Vector-boson pair production at the LHC

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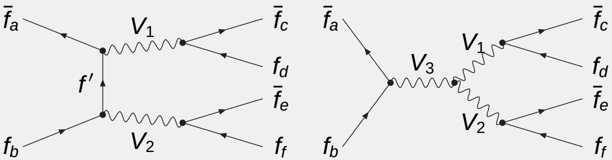
Outline

Introduction – Weak-Boson Pair Production at the LHC

Vector-boson pair production: $\mathrm{pp} \to \mathrm{WW/ZZ/WZ} \to 4l$

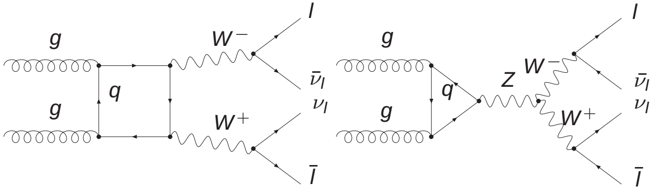


- ZZ/WW/$\gamma\gamma$ production important irreducible background to inclusive SM Higgs-boson production
- Probe non-abelian structure of the Standard Model (SM) at high energies
- Search for anomalous couplings
- Backgrounds to new-physics searches, i.e. leptons + $\not{E}_\mathrm{T}$ signatures
  $\longrightarrow$ SUSY-particle pair production

QCD Corrections

Extensive study of production of WW, WZ, ZZ, Wγ, Zγ, $\gamma\gamma$ at NLO QCD [Campbell, Ellis, Williams '05; Campbell, Ellis '99; Dixon, Kunszt, Signer '98]

- Results matched with parton showers $\oplus$ combined with soft gluon resummation
  [Nason, Ridolfi '06; Frixione, Webber '06; Grazzini '06; Dawson, Lewis, Zeng '13]
- On-shell leptonic decays of the vector bosons taken into account (narrow-width approximation) retaining all spin information
- Corrections dominated by the $q\bar{q}$ channels
  - Significant contributions of the channels $gg \to V_1V_2 \sim 10\,\%$ to LO, although formally at $\mathcal{O}(\alpha_s^2)$ [Glover, van der Bij '89; Kao, Dicus '91; Duhrssen et al. '05]



  - Even larger corrections of 30% if event selection for Higgs searches is applied [Binoth et al. '06]

QCD at NNLO

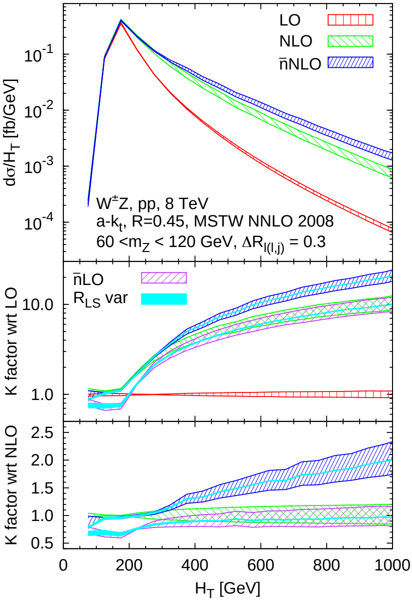
Huge NLO K-factors at high p_{T} in V-pair production, large residual uncertainties

$\rightarrow$ need NNLO for accurate predictions!

✓ NNLO QCD for pp$\rightarrow \gamma\gamma$ known fully differentially [Bern, de Freitas, Dixon '01; Catani, Cieri, de Florian, Ferrera, Grazzini '12]

- Two-loop matrix elements known for $V\gamma$ [Gehrmann, Tancredi, Weihs, '12/13] and WW (high-energy approx.) [Chachamis, Czakon, Eiras '08]
  Recently: Two-loop master integrals for $q\bar{q} \rightarrow VV$ (planar topologies) [Gehrmann, Tancredi, Weihs '13]
- pp$\rightarrow VV+$jet known at NLO [Dittmaier, Kallweit, Uwer '08; Campanario, Englert, Spannowsky, Zeppenfeld '09/10/11, . . .]
- **Missing:**
  - non-planar topologies at 2 loop
  - double-real radiation with 2 soft/collinear partons
  - pp$\rightarrow VV$+jet at one loop with one soft/collinear parton
- Approximate NNLO result for WZ production provided recently [Campanario, Sapeta '12] using the `LoopSim` method [Rubin, Salam, Sapeta '10] (**caveat:** only reliable at high p_{T})

QCD at NNLO – Numerical Results

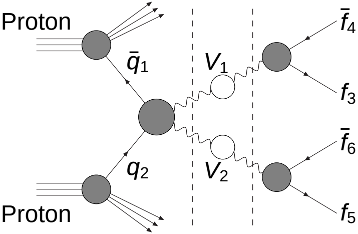


[Campanario, Sapeta [arXiv:1209.4595v1]]

- Huge NLO K-factors at high p_{T}
- Prominent shift going from NLO to n̄NLO (larger than NLO scale uncertainty!)
- Reduction of scale uncertainty
- Residual uncertainty due to missing NNLO terms small!
- Low p_{T}: method does not work, full NNLO still needed!

Electroweak Corrections – Theory Status

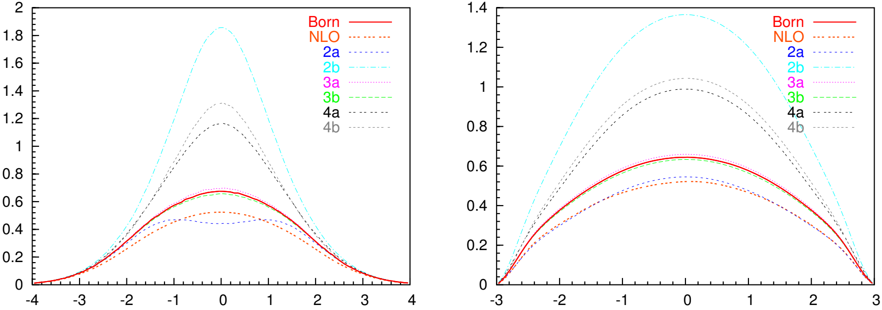
- $\mathcal{O}(\alpha)$ high-energy approximation known for all channels, vector bosons treated in pole-approximation $\rightarrow$ final-state leptons phenomenologically accessible [Accomando et al. '02–'06]



- Full $\mathcal{O}(\alpha)$ corrections known for Wγ and Zγ production in single-pole approximation [Accomando, Denner, Maier '05]
- NNLL effects at two loops published for the WW channel [Kühn, Metzler, Uccirati, Penin '11]
- **We have calculated the full one-loop corrections to on-shell *VV* production at the LHC [Bierweiler, TK, Kühn, Uccirati '12/13]**
- Detailed NLO analysis of massive V-pair production [Baglio, Le Duc Ninh, Weber '13]

Consider anomalous WWZ coupling in $\mathrm{pp} \to W^{\pm}Z \to l\bar{\nu}_l l'\bar{l}'$ at LHC14:
($p_{\mathrm{T},l} > 70$ GeV) [Accomando, Kaiser [arXiv:hep-ph/0511088]],

$\Delta y(Z,l)$ and $y(Z)$ distribution



- Significant distortion of distributions through aTGCs and EW corrections
- EW corrections may be misinterpreted as signal of aTGCs
  $\longrightarrow$ EW corrections have to be included in aTGC analysis!

