One-Loop Kähler Metric of Calabi-Yau Orientifolds

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Karlstad University, Sweden

M.B., Haack, Kang ’11
M.B., Conlon, Marsh, Witkowski ’12
M.B., Haack, Kang, Sjörs ’13
...

EPS-HEP, Stockholm, July 2013

Plan

- outline string models in cosmology and phenomenology that seem sensitive to loop corrections

- review of quantum string effective actions of orientifolds with D-branes

- recent work on string one-loop contributions to moduli space metrics

Ultraviolet sensitivity

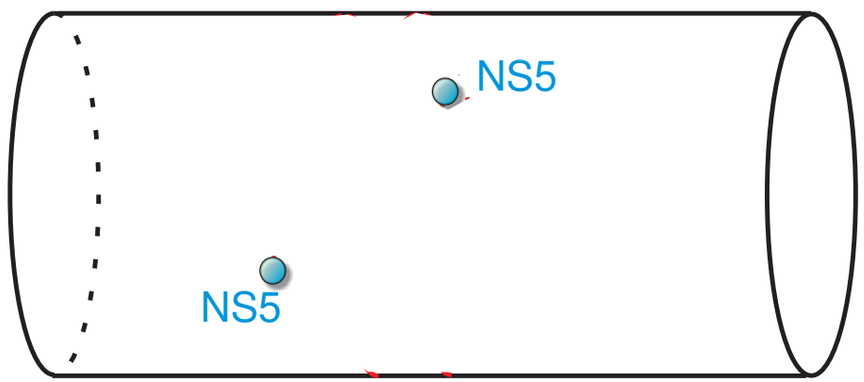
("effects of high-energy physics make a difference")

Example: leading-order vanishing due to symmetry,
small corrections become important

- approximate shift symmetry in inflationary cosmology
- approximate flavor universality in phenomenology

e.g. Assassi, Baumann, Green, McAllister '13

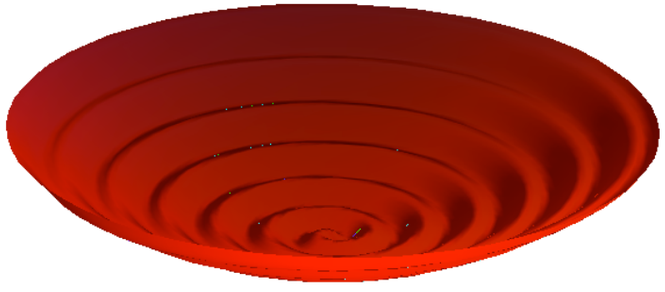
Cosmology: axion monodromy



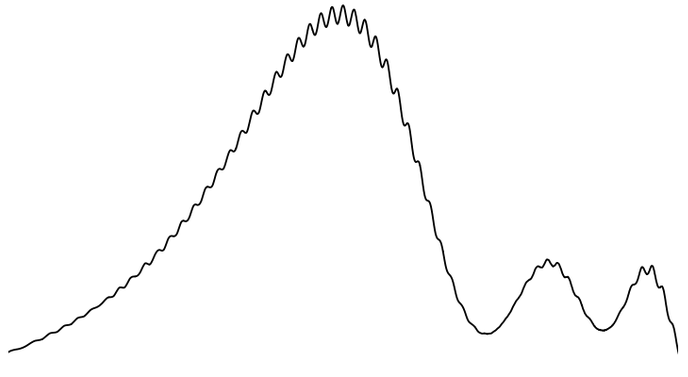
Silverstein, Westphal ’08 +McAllister ’08
M.B., Pajer, Sjörs ’09
...
Pajer, Peloso ’13

in KKLT stabilized background Kachru, Kallosh, Linde, Trivedi ’03

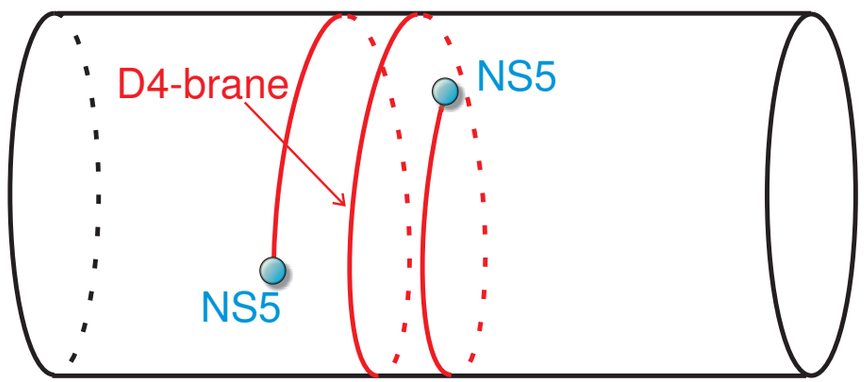
$$V(r,\theta) = cr^p + \Lambda^4\left(1-\cos\left(\frac{r}{f_r}-\frac{\theta}{f_\theta}\right)\right)$$



sensitive to nonperturbative corrections to Kähler potential



Cosmology: axion monodromy



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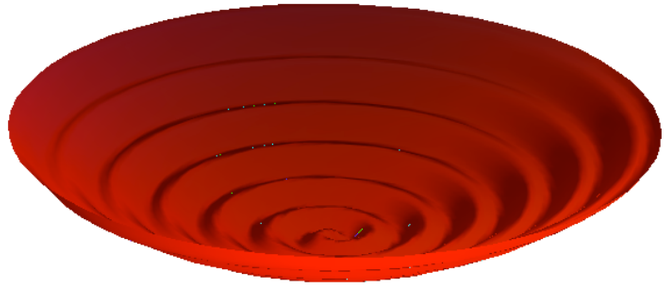

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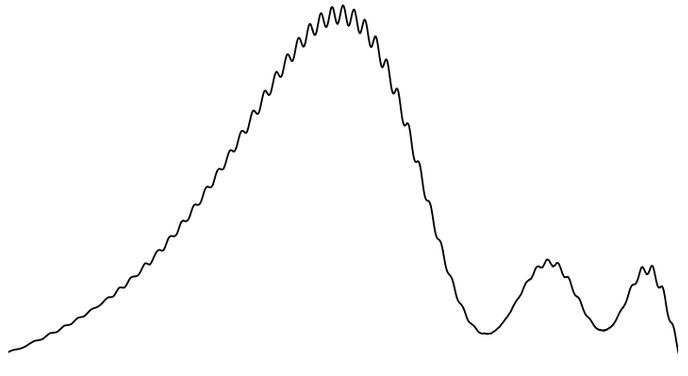
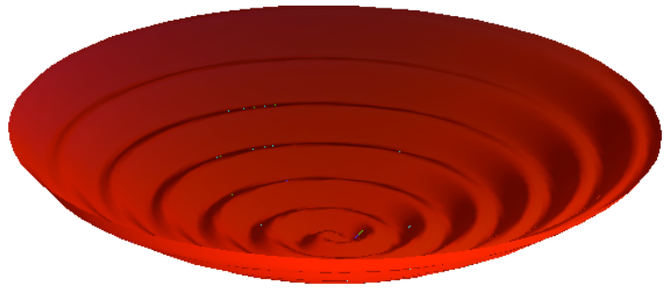


Cosmology: axion monodromy



Silverstein, Westphal ’08 +McAllister ’08
M.B., Pajer, Sjörs ’09
...
Pajer, Peloso ’13

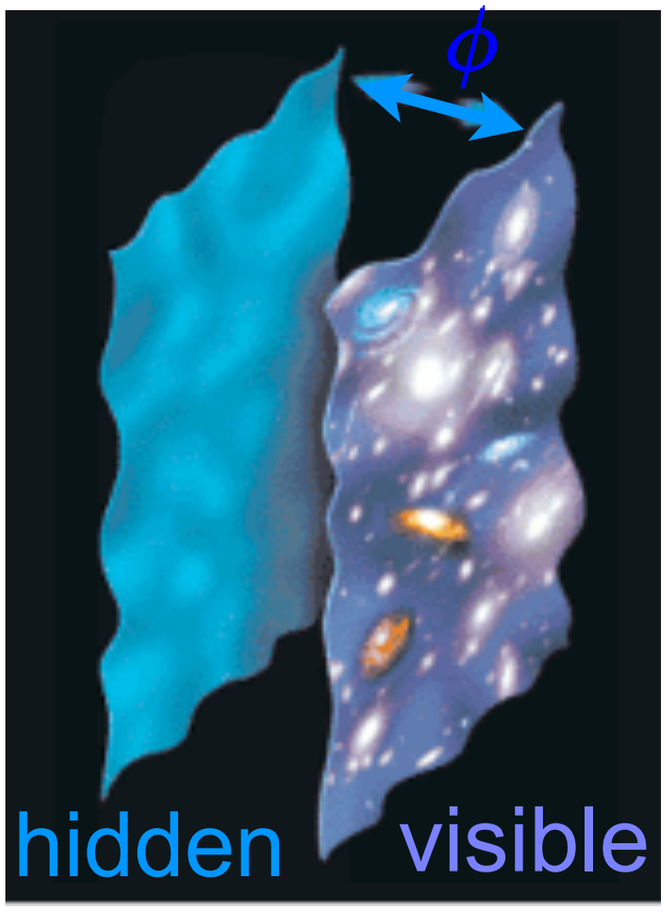
“The models most compatible with the Planck data in the set considered here are the two interesting axion monodromy potentials [McAllister et al] which are motivated by inflationary model building in the context of string theory”

Planck collaboration, ’13

Fun! But: small-field models fit the data better, until there is polarization.

Cosmology: brane inflation

D-brane position scalar is the inflaton (small-field) in KKLT stabilized background



Dvali, Tye ’98

...

M.B., Haack, Körs ’04

Baumann, Dymarsky, Klebanov, Maldacena, McAllister, Murugan ’06

...

Baumann, Dymarsky, Klebanov, McAllister, Steinhardt ’07

...

see also Marsh, McAllister, Pajer, Wrase ’13

sensitive to loop-induced dependence of stabilizing superpotential on D-brane moduli: “a delicate universe”

Phenomenology: gaugino masses

Becker, Becker, Haack, Louis ’02
Balasubramanian, Berglund, Conlon, Quevedo ’05

“KKLT v1.1”: the Large Volume Scenario.
The soft supersymmetry breaking terms due to flux display leading-order cancellations...

Berg, Haack, Pajer ’07

...but due to *further* cancellations, soft terms are not sensitive to perturbative corrections to Kähler potential!

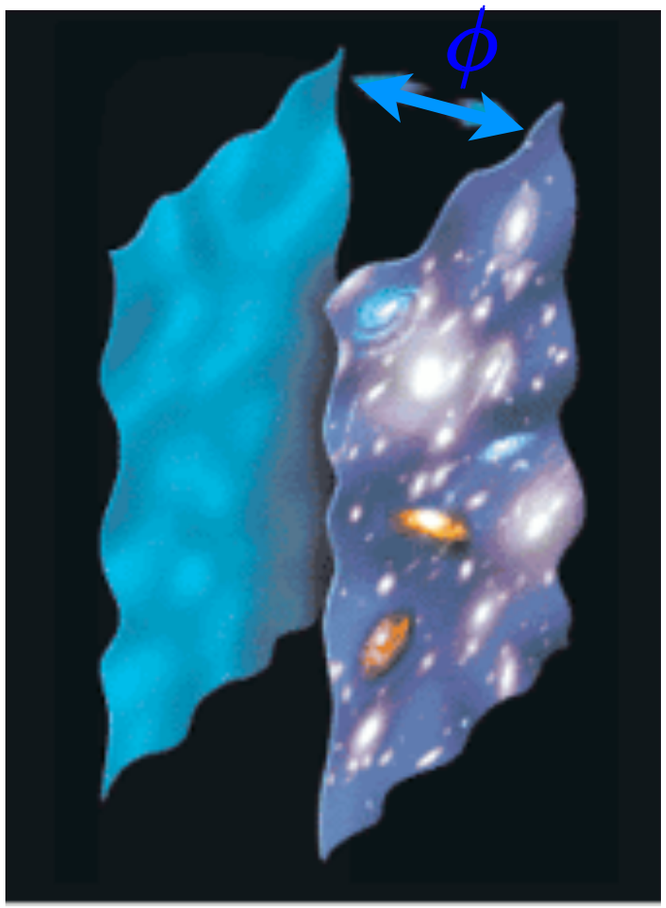
Cicoli, Conlon, Quevedo ’07, ...

called “extended no-scale structure”

Phenomenology: flavor physics

Randall, Sundrum ’98

brane models strongly constrain soft terms: “sequestering”
supposed to solve supersymmetry flavor problem

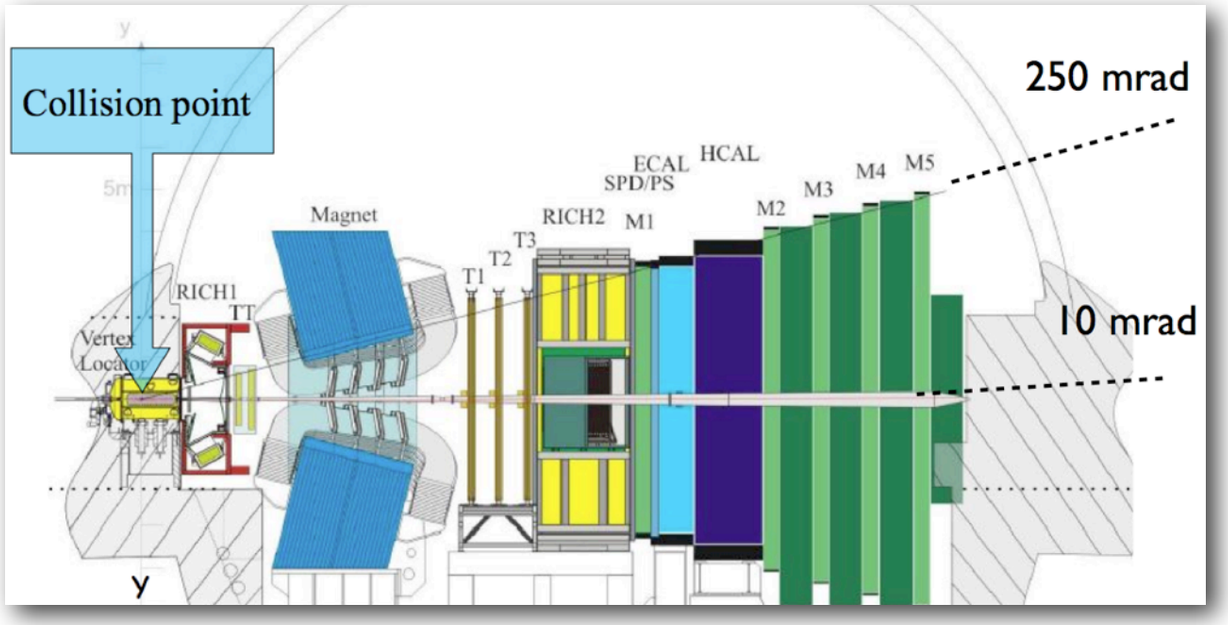


$(f = -3M_P^2 e^{-\frac{K}{3M_P^2}})$

$$f = f_{\rm hid} + f_{\rm vis}$$
$$W = W_{\rm hid} + W_{\rm vis} \quad \Rightarrow \quad A_{ijk} = 0\ , \quad m^2_{i\bar{j}} = 0$$

“clears the way” for anomaly mediation

Phenomenology: flavor physics



LHCb experiment

LHCb collaboration, 1211.2674

studies rare decays, e.g. $B_s \to \mu\mu$ (observed Nov 2012)

Blumenhagen, Conlon et al '09
M.B., Marsh, McAllister, Pajer '10
M.B. Conlon, Marsh Witkowski '12

sequestering in Large Volume Scenario
is sensitive to nonperturbative corrections to superpotential:
limits on rare decays produce strong constraints on the compactification volume

