

HIGGSOLOGY: THEORY AND PRACTICE

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PHENO 2013: MAY 6-8

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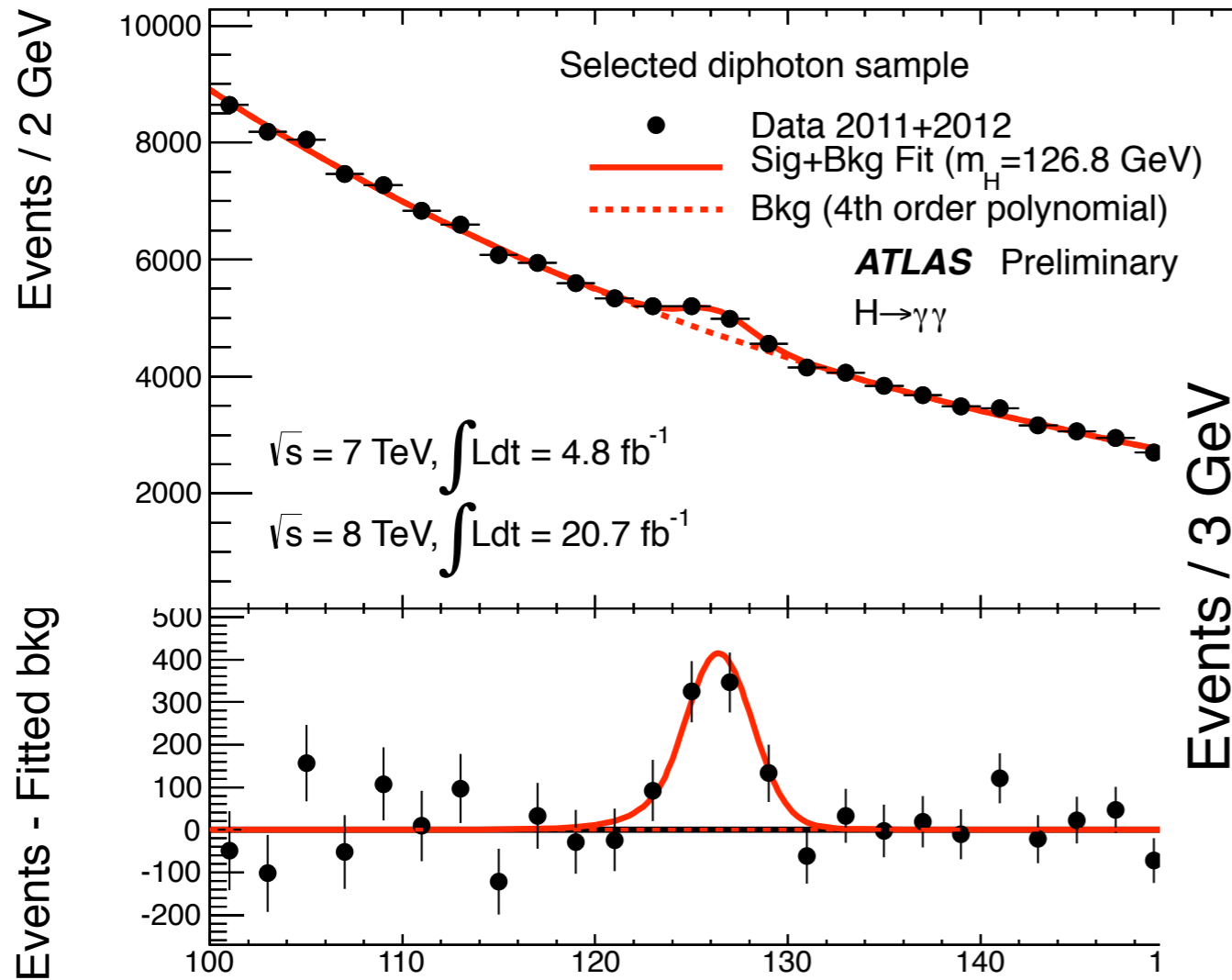
*Look for new physics
where the rivers meet.*

<http://indico.cern.ch/event/pheno13>

Organizers: Cindy Cercone, Neil Christensen, Ayres Freitas, Tao Han (chair), Adam Leibovich, Joshua Sayre, Susanne Westhoff
Program Advisors: Vernon Barger, Lisa Everett, Kaoru Hagihara, JoAnne Hewett, Xerxes Tata, Dieter Zeppenfeld

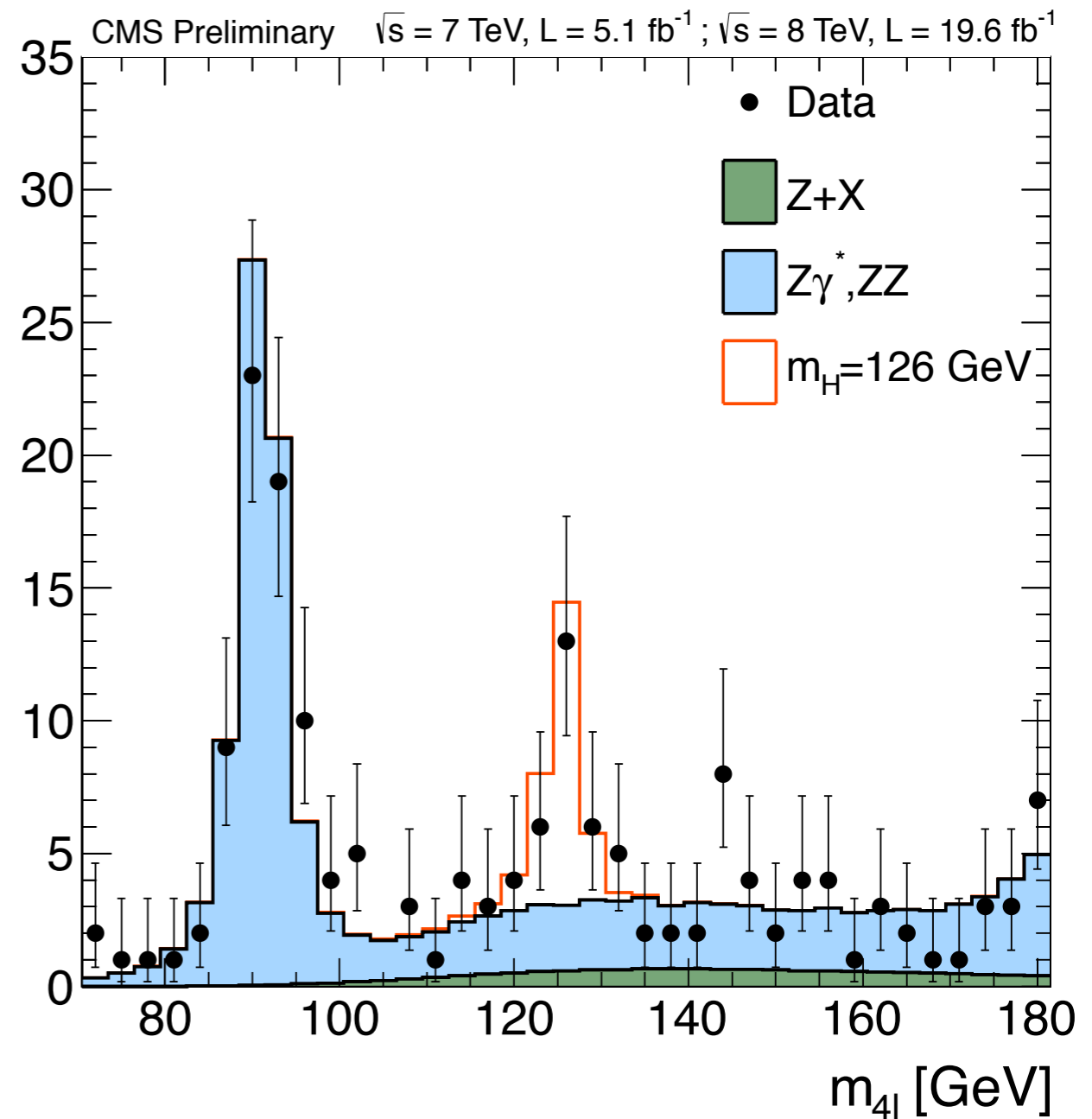
Pheno Symposia
are supported by
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IT IS ONE OF THE MOST EXCITING TIMES:



Events - Fitted bkg

Events / 3 GeV



Tao Han

ATLAS Preliminary

W,Z H → bb

$\sqrt{s} = 7 \text{ TeV}: \int L dt = 4.7 \text{ fb}^{-1}$

$\sqrt{s} = 8 \text{ TeV}: \int L dt = 13 \text{ fb}^{-1}$

H → ττ

$\sqrt{s} = 7 \text{ TeV}: \int L dt = 4.6 \text{ fb}^{-1}$

$\sqrt{s} = 8 \text{ TeV}: \int L dt = 13 \text{ fb}^{-1}$

H → WW^(*) → lvlv

$\sqrt{s} = 7 \text{ TeV}: \int L dt = 4.6 \text{ fb}^{-1}$

$\sqrt{s} = 8 \text{ TeV}: \int L dt = 20.7 \text{ fb}^{-1}$

H → γγ

$\sqrt{s} = 7 \text{ TeV}: \int L dt = 4.8 \text{ fb}^{-1}$

$\sqrt{s} = 8 \text{ TeV}: \int L dt = 20.7 \text{ fb}^{-1}$

H → ZZ^(*) → 4l

$\sqrt{s} = 7 \text{ TeV}: \int L dt = 4.6 \text{ fb}^{-1}$

$\sqrt{s} = 8 \text{ TeV}: \int L dt = 20.7 \text{ fb}^{-1}$

Combined

$\mu = 1.30 \pm 0.20$

$\sqrt{s} = 7 \text{ TeV}: \int L dt = 4.6 - 4.8 \text{ fb}^{-1}$

$\sqrt{s} = 8 \text{ TeV}: \int L dt = 13 - 20.7 \text{ fb}^{-1}$

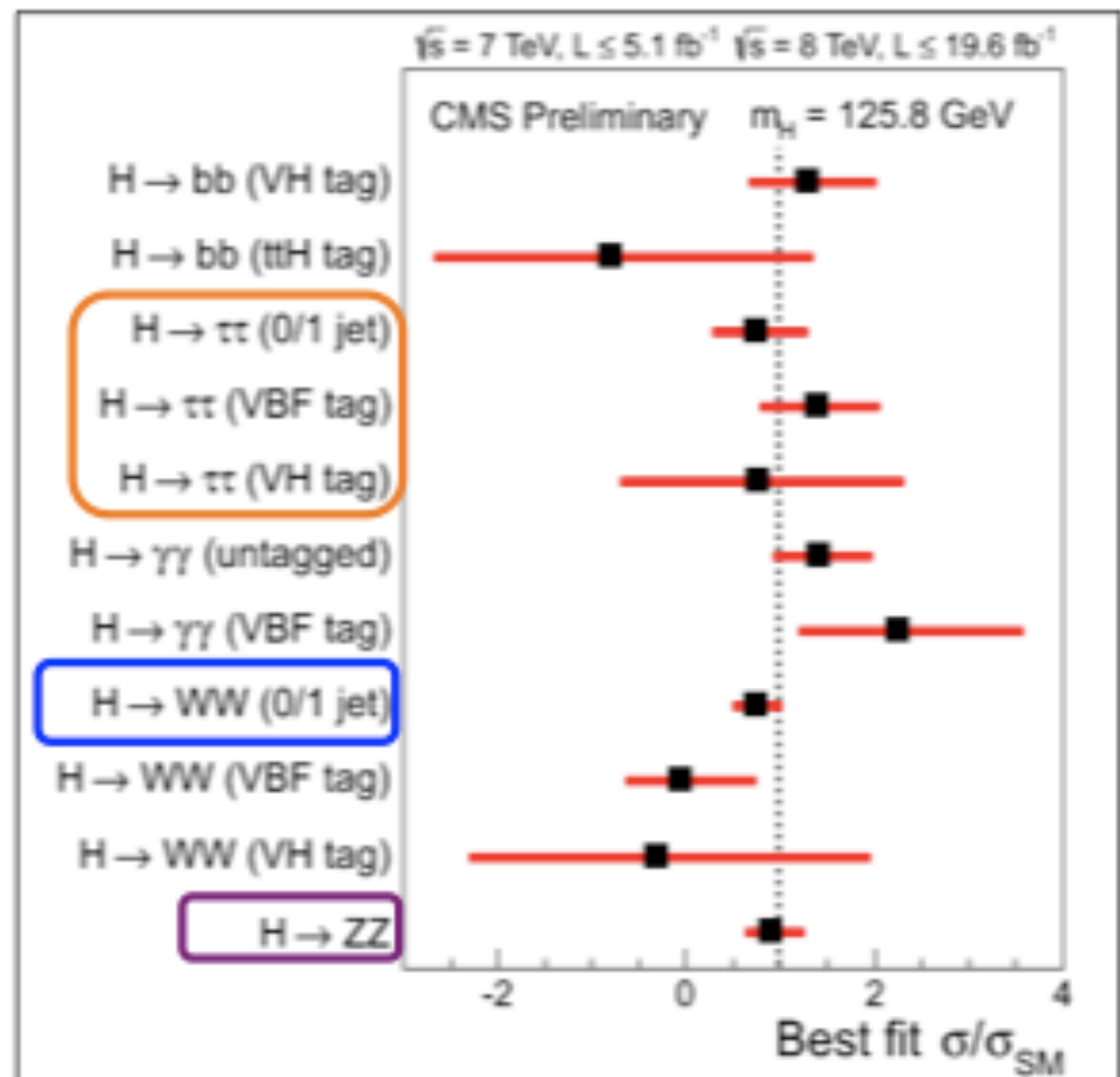
$m_H = 125.5 \text{ GeV}$

-1 0 +1
Signal strength (μ)

the observed boson: $125.8 \pm 0.6 \text{ GeV}$

$m_H = 125.5 \pm 0.2 \text{ (stat)}^{+0.5}_{-0.6} \text{ (sys)} \text{ GeV}$

$\mu = 1.43 \pm 0.16 \text{ (stat)} \pm 0.14 \text{ (sys)}$





Mosaic of the CMS and ATLAS detectors (as in 2007), part of the Large Hadron Collider at CERN. In 2012, research teams used these detectors to fingerprint decay products from the long-sought Higgs boson and determine its mass, successfully testing a key prediction of the standard model of particle physics.

Photos: Maximilien Brice and Claudia Marcelloni/CERN



Fabiola Gianotti, ATLAS spokesperson
Runner-up of 2012 Person of the year

This discovery opens up a new era in HEP!

In these Lectures, I wish to convey to you:

- This is truly an “LHC Revolution”, ever since the “November Revolution” in 1974 for the J/ψ discovery!
- It strongly argues for new physics beyond the Standard Model (BSM).

Outline

Lecture I: Higgs Sector in the SM

A. The Higgs Mechanism

1. A historic count
2. The spontaneous symmetry breaking
3. The Goldstone Theorem
4. The Higgs mechanism

B. The Higgs Boson Interactions

1. The Standard Model
2. The Higgs boson interactions

Lecture II: Higgs Physics at Colliders

A. Higgs Boson Decay

1. Decay to fermions
2. Decay WW, ZZ
3. Decay through loops

B. Higgs Boson Production at the LHC

1. The leading channels
2. The search strategies
3. Signal characteristics

C. Higgs Boson Production at e^+e^- colliders

D. Higgs Boson Production at a muon collider

Lecture III: Higgs and Beyond

-- Motive for Physics Beyond SM

A. A Weakly Coupled Scalar?

1. The Higgs mechanism \neq Higgs boson!
2. Why not a heavier, broader Higgs?

B. SM Higgs Sector at Higher Energies

1. Triviality bound
2. Vacuum stability
3. Naturalness

C. New Physics associated with the Higgs

1. Supersymmetry
2. Extended Higgs sector
3. Composite Higgs
4. Coupling deviations from SM

Lect I. Higgs Sector in the SM

A. The Higgs Mechanism

1. A historical Count:

(a). Deep Root in QED

Maxwell Equations →

Lorentz invariance, U(1) Gauge Invariance

Although the electromagnetic fields in $\mathbf{E}(\mathbf{x},t)$, $\mathbf{B}(\mathbf{x},t)$ seem adequate for all practical purposes, the introduction of co-variant vector potential $A_\mu(\mathbf{x},t)$ is viewed as revolutionary!‡

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- 1). Lorentz/Local Gauge invariance **manifest**.
- 2). Classically, geometrical interpretation: fiber bundles...
- 3). Quantum-mechanically, wave function for the EM field.

‡ Still with dispute: physical? redundancy?





Dirac's relativistic theory:
Lorentz/Local gauge invariant

→ antiparticle e^+

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m_e)\psi$$

$$D_\mu = \partial_\mu + ieA_\mu$$

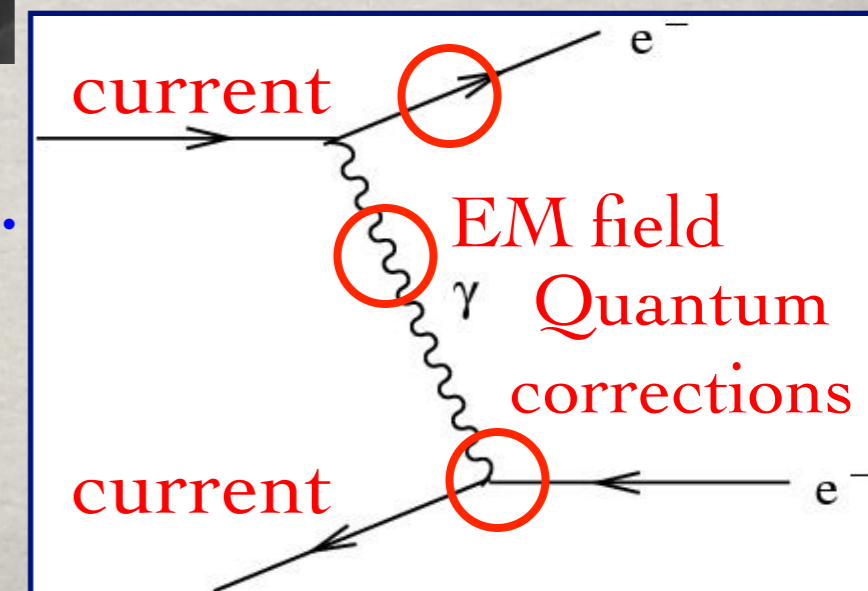
Feynman/Schwinger/Tomonaga

→ Renormalization



* The perturbation theory, thus Feynman diagram approach, the most successful aspect.

* QED becomes the most accurate theory in science.



Warmup Exercise 1:

For charge scalar field ϕ^\pm , construct the locally $U(1)_{\text{em}}$ gauge invariant Lagrangian and derive the Feynman rules for its EM interactions.

Sketch a calculation for the differential and total cross section for the process:

$$e^+ e^- \rightarrow \phi^+ \phi^-$$

(b). Build up the Weak Interactions

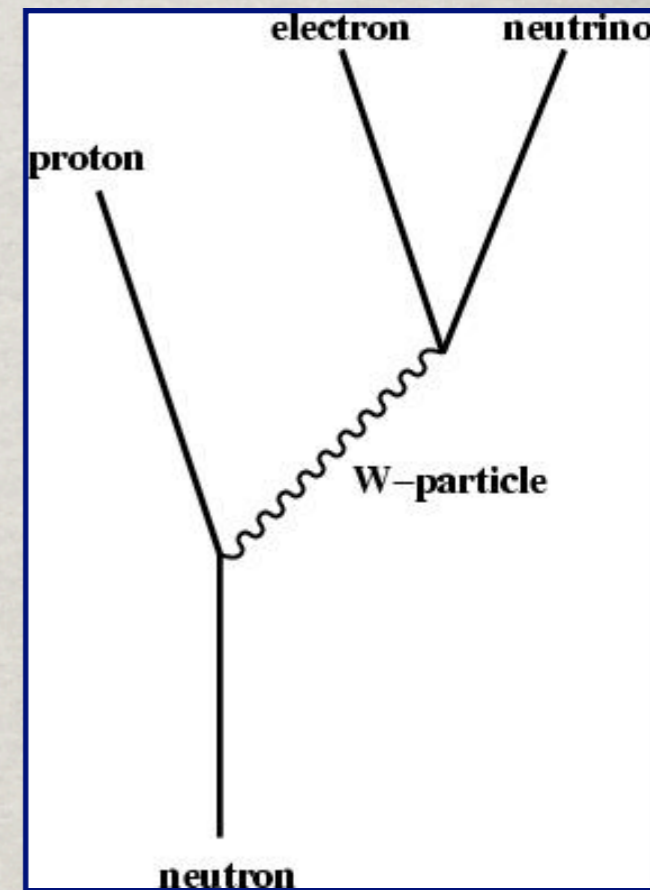
beta decay $n \rightarrow p^+ e^- \nu \rightarrow$ Charged current interaction: W^\pm

$\nu N \rightarrow \nu N \rightarrow$ Neutral current interaction: Z^0

$$-\mathcal{L}_{eff}^{CC} = \frac{G_F}{\sqrt{2}} J_W^\mu J_{W\mu}^\dagger, \quad -\mathcal{L}_{eff}^{NC} = \frac{G_F}{\sqrt{2}} J_Z^\mu J_{Z\mu}.$$
$$J_\lambda^{(\pm)} = \sum_i \bar{\psi}_i \tau_\pm \gamma_\lambda (1 - \gamma_5) \psi_i,$$

Fermi was inspired by the EM current-current interactions to construct the weak interaction.

