

Lattice QCD - Progress and Prospects

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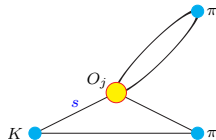
Lattice 2013: Mainz July 29th - August 3rd 2013

The 31st International Symposium on Lattice Field Theory, starting next week, has 510 participants and the following parallel sessions:

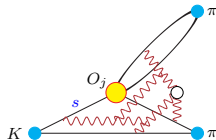
- 1 [QCD at] Non-zero temperature and density
 - 2 Theoretical developments
 - 3 Standard Model parameters and renormalization
 - 4 Algorithms and machines
 - 5 Vacuum structure and confinement
 - 6 Physics beyond the standard model
 - 7 Hadron spectroscopy and interactions
 - 8 Hadron Structure
 - 9 Weak decays and matrix elements
 - 10 Chiral symmetry
 - 11 Applications beyond QCD
 - 12 Coding efforts
- In this talk I focus on lattice computations contributing to precision flavour physics.

The rôle of flavour physics

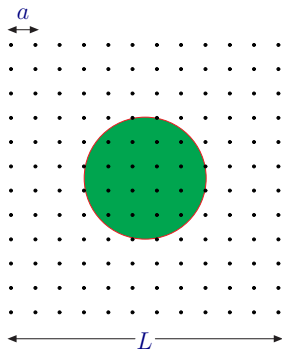
- (Precision) Flavour physics is a key tool in exploring the limits of the Standard Model of Particle Physics and in searches for new physics.
- It is complementary to high-energy experiments (most notably the LHC).
 - If, as expected/hoped the LHC experiments discover new elementary particles BSM, then precision flavour physics will be necessary to understand the underlying framework.
 - The discovery potential of precision flavour physics should also not be underestimated.
 - Precision flavour physics requires control of hadronic effects for which lattice QCD simulations are essential.



means



Lattice QCD

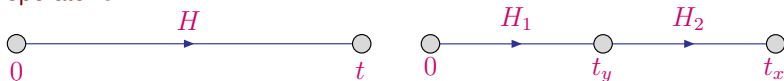


- Lattice phenomenology starts with the evaluation of correlation functions of the form:

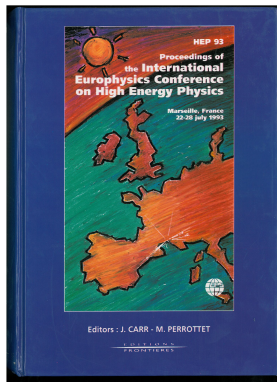
$$\langle 0 | O(x_1, x_2, \dots, x_n) | 0 \rangle = \frac{1}{Z} \int [dA_\mu] [d\psi] [d\bar{\psi}] e^{-S} O(x_1, x_2, \dots, x_n),$$

where $O(x_1, x_2, \dots, x_n)$ is a multilocal operator composed of quark and gluon fields and Z is the partition function.

- The physics which can be studied depends on the choice of the multilocal operator O .



- The functional integral is performed by discretising space-time and using Monte-Carlo Integration.



Lattice Phenomenology

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ABSTRACT

A review of the status of lattice simulations in particle physics phenomenology is presented. Recent computations of leptonic decay constants of light and heavy mesons, and of the Isgur-Wise function relevant for semi-leptonic decays of B-mesons, are discussed in some detail. Calculations of other quantities are briefly outlined. The systematic errors inherent in lattice simulations, and procedures to reduce and control them, are described.

Outline of Talk

- 1 Introduction
 - 1a Introducing the Flavour Physics Lattice Averaging Group (FLAG)
- 2 (Selected) Physics of light flavours
 - 2a *Standard Quantities*: quark masses, B_K , V_{us} and V_{ud} .
 - 2b Mini-puzzles: g_A and $\langle x \rangle_{u-d}$.
 - 2c Isospin Breaking Effects.
- 3 Extending the range of lattice calculations:
 - 3a $K \rightarrow \pi\pi$ Decays
 - 3b Prospects for the calculation of Δm_K and rare kaon decay amplitudes.
- 4 Heavy quark physics
 - 4a B -physics
 - 4b D -physics
- 5 Summary and Conclusions

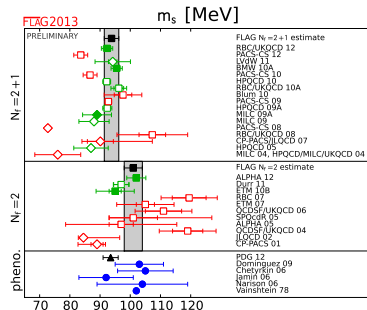
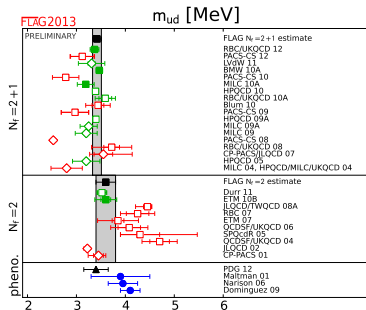
1a Flavour Physics Lattice Averaging Group (FLAG)