General D=4, N=1 effective theory of gravity+moduli+gauge fields

open and closed string moduli

$$\mathcal{L}_{\rm eff} = \frac{1}{2\kappa^2}R - K_{\Phi_i\bar{\Phi}_{\bar{\jmath}}}\partial_\mu\Phi^i\partial^\mu\bar{\Phi}^{\bar{\jmath}} - \frac{1}{g^2(\Phi)}\mathrm{tr}_a F^2$$

$$-V(\Phi) + \text{corrections}$$

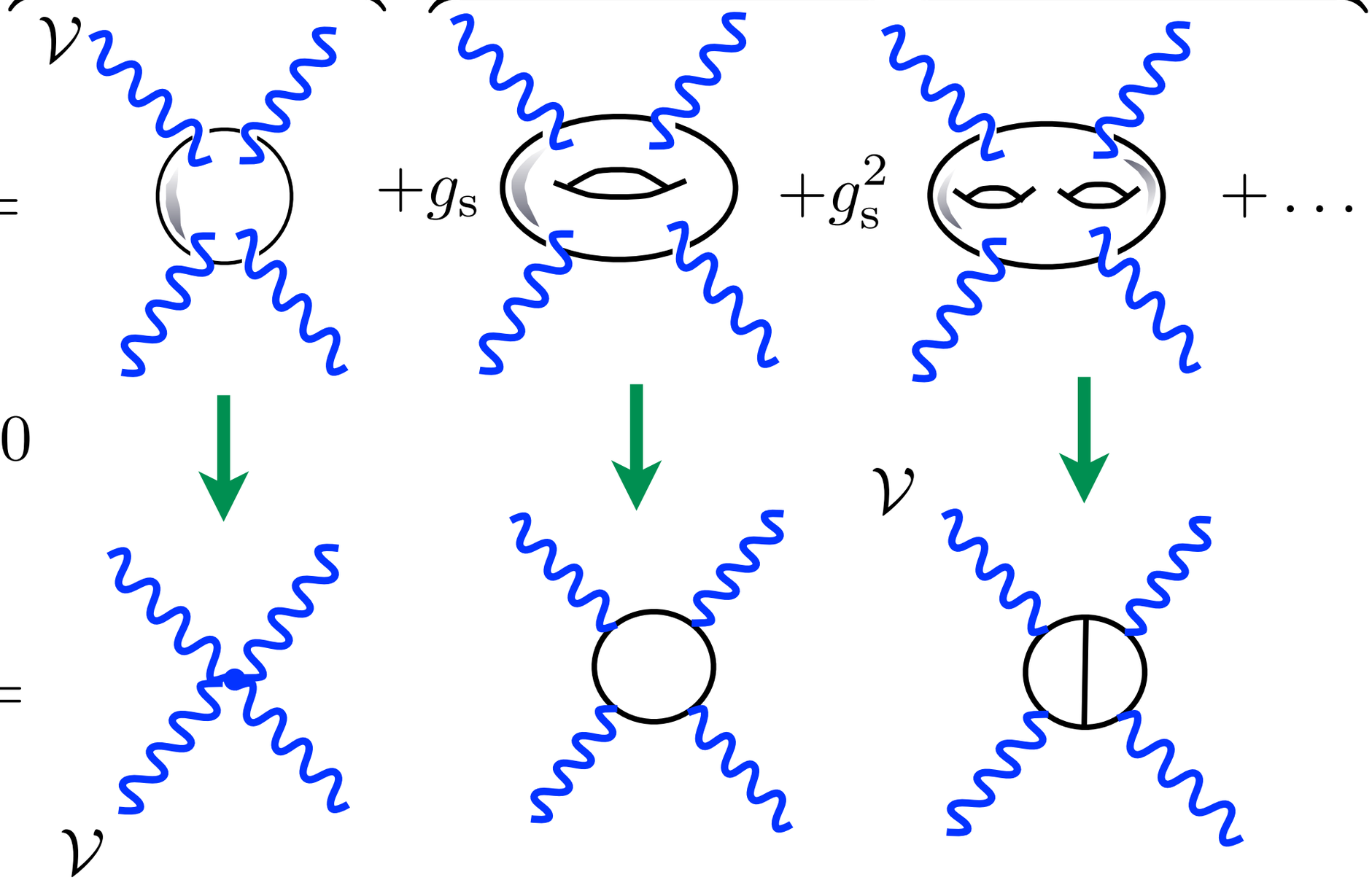
$$K_{\Phi_i\bar{\Phi}_{\bar{\jmath}}} = \partial_{\Phi_i}\partial_{\bar{\Phi}_{\bar{\jmath}}}K$$

Can't always think "negligible"
even when numerically small
at a point in moduli space (i.e. at fixed Φ^i).

