Vector-boson pair production at the LHC

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Vector-boson pair production: \( pp \rightarrow WW/ZZ/WZ \rightarrow 4l \)

- ZZ/WW/\( \gamma\gamma \) production important irreducible background to inclusive SM Higgs-boson production
- Probe non-abelian structure of the Standard Model (SM) at high energies
- Search for anomalous couplings
- Backgrounds to new-physics searches, i.e. leptons + \( E_T \) signatures → SUSY-particle pair production
Extensive study of production of WW, WZ, ZZ, Wγ, Zγ, γγ at NLO QCD \cite{Campbell, Ellis, Williams '05; Campbell, Ellis '99; Dixon, Kunszt, Signer '98}

- Results matched with parton showers $\oplus$ combined with soft gluon resummation \cite{Nason, Ridolfi '06; Frixione, Webber '06; Grazzini '06; Dawson, Lewis, Zeng '13}
- On-shell leptonic decays of the vector bosons taken into account (narrow-width approximation) retaining all spin information
- Corrections dominated by the $q\bar{q}$ channels
  - Significant contributions of the channels $gg \rightarrow V_1 V_2 \sim 10\%$ to LO, although formally at $\mathcal{O}(\alpha_s^2)$ \cite{Glover, van der Bij '89; Kao, Dicus '91; Duhrssen et al. '05}
- Even larger corrections of 30% if event selection for Higgs searches is applied \cite{Binoth et al. '06}
Huge NLO $K$-factors at high $p_T$ in $V$-pair production, large residual uncertainties
→ need NNLO for accurate predictions!

- NNLO QCD for $pp \rightarrow \gamma\gamma$ known fully differentially [Bern, de Freitas, Dixon '01; Catani, Cieri, de Florian, Ferrera, Grazzini '12]
- Two-loop matrix elements known for $V\gamma$ [Gehrmann, Tancredi, Weihs, '12/13] and WW (high-energy approx.) [Chachamis, Czakon, Eiras '08]
- Recently: Two-loop master integrals for $q\bar{q} \rightarrow VV$ (planar topologies) [Gehrmann, Tancredi, Weihs '13]
- $pp \rightarrow VV+\text{jet}$ known at NLO [Dittmaier, Kallweit, Uwer '08; Campanario, Englert, Spannowsky, Zeppenfeld '09/10/11, ...]
- Missing:
  - non-planar topologies at 2 loop
  - double-real radiation with 2 soft/collinear partons
  - $pp \rightarrow VV+\text{jet}$ at one loop with one soft/collinear parton
- Approximate NNLO result for $WZ$ production provided recently [Campanario, Sapeta '12] using the LoopSim method [Rubin, Salam, Sapeta '10] (caveat: only reliable at high $p_T$)
Huge NLO $K$-factors at high $p_T$

Prominent shift going from NLO to $\bar{n}$NLO (larger than NLO scale uncertainty!)

Reduction of scale uncertainty

Residual uncertainty due to missing NNLO terms small!

Low $p_T$: method does not work, full NNLO still needed!
$O(\alpha)$ high-energy approximation known for all channels, vector bosons treated in pole-approximation → final-state leptons phenomenologically accessible [Accomando et al. ’02–’06]

Full $O(\alpha)$ corrections known for $W\gamma$ and $Z\gamma$ production in single-pole approximation [Accomando, Denner, Maier ’05]

NNLL effects at two loops published for the WW channel [Kühn, Metzler, Uccirati, Penin ’11]

We have calculated the full one-loop corrections to on-shell $VV$ production at the LHC [Bierweiler, TK, Kühn, Uccirati ’12/13]

Detailed NLO analysis of massive V-pair production [Baglio, Le Duc Ninh, Weber ’13]
Consider anomalous WWZ coupling in $pp \rightarrow W^{\pm}Z \rightarrow l\bar{l}l'^{-}\overline{l}'$ at LHC14: ($p_T, l > 70$ GeV) [Accomando, Kaiser [arXiv:hep-ph/0511088]],

$\Delta y(Z, l)$ and $y(Z)$ distribution

- Significant distortion of distributions through aTGCs and EW corrections
- EW corrections may be misinterpreted as signal of aTGCs
  $\rightarrow$ EW corrections have to be included in aTGC analysis!
NLO Electroweak Corrections – Our Strategy

1. Only consider on-shell vector bosons ⊕ include all mass effects
2. Include leptonic decays → physical final states phenomenologically accessible

Setup

- **Renormalization:**
  On-shell scheme \((G_\mu, M_W, M_Z)\) to obtain UV finite

