Sterile neutrinos from low energy to GUT scale

Max-Planck-Institut
für Kernphysik

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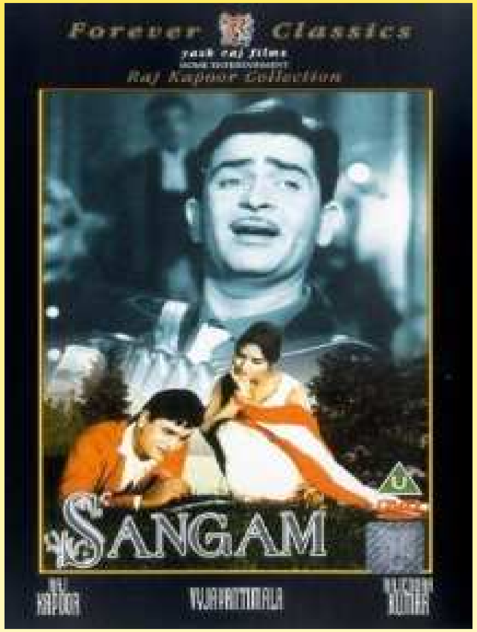
25/03/13

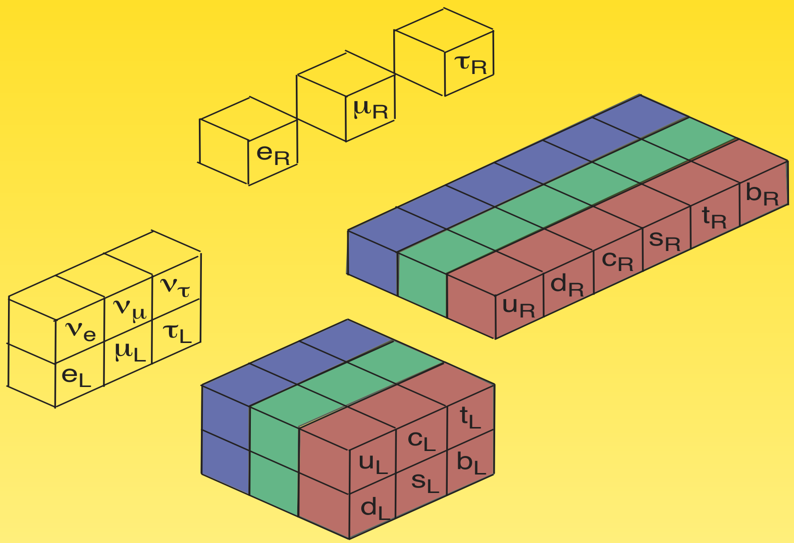


Outline

- General aspects and phenomenology
  - What is a neutrino?
  - What is a sterile neutrino?
  - What is its mass?
  - 3 (4) well motivated scales and their phenomenology
    - eV
    - keV
    - (TeV)
    - heavy
    - νMSM
- Models for light sterile neutrinos: 3 ways to make them light

Introduction

Standard Model of Elementary Particle Physics: $SU(3)_C \times SU(2)_L \times U(1)_Y$



Species	#	$\sum$
Quarks	10	10
Leptons	3	13
Charge	3	16
Higgs	2	18

18 free parameters...

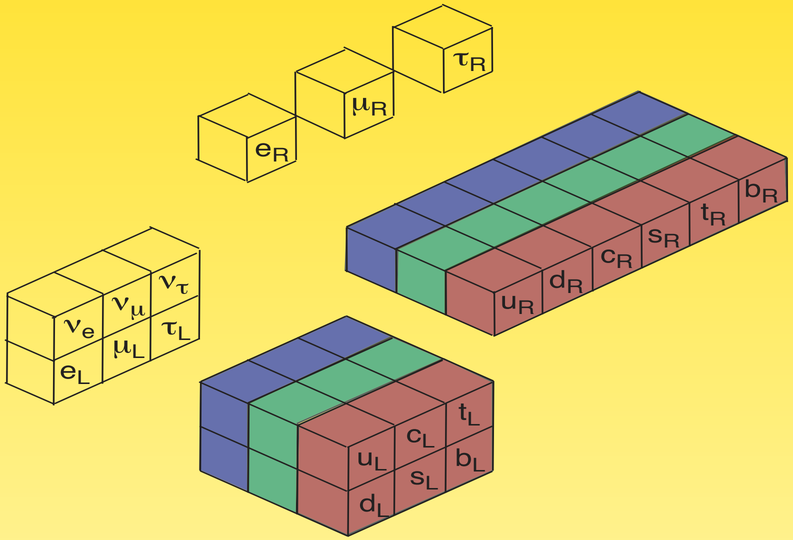
+ Gravitation

+ Dark Energy

+ Dark Matter

+ Baryon Asymmetry

Standard Model of Elementary Particle Physics: $SU(3)_C \times SU(2)_L \times U(1)_Y$



Species	#	$\sum$
Quarks	10	10
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+ Neutrino Mass m_ν

Standard Model of Particle Physics

add neutrino mass matrix m_ν

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Standard Model of Particle Physics

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$\longrightarrow$

Species	$\#$	$\sum$
Quarks	10	10
Leptons	12 (10)	22 (20)
Charge	3	25 (23)
Higgs	2	27 (25)

Standard Model* of Particle Physics

add neutrino mass matrix m_ν

Species	#	$\sum$
Quarks	10	10
Leptons	3	13
Charge	3	16
Higgs	2	18

$\longrightarrow$

Species	#	$\sum$
Quarks	10	10
Leptons	12 (10)	22 (20)
Charge	3	25 (23)
Higgs	2	27 (25)

And: a new energy scale besides Higgs VEV?

Status of Neutrino Physics

Neutrinos. . .

- . . .have mass
- . . .mix

lecture by de Gouvea

Standard Neutrino Physics

Assume neutrinos to be Majorana particles:

$$\mathcal{L}_M = \frac{1}{2}\,\overline{\nu_L}\,m_\nu\,\nu_L^c + h.c.$$

diagonalized by

$$m_\nu = U^*\,m_\nu^{\rm diag}\,U^\dagger$$

with PMNS matrix

$$U = \left(\begin{array}{ccc} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}\,s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\,e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}\,s_{13}e^{i\delta} & c_{23}c_{13} \end{array} \right) P$$

Majorana phases in $P = {\rm diag}(1, e^{i\alpha}, e^{i(\beta+\delta)})$

PMNS matrix

3 families: $U = R_{23}\,\tilde{R}_{13}\,R_{12}\,P$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\,e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\,e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P$$

$$= \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{-i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{-i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{-i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{-i\delta} & c_{23}\,c_{13} \end{pmatrix} P$$

with $P = \mathrm{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)})$

Interpretation in 3 Neutrino Framework

assume $\Delta m^2_{21} \ll \Delta m^2_{31} \simeq \Delta m^2_{32}$ and small θ_{13}:

- atmospheric and accelerator neutrinos: $\Delta m^2_{21} L/E \ll 1$

$$P(\nu_\mu \to \nu_\tau) \simeq \sin^2 2\theta_{23} \, \sin^2 \frac{\Delta m^2_{31}}{4\,E} \, L$$

- solar and KamLAND neutrinos: $\Delta m^2_{31} L/E \gg 1$

$$P(\nu_e \to \nu_e) \simeq 1 - \sin^2 2\theta_{12} \, \sin^2 \frac{\Delta m^2_{21}}{4\,E} \, L$$

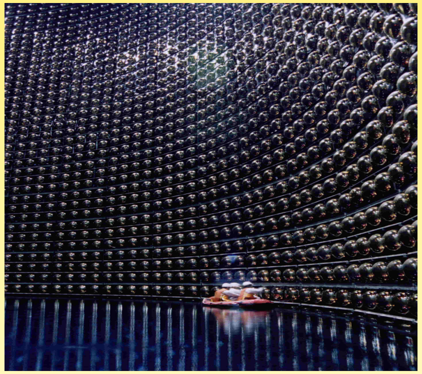
- short baseline reactor neutrinos: $\Delta m^2_{21} L/E \ll 1$

$$P(\nu_e \to \nu_e) \simeq 1 - \sin^2 2\theta_{13} \, \sin^2 \frac{\Delta m^2_{31}}{4\,E} \, L$$

$$U = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{-i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{-i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{-i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{-i\delta} & c_{23}\,c_{13} \end{pmatrix} =$$

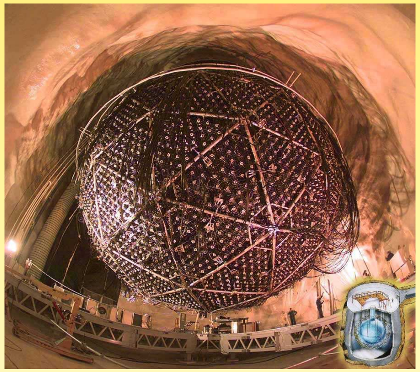
$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric and LBL accelerator}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}\,e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\,e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{SBL reactor}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar and LBL reactor}}$$

atmospheric and LBL accelerator



SBL reactor

solar and LBL reactor



$$U = \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{i\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{-i\delta} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{-i\delta} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{-i\delta} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{-i\delta} & c_{23}\, c_{13} \end{pmatrix} =$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\substack{\text{atmospheric and} \\ \text{LBL accelerator}}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{SBL reactor}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\substack{\text{solar and} \\ \text{LBL reactor}}}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(\sin^2\theta_{23} = \tfrac{1}{2}) \qquad (\sin^2\theta_{13} = 0) \qquad (\sin^2\theta_{12} = \tfrac{1}{3})$$

$$\Delta m^2_{\text{A}} \qquad \Delta m^2_{\text{A}} \qquad \Delta m^2_{\odot}$$

Tri-bimaximal Mixing?

$$U \stackrel{?}{=} \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

still good at zeroth order $\leftrightarrow V \simeq \mathbb{1}$

mass matrix

$$(m_\nu)_{\rm TBM} = \begin{pmatrix} A & B & B \\ \cdot & \frac{1}{2}(A+B+D) & \frac{1}{2}(A+B-D) \\ \cdot & \cdot & \frac{1}{2}(A+B+D) \end{pmatrix}$$

$$A = \frac{1}{3}\left(2\,m_1 + m_2\,e^{-2i\alpha}\right)\,, \quad B = \frac{1}{3}\left(m_2\,e^{-2i\alpha} - m_1\right)\,, \quad D = m_3\,e^{-2i\beta}$$

$\Rightarrow$ Flavor symmetries...

$$U = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{i\delta} & c_{23}\,c_{13} \end{pmatrix} =$$

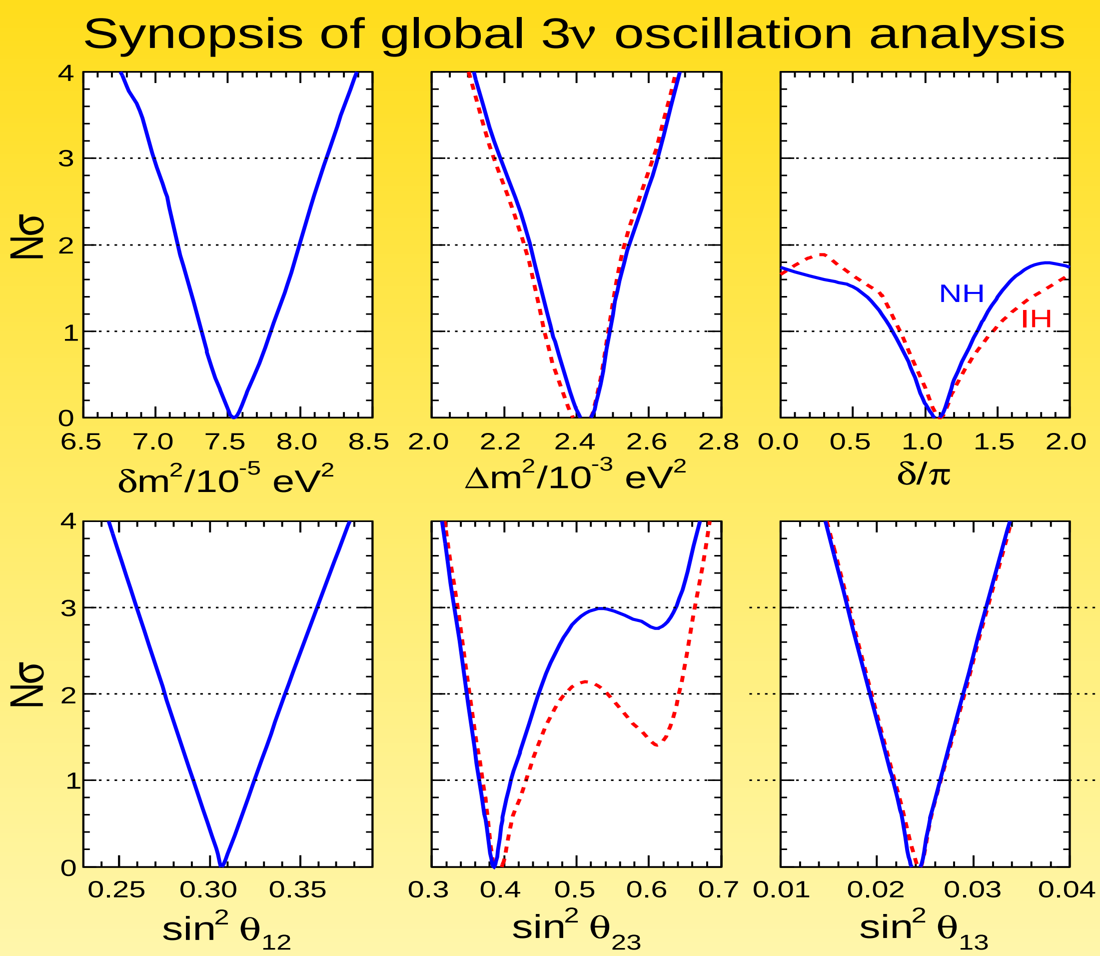
$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\substack{\text{atmospheric and} \\ \text{LBL accelerator}}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}\,e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\,e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{SBL reactor}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\substack{\text{solar and} \\ \text{LBL reactor}}}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix} + \epsilon \qquad \begin{pmatrix} 1 & 0 & \epsilon\,e^{-i\delta} \\ 0 & 1 & 0 \\ -\epsilon\,e^{i\delta} & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 1 \end{pmatrix} + \epsilon$$

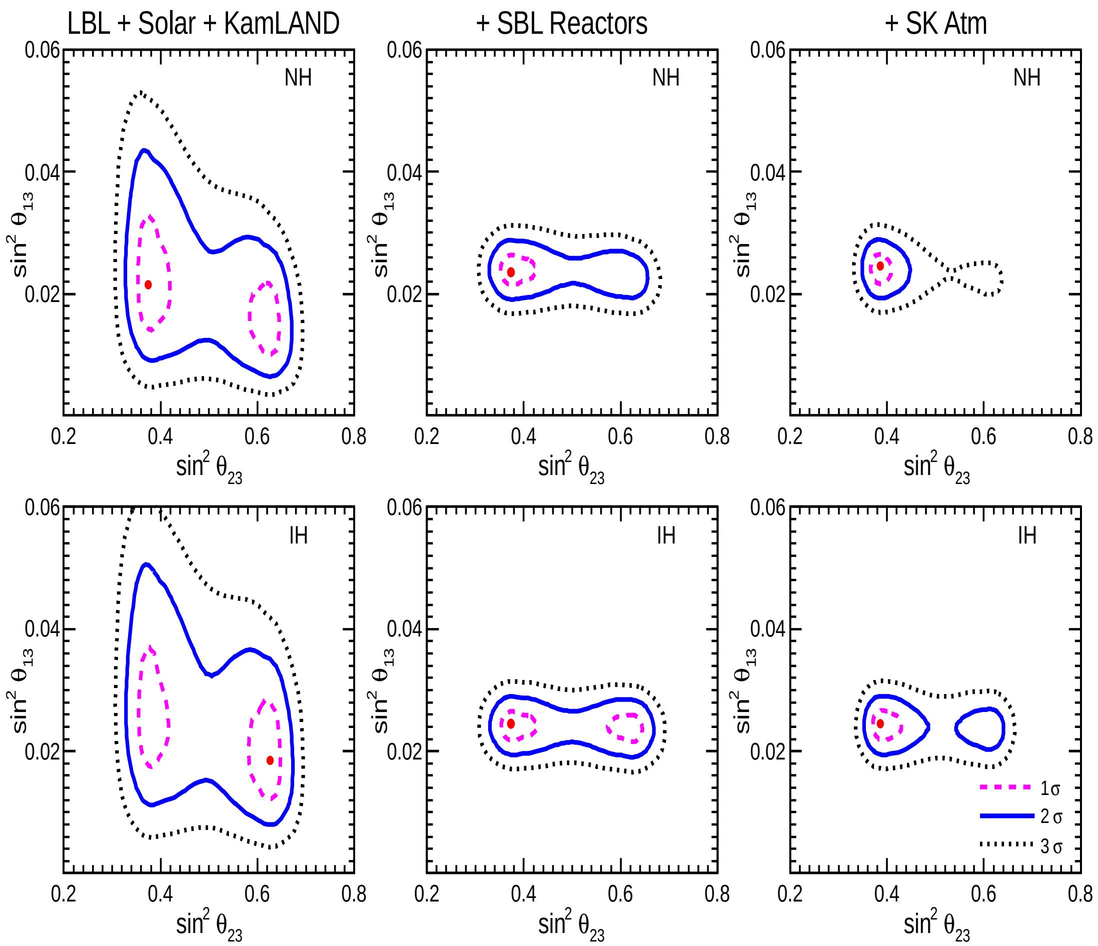
$$(\sin^2\theta_{23} \simeq \tfrac{1}{2}) \qquad (\sin^2\theta_{13} = \epsilon^2) \qquad (\sin^2\theta_{12} \simeq \tfrac{1}{3})$$

$$\Delta m^2_{\mathrm{A}} \qquad \Delta m^2_{\mathrm{A}} \qquad \Delta m^2_{\odot}$$

Status of global fits



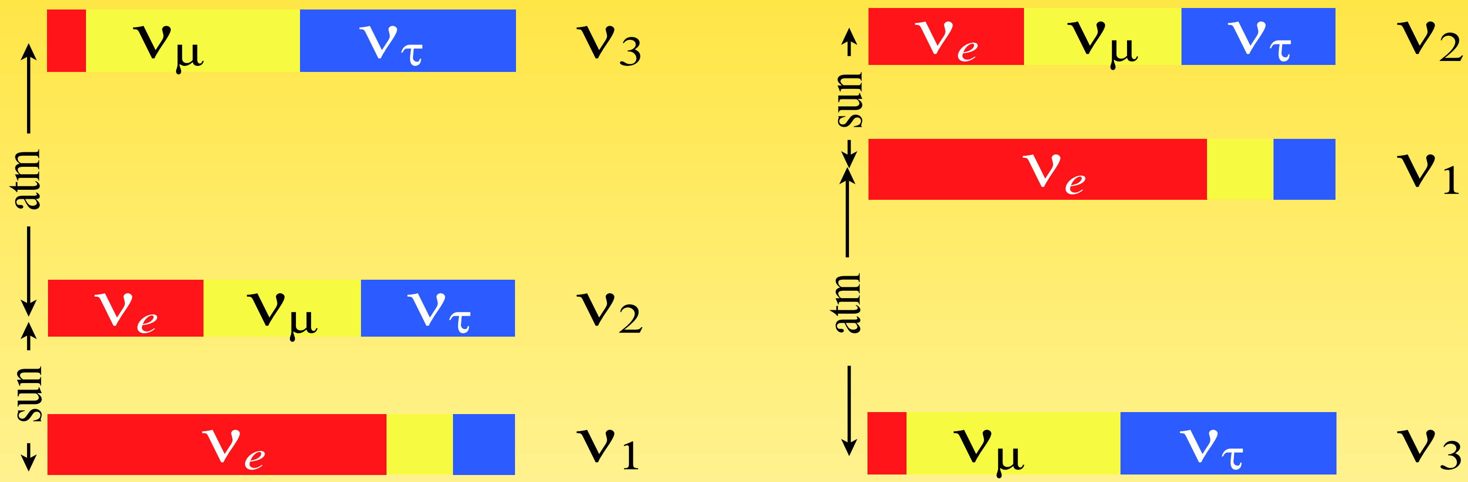
Fogli, Lisi *et al.*, June 2012



Fogli, Lisi *et al.*, June 2012

Unknown Parameters

- CP phase?
- octant?
- mass ordering?