String perturbation theory: two expansions

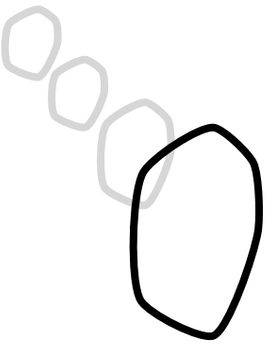
(semi-)classical

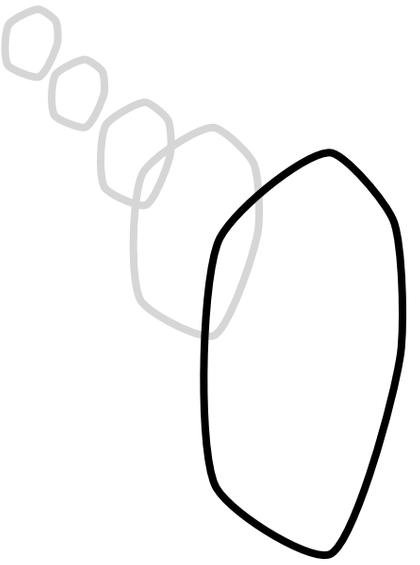
quantum

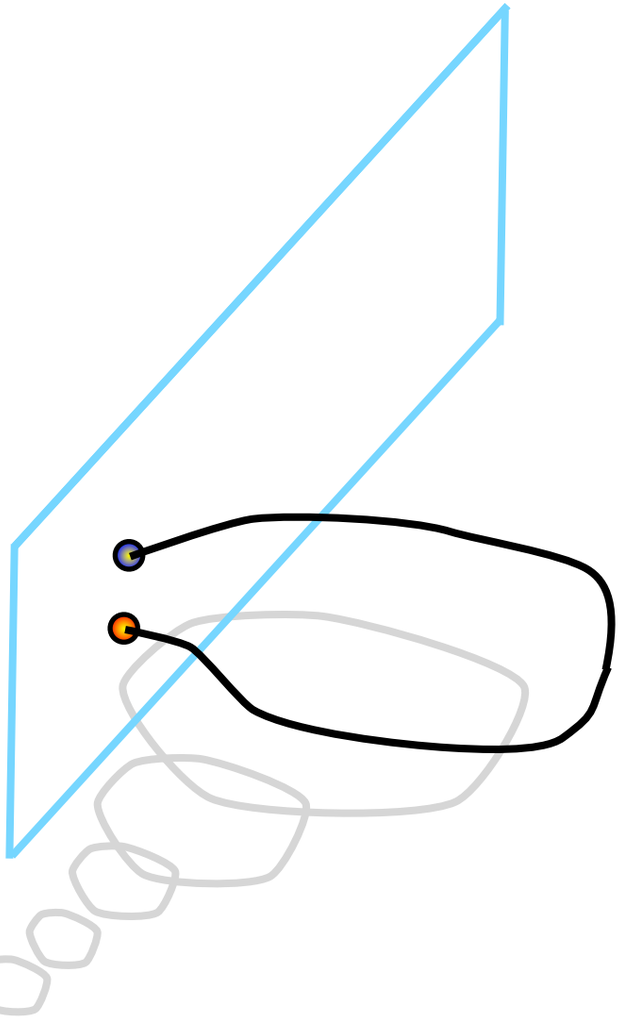
$\langle \mathcal{V}^4 \rangle =$ 

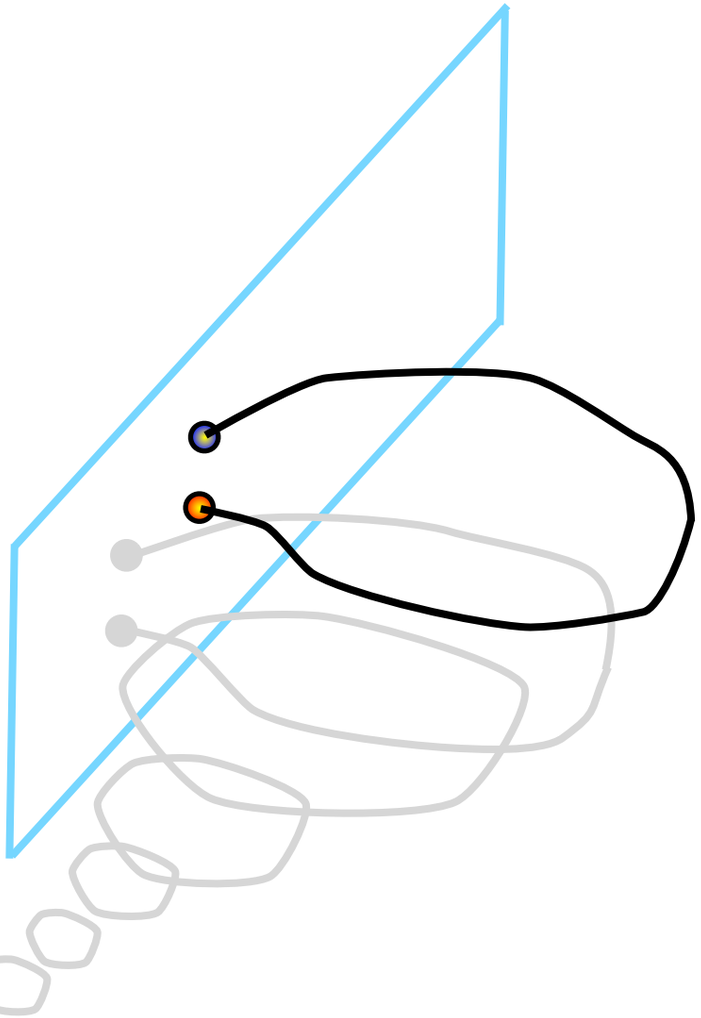
$E^2\alpha' \to 0$

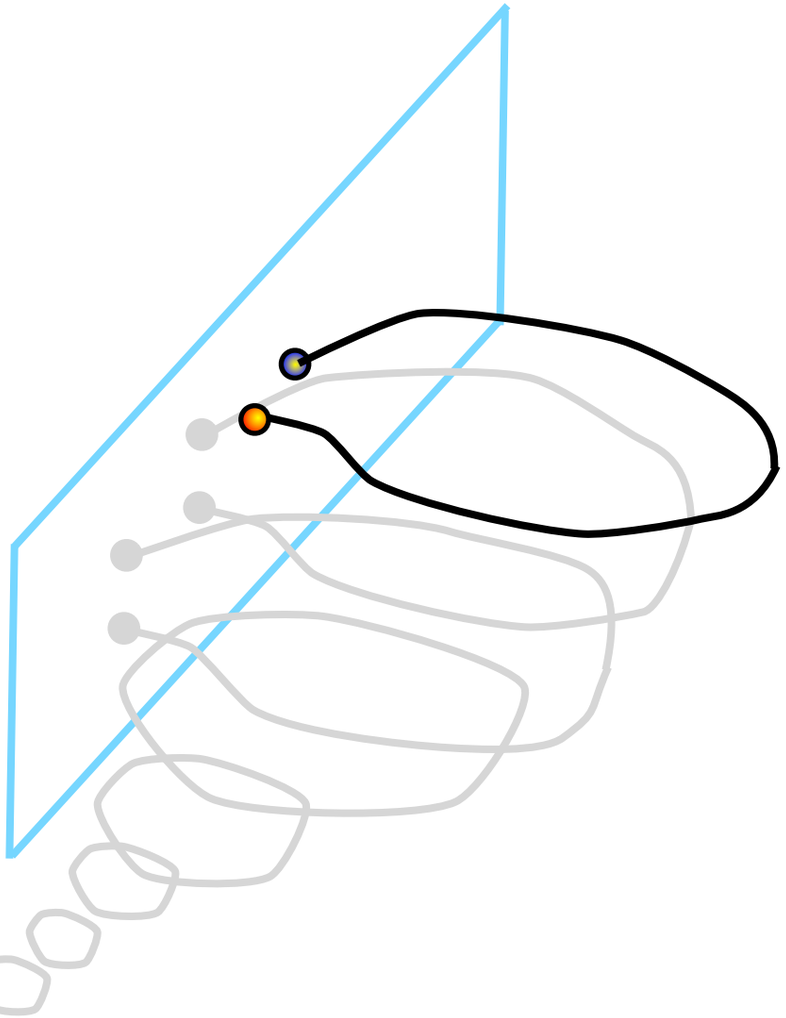
$\langle \mathcal{V}^4 \rangle =$

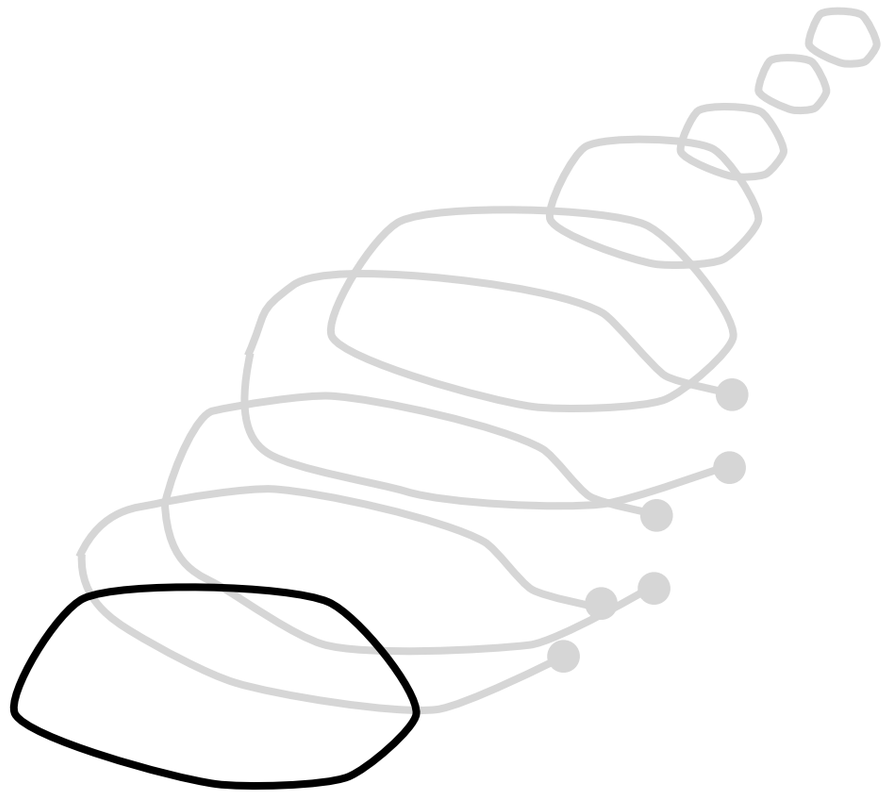


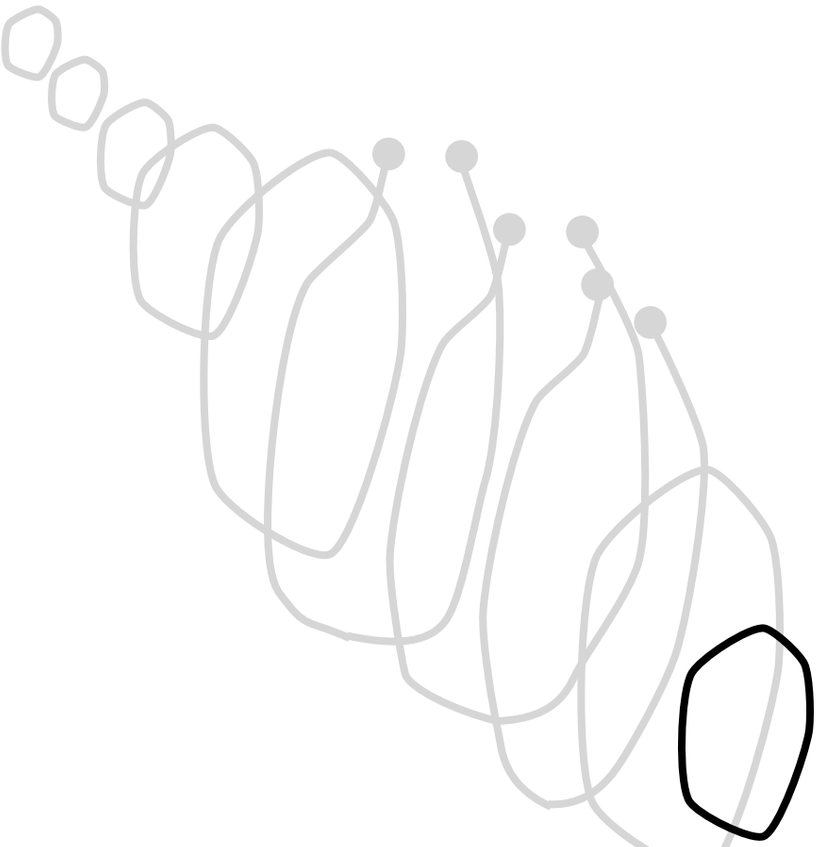


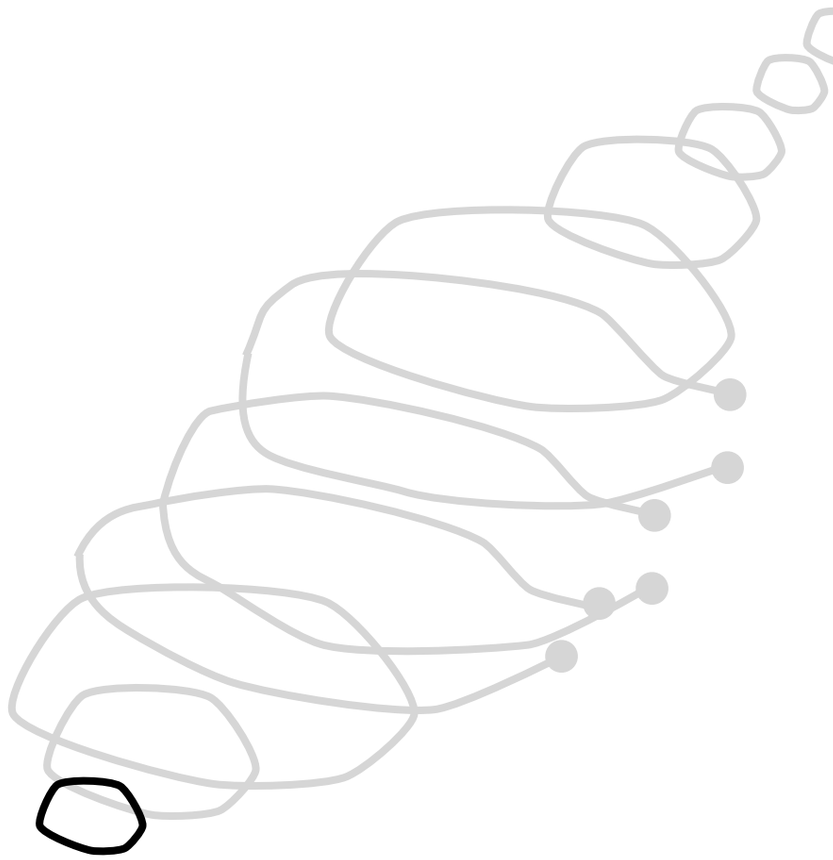


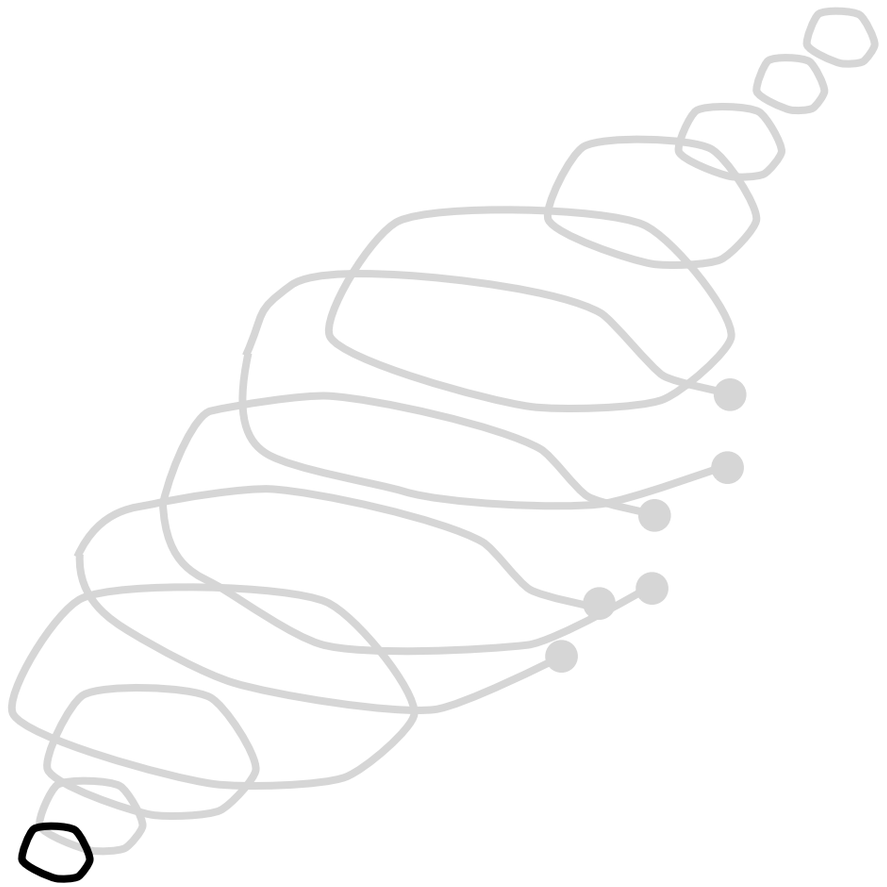


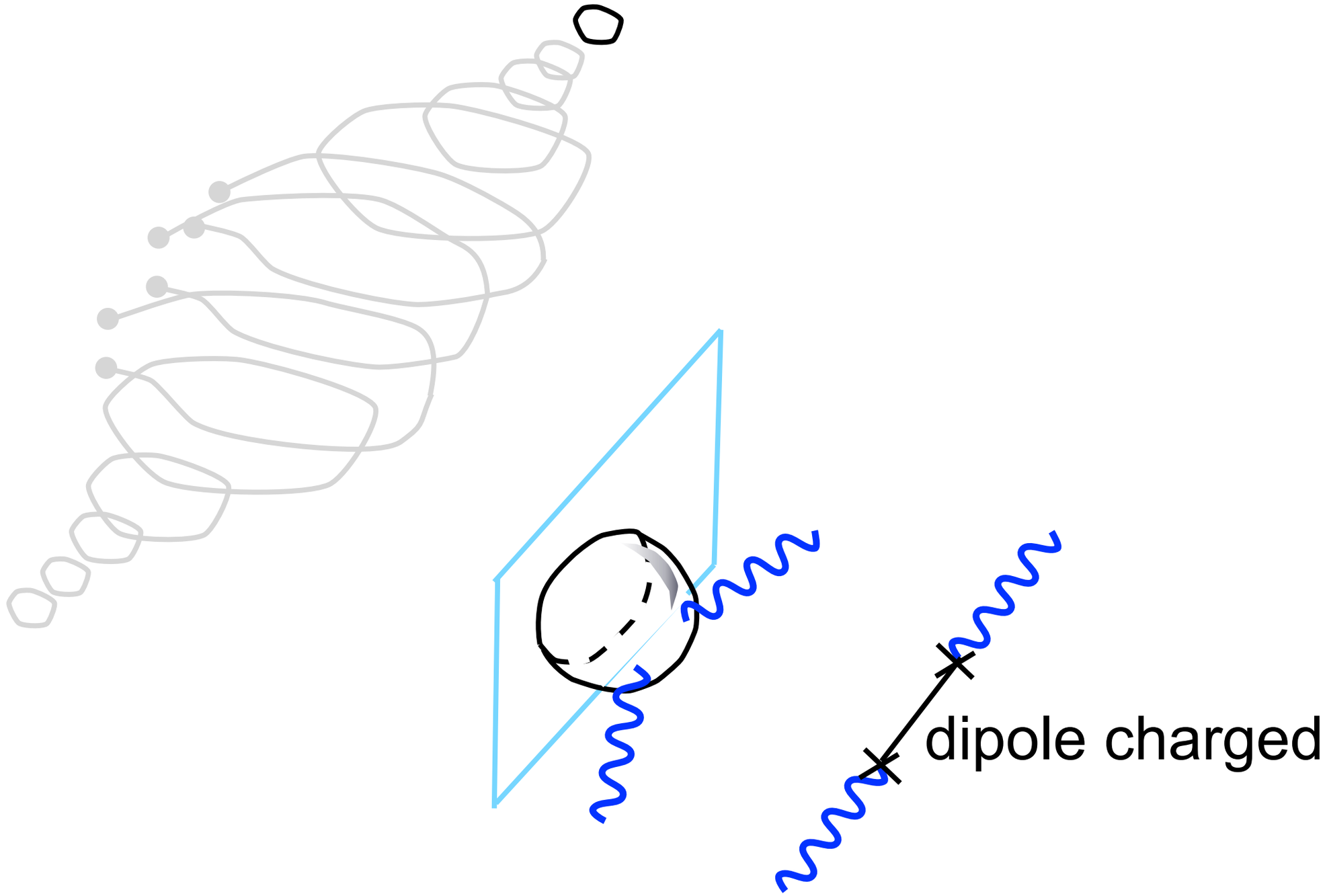




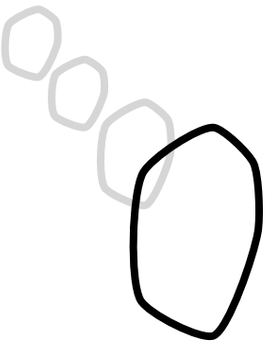


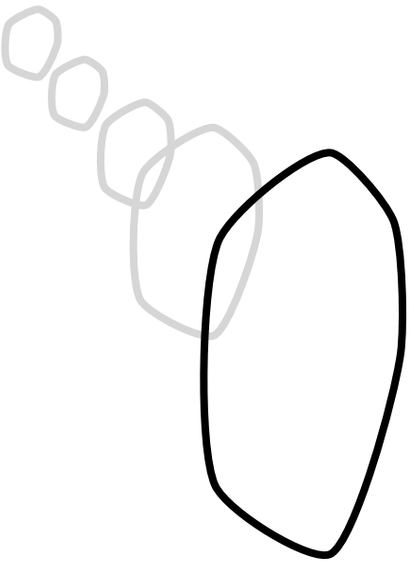


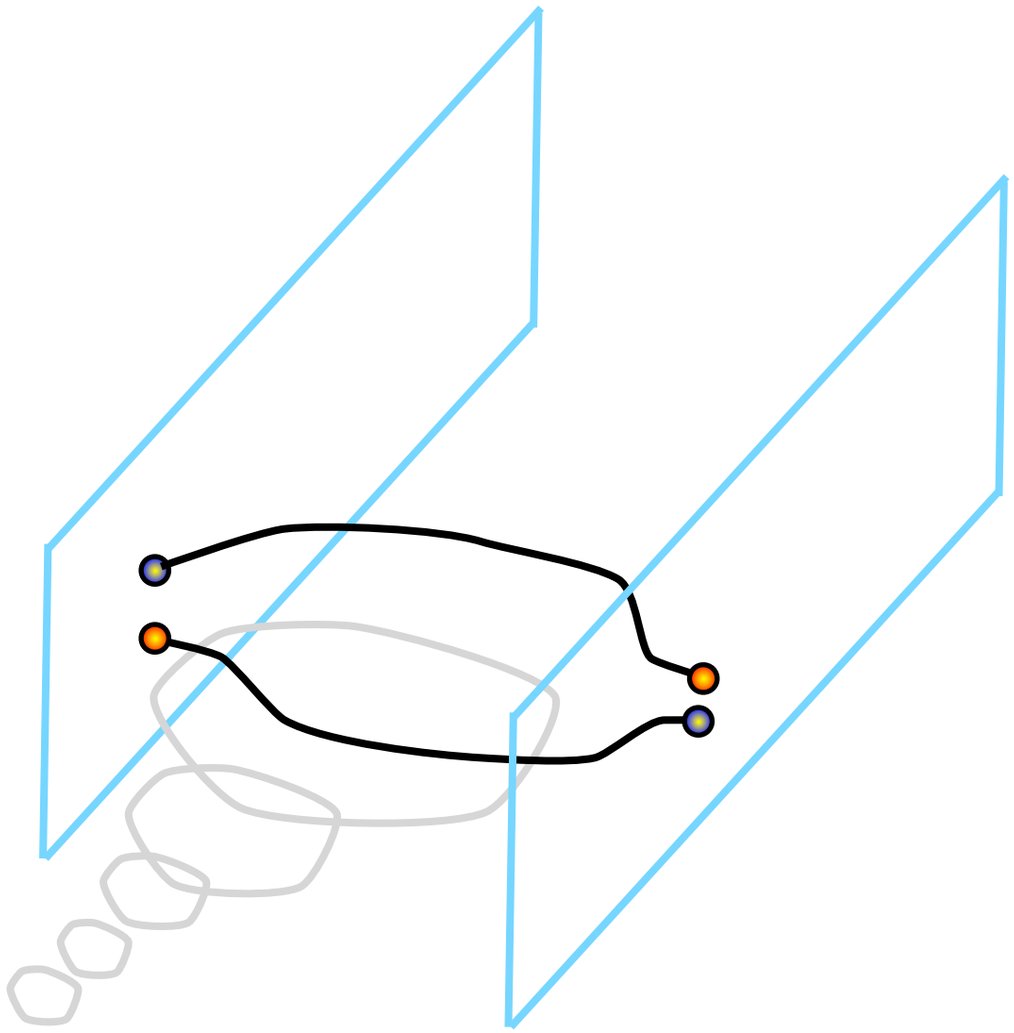


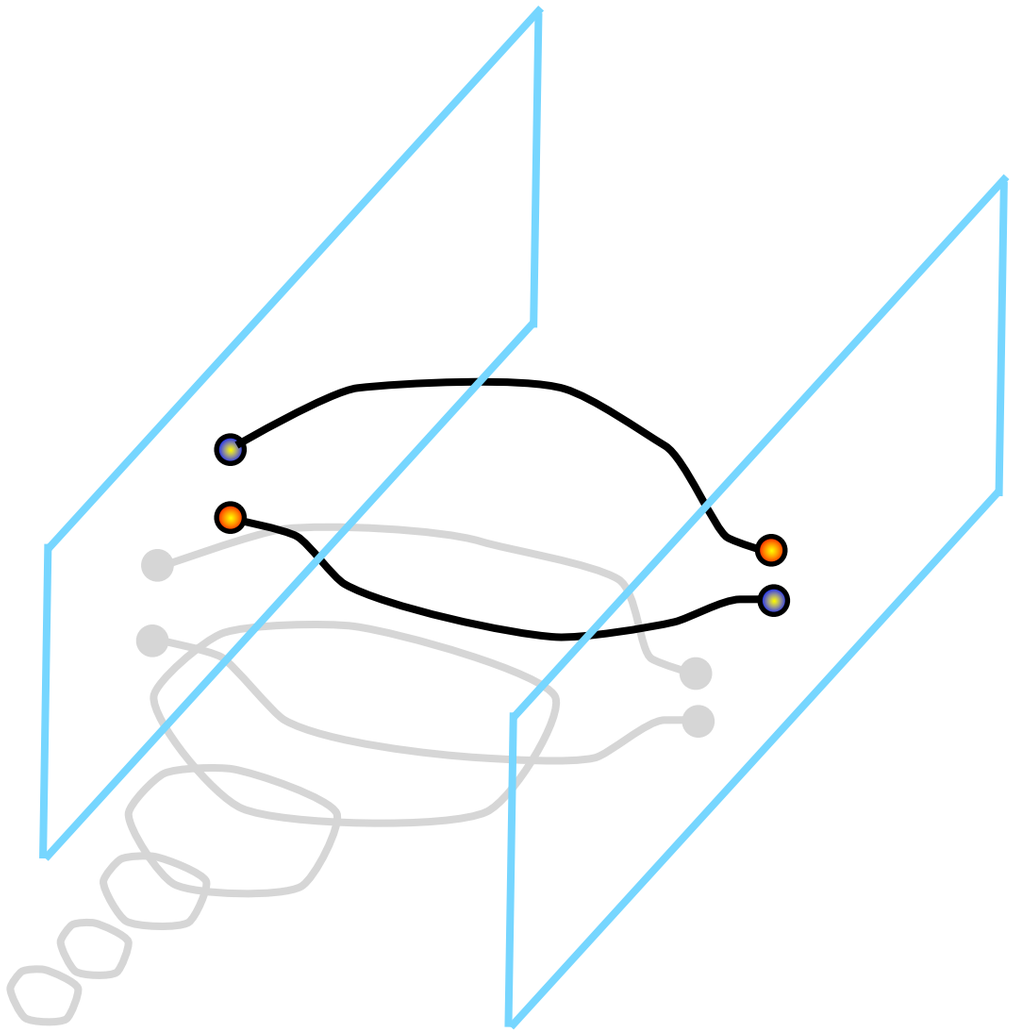


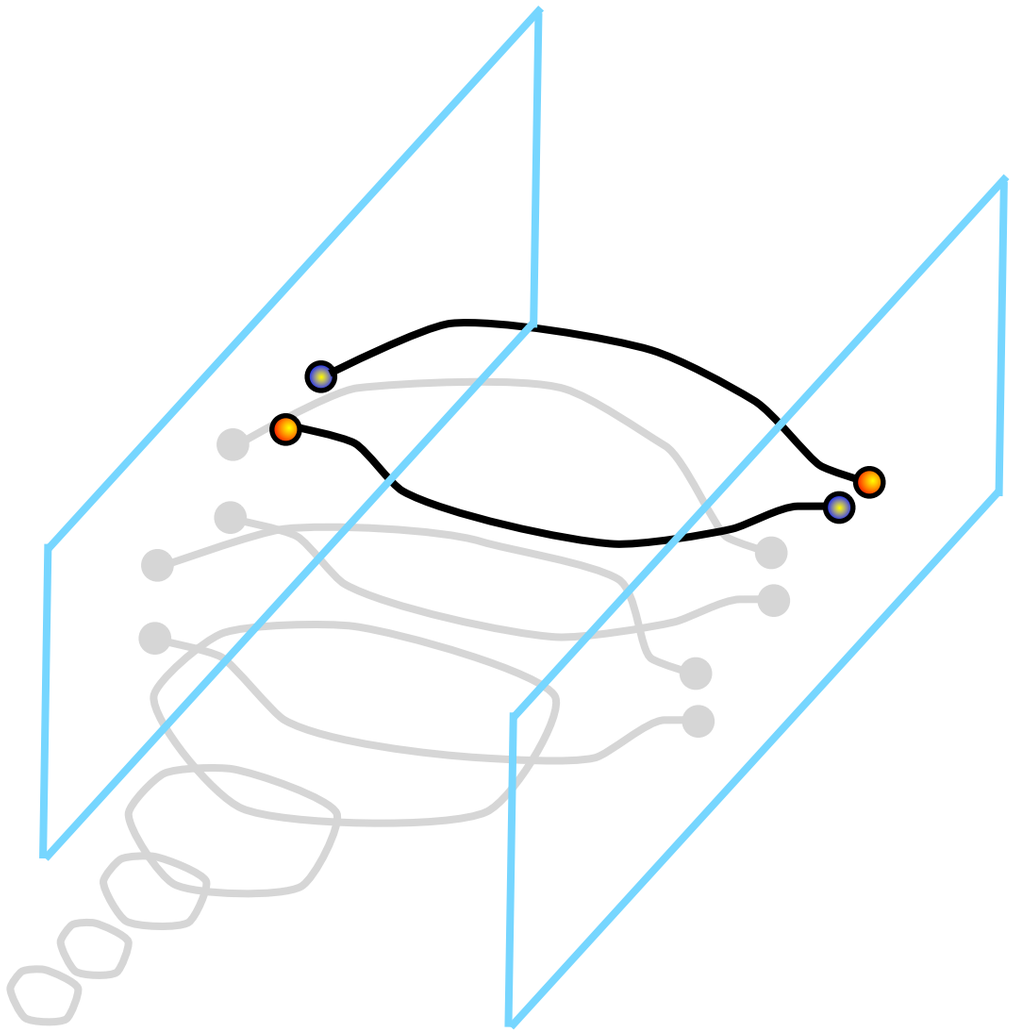
(semi-)classical, string tree level

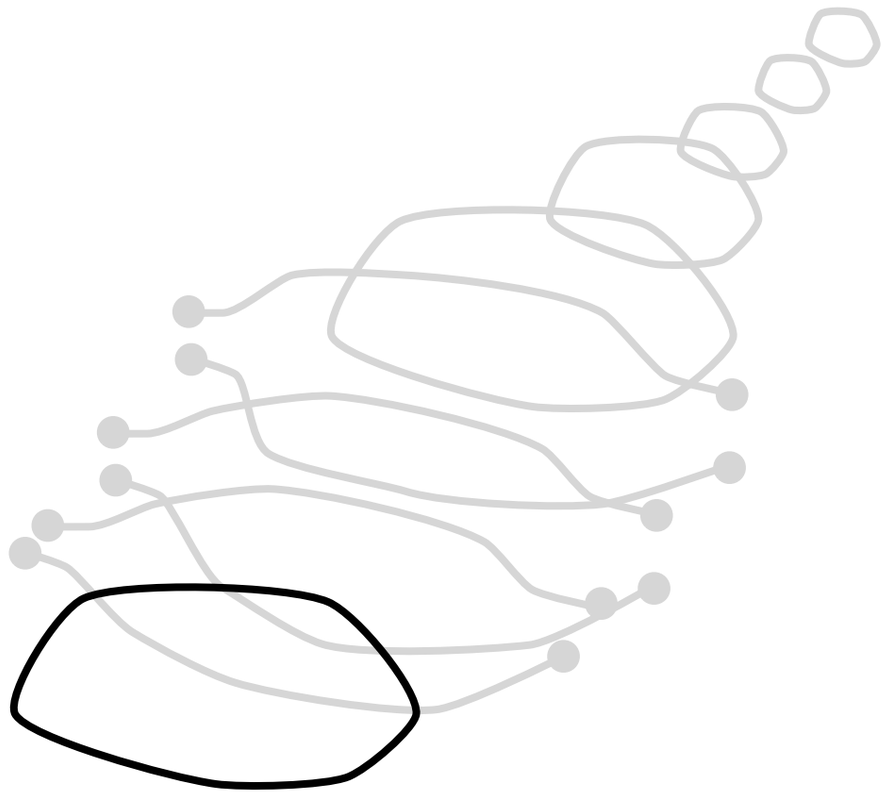