- **Virtual corrections:**
  IR divergent (regularized by \(m_\gamma, m_q\)), compensated by

- **Real radiation:**
  remaining collinear singularities to be absorbed in PDFs

- **Practical implementation:**
  use MSTW2008LO PDFs [Martin et al. ’09]
  (impact of QED and factorization scheme small, in general sub-percent)
Numerical Results for WW/WZ/ZZ/γγ Production

LHC at 8 TeV, default cuts: $p_{T,V} > 15$ GeV, $|y_V| < 2.5$

- **LO:** Drastically decreasing cross sections for large $p_T$
- **NLO:**
  - Full result, i.e. virtual, soft, hard, and collinear photons included
  - All mass effects included
  - Corrections largest for ZZ, smallest for $\gamma\gamma$

W-Pair production at Tree-Level

- Partonic LO contributions at $\mathcal{O}(\alpha^2)$

- Photon-induced contributions at $\mathcal{O}(\alpha^2)$

- Adopt MRST2004qed PDF set [Martin et al. '05]
  - 😞 no error estimate
  - 😞 no input from data for photon PDF

- Potentially large contribution at high invariant masses!
- **Now possible:** Comparison to NNPDF2.3qed [Carrazza '13], determined from DIS data
Significant discrepancy between \textsc{mrst2004qed} and \textsc{nnpdf2.3qed} for photon PDFs.

\[ \gamma\gamma \rightarrow WW \text{ at LHC14:} \] Huge relative corrections at high invariant masses

- Different predictions for \textsc{mrst2004qed} (+20\%) and \textsc{nnpdf2.3qed} (+70\%)
- Huge error (\(\sim \pm 50\\%\)) on \textsc{nnpdf2.3qed} cross section

\[ \rightarrow \] further constrain photon PDFs through LHC WW production and DY [Carrazza ’13 [arXiv: 1307.1131]]

- Potentially large effects from \(q\gamma\) channels [Baglio, Le Duc Ninh, Weber ’13]
Leptonic Decays – ZZ Production

Purely weak corrections well defined in ZZ production
→ contributions of QED in general below 1%

- compute purely weak corrections to \( pp \rightarrow (Z/\gamma^*)(Z/\gamma^*) \rightarrow e^+ e^- \mu^+ \mu^- \)
- **LO:** full calculation, non-resonant and off-shell effects included.
  - naive fixed-width scheme
  - Complex-Mass Scheme (CMS) [Denner, Dittmaier, Roth, Wieders 2005]
- **NLO:** Two different approaches, including full spin correlations
  - Double-Pole Approximation (DPA): only doubly resonant contributions included, finite width taken into account
    (On-shell projection, **caveat:** non-factorizable corrections neglected)
  - Narrow-Width Approximation (NWA): particles strictly on shell
    \[
    \frac{1}{(Q^2 - M^2)^2 + M^2 \Gamma^2} \rightarrow \frac{\pi}{M \Gamma} \delta(Q^2 - M^2),
    \]
    valid if \( \Gamma/M \rightarrow 0 \).
Numerical Results: $pp \rightarrow ZZ \rightarrow e^+e^−μ^+μ^−$

<table>
<thead>
<tr>
<th>$M_{\text{inv}}^{\text{cut}}(4l)/\text{GeV}$</th>
<th>$\sigma_{\text{naive}}^{\text{LO}}/\text{pb}$</th>
<th>$\sigma_{\text{CMS}}^{\text{LO}}/\text{pb}$</th>
<th>$\sigma_{\text{DPA}}^{\text{LO}}/\text{pb}$</th>
<th>$\sigma_{\text{NWA}}^{\text{LO}}/\text{pb}$</th>
<th>$\delta_{\text{DPA weak}}/%$</th>
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<td>$−29.7$</td>
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**LHC14, standard leptonic cuts**

- **LO**: DPA works well, NWA: discrepancy of 5–10%
- **NLO**: Good agreement ($\sim 1\%$) with $K$-factors obtained in Sudakov approximation [Accomando, Denner, Kaiser 2004]

→ QED contributions (real-photon radiation, photon loops, non-factorizable contributions, corrections to Z-boson decay) only at the 1% level

**Conclusion**: weak $K$-factors of hard process sufficient to describe resonant 4-lepton production at reasonable accuracy
Monte Carlo Implementation in HERWIG++

Our strategy

**Factorization** of EW and QCD corrections:

\[ d\sigma_{QCD \times EW} = K_{\text{weak}}(\hat{s}, \hat{t}) \times d\sigma_{QCD} \]