- Most of the compilations in this talk are taken from the current results of the FLAG collaboration:
S. Aoki, Y. Aoki, C. Bernard, T. Blum, G. Colangelo, M. Della Morte, S. Dür, A. El Khadra, H. Fukaya, A. Jüttner, R. Horsley, T. Kaneko, J. Laiho, L. Lellouch, H. Leutwyler, V. Lubicz, E. Lunghi, S. Necco, T. Onogi, C. Pena, C. Sachrajda, J. Shigemitsu, S. Simula, S. Sharpe, R. Sommer, R. Van de Water, A. Vladikas, U. Wenger, H. Wittig.
246-page web update released for Lattice 2013; full paper in the autumn.
- This is the continuation of the work of the Flavianet Lattice Averaging Group: “Review of lattice results concerning low energy particle physics,”
G. Colangelo, S. Durr, A. Juttner, L. Lellouch, H. Leutwyler, V. Lubicz, S. Necco, C. T. Sachrajda, S. Simula, A. Vladikas, U. Wenger, H. Wittig, [arXiv:1011.4408](https://arxiv.org/abs/1011.4408)
- Motivation - to present to the wider community an *average* of lattice results for important quantities obtained after a critical expert review.
- Danger - original papers (particularly those which pioneer new techniques) do not get cited appropriately by the community.

2. Selected results from the light-meson physics (FLAG)

Quark Masses: From $N_f = 2 + 1$ simulations, \overline{MS} at $\mu = 2 \text{ GeV}$

$$\frac{m_u + m_d}{2} = (3.42 \pm 0.09) \text{ MeV} \quad m_s = (93.8 \pm 2.4) \text{ MeV} \quad \frac{m_s}{m_{ud}} = 27.5 \pm 0.4$$



- Filled green points used in the averages.
- Diamonds = Pert. Normalization; Squares = Non-Pert. Renormalization.
- Precision is now such that electromagnetic and other isospin breaking effects should be included. (I will briefly return to this.)
- This is necessary for determinations of m_u and m_d separately.

ϵ_K and Neutral Kaon Mixing

$$\epsilon_K = \frac{A(K_L \rightarrow (\pi\pi)_{I=0})}{A(K_S \rightarrow (\pi\pi)_{I=0})} = e^{i\phi_\epsilon} \sin \phi_\epsilon \left[\frac{\text{Im} \langle \bar{K}^0 | H_W^{\Delta S=2} | K^0 \rangle}{\Delta m_K} + \text{L.D. effects} \right]$$

where

Buras, Guadagnoli, arXiv:0805.3887

$$\begin{aligned} |\epsilon_K| &= 2.228(11) \times 10^{-3} \\ \phi_\epsilon &= \arctan \frac{\Delta m_K}{\Delta \Gamma_K / 2} = 43.52(5)^\circ \\ \Delta m_K &= m_{K_L} - m_{K_S} = 3.4839(59) \times 10^{-12} \text{ MeV} \\ \Delta \Gamma_K &= \Gamma_S - \Gamma_L = 7.3382(33) \times 10^{-15} \text{ GeV.} \end{aligned}$$

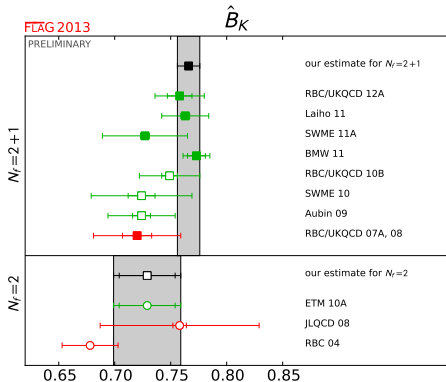
- It is conventional to present the short-distance contribution in terms of the B_K parameter:

$$\langle \bar{K}^0 | H_W^{\Delta S=2} | K^0 \rangle \propto \langle \bar{K}^0 | (\bar{s}\gamma^\mu (1 - \gamma^5)d) (\bar{s}\gamma_\mu (1 - \gamma^5)d) | K^0 \rangle \equiv \frac{8}{3} f_K^2 m_K^2 B_K(\mu).$$

- Lattice calculations of B_K have been performed since the mid 1980s. The precision is now such that the $O(5\%)$ long-distance (LD) effects have to be considered.

Buras, Guadagnoli, Isidori arXiv:1002.3612

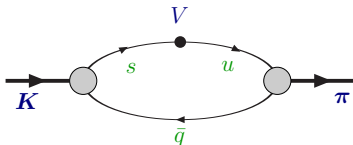
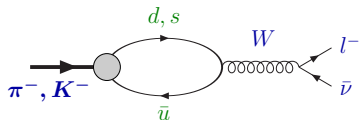
Results for B_K (FLAG)



- FLAG-2 quote from simulations with $N_f = 2 + 1$:

$$\hat{B}_K = 0.766(10) \text{ corresponding to } B_K^{\overline{\text{MS}}}(2 \text{ GeV}) = 0.560(7).$$

- The FLAG-1 result was $\hat{B}_K = 0.738(20)$ and in 1993 I quoted a summary $\hat{B}_K = 0.8(2)$.
M. Lusignoli, L. Maiani, G. Martinelli and L. Reina, Nucl.Phys. B369 (1992) 139
- The dominant contribution to $\varepsilon_K \propto |V_{cb}|^4$ and PDG(2012) quote $|V_{cb}| = (40.9 \pm 1.1) \times 10^{-3}$ error on B_K is no longer the dominant one.