NLO Electroweak Corrections – Our Strategy

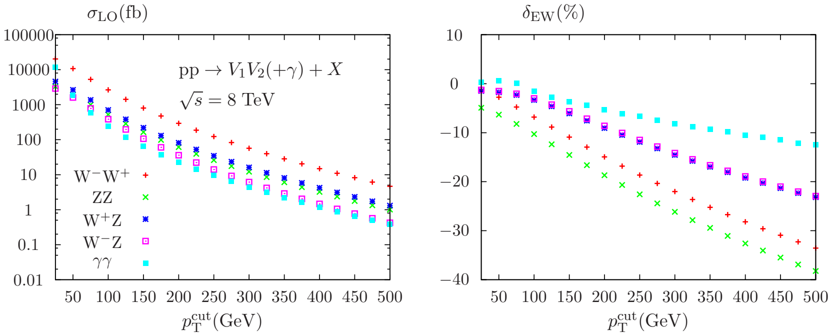
1. Only consider on-shell vector bosons $\oplus$ include all mass effects
2. Include leptonic decays $\rightarrow$ **physical final states phenomenologically accessible**

Setup

- **Renormalization:**
  On-shell scheme (G_μ, M_W, M_Z) to obtain UV finite
- **Virtual corrections:**
  IR divergent (regularized by m_γ, m_q), compensated by
- **Real radiation:**
  remaining collinear singularities to be absorbed in PDFs
- **Practical implementation:**
  use `MSTW2008LO` PDFs [Martin et al. '09]
  (impact of QED and factorization scheme small, in general sub-percent)

Numerical Results for WW/WZ/ZZ/$\gamma\gamma$ Production

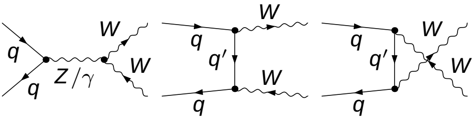
LHC at 8 TeV, default cuts: $p_{\mathrm{T},V} > 15$ GeV, $|y_V| < 2.5$



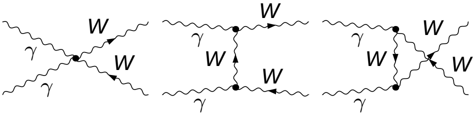
- **LO:** Drastically decreasing cross sections for large p_T
- **NLO:**
  - Full result, i.e. virtual, soft, hard, and collinear photons included
  - All mass effects included
  - Corrections largest for ZZ, smallest for $\gamma\gamma$
- Results published in arXiv:1208.3147, arXiv:1208.3404

W-Pair production at Tree-Level

- **Partonic LO contributions at** $\mathcal{O}(\alpha^2)$



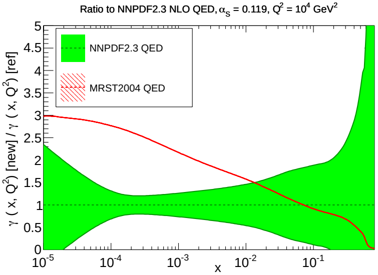
- **Photon-induced contributions at** $\mathcal{O}(\alpha^2)$

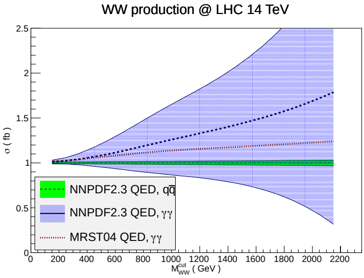


- Adopt `MRST2004qed` PDF set [Martin et al. '05]
  - ☹ no error estimate
  - ☹ no input from data for photon PDF
- Potentially large contribution at high invariant masses!
- **Now possible:** Comparison to **NNPDF2.3qed** [Carrazza '13], determined from DIS data

$\gamma\gamma \to$ WW – Numerical Results & Uncertainties

[Plots by Juan Rojo (thanks!)]





- Significant discrepancy between `MRST2004qed` and `NNPDF2.3qed` for photon PDFs
- $\gamma\gamma \to$ **WW at LHC14:** Huge relative corrections at high invariant masses
  - Different predictions for `MRST2004qed` (+20%) and `NNPDF2.3qed` (+70%)
  - Huge error ($\sim \pm 50\%$) on `NNPDF2.3qed` cross section
  - $\rightarrow$ further constrain photon PDFs through LHC WW production and DY [Carrazza '13 [arXiv: 1307.1131]]
  - Potentially large effects from $q\gamma$ channels [Baglio, Le Duc Ninh, Weber '13]

Purely weak corrections well defined in ZZ production
$\rightarrow$ contributions of QED in general below 1%

- compute purely weak corrections to $\mathrm{pp} \rightarrow (Z/\gamma^*)(Z/\gamma^*) \rightarrow e^+e^-\mu^+\mu^-$
- **LO:** full calculation, non-resonant and off-shell effects included.
  - naive fixed-width scheme
  - Complex-Mass Scheme (CMS) [Denner, Dittmaier, Roth, Wieders 2005]
- **NLO:** Two different approaches, including full spin correlations
  - Double-Pole Approximation (DPA): only doubly resonant contributions included, finite width taken into account
    (On-shell projection, **caveat:** non-factorizable corrections neglected)
  - Narrow-Width Approximation (NWA): particles strictly on shell

$$\frac{1}{(Q^2 - M^2)^2 + M^2\Gamma^2} \rightarrow \frac{\pi}{M\Gamma}\delta(Q^2 - M^2)\,,$$

valid if $\Gamma/M \rightarrow 0$.

Numerical Results: $\mathrm{pp} \to \mathrm{ZZ} \to e^+e^-\mu^+\mu^-$

$\mathrm{pp} \to (\mathrm{Z}/\gamma^*)(\mathrm{Z}/\gamma^*) + X \to \mathrm{e}^+\mathrm{e}^-\mu^+\mu^- + X,\ \lvert\Delta y_{\mathrm{ZZ}}\rvert < 3$					
$M_{\mathrm{inv}}^{\mathrm{cut}}(4l)/\mathrm{GeV}$	$\sigma_{\mathrm{LO}}^{\mathrm{naive}}/\mathrm{pb}$	$\sigma_{\mathrm{LO}}^{\mathrm{CMS}}/\mathrm{pb}$	$\sigma_{\mathrm{LO}}^{\mathrm{DPA}}/\mathrm{pb}$	$\sigma_{\mathrm{LO}}^{\mathrm{NWA}}/\mathrm{pb}$	$\delta_{\mathrm{weak}}^{\mathrm{DPA}}/\%$
LHC14					
500	0.326×10^{-3}	0.326×10^{-3}	0.319×10^{-3}	0.343×10^{-3}	-15.9
600	0.168×10^{-3}	0.168×10^{-3}	0.164×10^{-3}	0.177×10^{-3}	-19.3
700	0.962×10^{-4}	0.962×10^{-4}	0.941×10^{-4}	1.017×10^{-4}	-22.3
800	0.587×10^{-4}	0.587×10^{-4}	0.575×10^{-4}	0.621×10^{-4}	-24.9
900	0.374×10^{-4}	0.374×10^{-4}	0.367×10^{-4}	0.397×10^{-4}	-27.4
1000	0.247×10^{-4}	0.247×10^{-4}	0.242×10^{-4}	0.262×10^{-4}	-29.7

LHC14, standard leptonic cuts

- **LO:** DPA works well, NWA: discrepancy of 5–10%
- **NLO:** Good agreement ($\sim 1\%$) with K-factors obtained in Sudakov approximation [Accomando, Denner, Kaiser 2004]

$\longrightarrow$ QED contributions (real-photon radiation, photon loops, non-factorizable contributions, corrections to Z-boson decay) only at the 1% level

Conclusion: weak K-factors of hard process sufficient to describe resonant 4-lepton production at reasonable accuracy

Monte Carlo Implementation in HERWIG++

Our strategy

Factorization of EW and QCD corrections:

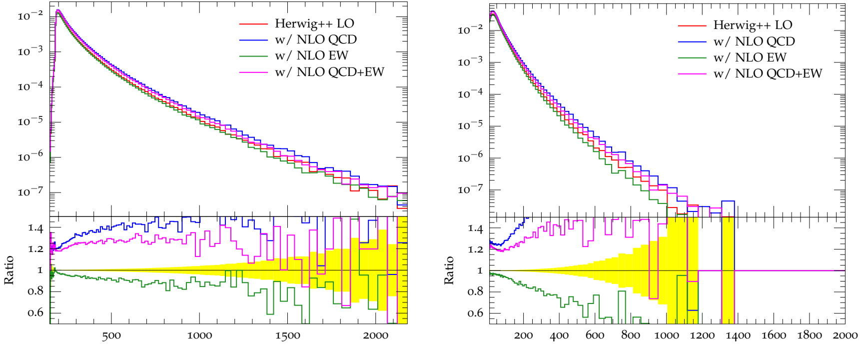
$$\mathrm{d}\sigma_{\mathrm{QCD}\times\mathrm{EW}} = K_{\mathrm{weak}}(\hat{s},\hat{t}) \times \mathrm{d}\sigma_{\mathrm{QCD}}$$

σ_{QCD}: best prediction available for QCD-corrected cross section

- Assumption: bulk of EW effects properly described by weak K-factor $K_{\mathrm{weak}}(\hat{s},\hat{t})$ derived from $2 \to 2$ process.
- FSR included in YFS formalism (SOPHTY) [Hamilton, Richardson 2006] (only dressed leptons)
- $K_{\mathrm{weak}}(\hat{s},\hat{t})$ computed once and for all, data provided as grid files.
- **Some caveats:**
    - factorization assumption only sensible without additional hard jets; $\to$ EW corrections to ZZ+jet would have to be included in this configuration.
    - Ansatz does not include corrections to non-resonant or off-shell contributions.