\(\sigma_{QCD}\): best prediction available for QCD-corrected cross section

- **Assumption**: bulk of EW effects properly described by weak \( K \)-factor \( K_{\text{weak}}(\hat{s}, \hat{t}) \) derived from \( 2 \rightarrow 2 \) process.
- **FSR included** in YFS formalism (SOPHTY) [Hamilton, Richardson 2006] (only dressed leptons)
- \( K_{\text{weak}}(\hat{s}, \hat{t}) \) computed once and for all, data provided as grid files.
- **Some caveats:**
  - factorization assumption only sensible without additional hard jets; \( \rightarrow \) EW corrections to ZZ+jet would have to be included in this configuration.
  - Ansatz does not include corrections to non-resonant or off-shell contributions.
Simulation for $pp \rightarrow ZZ \rightarrow e^+e^- \mu^+\mu^- + X$ at 8 TeV, $M_{ZZ}$ and $p_{T,Z}$ distributions

- **Standard** *Herwig++* setup used
  (v2.6.2, with simple add-on for EW corrections, 10M events), ZZ at NLO QCD matched with parton showers, hadronization included, underlying event switched off

- **huge QCD corrections at large** $p_{T,Z}$, factorized ansatz not justified
  → jet veto, cut on $p_{T,ZZ}$
Problem:
- In WW and WZ production no gauge-invariant separation of dominant weak corrections and QED possible
- QED contributions inevitably lead to IR singularities
- Real radiation has to be included:
  numerical integration has to deal with singular integrands, check cancellation of divergences, check that slicing cuts drop out, ... 
- Finally, QED effects at the level of 1% ($\alpha/\pi$).

Possible solution:
- V + E approximation: Endpoint from subtraction contributions $\oplus$ virtual corrections gives IR finite result [Dittmaier 1999]
- Completely avoid computation of real photon radiation
Fantastic approximation of full result (better than 1% in WW, WZ production)
Approximation works well at high \( p_T \), high invariant masses and near threshold.
NNLO EW corrections at the level of 5–10% at high \( p_T \) [Kühn, Metzler, Penin, Uccirati '11]

Conclusion: Corresponding \( K \)-factor should be used for MC implementation.
4-Lepton Production – Test of our Approach

\[ pp \rightarrow (W^+ \rightarrow )e^+\nu_e (Z \rightarrow )\mu^-\mu^+ \text{ at LHC14, standard event-selection cuts (Preliminary results!)} \]

- **LO**: NWA and DPA work at the level of \( \pm 5\% \)
- **NLO:**
  - \( \delta_{\text{EW}} \): Full NLO EW corrections to production process in NWA, spin correlations for decay process included
  - \( \delta_{\text{V+E}} \): LO in NWA multiplied with \( K_{\text{EW}}(\hat{s}, \hat{t}) \) (unpolarized \( K \)-factor used!)
  - Good agreement at the 1\% level for relative corrections
  - Spin correlations well reproduced!
Simulation for $pp \rightarrow (W^+ \rightarrow e^+\nu_e) (W^- \rightarrow \mu^-\bar{\nu}_\mu) + X$ at 8 TeV, $M_{WW}$ and $p_{T,W}$ distributions

- **Standard Herwig++ setup used**
  (v2.6.2, with simple add-on for EW corrections, 10M events), WW at LO QCD + parton shower, hadronization included, underlying event switched off

- V+E approximate results consistent with arXiv:1208.3147
✓ Precise predictions for vector-boson pair production at NLO QCD & EW exist.
✓ Approximate results at NNLO available $\rightarrow$ large corrections at high $p_T$, reduction of residual theoretical uncertainties

- Photon-induced contributions potentially large $\rightarrow$ further constrain photon PDFs using LHC data

✓ We have computed the full EW corrections to $pp \rightarrow VV$ at hadron colliders
  - Leptonic decays have been implemented for WW, ZZ, WZ production, including spin correlations

- EW corrections to ZZ production will be implemented in the ATLAS analysis of the 8 TeV data set $\rightarrow$ anomalous gauge couplings
- We have proposed a straight-forward MC implementation in the HERWIG++ setup, relying on $2 \rightarrow 2 K$-factors.
  - Claim: predictions match the “true” NLO EW result at the level of a few %.
  - QCD uncertainties (PDFs, hadronization, missing higher orders, . . .) presumably much larger
  - Approach could easily be applied to $V$+jet, $t\bar{t}$ production in the future.

Thank you!