V_{ud} and V_{us}


- For leptonic decays of a pseudoscalar meson P , all QCD effects are contained in a single constant, f_P , the (leptonic) decay constant:

$$\langle 0 | \bar{s}(\bar{d}) \gamma^\mu \gamma^5 u | P(p) \rangle \equiv i f_P p^\mu.$$

- For $K_{\ell 3}$ decays QCD effects are contained in form factors e.g. for $B \rightarrow \pi$ decays:

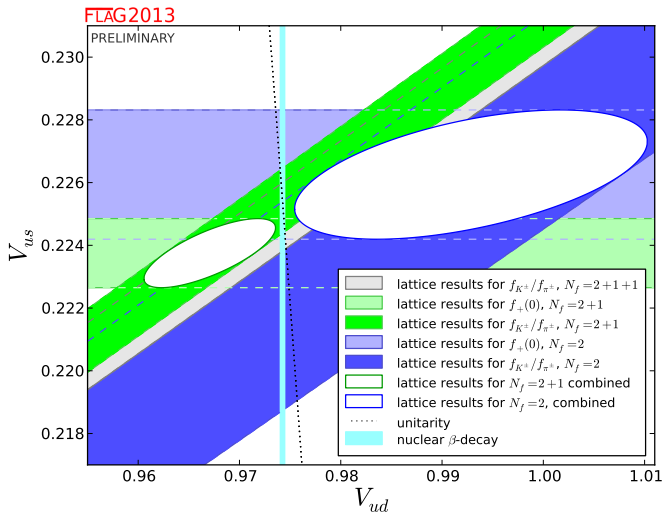
$$\langle \pi(p_\pi) | \bar{s} \gamma_\mu u | K(p_K) \rangle = f_0(q^2) \frac{m_K^2 - m_\pi^2}{q^2} q_\mu + f_+(q^2) \left[(p_\pi + p_K)_\mu - \frac{m_K^2 - m_\pi^2}{q^2} q_\mu \right]$$

where $q \equiv p_B - p_\pi$.

- We start with the very precise experimental data:

$$\frac{|V_{us}|}{|V_{ud}|} \frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.2758(5) \quad \text{and} \quad |V_{us}| f_+(0) = 0.2163(5).$$

The job of the lattice community is therefore to calculate f_K/f_π and $f_+(0)$.

V_{ud} and V_{us} (FLAG)


V_{ud} and V_{us} (FLAG)

$$(N_f = 2 + 1)$$

- For leptonic decays:

$$\frac{f_{K^\pm}}{f_{\pi^\pm}} = 1.192(5) \quad \Rightarrow \quad \frac{|V_{us}|}{|V_{ud}|} = 0.2314(11).$$

- For $K_{\ell 3}$ decays,

$$f_+(0) = 0.9667(23)(33) \quad \Rightarrow \quad |V_{us}| = 0.2238(7)(8).$$

- Unitarity of the first row of the CKM Matrix (check on quark-lepton universality of the effective G_F):

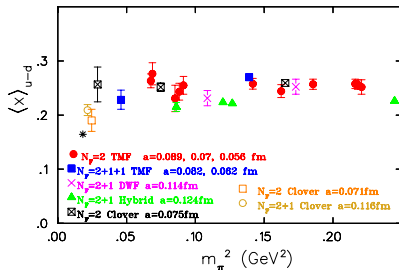
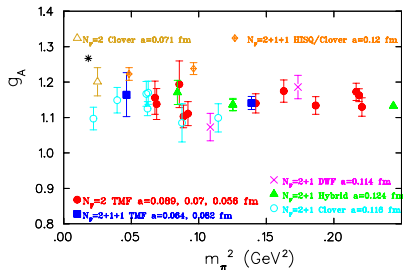
$$\begin{aligned} |V_u|^2 &\equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \simeq |V_{ud}|^2 + |V_{us}|^2 \\ &= 0.985(13) \quad (N_f = 2 + 1 \text{ results for } f_K/f_\pi \text{ and } f_+(0)) \\ &= 0.9992(6) \quad (N_f = 2 + 1 \text{ results for } f_+(0) \text{ and } |V_{ud}| = 0.97425(22)) \\ &\hspace{15em} \text{J.C.Hardy \& I.S.Towner, arXiv:0812.1202} \\ &= 1.0000(6) \quad (N_f = 2 + 1 \text{ results for } f_K/f_\pi \text{ and } |V_{ud}| = 0.97425(22)) \end{aligned}$$

- There is very little room for any discrepancy of $|V_u|^2 = 1$.

2b A couple of mini(?)-puzzles

- Is everything completely under control in computations of standard quantities?

Almost, but there are a few puzzles, particularly in baryon physics.



Courtesy of C.Alexandrou; ETMC, arXiv:1303.5979

- The lattice results are in good agreement with each other, but as $m_\pi \rightarrow m_\pi^{\text{phys}}$ g_A appears to be low and $\langle x \rangle_{u-d}$ is perhaps too high?
- The leading candidates for this discrepancy are finite-volume effects (or perhaps contamination by excited states or finite-T effects).
- Much work is being done to understand the systematic errors in these quantities.

2c Isospin Breaking (IB) Effects

- 1% precision in QCD simulations \Rightarrow IB effects ($m_u \neq m_d$ and electromagnetism) must be included.
- Until recently all the calculations were performed in the isospin limit without electromagnetism, but now the explicit calculations of IB-effects are included.
- For the evaluation of m_u and m_d separately, some results have been obtained by combining isospin-symmetric computations with continuum phenomenology.
- Defining $\Delta_\pi \equiv m_{\pi^+}^2 - m_{\pi^0}^2$ and $\Delta_P^\gamma \equiv m_P^2 - \hat{m}_P^2$ (\hat{m}_P is the mass of P in QCD alone), violations of Dashen's Theorem can be parametrized:

$$\Delta_{K^+}^\gamma - \Delta_{K^0}^\gamma - \Delta_{\pi^+}^\gamma + \Delta_{\pi^0}^\gamma = \varepsilon \Delta_\pi$$

(ε does not parametrize all IB effects.)