HERWIG++ Analysis for ZZ Production – Preliminary!

Simulation for $pp \to ZZ \to e^+e^-\mu^+\mu^- + X$ at 8 TeV, M_{ZZ} and $p_{T,Z}$ distributions



- Standard Herwig++ setup used
  (v2.6.2, with simple add-on for EW corrections, 10M events), ZZ at NLO QCD matched with parton showers, hadronization included, underlying event switched off
- huge QCD corrections at large $p_{T,Z}$, factorized ansatz not justified
  $\to$ jet veto, cut on $p_{T,ZZ}$

The QED Issue

Problem:

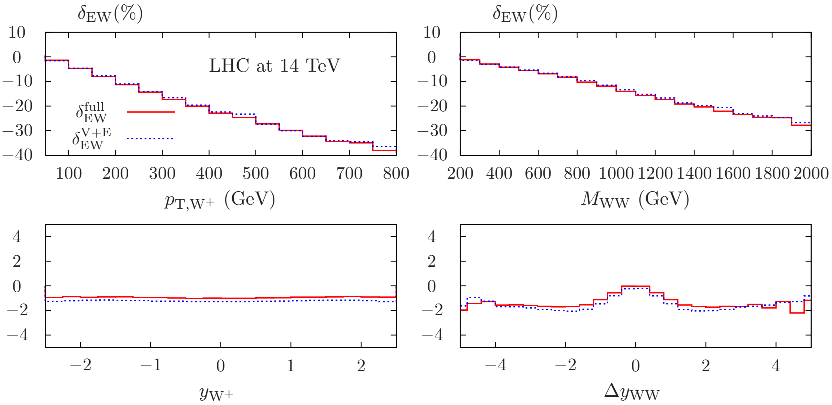
- In WW and WZ production no gauge-invariant separation of dominant weak corrections and QED possible
- QED contributions inevitably lead to IR singularities
- Real radiation has to be included:
  numerical integration has to deal with singular integrands, check cancellation of divergences, check that slicing cuts drop out, . . .
- Finally, QED effects at the level of 1% (α/π).

Possible solution:

- V + E approximation: Endpoint from subtraction contributions $\oplus$ virtual corrections gives IR finite result [Dittmaier 1999]
- Completely avoid computation of real photon radiation

V + E Approximation

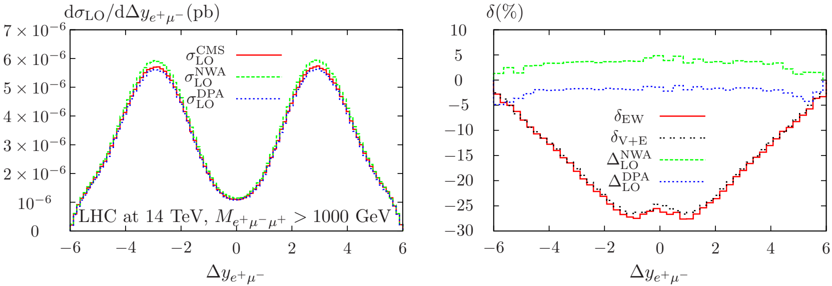
On-shell W-Pair production at LHC14, default cuts



- Fantastic approximation of full result (better than 1% in WW, WZ production)
- Approximation works well at high p_{T}, high invariant masses and near threshold.
- NNLO EW corrections at the level of 5–10% at high p_{T} [Kühn, Metzler, Penin, Uccirati '11]

Conclusion: Corresponding *K*-factor should be used for MC implementation.

$pp \to (W^+ \to) e^+ \nu_e\ (Z \to) \mu^- \mu^+$ at LHC14, standard event-selection cuts (Preliminary results!)

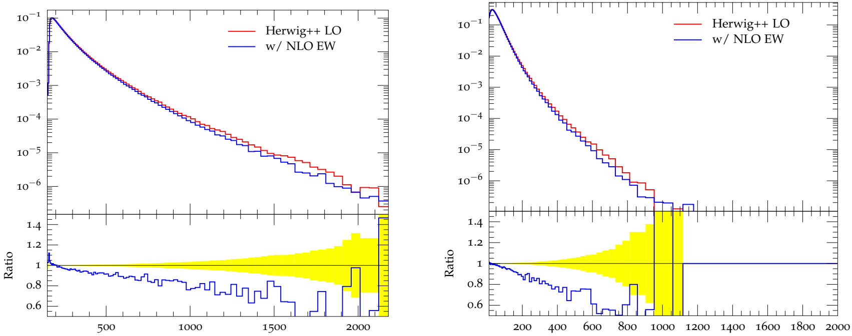


- **LO:** NWA and DPA work at the level of $\pm 5\%$
- **NLO:**
  - $\boldsymbol{\delta_{EW}}$**:** Full NLO EW corrections to production process in NWA, spin correlations for decay process included
  - $\boldsymbol{\delta_{V+E}}$**:** LO in NWA multiplied with $K_{EW}(\hat{s}, \hat{t})$ (unpolarized K-factor used!)

→ Good agreement at the 1% level for relative corrections

→ Spin correlations well reproduced!

Simulation for $pp \to (W^+ \to)e^+\nu_e\ (W^- \to)\mu^-\bar{\nu}_\mu + X$ at 8 TeV, M_{WW} and $p_{T,W}$ distributions



- Standard Herwig++ setup used
  (v2.6.2, with simple add-on for EW corrections, 10M events), WW at LO QCD $\oplus$ parton shower, hadronization included, underlying event switched off
- V+E approximate results consistent with arXiv:1208.3147

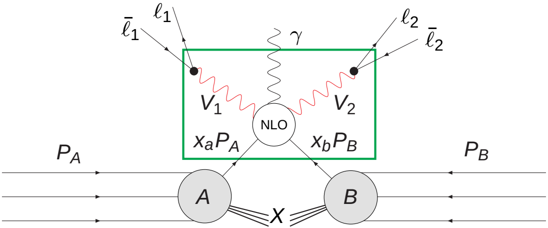
Summary & Outlook

- ✓ Precise predictions for vector-boson pair production at NLO QCD & EW exist.
- ✓ Approximate results at NNLO available → large corrections at high p_{T}, reduction of residual theoretical uncertainties
- Photon-induced contributions potentially large → further constrain photon PDFs using LHC data
- ✓ We have computed the full EW corrections to $\mathrm{pp} \to VV$ at hadron colliders
  - Leptonic decays have been implemented for WW, ZZ, WZ production, including
  - spin correlations
- EW corrections to ZZ production will be implemented in the ATLAS analysis of the 8 TeV data set → anomalous gauge couplings
- We have proposed a straight-forward MC implementation in the `HERWIG++` setup, relying on $2 \to 2$ K-factors.
  - Claim: predictions match the “true” NLO EW result at the level of a few %.
  - QCD uncertainties (PDFs, hadronization, missing higher orders, ...) presumably much larger
  - Approach could easily be applied to V+jet, $\mathrm{t\bar{t}}$ production in the future.

Thank you!