Tobias Kasprzik (KIT)
Reminder: Calculation of Hadronic Cross Sections

Schematic illustration for
\[ \text{pp} \rightarrow V_1 V_2 (+\gamma) + X \]
\[ \rightarrow \ell_1 \ell_2 \bar{\ell}_1 \bar{\ell}_2 (+\gamma) + X \]

Hadronic cross sections

\[
\frac{d\sigma_{AB}(p_A, p_B)}{d^4p_A d^4p_B} = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b \frac{f_{a/A}(x_a, \mu_F) f_{b/B}(x_b, \mu_F)}{x_a x_b} \frac{d\hat{\sigma}_{ab}^{\text{NLO}}(p_a, p_b, \mu_F, \mu_R)}{d^4p_a d^4p_b} \\
\times \mathcal{F}^{(4\ell+\gamma)}(\{\mathcal{O}_{FS}\}), \quad p_{\{a,b\}}^\mu = x_{\{a,b\}} P_{\{a,b\}}^\mu
\]

- Dependence on \( \mu_R, \mu_F \) reduced by inclusion of higher perturbative orders
- \( \mathcal{F}^{(4\ell+\gamma)} \) incorporates definition of observables + phase-space cuts
Reminder: Calculation of Hadronic Cross Sections

Schematic illustration for
\[ pp \rightarrow V_1 V_2 (+\gamma) + X \]
\[ \rightarrow \ell_1 \ell_2 \bar{\ell}_1 \bar{\ell}_2 (+\gamma) + X \]

Hadronic cross sections

\[
d\sigma_{AB}(p_A, p_B) = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b \; f_{a/A}(x_a, \mu_F) f_{b/B}(x_b, \mu_F) \; d\hat{\sigma}_{ab}^{NLO}(p_a, p_b, \mu_F, \mu_R) \\
\times F^{(4\ell+\gamma)}(\{O_{FS}\}), \quad p_{\{a,b\} \mu} = x_{\{a,b\}} P_{\{A,B\} \mu}
\]

NLO partonic cross section:

\[
\hat{\sigma}_{ab}^{NLO} = \hat{\sigma}_{ab}^{LO} + \hat{\sigma}_{ab}^{\text{virt}} + \hat{\sigma}_{ab}^{\text{real}}
\]
Numerical Results for WW/WZ/ZZ/γγ Production

LHC at 14 TeV, default cuts: $p_{T,V} > 15 \text{ GeV}, |y_V| < 2.5$

- **LO:** Drastically decreasing cross sections for large $p_T$
- **NLO:**
  - Full result, i.e. virtual, soft, hard, and collinear photons included
  - corrections largest for ZZ, smallest for $\gamma\gamma$
Numerical Results (II) for WW/WZ/ZZ/γγ Production

LHC at 14 TeV, high-energy cuts: \( p_T, \nu > 15 \) GeV, \(|y_\nu| < 2.5\), \( M_{VV} > 1000 \) GeV

- significant distortion of rapidity distributions at large invariant masses
- Corrections could be misinterpreted as signal of anomalous couplings.
Double-Pole Approximation (DPA)

- **Lowest order**: Amplitude given as a product of on-shell (OS) production amplitude $\otimes$ on-shell decay amplitude $\otimes$ Breit–Wigner:

$$M_{\text{Born,DPA}}^{\bar{q}_1 q_2 \rightarrow V_1 V_2 \rightarrow 4f} = \frac{1}{k_1^2 - M_1^2 + iM_1 \Gamma_1} \frac{1}{k_2^2 - M_2^2 + iM_2 \Gamma_2} \times \sum_{\lambda_1, \lambda_2} M_{\text{Born}}^{\bar{q}_1 q_2 \rightarrow V_1, \lambda_1 V_2, \lambda_2} M_{\text{Born}}^{V_1, \lambda_1 \rightarrow f_3 \bar{t}_4} M_{\text{Born}}^{V_2, \lambda_2 \rightarrow f_5 \bar{t}_6}$$

- Use OS-projected momenta $\hat{k}$ [Denner, Dittmaier, Roth, Wackeroth 2000] in the OS matrix elements:

$$\hat{k}_{1,0} = \frac{1}{2} \sqrt{\hat{s}}, \quad \hat{k}_1 = \frac{k_1}{|k_1|} \beta_w \frac{\sqrt{\hat{s}}}{2}, \quad \ldots$$

- **NLO**: EW corrections consist of factorizable and non-factorizable contributions, e.g.

$$M_{\text{fact}} = \frac{R(k_1, k_2, \theta)}{(k_1^2 - M_1^2 + iM_1 \Gamma_1) (k_2^2 - M_2^2 + iM_2 \Gamma_2)}$$

