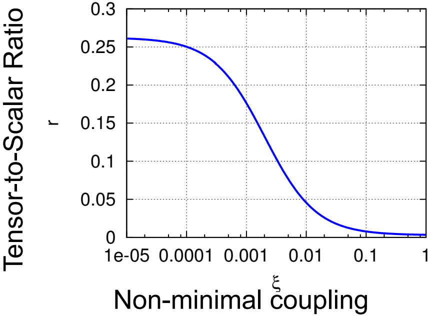
flattens the potential

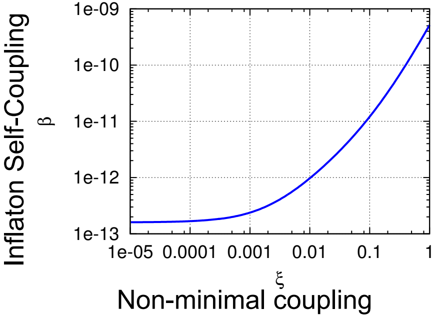
$$V(\varphi) \to \hat{V}(\varphi) = \frac{V(\varphi)}{\left(1 + \xi\varphi^2/M_P^2\right)^2}$$



(Change of the field $\frac{d\chi}{d\varphi} = \sqrt{\frac{1+(\xi+6\xi^2)\varphi^2/M_P^2}{(1+\xi\varphi^2/M_P^2)^2}}$ is also needed)

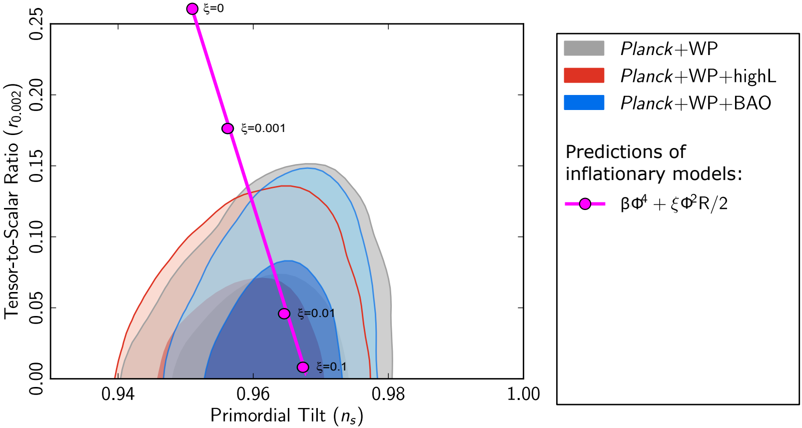
The tensor perturbations are suppressed, inflaton self-coupling β is increased





[Tsujikawa, Gumjudpai'04, FB'08, Okada, Rehman, Shafi'10]

φ^4 inflation is compatible with observations for non-minimal coupling $\xi \gtrsim 0.003$



SM + Light Inflaton coupled in the Higgs sector only

$$\mathcal{L} = \mathcal{L}_{SM} + \alpha H^\dagger H \varphi^2 + \frac{\beta}{4}\varphi^4 + \frac{\xi\varphi^2}{2} R$$

Standard Model | Interaction | Inflationary sector

Inflaton mass depends on interaction strength: $m_\chi = m_h \sqrt{\beta/2\alpha}$

Specifically: the Higgs-inflaton scalar potential is

$$V(H,\varphi) = \lambda\left(H^\dagger H - \frac{\alpha}{\lambda}\varphi^2\right)^2 + \frac{\beta}{4}\varphi^4 - \frac{1}{2}\mu^2\varphi^2 + V_0$$

We assumed here, that the scale invariance is broken *in the inflaton sector only*

[Shaposhnikov, Tkachev'06, Anisimov, Bartocci, FB'09, FB, Gorbunov'10,13]

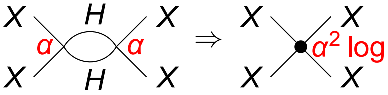
All constants of the model are bound from cosmology