- Some groups take ε from phenomenological studies (e.g. MILC), others introduce quenched electromagnetism as a $U(1)$ degree of freedom (e.g. RBC, BMW) or by performing perturbation theory in α (RM123):

$$\varepsilon = 0.7(3), \quad m_u = 2.16(9)(7) \text{ MeV}, \quad m_d = 4.68(14)(7) \text{ MeV}, \quad \frac{m_u}{m_d} = 0.46(2)(2)$$

$\overline{\text{MS}}$ masses at 2 GeV, FLAG estimates

- Evaluating IB effects will (continue to) be a major activity in the coming period.

3a $K \rightarrow \pi\pi$ Decays

- The RBC-UKQCD collaboration has performed the first calculation of the $K \rightarrow (\pi\pi)_{I=2}$ amplitude A_2 : arXiv:1111.1699, arXiv:1206.5142

$$\text{Re}A_2 = (1.381 \pm 0.046_{\text{stat}} \pm 0.258_{\text{syst}}) 10^{-8} \text{ GeV}$$

$$\text{Im}A_2 = -(6.54 \pm 0.46_{\text{stat}} \pm 1.20_{\text{syst}}) 10^{-13} \text{ GeV}.$$

- The result for $\text{Re}A_2$ agrees well with the experimental value of $1.479(4) \times 10^{-8} \text{ GeV}$ obtained from K^+ decays .
- $\text{Im}A_2$ is unknown so that our result provides its first direct determination.
- Combining our result for $\text{Im}A_2$ with the experimental results for $\text{Re}A_2$, $\text{Re}A_0 = 3.3201(18) \cdot 10^{-7} \text{ GeV}$ and ϵ'/ϵ we obtain:

$$\frac{\text{Im}A_0}{\text{Re}A_0} = -1.61(19)_{\text{stat}}(20)_{\text{syst}} \times 10^{-4}.$$

(Of course, we wish to confirm this directly.)

$$\frac{\text{Im}A_0}{\text{Re}A_0} = \frac{\text{Im}A_2}{\text{Re}A_2} - \frac{\sqrt{2}|\epsilon|}{\omega} \frac{\epsilon'}{\epsilon}$$

$$-1.61(19)_{\text{stat}}(20)_{\text{syst}} \times 10^{-4} = -4.42(31)_{\text{stat}}(89)_{\text{syst}} \times 10^{-5} - 1.16(18) \times 10^{-4}.$$

$K \rightarrow \pi\pi$ Decays (cont.)

$$\operatorname{Re}A_2 = (1.381 \pm 0.046_{\text{stat}} \pm 0.258_{\text{syst}}) 10^{-8} \text{ GeV}$$

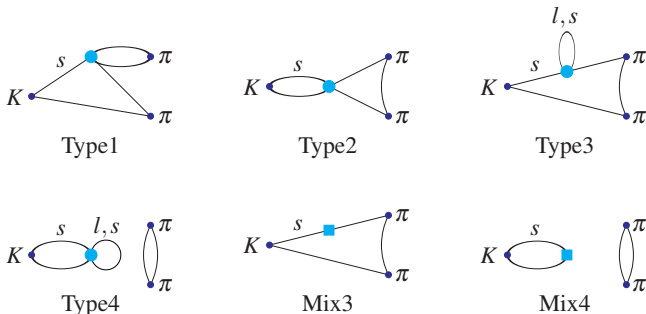
$$\operatorname{Im}A_2 = -(6.54 \pm 0.46_{\text{stat}} \pm 1.20_{\text{syst}}) 10^{-13} \text{ GeV}.$$

arXiv:1111.1699, arXiv:1206.5142

- Currently the error is dominated by *lattice artefacts*, since the calculation was performed at a single, rather coarse, lattice spacing.
 - Preliminary results at two finer lattice spacings will be presented at Lattice 2013 next week \Rightarrow this uncertainty will be reduced very significantly.
- For two-pion states, control of finite-volume and rescattering effects becomes particularly important. M.Lüscher, ...
 - For hadronic B-decays, for which inelastic intermediate states are important, we do not even know how to formulate a possible computation.
- In a finite volume, with the kaon at rest, the two-pion spectrum is discrete, with the ground state corresponding to both pions at rest.
 - $E_{\pi\pi} = m_K$ requires an excited state (possible in principle, but difficult).
 - To avoid this, we use the Wigner-Eckart theorem, relating $K^+ \rightarrow \pi^+\pi^0$ to $K^+ \rightarrow \pi^+\pi^+$ matrix elements and impose antiperiodic boundary conditions on the u quark and periodic for the remaining fields.
 - In particular with our choice of volume, the energy of the state $|\pi^+(\pi/L, \pi/L, 0)\pi^+(-\pi/L, -\pi/L, 0)\rangle \simeq m_K$.

$K \rightarrow \pi\pi$ Decays (cont.)

- The calculation is much more difficult for the $K \rightarrow (\pi\pi)_{I=0}$ amplitude A_0 :
 - The presence of disconnected diagrams:



- The efficient evaluation of disconnected diagrams is a major area of research in the lattice community.
- Breaking I -invariance by different boundary conditions for u and d quarks fatal. Even without interactions, $|\pi^+\pi^-\rangle$ and $|\pi^0\pi^0\rangle$ have different energies.

$K \rightarrow \pi\pi$ Decays (cont.)