Caution: Gauge invariance!
EW corrections to $pp \rightarrow W^+W^- \rightarrow \nu_e e^+ \mu^- \bar{\nu}_\mu$ (DPA)

- Standard LHC event selection cuts applied to final-state leptons and missing transverse momentum; additionally $M_{e^+\mu^-} > 500$ GeV required
- Large negative corrections at large transverse momenta
- Substantial negative corrections to inclusive observables
- Error due to DPA about 10% in the relative corrections
- EW corrections significantly larger than experimental error throughout the whole energy range (for $L \sim 30$ fb$^{-1}$)

Simple LL ansatz for $f_{\gamma/p}(x, Q_0^2)$

$$f_{\gamma/p}(x, Q_0^2) = \frac{\alpha}{2\pi} \left[ \frac{4}{9} \ln \left( \frac{Q_0^2}{m_u^2} \right) f_{u/p,v}(x, Q_0^2) + \frac{1}{9} \ln \left( \frac{Q_0^2}{m_d^2} \right) f_{d/p,v}(x, Q_0^2) \right] \otimes \frac{1 + (1 - x)^2}{x}$$

Running of $f_{q/p}(x, Q^2)$ at $O(\alpha)$ affected by photon PDFs!

$$\frac{\partial f_{q/p}(x, \mu^2)}{\partial \ln \mu^2} = \frac{\alpha}{2\pi} \int_x^1 \frac{dy}{y} \left[ P_{qq}(y) Q_q^2 f_{q/p}(x/y, \mu^2) + P_{q\gamma}(y) Q_q^2 f_{\gamma/p}(x/y, \mu^2) \right]$$

Momentum conservation

$$\int_0^1 dx \ x \left[ \sum_q f_{q/p}(x, \mu^2) + f_{g/p}(x, \mu^2) + f_{\gamma/p}(x, \mu^2) \right] = 1$$

$\Rightarrow$ QED effects on $f_{q/p}(x, \mu^2)$ small!

$\Rightarrow$ Still large conceptual uncertainties in $f_{\gamma,0}$
Consider the DIS process

\[ e p \rightarrow e \gamma + X \]

with high-\( p_T \) back-to-back \( e, \gamma \) in the final state

\[
\sigma(e p \rightarrow e \gamma + X) = \int dx^\gamma f_{\gamma/p}(x^\gamma, \mu^2) \hat{\sigma}(e \gamma \rightarrow e \gamma),
\]

related to Compton scattering

\[
x^\gamma = \frac{E^\gamma_T E_e \exp(\eta^\gamma)}{2E_p E_e - E^\gamma_T E_e \exp(-\eta^\gamma)}
\]

\( f_{\gamma/p}(x^\gamma, \mu^2) \) could be in principle extracted from HERA data!
EW Input Schemes – Definition of \( \alpha \)

- \( \alpha(0) \): On-shell definition in the Thomson-limit (zero momentum transfer)

\[
\bar{u}(p) \Gamma_\mu^{Ae\bar{e}}(p, p) u(p) \bigg|_{p^2=m_e^2} = e(0) \bar{u}(p) \gamma_\mu u(p), \quad \alpha(0) = e(0)^2 / 4\pi
\]

- \( \alpha(M_Z) \) obtained via renormalization-group running from 0 to weak scale \( M_Z \)

\[
\alpha(M_Z) = \frac{\alpha(0)}{1 - \Delta \alpha(M_Z)}, \quad \Delta \alpha(M_Z) = \Pi_{f\neq t}^{AA}(0) - \text{Re} \, \Pi_{f\neq t}^{AA}(M_Z^2)
\]

- \( \alpha_{G_\mu} \) defined through the Fermi constant related to the muon lifetime

\[
\alpha_{G_\mu} = \frac{\sqrt{2} G_\mu M_W^2 S_W^2}{\pi} = \frac{\alpha(0)}{1 - \Delta r}
\]

\( \Delta r \) includes corrections to muon lifetime not contained in QED-improved Fermi model

- light-fermion mass logs contained in \( \Pi_{f\neq t}^{AA}(0) \) resummed in effective couplings \( \alpha(M_Z) \) and \( \alpha_{G_\mu} \)
Virtual EW Corrections to \( pp \rightarrow W^- W^+ + X \)

One-loop contributions at \( \mathcal{O}(\alpha^3) \)