- mass value?
- Dirac/Majorana?
- unitarity, NP

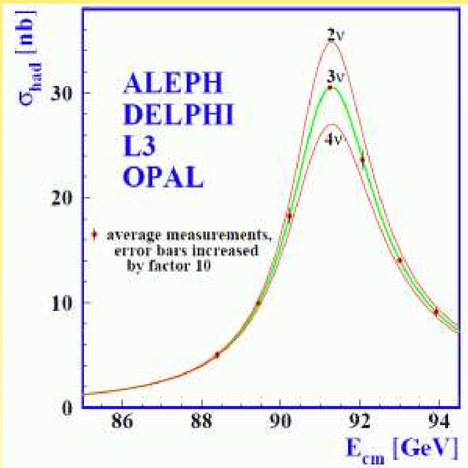
Lessons

- consistent picture emerging
- there are 3 generation effects!
- about 2σ hint for $\theta_{23} < \pi/4$
- about 1σ hint for $\delta \neq 0$
- no hint for hierarchy
- future program of LBL experiments to pin down, make more precise
- all is well...?

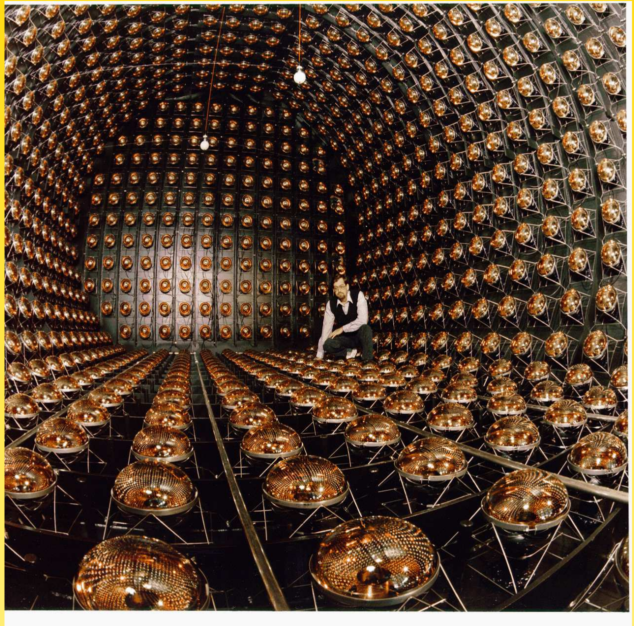
Light Sterile Neutrinos

$\leftrightarrow$ is there an additional **sterile** state at $\Delta m^2 \lesssim$ eV ?

- LSND ($\overline{\nu}_\mu \to \overline{\nu}_e$)
- MiniBooNE ($\overline{\nu}_\mu \to \overline{\nu}_e$ and $\nu_\mu \to \nu_e$)
- Gallium anomaly ($\nu_e \to \nu_e$)
- reactor anomaly ($\overline{\nu}_e \to \overline{\nu}_e$)
- cosmology and astroparticle physics



Motivation for Sterile Neutrinos: LSND



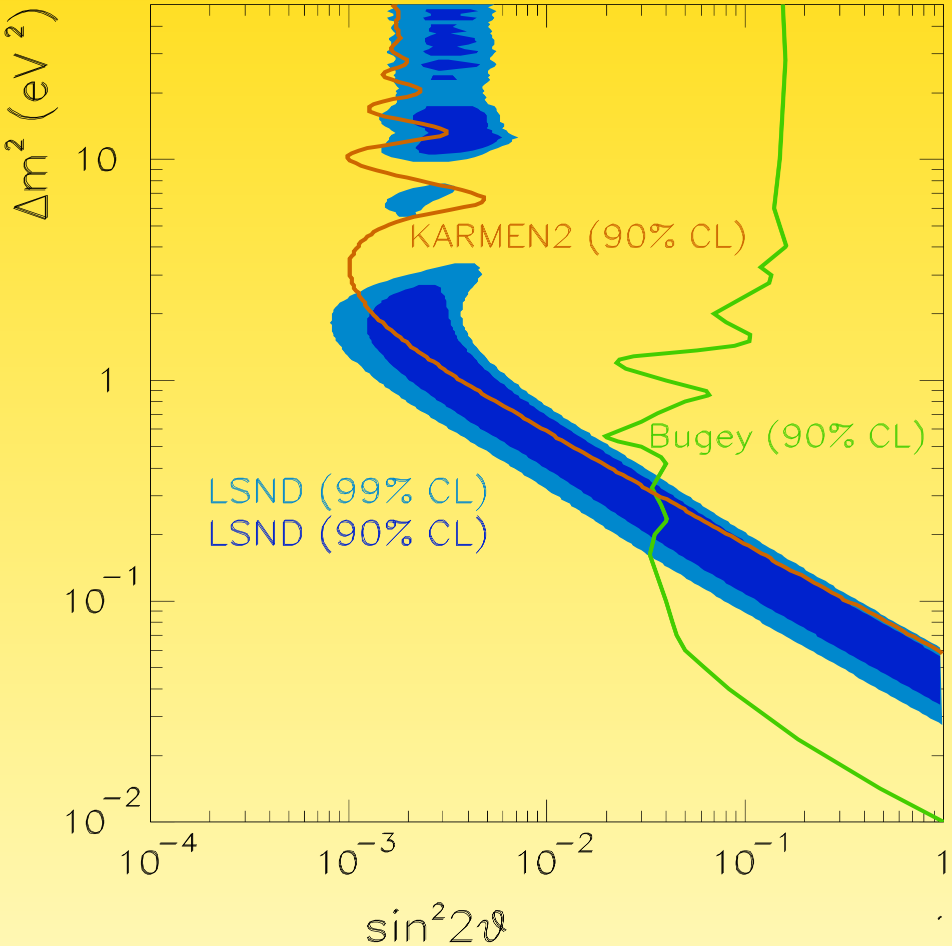
LSND: π^+ from an 800 MeV p-beam: $\pi^+ \to \nu_\mu \, \overline{\nu}_\mu \, \nu_e$

and look for $\overline{\nu}_e$ doing inverse β decay

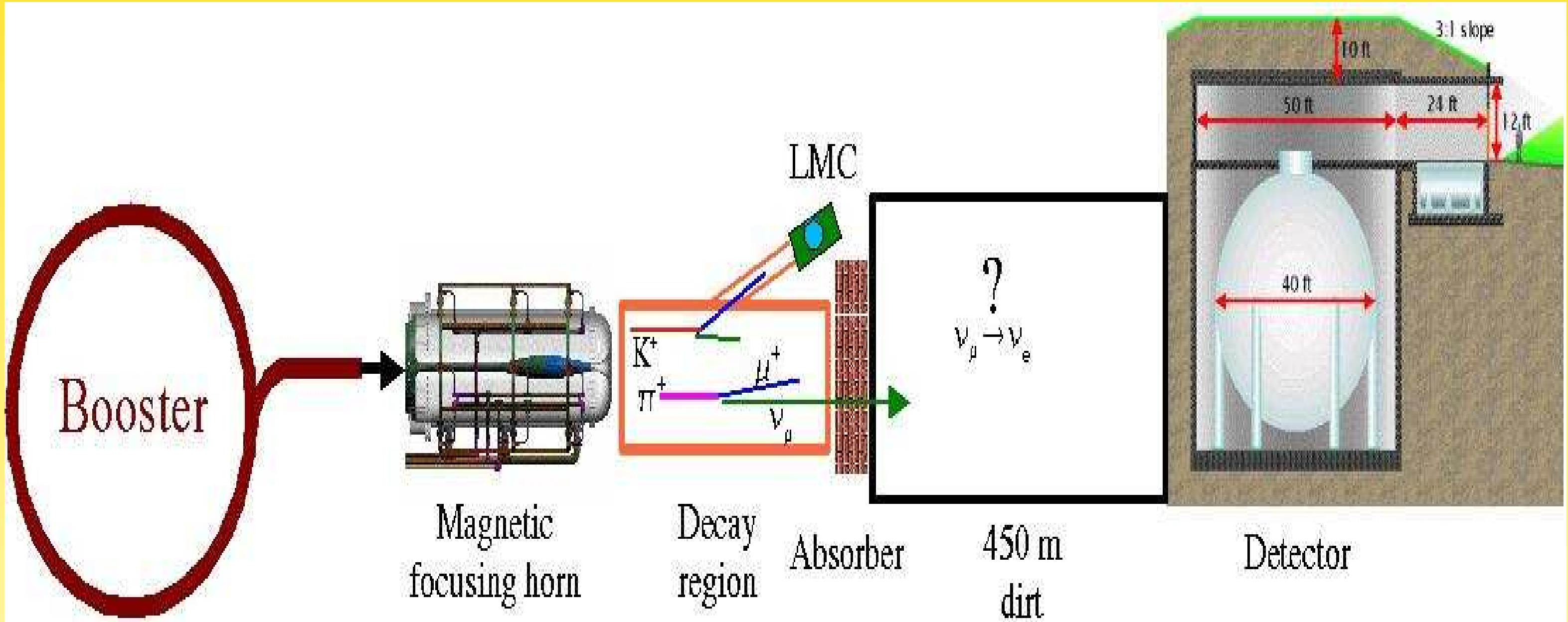
excess: $87.9 \pm 22.4 \pm 6.0 \ (3.8\sigma)$

$$P(\overline{\nu}_\mu \to \overline{\nu}_e) = 0.0264 \pm 0.0067 \pm 0.0045$$

Motivation for Sterile Neutrinos: LSND

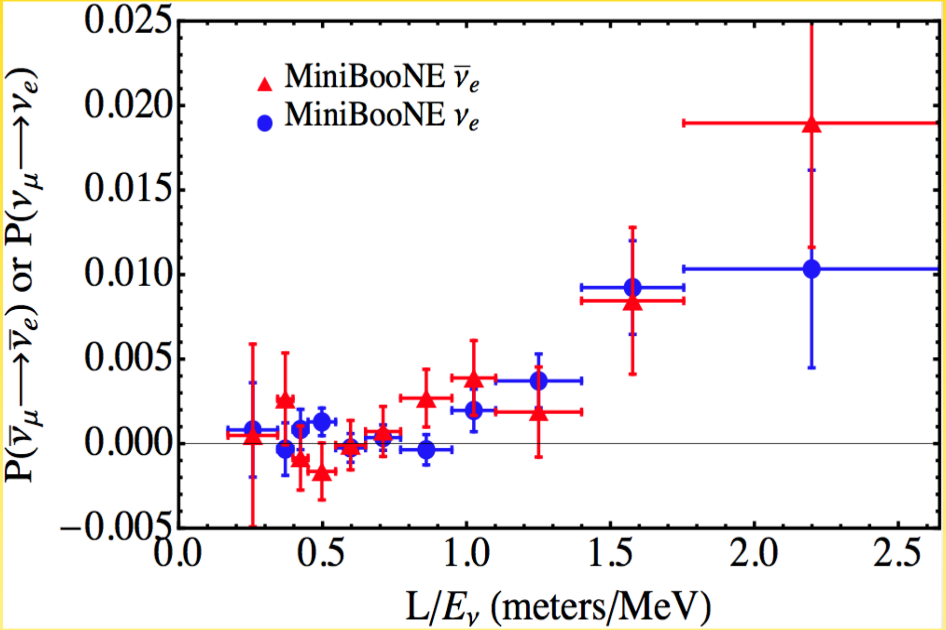


Motivation for Sterile Neutrinos: MiniBooNE



Motivation for Sterile Neutrinos: MiniBooNE

to cut it short: appearence has excess of events: 3.8σ (sic!)



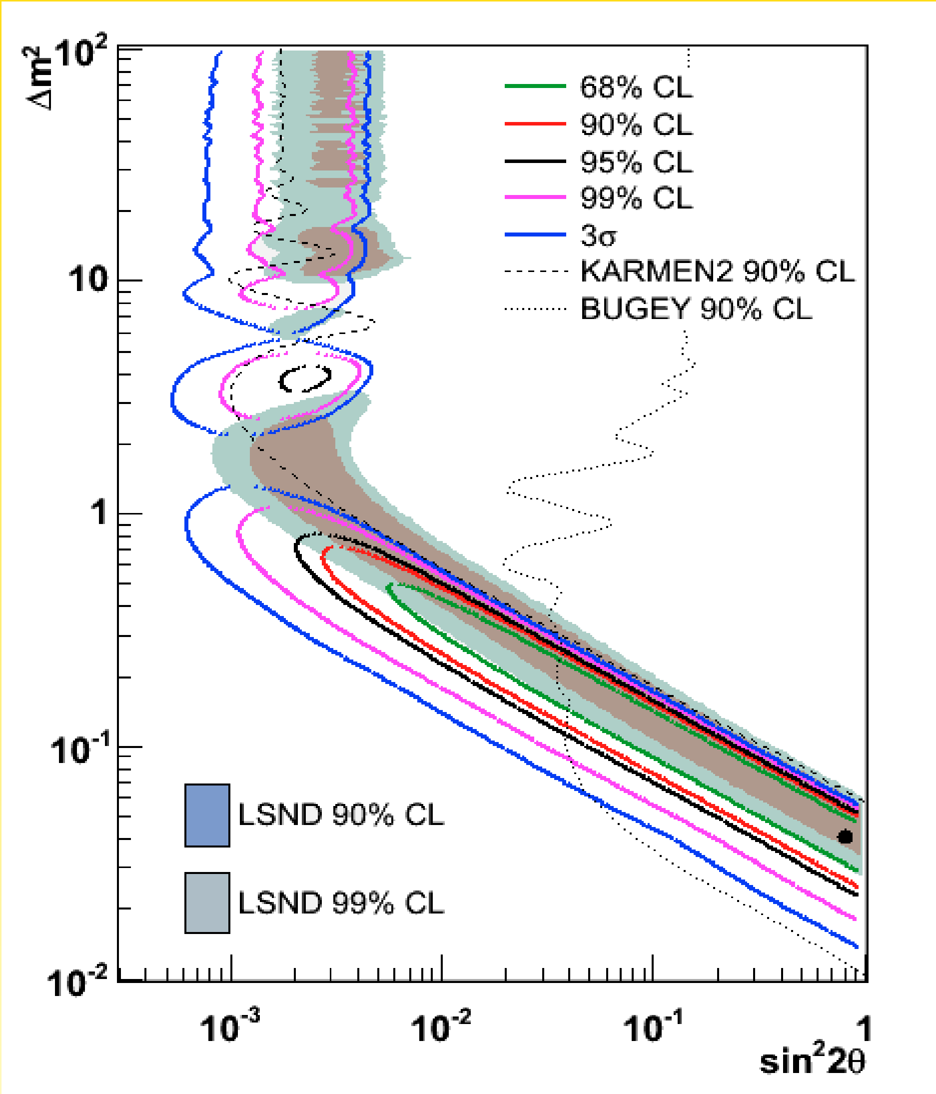
ν and $\bar{\nu}$ data consistent (scenarios with two steriles not necessary anymore)

ν with 3.0σ, but tension between events > 200 MeV and > 475 MeV

[P(bf) $= 6 \to 42\%$]

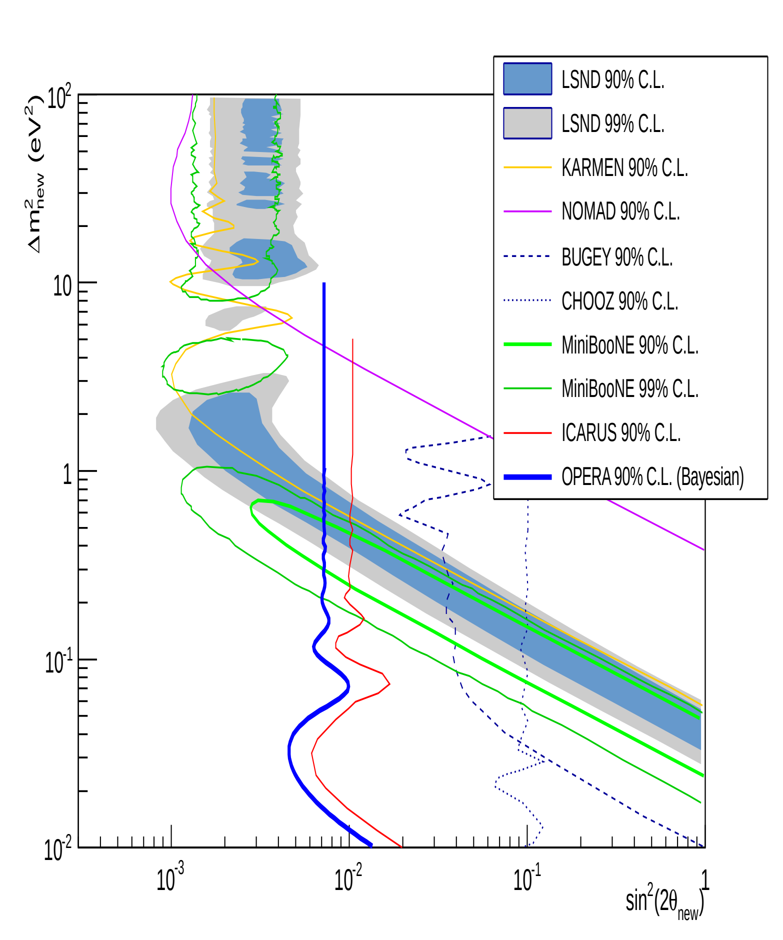
$\bar{\nu}$: 2.5σ, no tension between low and high energy events

Motivation for Sterile Neutrinos: MiniBooNE



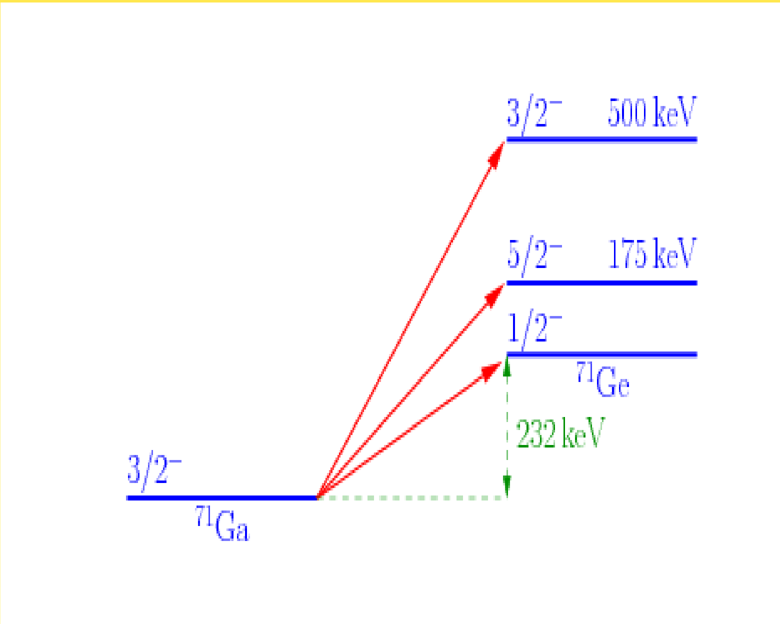
Checking Sterile Neutrinos: LBL

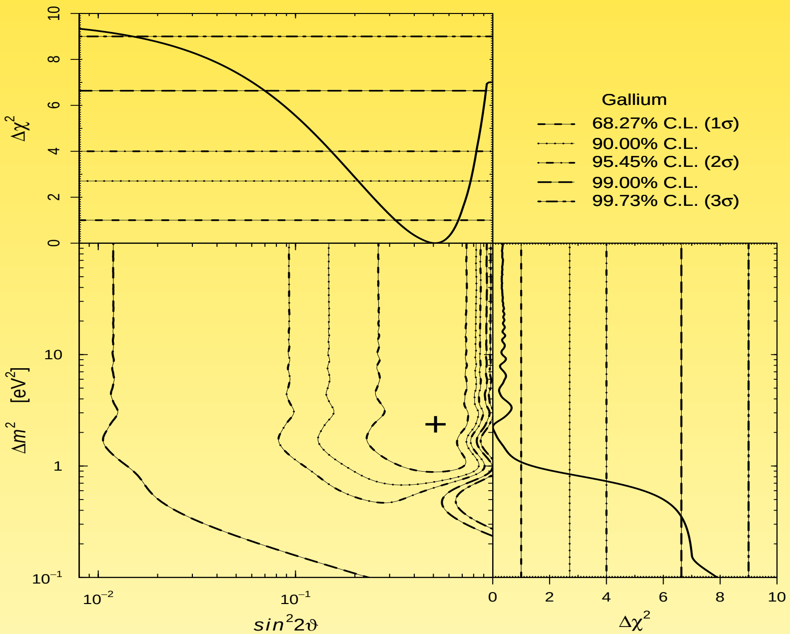
$P(\nu_\mu \to \nu_e) \simeq 0.5 \sin^2 2\theta$ with large L/E (CNGS)



Motivation for Sterile Neutrinos: Gallium Anomaly

- calibration of Gallium experiments with ^{51}Cr and ^{37}Ar sources
- (should know the calibration source and detection process very well when you do calibration. . .)
- $E \simeq 0.7$ MeV and $L \simeq 1$ m; resulted in $R = 0.86 \pm 0.05$ (2.8σ)