Reminder: Calculation of Hadronic Cross Sections

Schematic illustration for
$\mathrm{pp} \to V_1 V_2 (+\gamma) + X$
$\to \ell_1 \ell_2 \bar{\ell}_1 \bar{\ell}_2 (+\gamma) + X$

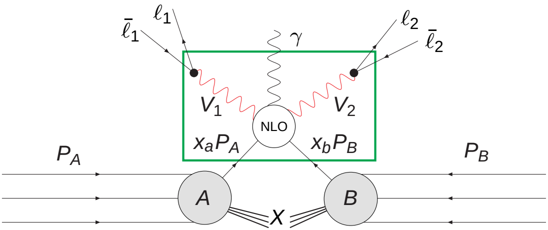


Hadronic cross sections

$$\mathrm{d}\sigma_{AB}(p_A, p_B) = \sum_{a,b} \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \, f_{a/A}(x_a, \mu_\mathrm{F}) \, f_{b/B}(x_b, \mu_\mathrm{F}) \, \mathrm{d}\hat{\sigma}^{\mathrm{NLO}}_{ab}(p_a, p_b, \mu_\mathrm{F}, \mu_\mathrm{R})$$
$$\times \mathcal{F}^{(4\ell+\gamma)}(\{\mathcal{O}_\mathrm{FS}\}), \qquad p^\mu_{\{a,b\}} = x_{\{a,b\}} P^\mu_{\{A,B\}}$$

- Dependence on $\mu_\mathrm{R}, \mu_\mathrm{F}$ reduced by inclusion of higher perturbative orders
- $\mathcal{F}^{(4\ell+\gamma)}$ incorporates definition of observables + phase-space cuts

Schematic illustration for
$\mathrm{pp} \to V_1 V_2 (+\gamma) + X$
$\to \ell_1 \ell_2 \bar{\ell}_1 \bar{\ell}_2 (+\gamma) + X$



Hadronic cross sections

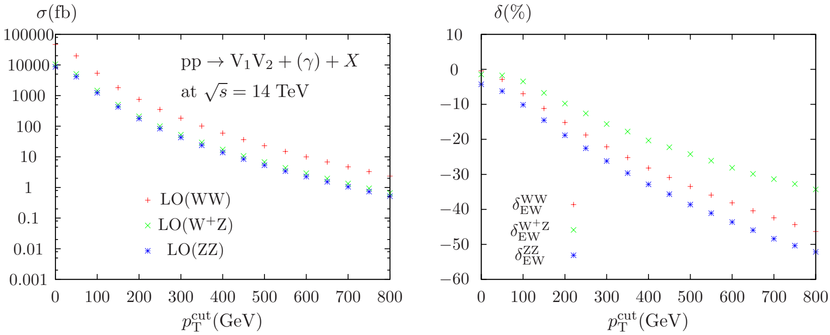
$$\mathrm{d}\sigma_{AB}(p_A, p_B) = \sum_{a,b} \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b\, f_{a/A}(x_a, \mu_\mathrm{F})\, f_{b/B}(x_b, \mu_\mathrm{F})\, \mathrm{d}\hat{\sigma}^{\mathrm{NLO}}_{ab}(p_a, p_b, \mu_\mathrm{F}, \mu_\mathrm{R})$$
$$\times\, \mathcal{F}^{(4\ell+\gamma)}(\{\mathcal{O}_\mathrm{FS}\}), \qquad p^\mu_{\{a,b\}} = x_{\{a,b\}} P^\mu_{\{A,B\}}$$

NLO partonic cross section:

$$\hat{\sigma}^{\mathrm{NLO}}_{ab} = \hat{\sigma}^{\mathrm{LO}}_{ab} + \hat{\sigma}^{\mathrm{virt}}_{ab} + \hat{\sigma}^{\mathrm{real}}_{ab}$$

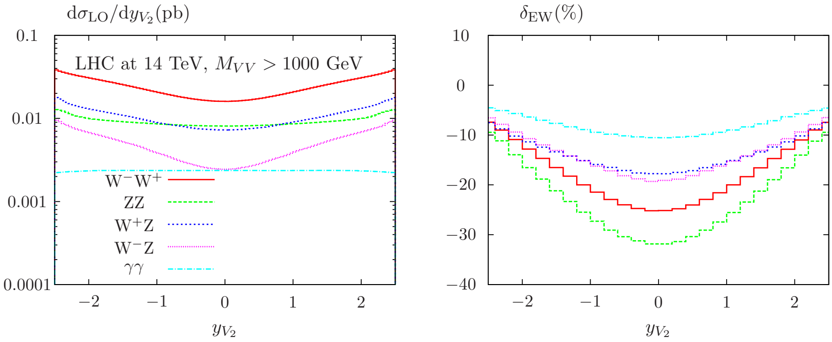
Numerical Results for WW/WZ/ZZ/$\gamma\gamma$ Production

LHC at 14 TeV, default cuts: $p_{\mathrm{T},V} > 15$ GeV, $|y_V| < 2.5$



- **LO:** Drastically decreasing cross sections for large p_T
- **NLO:**
  - Full result, i.e. virtual, soft, hard, and collinear photons included
  - corrections largest for ZZ, smallest for $\gamma\gamma$

LHC at 14 TeV, high-energy cuts: $p_{T,V} > 15$ GeV, $|y_V| < 2.5$, $M_{VV} > 1000$ GeV



- significant distortion of rapidity distributions at large invariant masses
- Corrections could be misinterpreted as signal of anomalous couplings.

Double-Pole Approximation (DPA)

- **Lowest order:** Amplitude given as a product of on-shell (OS) production amplitude ⊗ on-shell decay amplitude ⊗ Breit–Wigner:

$$\mathcal{M}^{\bar{q}_1 q_2 \to V_1 V_2 \to 4f}_{\text{Born,DPA}} = \frac{1}{k_1^2 - M_1^2 + \mathrm{i} M_1 \Gamma_1} \frac{1}{k_2^2 - M_2^2 + \mathrm{i} M_2 \Gamma_2} \times \sum_{\lambda_1, \lambda_2} \mathcal{M}^{\bar{q}_1 q_2 \to V_{1,\lambda_1} V_{2,\lambda_2}}_{\text{Born}} \mathcal{M}^{V_{1,\lambda_1} \to f_3 \bar{f}_4}_{\text{Born}} \mathcal{M}^{V_{2,\lambda_2} \to f_5 \bar{f}_6}_{\text{Born}}$$

- Use OS-projected momenta $\hat{k}$ [Denner, Dittmaier, Roth, Wackeroth 2000] in the OS matrix elements:

$$\hat{k}_{1,0} = \frac{1}{2}\sqrt{\hat{s}}, \quad \hat{\mathbf{k}}_1 = \frac{\mathbf{k}_1}{|\mathbf{k}_1|} \beta_{\mathrm{W}} \frac{\sqrt{\hat{s}}}{2}, \quad \ldots$$

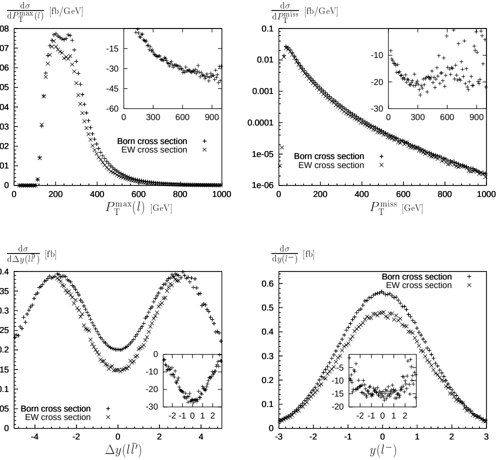
- **NLO:** EW corrections consist of factorizable and non-factorizable contributions, e.g.

$$\mathcal{M}_{\text{fact}} = \frac{R(k_1, k_2, \theta)}{(k_1^2 - M_1^2 + \mathrm{i} M_1 \Gamma_1)\,(k_2^2 - M_2^2 + \mathrm{i} M_2 \Gamma_2)}$$

  Caution: Gauge invariance!