- On-shell renormalization of SM parameters
- We use the Fermi scheme to calculate the loop corrections.
  - universal corrections to \( \Delta r \) absorbed in effective LO coupling
- \( V_{ij}^{\text{CKM}} = \delta_{ij} \) within the loops → no renormalization of \( V_{ij}^{\text{CKM}} \)
Real EW Corrections – Infrared Singularities

Real photon radiation at $\mathcal{O}(\alpha^3)$ (generic diagrams): $q\bar{q} \rightarrow W^- W^+ + \gamma$

- **Soft singularities** due to soft photons
- **Initial-state collinear singularities** due to collinear photon radiation off initial-state quarks $\rightarrow$ renormalization of PDFs
- Introduce small **quark mass** $m_q$ and infinitesimal **photon mass** $\lambda$ to regularize divergences $\rightarrow$ results exhibit unphysical $\ln m_q$ and $\ln \lambda$ terms

**Apply phase-space slicing for numerically-stable evaluation of phase-space integral**
Two-cut-off phase-space slicing

- Definition of bremsstrahlung phase space:

\[ \sigma_{real} = \int d\text{PS}(W^- W^+ \gamma) |M^\gamma|^2 \]

- Phase-space decomposition:

\[ \sigma_{real} = \sigma_{hard} + \sigma_{soft} + \sigma_{coll} \]
Phase-Space Slicing

**Soft limit:** $E_\gamma < \Delta E \ll M_W$

$$
\sigma_{\text{soft}}(\Delta E) = -\sigma_{\text{LO}} \left[ \frac{e^2}{(2\pi)^3} \int_{|k_\gamma|<\Delta E} \frac{d^3k_\gamma}{2\sqrt{k_\gamma^2 + \lambda^2}} \sum_{ij} \pm Q_i Q_j(p_i p_j) \right]
$$

**Collinear limit:** $\theta_{q\gamma} < \Delta \theta \ll 1$, $E_\gamma > \Delta E$

$$
\sigma_{\text{coll},q}(\Delta E, \Delta \theta) = \frac{\alpha Q_q^2}{2\pi} \int_0^{1-2\Delta E/\sqrt{\hat{s}}} dz \left( \frac{1+z^2}{1-z} \left( \ln \frac{\hat{s}(\Delta \theta)^2}{4m_q^2} - \frac{2z}{1+z^2} \right) \sigma_{\text{LO}}(z\hat{s}) \right)
$$

**Hard bremsstrahlung:** $\theta_{q\gamma} > \Delta \theta$, $E_\gamma > \Delta E$; numerical evaluation of $\sigma_{\text{hard}}(\Delta E, \Delta \theta)$ without regulators

Numerical result independent of $\ln \Delta E$ and $\ln \Delta \theta$

$\ln m_q$ and $\ln \lambda$ terms cancel in the sum $\sigma_{\text{virt}} + \sigma_{\text{soft}} + \sigma_{\text{coll}}$ in infrared-safe observables
A problem with unstable particles

Naive implementation of finite width in gauge-boson propagator:

\[
\frac{-ig^{\mu\nu}}{q^2 - M_W^2 + i\epsilon} \rightarrow \frac{-ig^{\mu\nu}}{q^2 - M_W^2 + iM_W\Gamma_W}
\]

\(\Gamma_W\) includes Dyson summation of self energies, mixing of perturbative orders

\[\rightarrow \text{might destroy gauge invariance (even at leading order!)}\]
A problem with unstable particles

Naive implementation of finite width in gauge-boson propagator:

\[ \frac{-i g^{\mu\nu}}{q^2 - M_W^2 + i\epsilon} \rightarrow \frac{-ig^{\mu\nu}}{q^2 - M_W^2 + iM_W \Gamma_W} \]

\( \Gamma_W \) includes Dyson summation of self energies, mixing of perturbative orders → might destroy gauge invariance (even at leading order!)

→ CMS universal solution that
  - respects gauge invariance
  - is valid in all phase-space regions

Straightforward implementation:

- **LO:** \( M_V^2 \rightarrow \mu_V^2 = M_V^2 - iM_V \Gamma_V \), \( \cos^2 \Theta_W = \frac{\mu_W^2}{\mu_Z^2} \), \( V = W, Z \)
- **NLO:**
  - Complex renormalization: \( \mathcal{L}_0 \rightarrow \mathcal{L} + \delta \mathcal{L}, \) bare (real) Lagrangian unchanged!
  - Evaluate loop integrals with complex masses
"Real Radiation" of Massive Vector Bosons

**Low energies:** Phase-space and perturbative suppression of $pp \rightarrow V_1 V_2 + (W/Z)$
$\Rightarrow$ contribution below 1%

**High energies:** Logarithmic enhancement of additional soft/collinear $W$- or $Z$-boson radiation
$\Rightarrow$ Investigation of $V_1 V_2 + W/Z$ production as background to $V$ pairs at large $p_T$, $M_{VV}$
- invisible decay of $Z \rightarrow \nu\bar{\nu}$
- collinear emission
- ...