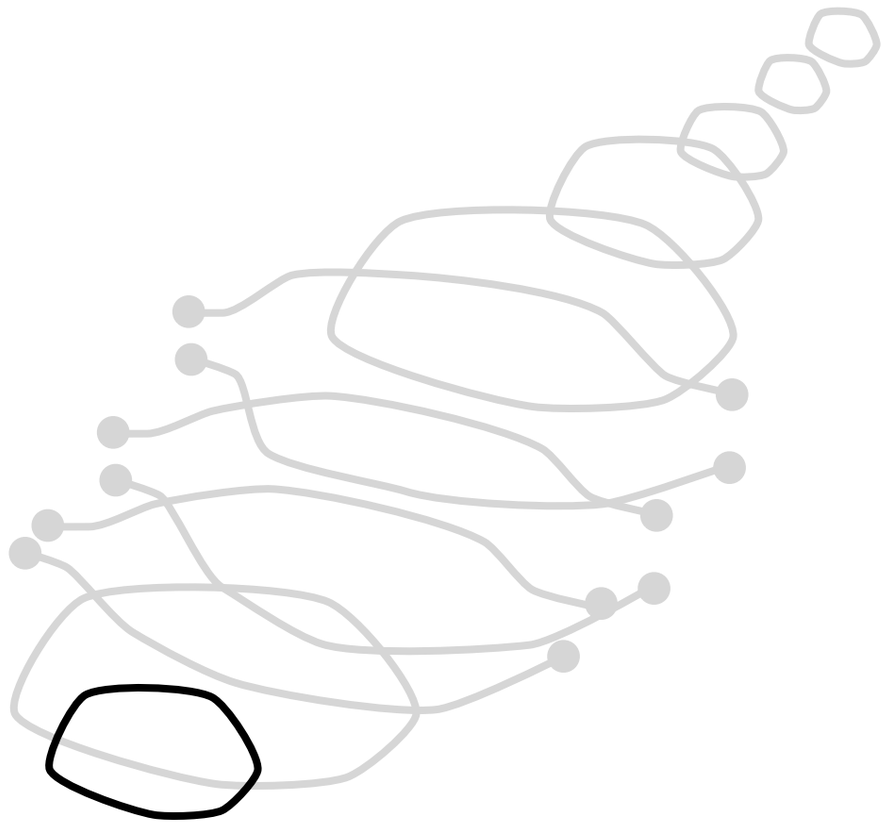


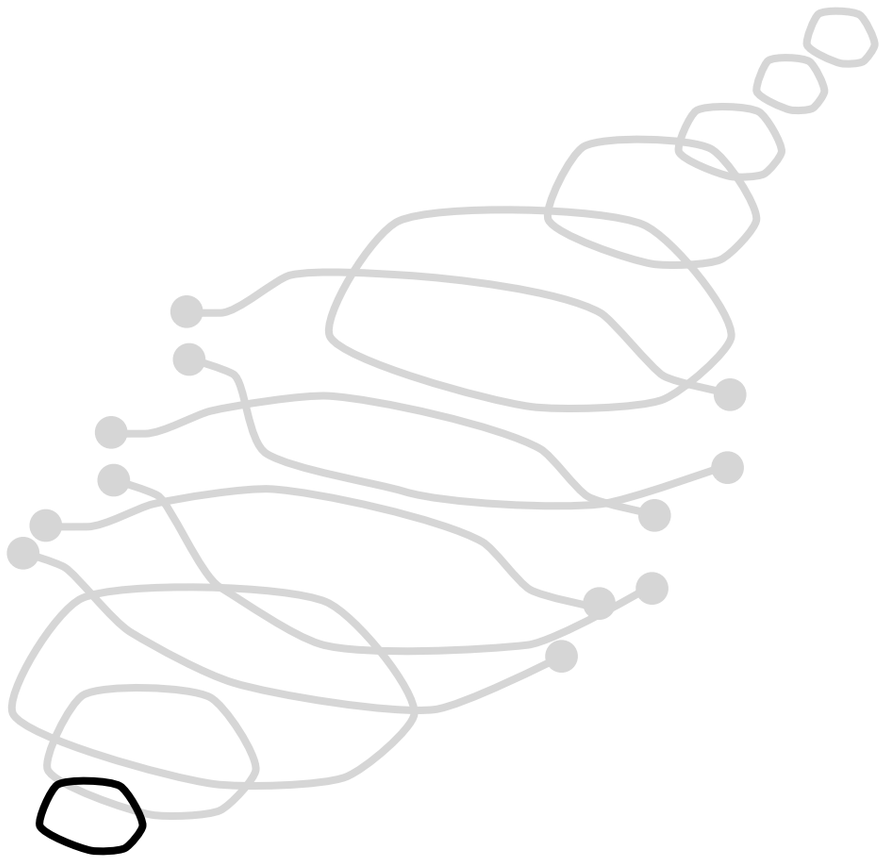


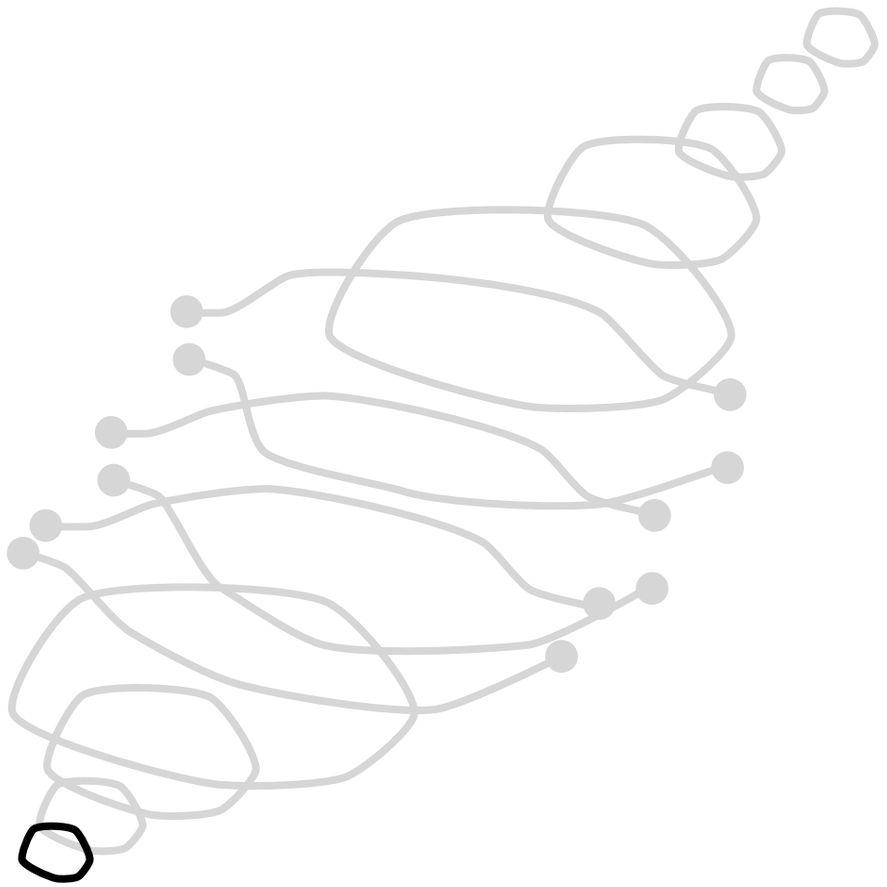


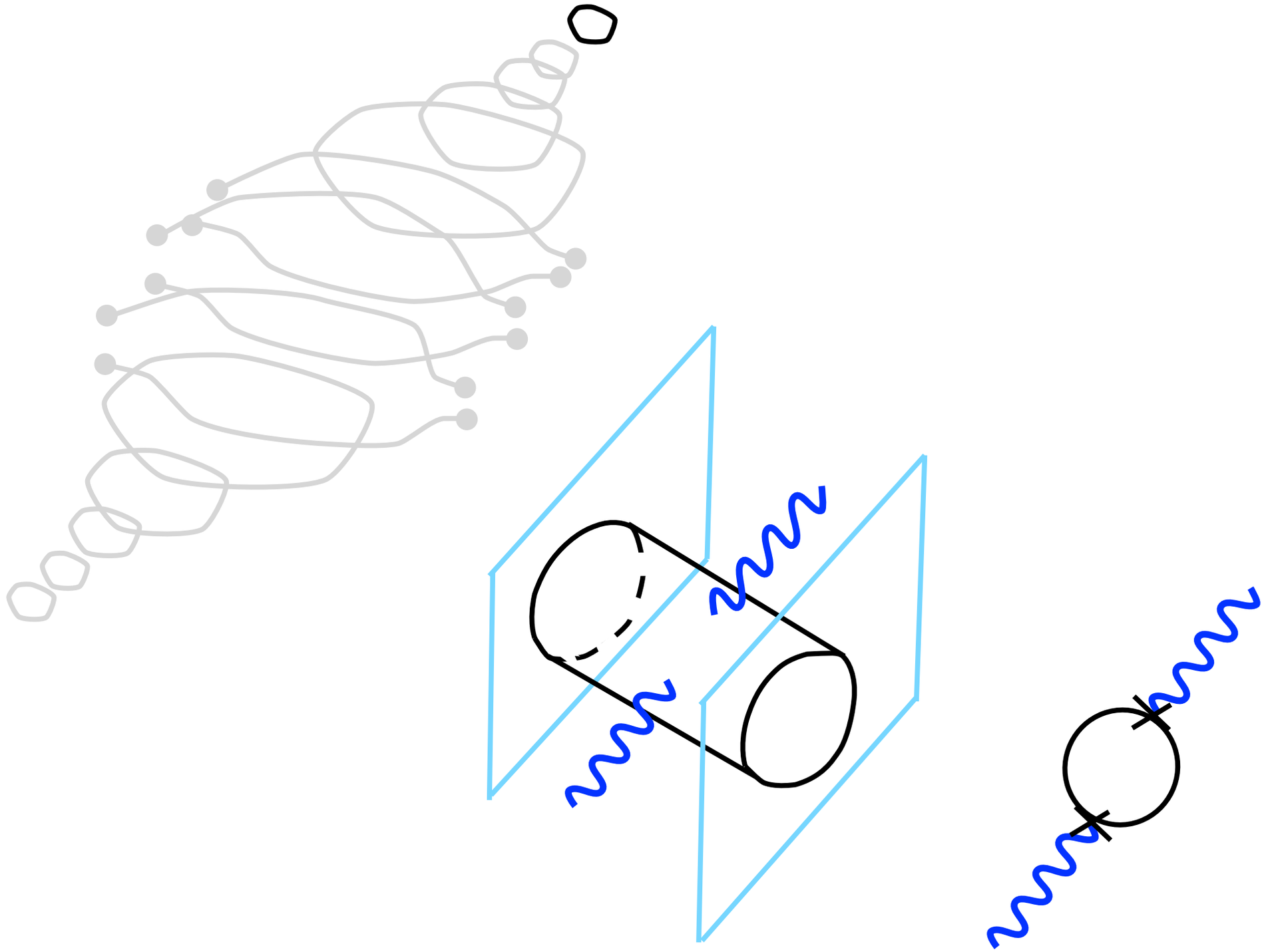










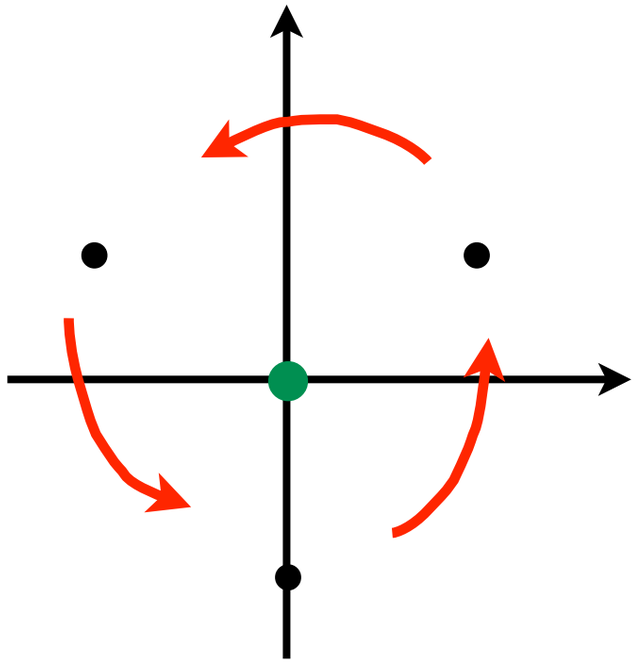


quantum, string one-loop

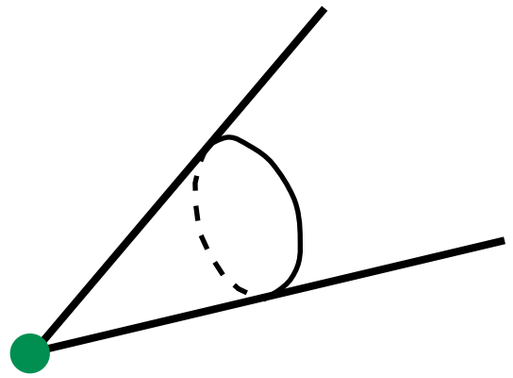
Simple models for extra dimensions

Orbifold: Identify under spatial rotation

$\mathbb{Z}_3$



makes cone



Orientifold: Identify under worldsheet reflection

$$\Omega \left| \right\rangle = \left| \right\rangle$$

$$\frac{1 \pm \Omega}{2} \left| \right\rangle$$ unoriented eigenstate

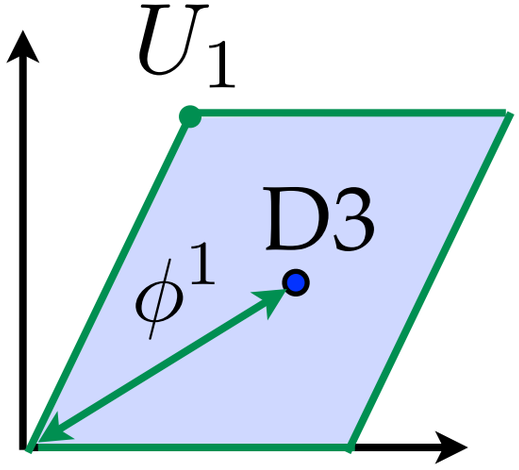
Simple models for extra dimensions

sample $\mathcal{N}=1$ $\quad \mathbb{T}^6/\mathbb{Z}_6'$

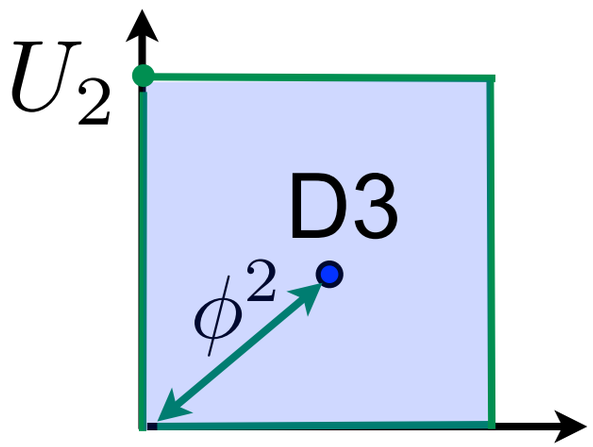
orientifolds: $\quad \mathbb{T}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$