- RBC-UKQCD have computed A_0 with the two pions at rest and with unphysical masses, finding e.g. [arXiv:1106.2714](https://arxiv.org/abs/1106.2714), Qi Liu Columbia Un.Thesis

$$\frac{\text{Re}A_0}{\text{Re}A_2} = 9.1 \pm 2.1 \quad 877 \text{ MeV kaon decaying into two } 422 \text{ MeV pions}$$
$$\frac{\text{Re}A_0}{\text{Re}A_2} = 12.0 \pm 1.7 \quad 662 \text{ MeV kaon decaying into two } 329 \text{ MeV pions}$$

- Whilst both these results are obtained at unphysical kinematics and are different from the physical value of 22.5, it is nevertheless interesting to understand the origin of these enhancements.
- 99% of the contribution to the real part of A_0 and A_2 come from the matrix elements of the current-current operators.
- For a calculation of ϵ'/ϵ at physical kinematics, RBC-UKQCD are developing G-parity boundary conditions (estimate timescale ~ 2 years).

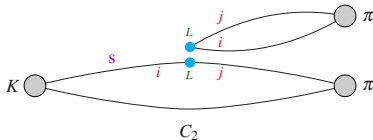
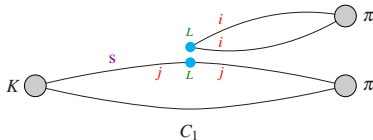
“Emerging understanding of the $\Delta I = \frac{1}{2}$ rule from Lattice QCD”

RBC-UKQCD Collaboration, arXiv:1212.1474

- $\text{Re}A_2$ is dominated by a simple operator:

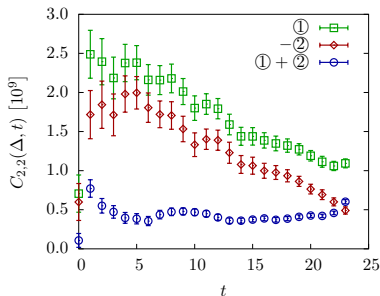
$$O_{(27,1)}^{3/2} = (\bar{s}^i d^i)_L \{ (\bar{u}^j u^j)_L - (\bar{d}^j d^j)_L \} + (\bar{s}^i u^i)_L (\bar{u}^j d^j)_L$$

and two diagrams:

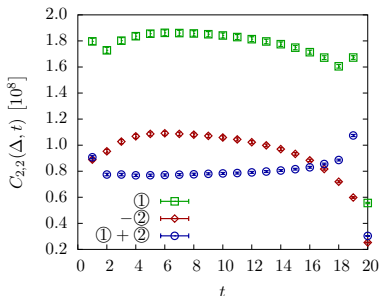


- $\text{Re}A_2$ is proportional to $C_1 + C_2$.
- The contribution to $\text{Re}A_0$ from Q_2 is proportional to $2C_1 - C_2$ and that from Q_1 is proportional to $C_1 - 2C_2$ with the same overall sign.
- Colour counting might suggest that $C_2 \simeq \frac{1}{3}C_1$.
- We find instead that $C_2 \approx -C_1$ so that A_2 is significantly suppressed!
- We believe that the strong suppression of $\text{Re}A_2$ and the (less-strong) enhancement of $\text{Re}A_0$ is a major factor in the $\Delta I = 1/2$ rule.

Evidence for the Suppression of $\text{Re} A_2$



Physical Kinematics



$m_\pi \simeq 330$ MeV at threshold.

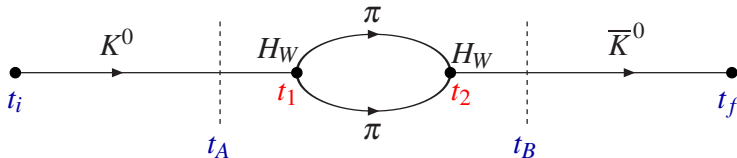
- Notation $\textcircled{i} \equiv C_i$, $i = 1, 2$.
- Of course before claiming a quantitative understanding of the $\Delta I = 1/2$ rule we need to compute $\text{Re} A_0$ at physical kinematics and reproduce the experimental value of 22.5.
- Much early phenomenology was based on the vacuum insertion approach; although the qualitative picture we find had been suggested by Bardeen, Buras and Gerard in 1987.

3b Long Distance Effects in Kaon Physics

- RBC-UKQCD collaboration is developing the theoretical and computational framework for long-distance effects in $\Delta m_K = m_{K_L} - m_{K_S}$ and in rare-kaon decays.
- This requires the evaluation of correlation functions of the form:

$$\int d^4x \int d^4y \langle h_2 | T \{ O_1(x) O_2(y) \} | h_1 \rangle,$$

where $h_{1,2}$ are hadrons and $O_{1,2}$ are composite local operators. For example in Δm_K the correlation function can be represented as:

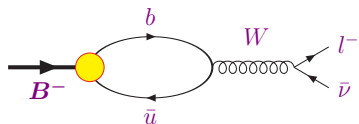


and exploratory results have been presented in [N.H.Christ et al., arXiv:1212.5931](#).

- The use of $T = t_B - t_A$ dependence to isolate the relevant contribution has had to be understood.
- Finite-volume effects have had to be understood.
- GIM mechanism has to be invoked to cancel quadratic UV divergences.
- For rare-kaon decays, such as $K_S \rightarrow \pi^0 \ell^+ \ell^-$, $O_1 = H_W$ and $O_2 = J_{em}$.
- This will be a major area of our research in the coming period.