EW corrections to $\mathrm{pp} \to \mathrm{W}^+\mathrm{W}^- \to \nu_\mathrm{e}\mathrm{e}^+\mu^-\bar{\nu}_\mu$ (DPA)

- Standard LHC event selection cuts applied to final-state leptons and missing transverse momentum; additionally $M_{\mathrm{e}^+\mu^-} > 500$ GeV required
- Large negative corrections at large transverse momenta
- Substantial negative corrections to inclusive observables
- Error due to DPA about 10% in the relative corrections
- **EW corrections significantly larger than experimental error throughout the whole energy range (for $L \sim 30\ \mathrm{fb}^{-1}$)**



[Accomando, Denner, Kaiser: arXiv:0409247 [hep-ph]]

Photon PDFs (MRST2004QED)

- Simple LL ansatz for $f_{\gamma/\mathrm{p}}(x, Q_0^2)$

$$f_{\gamma/\mathrm{p}}(x, Q_0^2) = \frac{\alpha}{2\pi}\left[\frac{4}{9}\ln\left(\frac{Q_0^2}{m_u^2}\right) f_{u/\mathrm{p,v}}(x, Q_0^2) + \frac{1}{9}\ln\left(\frac{Q_0^2}{m_d^2}\right) f_{d/\mathrm{p,v}}(x, Q_0^2)\right] \otimes \frac{1+(1-x)^2}{x}$$

- Running of $f_{q/\mathrm{p}}(x, Q^2)$ at $\mathcal{O}(\alpha)$ affected by photon PDFs!

$$\frac{\partial f_{q/\mathrm{p}}(x,\mu^2)}{\partial \ln \mu^2} = \frac{\alpha}{2\pi}\int_x^1 \frac{\mathrm{d}y}{y}\left[P_{qq}(y)\, Q_q^2\, f_{q/\mathrm{p}}(x/y,\mu^2) + P_{q\gamma}(y)\, Q_q^2\, f_{\gamma/\mathrm{p}}(x/y,\mu^2)\right]$$

- Momentum conservation

$$\int_0^1 \mathrm{d}x\, x\left[\sum_q f_{q/\mathrm{p}}(x,\mu^2) + f_{g/\mathrm{p}}(x,\mu^2) + f_{\gamma/\mathrm{p}}(x,\mu^2)\right] = 1$$

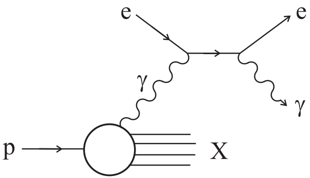
$\Rightarrow$ QED effects on $f_{q/\mathrm{p}}(x, \mu^2)$ small!
$\Rightarrow$ Still large conceptual uncertainties in $f_{\gamma,0}$

Measure Photon PDFs?

Consider the DIS process

$$e\mathrm{p} \to e\gamma + X$$

with high-p_T back-to-back e, γ in the final state



$$\sigma(e\mathrm{p} \to e\gamma + X) = \int \mathrm{d}x^\gamma f_{\gamma/\mathrm{p}}(x^\gamma, \mu^2)\hat{\sigma}(e\gamma \to e\gamma) \ ,$$

related to Compton scattering

- $x^\gamma = \frac{E_\mathrm{T}^\gamma E_e \exp(\eta^\gamma)}{2E_\mathrm{p}E_e - E_\mathrm{T}^\gamma E_e \exp(-\eta^\gamma)}$
- $f_{\gamma/\mathrm{p}}(x^\gamma, \mu^2)$ could be in principle extracted from HERA data!

EW Input Schemes – Definition of α

- $\alpha(0)$: On-shell definition in the Thomson-limit (zero momentum transfer)

$$\bar{u}(p)\Gamma_{\mu}^{Ae\bar{e}}(p,p)u(p)\Big|_{p^2=m_e^2} = e(0)\bar{u}(p)\gamma_{\mu}u(p)\,,\, \alpha(0) = e(0)^2/4\pi$$

- $\alpha(M_\mathrm{Z})$ obtained via renormalization-group running from 0 to weak scale M_Z

$$\alpha(M_\mathrm{Z}) = \frac{\alpha(0)}{1-\Delta\alpha(M_\mathrm{Z})}\,, \quad \Delta\alpha(M_\mathrm{Z}) = \Pi_{f\neq t}^{AA}(0) - \mathrm{Re}\,\Pi_{f\neq t}^{AA}(M_\mathrm{Z}^2)$$

- α_{G_μ} defined through the Fermi constant related to the muon lifetime

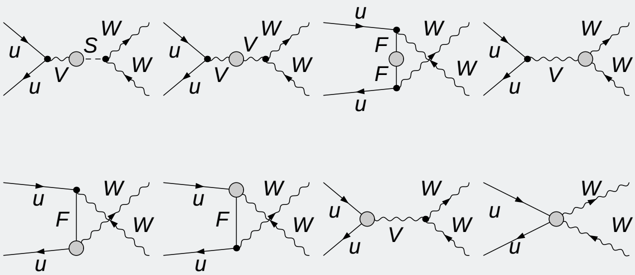
$$\alpha_{G_\mu} = \frac{\sqrt{2}G_\mu M_\mathrm{W}^2 s_\mathrm{w}^2}{\pi} = \frac{\alpha(0)}{1-\Delta r}$$

  Δr includes corrections to muon lifetime not contained in QED-improved Fermi model

- **light-fermion mass logs contained in $\Pi_{f\neq t}^{AA}(0)$ resummed in effective couplings $\alpha(M_\mathrm{Z})$ and α_{G_μ}**

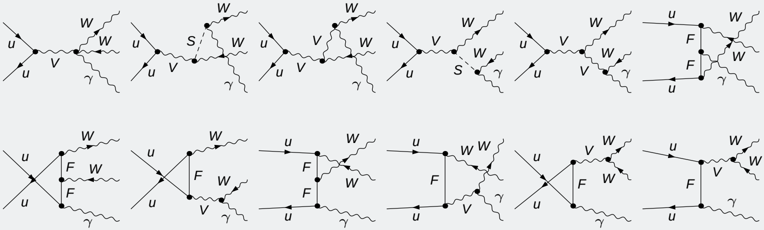
Virtual EW Corrections to $\mathrm{pp} \to \mathrm{W}^-\mathrm{W}^+ + X$

One-loop contributions at $\mathcal{O}(\alpha^3)$



- On-shell renormalization of SM parameters
- We use the Fermi scheme to calculate the loop corrections.
  $\rightarrow$ universal corrections to Δr absorbed in effective LO coupling
- $V_{ij}^{\mathrm{CKM}} = \delta_{ij}$ within the loops $\rightarrow$ no renormalization of V_{ij}^{CKM}

Real photon radiation at $\mathcal{O}(\alpha^3)$ (generic diagrams): $q\bar{q} \rightarrow \mathrm{W}^-\mathrm{W}^+ + \gamma$



- Soft singularities due to soft photons
- Initial-state collinear singularities due to collinear photon radiation off initial-state quarks $\longrightarrow$ renormalization of PDFs
- Introduce small **quark mass** m_q and infinitesimal **photon mass** λ to regularize divergences $\longrightarrow$ results exhibit unphysical $\ln m_q$ and $\ln \lambda$ terms

Apply phase-space slicing for numerically-stable evaluation of phase-space integral

Phase-Space Slicing

Two-cut-off phase-space slicing

- Definition of bremsstrahlung phase space:

$$\sigma_{\text{real}} = \int \mathrm{dPS}(\mathrm{W}^-\mathrm{W}^+\gamma)|\mathcal{M}^\gamma|^2$$

- Phase-space decomposition:

$$\sigma_{\text{real}} = \sigma_{\text{hard}} + \sigma_{\text{soft}} + \sigma_{\text{coll}}$$

Phase-Space Slicing

- **Soft limit:** $E_\gamma < \Delta E \ll M_\mathrm{W}$

$$\sigma_\mathrm{soft}(\Delta E) = -\sigma_\mathrm{LO}\left[\frac{e^2}{(2\pi)^3}\int_{|\mathbf{k}_\gamma|<\Delta E}\frac{\mathrm{d}^3\mathbf{k}_\gamma}{2\sqrt{\mathbf{k}_\gamma^2+\lambda^2}}\sum_{ij}\frac{\pm Q_i Q_j (p_i p_j)}{(p_i k_\gamma)(p_j k_\gamma)}\right]$$