$(Z^1 = X^4 + \bar{U} X^5)$

$\Theta Z^1 = e^{2\pi i v_1} Z^1$ $\quad$ $\Theta Z^2 = e^{2\pi i v_2} Z^2$ $\quad$ $\Theta Z^3 = e^{2\pi i v_3} Z^3$



D7 wraps



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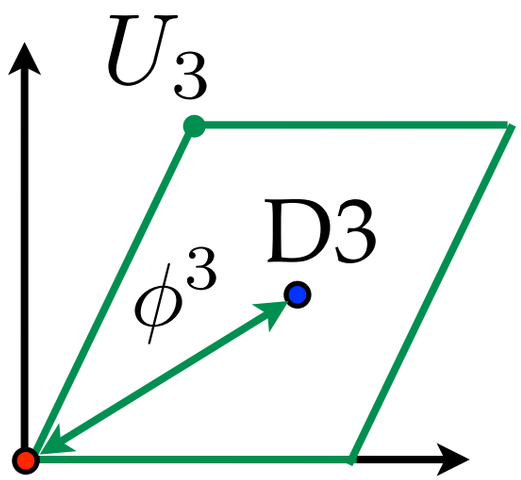


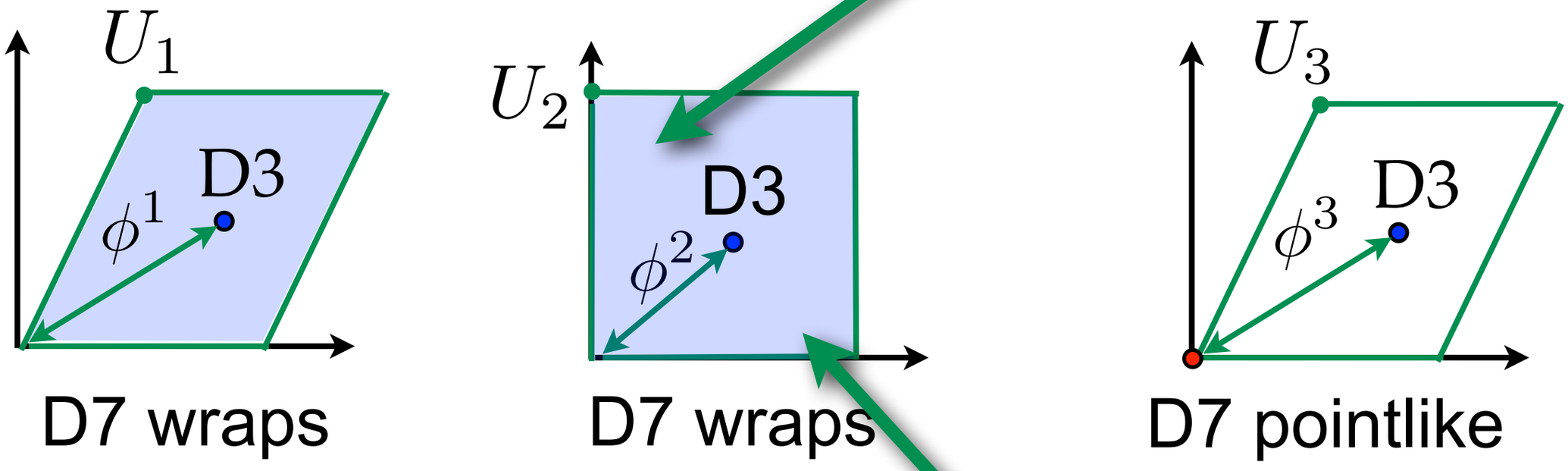
D7 pointlike

$$\mathbb{Z}_6' : (v_1, v_2, v_3) = \left(\frac{1}{6}, -\frac{1}{2}, \frac{1}{3}\right)$$

Θ	Θ^2
"N=1 sector"	"N=2 sector"
"completely twisted"	"partially twisted"

$$(Z^1 = X^4 + \bar{U} X^5)$$

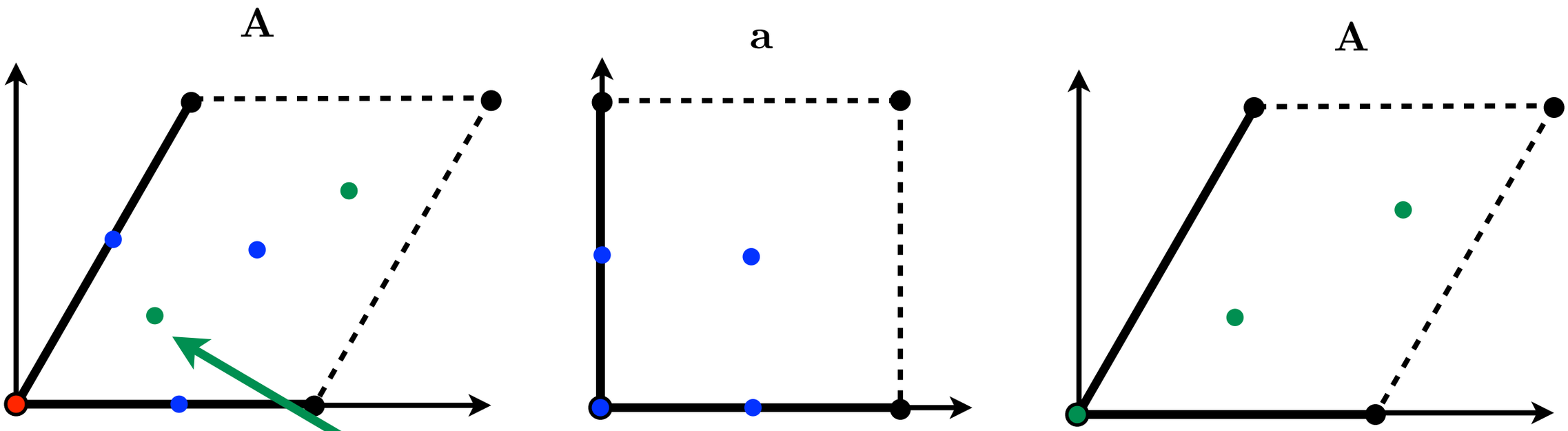
$$\Theta Z^1 = e^{2\pi i v_1} Z^1 \qquad \Theta Z^2 = e^{2\pi i v_2} Z^2 \qquad \Theta Z^3 = e^{2\pi i v_3} Z^3$$



D7 wraps D7 wraps D7 pointlike

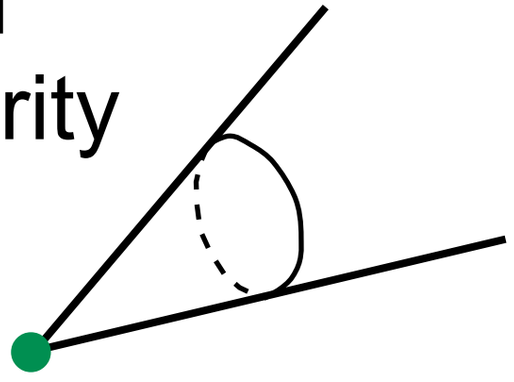
$$\mathbb{Z}_6' : (v_1, v_2, v_3) = \left(\frac{1}{6}, -\frac{1}{2}, \frac{1}{3} \right)$$

Singularities of T^6/Z_6'

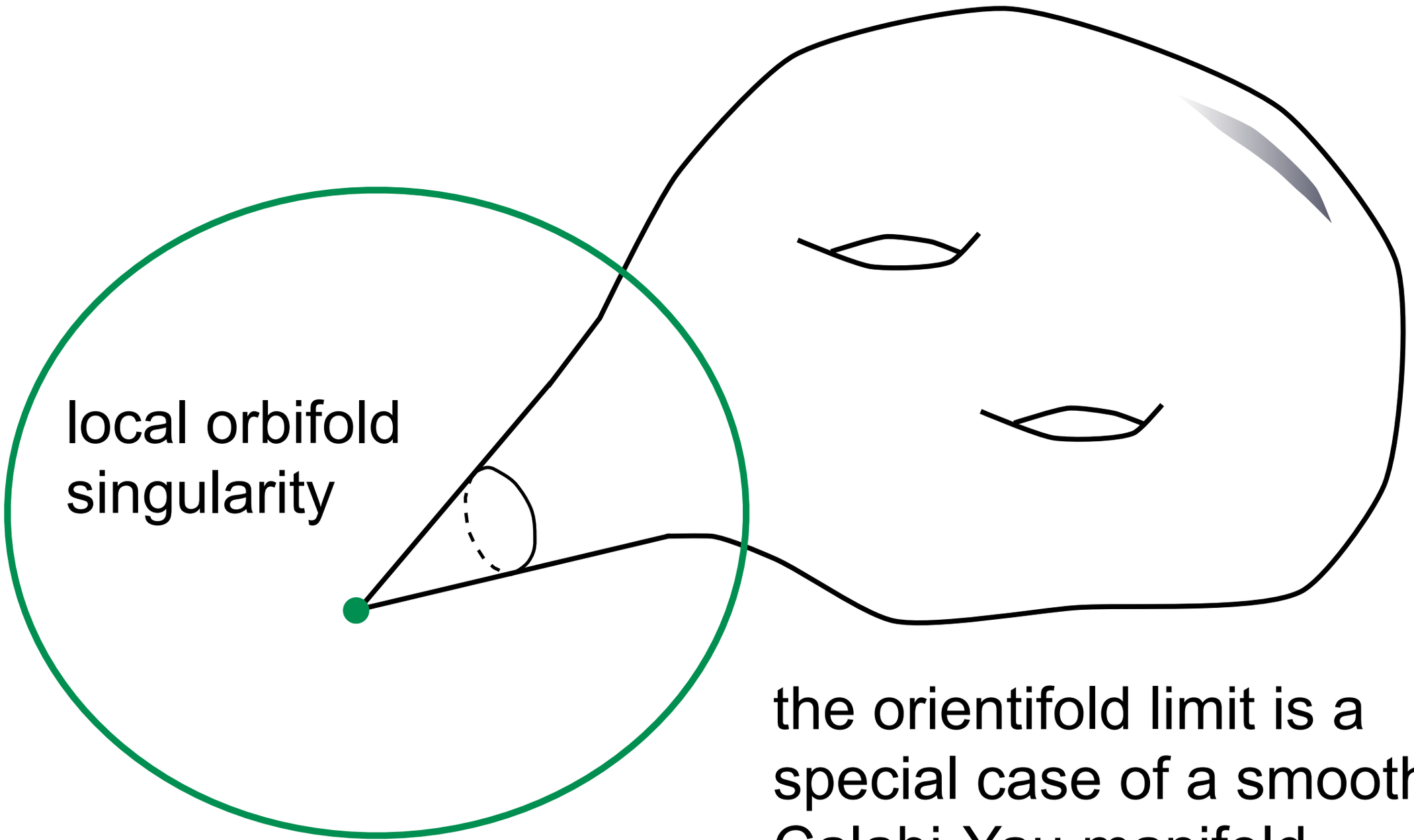


orientifold
singularity

orbifold
singularity



Limit of Calabi-Yau orientifold



the orientifold limit is a special case of a smooth Calabi-Yau manifold.

Review: one-loop effective action

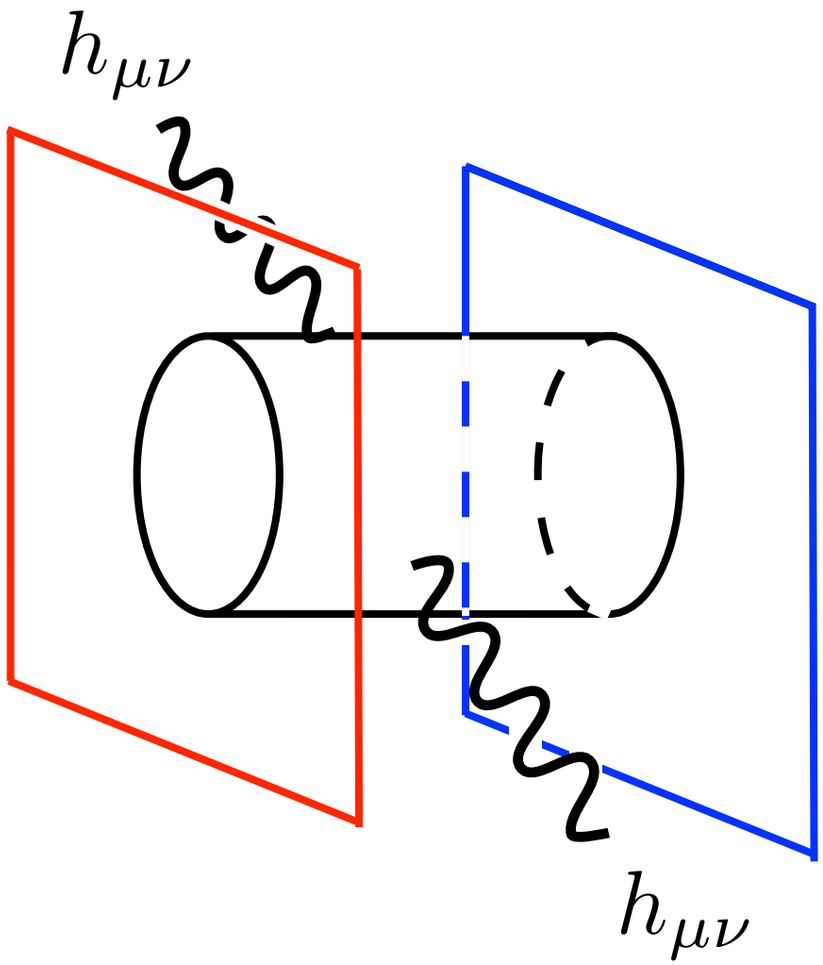
$$\mathcal{L}_{\text{eff}} = \frac{1}{2\kappa^2} R - K_{\Phi_i \bar{\Phi}_{\bar{\jmath}}} \partial_\mu \Phi^i \partial^\mu \bar{\Phi}^{\bar{\jmath}} - \frac{1}{g^2(\Phi)} \text{tr}_a F^2$$

$$- V(\Phi) + \text{corrections}$$

- partially twisted: *g, K* corrections
- completely twisted: *g, K* corrections

First: external one-loop closed string two-point function

Pasquinucci ’97



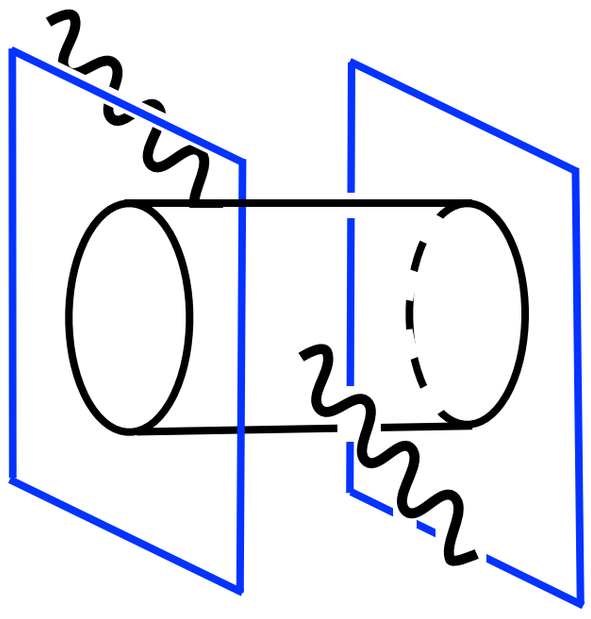
partition function

$$\sim \int \frac{dt}{t^4} \sum_{\vec{\alpha}} \mathcal{Z}^{\vec{\alpha}} \langle V_h V_h \rangle_{\gamma}^{\vec{\alpha}}$$

each vertex operator has 0+2+4 fermions, so contractions contain at most 8 fermions, and the more fermions, the more explicit powers of momentum *p*