(parity violation \rightarrow $V-A$ interactions)



The fact $G_F = (300 \text{ GeV})^{-2}$ implies that:

1. A new mass scale to show up at $O(100 \text{ GeV})$.
2. Partial-wave Unitarity requires new physics below $E < 300 \text{ GeV}$

Exercise 2:

Assume that the $\nu e \rightarrow \nu e$ scattering amplitude to be

$$M = G_F E_{\text{cm}}^2$$

estimate the unitarity bound on the c.m. energy.

Partial wave expansion:

$$a_{I\ell}(s) = \frac{1}{64\pi} \int_{-1}^1 d\cos\theta P_\ell(\cos\theta) \mathcal{M}^I(s, t)$$

Partial wave unitarity:

$$\text{Im}(a_{I\ell}) = |a_{I\ell}|^2 < 1, \quad \text{Re}(a_{I\ell}) < \frac{1}{2}$$

(c). Idea of Unification:
Within a frame work of relativistic,
quantum, gauge field theory

PARTIAL-SYMMETRIES OF WEAK INTERACTIONS

SHELDON L. GLASHOW †

Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark

Received 9 September 1960

Abstract: Weak and electromagnetic interactions of the leptons are examined under the hypothesis that the weak interactions are mediated by vector bosons. With only an isotopic triplet of leptons coupled to a triplet of vector bosons (two charged decay-intermediaries and the photon) the theory possesses no partial-symmetries. Such symmetries may be established if additional vector bosons or additional leptons are introduced. Since the latter possibility yields a theory disagreeing with experiment, the simplest partially-symmetric model reproducing the observed electromagnetic and weak interactions of leptons requires the existence of at least four vector-boson fields (including the photon). Corresponding partially-conserved quantities suggest leptonic analogues to the conserved quantities associated with strong interactions: strangeness and isobaric spin.



The birth of the Standard Model:

VOLUME 19, NUMBER 21

PHYSICAL REVIEW LETTERS

20 NOVEMBER 1967

¹¹ In obtaining the expression (11) the mass difference between the charged and neutral has been ignored.

¹² M. Ademollo and R. Gatto, *Nuovo Cimento* **44A**, 282 (1967).

bra is slightly larger than that (0.23%) obtained from the ρ -dominance model of Ref. 2. This seems to be true also in the other case of the ratio $\Gamma(\eta \rightarrow \pi^+\pi^-\gamma)/\Gamma(\eta \rightarrow \pi^+\pi^-\gamma)$ calculated in Refs. 12 and 14.

J. M. Brown and P. Singer, *Phys. Rev. Letters* **8**, 100 (1962).

A MODEL OF LEPTONS*

Steven Weinberg†

A MODEL OF LEPTONS*

Leptons interact only with photons, and with the intermediate bosons that presumably mediate weak interactions. What could be more natural than to unite¹ these spin-one bosons into a multiplet of gauge fields? Standing in the way of this synthesis are the obvious differences in the masses of the photon and intermediate meson, and in their couplings. We might hope to understand these differences

†
Physics Department,
Cambridge, Massachusetts
02138 (1967)

on a right-handed singlet

$$R = [\frac{1}{2}(1-\gamma_5)]e.$$

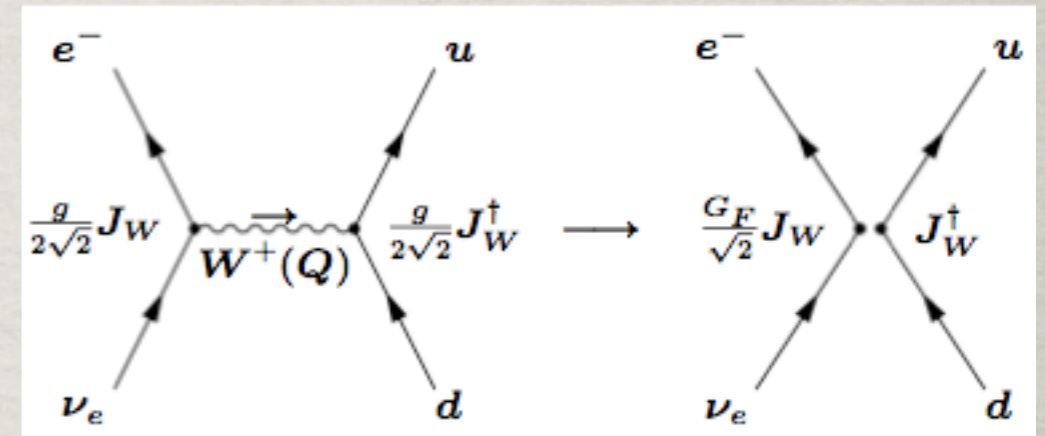


The EW Unification I: Couplings

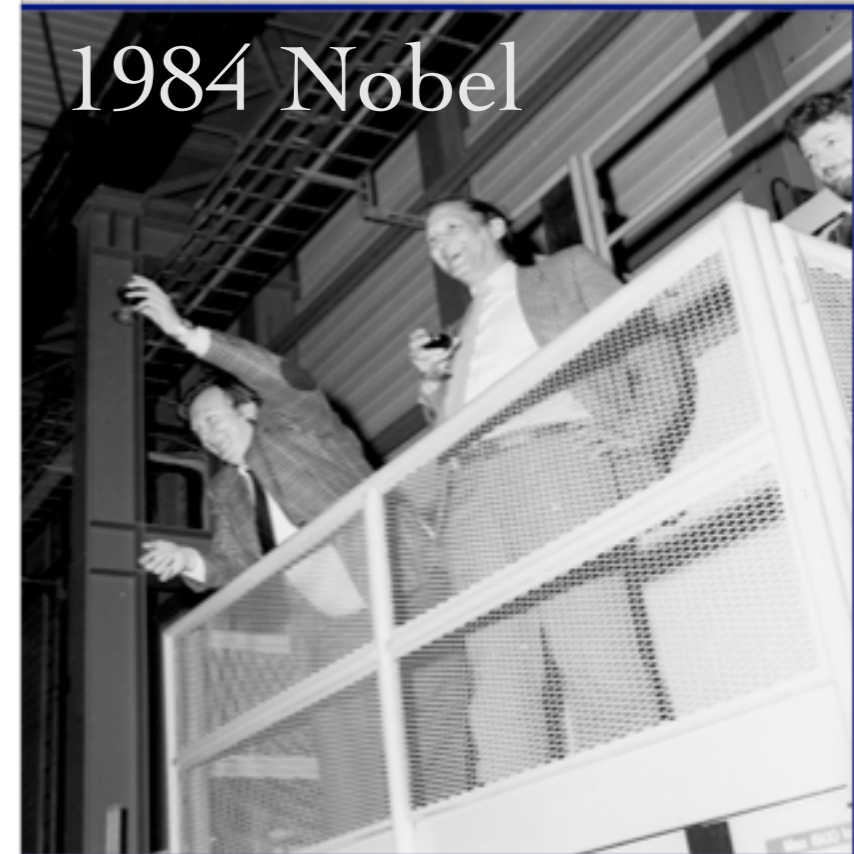
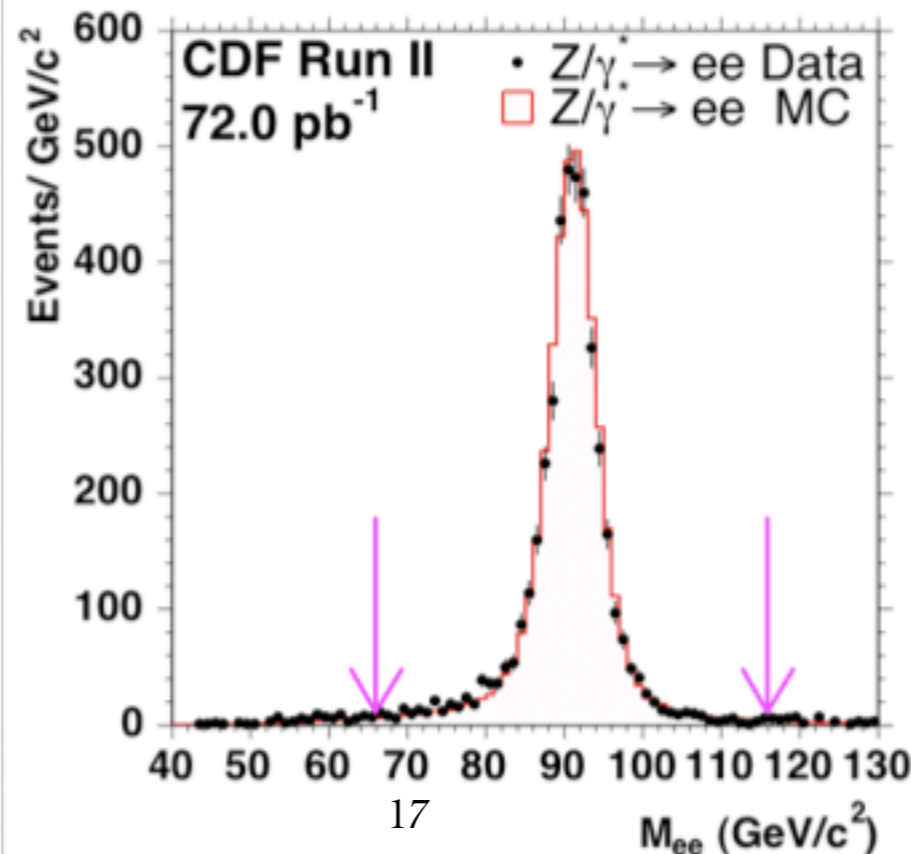
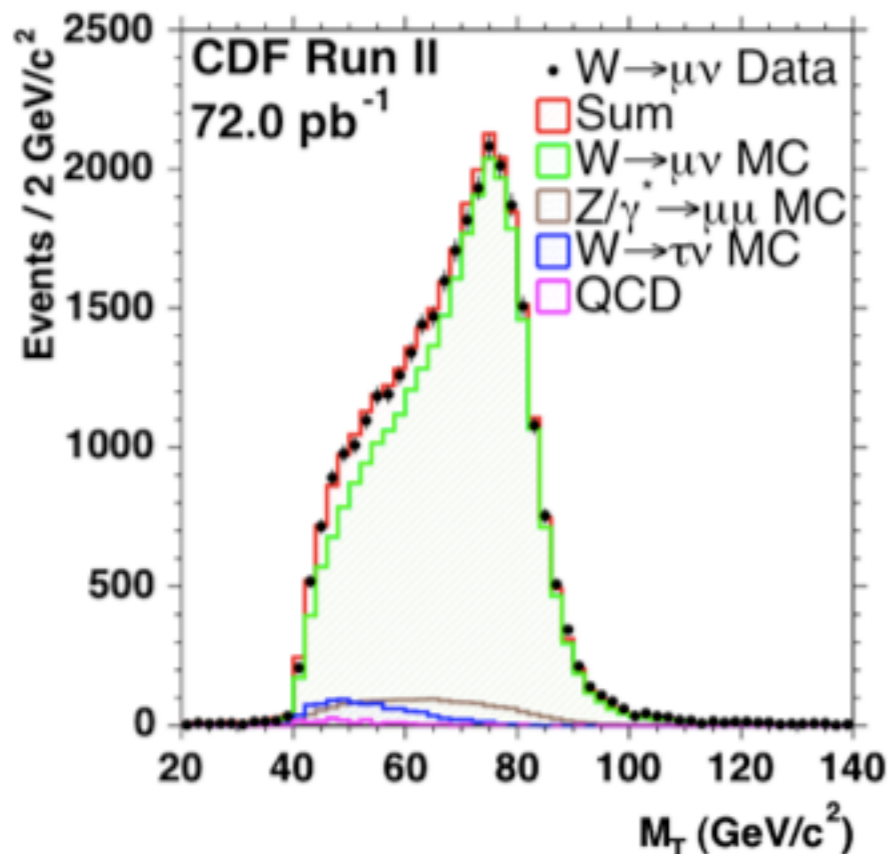
$SU(2)_L \otimes U(1)_Y$ interactions.