4a B-Physics

- The b -quark is light-enough to be produced copiously and heavy enough to have a huge number of possible decay channels.
- In addition to the lattice systematics already discussed, we now have to deal with the fact that $m_b a \gtrsim 1$.
- Most approaches rely on effective theories and invest a considerable effort in matching the effective theory to QCD.
 - Heavy Quark Effective Theory (expansion in $\frac{\Lambda_{\text{QCD}}}{m_B}$).
 - Nonrelativistic QCD (expansion in the quark's velocity).
 - Relativistic Heavy Quarks ("Fermilab Approach" and extensions).
A. El Khadra, A. Kronfeld and P. Mackenzie, hep-lat/9604004
- Some groups also extrapolate results from the charm to the bottom region, using scaling laws where applicable and possibly using results in the static limit.
- There are far fewer calculations in heavy-quark physics, so less opportunity to check for consistency of different approaches.
This is not a criticism of those who have done the calculations but of those of us who have not!
- Unfortunately we do not know (yet?) how to compute non-leptonic B -decays ($B \rightarrow \pi\pi$, $B \rightarrow \pi K$ etc).



All QCD effects are contained in a single constant, f_B , the B-meson's (leptonic) decay constant:

$$\langle 0 | \bar{b} \gamma^\mu \gamma^5 u | B(p) \rangle \equiv i f_B p^\mu.$$

- The FLAG compilation for $N_f = 2 + 1$ simulations:

$$f_B = (190.5 \pm 4.2) \text{ MeV} \quad f_{B_s} = (227.7 \pm 4.5) \text{ MeV} \quad \frac{f_{B_s}}{f_B} = 1.202 \pm 0.022.$$

- CTS at EPS 1993: $f_B = 180 \pm 40 \text{ MeV}$ and "Lattice simulations typically give a result 10-20% larger for B_s and D_s than for f_B and f_D ".
- (In principle at least) Experimental measurements of $B(B \rightarrow \tau \nu_\tau)$ by Belle and BABAR + Lattice determinations of $f_B \Rightarrow V_{ub}$. However there are still $\sim 2\sigma$ experimental questions:

PDG 2012

$$B(B^+ \rightarrow \tau^+ \nu_\tau) = (1.65 \pm 0.34) \times 10^{-4}$$

$$\Rightarrow |V_{ub}| = (5.07 \pm 0.52 \pm 0.11) \times 10^{-3} \quad (N_f = 2 + 1)$$

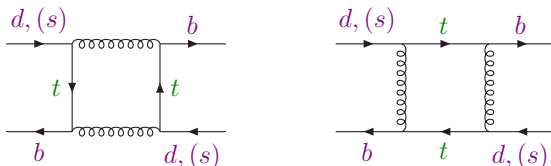
Belle (arXiv : 1208.4678)

$$B(B^+ \rightarrow \tau^+ \nu_\tau) = (0.72_{-0.25}^{+0.27} \pm 0.11) \times 10^{-4}$$

$$\Rightarrow |V_{ub}| = (3.35 \pm 0.65 \pm 0.07) \times 10^{-3} \quad (N_f = 2 + 1)$$

- At present this is not the most competitive way to determine V_{ub} .

Neutral B -meson mixing



- For the SU(3)-breaking parameter ξ , FLAG take the result of the FNAL/MILC collaboration as currently the best result: FNAL/MILC, arXiv:1205.7013

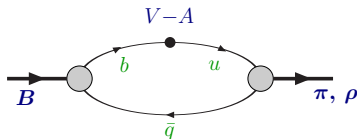
$$\xi^2 \equiv \frac{\langle \bar{B}_s^0 | (\bar{b}\gamma^\mu (1 - \gamma^5)s) (\bar{b}\gamma_\mu (1 - \gamma^5)s) | B^0 \rangle}{\langle \bar{B}_s^0 | (\bar{b}\gamma^\mu (1 - \gamma^5)s) (\bar{b}\gamma_\mu (1 - \gamma^5)s) | B^0 \rangle} = 1.268(63).$$

- Combining this result with experimental values of Δm_d and $\Delta m_s \Rightarrow$

$$\left| \frac{V_{td}}{V_{ts}} \right| = 0.216 \pm 0.011. \quad \text{FNAL/MILC, arXiv:1205.7013}$$

- For generic BSM theories, there are 5 $\Delta B = 2$ operators (and 5 $\Delta S = 2$ operators for neutral kaon mixing) whose matrix elements can be computed in a similar way.

Semileptonic $B \rightarrow \pi, \rho$ Decays



QCD effects are contained in form factors
 e.g. for $B \rightarrow \pi$ decays:

$$\langle \pi(p_\pi) | \bar{b} \gamma_\mu u | B(p_B) \rangle = f_0(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q_\mu$$

$$+ f_+(q^2) \left[(p_\pi + p_B)_\mu - \frac{m_B^2 - m_\pi^2}{q^2} q_\mu \right]$$

where $q \equiv p_B - p_\pi$.

- For B -decays, in order to avoid lattice artefacts, the momentum of the π or ρ is limited \Rightarrow get results only at large values of q^2 .
- Thus V_{ub} can only be obtained directly by combining the lattice results with a subset of the experimental data:

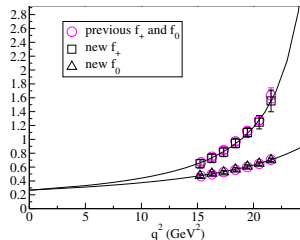
$$\Delta\zeta(q_1^2, q_2^2) = \frac{1}{|V_{ub}|^2} \int_{q_1^2}^{q_2^2} dq^2 \frac{d\Gamma}{dq^2}.$$

- The lattice results can be combined with theoretically motivated parametrisations for the form factors, including perhaps constraints from analyticity and other general properties of field theory, to extend the range of the predictions. (Not discussed here.)