K: zero correction from <u>untwisted</u> strings

Pasquinucci ’97



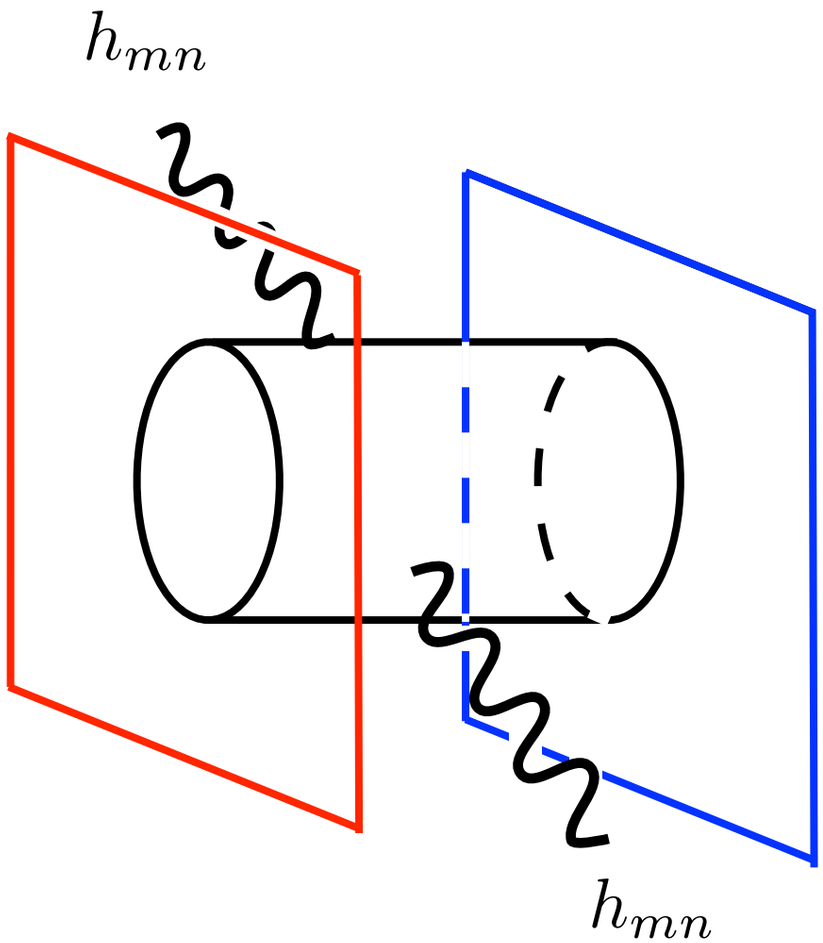
$\mathcal{A}$

N=4 supersymmetry, can be performed in 10 dimensions

gives higher curvature correction from 8-fermion contraction, but this is p^4, so no Kähler correction.

Internal (orbifold-charged) one-loop closed string two-point function

ABFPT '96, ..., M.B., Haack, Körs '05



partition function

$$\sim \int \frac{dt}{t^4} \sum_{\vec{\alpha}} \mathcal{Z}^{\vec{\alpha}} \langle V_T V_{\bar{T}} \rangle_{\gamma}^{\vec{\alpha}}$$

$$V_T = e_{i\bar{i}} (\partial Z^i + i(p \cdot \psi)\Psi^i)(\bar{\partial} Z^{\bar{i}} + i(p \cdot \tilde{\psi})\tilde{\Psi}^i) e^{ip \cdot X}$$

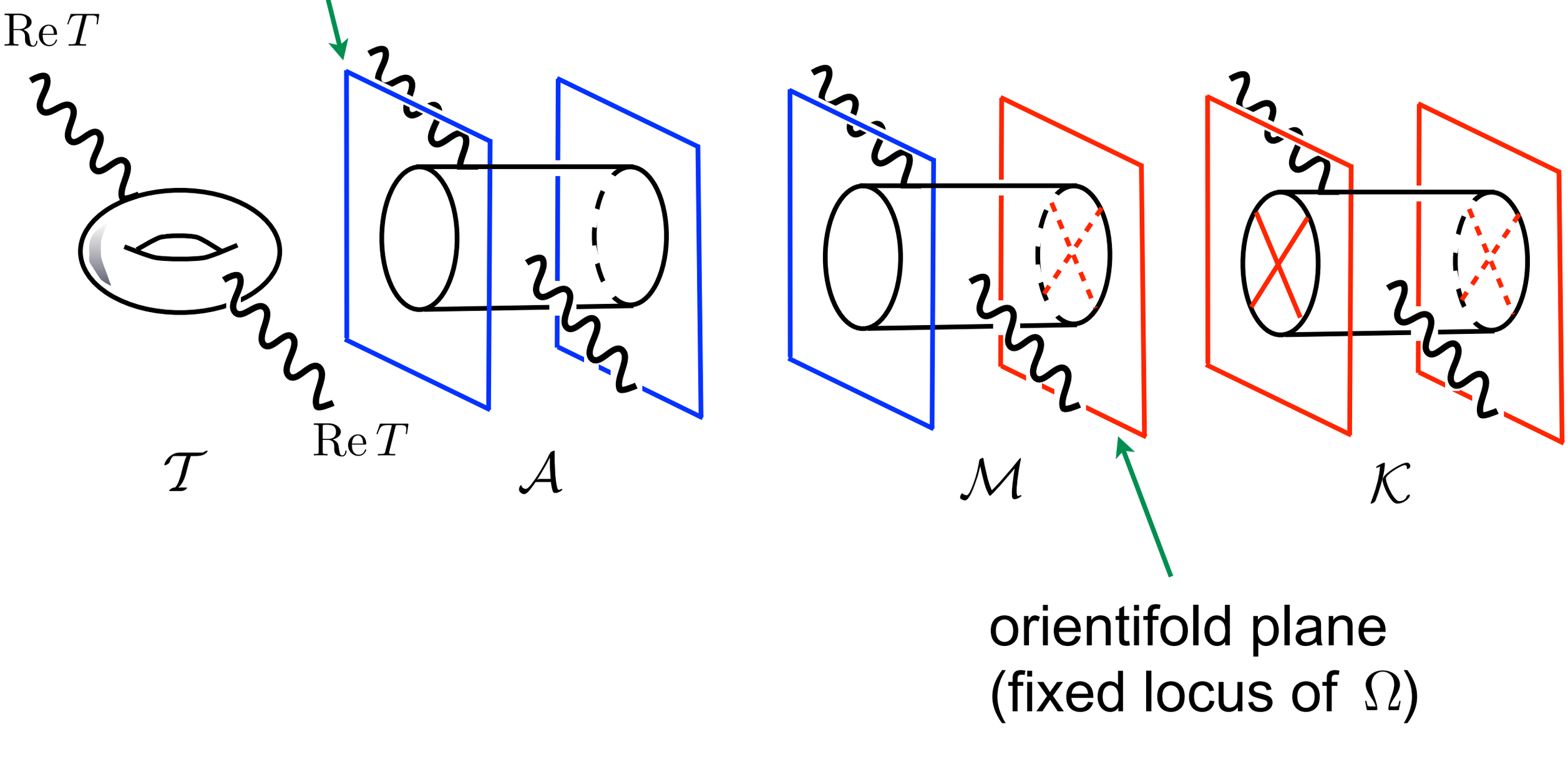
worldsheet bosons

worldsheet fermions

K: partially twisted strings

...
M.B., Haack, Körs '05

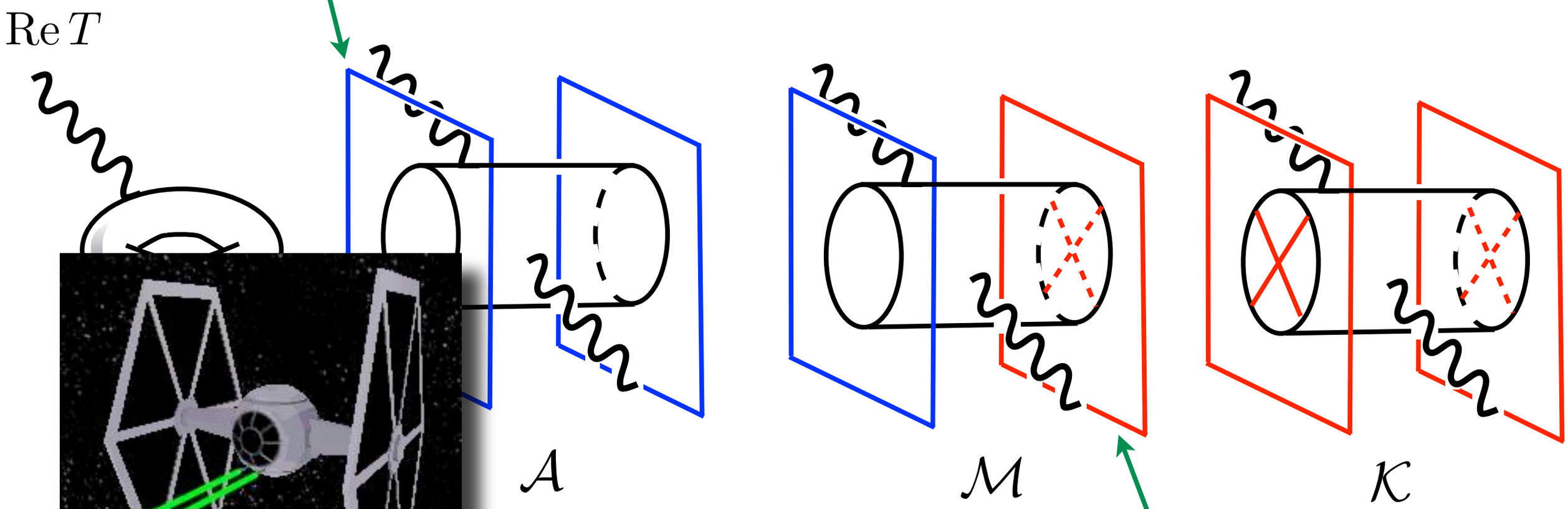
brane at arbitrary position ϕ

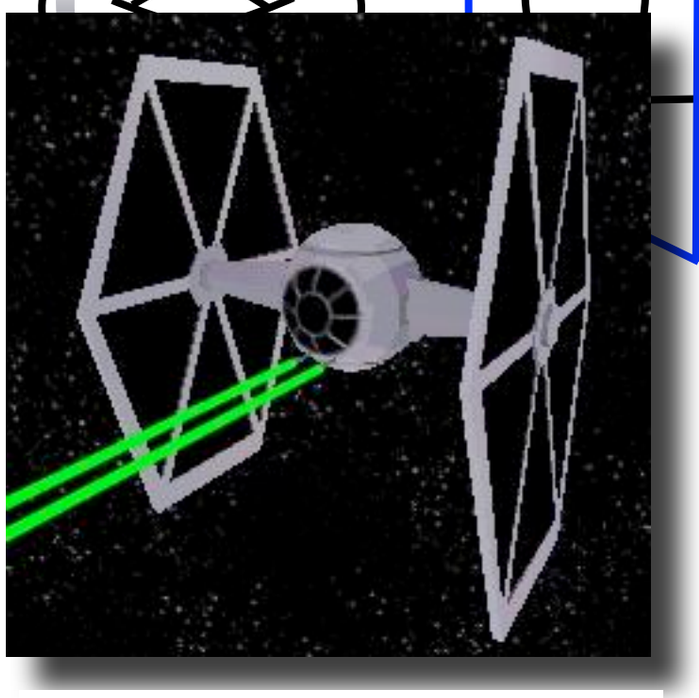


K: partially twisted strings

...
M.B., Haack, Körs '05

brane at arbitrary position ϕ





TIE Fighter (Star Wars)

K: partially twisted strings

M.B., Haack, Körs, '05

Huge simplifications due to N=2 supersymmetry

- 4-fermion term can now contribute, but supersymmetry makes total integrand *constant*!

- in particular, there are no vertex collision poles

that could allow 8-fermion term (naively p^4) to contribute

K: partially twisted strings

M.B., Haack, Körs, '05

"integrate" one-loop corrected Kähler metric to get one-loop corrected Kähler potential:

$$K = -\ln\left((S+\bar{S})(T+\bar{T})(U+\bar{U})\right)$$
$$-\ln\left(1-\frac{1}{8\pi}\sum_i \frac{N_i(\phi_i+\bar{\phi}_i)^2}{(T+\bar{T})(U+\bar{U})} - \frac{1}{128\pi^6}\sum_i \frac{\mathcal{E}_2(\phi_i,U)}{(S+\bar{S})(T+\bar{T})}\right)$$

sum over images of $E_2(\phi_i, U)$

$$E_2(\phi,U) = \sum_{(n,m)\neq(0,0)} \frac{\mathrm{Re}(U)^2}{|n+mU|^4}\exp\left(2\pi i\frac{\phi(n+m\bar{U})+\bar{\phi}(n+mU)}{U+\bar{U}}\right)$$

partially twisted strings: summary

Efforts by many people:

- moduli-dependent gauge coupling: one-loop order

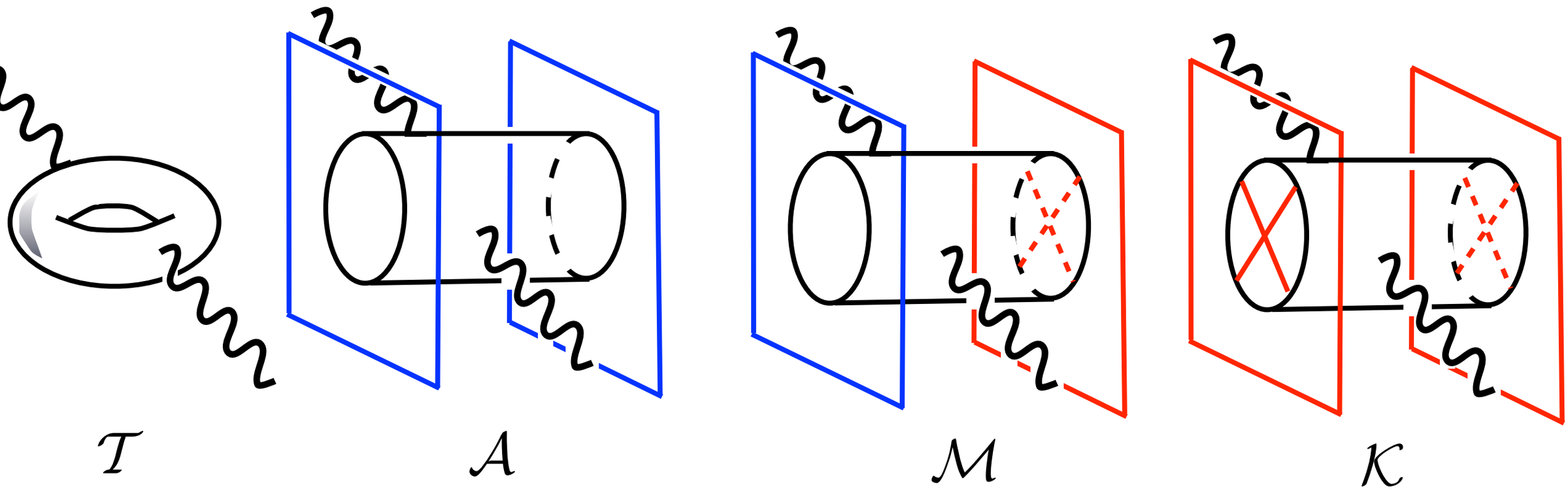
$$g(\Phi, \bar{\Phi}) = g(S, \bar{S}, T, \bar{T}, U, \bar{U}, \phi, \bar{\phi})$$

- Kähler potential: one-loop order

$$K(\Phi, \bar{\Phi}) = K(S, \bar{S}, T, \bar{T}, U, \bar{U}, \phi, \bar{\phi})$$

K: <u>completely twisted</u> for closed strings – no angles yet!