$e = g \sin \theta_W$ coupling unification

$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$ short – range scale.



The EW scale is fully open up:



The EW Unifcation II: Particle representation

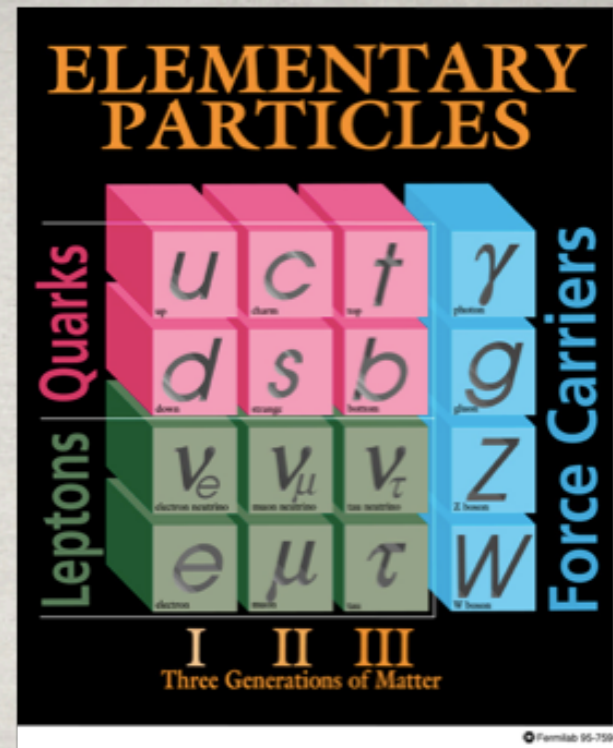
- Simple structure and particle contents:

Leptons:

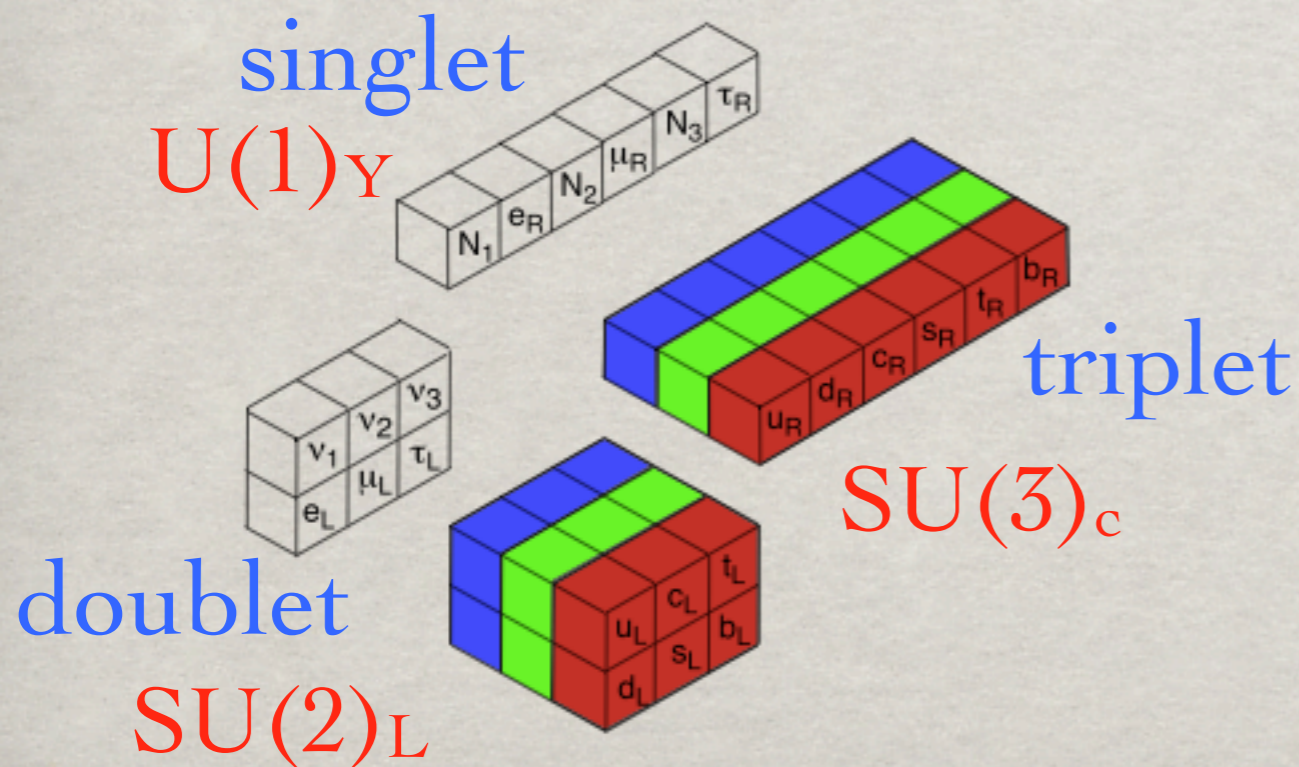
$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \quad e_R, \mu_R, \tau_R, \quad (\nu'_R \text{ s ?})$$

Quarks:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s \end{pmatrix}_L, \quad \begin{pmatrix} t \\ b \end{pmatrix}_L, \quad u_R, d_R, c_R, s_R, t_R, b_R$$



(1979 Nobel)



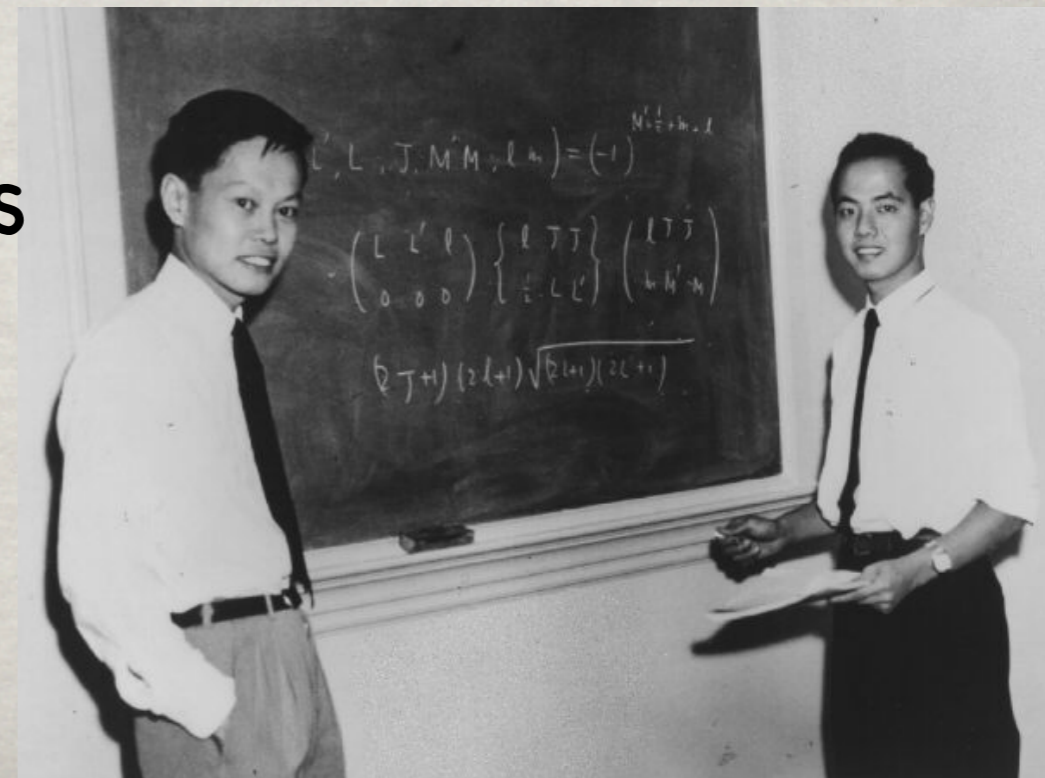
BUT, The local gauge symmetry prevents gauge bosons from acquiring masses!

$$\frac{1}{2}M_A^2 A_\mu A^\mu \rightarrow \frac{1}{2}M_A^2 \left(A_\mu - \frac{1}{e}\partial_\mu \alpha\right) \left(A^\mu - \frac{1}{e}\partial^\mu \alpha\right) \neq \frac{1}{2}M_A^2 A_\mu A^\mu$$

Worse, chiral fermion masses also forbidden by gauge symmetry!

$$-m_e \bar{e}e = -m_e \bar{e} \left(\frac{1}{2}(1 - \gamma_5) + \frac{1}{2}(1 + \gamma_5) \right) e = -m_e (\bar{e}_R e_L + \bar{e}_L e_R)$$

“The Left- and right-chiral electrons carry different Weak charges”
(1957 Noble Prize)



Pauli's Criticism:

An Anecdote by Yang: SU(2) gauge symmetry

Wolfgang Pauli (1900-1958) was spending the year in Princeton, and was deeply interested in symmetries and interactions.... Soon after my seminar began, when I had written on the blackboard,



$$(\partial_\mu - i\epsilon\mathbf{B}_\mu)\psi$$

Pauli asked, "What is the mass of this field \mathbf{B}_μ ?" I said we did not know. Then I resumed my presentation but soon Pauli asked the same question again. I said something to the effect that it was a very complicated problem, we had worked on it and had come to no definite conclusions. I still remember his repartee: "That is not sufficient excuse". I was so taken aback that I decided, after a few moments' hesitation, to sit down. There was general embarrassment. Finally Oppenheimer, who was chairman of the seminar, said "We should let Frank proceed". I then resumed and Pauli did not ask any more questions during the seminar.