- **Collinear limit:** $\theta_{q\gamma} < \Delta\theta \ll 1, \quad E_\gamma > \Delta E$

$$\sigma_{\mathrm{coll},q}(\Delta E, \Delta\theta) = \frac{\alpha Q_q^2}{2\pi}\int_0^{1-2\Delta E/\sqrt{\hat{s}}}\mathrm{d}z\,\frac{(1+z^2)}{1-z}\left(\ln\frac{\hat{s}(\Delta\theta)^2}{4m_q^2} - \frac{2z}{1+z^2}\right)\sigma_\mathrm{LO}(z\hat{s})$$

- **Hard bremsstrahlung:** $\theta_{q\gamma} > \Delta\theta, \quad E_\gamma > \Delta E$;
  numerical evaluation of $\sigma_\mathrm{hard}(\Delta E, \Delta\theta)$ without regulators
- Numerical result independent of $\ln\Delta E$ and $\ln\Delta\theta$

$\ln m_q$ and $\ln\lambda$ terms cancel in the sum $\sigma_\mathrm{virt} + \sigma_\mathrm{soft} + \sigma_\mathrm{coll}$ in infrared-safe observables

The Complex-Mass Scheme (CMS) for Unstable Particles

[Denner, Dittmaier, Roth, Wieders 2005]

A problem with unstable particles

Naive implementation of finite width in gauge-boson propagator:

$$\frac{-\mathrm{i}g^{\mu\nu}}{q^2 - M_\mathrm{W}^2 + \mathrm{i}\epsilon} \rightarrow \frac{-\mathrm{i}g^{\mu\nu}}{q^2 - M_\mathrm{W}^2 + \mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W}}$$

Γ_W includes Dyson summation of self energies, mixing of perturbative orders

$\rightarrow$ **might destroy gauge invariance (even at leading order!)**

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Γ_W includes Dyson summation of self energies, mixing of perturbative orders
$\rightarrow$ **might destroy gauge invariance (even at leading order!)**

$\rightarrow$ **CMS universal solution that**
- respects gauge invariance
- is valid in all phase-space regions

Straightforward implementation:

- **LO:** $M_\mathrm{V}^2 \rightarrow \mu_\mathrm{V}^2 = M_\mathrm{V}^2 - \mathrm{i}M_\mathrm{V}\Gamma_\mathrm{V}$, $\cos^2\Theta_\mathrm{W} = \frac{\mu_\mathrm{W}^2}{\mu_\mathrm{Z}^2}$, $\mathrm{V} = \mathrm{W}, \mathrm{Z}$
- **NLO:**
  - Complex renormalization: $\mathcal{L}_0 \rightarrow \mathcal{L} + \delta\mathcal{L}$, **bare (real) Lagrangian unchanged!**
  - Evaluate loop integrals with complex masses

"Real Radiation" of Massive Vector Bosons

Low energies: Phase-space and perturbative suppression of $\mathrm{pp} \to V_1 V_2 + (\mathrm{W}/\mathrm{Z})$
$\Rightarrow$ contribution below 1%

High energies: Logarithmic enhancement of additional soft/collinear W- or Z-boson radiation
$\Rightarrow$ Investigation of $V_1 V_2 + \mathrm{W}/\mathrm{Z}$ production as background to V pairs at large p_{T}, M_{VV}

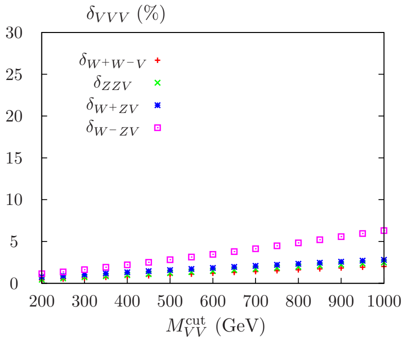
- invisible decay of $Z \to \nu\bar{\nu}$
- collinear emission
- ...

Simplified approach (details depend on experimental analysis), e.g. W-Pair production:

1. Include $\mathrm{pp} \to \mathrm{W}^- \mathrm{W}^+ \mathrm{Z}$ with totally inclusive Z
2. Include $\mathrm{pp} \to \mathrm{W}^- \mathrm{W}^+ \mathrm{W}^{\pm}$; treat $W^{\pm}$ with lowest p_{T} totally inclusively

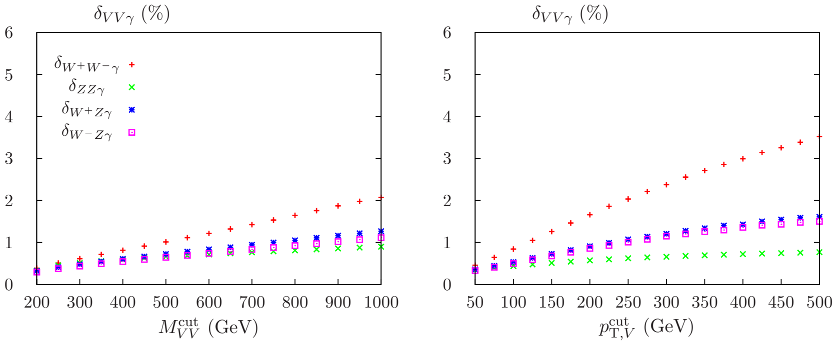
Numerical Results (I) – "Real Radiation" of W/Z

LHC at 8 TeV, default cuts: $p_{\mathrm{T},V} > 15$ GeV, $|y_V| < 2.5$



- corrections below 5% even for large transverse momenta and invariant masses
- corrections to W^-Z production enhanced due to W^-ZW^+ final states (PDFs!)

LHC at 8 TeV, default cuts: $p_{\mathrm{T},V} > 15$ GeV, $|y_V| < 2.5$, $p_{\mathrm{T},\gamma} > 15$ GeV, $|y_\gamma| < 2.5$



- real radiation of hard photons marginal ($< 2\%$)
  $\rightarrow$ neglect in MC implementation
- corrections largest for WW production

Virtual corrections computed in the `FeynArts/FormCalc/LoopTools(FF)` framework [(FA): Küblbeck, Böhm, Denner 1990; (FC,LT): Hahn, Pérez Victoria 1999; Hahn 2001; (FF): van Oldenborgh, Vermaseren 1990]

1. `FeynArts-3.5:`
   - Automatic generation of diagrams
   - Calculation of amplitudes
2. `FormCalc-6.1:`
   - Algebraical simplification of amplitudes, introduction of tensor coefficients
   - Analytical calculation of squared amplitudes
   - Spin-, colour- and polarization sums
   - Generation of `Fortran` code
3. `LoopTools-2.5:`
   - Numerical Passarino–Veltman reduction within `Fortran`
   - Numerically-stable evaluation of scalar integrals

Bremsstrahlung amplitudes computed with `FeynArts/FeynCalc` $\oplus$ `Madgraph` [Alwall et al.], numerical phase-space integration within `Fortran` using the `Vegas` algorithm

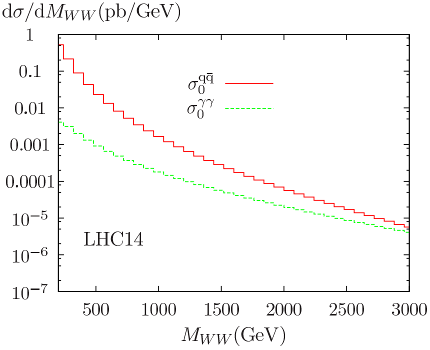
ZZ Production: Polarizations

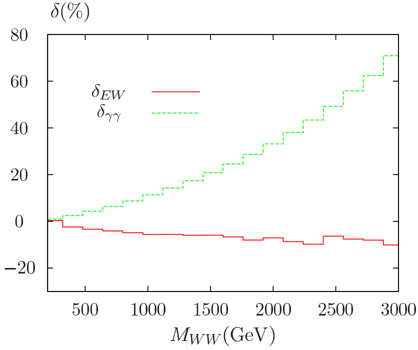
$pp \longrightarrow ZZ + X$					
ZZ polarizations	summed	LL	L+	++	+−
LHC8					
σ_{LO}/pb $\delta\sigma_{weak}/pb$	3.810 −0.179(−0.155)	0.223 −0.009(−0.008)	0.396 −0.016(−0.014)	10^{-1} × [0.559 −0.002(−0.002)]	2.676 −0.134(−0.117)
$p_{T,Z} > 500$ GeV σ_{LO}/pb $\delta\sigma_{weak}/pb$	10^{-2} × [0.101 −0.039(−0.030)]	10^{-7} × [0.202 −0.975(+0.748)]	10^{-5} × [0.779 −0.204(−0.157)]	10^{-8} × [0.504 −0.895(−0.425)]	10^{-3} × [0.996 −0.383(−0.293)]
$p_{T,Z} > 1000$ GeV σ_{LO}/pb $\delta\sigma_{weak}/pb$	10^{-5} × [0.919 −0.557(−0.387)]	10^{-10} × [0.121 −2.599(+14.909)]	10^{-7} × [0.231 −0.098(−0.070)]	10^{-11} × [0.303 −1.742(+1.043)]	10^{-5} × [0.915 −0.555(−0.387)]