CMB normalization sets $\beta(\xi)$

$$\beta = \frac{3\pi^2\,\Delta^2_{\mathcal{R}}}{2}\,\frac{(1+6\xi)(1+6\xi+8(N+1)\xi)}{(1+8(N+1)\xi)(N+1)^3}$$

$\alpha \lesssim \beta^2$ (mass lower bound)

Inflation is not spoiled by the radiative corrections



CMB tensor modes bound ξ

$$r = \frac{16(1+6\xi)}{(N+1)(1+8(N+1)\xi)} \lesssim 0.15$$

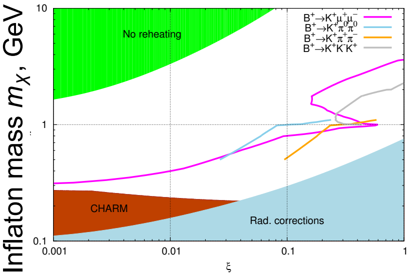
$\alpha > 10^{-7}$ (mass upper bound)

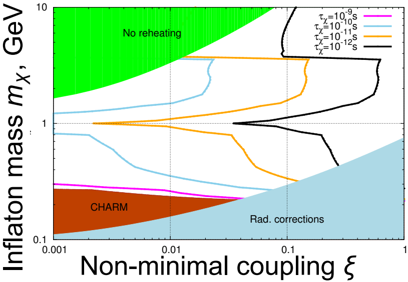
Sufficient reheating

- After inflation: empty & cold
- Needed: hot, $T_r \gtrsim 150\,\mathrm{GeV}$ (to get baryogenesis)

[Anisimov, Bartocci, FB'09]

Experimental searches are possible





Behaves as light "Higgs" boson, suppresed by $\theta = \sqrt{2\beta}v/m_\chi$

- Created in meson decays
- Decays: KK, $\pi\pi$, $\mu\mu$, ee, …
- Interacts with media: extremely weakly

Search (LHCb, Belle)

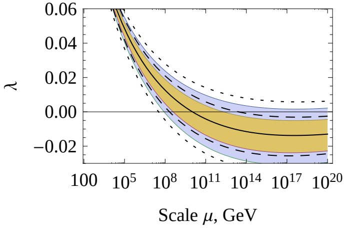
- Events with offset vertices in B decays
- Peaks in Daltiz plot of three body B decays

Another prediction: The Higgs boson can not be light

Inflation proceeds along $H^\dagger H = \frac{\alpha}{\lambda} X^2$

- The Higgs self-coupling λ: must be positive up to inflationary scales

Higgs mass M_h=125.3±0.6 GeV



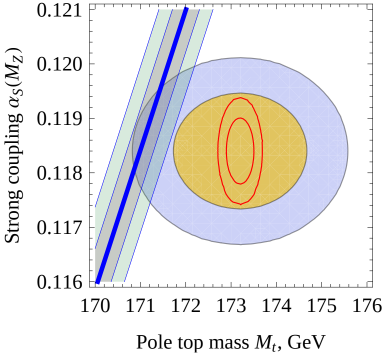
Mass for $\lambda(\mu) = \beta_\lambda(\mu) = 0$ (boundary situation)

$$M_{\rm min} = \left[129.3 + \frac{M_t - 173.2\text{GeV}}{0.95\text{GeV}} \times 1.9 - \frac{\alpha_s - 0.1184}{0.0007} \times 0.6\right] \text{GeV}$$

FB, Kalmykov, Kniehl, Shaposhnikov'12, Degrassi et.al'12, Buttazzo et.al'13

LHC Higgs mass is compatible at 2σ with stable vacuum

Higgs mass M_h=125.3±0.6 GeV



Main uncertainties

- Determination of $\overline{\text{MS}}$ y_t
  - Experimental M_t
  - Extraction of $\overline{\text{MS}}$ mass/Yukawa
- Strong coupling constant
- Higgs mass

$$M_h > M_{\rm min} = \left[129.3 + \frac{y_t(M_t) - 0.9361}{0.0055} \times 1.9 - \frac{\alpha_s - 0.1184}{0.0007} \times 0.6\right] \text{GeV}$$