Wolfgang Pauli and C. N. Yang

2. The Spontaneous Symmetry Breaking -- Nature May Not be THAT Symmetric:

“The Lagrangian of the system may display an symmetry, but the ground state does not respect the same symmetry.”

Known Example: Ferromagnetism

Above a critical temperature, the system is symmetric, magnetic dipoles randomly oriented. Below a critical temperature, the ground state is a completely ordered configuration in which all dipoles are ordered in some arbitrary direction,

$$SO(3) \rightarrow SO(2)$$



Domains Before Magnetization



Domains After Magnetization

Known Example: QCD condensation

Consider the two-flavor massless QCD:

$$-\frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu} - \sum_{u,d} (\bar{q}_L \gamma^\mu D_\mu q_L + \bar{q}_R \gamma^\mu D_\mu q_R)$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow (U_L \gamma_L + U_R \gamma_R) \begin{pmatrix} u \\ d \end{pmatrix} \Rightarrow SU(2)_L \otimes SU(2)_R$$

QCD below Λ_{QCD} becomes strong and forms condensate: $\langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \sim v^3$

$$SU(2)_L \otimes SU(2)_R \Rightarrow SU(2)_V, \text{ thus } U_L = U_R.$$

Chiral symmetry breaking to iso-spin.

The concept of SSB: profound, common.

Y. Nambu was the first one to have formulated the spontaneous symmetry breaking in a relativistic quantum field theory (1960).

He is the one to propose the understanding of the nucleon mass by dynamical chiral symmetry breaking: The Nambu-Jona-Lasinio Model.



2008 Nobel Prize in physics: "for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"

Be aware of the difference between the dynamical mass for baryons (you and me) and that of elementary particles by the Higgs mechanism.

Exercise 3:

Find (or make up) other examples for spontaneous symmetry breaking.

Also, think about the relations between the fundamental theoretical formalisms (Newton's Law; Maxwell Equations; Einstein Equation; Lagrangians...) and specific states for a given system (initial and boundary conditions of a system).

3. The Goldstone Theorem

-- A show stopper or helper?

“If a continuous symmetry of the system is spontaneously broken, then there will appear a massless degree of freedom, called the Nambu-Goldstone boson.”

$$\text{Symmetry: } [Q, H] = QH - HQ = 0$$

$$\text{Vacuum state: } H|0\rangle = E_{\min}|0\rangle \quad \text{But: } Q|0\rangle \neq 0 = |0'\rangle$$

$$(QH - HQ)|0\rangle = 0 = (E_{\min} - H)|0'\rangle,$$
$$\text{thus: } H|0'\rangle = E_{\min}|0'\rangle$$

There is a new, non-symmetric state $|0'\rangle$, that has a degenerate energy with vacuum $|0\rangle$, thus massless: the Nambu-Goldstone boson.

The Goldstone Theorem (continued)

Broken Symmetries*

JEFFREY GOLDSTONE

Trinity College, Cambridge University, Cambridge, England

AND

ABDUS SALAM AND STEVEN WEINBERG†

Imperial College of Science and Technology, London, England

(Received March 16, 1962)

Some proofs are presented of Goldstone's conjecture, that if there is continuous symmetry transformation under which the Lagrangian is invariant, then either the vacuum state is also invariant under the transformation, or there must exist spinless particles of zero mass.

Properties of the Nambu-Goldstone boson:

1. Massless, gapless in spectrum

3. Decouple at low energies:

$$\langle G | Q | 0 \rangle \neq 0, \quad \langle G(p) | j^\mu(x) | 0 \rangle \sim e^{-ipx} p^\mu v$$

A illustrative (Goldstone's original) Model:

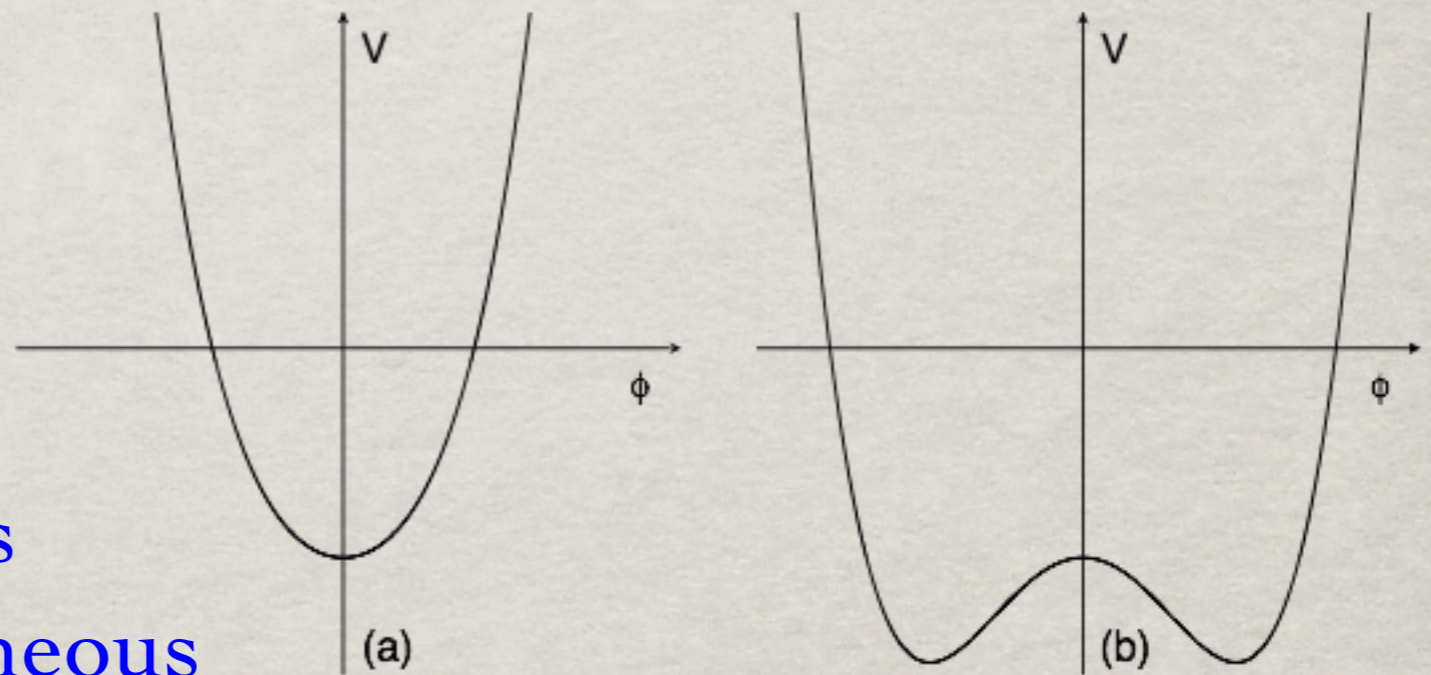
(a). Background complex scalar field Φ :

$$\mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi - V(\phi^* \phi) \qquad V = \frac{\lambda}{4} \left(\phi^* \phi - \frac{\mu^2}{\lambda} \right)^2$$

Invariant under a U(1)
global transformation:

$$\phi \rightarrow e^{i\alpha} \phi$$

For $\mu^2 > 0$, the vacuum is
shifted, and thus spontaneous
symmetry breaking.



$$v = \langle 0 | \phi | 0 \rangle = \mu / \sqrt{\lambda}.$$

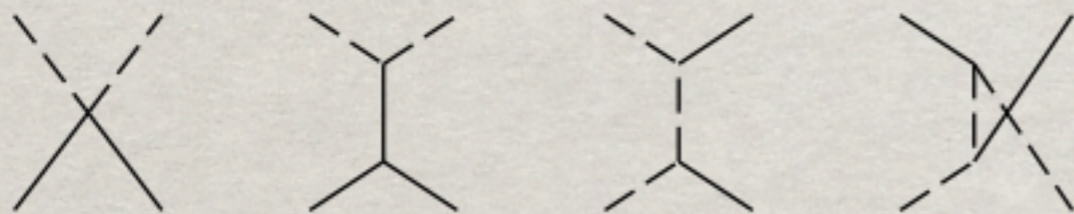
Particle spectrum:[§]

Shift: $R = \sqrt{2} \operatorname{Re}(\phi - v), \quad I = \sqrt{2} \operatorname{Im}\phi,$

Then:
$$\mathcal{L} = \frac{1}{2} \partial^\mu R \partial_\mu R - \frac{\lambda v^2}{2} R^2 - \frac{\lambda \mu}{2\sqrt{2}} R^3 - \frac{\lambda}{16} R^4$$

$$+ \frac{1}{2} \partial^\mu I \partial_\mu I - \frac{\lambda \mu}{2\sqrt{2}} R I^2 - \frac{\lambda}{16} (R^2 I^2 + I^4)$$

- * R is a massive scalar: $M_R = \sqrt{\lambda} v.$
- * I is massless, interacting.
- * Though not transparent, it can be verified:[§]



$\mathcal{M}(RI \rightarrow RI)|_{p \rightarrow 0} \rightarrow 0!$

I does decouple at low energies!

Exercise 4: Show this result by an explicit calculation.

[§] C. Burgges, hep-ph/9812468

(b). Field Φ Re-definition:

+ C. Burgges, hep-ph/9812468

Weinberg's 1st Law of Theoretical Physics⁺:

“You can use whatever variables you like. But if you used the wrong one, you'd be sorry.”

Define:

$$\phi(x) = \chi(x) e^{i\theta(x)},$$

$$\mathcal{L} = -\partial_\mu \chi \partial^\mu \chi - \chi^2 \partial_\mu \theta \partial^\mu \theta - V(\chi^2).$$

(this is like from the rectangular *form* to the *polar form*.)