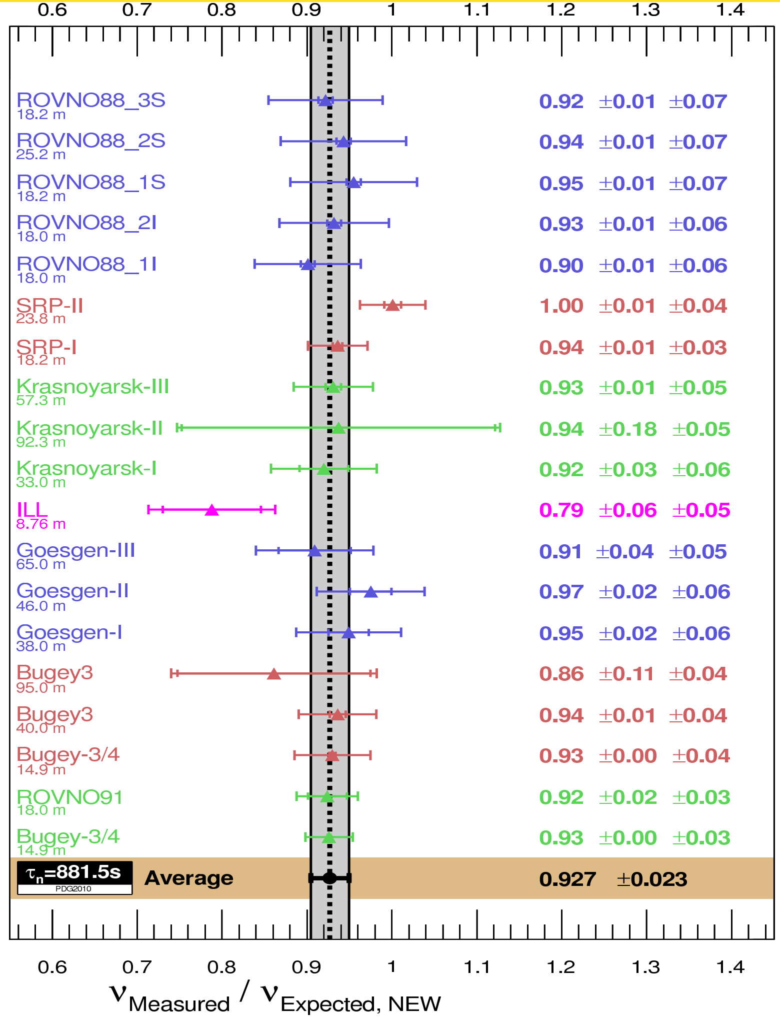


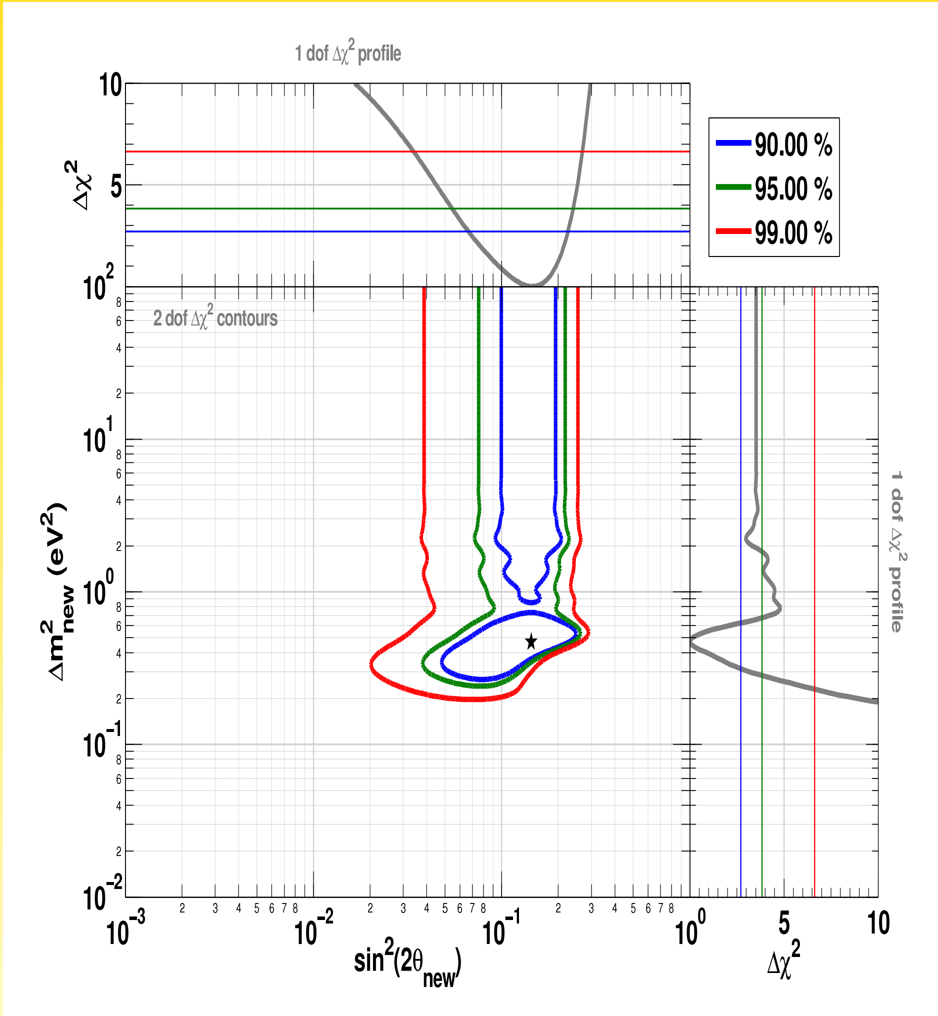


Giunti

Motivation for Sterile Neutrinos: Reactor Anomaly

Overall result (`Mention` *et al.*; `Huber`):





Sterile Neutrinos: Disapperance vs. Appearence

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

$$P(\nu_\mu \to \nu_e) = 4\,|U_{e4}|^2\,|U_{\mu 4}|^2\,\sin^2\frac{\Delta m^2_{41}}{4E}L$$

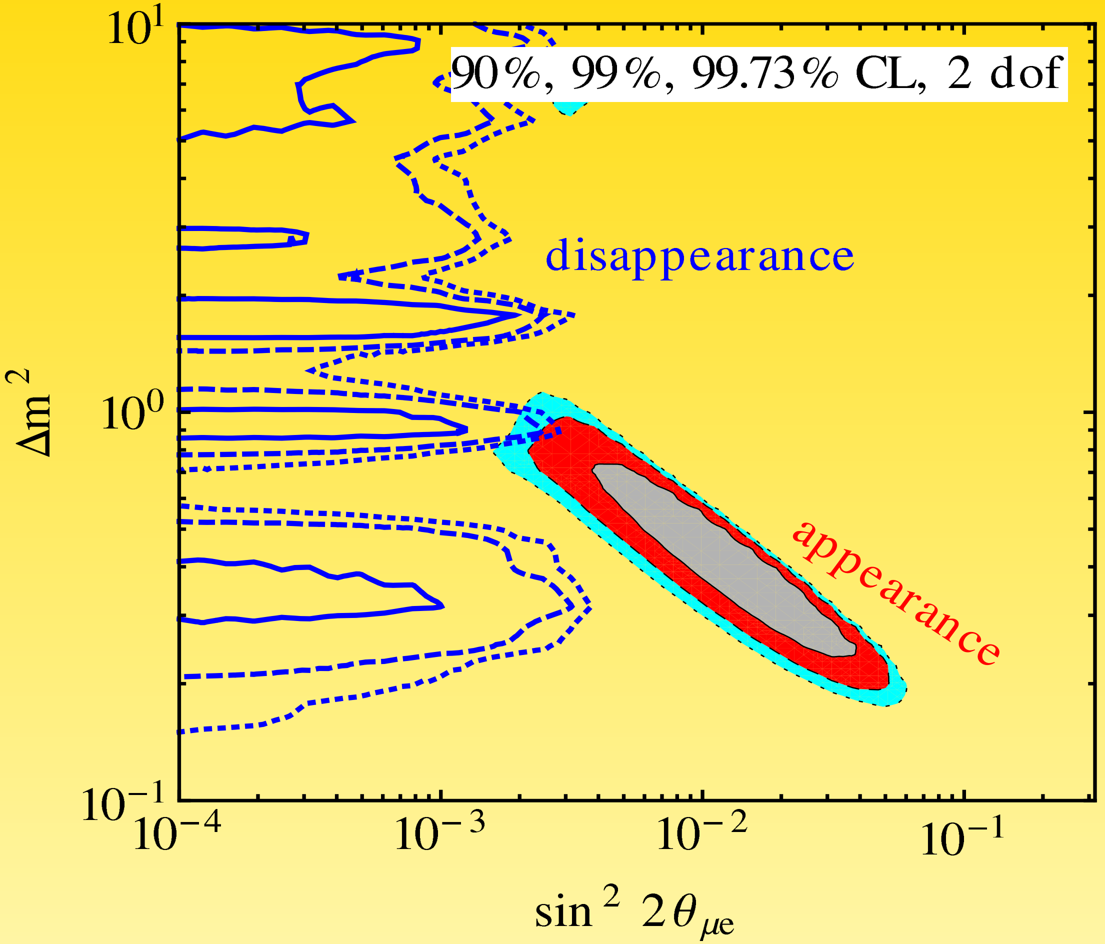
$$P(\nu_e \to \nu_e) = 1 - 4\,|U_{e4}|^2(1 - |U_{e4}|^2)\,\sin^2\frac{\Delta m^2_{41}}{4E}L$$

$$P(\nu_\mu \to \nu_\mu) = 1 - 4\,|U_{\mu 4}|^2(1 - |U_{\mu 4}|^2)\,\sin^2\frac{\Delta m^2_{41}}{4E}L$$

$\Rightarrow$ if $\nu_\mu \to \nu_e$ appearence, then both $\nu_e \to \nu_e$ and $\nu_\mu \to \nu_\mu$ disappearence

$\Leftrightarrow$ tension between appearence and disappearence data...

Sterile Neutrinos: Disapperance vs. Appearence

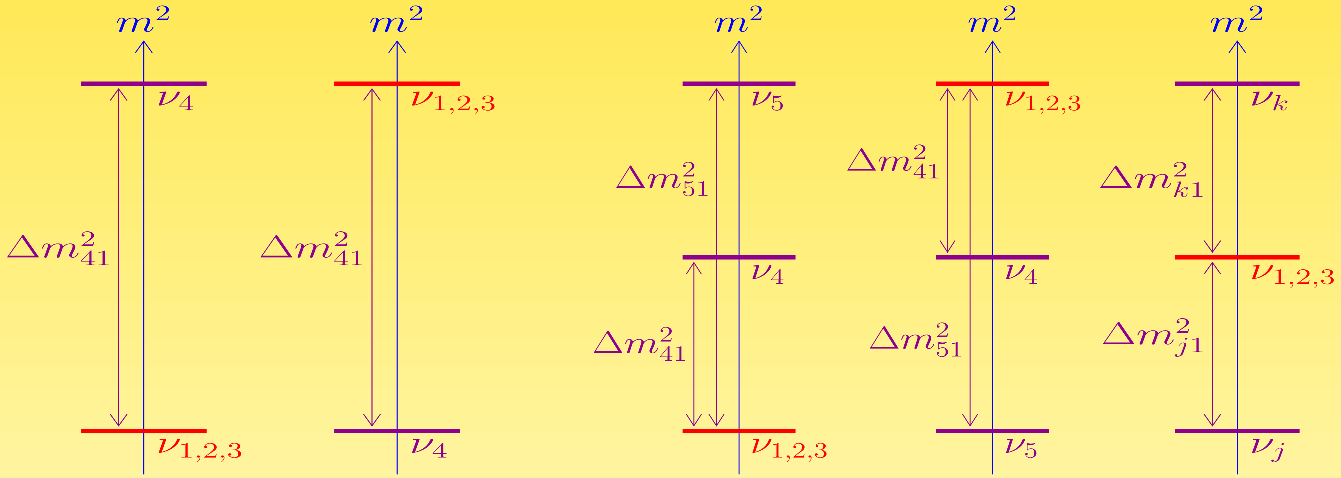


some overlap at 99 % CL

Kopp, Machado, Maltoni, Schwetz

Sterile Neutrinos: more than one?

- does not help tension between appearence and disappearence
- mass ordering? 3+1, 1+3, 3+2, 2+3, 1+3+1



Sterile Neutrinos: Typical Values

	Δm^2_{41} [eV^2]	$\lvert U_{e4}\rvert$	$\lvert U_{\mu 4}\rvert$	Δm^2_{51} [eV^2]	$\lvert U_{e5}\rvert$	$\lvert U_{\mu 5}\rvert$	$\gamma_{\mu e}$
3+1	0.93	0.15	0.17				
3+2	0.47	0.13	0.15	0.87	0.14	0.13	-0.15π
1+3+1	-0.87	0.15	0.13	0.47	0.13	0.17	0.06π

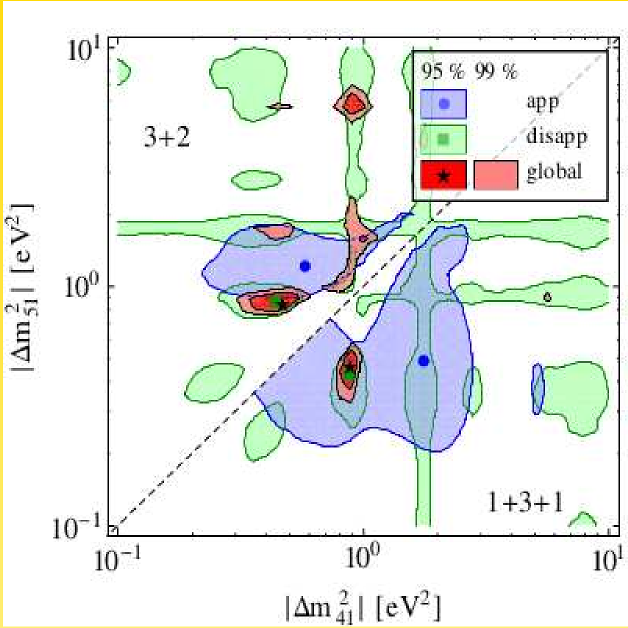
Kopp, Machado, Maltoni, Schwetz

$$P^{\rm SBL}_{\nu_\alpha \to \nu_\beta} = 4\,|U_{\alpha 4}|^2 |U_{\beta 4}|^2 \, \sin^2 \phi_{41} + 4\,|U_{\alpha 5}|^2 |U_{\beta 5}|^2 \, \sin^2 \phi_{51}$$
$$+ \, 8\,|U_{\alpha 4} U_{\beta 4} U_{\alpha 5} U_{\beta 5}| \, \sin \phi_{41} \sin \phi_{51} \cos(\phi_{54} - \gamma_{\alpha\beta})$$

$$\phi_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E} \, , \qquad \gamma_{\alpha\beta} \equiv \arg \left(I_{\alpha\beta 54} \right) \, , \qquad I_{\alpha\beta ij} \equiv U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}$$

both $\Delta m^2_{51} > 0$ and $\Delta m^2_{41} > 0$: 3+2

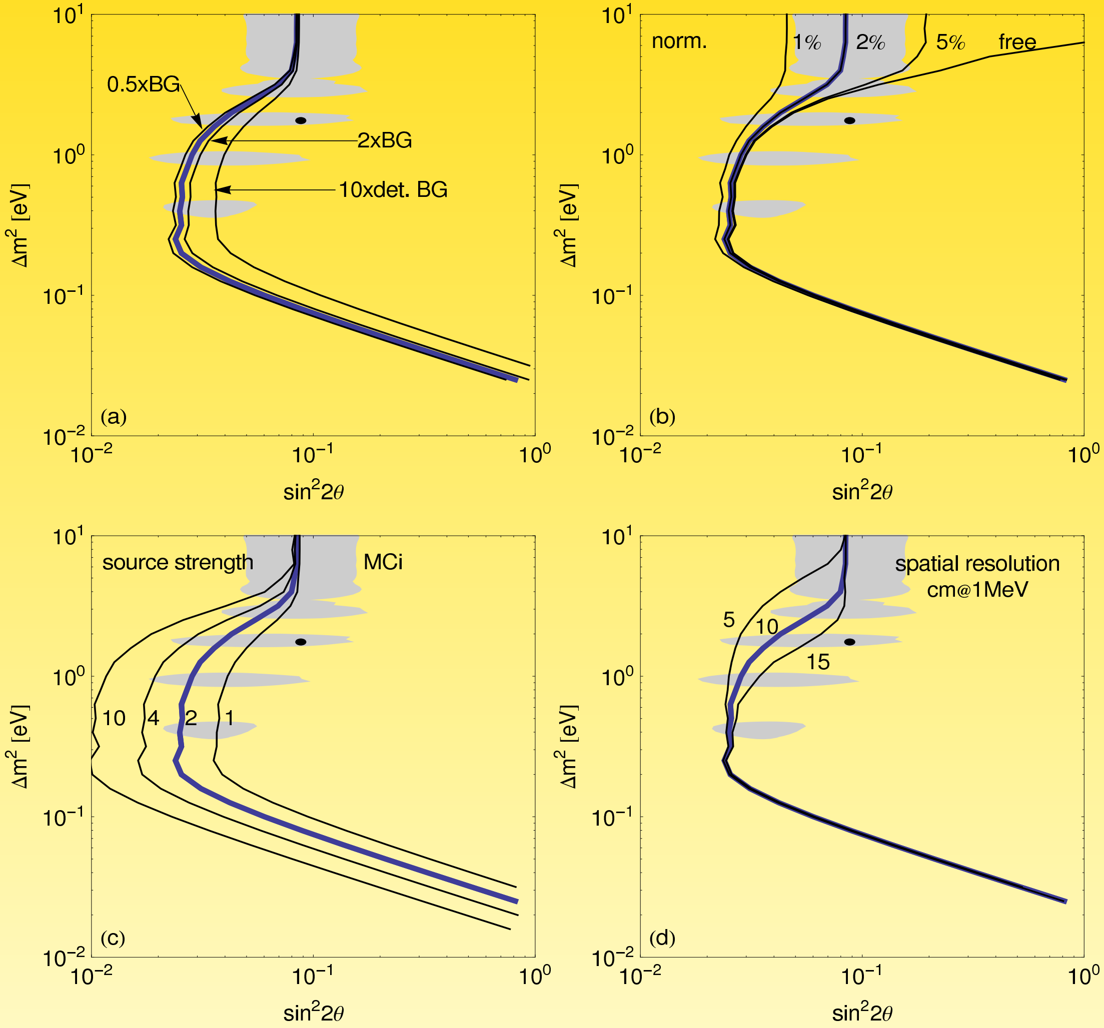
one of them negative: 1 + 3 + 1



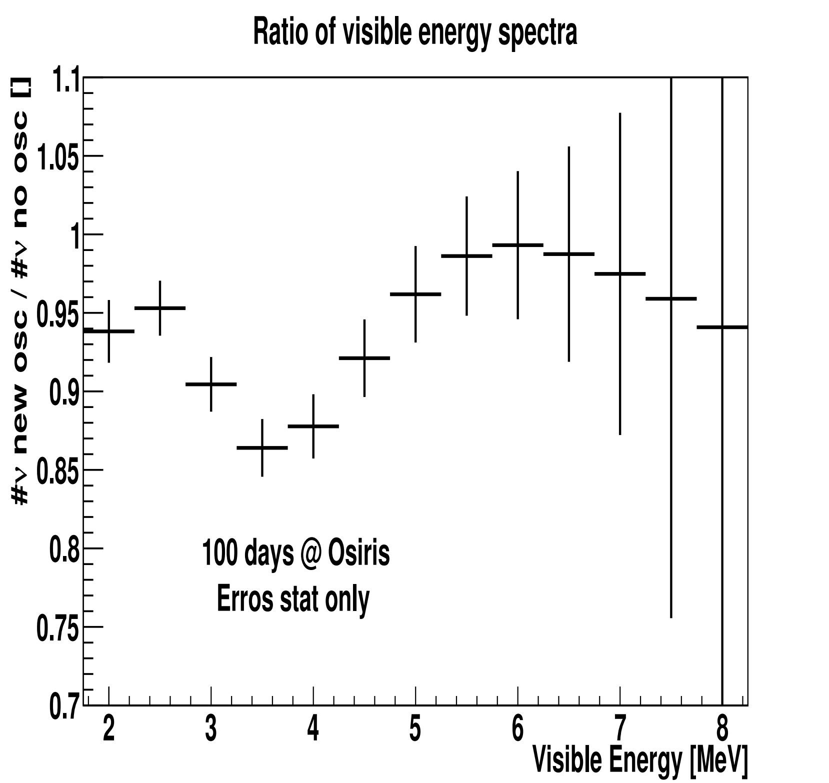
	$\chi^2_{\rm min}$/dof	GOF	$\chi^2_{\rm PG}$/dof	PG	$\chi^2_{\rm app,glob}$	$\Delta\chi^2_{\rm app}$	$\chi^2_{\rm dis,glob}$
3+1	$712/(689-9)$	19%	18.0/2	1.2×10^{-4}	95.8/68	7.9	616/621
3+2	$701/(689-14)$	23%	25.8/4	3.4×10^{-5}	92.4/68	19.7	609/621
1+3+1	$694/(689-14)$	30%	16.8/4	2.1×10^{-3}	82.4/68	7.8	611/621