Outline

With large non-minimal coupling no new particles are needed

Standard Model Higgs boson itself can be used as inflaton

Scalar part of the (Jordan frame) action

$$S_J = \int d^4x\sqrt{-g}\left\{ -\frac{M_P^2}{2}R - \xi\frac{h^2}{2}R + g_{\mu\nu}\frac{\partial^\mu h\partial^\nu h}{2} - \frac{\lambda}{4}(h^2-v^2)^2 \right\}$$

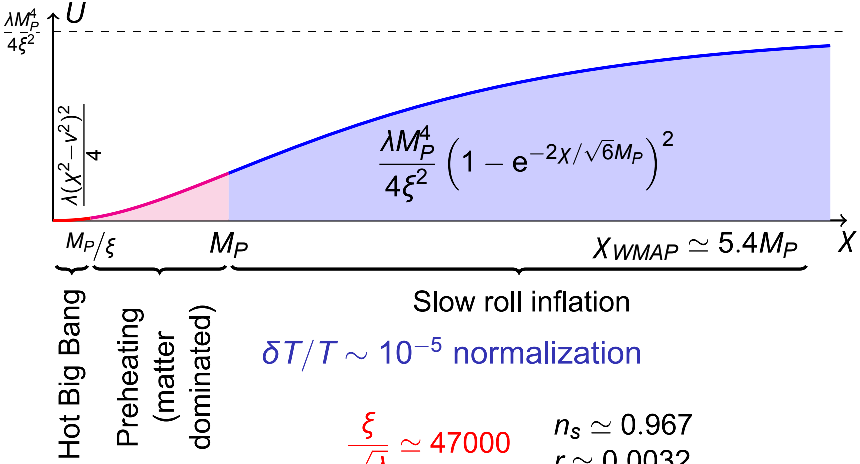
- h is the Higgs field; $M_P \equiv \frac{1}{\sqrt{8\pi G_N}} = 2.4 \times 10^{18}$ GeV
- large ξ allows for large λ

$$\frac{\xi}{\sqrt{\lambda}} \simeq 47000$$

- SM higgs vev $v \ll M_P/\sqrt{\xi}$

[FB, Shaposhnikov'08]

Potential – different stages of the Universe



$\delta T/T \sim 10^{-5}$ normalization

$$\frac{\xi}{\sqrt{\lambda}} \simeq 47000$$

$n_s \simeq 0.967$
$r \simeq 0.0032$

Reheating is very effective for the Higgs boson

Universe Evolution

- $h > M_P/\sqrt{\xi}$: Inflation
- $h \lesssim M_p/\sqrt{\xi}$: Matter dominated expansion with higgs oscillations
- $h \lesssim M_P/\xi$: Radiative dominated expansion
  - I.e. lower bound on reheating temperature

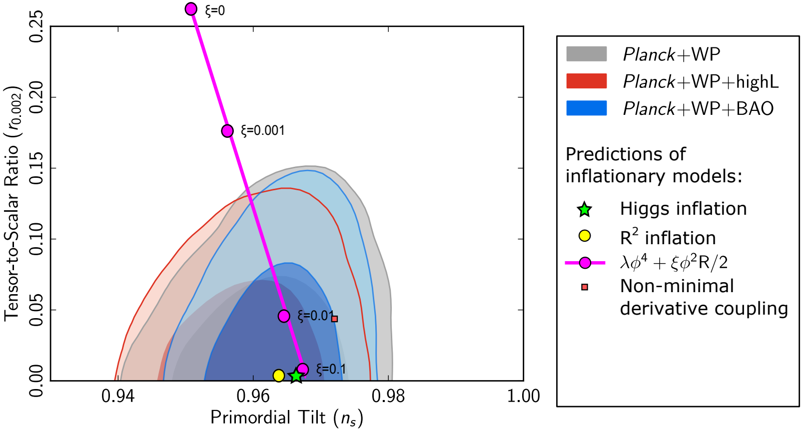
$$T_r \gtrsim \left(\frac{15}{2\pi^2 g_* \lambda}\right)^{1/4} \frac{M_p}{47000} \gtrsim 10^{13}\,\text{GeV}$$