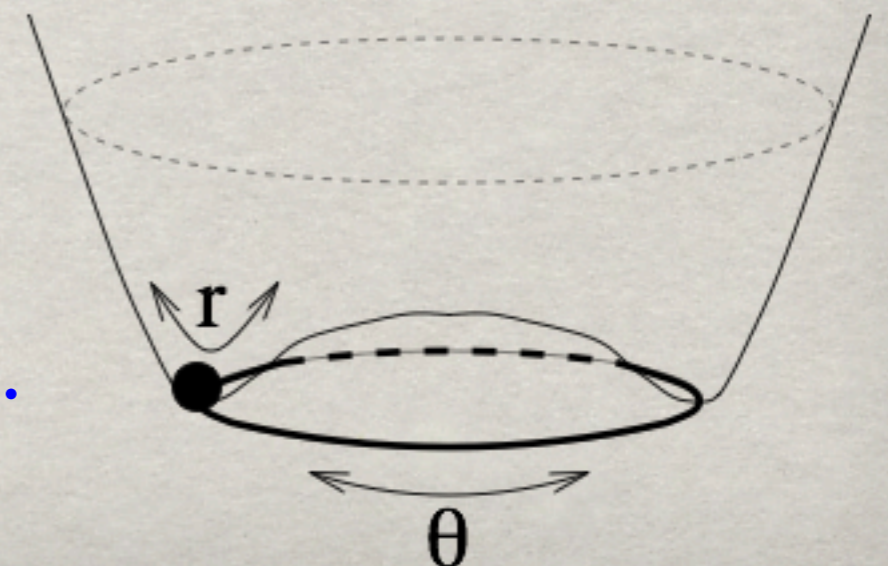
We then see that:

- * the θ field is only derivatively coupled, and thus decoupled at low energies
- * the θ field respects an inhomogeneous transformation

$$\theta \rightarrow \theta + \alpha, \quad \phi = v e^{i\theta(x)}$$

a phase rotation from the vacuum:

- * the $\chi(x)$ is massive radial excitation.



Known example: Chiral symmetry breaking

$$-\frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu} - \sum_{u,d} (\bar{q}_L \gamma^\mu D_\mu q_L + \bar{q}_R \gamma^\mu D_\mu q_R)$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow (U_L \gamma_L + U_R \gamma_R) \begin{pmatrix} u \\ d \end{pmatrix} \Rightarrow SU(2)_L \otimes SU(2)_R$$

QCD breaks the chiral symmetry dynamically:

$$SU(2)_L \otimes SU(2)_R \Rightarrow SU(2)_V, \text{ thus } U_L = U_R.$$

$(\mathbf{3}+\mathbf{3}) - \mathbf{3} = \mathbf{3}$ Goldstone bosons: π^+, π^-, π^0

In the non-linear formulation of the Chiral Lagrangian for the Goldstone bosons:

$$\phi = \frac{v}{\sqrt{2}} \exp(i\vec{\tau} \cdot \vec{\pi}/v) \equiv \frac{v}{\sqrt{2}} U, \quad \mathcal{L} = \frac{v^2}{4} \text{Tr}(\partial^\mu U \partial_\mu U)$$

necessarily derivative coupling.

The pion-pion scattering:

$$\pi_i + \pi_j \rightarrow \pi_k + \pi_\ell \quad (i, j, k, \ell = 1, 2, 3)$$

$$A(s, t, u) = \frac{s}{v^2}.$$

in accordance with the Low Energy theorem.

Chiral perturbation theory agrees well with the pion-pion scattering data,[¶] supporting the Goldstone nature.

Exercise 5: Linearize the Chiral Lagrangian for $\pi\pi$ interaction and calculate one scattering amplitude.[¶]

[¶] J. Donoghue et al., Dynamics of the SM.

“Pseudo-Nambu-Goldstone Bosons”

When a continuous symmetry is broken both explicitly AND spontaneously, and if the effect of the explicit breaking is much smaller than the SSB, then the Goldstone are massive, governed by the explicit breaking, thus called:

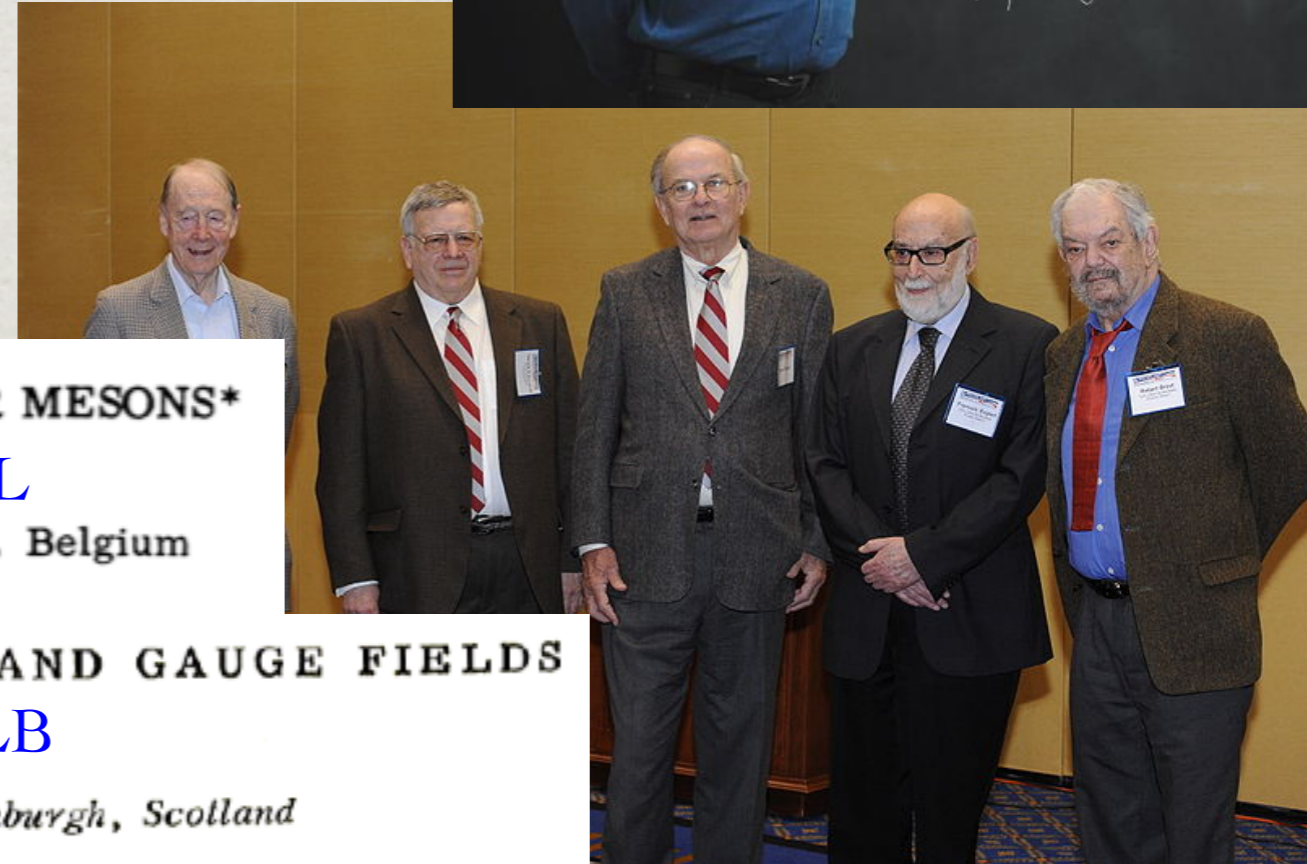
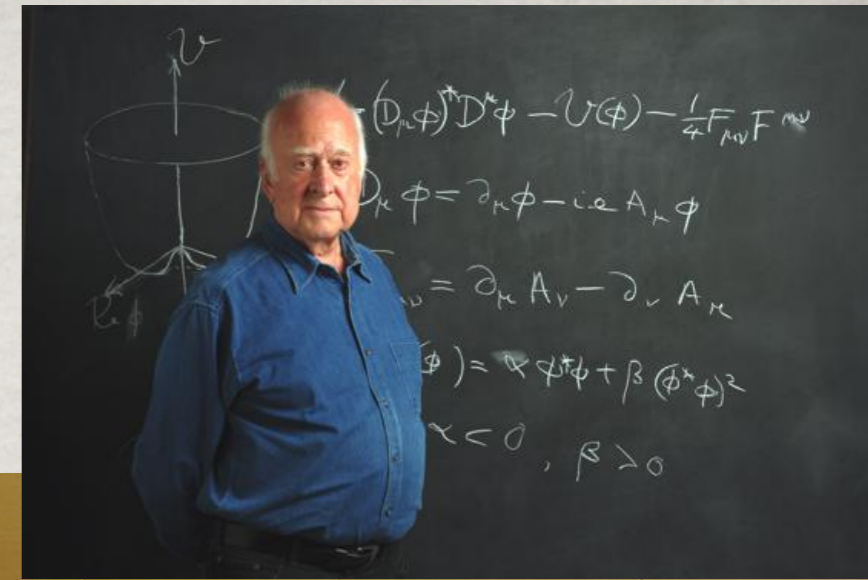
“Pseudo-Nambu-Goldstone bosons”.

The pions are NOT massless, due to explicit symmetry breaking. They are “Pseudo-Nambu-Goldstone bosons”.

Except the photon, no massless boson (a long-range force carrier) has been seen in particle physics!

4. The Magic in 1964: The “Higgs Mechanism”

“If a LOCAL gauge symmetry is spontaneously broken, then the gauge boson acquires a mass by absorbing the Goldstone mode.”



BROKEN SYMMETRY AND THE MASS OF GAUGE VECTOR MESONS*

F. Englert and R. Brout

PRL

Faculté des Sciences, Université Libre de Bruxelles, Bruxelles, Belgium

(Received 26 June 1964)

BROKEN SYMMETRIES, MASSLESS PARTICLES AND GAUGE FIELDS

PLB

P. W. HIGGS

Tait Institute of Mathematical Physics, University of Edinburgh, Scotland

Received 27 July 1964

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

PRL

Peter W. Higgs

Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland

(Received 31 August 1964)

GLOBAL CONSERVATION LAWS AND MASSLESS PARTICLES*

G. S. Guralnik,[†] C. R. Hagen,[‡] and T. W. B. Kibble PRL

Department of Physics, Imperial College, London, England

(Received 13 October 1964)

A illustrative (original) Model:[¶]

$$\mathcal{L} = |\mathcal{D}^\mu \phi|^2 - \mu^2 |\phi|^2 - |\lambda| (\phi^* \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu},$$

where

$$\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$$

is a complex scalar field⁴ and as usual

$$\mathcal{D}_\mu \equiv \partial_\mu + iqA_\mu$$

and

$$F_{\mu\nu} \equiv \partial_\nu A_\mu - \partial_\mu A_\nu.$$

The Lagrangian (5.3.1) is invariant under U(1) rotations

$$\phi \rightarrow \phi' = e^{i\theta} \phi$$

and under the local gauge transformations

$$\begin{aligned} \phi(x) &\rightarrow \phi'(x) = e^{iq\alpha(x)} \phi(x), \\ A_\mu(x) &\rightarrow A'_\mu(x) = A_\mu(x) - \partial_\mu \alpha(x). \end{aligned}$$

[¶] C. Quigg, Gauge Theories of the Strong ...