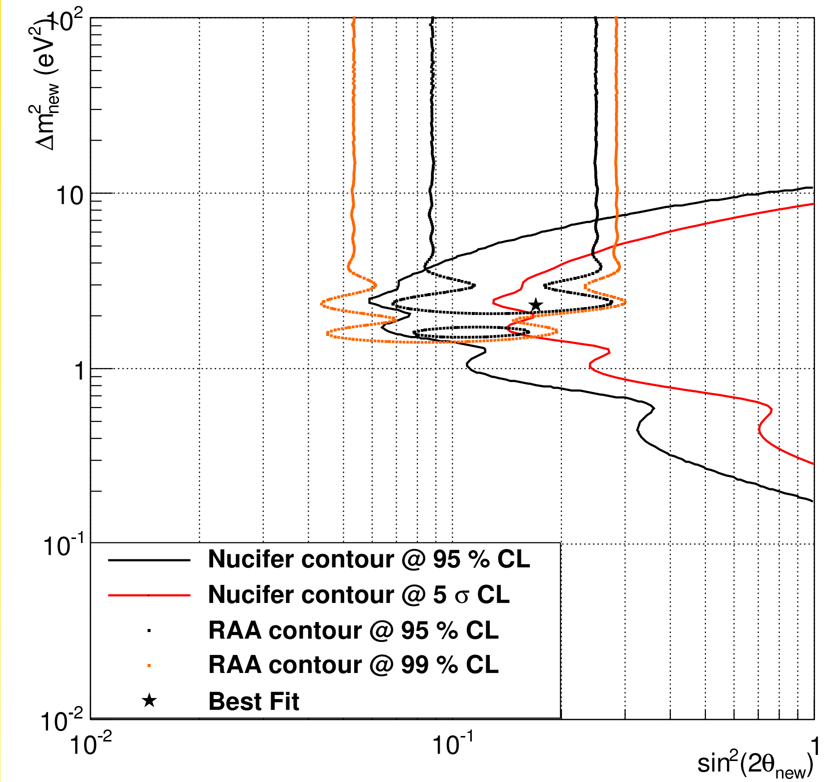
Ruling out Sterile Neutrinos: Oscillations

- SNO+Cr



- Nucifer


Ratio of visible energy spectra
#ν new osc / #ν no osc []
100 days @ Osiris
Erros stat only
Visible Energy [MeV]


Δm^2_{new} (eV^2)
$\sin^2(2\theta_{new})$
Nucifer contour @ 95 % CL
Nucifer contour @ 5 σ CL
RAA contour @ 95 % CL
RAA contour @ 99 % CL
Best Fit

Motivation for Sterile Neutrinos: Cosmology

Model	Data	N_{eff}
N_{eff}	**W-5+BAO+SN+H_0**	$4.13^{+0.87(+1.76)}_{-0.85(-1.63)}$
	W-5+LRG+H_0	$4.16^{+0.76(+1.60)}_{-0.77(-1.43)}$
	W-5+CMB+BAO+XLF+f_{gas}+H_0	$3.4^{+0.6}_{-0.5}$
	W-7+BAO+H_0	$4.34^{+0.86}_{-0.88}$
	W-7+LRG+H_0	$4.25^{+0.76}_{-0.80}$
	W-7+ACT	5.3 ± 1.3
	W-7+ACT+BAO+H_0	4.56 ± 0.75
	W-7+SPT	3.85 ± 0.62
	W-7+SPT+BAO+H_0	3.85 ± 0.42
	W-7+ACT+SPT+LRG+H_0	$4.08^{(+0.71)}_{(-0.68)}$
	W-7+ACT+SPT+BAO+H_0	3.89 ± 0.41
N_{eff}+f_ν	**W-7+CMB+BAO+H_0**	$4.47^{(+1.82)}_{(-1.74)}$
	W-7+CMB+LRG+H_0	$4.87^{(+1.86)}_{(-1.75)}$
N_{eff}+Ω_k	**W-7+BAO+H_0**	4.61 ± 0.96
	W-7+ACT+SPT+BAO+H_0	4.03 ± 0.45
N_{eff}+Ω_k+f_ν	**W-7+ACT+SPT+BAO+H_0**	4.00 ± 0.43
N_{eff}+f_ν+w	**W-7+CMB+BAO+H_0**	$3.68^{(+1.90)}_{(-1.84)}$
	W-7+CMB+LRG+H_0	$4.87^{(+2.02)}_{(-2.02)}$
N_{eff}+Ω_k+f_ν+w	**W-7+CMB+BAO+SN+H_0**	$4.2^{+1.10(+2.00)}_{-0.61(-1.14)}$
	W-7+CMB+LRG+SN+H_0	$4.3^{+1.40(+2.30)}_{-0.54(-1.09)}$

Motivation for Sterile Neutrinos: Cosmology

Model	Data	N_{eff}
η+N_{eff}	η_{CMB}+Y_p+D/H	$3.8^{(+0.8)}_{(-0.7)}$
	η_{CMB}+Y_p+D/H	$< (4.05)$
	Y_p+D/H	3.85 ± 0.26
		3.82 ± 0.35
		3.13 ± 0.21
η+N_{eff}, $(\Delta N_{eff} \equiv N_{eff} - 3.046 \geq 0)$	η_{CMB}+D/H	3.8 ± 0.6
	η_{CMB}+Y_p	$3.90^{+0.21}_{-0.58}$
	Y_p+D/H	$3.91^{+0.22}_{-0.55}$

Motivation for Sterile Neutrinos: Cosmology

sum of neutrino masses also affected

Model	Observables	$\sum m_i$ [eV]
$o\omega\mathrm{CDM} + \Delta N_{\mathrm{rel}} + m_\nu$	CMB+HO+SN+BAO	≤ 1.5
$o\omega\mathrm{CDM} + \Delta N_{\mathrm{rel}} + m_\nu$	CMB+HO+SN+LSSPS	≤ 0.76
$\Lambda\mathrm{CDM} + m_\nu$	CMB+H0+SN+BAO	≤ 0.61
$\Lambda\mathrm{CDM} + m_\nu$	CMB+H0+SN+LSSPS	≤ 0.36
$\Lambda\mathrm{CDM} + m_\nu$	CMB (+SN)	≤ 1.2
$\Lambda\mathrm{CDM} + m_\nu$	CMB+BAO	≤ 0.75
$\Lambda\mathrm{CDM} + m_\nu$	CMB+LSSPS	≤ 0.55
$\Lambda\mathrm{CDM} + m_\nu$	CMB+H0	≤ 0.45

tension between oscillations ($m_s \simeq$ eV) and cosmology $m_s \lesssim$ eV

Ruling out Sterile Neutrinos: Cosmology

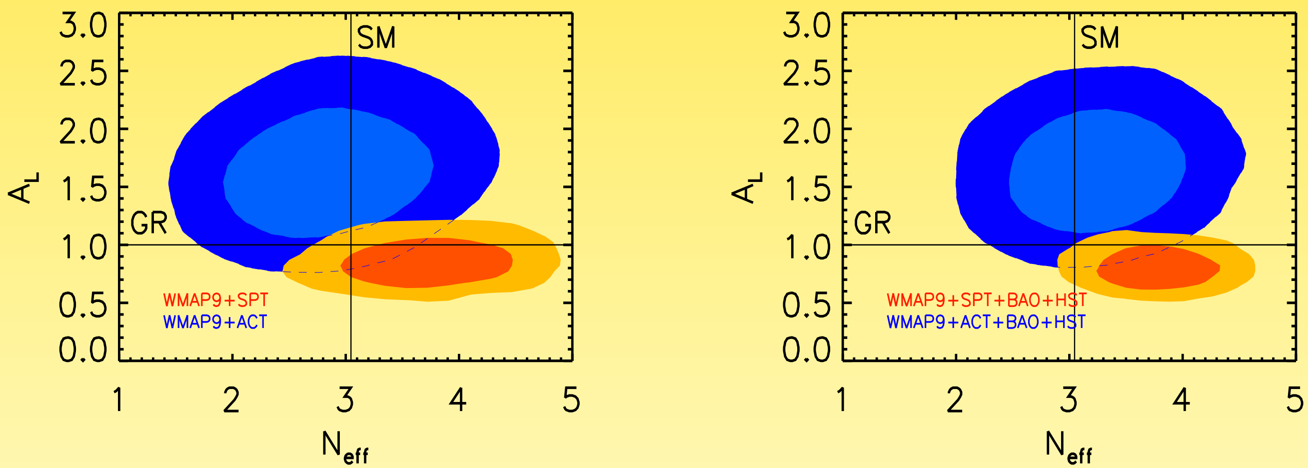
Probe	Pot. sensitivity [eV] (short term)	Pot. sensitivity [eV] (long term)
CMB	0.4–0.6	0.4
CMB with lensing	0.1–0.15	0.04
CMB + Galaxy Distribution	0.2	0.05–0.1
CMB + Lensing of Galaxies	0.1	0.03–0.04
CMB + Lyman-α	0.1–0.2	Unknown
CMB + Galaxy Clusters	–	0.05
CMB + 21 cm	–	0.0003–0.1

plus $\Delta N_\nu = 0.2$ from Planck

Planck Panic

- WMAP-9, arXiv:1212.5226v1: $N_{\rm eff} = 3.26 \pm 0.35$
- WMAP-9, arXiv:1212.5226v2: $N_{\rm eff} = 3.84 \pm 0.40$

  comments: "slight correction to Neff for case with BAO"...
- ACT, arXiv:1301.0824: $N_{\rm eff} = 2.79 \pm 0.56$, $\Sigma < 0.39$ eV
- SPT, arXiv:1212.6267: $N_{\rm eff} = 3.62 \pm 0.48$, $\Sigma = (0.32 \pm 0.11)$ eV (sic!)



Melchiorri *et al.*, 1301.7343

WMAP 9 year results!

Number of neutrino species (68% CL)

$N_\nu = 3.89 \pm 0.67$	WMAP + eCMB; $Y_{\rm He}$ fixed
$N_\nu = 3.26 \pm 0.35$	WMAP + eCMB + BAO $+H_0$; $Y_{\rm He}$ fixed
$N_\nu = 2.83 \pm 0.38$	WMAP + eCMB + BAO $+H_0$

Sum of masses (95% CL)

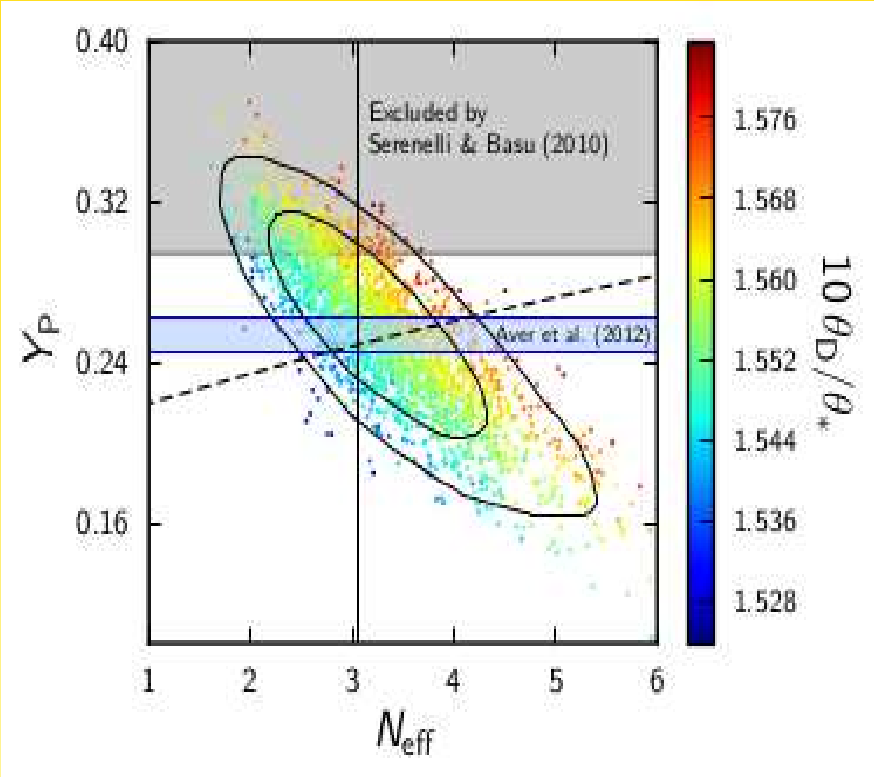
$\Sigma m_i \leq 1.3\,\mathrm{eV}$	WMAP
$\Sigma m_i \leq 0.44\,\mathrm{eV}$	WMAP + eCMB + BAO $+ H_0$

Got Plancked?

1303.5076: "There is no evidence for additional neutrino-like relativistic particles beyond the three families of neutrinos in the standard model."

$$N_{\rm eff} = \ldots$$

- $\ldots 3.36^{+0.68}_{-0.64}$ from P + WP + high l
- $\ldots 3.30^{+0.54}_{-0.51}$ from P + WP + high l + BAO
- $\ldots 3.62^{+0.50}_{-0.48}$ from P + WP + high l + H_0
- $\ldots 3.52^{+0.48}_{-0.45}$ from P + WP + high l + BAO + H_0
- $\ldots 3.33^{+0.59}_{-0.83}$ from P + WP + high l + Y_p



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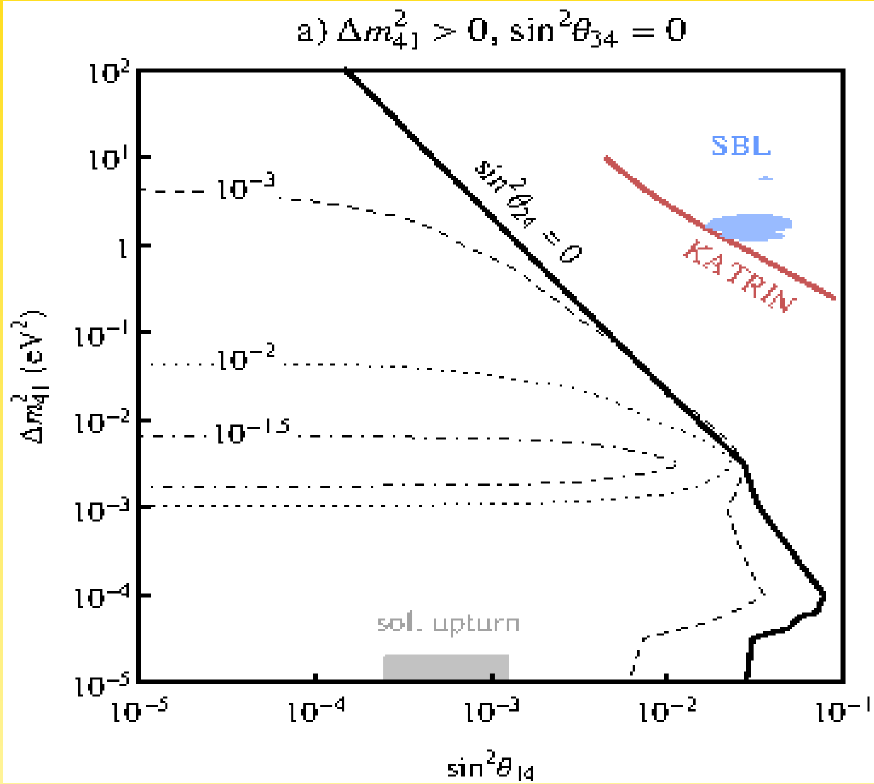

1303.5368

Light Sterile Neutrinos: A White Paper

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Phenomenology of eV steriles: β-decays

with non-zero U_{e4} and m_4:

- Kurie-plot experiments:

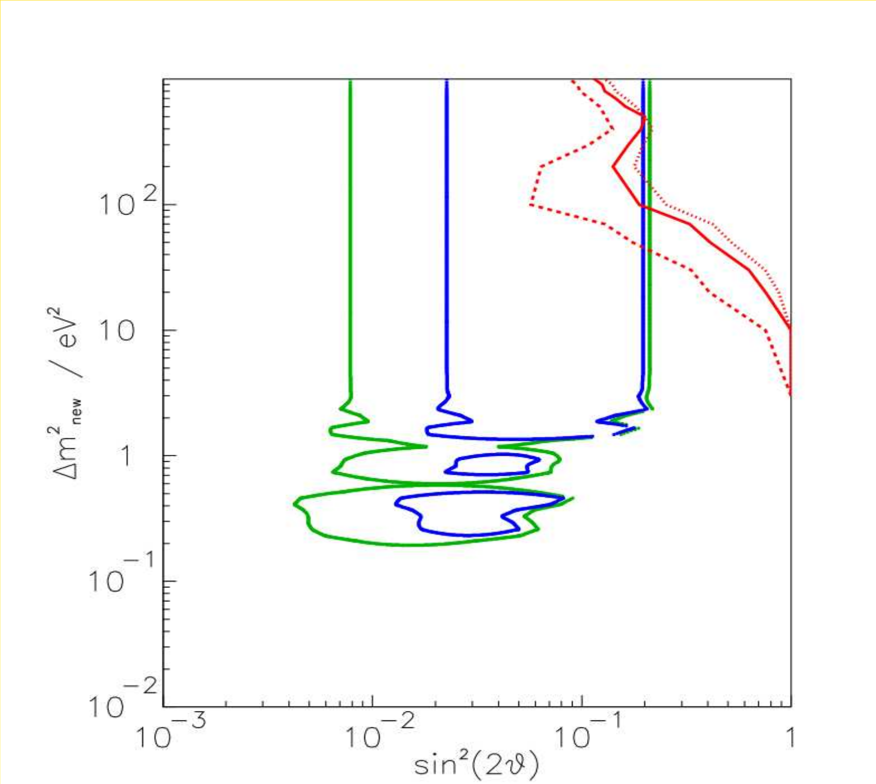
$$m_\beta^2 = |U_{e1}|^2\, m_1^2 + |U_{e2}|^2\, m_2^2 + |U_{e3}|^2\, m_3^2 + |U_{e4}|^2\, m_4^2 \leq (2.2\,\mathrm{eV})^2$$

- neutrino-less double beta decay experiments:

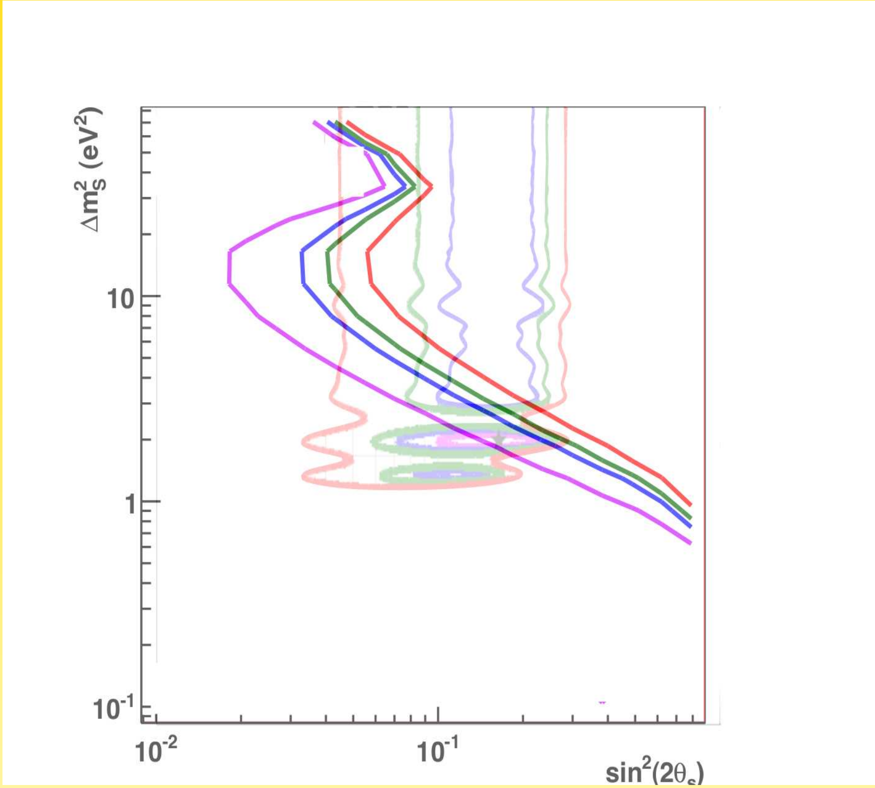
$$|m_{ee}| = \left|U_{e1}^2\, m_1 + U_{e2}^2\, m_2 + U_{e3}^2\, m_3 + U_{e4}^2\, m_4\right| \leq 0.3\,\mathrm{eV}$$