More careful analysis may lead to higher temperatures

- Production of heavy gauge bosons when *h* crosses zero
  - Annihilation of gauge bosons into light relativistic fermions
- Production of higgs excitations at zero crossings

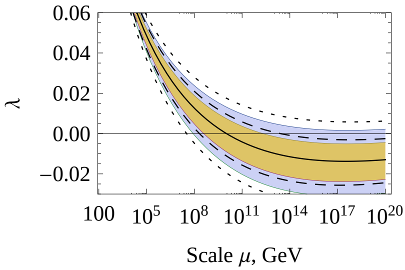
FB, Gorbunov, Shaposhnikov'08, Garcia-Bellido'08

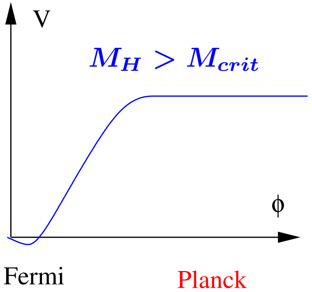
Higgs Inflation – nice in the center of the allowed region

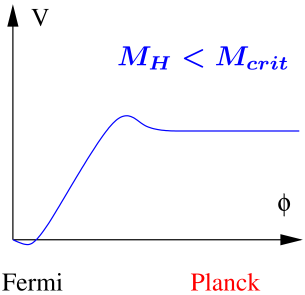


Higgs can not be too light!

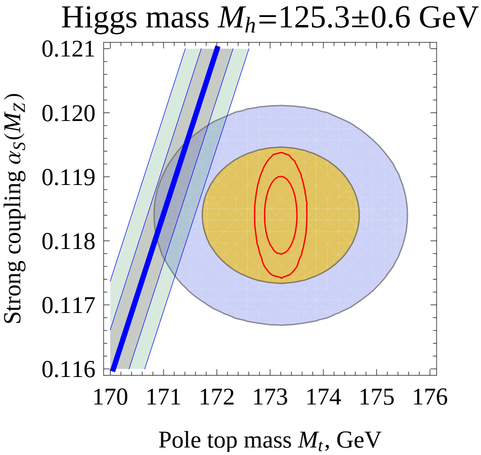
Higgs mass M_h=125.3±0.6 GeV







LHC Higgs mass is compatible at 2σ with Higgs Inflation



$$M_h > M_{\rm min} = \left[129.3 + \frac{y_t(M_t) - 0.9361}{0.0055} \times 1.9 - \frac{\alpha_s - 0.1184}{0.0007} \times 0.6\right] \text{GeV}$$

FB, Kalmykov, Kniehl, Shaposhnikov'12, Degrassi et.al'12, Buttazzo et.al'13

Outline

Modifying the gravity action gives inflation

Another way to get inflation in the SM

The first working inflationary model

[Starobinsky'80]

The gravity action gets higher derivative terms

$$S_J = \int d^4x \sqrt{-g} \left\{ -\frac{M_P^2}{2} R + \frac{\zeta^2}{4} R^2 \right\} + S_{SM}$$

Conformal transformation

conformal transformation (change of variables)

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \; , \qquad \Omega^2 \equiv \exp\left(\frac{\chi(x)}{\sqrt{6}M_P}\right)$$

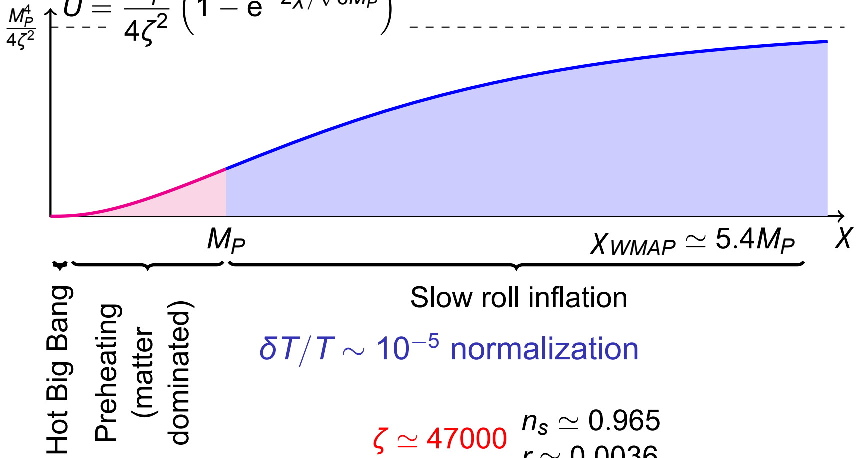
$\chi(x)$ – new field (d.o.f.) "scalaron"