M.B., Haack, Kang, Sjörs ’13



K: completely twisted strings

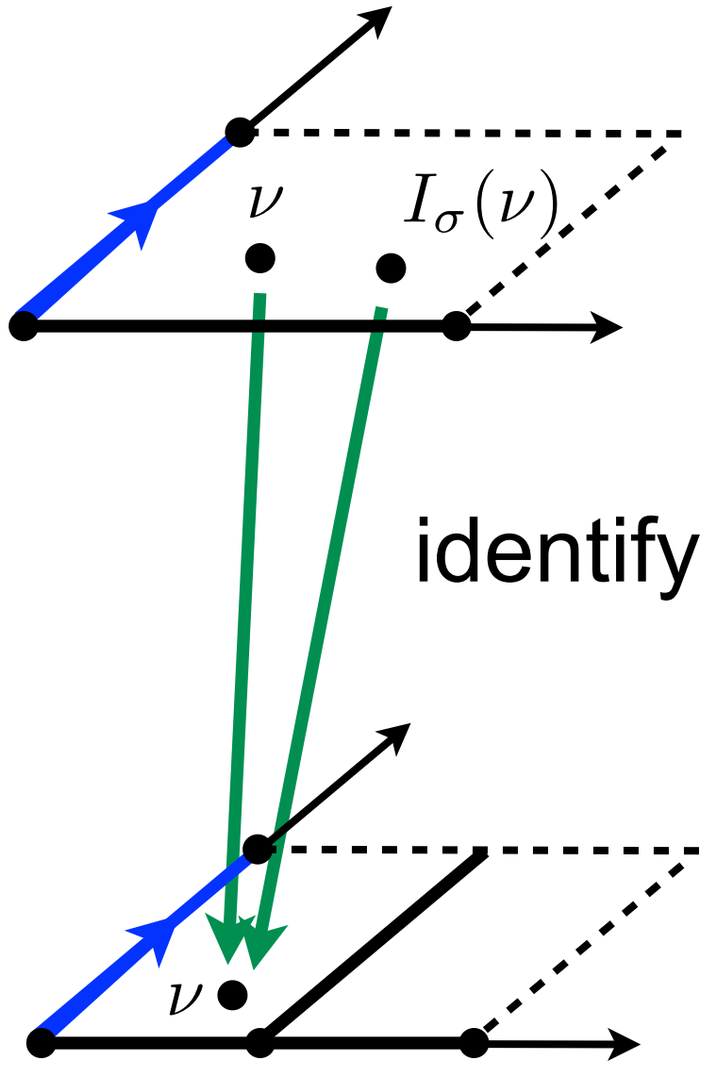
M.B., Haack, Kang, Sjörs, '13

No big simplifications from to N=1 supersymmetry

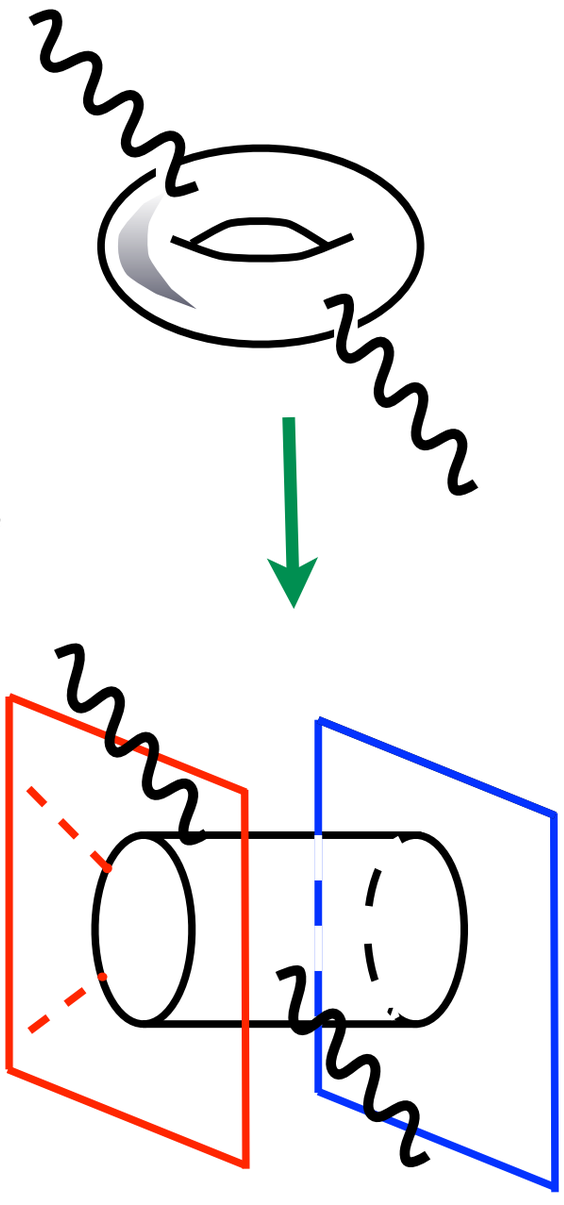
- total integrand not constant

- vertex collision poles allow 8-fermion term (naively p^4) to contribute

Worldsheets by identification



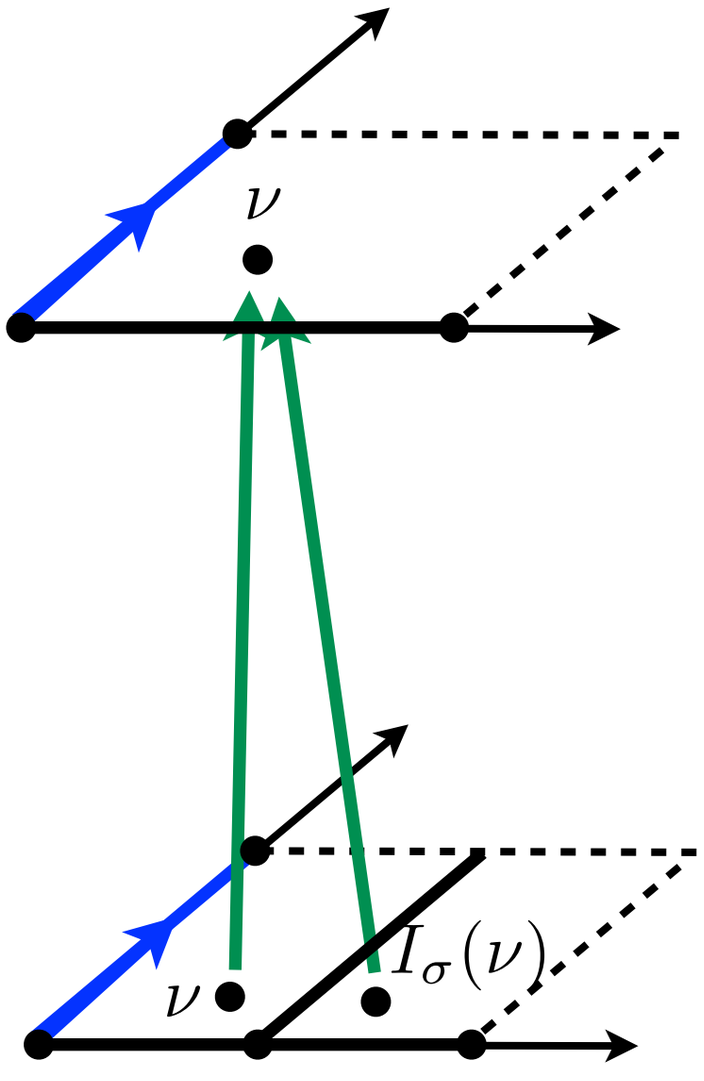
identify $I_{\mathcal{A}} = 1 - \bar{\nu}$

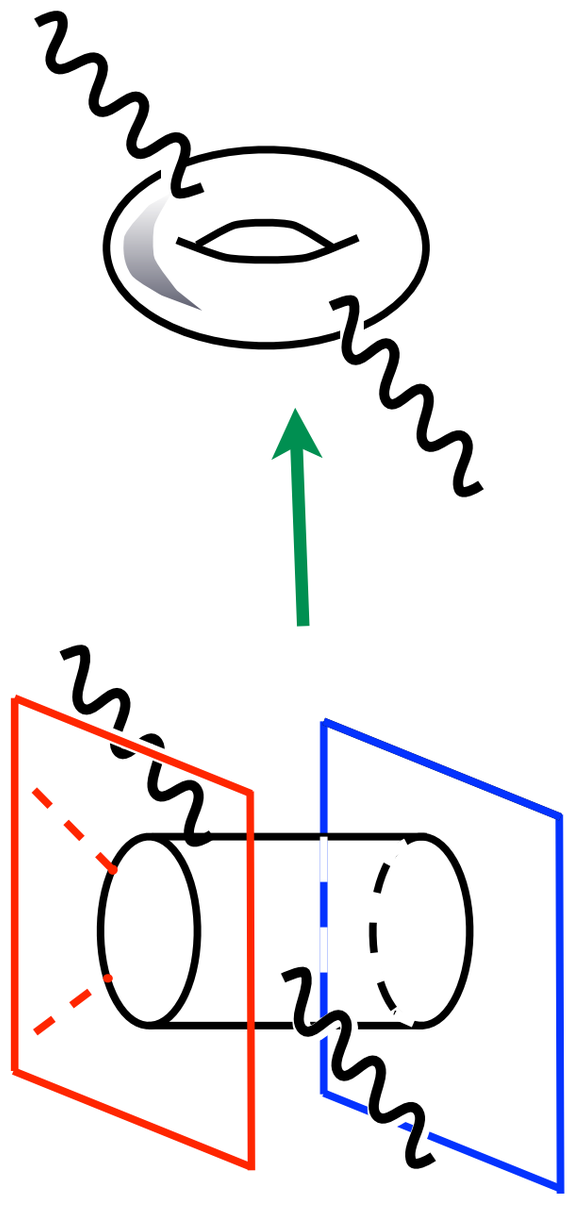


fixed line = boundary

Lifting to covering torus

$$\int_{\sigma} d^2\nu \; (f(\nu) + f(I_\sigma(\nu)) = \int_{\mathcal{T}} d^2\nu \; f(\nu)$$





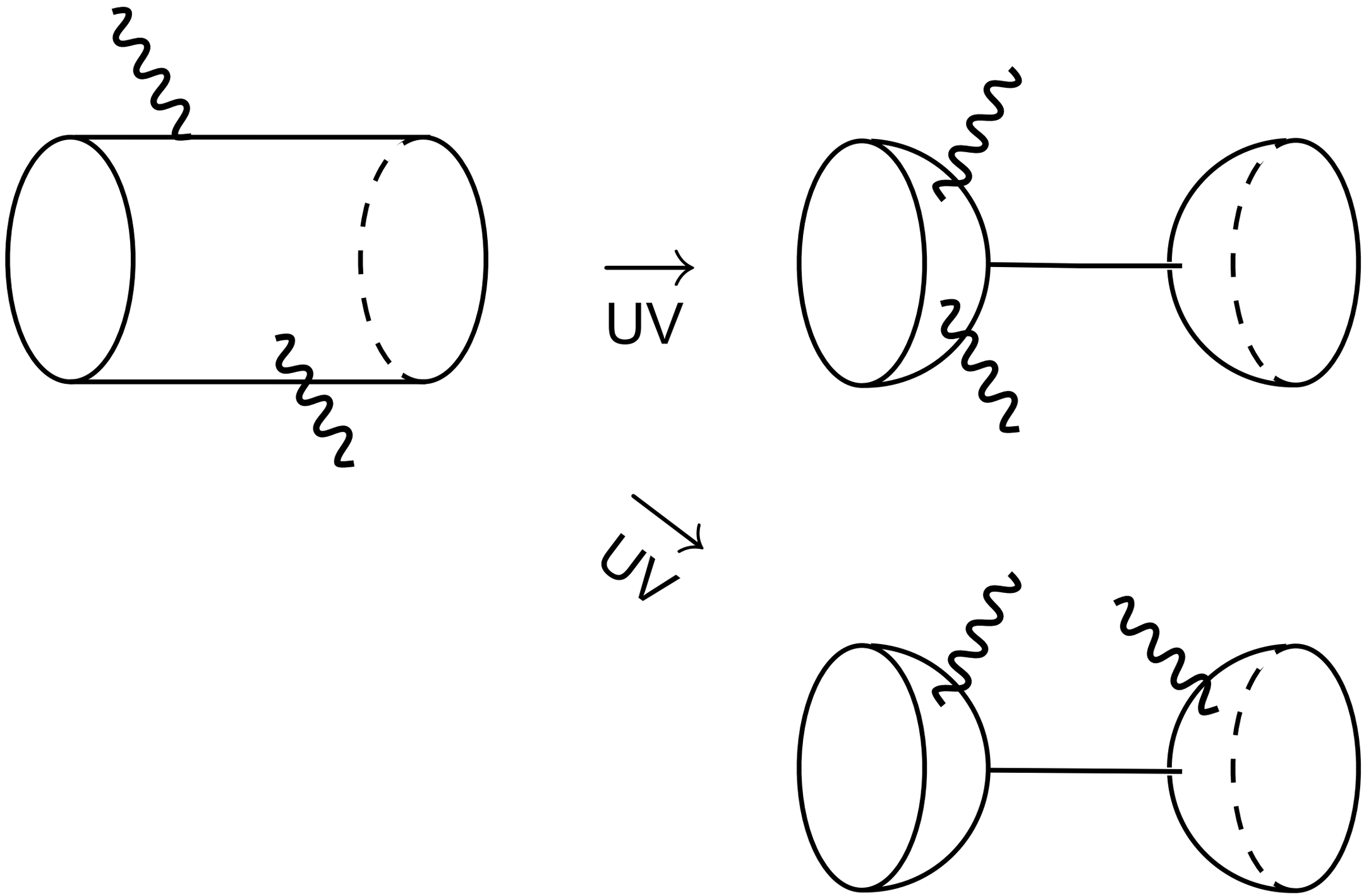
K: completely twisted for closed strings – no angles yet!

- lift to covering torus (by previous slide)
- perform spin structure sum
- perform torus integral with zeta function regularization

$$\int_{\mathcal{T}} d^2\nu \, G^F_{(1/2,1/2+\gamma)}(\nu,\tau)\overline{\partial_\nu G^F_{(1/2,1/2+\gamma)}(\nu,\tau)}$$

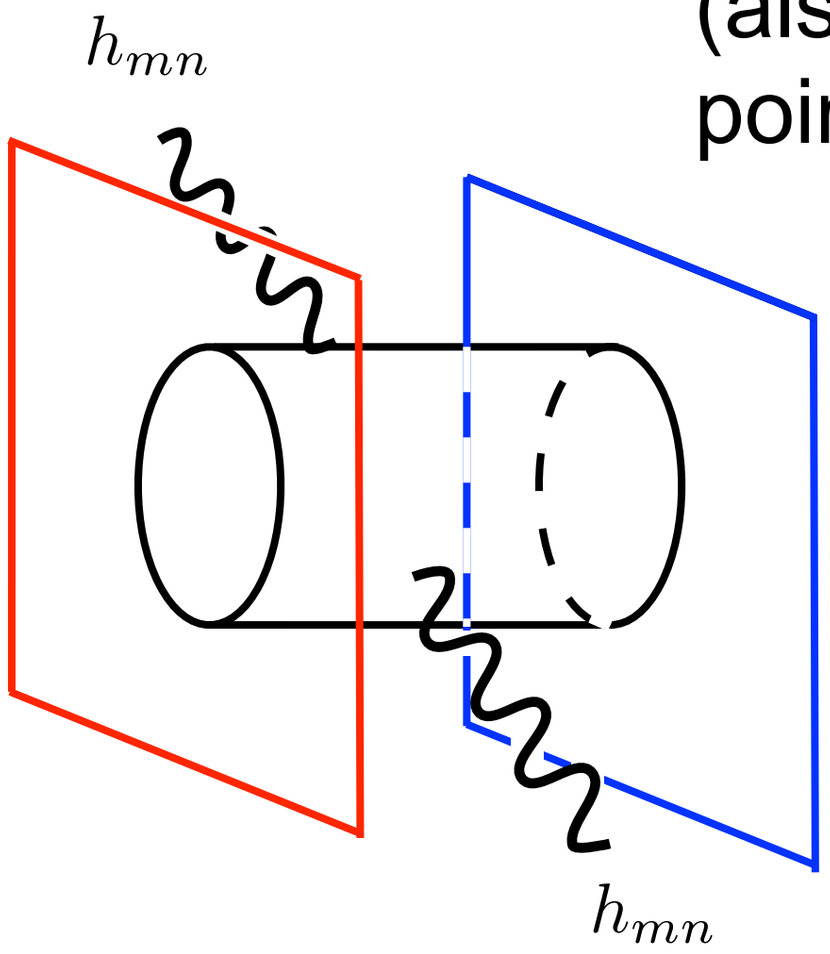
orbifold twist (rational number)

UV finiteness



Zeta function regularization

(also did: without regularization,
point splitting regularization)



plane wave expansion,
integrate over positions

result at order p^2:

$$\lim_{s\to 0} \frac{2(-1)^s \pi}{s} \partial_\gamma \left[\sum_{m,n} \frac{1}{((m+\gamma)^2 + 2(m+\gamma)n\tau_1 + n^2|\tau|^2)^s} \right]$$

K: completely twisted for closed strings

so, gives derivative of this function

$$\sum_{m,n} \frac{1}{((m+\gamma)^2 + 2(m+\gamma)n\tau_1 + n^2|\tau|^2)^s} = \zeta(s, \vec{0}, \vec{\gamma}, G)$$

- use *reflection formula* for twisted Eisenstein series

- finally perform integral over worldsheet modulus

Automorphic form: reflection formula

$$\sum_{m^1,m^2}{}' \frac{\tau_2^s}{|m^1+\gamma^1+\tau(m^2+\gamma^2)|^{2s}} \to \frac{\Gamma(1-s)}{\Gamma(s)}\pi^{2s-1} \sum_{m^1,m^2}{}' \frac{\tau_2^{1-s} e^{2\pi i \vec{m}\cdot\vec{\gamma}}}{|m^2+\tau m^1|^{2-2s}} \,.$$

formally:
partition function of
twisted boson on torus

formally:
torus “Green’s function”
but in the twist

e.g. Siegel

Integrate worldsheet moduli (τ)