A illustrative (original) Model:¶

After the EWSB, parameterized in terms of

$$\langle \phi \rangle_0 = v/\sqrt{2}, \quad \phi = e^{i\zeta/v} (v + \eta)/\sqrt{2} \\ \approx (v + \eta + i\zeta)/\sqrt{2}.$$

Then the Lagrangian appropriate for the study of small oscillations is

$$\mathcal{L}_{\text{so}} = \frac{1}{2} [(\partial_\mu \eta)(\partial^\mu \eta) + 2\mu^2 \eta^2] + \frac{1}{2} [(\partial_\mu \zeta)(\partial^\mu \zeta)] \\ - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \underline{qvA_\mu(\partial^\mu \zeta)} + \frac{q^2 v^2}{2} A_\mu A^\mu + \dots$$

The gauge field acquires a mass, mixes with the Goldstone boson.

Upon diagonalization: $\frac{q^2 v^2}{2} \left(A_\mu + \frac{1}{qv} \partial_\mu \zeta \right) \left(A^\mu + \frac{1}{qv} \partial^\mu \zeta \right),$

a form that pleads for the gauge transformation

$$A_\mu \rightarrow A'_\mu = A_\mu + \frac{1}{qv} \partial^\mu \zeta,$$

which corresponds to the phase rotation on the scalar field

$$\phi \rightarrow \phi' = e^{-i\zeta(x)/v} \phi(x) = (v + \eta)/\sqrt{2}.$$

the resultant Lagrangian is then:

$$\mathcal{L}_{\text{so}} = \frac{1}{2}[(\partial_\mu \eta)(\partial^\mu \eta) + 2\mu^2 \eta^2] - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2 v^2}{2}A'_\mu A'^\mu$$

- an η -field, with $(\text{mass})^2 = -2\mu^2 > 0$; the Higgs boson!
 - a massive vector field A'_μ , with mass = qv
 - no ζ -field.
- By virtue of a gauge choice - the unitary gauge, the ζ -field disappears in the spectrum: a massless photon “swallowed” the massless NG boson!

Degrees of freedom count:

Before EWSB:

After:

2 (scalar)+2 (gauge pol.); 1 (scalar)+3 (gauge pol.)

- Two problems provide cure for each other!
massless gauge boson + massless NG boson
→ massive gauge boson + no NG boson

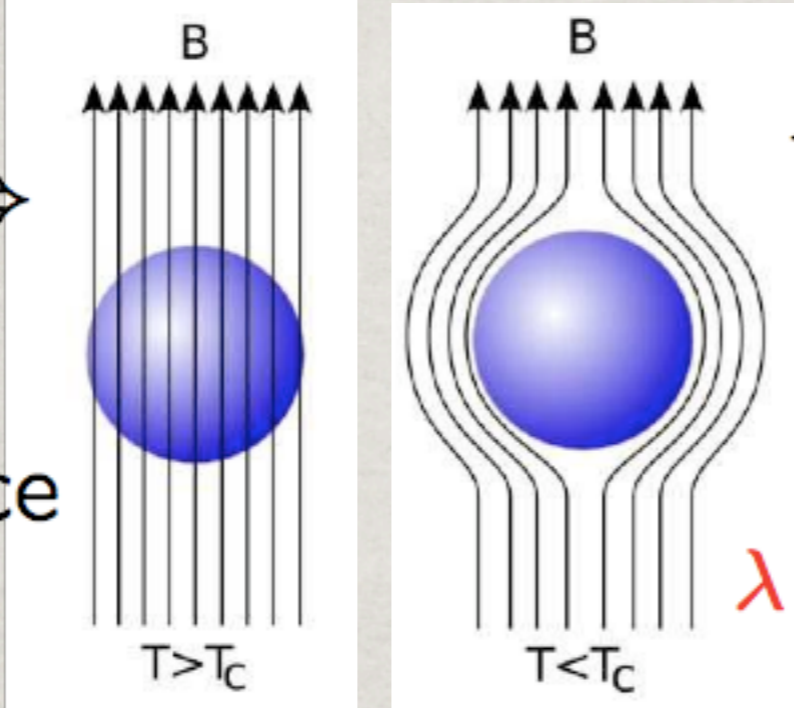
This is truly remarkable!

Known example: Superconductivity

Normal phase \Rightarrow

$E^2 = p^2 c^2$

Long-range force



$T > T_c$ $T < T_c$

\Leftarrow Superconducting phase

$E^2 = p^2 c^2 + m^2 c^4$

gap leads to $\sim \exp(-r/\lambda)$

$\lambda \sim m^{-1}$ penetration depth

In “conventional” electro-magnetic superconductivity:

$m_\gamma \sim m_e/1000$, $T_c^{em} \sim \mathcal{O}(\text{few } K)$. BCS theory.

In “electro-weak superconductivity”:

$m_w \sim G_F^{-\frac{1}{2}} \sim 100 \text{ GeV}$, $T_c^w \sim 10^{15} K!$

True understanding was the work of many hands, most notably:†

- 1960: Nambu formulated spontaneous symmetry breaking for chiral fermions to dynamically generate the nucleon mass (Nambu-Jona-Lasinio model)
- 1961, 1962: Goldstone theorem challenged the implementation of spontaneous symmetry breaking for gauge symmetry: No experimental observation for a massless Goldstone boson.
- 1963: Anderson conjectured a non-relativistic version of a massive Goldstone mode, the “plasmon” in superconductor.
- 1964: Englert+Brout; Higgs; Guralnik+Hagen+Kibble showed the U(1) photon mass generation mechanism, evading the Goldstone theorem in locally gauge invariant theory.§

† Univ. of Edinburgh, Peter Higgs and the Higgs Boson.

§ Sidney Coleman:

(in 1989) that they “had been looking forward to tearing apart this idiot who thought he could get around the Goldstone theorem”.

“Evading the Goldstone Theorem” continues †

- 1964: Higgs (PRL) first commented on the spin-zero boson, in the revised version (upon Nambu’s request to compare with the other’s works) ¶ † Peter Higgs: My Life as a Boson.
- 1966: Higgs (PRD) laid out the scalar scattering/decay in an Abelian $U(1)$ model.‡

¶ It is worth noting that an essential feature of the type of theory which has been described in this note is the prediction of incomplete multiplets of scalar and vector bosons.⁸ It is to be expected that this feature will appear also in theories in which the symmetry-breaking scalar fields are not elementary dynamic variables but bilinear combinations of Fermi fields.⁹

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Spontaneous Symmetry Breakdown without Massless Bosons*

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- 1967: Weinberg (PRL) laid out the fermion mass generation, formulated the $SU(2)_L \times U(1)_Y$ SM.

† Univ. of Edinburgh, Peter Higgs and the Higgs Boson.

As for the name ...

1972: Ben Lee (Rochester Conf. at FNAL) named “Higgs boson” and the “Higgs mechanism”.[§]

[§] Peter Higgs: *My Life as a Boson*.



As to my responsibility for the name “Higgs boson,” because of a mistake in reading the dates on these three earlier papers, I thought that the earliest was the one by Higgs, so in my 1967 paper I cited Higgs first, and have done so since then. Other physicists apparently have followed my lead. But as Close points out, the earliest paper of the three I cited was actually the one by Robert Brout and François Englert. In extenuation of my mistake, I should note that Higgs and Brout and Englert did their work independently and at about the same time, as also did the third group (Gerald Guralnik, C.R. Hagen, and Tom Kibble). But the name “Higgs boson” seems to have stuck. ↩

Recollection:

The Symmetry

QED (1950's)



Yang-Mills Theory
[Yang, Mills, 1954]



EW unification theory
[Glashow, 1960]

The Breaking

Spontaneous symmetry breaking
in particle physics [Nambu,
1960; Goldstone, 1962]



Prediction of the existence of
massive Higgs boson [Higgs,
1964]

Higgs mechanism + EW unification theory
[Weinberg, 1967; Salam, 1968]

B. Higgs Boson Interactions

1. The SM Lagrangian: $\mathcal{L}_{SU(2)\times U(1)} = \mathcal{L}_{gauge} + \mathcal{L}_\phi + \mathcal{L}_f + \mathcal{L}_{Yuk}$.

The gauge part is

Pure gauge sector:

$$\mathcal{L}_{gauge} = -\frac{1}{4}W_{\mu\nu}^i W^{\mu\nu i} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu},$$

The scalar part of the Lagrangian is

The Higgs: $\mathcal{L}_\phi = (D^\mu \phi)^\dagger D_\mu \phi - V(\phi)$ $D_\mu \phi = \left(\partial_\mu + ig \frac{\tau^i}{2} W_\mu^i + \frac{ig'}{2} B_\mu \right) \phi,$

$$V(\phi) = +\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2.$$

$$\phi = \frac{1}{\sqrt{2}} e^{i \sum \xi^i L^i} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix}$$

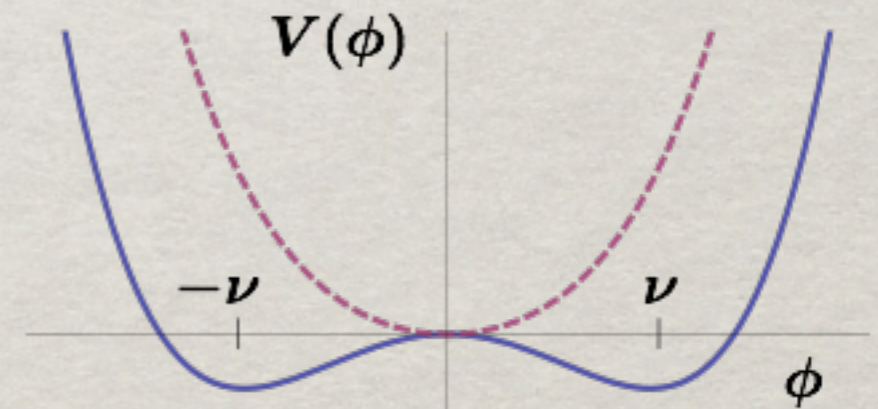
$$\phi \rightarrow \phi' = e^{-i \sum \xi^i L^i} \phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix}$$

$$\mathcal{L}_\phi = (D^\mu \phi)^\dagger D_\mu \phi - V(\phi)$$

$$= \underline{M_W^2 W^{\mu+} W_\mu^-} \left(1 + \frac{H}{\nu}\right)^2 + \frac{1}{2} \underline{M_Z^2 Z^\mu Z_\mu} \left(1 + \frac{H}{\nu}\right)^2$$

$$+ \frac{1}{2} (\partial_\mu H)^2 - V(\phi).$$

$$V(\phi) = -\frac{\mu^4}{4\lambda} - \underline{\mu^2 H^2} + \lambda \nu H^3 + \frac{\lambda}{4} H^4.$$



$$\nu = (-\mu^2/\lambda)^{1/2}$$

$$M_H^2 = -2\mu^2 = 2\lambda\nu^2$$

The Fermions:§

$$\mathcal{L}_f = \sum_{m=1}^F (\bar{q}_{mL}^0 i \not{D} q_{mL}^0 + \bar{l}_{mL}^0 i \not{D} l_{mL}^0 + \bar{u}_{mR}^0 i \not{D} u_{mR}^0 + \bar{d}_{mR}^0 i \not{D} d_{mR}^0 + \bar{e}_{mR}^0 i \not{D} e_{mR}^0 + \bar{\nu}_{mR}^0 i \not{D} \nu_{mR}^0)$$

$$D_\mu q_{mL}^0 = \left(\partial_\mu + \frac{ig}{2} \vec{\tau} \cdot \vec{W}_\mu + \frac{ig'}{6} B_\mu \right) q_{mL}^0 \quad D_\mu u_{mR}^0 = \left(\partial_\mu + \frac{2ig'}{3} B_\mu \right) u_{mR}^0$$

$$D_\mu l_{mL}^0 = \left(\partial_\mu + \frac{ig}{2} \vec{\tau} \cdot \vec{W}_\mu - \frac{ig'}{2} B_\mu \right) l_{mL}^0 \quad D_\mu d_{mR}^0 = \left(\partial_\mu - \frac{ig'}{3} B_\mu \right) d_{mR}^0$$

$$D_\mu e_{mR}^0 = (\partial_\mu - ig' B_\mu) e_{mR}^0$$

$$D_\mu \nu_{mR}^0 = \partial_\mu \nu_{mR}^0,$$

Gauge invariant, massless.