- small transverse momenta: 70% from $(+-)$, similar K-factors for all polarizations.
- large transverse momenta: 99% from $(+-)$, other contributions negligible.
- **Note:** One-loop squared term (given in brackets) contributes at $\sim 10\%$
  $\rightarrow$ large uncertainties due to missing EW higher orders.

Conclusion: One K-factor sufficient to describe polarized ZZ production

No cuts



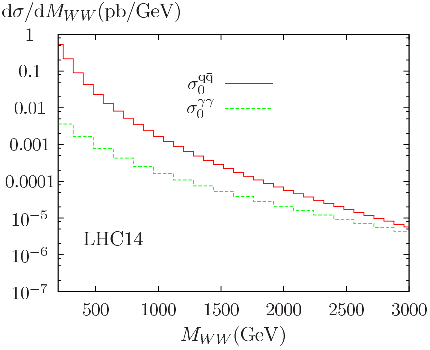


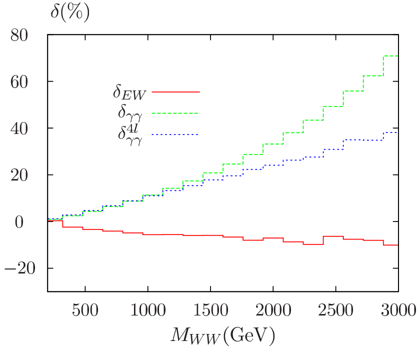
- LO cross section dominated by $q\bar{q}$ contributions
- Rapid decrease of cross section for increasing invariant masses

- EW corrections small even for large values of M_{WW}
- Large contributions (+80%!) from $\gamma\gamma \to WW$ at high invariant masses

$\Rightarrow$ Leptonic decays?

LHC acceptance cuts





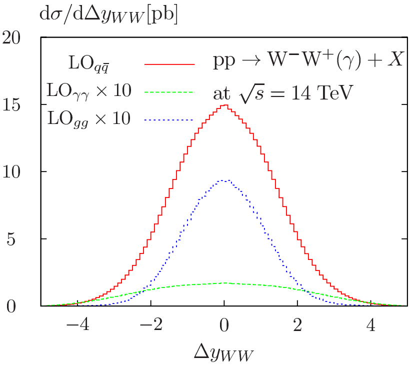
- LO cross section dominated by $q\bar{q}$ contributions
- Rapid decrease of cross section for increasing invariant masses

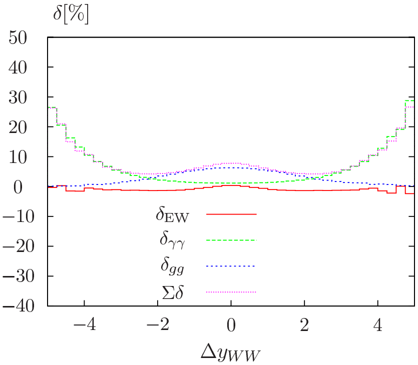
- Employ LHC cuts on decay products: $p_{T,l} > 20$ GeV, $|y_l| < 3$, $p_{T,\text{miss}} > 25$ GeV

$\Rightarrow$ relative effect of $\gamma\gamma \to WW$ reduced by factor 2 at large M_{WW}

EW Corrections to $\mathrm{pp} \to \mathrm{W}^-\mathrm{W}^+$ – Numerical Results

Default cuts: $p_{\mathrm{T,W^\pm}} > 15$ GeV, $y_{\mathrm{W^\pm}} < 2.5$

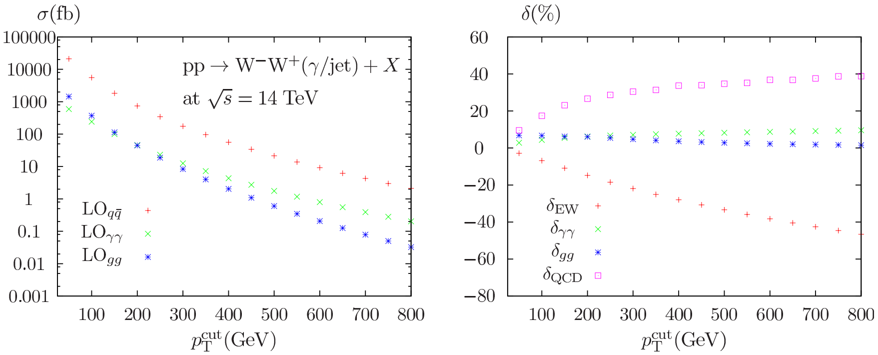




- WW production dominated by events near threshold, isotropic production at small Δy_{WW}
- 5% increase of cross section by gg channel

- EW corrections at the percent level
- Sizable contributions from $\gamma\gamma$ at large $|\Delta y_{WW}|$

Default cuts: $p_{T,W^\pm} > 15$ GeV, $|y_{W^\pm}| < 2.5$

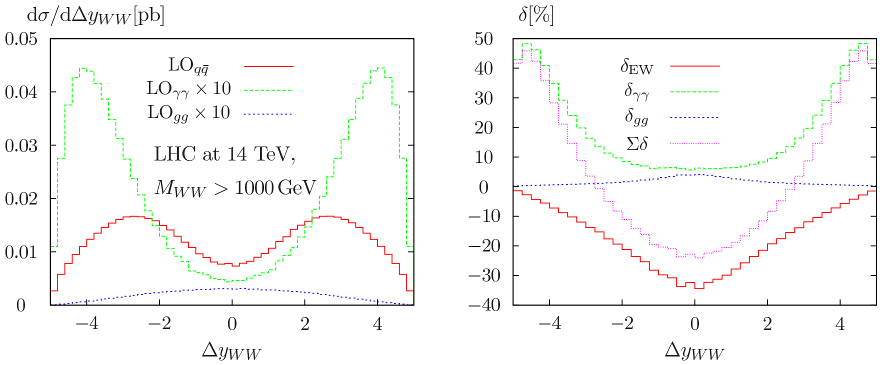


- assume $\int \mathcal{L}dt = 200\ \text{fb}^{-1}$
  $\Rightarrow$ 1000 WW events with $p_T > 500$ GeV
- decreasing admixture of gg, increasing admixture of $\gamma\gamma$
- large admixture of $\gamma\gamma$ (10%!)
- large negative EW corrections (-45%), comparable to QCD corrections

No compensation between $\gamma\gamma \to WW$ and weak corrections!
$\Longrightarrow$ Different angular distributions!