Phenomenology of eV steriles

Neutrino mass observables: β decays



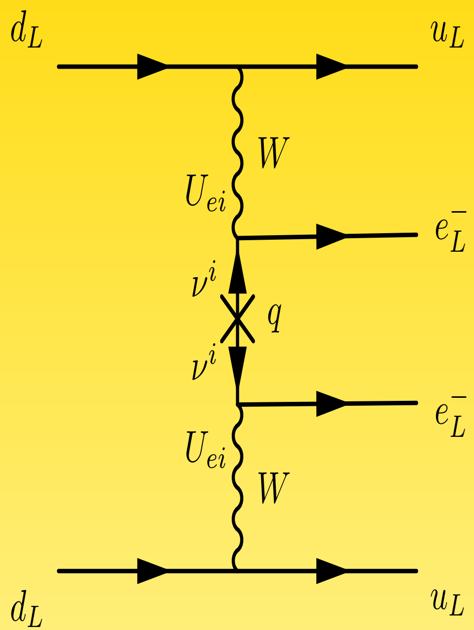
Mainz



Sejersen Riis, Hannestad;

Formaggio, Barret

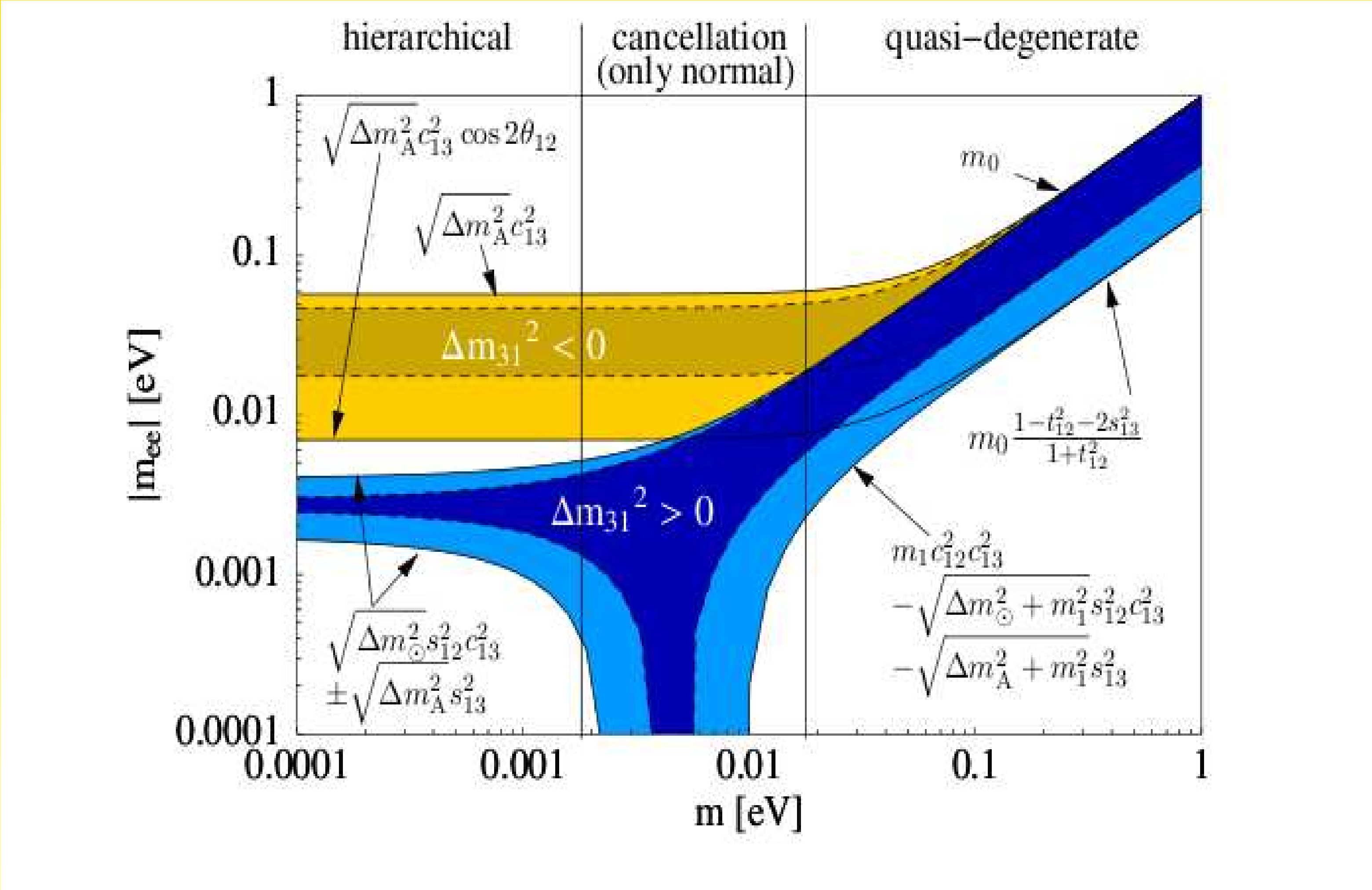
Neutrinoless double beta decay: $nn \to pp\, e^- e^-$



Amplitude proportional to

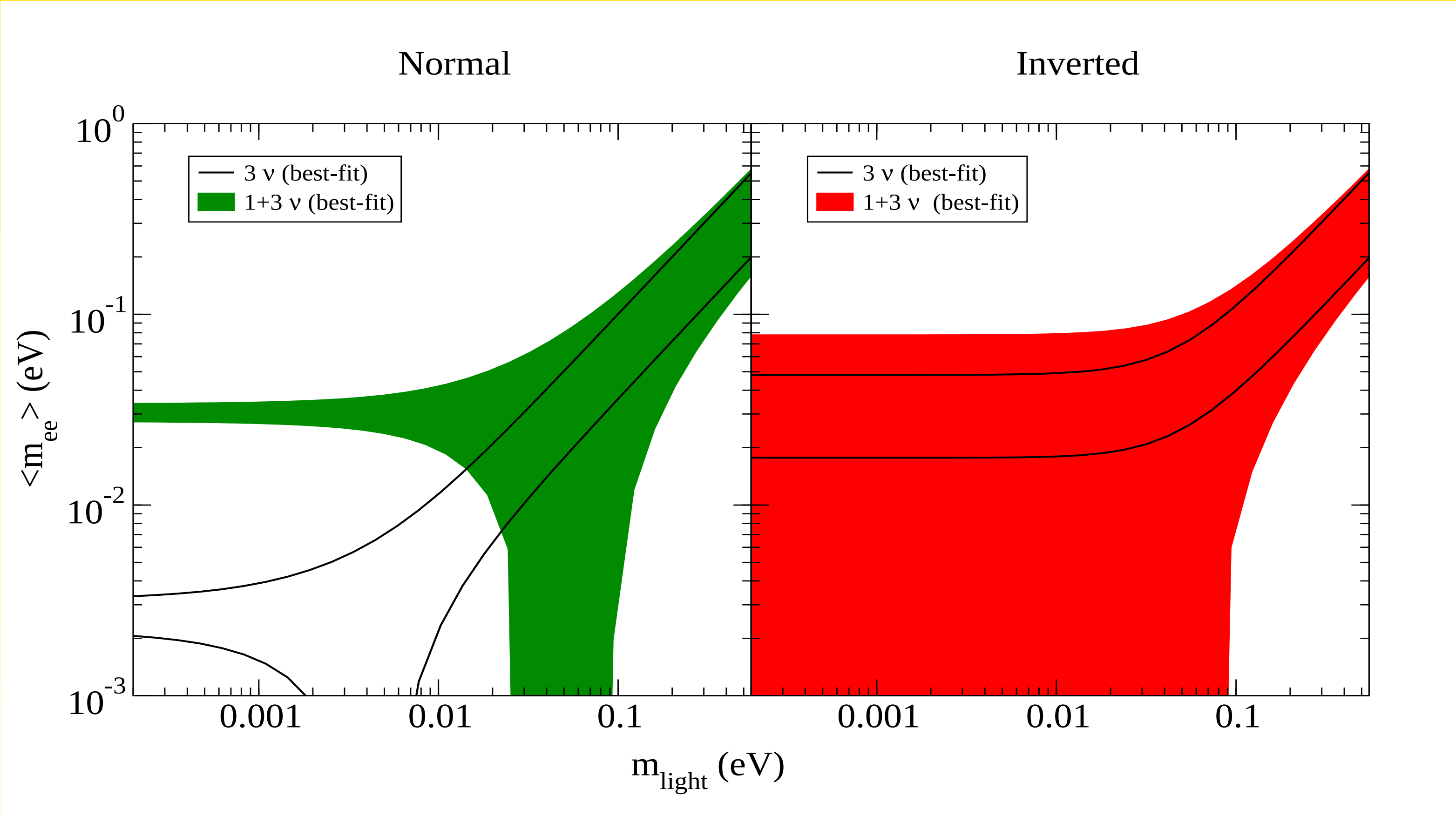
$$\frac{U_{ei}^2\, m_i}{q^2 - m_i^2} \propto \begin{cases} U_{ei}^2\, m_i & q^2 \gg m_i^2 \quad \text{light neutrinos} \\ \dfrac{U_{ei}^2}{m_i} & q^2 \ll m_i^2 \quad \text{heavy neutrinos} \end{cases}$$

The usual plot for double beta decay. . .



The usual plot for double beta decay. . .

. . . gets completely turned around!



Barry, W.R., Zhang

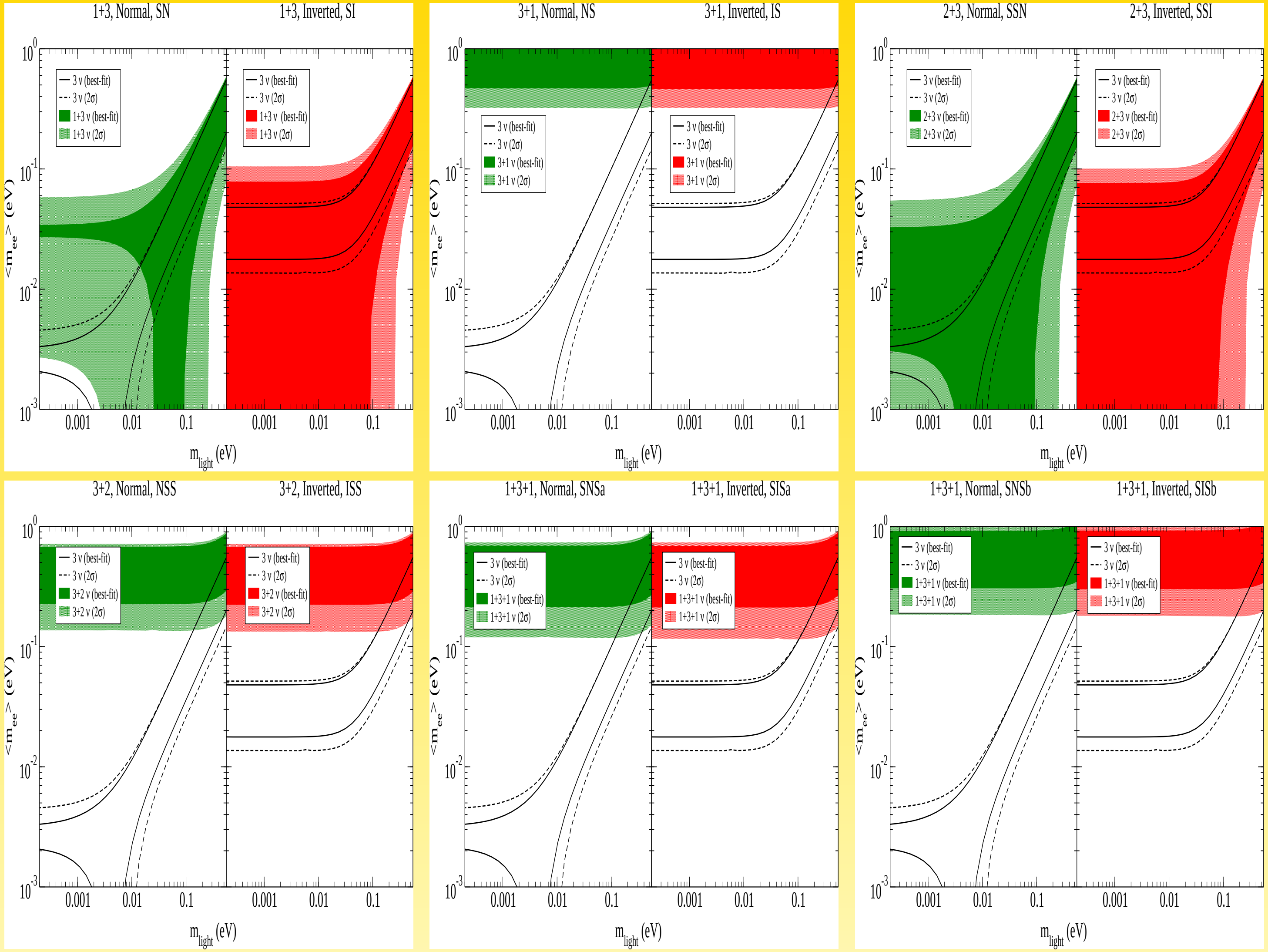
Sterile Neutrinos and $0\nu\beta\beta$

- recall $|m_{ee}|_{\rm NH}^{\rm act}$ can vanish and $|m_{ee}|_{\rm IH}^{\rm act} \sim 0.03$ eV cannot vanish
- $|m_{ee}| = | \underbrace{|U_{e1}|^2 m_1 + |U_{e2}|^2 m_2 \, e^{2i\alpha} + |U_{e3}^2| m_3 \, e^{2i\beta}}_{m_{ee}^{\rm act}} + \underbrace{|U_{e4}|^2 m_4 \, e^{2i\Phi_1}}_{m_{ee}^{\rm st}} |$
- $\Delta m^2_{\rm st} \simeq 1.8$ eV^2 and $|U_{e4}| \simeq 0.13$
- sterile contribution to $0\nu\beta\beta$:

$$|m_{ee}|^{\rm st} \simeq \sqrt{\Delta m^2_{\rm st}} \, |U_{e4}|^2 \simeq 0.03 \text{ eV} \left\{ \begin{array}{c} \gg |m_{ee}|^{\rm act}_{\rm NH} \\ \simeq |m_{ee}|^{\rm act}_{\rm IH} \end{array} \right.$$

- $\Rightarrow$ $|m_{ee}|_{\rm NH}$ cannot vanish and $|m_{ee}|_{\rm IH}$ can vanish!

`Barry, W.R., Zhang`



Barry, W.R., Zhang

Other Sterile Neutrinos

- very light $\ll$ eV ($\leftrightarrow$ solar neutrinos)
- keV ($\leftrightarrow$ Warm Dark Matter)
- $10^{10} \ldots 10^{15}$ GeV ($\leftrightarrow$ GUTs, leptogenesis)
- [TeV ($\leftrightarrow$ LHC)]

What is a sterile neutrino?

SM contains 3 active neutrinos with isospin $\frac{1}{2}$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L , \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L , \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

their anti-particles (CP-partners; $\nu \to \nu^c$) are also active:

$$\begin{pmatrix} e^+ \\ \bar{\nu}_e \end{pmatrix}_R , \quad \begin{pmatrix} \mu^+ \\ \bar{\nu}_\mu \end{pmatrix}_R , \quad \begin{pmatrix} \tau^+ \\ \bar{\nu}_\tau \end{pmatrix}_R$$

the $(\nu_{e,\mu,\tau})_L$ and $(\bar{\nu}_{e,\mu,\tau})_R$ take part in weak interactions = couple to W, Z

What is a sterile neutrino?

- add a fourth state to the game, but don't give it isospin!

  $\Rightarrow$ **a sterile neutrino** ν_s

- a sterile neutrino ν_s does NOT take part in weak interactions = does NOT couple to W, Z

- can mix with active neutrinos

- can couple to Higgs

- can couple to BSM physics

we discuss N_R, the right-handed neutrino of the seesaw mechanism, and assume that it is Majorana, and assume that no other New Physics is there

Formalism

$$\mathcal{L} = \frac{1}{2}(\bar{\nu}_L,\, \bar{N}_R^c) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}$$

6×6 mass matrix diagonalized by

$$\mathcal{U}_\nu \simeq \begin{pmatrix} 1 - \frac{1}{2}BB^\dagger & B \\ -B^\dagger & 1 - \frac{1}{2}B^\dagger B \end{pmatrix} \begin{pmatrix} U & 0 \\ 0 & V_R \end{pmatrix} \quad \text{with } B = m_D M_R^{-1}$$

light neutrino mass matrix:

$$m_\nu = -m_D\, M_R^{-1}\, m_D^T = U\, \text{diag}(m_1, m_2, m_3)\, U^T$$

heavy neutrino mass matrix:

$$M_R = V_R\, \text{diag}(M_1, M_2, M_3)\, V_R^T$$

N_R is a *sterile neutrino*

What is the mass of a Sterile Neutrino?

total mass term for active neutrinos and sterile neutrino(s):

$$\mathcal{L} = \frac{1}{2}(\bar{\nu}_L,\, \bar{N}_R^c) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}$$

Question: WHAT IS THE SCALE OF M_R?

What is the mass of a Sterile Neutrino?

total mass term for active neutrinos and sterile neutrino(s):

$$\mathcal{L} = \frac{1}{2}(\bar{\nu}_L,\, \bar{N}_R^c) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}$$

Question: WHAT IS THE SCALE OF M_R?

answer: we don't know. . .

What is the mass of a Sterile Neutrino?

total mass term for active neutrinos and sterile neutrino(s):

$$\mathcal{L} = \frac{1}{2}(\bar{\nu}_L,\, \bar{N}_R^c) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}$$

Question: WHAT IS THE SCALE OF M_R?

answer: we don't know. . .

two good ideas for M_R:

What is the mass of a Sterile Neutrino?

total mass term for active neutrinos and sterile neutrino(s):

$$\mathcal{L} = \frac{1}{2}(\bar{\nu}_L,\, \bar{N}_R^c) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}$$

Question: WHAT IS THE SCALE OF M_R?

answer: we don't know. . .

two good ideas for M_R:

- SM singlet, not protected by v, hence GUT-scale, or $B - L$ breaking scale, or Planck-scale $\Rightarrow$ naturally large
-

What is the mass of a Sterile Neutrino?

total mass term for active neutrinos and sterile neutrino(s):

$$\mathcal{L} = \frac{1}{2}(\bar{\nu}_L, \, \bar{N}_R^c) \left(\begin{array}{cc} 0 & m_D \\ m_D^T & M_R \end{array} \right) \left(\begin{array}{c} \nu_L^c \\ N_R \end{array} \right)$$

Question: WHAT IS THE SCALE OF M_R?

answer: we don't know. . .

two good ideas for M_R:

- SM singlet, not protected by v, hence GUT-scale, or $B-L$ breaking scale, or Planck-scale $\Rightarrow$ naturally large
- if M_R is zero, symmetry of the Lagrangian is enlarged $\Rightarrow$ naturally small

What is the mass of a Sterile Neutrino?

total mass term for active neutrinos and sterile neutrino(s):

$$\mathcal{L} = \frac{1}{2}(\bar{\nu}_L,\, \bar{N}_R^c) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}$$

Question: WHAT IS THE SCALE OF M_R?

answer: we don't know...

two good ideas for M_R:

- SM singlet, not protected by v, hence GUT-scale, or $B-L$ breaking scale, or Planck-scale $\Rightarrow$ naturally large
- if M_R is zero, symmetry of the Lagrangian is enlarged $\Rightarrow$ naturally small

so, what now?

What is the mass of a Sterile Neutrino?

$$\mathcal{L} = \frac{1}{2}(\bar{\nu}_L,\, \bar{N}_R^c) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}$$

special cases:

- $m_D = 0$; **pure Majorana case**
- $M_R = 0$; **pure Dirac case**
- $M_R \gg m_D$; **seesaw case**
- $m_D \gg M_R$; **pseudo-Dirac case**
- $M_D \sim M_R$; **ugly case**

What is the mass of a Sterile Neutrino?

The seesaw limit $M_R \gg m_D$

$$m_\nu = \frac{m_D^2}{M_R}$$

does this fix everything?