Semileptonic $B \rightarrow \pi, \rho$ Decays Cont.

The (peer-reviewed) published values for the form factors are relatively old:

Collaboration	Reference	$\Delta\zeta$ ps $^{-1}$
FNAL/MILC	arXiv:0811.3640	$2.21^{+0.47}_{-0.42}$
HPQCD	hep-lat/0601021	$2.07(41)(39)$



HPQCD

- The two collaborations use overlapping sets of rooted staggered ensembles, but different treatments of the heavy quarks (HPQCD use NRQCD and FNAL/MILC use the FNAL approach). Assuming (conservatively) a 100% correlation FLAG quote

$$\Delta\zeta(16\text{GeV}^2, q_{\text{max}}^2) = 2.16(50)\text{ps}^{-1}$$

- FLAG, perform a detailed analysis, finding a preferred parametrization and quote

$$\text{Lattice + BABAR} \quad |V_{ub}| = 3.37(20) \times 10^{-3}$$

$$\text{Lattice + Belle} \quad |V_{ub}| = 3.47(22) \times 10^{-3}.$$

Semileptonic $B \rightarrow \pi, \rho$ Decays Cont.

- FLAG, perform a detailed analysis, finding a preferred parametrization and quote

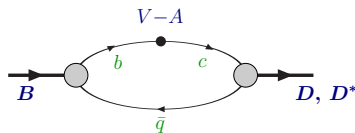
$$\text{Lattice + BABAR} \quad |V_{ub}| = 3.37(20) \times 10^{-3}$$

$$\text{Lattice + Belle} \quad |V_{ub}| = 3.47(22) \times 10^{-3}.$$

- Assuming (not assuming) unitarity PDG quote $|V_{ub}| = 3.51_{-0.14}^{+0.15} \times 10^{-3}$
 $(|V_{ub}| = (4.15 \pm 0.49) \times 10^{-3})$.
- The issue is the tension with the inclusive determination
 $|V_{ub}| = (4.41 \pm 0.15_{-0.19}^{+0.15}) \times 10^{-3}$. This has very different systematics and cannot be studied in lattice simulations.
- The evaluation of $f_+(q^2)$ and $f_0(q^2)$ and the subsequent determination of V_{ub} is clearly a major priority for lattice simulations and several collaborations have made conference presentations about their current work:

Collaboration	Approach	Reference
Alpha	HQET	arXiv:1210.3478 , arXiv:1211.6327
HPQCD	NRQCD	arXiv:1210.6992
RBC/UKQCD	Relativistic Heavy Quarks	arXiv:1211.0956
FNAL/MILC	Fermilab	arXiv:1211.1390

- We can look forward to many new results for the form factors.
 Next week at the Lattice Conference?

$B \rightarrow D, D^*$ Semileptonic Decays and Determination of V_{cb} .


- V_{cb} is known more precisely than V_{ub} , because the experimental cuts are at higher energies where the OPE is more reliable and heavy-quark symmetry \Rightarrow the form factors are close to 1.

- V_{cb} is not known well enough however! (For example $\epsilon_K \propto |V_{cb}|^4$.)
- Work in this area has been led by the FNAL/MILC collaboration, using the Fermilab approach and staggered fermions. [arXiv:0808.2519](https://arxiv.org/abs/0808.2519), [arXiv:1011.2166](https://arxiv.org/abs/1011.2166)
- Defining $\omega = v_B \cdot v_{D^*}$,

$$\frac{d\Gamma_{B^- \rightarrow D^{0^*} \ell \bar{\nu}}}{d\omega} \propto |V_{cb}|^2 |\mathcal{F}(\omega)|^2,$$

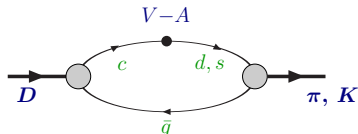
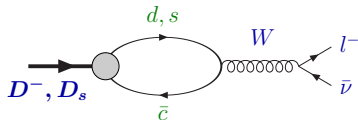
and at zero recoil FNAL/MILC find

$$\mathcal{F}(1) = 0.9017(51)(156) = 0.9017(51)(156) \Rightarrow |V_{cb}| = 39.55(72)(50) \times 10^{-3}.$$

FLAG

- For inclusive $B \rightarrow X_c \ell \nu$ decays, PDG quote $|V_{cb}|^{\text{incl}} = (41.9 \pm 0.7) \times 10^{-3}$.
- New lattice calculations of $\mathcal{F}(\omega)$ are being performed \Rightarrow improved determination of $|V_{cb}|$ in the next 1-2 years.