Resulting action (Einstein frame action)

$$S_E = \int d^4x\sqrt{-\hat{g}}\left\{-\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu\chi\partial^\mu\chi}{2} - \frac{M_P^4}{4\zeta^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2\right\}$$

Inflationary potential

$$U = \frac{M_P^4}{4\zeta^2}\left(1 - e^{-2\chi/\sqrt{6}M_P}\right)^2$$



$\delta T/T \sim 10^{-5}$ normalization

$\zeta \simeq 47000$ $\quad n_s \simeq 0.965$, $r \simeq 0.0036$

Reheating is due to the Planck suppressed terms

Einstein frame action – χ interactions are M_P suppressed

$$S_E^{\text{scalar}} = \int d^4x \left\{ \frac{1}{2}\Omega^{-2}\partial(\Omega\hat{\varphi})\partial(\Omega\hat{\varphi}) - \frac{m_\varphi^2}{2}\Omega^{-2}\hat{\varphi}^2 \right\}$$

$$S_E^{\text{fermion}} = \int d^4x \left\{ i\bar{\hat{\psi}}\not{D}\hat{\psi} - m_\psi\Omega^{-1}\bar{\hat{\psi}}\hat{\psi} \right\} \quad \text{where } \Omega^2 \equiv \exp\left(\frac{\chi(x)}{\sqrt{6}M_P}\right)$$

Reheating temperature from the scalaron decay

$$T_r \approx 3.5\times 10^{-2} g_*^{-1/4}\sqrt{\frac{N_s}{\zeta}} \approx 3.1\times 10^9\ \text{GeV}$$

May be even smaller, if the Higgs boson is coupled conformally

Gorbunov, Panin'10, Gorbunov, Tokareva'12

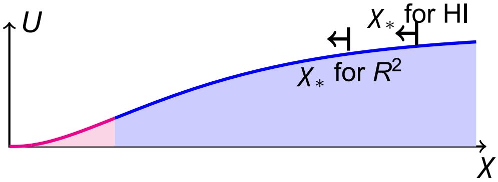
Different T_r means different moments of horizon exit

- Hubble at the Horizon exit $H_* = \frac{k}{a_0}\frac{a_0}{a_r}\frac{a_r}{a_e}e^{N_*}$

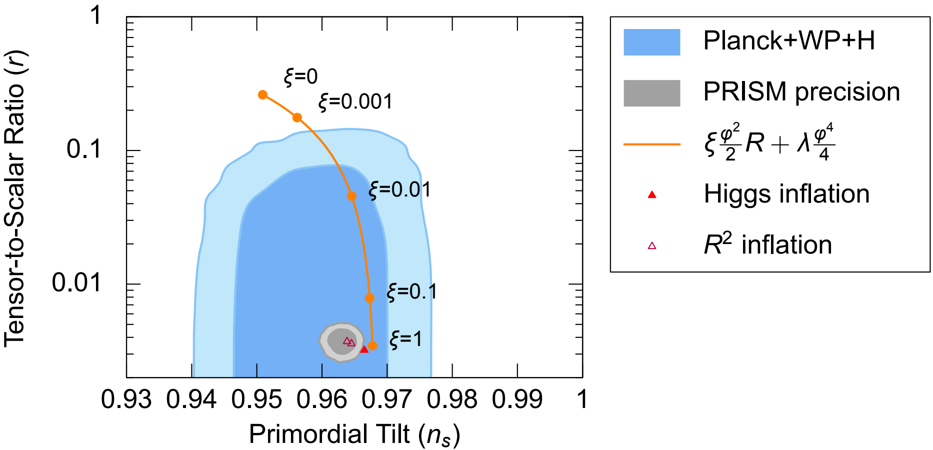
$$\frac{a_r}{a_0} = \left(\frac{g_0}{g_r}\right)^{1/3}\frac{T_0}{T_r}, \qquad \frac{a_r}{a_e} = \left(\frac{V_e}{g_r\frac{\pi^2}{30}T_r^4}\right)^{1/3}$$