No, multiply m_D with x and M_R with x^2: leaves m_ν invariant

stay in the seesaw limit $M_R \gg m_D$ from now on

Formalism

$$\mathcal{L} = \frac{1}{2}(\bar{\nu}_L,\, \bar{N}_R^c) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}$$

6×6 mass matrix diagonalized by

$$\mathcal{U}_\nu \simeq \begin{pmatrix} 1 - \frac{1}{2}BB^\dagger & B \\ -B^\dagger & 1 - \frac{1}{2}B^\dagger B \end{pmatrix} \begin{pmatrix} U & 0 \\ 0 & V_R \end{pmatrix}$$

3 active neutrinos mix with each other through

$N \equiv U\left(1 - \frac{1}{2}BB^\dagger\right)$ with $B = m_D M_R^{-1}$

3 active neutrinos mix with sterile neutrinos via

$$\theta_{\alpha i} = (m_D\, M_R^{-1}\, V_R)_{\alpha i} = \frac{[m_D V_R^*]_{\alpha i}}{M_i} = \mathcal{O}(\sqrt{m_\nu/M_R})$$

Formalism

Immediate consequences:

- unitarity violation of PMNS matrix of order $(m_D/M_R)^2$

$$\left|\frac{1}{2}BB^\dagger\right| < \begin{pmatrix} 4.0\times10^{-3} & 1.2\times10^{-5} & 3.2\times10^{-3} \\ \cdot & 1.6\times10^{-3} & 2.1\times10^{-3} \\ \cdot & \cdot & 5.3\times10^{-3} \end{pmatrix}$$

- Lepton flavor violation

$$\mathrm{BR}(\mu\to e\gamma) \propto \left|N^*_{\mu i}\,N_{ei}\,f(m_i/m_W) + \theta^*_{\mu i}\,\theta_{ei}\,g(M_i/m_W)\right|^2 \lesssim 1.1\times10^{-8}$$

- neutrinoless double beta decay

$$\sum N_{ei}^2\,m_i \lesssim 0.3\ \mathrm{eV} \text{ and } \sum \frac{\theta_{ei}^2}{M_i} \lesssim 2\times10^{-8}\ \mathrm{GeV}^{-1}$$

Seesaw parameters and sterile neutrinos: eV scale

- 3+2 scenario: m_ν is 5×5 matrix, with a total of 5 masses, 9 mixing angles, 6 Dirac and 4 Majorana phases, 24 parameters
- seesaw with 2 singlet neutrinos has 11 parameters

But: no problem, seesaw fits work as well

Donini *et al.*; Blennow, Fernandez-Martinez; Fan, Langacker

Sterile Neutrinos, Seesaw and $0\nu\beta\beta$

- if the eV-steriles are from seesaw: individual cancellations in flavor symmetry models, e.g.:

$$U_{e2}^2 \, m_2 + U_{e4}^2 \, m_4 = 0$$

- if seesaw scale is below 100 MeV ("Mini-Seesaw"): No double beta decay!

$$\sum_{i=1}^{6} U_{ei}^2 \, m_i = 0 \text{ since } \mathcal{M} = \left(\begin{array}{cc} 0 & m_D \\ m_D^T & M_R \end{array} \right) = U \left(\begin{array}{cc} m_\nu^{\rm diag} & 0 \\ 0 & M_R^{\rm diag} \end{array} \right) U^T$$

keV steriles as Warm Dark Matter

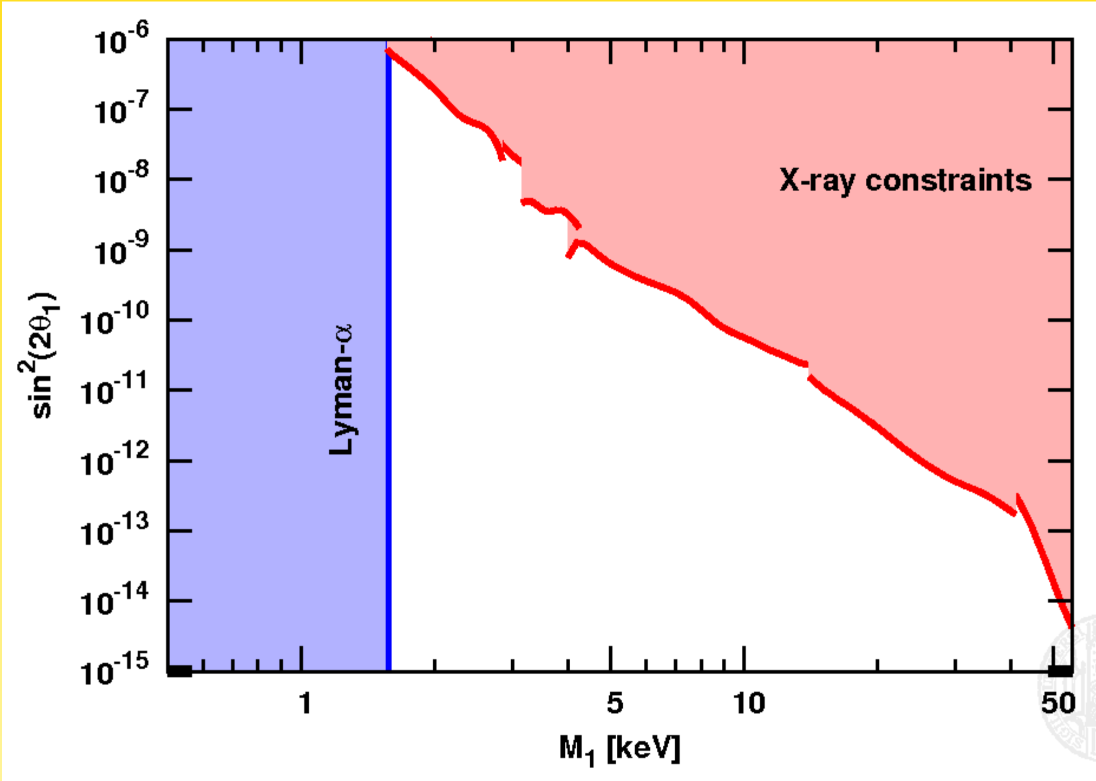
$\rightarrow$ WDM has same large scale structure formation as CDM, but suppresses small scale formations

$\Rightarrow$ predicts less cuspy (=smoother) DM profiles, and less dwarf satellites

keV sterile is excellent candidate

parameters: mass M_1 and mixing θ

- X-ray searches $\Gamma \sim G_F^2 M_1^5 \, \theta^2$
- Ly-α: structure formation at low scales $\sim$ MPc
- Tremaine-Gunn
- $\tau \sim \tau_{\rm U}$
- etc.



$m_\nu = \theta^2 M \Rightarrow$ one massless active neutrino! **(unless strong cancellations)**

keV WDM

Production mechanism

- produced from non-resonant (Dodelson-Widrow) or **resonant (Shi-Fuller) with lepton asymmetries** mixing with SM neutrinos
- thermally produced and then diluted
- non-thermally produced from BSM physics

TeV seesaw

naively, $m_\nu = m_D^2/M_R$ and mixing m_D/M_R

$\Rightarrow$ TeV neutrinos have mixing of order 10^{-7}

But, matrices are involved. . .e.g. (Kersten, Smirnov)

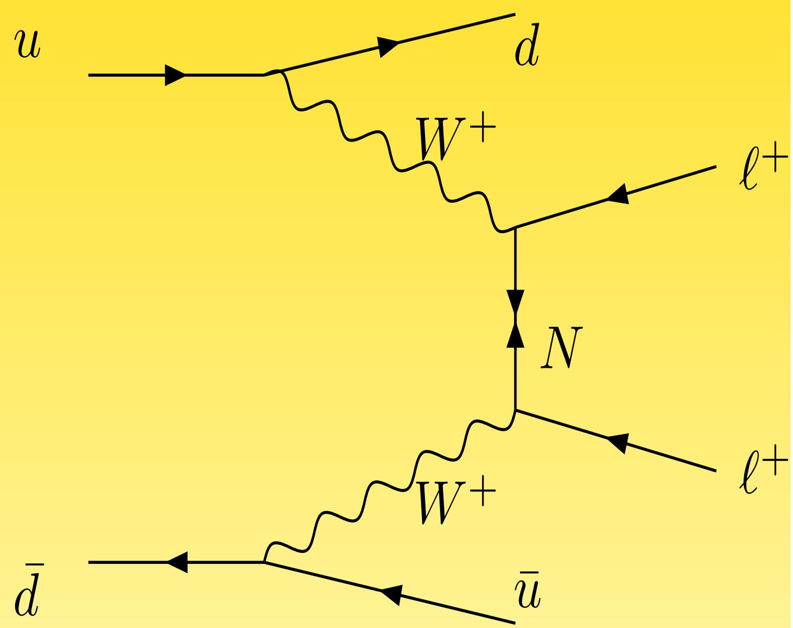
$$m_D = v \begin{pmatrix} h_1 & h_2 & h_3 \\ \omega\, h_1 & \omega\, h_2 & \omega\, h_3 \\ \omega^2\, h_1 & \omega^2\, h_2 & \omega^2\, h_3 \end{pmatrix} \text{ and } M_R = M_0 \mathbb{1}$$

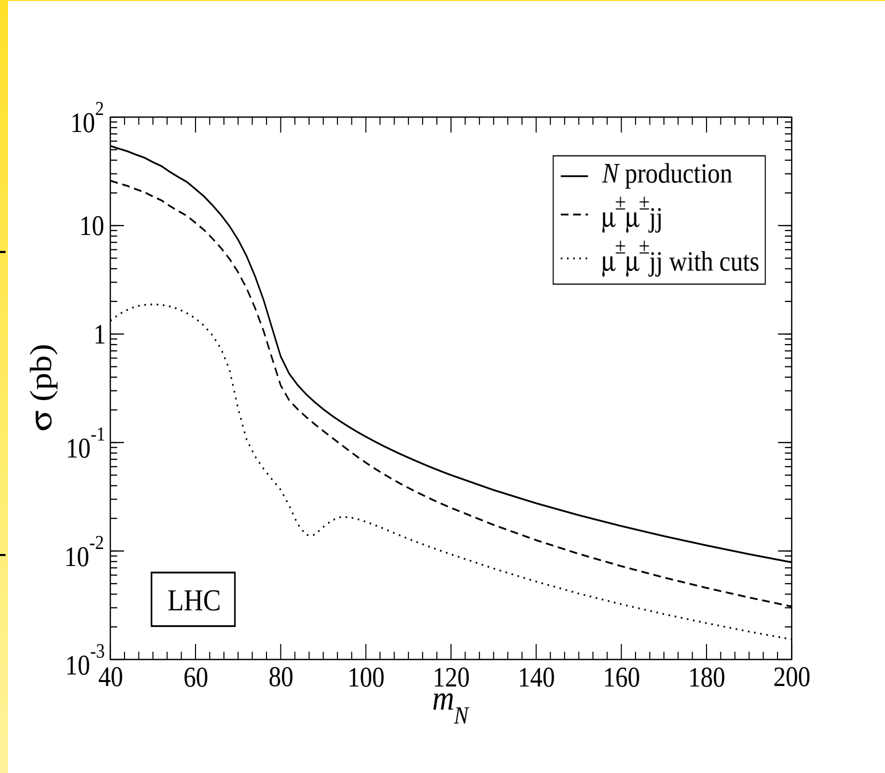
gives $m_\nu = 0$, add (very) small corrections

first pointed out: Korner, Pilaftsis, Schilcher (1993)

works with $Y_\nu = \mathcal{O}(1)$, mixing $m_D/M_R = \mathcal{O}(0.1)$ and $M_0 \lesssim$ TeV!

TeV seesaw





at most $M_i \leq 400$ GeV

Han, Zhang; del Aguila, Aguilar-Saavedra, Pittau

TeV scale seesaw with sizable mixing

$$M_D = m \begin{pmatrix} f\epsilon^2 & 0 & 0 \\ 0 & g\epsilon & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad M_R^{-1} = M^{-1} \begin{pmatrix} a & b & k \\ b & c & d\epsilon \\ k & d\epsilon & e\epsilon^2 \end{pmatrix}$$

M/GeV	m/MeV	ϵ	a	k	b	c	d	e	f	g
5.00	0.935	0.02	1.00	1.35	0.90	1.4576	0.7942	0.2898	0.0948	0.485

gives successful m_ν and for double beta decay:

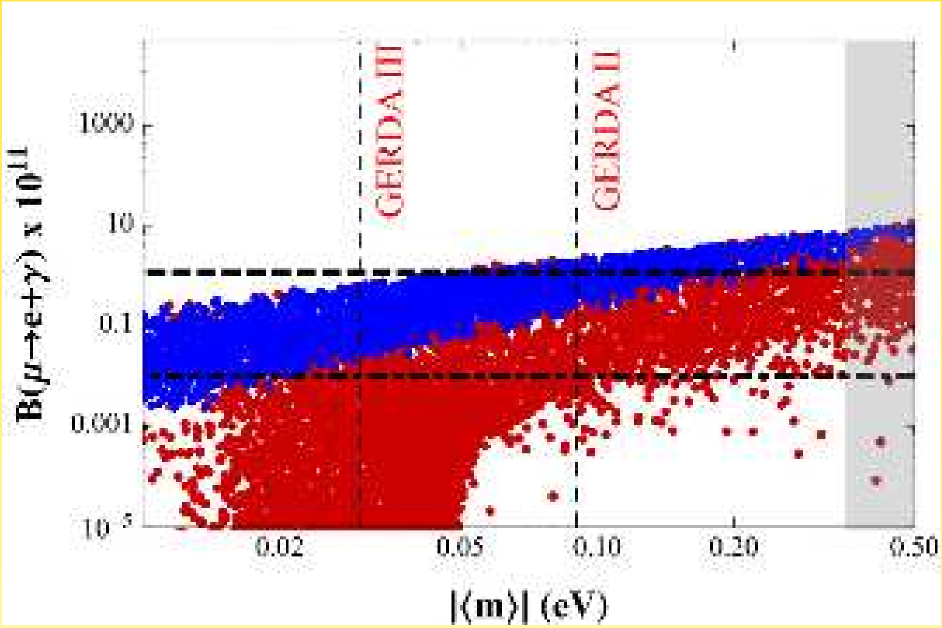
$$\frac{T_{1/2}(\text{light})}{T_{1/2}(\text{heavy})} \simeq 10^4$$

Mitra, Senjanovic, Vissani

TeV scale seesaw with sizable mixing

Casas-Ibarra Parametrization

$$m_D = iU\,\sqrt{m_\nu^{\rm diag}}\,R\,\sqrt{M_R^{\rm diag}}\,V_R^T$$



Ibarra, Molinaro, Petcov

$10^9 \ldots 10^{15}$ GeV: The case of very heavy $M_R \ldots$

. . . gives correct neutrino masses for $m_D \simeq v$

. . . gives successful thermal leptogenesis (`lecture by Ibarra`)

. . . is a generic GUT prediction

this is the scale where one would expect M_R

$10^9 \ldots 10^{15}$ GeV: The case of very heavy $M_R \ldots$

…gives correct neutrino masses for $m_D \simeq v$

…gives successful thermal leptogenesis (`lecture by Ibarra`)

…is a generic GUT prediction

this is the scale where one would expect M_R

Recall: theorists also expected small neutrino mixing…

Phenomenology of heavy singlets

recall: for small quartic Higgs coupling $\lambda = m_h/(v\sqrt{2})$ is driven to negative values by top Yukawa:

$$\beta_\lambda \propto -24\,\mathrm{Tr}\left(Y_u^\dagger Y_u\right)^2 \Rightarrow m_h \geq f(\Lambda)$$

vacuum stability bound

currently unclear situation:

- could be $\lambda(M_{\mathrm{Pl}}) = 0$
- vacuum could be stable
- vacuum could be unstable
- vacuum could be metastable

(Holthausen, Lim, Lindner; Bezrukov *et al.*; Strumia *et al.*; Masina)

strong dependence on top mass, treshold corrections, α_s

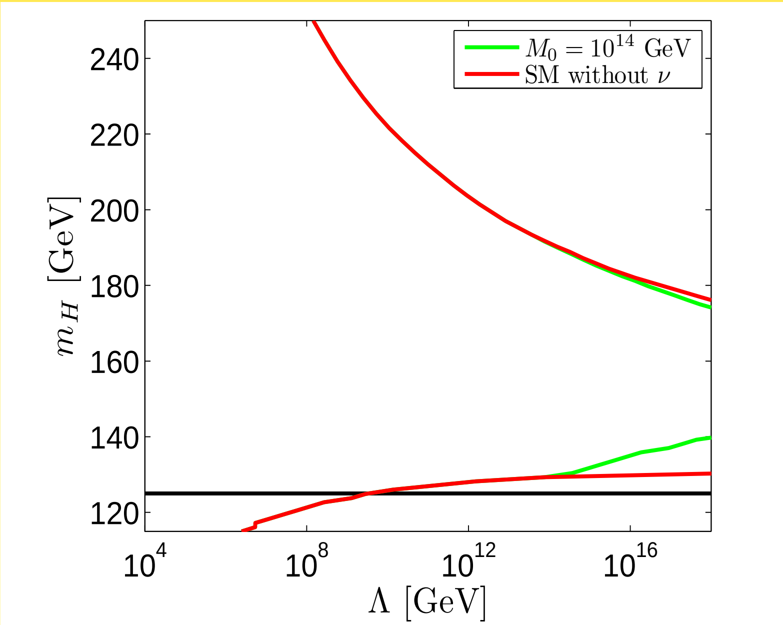
Phenomenology of heavy singlets

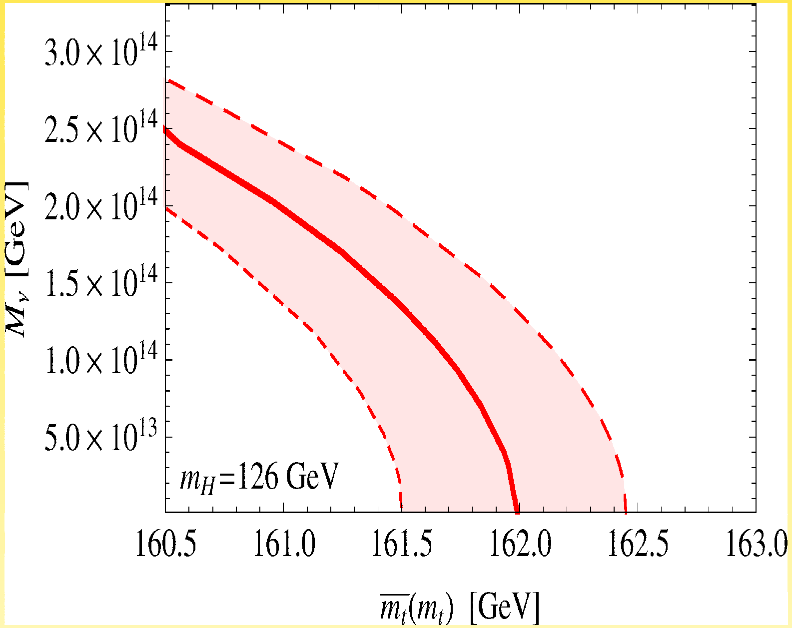
often overlooked: Dirac Yukawa $\bar{\nu}_L\, Y_\nu\, N_R$ contribution to λ:

$$\Delta\beta_\lambda = -8\,\mathrm{Tr}\left(Y_\nu^\dagger Y_\nu\right)^2$$

Casas *et al.*; Strumia *et al.*

makes vacuum stability condition worse!





naively, if M_R goes down, Y_ν goes down and effect is negligible

Higgs physics and sterile neutrinos (W.R., Zhang)

$$m_\nu = v^2 Y_\nu^T M_R^{-1} Y_\nu \quad \text{with} \quad Y_\nu = \frac{1}{v}\sqrt{M_R^{\rm diag}}\, R \sqrt{m_\nu^{\rm diag}}\, U^\dagger$$

useful parametrization:

$$R = O e^{\mathrm{i}A} \quad \text{with} \quad A = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$$

degenerate heavy and light neutrinos:

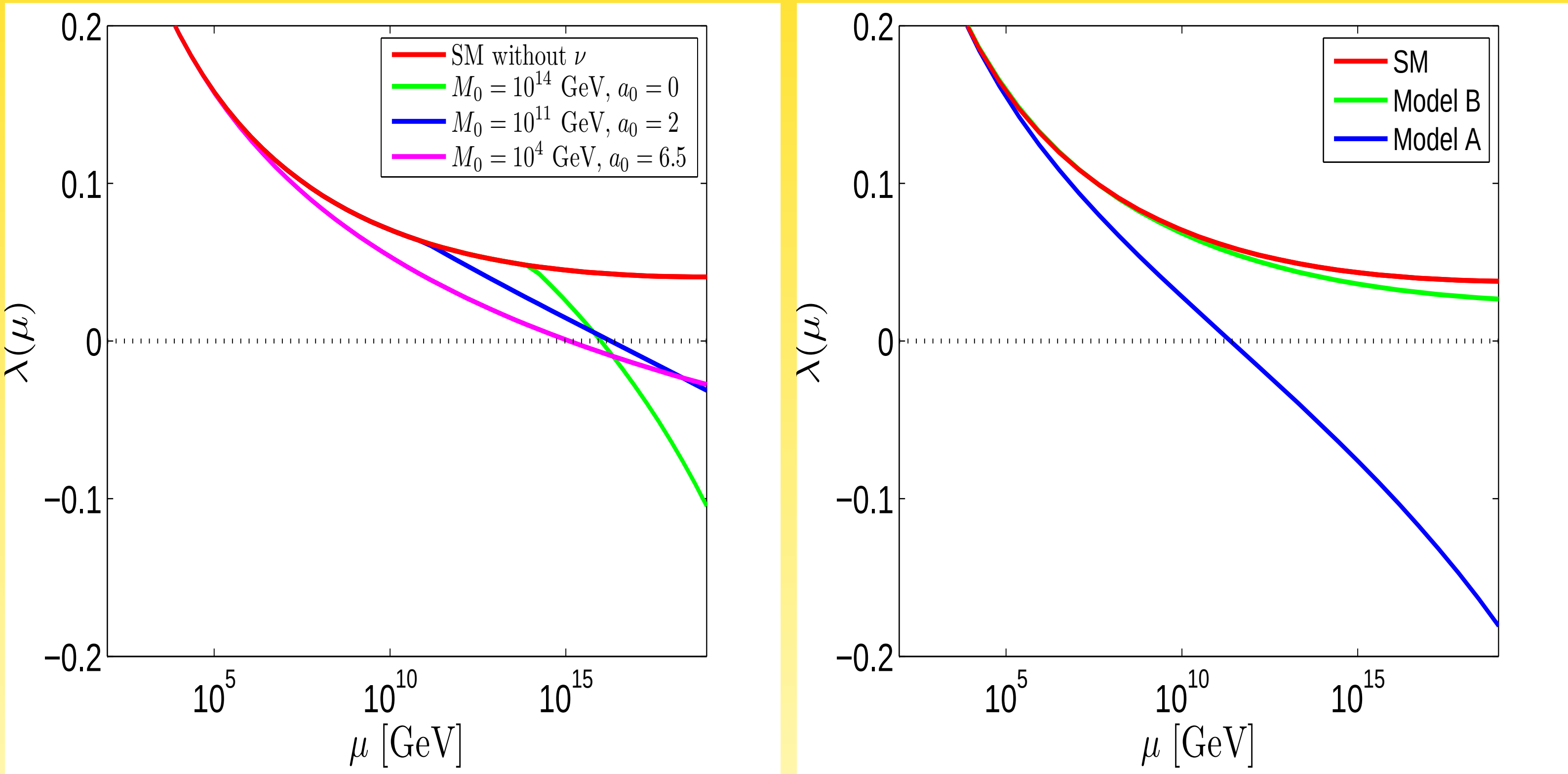
$$\mathrm{tr}\left(Y_\nu^\dagger Y_\nu\right) \simeq \frac{M_0 m_0}{v^2}\left(1 + 2\cosh r\right) \quad \text{with} \quad r = 2\sqrt{a^2+b^2+c^2}$$

for instance, $M_0 = 1\ \mathrm{TeV}$, $m_0 = 0.1\ \mathrm{eV}$, $r = 25$ gives $\mathrm{tr}\left(Y_\nu^\dagger Y_\nu\right) = \mathcal{O}(0.1)$

(compare to naive estimate $Y_\nu \simeq m_0 M_0 / v^2 \sim 10^{-11}$)

Higgs physics and sterile neutrinos

if neutrinos are made accessible at colliders, Dirac Yukawa is large even for TeV neutrinos $\Rightarrow$ influences vacuum stability bound



W.R., Zhang

Phenomenology of (high scale) Leptogenesis

little

(would expect leptonic CP violation and neutrinoless double beta decay)

But note:

- bread and butter leptogenesis requires $M_1 \gtrsim 10^9$ GeV
- *resonant* leptogenesis works even at weak scale
- *flavor oscillation* of sterile neutrinos with mass around few GeV

3 well motivated scales

there are three well-motivated mass values of M_R:

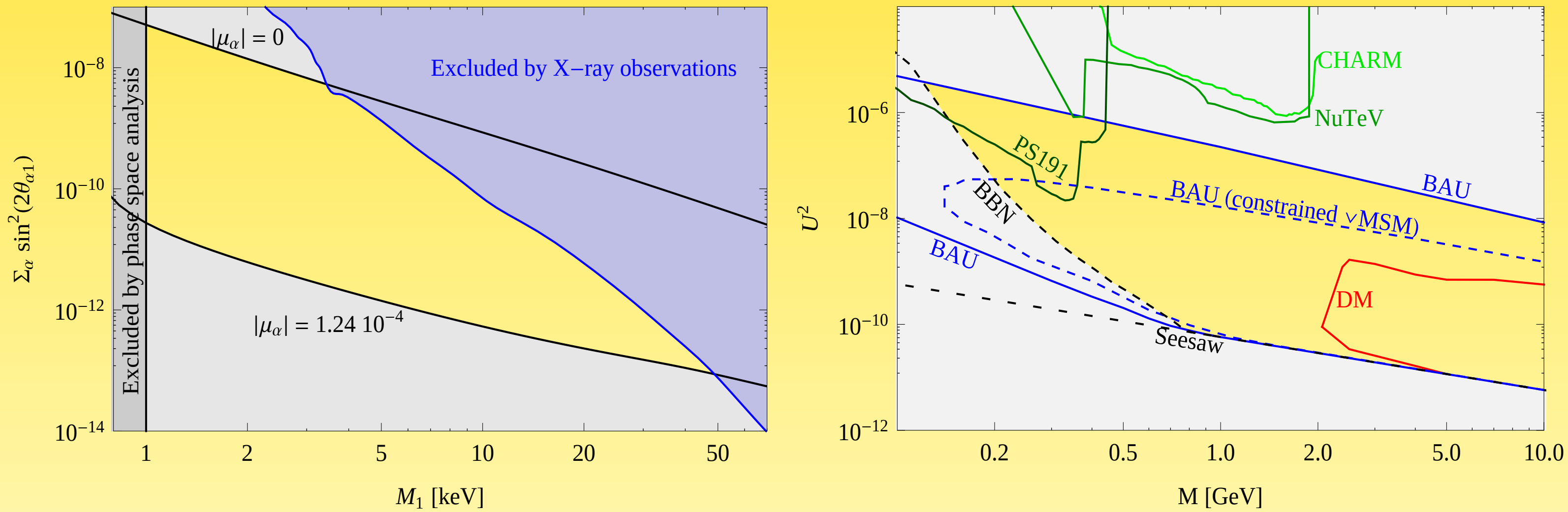
- eV
- keV
- $\gtrsim 10^9$ GeV

what if all three are there?

N_1	N_2	N_3	BAU	eV-anomalies	DM
eV	GUT	GUT	✓	✓	–
eV	keV	GUT	–	✓	✓
keV	GUT or GeV	GUT or GeV	✓	–	✓

νMSM

- no new scale beyond v and Planck scale
- no new particles except 3 right-handed neutrinos
  - one is keV and is Warm Dark Matter
  - two are few GeV, almost degenerate, and do leptogenesis via oscillations



Shaposhnikov *et al.*; Shaposhnikov *et al.*; Shaposhnikov *et al.*; Shaposhnikov *et al.*; Shaposhnikov *et al.*; Shaposhnikov *et al.*,...

νMSM

- $N_{2,3}$ produced thermally at $T \gtrsim T_{\rm EW}$
- oscillate and generate lepton asymmetry
- $\mu \simeq 10^{-10}$ at $T = T_{\rm EW}$
- $N_{2,3}$ freeze out, decay at $T \lesssim$ GeV and generate lepton asymmetry $\mu \simeq 10^{-7}$ at $T \simeq 100$ MeV
- resonant WDM production at $T \simeq 100$ MeV

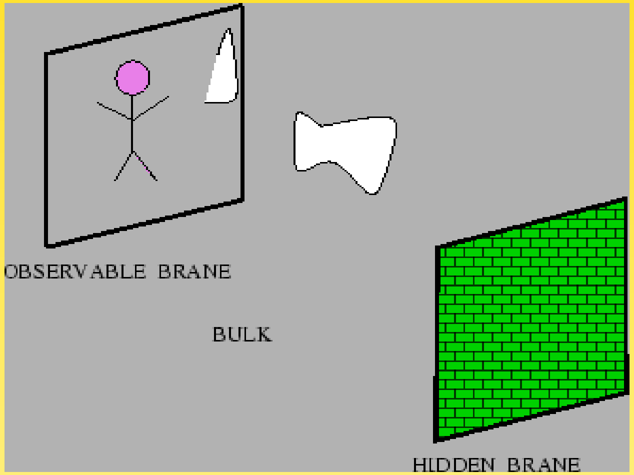
Models for light sterile neutrinos

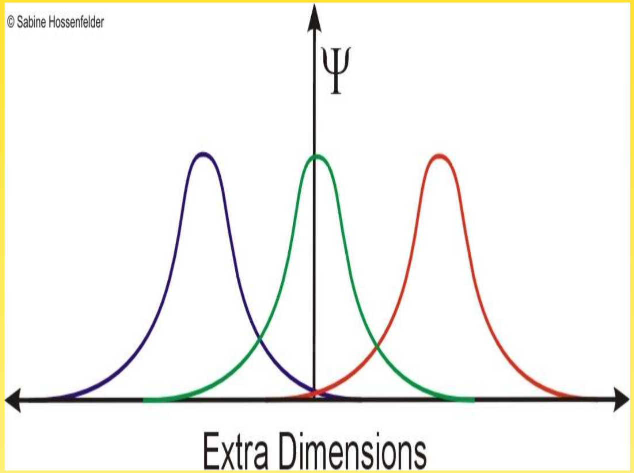
how to bring one (or all) of the singlet neutrinos down to (k)eV ?

- extra dimensions (Kusenko, Takahashi, Yanagida)
- zero mass plus corrections (Mohapatra; Shaposhnikov; Lindner, Merle, Niro; Araki, Li)
- Froggatt-Nielsen (Merle, Niro; Barry, W.R., Zhang)

Light sterile neutrinos from extra dimensions

localize one heavy neutrino N_1 on distant brane, separated from the SM brane, where we live





small wave function overlap between this field and the other ones

$$M_s \propto e^{-2m\,l}\ ,\quad m_D \propto e^{-m\,l} \Rightarrow m_D^2/M_R = \text{const}$$

(m mass of 5D spinor, l size of ED)

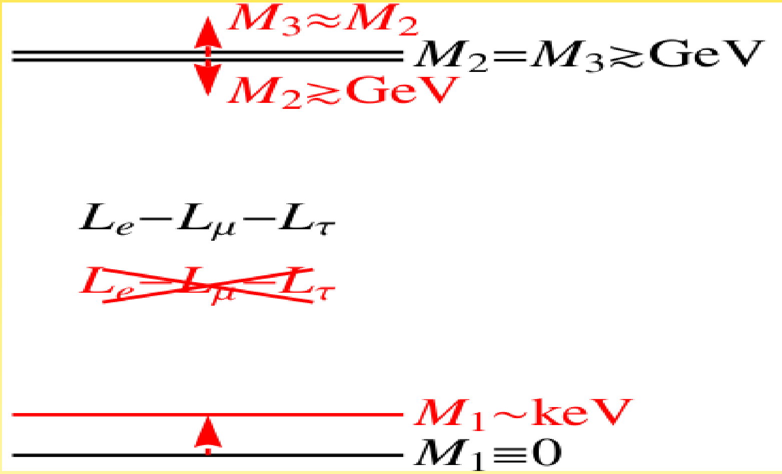
"Split seesaw"

`Kusenko, Takahashi, Yanagida`

Light sterile neutrinos from slightly broken flavor symmetry

introduce flavor symmetry leading to one massless neutrino, e.g.

$$M_R^{L_e-L_\mu-L_\tau} = \begin{pmatrix} 0 & a & b \\ \cdot & 0 & 0 \\ \cdot & \cdot & 0 \end{pmatrix} \Rightarrow M_1 = 0 \,, \quad M_{2,3} = \pm\sqrt{a^2+b^2}$$



small breaking to lift M_1

Mohapatra; Shaposhnikov; Lindner, Merle, Niro; Araki, Li

Froggatt-Nielsen mechanism

effective theory

introduce new scalar field θ with charge -1 under new $U(1)$; acquires VEV $\langle\theta\rangle$

L_e and Φ have charge 0, e_R has charge 4, thus the term

$$\overline{L_e}\,\Phi\,e_R\frac{\theta^4}{\Lambda^4} \to \overline{L_e}\,\Phi\,e_R\frac{\langle\theta\rangle^4}{\Lambda^4}$$

is allowed

tau mass can go as $\overline{L_\tau}\,\Phi\,\tau_R$

With $\langle\theta\rangle/\Lambda \simeq \lambda$: $m_e \simeq \lambda^4\,m_\tau$

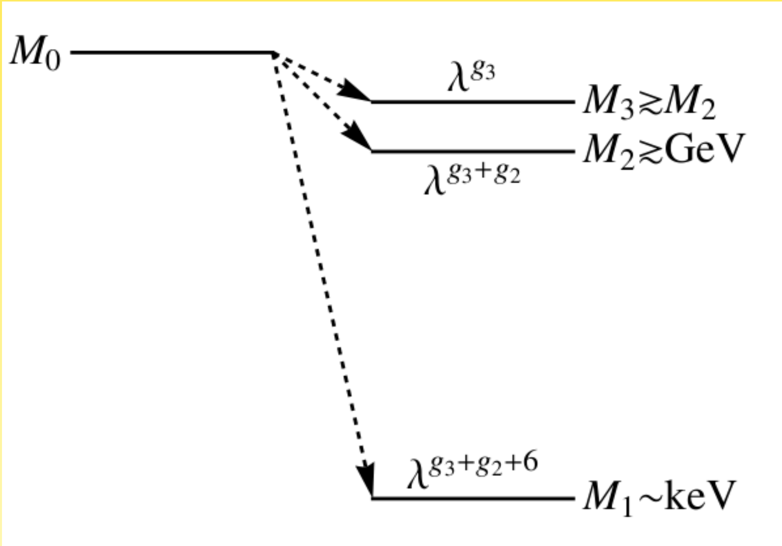
Light sterile neutrinos from Froggatt-Nielsen

introduce new $U(1)$ and field Θ with charge -1

N_R has charge m and ν_L has charge n:

$$m_D\, \bar{\nu}_L\, N_R \left(\frac{\Theta}{\Lambda}\right)^{n+m} + M_R\, \bar{N}_R^c\, N_R \left(\frac{\Theta}{\Lambda}\right)^{2m} , \qquad \frac{\Theta}{\Lambda} \simeq \lambda$$

$\Rightarrow$ FN charge of N_R drops out in m_D^2/M_R



Merle, Niro; Barry, W.R., Zhang

Flavor Symmetries

	l	e^c	μ^c	τ^c	ν^c	$h_{u,d}$	ν_s
A_4	3	1	1"	1'	3	1	1
Z_3	ω	ω^2	ω^2	ω^2	ω^2	1	1
$U(1)$	0	4	2	0	0	0	6 (8)

active neutrino terms of order $llhh(\xi, \varphi')/\Lambda^2$

active-sterile terms of order $l\xi\varphi' h\nu_s\lambda^6/\Lambda^2$

sterile-sterile terms of order $\varphi^2\nu_s\nu_s\lambda^{12}/\Lambda$

generate tri-bimaximal mixing and mixing of order 0.1 with eV-steriles
(or 10^{-4} with keV)

Barry, W.R., Zhang

Flavor Symmetries

add ν_s and use FN to control magnitude of its mass

$$M_\nu^{4\times 4} = \begin{pmatrix} a + \frac{2d}{3} & -\frac{d}{3} & -\frac{d}{3} & e \\ \cdot & \frac{2d}{3} & a - \frac{d}{3} & e \\ \cdot & \cdot & \frac{2d}{3} & e \\ \cdot & \cdot & \cdot & m_s \end{pmatrix} \quad \text{with} \quad \begin{array}{ll} a, d \simeq 10^{-2}\,\text{eV} & \\ e/m_s \simeq 0.1 & (10^{-4}) \\ m_s \simeq \text{eV} & (\text{keV}) \end{array}$$

diagonalized by

$$U \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & \frac{e}{m_s} \\ 0 & 0 & 0 & \frac{e}{m_s} \\ 0 & 0 & 0 & \frac{e}{m_s} \\ 0 & -\frac{\sqrt{3}e}{m_s} & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -\frac{\sqrt{3}e^2}{2m_s^2} & 0 & 0 \\ 0 & -\frac{\sqrt{3}e^2}{2m_s^2} & 0 & 0 \\ 0 & -\frac{\sqrt{3}e^2}{2m_s^2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{3e^2}{2m_s^2} \end{pmatrix}$$

`Barry, W.R., Zhang`

A_4 Seesaw Model with light steriles (Barry, W.R., Zhang)

Field	L	e^c	μ^c	τ^c	$h_{u,d}$	φ	φ'	φ''	ξ	ξ'	ξ''	Θ	ν_1^c	ν_2^c	ν_3^c
$SU(2)_L$	2	1	1	1	2	1	1	1	1	1	1	1	1	1	1
A_4	$\underline{3}$	$\underline{1}$	$\underline{1}''$	$\underline{1}'$	$\underline{1}$	$\underline{3}$	$\underline{3}$	$\underline{3}$	$\underline{1}$	$\underline{1}'$	$\underline{1}$	$\underline{1}$	$\underline{1}$	$\underline{1}'$	$\underline{1}$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω^2	ω^2	ω	1	1	ω^2	ω	1
$U(1)$	-	3	1	0	-	-	-	-	-	-	-	-1	F_1	F_2	F_3

various possibilities for the FN-charges:

	F_1, F_2, F_3	Mass spectrum	$\lvert U_{\alpha 4}\rvert$	$\lvert U_{\alpha 5}\rvert$	m_{ee} NO	m_{ee} IO	Phenomenology
I	9, 10, 10	$M_{2,3} = \mathcal{O}(\text{eV})$	$\mathcal{O}(0.1)$	$\mathcal{O}(0.1)$	0	0	$3+2$ mixing
IIA	9, 10, 0	$M_2 = \mathcal{O}(\text{eV})$ $M_3 = \mathcal{O}(10^{11}\,\text{GeV})$	$\mathcal{O}(0.1)$	$\mathcal{O}(10^{-11})$	0	$\frac{2\sqrt{\Delta m_{\text{A}}^2}}{3}$	$3+1$ mixing
IIB	9, 0, 10	$M_2 = \mathcal{O}(10^{11}\,\text{GeV})$ $M_3 = \mathcal{O}(\text{eV})$	$\mathcal{O}(10^{-11})$	$\mathcal{O}(0.1)$	$\frac{\sqrt{\Delta m_{\odot}^2}}{3}$	$\frac{\sqrt{\Delta m_{\text{A}}^2}}{3}$	$3+1$ mixing
III	9, 5, 5	$M_{2,3} = \mathcal{O}(10\,\text{GeV})$	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-6})$	$\frac{\sqrt{\Delta m_{\odot}^2}}{3}$	$\sqrt{\Delta m_{\text{A}}^2}$	Leptogenesis

Final Remarks

Steriles have a number of consequences:

- oscillations
- astrophysics
- cosmology
- beta decays, neutrinoless double beta decay
- Higgs physics
- Lepton flavor vialation
- ...

would be extraordinary discovery!

Summary

- Are there sterile neutrinos?
-
-
-

Summary

- Are there sterile neutrinos? Maybe!
-
-
-

Summary

- Are there sterile neutrinos? Maybe!
- if there are steriles, are they light?
-
-

Summary

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Summary

- Are there sterile neutrinos? Maybe!
- if there are steriles, are they light? Maybe!
- experimental input necessary
-

Summary

- Are there sterile neutrinos? Maybe!
- if there are steriles, are they light? Maybe!
- experimental input necessary
- if (light) steriles necessary, we know what to do

Remarks

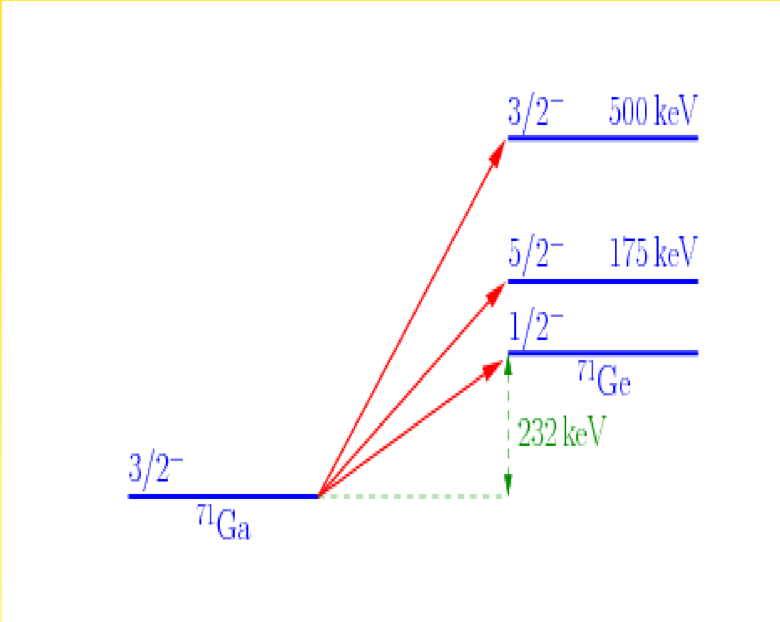
Steriles have a number of consequences:

- oscillations
- astrophysics
- cosmology
- beta decays, neutrinoless double beta decay
- Higgs physics
- Lepton flavor vialation
- . . .

would be extraordinary discovery!