**Simplified approach (details depend on experimental analysis),**
e.g. $W$-Pair production:
1. Include $pp \rightarrow W^- W^+ Z$ with totally inclusive $Z$
2. Include $pp \rightarrow W^- W^+ W^\pm$; treat $W^\pm$ with lowest $p_T$ totally inclusively
LHC at 8 TeV, default cuts: $p_{T,V} > 15$ GeV, $|y_V| < 2.5$

- corrections below 5% even for large transverse momenta and invariant masses
- corrections to $W^- Z$ production enhanced due to $W^- Z W^+$ final states (PDFs!)

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LHC at 8 TeV, default cuts: $p_{T,V} > 15$ GeV, $|y_V| < 2.5$, $p_{T,\gamma} > 15$ GeV, $|y_\gamma| < 2.5$

- real radiation of hard photons marginal ($< 2\%$)
  - → neglect in MC implementation
- corrections largest for WW production
**Virtual corrections** computed in the **FeynArts/FormCalc/LoopTools (FF) framework** ([FA]: Küblbeck, Böhm, Denner 1990; (FC,LT): Hahn, Pérez Victoria 1999; Hahn 2001; (FF): van Oldenborgh, Vermaseren 1990]

1. **FeynArts-3.5:**
   - Automatic generation of diagrams
   - Calculation of amplitudes

2. **FormCalc-6.1:**
   - Algebraical simplification of amplitudes, introduction of tensor coefficients
   - Analytical calculation of squared amplitudes
   - Spin-, colour- and polarization sums
   - Generation of Fortran code

3. **LoopTools-2.5:**
   - Numerical Passarino–Veltman reduction within Fortran
   - Numerically-stable evaluation of scalar integrals

**Bremsstrahlung amplitudes** computed with **FeynArts/FeynCalc ⊕ Madgraph** [Alwall et al.], numerical phase-space integration within Fortran using the **Vegas algorithm**
### ZZ Production: Polarizations

\[ \text{pp} \rightarrow \text{ZZ + X} \]

<table>
<thead>
<tr>
<th>ZZ polarizations</th>
<th>summed</th>
<th>LL</th>
<th>L+</th>
<th>++</th>
<th>+–</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{\text{LO}} / \text{pb} )</td>
<td>( \delta \sigma_{\text{weak}} / \text{pb} )</td>
<td>( 3.810 )</td>
<td>( -0.179 (-0.155) )</td>
<td>( 0.223 )</td>
<td>( -0.009 (-0.008) )</td>
</tr>
<tr>
<td>( \rho_{T,Z} &gt; 500 \text{ GeV} )</td>
<td>( \sigma_{\text{LO}} / \text{pb} )</td>
<td>( 10^{-2} \times [0.101] )</td>
<td>( -0.039 (-0.030) )</td>
<td>( 10^{-7} \times [0.202] )</td>
<td>( -0.975 (+0.748) )</td>
</tr>
<tr>
<td>( \rho_{T,Z} &gt; 1000 \text{ GeV} )</td>
<td>( \sigma_{\text{LO}} / \text{pb} )</td>
<td>( 10^{-5} \times [0.919] )</td>
<td>( -0.557 (-0.387) )</td>
<td>( 10^{-10} \times [0.121] )</td>
<td>( -2.599 (+14.909) )</td>
</tr>
</tbody>
</table>

- **Small transverse momenta:** 70% from (+–), similar \( K \)-factors for all polarizations.
- **Large transverse momenta:** 99% from (+–), other contributions negligible.
- **Note:** One-loop squared term (given in brackets) contributes at \( \sim 10\% \) → large uncertainties due to missing EW higher orders.