- E-folding number of the hirizon exit

$$N_* \simeq 57 - \frac{1}{3}\log\frac{10^{13}\,\text{GeV}}{T_r} \quad \Rightarrow \quad N_{HI} = 57.7, \qquad N_{R^2} = 54.4$$

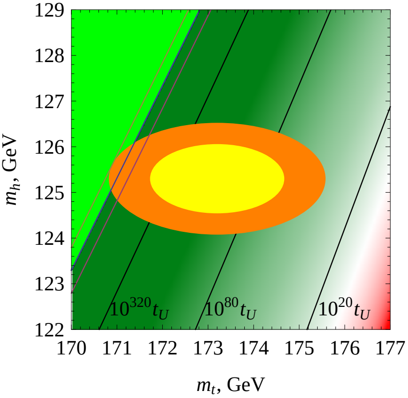


Different T_r – different CMB predictions



If the Higgs starts at electroweak vacuum, it just stays there

Even if the vacuum is metastable, it lives much longer than the Universe age



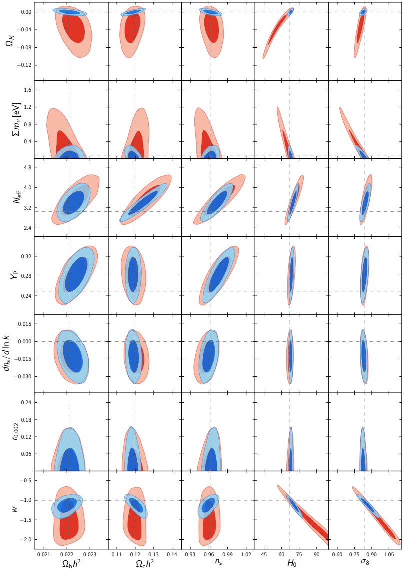
- Decay at hot stage after inflation – slightly stronger bound $m_h \gtrsim 116\,\text{GeV}$ Espinosa, Giudice, Riotto'07
- Even stronger bound for conformally coupled Higgs, $m_h \gtrsim 126.2 \pm \ldots$ Gorbunov, Tokareva'12

Outline

Conclusions

- Experiments say
  - Planck results are compatible with one field slow roll inflation with not very high energy scale
  - LHC results are compatible with Standard Model
- Simple inflationary models seem plausible
  - Higgs inflation
  - R^2 inflation
  - non-minimally coupled φ^4 inflation
- Crucial future experiments
  - CMB B-mode polarization – up to $r \sim 10^{-3}$
  - n_s running
  - Top quark mass
  - Higgs boson properties

One parameter extensions of ΛCDM



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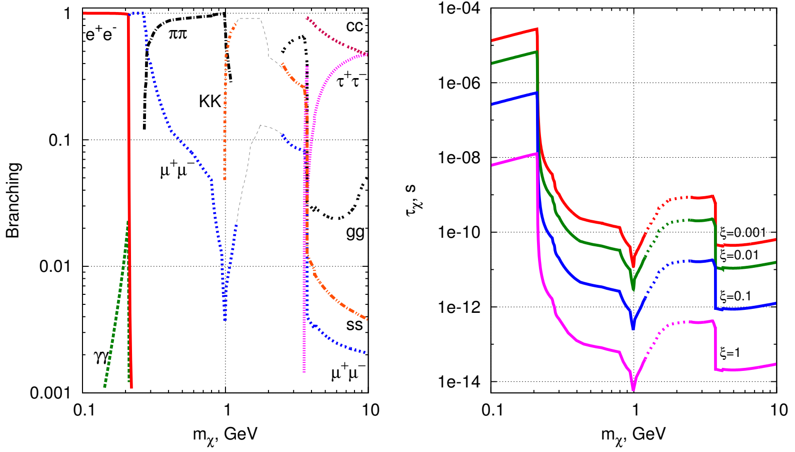
Inflaton decays and lifetime