Motivation for Sterile Neutrinos: Gallium Anomaly

- overestimate of detection process $\sigma(\nu_e + ^{71}\mathrm{Ga} \to e^- + ^{71}\mathrm{Ge})$?



- small contributions of excited states confirmed by $^{71}\mathrm{Ga}(^3\mathrm{He},\mathrm{t})^{71}\mathrm{Ge}$ measurements

Motivation for Sterile Neutrinos: Reactor Anomaly

- 200 MeV energy per fission, 6 neutrinos generated in β-decay chain $\Rightarrow 2 \times 10^{20}\,\nu$/s per GW thermal power
- U and Pu chains have $\mathcal{O}(10^2)$ nuclei, with $\mathcal{O}(10)$ branches each
- high energy part (shortest lifetime, i.e. least known) most important
- $\Leftrightarrow$ measurement of e^- spectrum at ILL
- sophisticated translation into $\overline{\nu}_e$ spectra, taking into account
  - new neutron lifetime ($\sigma_{\rm fission} \propto 1/\tau_n$)
  - corrections to Fermi theory
    - nuclear charge distribution ($\leftrightarrow$ QED corrections)
    - weak magnetism ($\leftrightarrow$ magn. moment and axial current interference)
    - off-equilibrium effects ($\leftrightarrow$ evolution of reactor)
    - radiative corrections
    - more branches

$$S_\beta(Z,A,E_e) = \underbrace{K}_{\text{Norm.}} \times \underbrace{\mathcal{F}(Z,A,E_e)}_{\text{Fermi function}} \times \underbrace{p_e E_e (E_e - E_0)^2}_{\text{Phase space}} \times \underbrace{\Big(1+\delta(Z,A,E_e)\Big)}_{\text{Correction}}$$

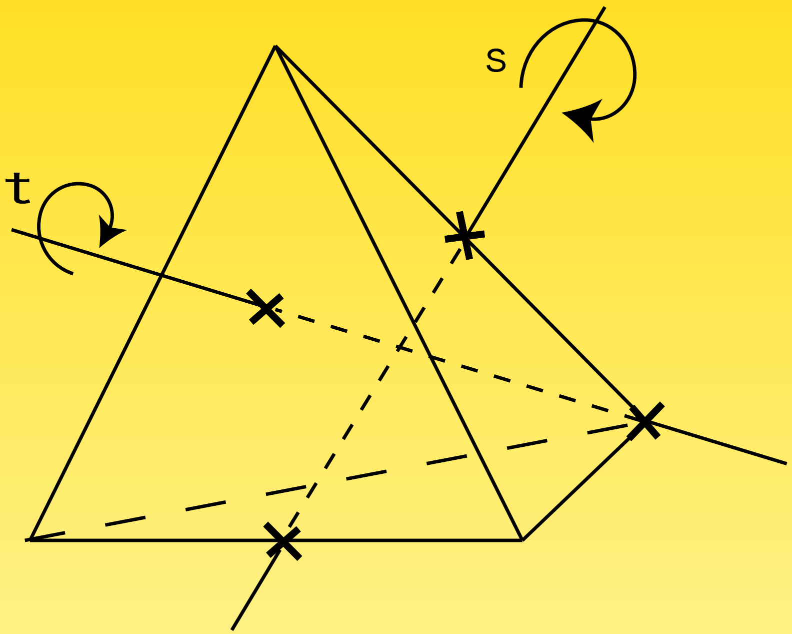
Origin of m_ν

$\mathcal{L}_M = \frac{1}{2}\,\overline{\nu_L}\,m_\nu\,\nu_L^c$ or $\mathcal{L}_D = \overline{\nu_L}\,m_\nu\,\nu_R$ are necessarily BSM

Ansatz	content	$\mathcal{L}$	m_ν	scale
"SM" (Dirac mass)	singlet	$y\overline{L}HN_R$	yv	$y = \mathcal{O}(10^{-12})$
"effective" (dim 5 operator)	new scale + LNF	$\frac{1}{\Lambda}\overline{L}HH^TL^c$	$\frac{v^2}{\Lambda}$	$\Lambda = \left(\frac{0.1\text{ eV}}{m_\nu}\right)10^{14}$ GeV
"direct" (Type II See-Saw)	Higgs triplet + LNV	$y\overline{L}\Delta L^c + \mu HH\Delta$	yv_T	$\Lambda = \frac{1}{y\mu}M_\Delta^2$
"indirect 1" (Type I see-saw)	Singlet + LNV	$y\overline{L}HN_R + \overline{N_R^c}M_RN_R$	$\frac{(yv)^2}{M_R}$	$\Lambda = \frac{1}{y}M_R$
"indirect 2" (Type III see-saw)	Fermion triplet + LNV	$y\overline{L}\Sigma H + \mathrm{Tr}\overline{\Sigma}M_\Sigma\Sigma$	$\frac{(yv)^2}{M_\Sigma}$	$\Lambda = \frac{1}{y}M_\Sigma$

focus here on type I see-saw mechanism

How to generate lepton mixing: Flavor Symmetries...



A_4 smallest group with irreducible $3 \leftrightarrow 3$ generations

$$3 \times 3 = 1 + 1' + 1'' + 3 + 3$$

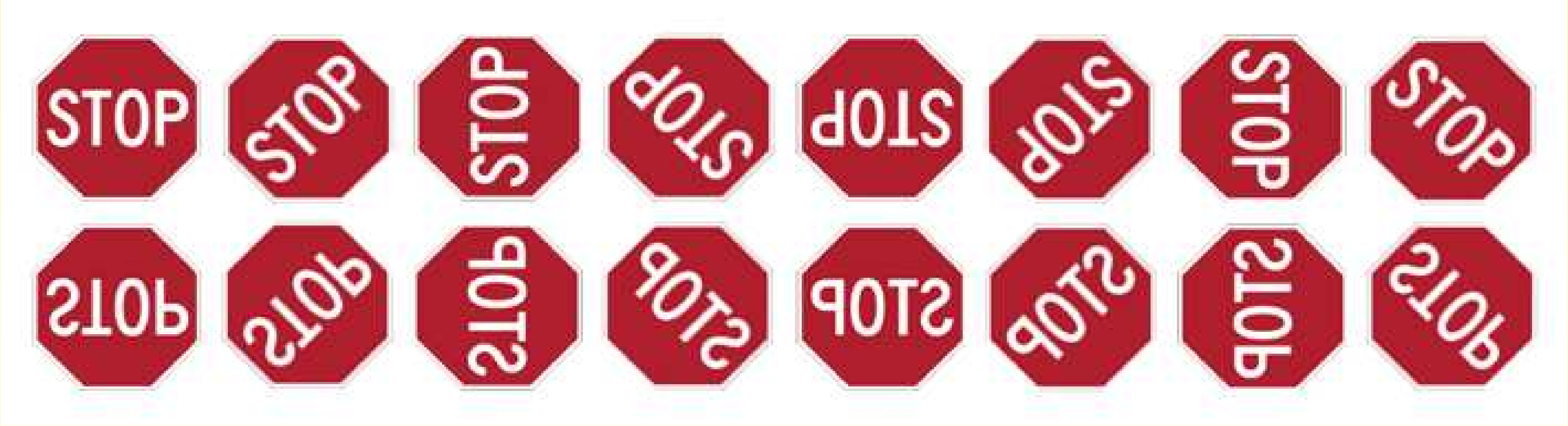
$$1 \times 1 = 1,\ 1' \times 1'' = 1,\ 1'' \times 1' = 1,\ 1' \times 1' = 1'',\ 1'' \times 1'' = 1',\ 3 \times 1 = 3, \ldots$$

Flavor Symmetries

$\longleftrightarrow$ new (Flavor-, horizontal) symmetries!?

- $U(1)$, $SU(2)$, $SU(3)$,...
- S_2, S_3,...
- A_4, D_4, D_5, D_{14}, $\mathcal{PSL}_2(7)$, $'T$,...
- $\Delta(27)$, $\Sigma(81)$,...

Often geometrical interpretation (e.g. dihedral groups D_8):



Which sort of group?

- discrete or continuous?

  want to avoid Goldstone bosons from broken symmetry

- Abelian or non-Abelian?

  want to explain existence of three generations or at least unify two of them

$\Rightarrow$ discrete non-Abelian flavor symmetry

A role model (Altarelli, Feruglio)

Field	l	e^c	μ^c	τ^c	$h_{u,d}$	φ	φ'	ξ	φ_0	φ'_0	ξ_0	θ
A_4	3	1	$1''$	$1'$	1	3	3	1	3	3	1	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	1	ω	ω	1
$U(1)_{\mathrm{FN}}$	0	4	2	0	0	0	0	0	0	0	0	-1
$U(1)_R$	1	1	1	1	0	0	0	0	2	2	2	0

- all 3 LH lepton doublets unified as $l = (L_e, L_\mu, L_\tau)$ with $L_e = (\nu_e, e)^T$
- Z_3 to separate charged leptons and neutrinos
- Froggatt-Nielsen to get charged lepton hierarchy
- φ_0 and φ'_0 acquire VEVs and break the symmetry ("VEV alignment")
- $U(1)_R$, φ_0, φ'_0 and ξ_0 to make the VEVs look the way they do ("driving fields")

Froggatt-Nielsen mechanism

effective theory

introduce new scalar field θ with charge -1 under new $U(1)$; acquires VEV $\langle\theta\rangle$

L_e and Φ have charge 0, e_R has charge 4, thus the term

$$\overline{L_e}\,\Phi\,e_R\frac{\theta^4}{\Lambda^4} \to \overline{L_e}\,\Phi\,e_R\frac{\langle\theta\rangle^4}{\Lambda^4}$$

is allowed

tau mass can go as $\overline{L_\tau}\,\Phi\,\tau_R$

With $\langle\theta\rangle/\Lambda \simeq \lambda$: $m_e \simeq \lambda^4\, m_\tau$

A role model (Altarelli, Feruglio)

Field	l	e^c	μ^c	τ^c	$h_{u,d}$	φ	φ'	ξ	φ_0	φ'_0	ξ_0	θ
A_4	3	1	$1''$	$1'$	1	3	3	1	3	3	1	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	1	ω	ω	1
$U(1)_{\rm FN}$	0	4	2	0	0	0	0	0	0	0	0	-1
$U(1)_R$	1	1	1	1	0	0	0	0	2	2	2	0

tau-lepton:

$$\begin{aligned}\mathcal{L} &= \frac{1}{\Lambda} y_\tau \tau^c (\varphi l)'' h_d \equiv y_\tau \tau^c (\varphi l)'' \\ &= y_\tau \tau^c (L_\mu \varphi_2 + L_e \varphi_3 + L_\tau \varphi_1)\end{aligned}$$

A role model (Altarelli, Feruglio)

Field	l	e^c	μ^c	τ^c	$h_{u,d}$	φ	φ'	ξ	φ_0	φ'_0	ξ_0	θ
A_4	3	1	$1''$	$1'$	1	3	3	1	3	3	1	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	1	ω	ω	1
$U(1)_{\rm FN}$	0	4	2	0	0	0	0	0	0	0	0	-1
$U(1)_R$	1	1	1	1	0	0	0	0	2	2	2	0

electron:

$$\begin{aligned}\mathcal{L} &= \frac{1}{\Lambda}\frac{\theta^4}{\Lambda^4} y_e e^c (\varphi l) h_d \equiv y_e e^c (\varphi l) \\ &= y_e e^c (L_e \varphi_1 + L_\mu \varphi_2 + L_\tau \varphi_3)\end{aligned}$$

A role model (Altarelli, Feruglio)

Field	l	e^c	μ^c	τ^c	$h_{u,d}$	φ	φ'	ξ	φ_0	φ'_0	ξ_0	θ
A_4	3	1	$1''$	$1'$	1	3	3	1	3	3	1	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	1	ω	ω	1
$U(1)_{\rm FN}$	0	4	2	0	0	0	0	0	0	0	0	-1
$U(1)_R$	1	1	1	1	0	0	0	0	2	2	2	0

Neutrinos:

$$\begin{aligned}\mathcal{L} &= \frac{1}{\Lambda^2} y_a \xi_0 (l h_u l h_u) \equiv y_a \xi_0 (ll) \\ &= y_a (L_e L_e + L_\tau L_\mu + L_\mu L_\tau)\end{aligned}$$

A role model (Altarelli, Feruglio)

Field	l	e^c	μ^c	τ^c	$h_{u,d}$	φ	φ'	ξ	φ_0	φ'_0	ξ_0	θ
A_4	3	1	$1''$	$1'$	1	3	3	1	3	3	1	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	1	ω	ω	1
$U(1)_{\rm FN}$	0	4	2	0	0	0	0	0	0	0	0	-1
$U(1)_R$	1	1	1	1	0	0	0	0	2	2	2	0

Neutrinos:

$$\mathcal{L} = \frac{1}{\Lambda^2} y_b (\varphi' l h_u l h_u) \equiv y_b (\varphi l l)$$

$$= \tfrac{1}{3} y_b \left[(2L_eL_e - L_\mu L_\tau - L_\tau L_\mu)\varphi'_1 + (2L_\tau L_\tau - L_e L_\mu - L_\mu L_e)\varphi'_2 + \right.$$

$$\left. (2L_\mu L_\mu - L_e L_\tau - L_\tau L_e)\varphi'_3\right]$$

A role model (Altarelli, Feruglio)

The mass matrices are

$$m_\ell = \begin{pmatrix} y_e\varphi_1 & y_e\varphi_2 & y_e\varphi_3 \\ y_\mu\varphi_2 & y_\mu\varphi_1 & y_\mu\varphi_3 \\ y_\tau\varphi_3 & y_\tau\varphi_2 & y_\tau\varphi_1 \end{pmatrix} \text{ and } m_\nu = \frac{1}{3}\begin{pmatrix} 3\xi_0 + 2\varphi'_1 & -\varphi'_2 & -\varphi'_3 \\ \cdot & 2\varphi'_3 & 3\xi_0 - \varphi'_1 \\ \cdot & \cdot & 2\varphi'_2 \end{pmatrix}$$

we arrange for "VEV alignment" $\langle\varphi\rangle = (v, 0, 0)$ and $\langle\varphi'\rangle = (v', v', v')$:

$$m_\ell = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \text{ and } m_\nu = \begin{pmatrix} a+2b & -b & -b \\ \cdot & 2b & a-b \\ \cdot & \cdot & 2b \end{pmatrix}$$

gives tri-bimaximal mixing

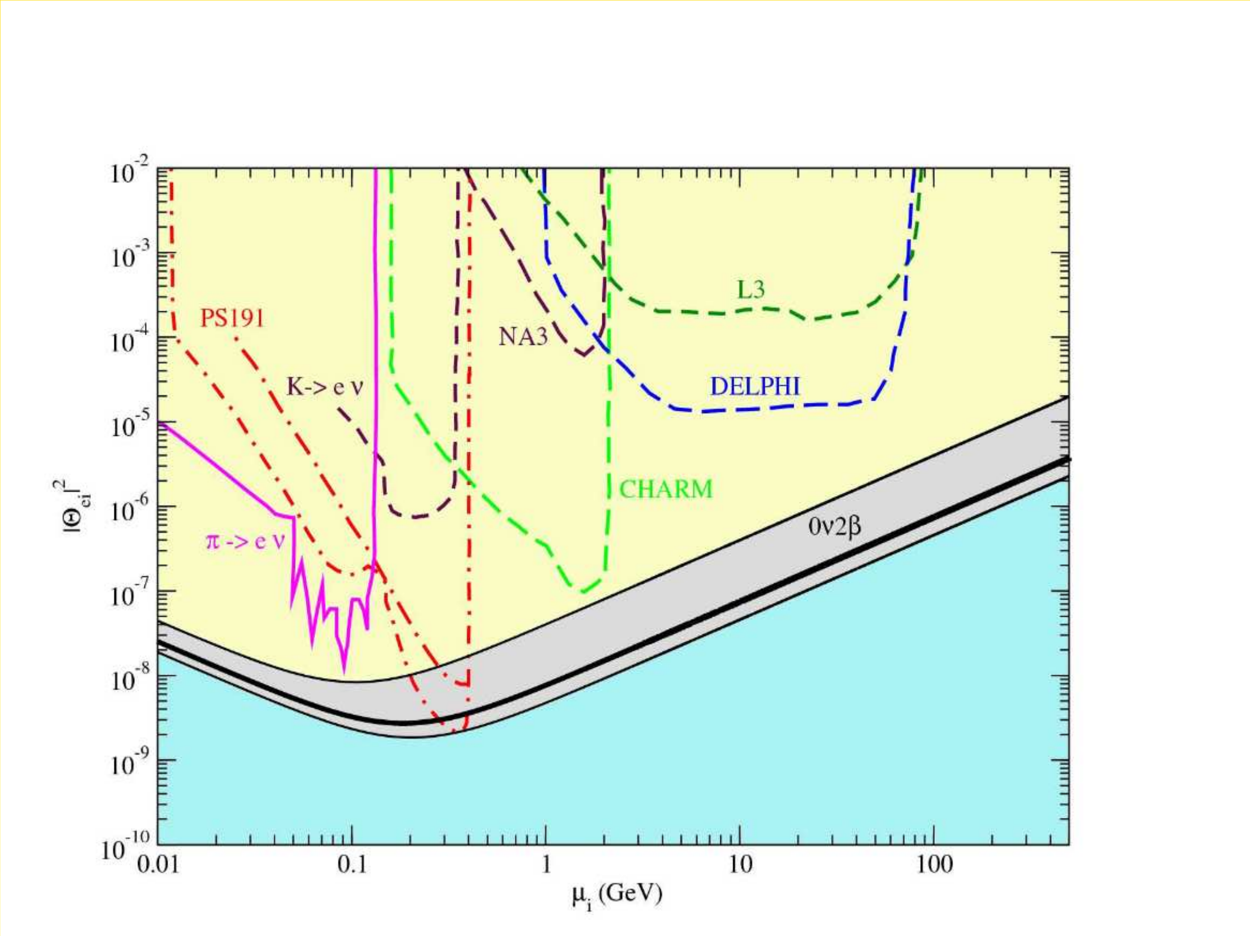
masses: $m_1 = a + 3b$, $m_2 = a$, $m_3 = -a + 3b$

Confused?

there are different transformations:

$(\nu_e)_L \xrightarrow{C} (\nu_e)_R$	charge conjugation
$(\nu_e)_L \xrightarrow{P} (\nu_e)_R$	parity conjugation
$(\nu_e)_L \xrightarrow{CP} (\bar{\nu}_e)_R$	CP transformation
$(\nu_e)_L \xrightarrow{\hat{\mathcal{C}}} (\bar{\nu}_e)_R$	particle-antiparticle transformation $\nu \to \nu^c$

Heavy neutrinos



Mitra, Senjanovic, Vissani

Non-maximal θ_{23}?

LBL accelerator experiments have octant-asymmetric amplitude (plus higher order terms with sensitivity to δ and sgn$(\Delta m^2_{\rm A})$)

$$P(\nu_\mu \to \nu_\mu) \propto \cos^2\theta_{13}\sin^2\theta_{23}\,(1 - \cos^2\theta_{13}\sin^2\theta_{23})$$
$$P(\nu_\mu \to \nu_e) \propto \cos^2\theta_{13}\sin^2\theta_{13}\sin^2\theta_{23}$$

MINOS and T2K disappearence data most important, preference for $\theta_{23} \neq \pi/4$

atmospheric data:

$$N_e - N_e^0 \propto (R\sin^2\theta_{23} - 1)\,f(\Delta m^2_{\rm A}, \theta_{13}) + (R\cos^2\theta_{23} - 1)\,g(\Delta m^2_\odot, \theta_{12})$$
$$-C\,\sin\theta_{13}\sin\theta_{23}\,\cos\theta_{23}\cos\delta$$

slight electron excess in sub-GeV atmospheric data sets easier explained by $\cos\delta = -1$ and $\theta_{23} < \pi/4$