However, a fermion mass must flip chirality:

$$m_f (\bar{f}_L f_R + \bar{f}_R f_L)$$

and thus not SM gauge invariant!

Need something like a doublet:

$$y_f (\bar{f}_1, f_2)_L \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}_L f_R$$

that's the Higgs doublet!

§ P. Langacker: TASI Lectures 2007.

The gauge invariant Yukawa interactions:

$$\mathcal{L}_{Yuk} = - \sum_{m,n=1}^F \left[\Gamma_{mn}^u \bar{q}_{mL}^0 \tilde{\phi} u_{nR}^0 + \Gamma_{mn}^d \bar{q}_{mL}^0 \phi d_{nR}^0 \right. \\ \left. + \Gamma_{mn}^e \bar{l}_{mn}^0 \phi e_{nR}^0 + \Gamma_{mn}^\nu \bar{l}_{mL}^0 \tilde{\phi} \nu_{nR}^0 \right] + h.c.,$$

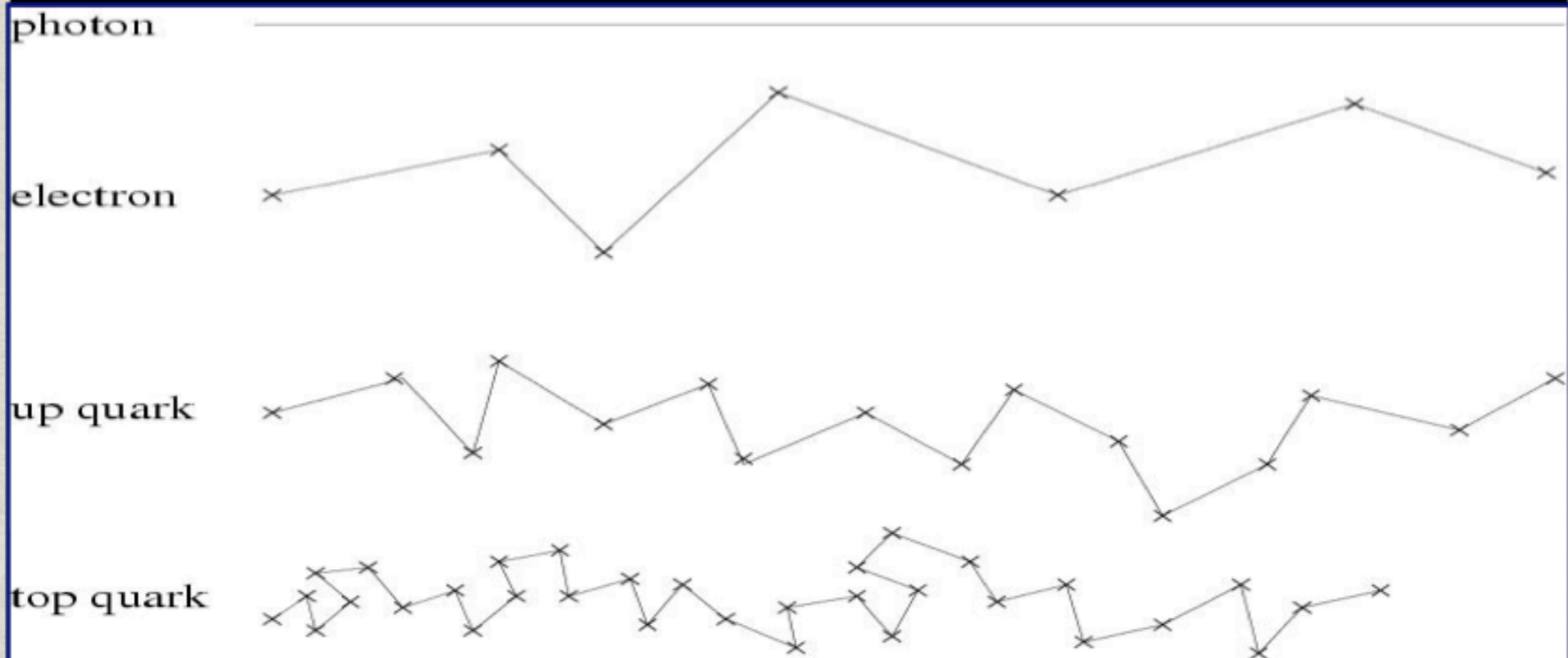
After the EWSB,

$$-\mathcal{L}_{Yuk} \rightarrow \sum_{m,n=1}^F \bar{u}_{mL}^0 \Gamma_{mn}^u \left(\frac{\nu + H}{\sqrt{2}} \right) u_{mR}^0 + (d, e, \nu) \text{ terms} \\ = \bar{u}_L^0 (M^u + h^u H) u_R^0 + (d, e, \nu) \text{ terms} + h.c.,$$

$$-\mathcal{L}_{Yuk} = \sum_i m_i \bar{\psi}_i \psi_i \left(1 + \frac{g}{2M_W} H \right) = \sum_i \underline{m_i \bar{\psi}_i \psi_i} \left(1 + \frac{H}{\nu} \right)$$

2. Higgs Boson Couplings:

Masses determined by interactions with vacuum:



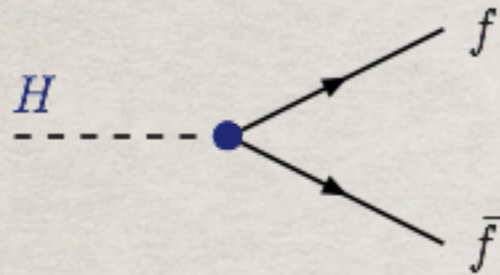
$$M_{W,Z} = \frac{1}{2}g_V v, \quad m_f = \frac{g_f}{\sqrt{2}} v, \quad v^{-2} = \sqrt{2} G_F.$$

Thus, where ever is mass, there will be H!

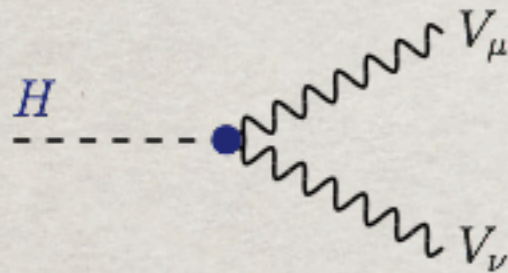
The Low-Energy-theorem:

$$m_i \rightarrow m_i \left(1 + \frac{H}{v}\right) \text{ for } p_H < v.$$

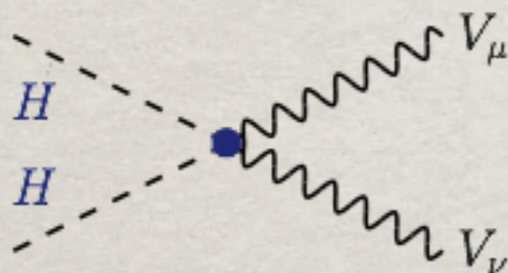
Feynman rules:



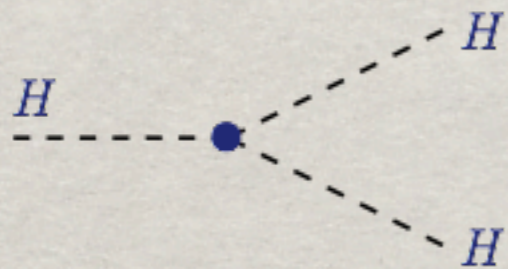
$$g_{Hff} = m_f/v = (\sqrt{2}G_\mu)^{1/2} m_f \quad \times (i)$$



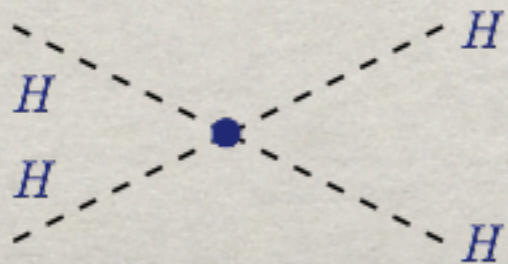
$$g_{HVV} = 2M_V^2/v = 2(\sqrt{2}G_\mu)^{1/2} M_V^2 \quad \times (-ig_{\mu\nu})$$



$$g_{HHVV} = 2M_V^2/v^2 = 2\sqrt{2}G_\mu M_V^2 \quad \times (-ig_{\mu\nu})$$



$$g_{HHH} = 3M_H^2/v = 3(\sqrt{2}G_\mu)^{1/2} M_H^2 \quad \times (i)$$



$$g_{HHHH} = 3M_H^2/v^2 = 3\sqrt{2}G_\mu M_H^2 \quad \times (i)$$

Exercise 6: Verify the above Feynman rules by invoking the low-energy theorem:

$$m_i \rightarrow m_i \left(1 + \frac{H}{v}\right) \text{ for } p_H < v.$$

Goldstone-boson Equivalence Theorem:

At high energies $E \gg M_W$, the longitudinally polarized gauge bosons behave like the corresponding Goldstone bosons. (They remember their origin!)

Caution: Very often, we say at high energies, $M_W \rightarrow 0$.

Rigorously speaking, we mean: $g, M_W \rightarrow 0$, but $M_W/g \rightarrow v/2$.

Exercise 7: Verify the Goldstone-boson Equivalence Theorem by examining the HWW vertex.

Hint: Use $\epsilon_L^\mu \rightarrow p_H^\mu / M_W$. It should give you HHH vertex.