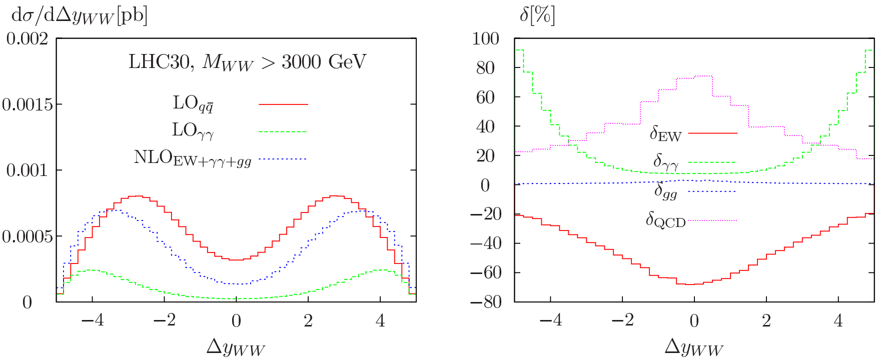
- $\sigma(\gamma\gamma \to WW) \to \frac{8\pi\alpha^2}{M_W^2}$ for large $\hat{s}$
  $\Rightarrow$ strong enhancement in forward & backward directions
- **weak corrections:**
  negative Sudakov logs for large $\hat{s}$ and $\hat{t}$
  $\Rightarrow$ negative corrections for large scattering angles
- gg small, isotropic
- implications for $\mathrm{d}\sigma/\mathrm{d}\Delta y_{WW}$ with $\Delta y_{WW} = y_{W^+} - y_{W^-}$
  (for fixed M_{WW} this corresponds to the angular distribution in the W-W rest frame!)

High-energy cuts: $p_{T,W^\pm} > 15$ GeV, $y_{W^\pm} < 2.5$, $M_{WW} > 1$ TeV



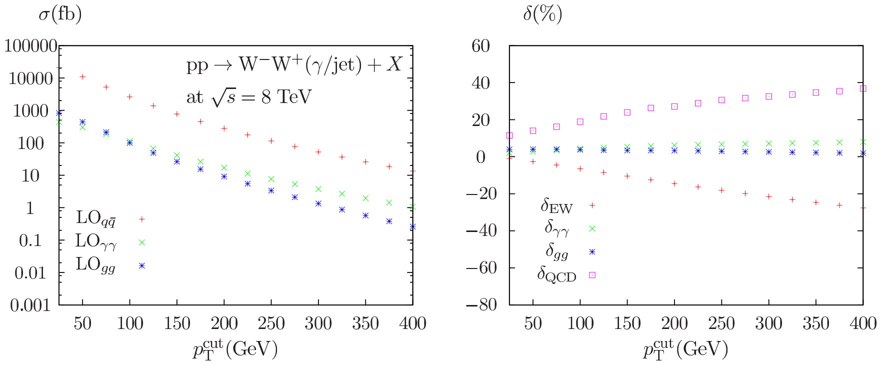
- WW production dominated by small scattering angles
- drastic forward-backward peaking of $\gamma\gamma \to WW$
- drastic distortion of angular distribution
- $\Sigma\delta$ varies from -30% and $+45\%$ for $M_{WW} > 1$ TeV!

Very-high-energy cuts: $p_{T,W^\pm} > 15$ GeV, $y_{W^\pm} < 2.5$, $M_{WW} > 3$ TeV

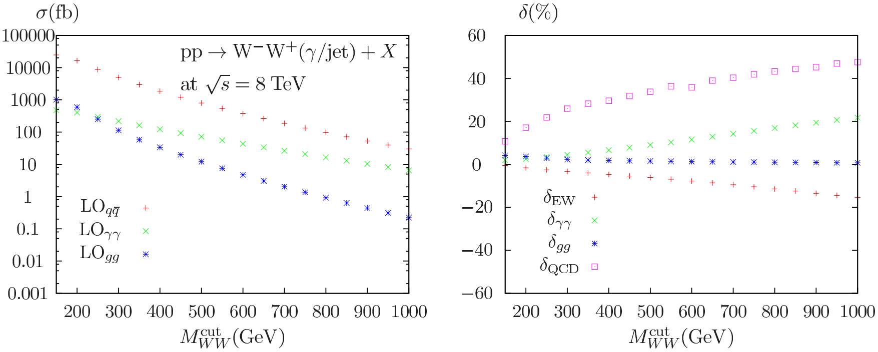


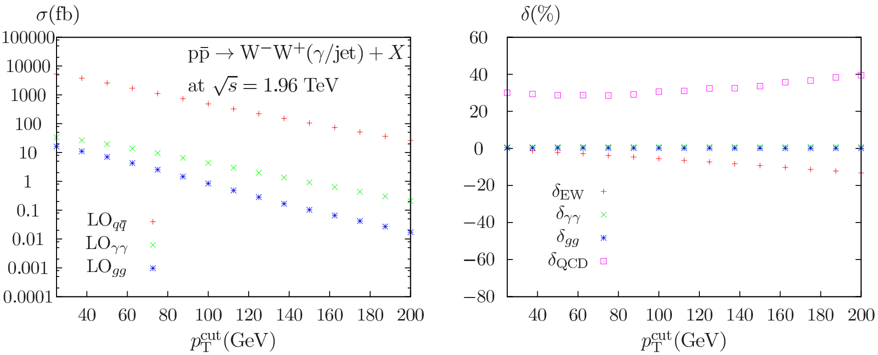
- NLO EW as important as QCD
- extreme distortion due to $\gamma\gamma$ (**caveat:** high uncertainty in photon PDFs)

Transverse-momentum distribution at the LHC8

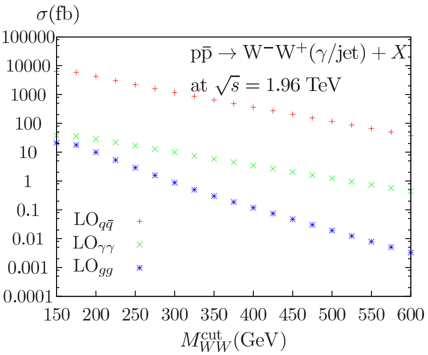


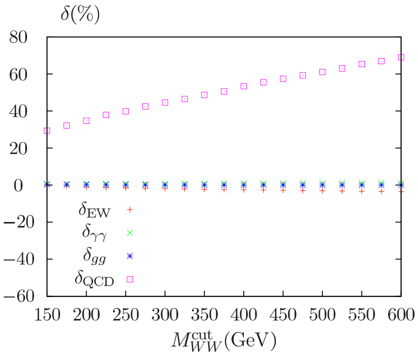
Invariant-mass distribution at the LHC8




σ(fb)
100000
10000
1000
100
10
1
0.1
0.01
0.001
0.0001
pp̄ → W⁻W⁺(γ/jet) + X
at √s = 1.96 TeV
LO_qq̄
LO_γγ
LO_gg
40 60 80 100 120 140 160 180 200
p_T^cut (GeV)
δ(%)
60
40
20
0
−20
−40
−60
−80
δ_EW
δ_γγ
δ_gg
δ_QCD
40 60 80 100 120 140 160 180 200
p_T^cut (GeV)

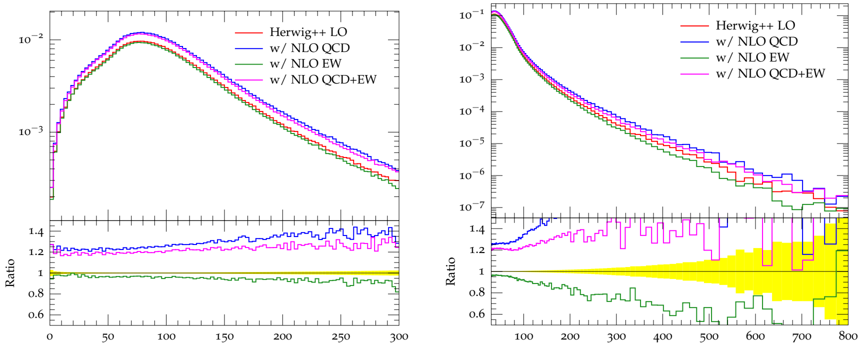
Invariant-mass distribution at the Tevatron





HERWIG++ Analysis for ZZ Production (II) – Preliminary!

Simulation for $\mathrm{pp} \to ZZ \to e^+e^-\mu^+\mu^- + X$ at 8 TeV, $M_{e^+\mu^-}$ and $p_{\mathrm{T},l}$ distributions



- Standard Herwig++ setup used (v2.6.2, with simple add-on for EW corrections, 10M events), ZZ at NLO QCD matched with parton showers, hadronization included, underlying event switched off
- Implementation seems to work fine