Coupled to everything proportional particle mass



Created in meson decays:

$$\mathrm{Br}(B \to \chi X_s) \simeq 10^{-6} \frac{\beta(\xi)}{1.5\times 10^{-13}} \frac{300\ \mathrm{MeV}}{m_\chi}^2$$

Cut off is background dependent!

Classical background → Quantum perturbations

$$\chi(x,t) = \bar{\chi}(t) + \delta\chi(x,t)$$

leads to background dependent suppression of operators of dim $n > 4$

$$\frac{\mathcal{O}_{(n)}(\delta\chi)}{[\Lambda_{(n)}(\bar{\chi})]^{n-4}}$$

Example

Potential in the inflationary region $\chi > M_P$:

$$U(\chi) = \frac{\lambda M_P^4}{4\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2$$

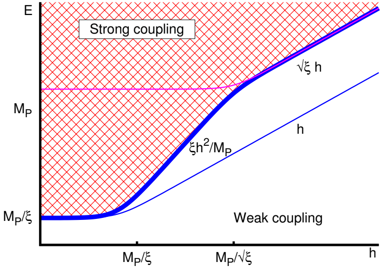
leads to operators of the form: $\frac{\mathcal{O}_{(n)}(\delta\chi)}{[\Lambda_{(n)}(\bar{\chi})]^{n-4}} = \frac{\lambda M_P^4}{\xi^2} e^{-\frac{2\bar{\chi}}{\sqrt{6}M_P}} \frac{(\delta\chi)^n}{M_P^n}$

Leading at high n to the cut-off

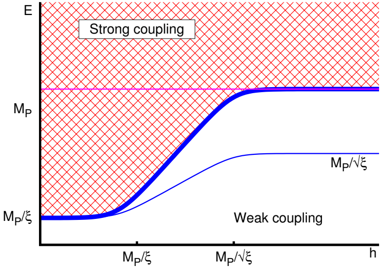
$$\Lambda \sim M_P$$

Cut-off grows with the field background

Jordan frame



Einstein frame



Relation between cut-offs in different frames:

$$\Lambda_{\text{Jordan}} = \Lambda_{\text{Einstein}} \Omega$$

Relevant scales

Hubble scale $H \sim \lambda^{1/2} \frac{M_P}{\xi}$

Energy density at inflation

$$V^{1/4} \sim \lambda^{1/4} \frac{M_P}{\sqrt{\xi}}$$

Reheating temperature $M_P/\xi < T_{\text{reheating}} < M_P/\sqrt{\xi}$

[FB, Sibiryakov, Shaposhnikov'10]

Shift symmetric UV completion allows to have effective theory during inflation

$$\begin{aligned}\mathcal{L} &= \frac{(\partial_\mu\chi)^2}{2} - U_0\left(1 + \sum u_n e^{-n\cdot\chi/M}\right) \\ &= \frac{(\partial_\mu\chi)^2}{2} - U_0\left(1 + \sum \frac{1}{k!}\left[\frac{\delta\chi}{M}\right]^k \sum n^k u_n e^{-n\cdot\bar{\chi}/M}\right)\end{aligned}$$

Effective action (from quantum corrections of loops of $\delta\chi$)

$$\mathcal{L}_{\text{eff}} = f^{(1)}(\chi)\frac{(\partial_\mu\chi)^2}{2} - U(\chi) + f^{(2)}(\chi)\frac{(\partial^2\chi)^2}{M^2} + f^{(3)}(\chi)\frac{(\partial\chi)^4}{M^4} + \cdots$$