**Conclusion:** One \( K \)-factor sufficient to describe polarized ZZ production.
No cuts

- LO cross section dominated by $q\bar{q}$ contributions
- Rapid decrease of cross section for increasing invariant masses
- EW corrections small even for large values of $M_{WW}$
- Large contributions (+80%) from $\gamma\gamma \rightarrow WW$ at high invariant masses
  $\Rightarrow$ Leptonic decays?
LO cross section dominated by $q\bar{q}$ contributions

Rapid decrease of cross section for increasing invariant masses

Employ LHC cuts on decay products:
$\rho_T, l > 20 \text{ GeV}, |y_l| < 3, \rho_T^{\text{miss}} > 25 \text{ GeV}$

$\Rightarrow$ relative effect of $\gamma\gamma \rightarrow WW$ reduced by factor 2 at large $M_{WW}$
**EW Corrections to \( pp \rightarrow W^- W^+ \) – Numerical Results**

**Default cuts:** \( \rho_{T,W^\pm} > 15 \text{ GeV}, y_{W^\pm} < 2.5 \)

- \( \Delta y_{WW} \) production dominated by events near threshold, isotropic production at small \( \Delta y_{WW} \)
- 5% increase of cross section by gg channel

EW corrections at the percent level
- Sizable contributions from \( \gamma \gamma \) at large \( |\Delta y_{WW}| \)
Default cuts: \( \rho_{T,W^\pm} > 15 \text{ GeV}, |y_{W^\pm}| < 2.5 \)

- Assume \( \int \mathcal{L} dt = 200 \text{ fb}^{-1} \)
  \( \Rightarrow \) 1000 WW events with \( \rho_T > 500 \) GeV
- Decreasing admixture of \( gg \), increasing admixture of \( \gamma\gamma \)
- Large admixture of \( \gamma\gamma \) (10%!) 
- Large negative EW corrections (-45%), comparable to QCD corrections
\( \mathcal{O}(\alpha) \)-Contributions to \( pp \rightarrow W^- W^+ \)

No compensation between \( \gamma\gamma \rightarrow WW \) and weak corrections!  
\( \Rightarrow \) Different angular distributions!

- \( \sigma(\gamma\gamma \rightarrow WW) \rightarrow \frac{8\pi \alpha^2}{M_W^2} \) for large \( \hat{s} \)  
  \( \Rightarrow \) strong enhancement in forward & backward directions

- **weak corrections:**
  - negative Sudakov logs for large \( \hat{s} \) and \( \hat{t} \)  
  \( \Rightarrow \) negative corrections for large scattering angles

- \( gg \) small, isotropic

- implications for \( d\sigma/d\Delta y_{WW} \) with \( \Delta y_{WW} = y_{W^+} - y_{W^-} \)  
  (for fixed \( M_{WW} \) this corresponds to the angular distribution in the W-W rest frame!)
**High-energy cuts:** $p^{T}_{T, W^\pm} > 15 \text{ GeV}$, $y_{W^\pm} < 2.5$, $M_{WW} > 1 \text{ TeV}$

- $W^- W^+$ production dominated by small scattering angles
- Drastic forward-backward peaking of $\gamma \gamma \rightarrow W^- W^+$
- Drastic distortion of angular distribution
- $\Sigma \delta$ varies from $-30\%$ and $+45\%$ for $M_{WW} > 1 \text{ TeV}$!
**Very-high-energy cuts:** $p_{T,W^\pm} > 15$ GeV, $y_{W^\pm} < 2.5$, $M_{WW} > 3$ TeV

- NLO EW as important as QCD
- Extreme distortion due to $\gamma\gamma$ (caveat: high uncertainty in photon PDFs)
Transverse-momentum distribution at the LHC

\[ \sigma(\text{fb}) \]

\[ pp \rightarrow W^- W^+ (\gamma/\text{jet}) + X \]
at \( \sqrt{s} = 8 \text{ TeV} \)

\[ p_T^{\text{cut}} (\text{GeV}) \]

\[ \delta(\%) \]
Invariant-mass distribution at the LHC8

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V-pair production at the LHC

18 July 2013, Stockholm
Transverse-momentum distribution at the Tevatron

\[ p\bar{p} \rightarrow W^-W^+(\gamma/jet) + X \]

at \( \sqrt{s} = 1.96 \) TeV

\[ \sigma(fb) \]

\[ p_T^{cut} (GeV) \]

\[ \delta(\%) \]

\[ p_T^{cut} (GeV) \]
Invariant-mass distribution at the Tevatron

\[ p\bar{p} \rightarrow W^- W^+(\gamma/\text{jet}) + X \]

at \( \sqrt{s} = 1.96 \text{ TeV} \)
Simulation for $pp \rightarrow ZZ \rightarrow e^+ e^- \mu^+ \mu^- + X$ at 8 TeV, $M_{e^+ \mu^-}$ and $p_T,l$ distributions

- **Standard Herwig++ setup used** (v2.6.2, with simple add-on for EW corrections, 10M events), ZZ at NLO QCD matched with parton showers, hadronization included, underlying event switched off
- Implementation seems to work fine