4b Charm Physics



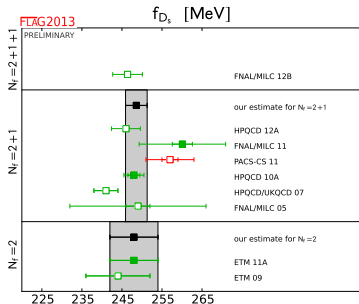
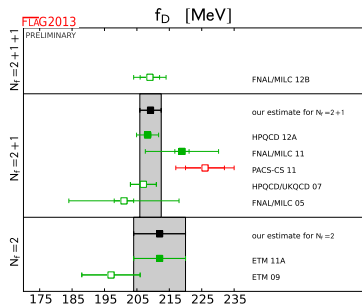
- The charm quark is somewhere in between *light* and *heavy*, i.e. typically $am_c \lesssim 1$.
- Some calculations are performed with the light-quark actions (increasingly this is the case) and some using one of the heavy approaches.
- FLAG $N_f = 2 + 1$ averages:

$$f_D = (209.2 \pm 3.3) \text{ MeV}, \quad f_{D_s} = (248.6 \pm 2.7) \text{ MeV}, \quad \frac{f_{D_s}}{f_D} = 1.187 \pm 0.012.$$

$$f_+^{D\pi}(0) = 0.666(29) \quad f_+^{DK}(0) = 0.747(19)$$

- Leptonic decays $\Rightarrow |V_{cd}| = 0.2218(35)(95)$ and $|V_{cs}| = 1.018(11)(21)$.
- Semileptonic decays $\Rightarrow |V_{cd}| = 0.2192(95)(45)$ and $|V_{cs}| = 0.9746(248)(67)$.

Results for decay constants (FLAG)



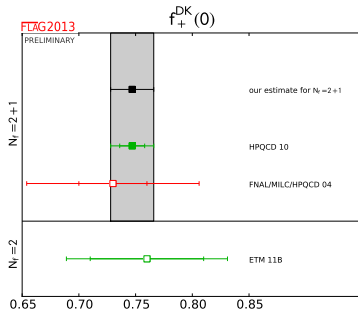
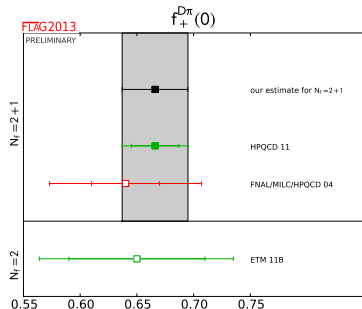
- In 2007-8 there was some excitement concerning f_{D_s} :

$$f_{D_s} = (277 \pm 9) \text{ MeV} \quad \text{compilation of experiments, Dobrescu \& Kronfeld, arXiv:0803.0512}$$

$$f_{D_s} = (241 \pm 3) \text{ MeV} \quad \text{HPQCD arXiv:0706.1726}$$

- This HPQCD result was updated to $(248.0 \pm 2.5) \text{ MeV}$ in [arXiv:1008.4018](#) and the experimental average has come down e.g. from the B -factories $f_{D_s} = (257.2 \pm 4.5) \text{ MeV}$. [A.Zupanc, arXiv:1301.7218](#)

Results for semileptonic form factors (FLAG)



- Far fewer computations than for the corresponding quantities in light-quark physics.
- A number of new calculations under way.

Conclusions



Kenneth Wilson
1936-2013
(1982 Nobel Laureate and
father of lattice QCD)

- We have now reached the era of $O(1\%)$ precision for many important quantities.
 - Improvement in precision will continue.
 - Isospin breaking effects, including electromagnetism, will increasingly be included.
 - I expect a major expansion of results in heavy-quark physics in the near future, generating competition among different approaches.
 - Technology for calculating *disconnected diagrams*, necessary e.g. for flavour singlets, will be fully developed.
- The range of computed quantities will continue to be extended (e.g. ε'/ε , rare-kaon decay amplitudes, ...)
- Lattice QCD has matured into the quantitative ab-initio method for computing non-perturbative strong-interaction effects in an increasing range of processes.

Supplementary Slide - $R(D)$

FNAL/MILC Collaboration have presented the first calculation of $R(D)$ which requires the evaluation of the scalar form factor:

$$R(D) = \frac{B(B \rightarrow D\tau\nu_\tau)}{B(B \rightarrow D\ell\nu_\ell)} = 0.316(12)(7)$$

[arXiv:1206.4992](https://arxiv.org/abs/1206.4992)

to be compared to the BaBar result

$$R(D) = \frac{B(B \rightarrow D\tau\nu_\tau)}{B(B \rightarrow D\ell\nu_\ell)} = 0.440(58)(42).$$

[arXiv:1205.5442](https://arxiv.org/abs/1205.5442)

Supplementary Slide - UTfit Input Values and SM predictions

- The CKMfitter and UTfit collaborations perform global unitarity triangle analyses.
- There is enough data to remove a quantity from the inputs and to make a prediction for the missing quantity e.g.

Observable	Input value	SM prediction	Pull
$\varepsilon_K \times 10^3$	2.23 ± 0.01	1.96 ± 0.20	1.4
$\Delta m_s \text{ ps}^{-1}$	17.69 ± 0.08	18.0 ± 1.3	< 1
$ V_{cb} \times 10^3$	41.0 ± 1.0	42.3 ± 0.9	< 1
$ V_{ub} \times 10^3$	3.82 ± 0.56	3.62 ± 0.14	< 1
$\text{Br}(\text{B}\tau\nu) \times 10^4$	1.67 ± 0.30	0.82 ± 0.08	2.7
$\sin 2\beta$	0.68 ± 0.02	0.81 ± 0.05	2.4
α	$91^\circ \pm 6^\circ$	$88^\circ \pm 4^\circ$	< 1
γ	$76^\circ \pm 11^\circ$	$68^\circ \pm 3^\circ$	< 1

C.Tarantino, "Lattice flavor physics with an eye to SuperB,"
PoS LATTICE 2012, 012 (2012).