All the divergences are absorbed in u_n and in $f^{(n)} \sim \sum f_l e^{-n\chi/M}$

UV completion requirement

Shift symmetry (or scale symmetry in the Jordan frame) is respected

$$\chi \mapsto \chi + \text{const}$$

Connection of inflationary and low energy physics requires more assumptions on the UV theory

$$\lambda U(\bar{\chi}+\delta\chi) = \lambda\left(U(\bar{\chi}) + \frac{1}{2}U''(\bar{\chi})(\delta\chi)^2 + \frac{1}{3!}U'''(\bar{\chi})(\delta\chi)^3 + \cdots\right)$$

in one loop: $\lambda U''(\bar{\chi})\bar{\Lambda}^2,\ \lambda^2(U''(\bar{\chi}))^2\log\bar{\Lambda}$,

in two loops: $\lambda U^{(IV)}(\bar{\chi})\bar{\Lambda}^4,\ \lambda^2(U''')^2\bar{\Lambda}^2,\ \lambda^3 U^{(IV)}(U'')^2(\log\bar{\Lambda})^2$,

No power law divergences are generated

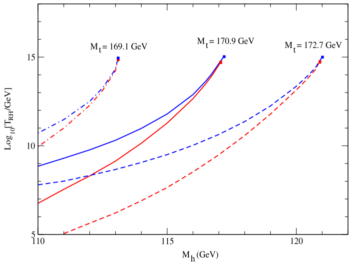
The loop corrections to the potential are arranged in a series in λ

$$U(\chi) = \lambda U_1(\chi) + \lambda^2 U_2(\chi) + \lambda^3 U_3(\chi) + \cdots$$

A rule to fix the finite parts of the counterterm functions $U_i(\chi)$

The SM vacuum should not decay at hot stage after inflation

The electroweak vacuum may decay at high temperature



[Espinosa, Giudice, Riotto'07]

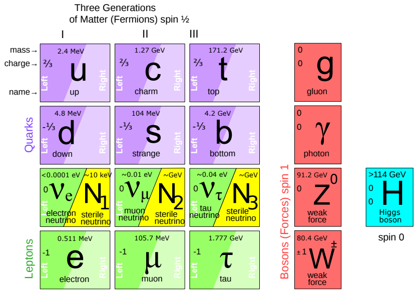
Reheating is due to M_P suppressed operators $\Rightarrow$ temperature is low $T_r \sim 10^7 - 10^9$ GeV

Higgs mass bounds in R^2 is weak

$$m_H > 116\,\text{GeV}$$

(superseded by LEP/LHC)

Dark matter – add νMSM and stir



Role of sterile neutrinos

N_1 (Warm) Dark Matter, $M_1 \sim 1$–$50\,\text{keV}$

$N_{2,3}$ Baryogenesys, $M_{2,3} \sim \ldots\text{GeV}$

http://arxiv.org/abs/hep-ph/0505013

Dark matter – add νMSM and stir

A νMSM inspired model with inflation χ

$$\mathcal{L} = (\mathcal{L}_{SM} + \bar{N}_I i\partial_\mu\gamma^\mu N_I - F_{\alpha I}\bar{L}_\alpha N_I \Phi - \frac{f_I}{2}\bar{N}_I^c N_I X + \text{h.c.}) + \frac{1}{2}(\partial_\mu X)^2 - V(\Phi, X)$$

$$\Omega_N = \frac{1.6 f(m_\chi)}{S} \cdot \frac{\beta}{1.5\times 10^{-13}} \cdot \left(\frac{M_1}{10\text{keV}}\right)^3 \cdot \left(\frac{100\,\text{MeV}}{m_\chi}\right)^3 ,$$

DM sterile neutrino mass bound

$$M_1 \lesssim 13 \cdot \left(\frac{m_\chi}{300\,\text{MeV}}\right)\left(\frac{S}{4}\right)^{1/3} \cdot \left(\frac{0.9}{f(m_\chi)}\right)^{1/3} \text{keV} .$$

[Shaposhnikov, Tkachev'06, FB, Gorbunov'10,13]