Integral to perform:

$$\int_0^\infty dy\, y^{s-1} \partial_\gamma \ln \vartheta_1(\gamma, iy)$$

twisted holomorphic Eisenstein series

Gaberdiel, Keller '09

call *I(s)*, reflection in *s* (again, but different!)

$$I(-1) = \frac{\pi^3}{24 \sin^2(\pi\gamma)} - \frac{\pi}{12}\psi'(\gamma)$$

(finite part)

AFP approach to tau integrals

Angelantonj, Florakis, Pioline ’12

- keeps T-duality manifest
- generalizes Rankin-Selberg-Zagier method

- Selberg-Poincaré series: seed $f(\tau) = \tau_2^{s-w/2} q^{-\kappa}$

difficult analytic continuation,
not eigenfunction of Laplacian

AFP approach to tau integrals

Angelantonj, Florakis, Pioline ’12

Avoid “unfolding”, keep T-duality manifest
generalizes Rankin-Selberg-Zagier method

- Niebur-Poincaré series: seed $f(\tau) = M_{s,w}(-\kappa\tau_2)e^{-2\pi i\kappa\tau_1}$

$$\int_{\mathcal{F}} d\mu\, \tau_2^{3/2}|\eta|^6 \frac{\hat{E}_2 E_4(\hat{E}_2 E_4 - 2E_6)}{\Delta} = -20\sqrt{2}$$

$$d\mu = \frac{d^2\tau}{\tau_2^2}$$

Summary

M.B., Haack, Kang ’11

- “string nonrenormalization theorem”
  for Kähler metric of D-brane moduli

M.B., Haack, Kang, Sjörs ’13

- computed finite constant addition
  to Kähler metric of closed string moduli

- developed/consolidated many techniques for
  amplitude calculations with D-branes and O-planes

- next set of calculations (angles/flux)
  can add moduli dependence

Outlook

Garcia-Etxebarria, Hayashi, Savelli, Shiu ’12
Grimm, Savelli, Weissenbacher ’13

- To be sure of Kähler metric, need α' reduction

- Some of the techniques are brute force:
  term-by-term integrations,
  invariances not manifest.
  AFP approach to tau integrals
  (Niebur-Poincaré series)
  leads the way to more general techniques

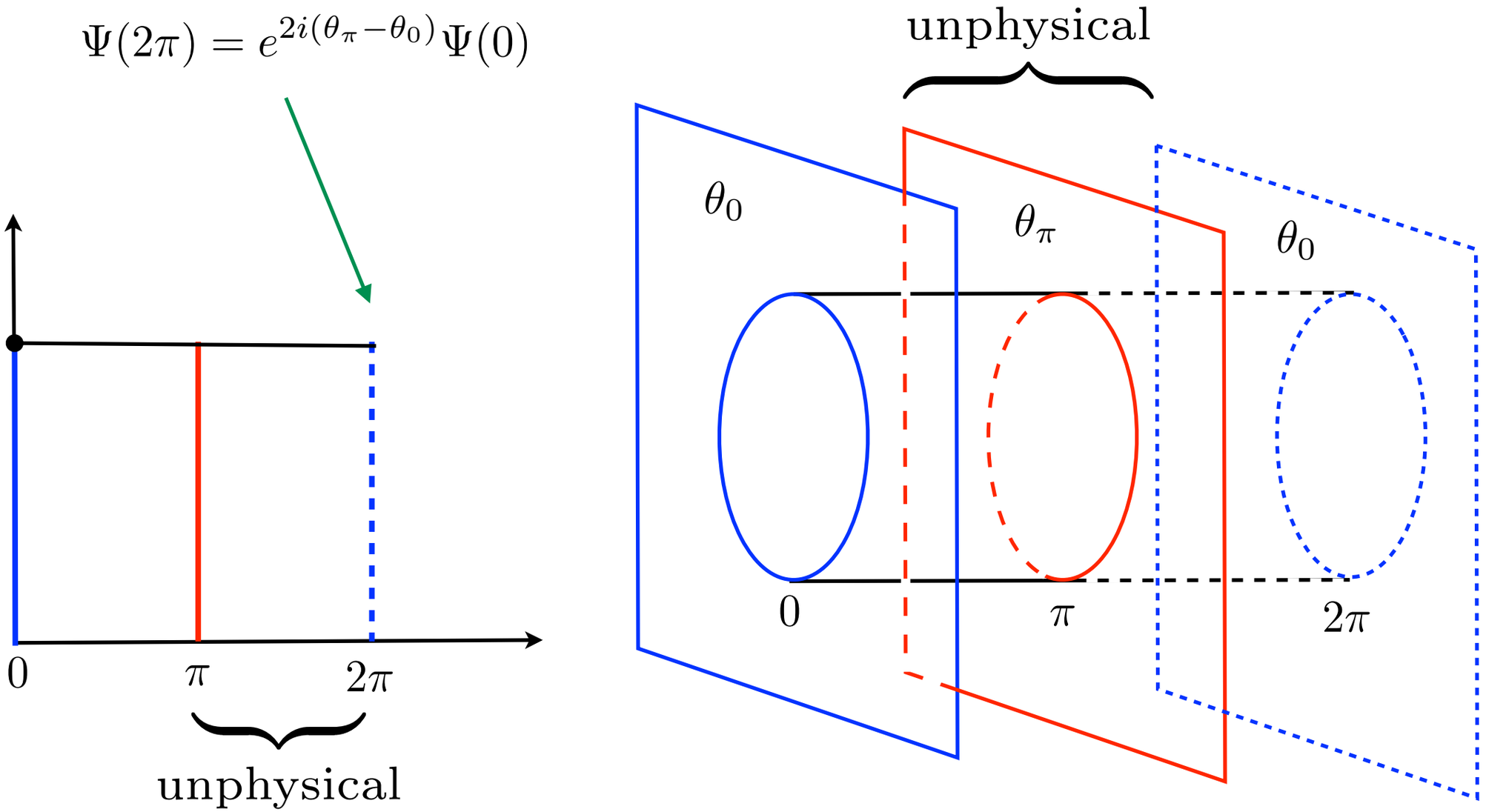
  Angelantonj, Florakis, Pioline ’12

- most interesting to consider flux in string background!

Thank you!

EXTRA SLIDES

Method of images

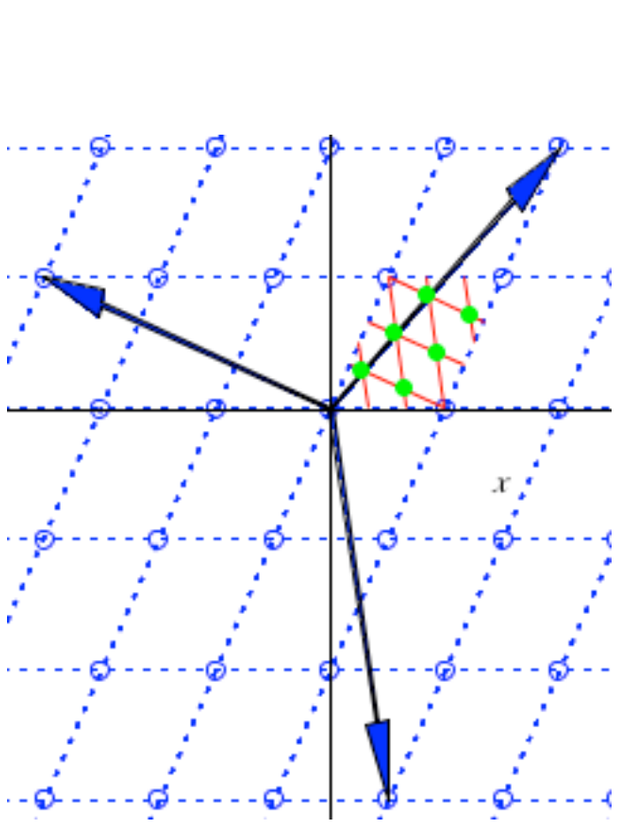


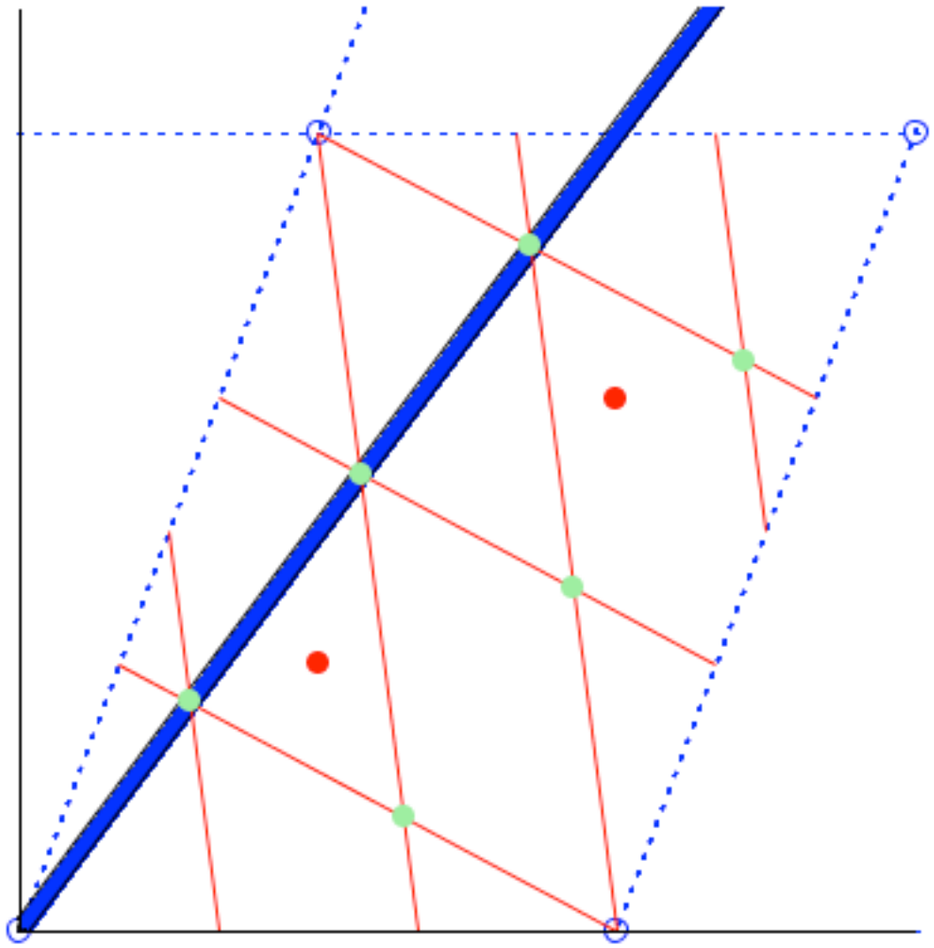
work on covering torus with twist of holomorphic fields

contrast: Bertolini, Billò, Lerda, Morales, Russo ’05

What string states can run in the loop?

Completely twisted states are localized at intersections of brane images





$$I = 7$$

Classical modular form

$$SL(2,\mathbb{Z})$$

$$\gamma \cdot \tau = \frac{a\tau + b}{c\tau + d}$$

$$f(\gamma \cdot \tau) = (c\tau + d)^w f(\tau)$$

$$f(\tau) = c(0) + \sum_{n=1}^{\infty} c(n) q^n$$

$$\varphi(s) = \sum_{n=1}^{\infty} \frac{c(n)}{n^s}$$

Dirichlet series

Classical modular form: reflection formula

$$q = e^{2\pi i \tau}$$

$$f(\tau) = c(0) + \sum_{n=1}^{\infty} c(n) q^n \qquad \varphi(s) = \sum_{n=1}^{\infty} \frac{c(n)}{n^s}$$

$$f(i/y) = (iy)^k f(iy)$$

$$(2\pi)^{-s}\Gamma(s)\varphi(s) = \int_0^{\infty} (f(iy) - c(0)) y^{s-1} dy$$

Classical modular form: reflection formula

$$q = e^{2\pi i \tau}$$

$$f(\tau) = c(0) + \sum_{n=1}^{\infty} c(n) q^n \qquad \varphi(s) = \sum_{n=1}^{\infty} \frac{c(n)}{n^s}$$

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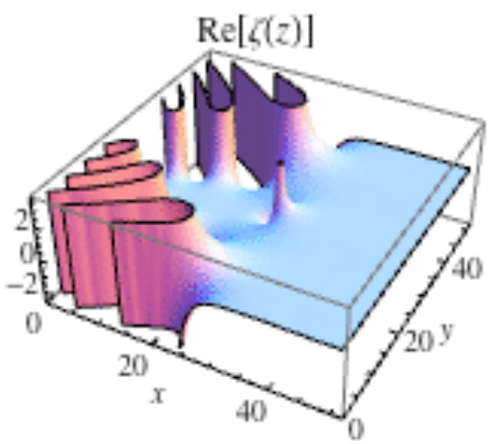
$$(2\pi)^{-s}\Gamma(s)\varphi(s) = \int_0^{\infty} (f(iy) - c(0)) y^{s-1} dy$$

$$(2\pi)^{-s}\Gamma(s)\varphi(s) = (-1)^{k/2}(2\pi)^{s-k}\Gamma(k-s)\varphi(k-s)$$

cf. Riemann zeta

$$f \to \vartheta_3$$

$$\pi^{-s}\Gamma\left(\frac{s}{2}\right)\zeta(s) = \pi^{\frac{s-1}{2}}\Gamma\left(\frac{1-s}{2}\right)\zeta(1-s)$$



Automorphic form: nonholomorphic Eisenstein series

seed $\qquad f(\tau) = \tau_2^s$

$$\mathrm{Im}\,(\gamma \cdot \tau) = \frac{\tau_2}{|c\tau + d|^2} \qquad \tau_2 = \mathrm{Im}\,\tau$$

$$E_s(\tau) = {\sum_{(n,m)}}' \frac{\tau_2^s}{|n + m\tau|^{2s}}$$

weight zero! e.g. Nakahara's book

Automorphic form: nonholomorphic Eisenstein series

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weight zero!

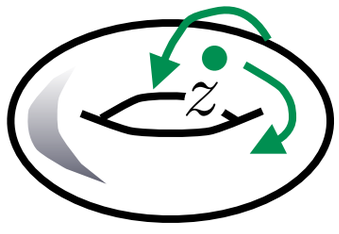
e.g. Nakahara's book

hyperbolic Laplacian

$$\overbrace{\tau_2^2(\partial_{\tau_1}^2+\partial_{\tau_2}^2)}^{\Delta}\tau_2^s = s(s-1)\tau_2^s$$

eigenfunction of Δ

Automorphic form: “generalized” nonholomorphic Eisenstein series



doubly periodic seed:

$$f(\tau) = \tau_2^s \, \exp\left(-2\pi i \frac{z_2}{\tau_2}\right)$$

$$z_2 = \mathrm{Im}\, z$$

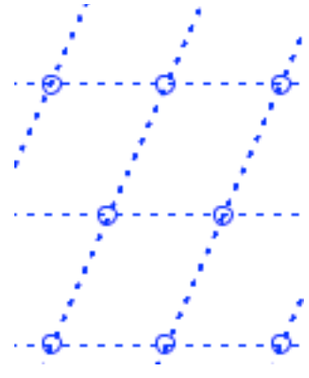
$$\tau_2 = \mathrm{Im}\, \tau$$

$$E_s(z,\tau) = \sum_{(n,m)}{}' \frac{\tau_2^s}{|n+m\tau|^{2s}} \exp\left(2\pi i \underbrace{\frac{z(n+m\bar{\tau}) - \bar{z}(n+m\tau)}{\tau - \bar{\tau}}}_{mx+ny}\right)$$

$$z = -x + \tau y$$

Worldsheet Green's functions

$$\frac{2}{\alpha'}\bar{\partial}_{\bar{z}}\partial_z G_B = -\delta^2(z_{12}) + \frac{1}{\mathrm{vol}}$$



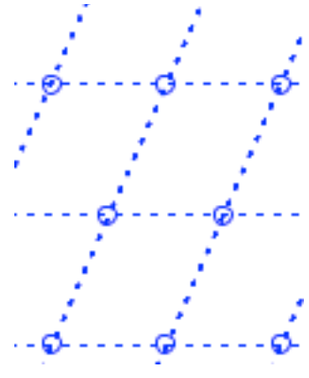
$$\begin{aligned} \omega &= m + n\tau \\ p &= \frac{i}{\tau_2}(n - m\tau) \end{aligned}$$

plane wave expansion

$$G_B(z,\tau) = \sum_{m,n}{}' \frac{1}{|p|^2} e^{2\pi i(p\bar{z}+\bar{p}z)} = \sum_{m,n}{}' \frac{\tau_2^2}{|n-m\tau|^2} e^{2\pi i(p\bar{z}+\bar{p}z)}$$

Worldsheet Green's functions

$$\frac{2}{\alpha'}\bar{\partial}_{\bar{z}}\partial_z G_B = -\delta^2(z_{12}) + \frac{1}{\mathrm{vol}}$$



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generalized nonholomorphic Eisenstein series E_1