3 - Mass-Induced Neutrino Flavor Oscillations

Neutrino Flavor change can arise out of several different mechanisms. The simplest one is to appreciate that, once **neutrinos have mass**, **leptons can mix**. This turns out to be the correct mechanism (certainly the dominant one), and **only** explanation that successfully explains **all** long-baseline data consistently.

Neutrinos with a well defined mass:

$$\nu_1, \nu_2, \nu_3, \ldots \qquad \text{with masses} \;\; m_1, m_2, m_3, \ldots$$

How do these states (neutrino mass eigenstates) relate to the neutrino flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$?

$$\nu_\alpha = U_{\alpha i}\nu_i \qquad \alpha = e, \mu, \tau, \quad i = 1, 2, 3$$

U is a unitary mixing matrix. I'll talk more about it later.

The Propagation of Massive Neutrinos

Neutrino mass eigenstates are eigenstates of the free-particle Hamiltonian:

$$|\nu_i\rangle = e^{-iE_i t}|\nu_i\rangle, \qquad E_i^2 - |\vec{p}_i|^2 = m_i^2$$

The neutrino flavor eigenstates are linear combinations of ν_i's, say:

$$\begin{aligned} |\nu_e\rangle &= \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle. \\ |\nu_\mu\rangle &= -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle. \end{aligned}$$

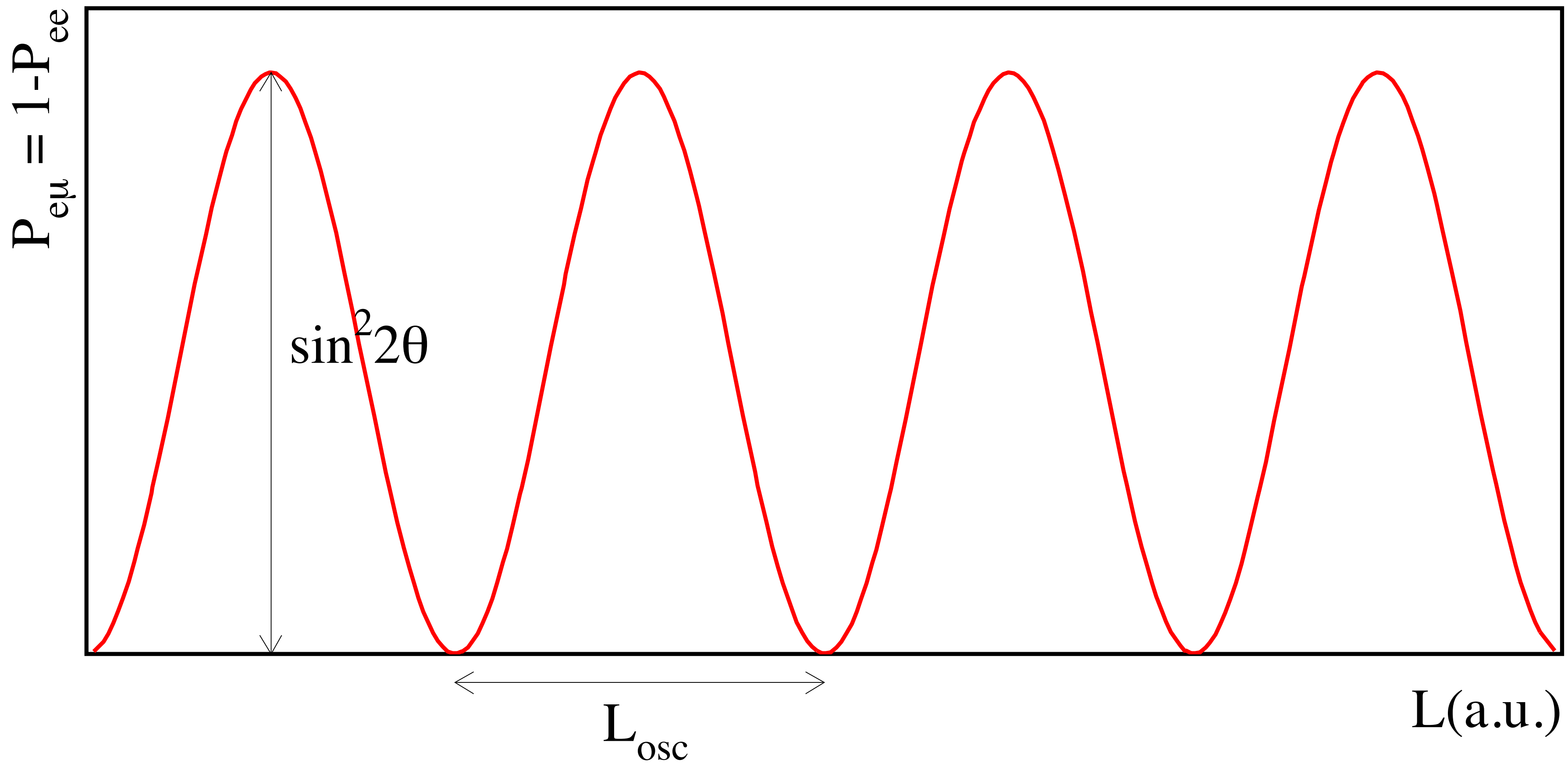
If this is the case, a state produced as a ν_e evolves in vacuum into

$$|\nu(t,\vec{x})\rangle = \cos\theta e^{-ip_1 x}|\nu_1\rangle + \sin\theta e^{-ip_2 x}|\nu_2\rangle.$$

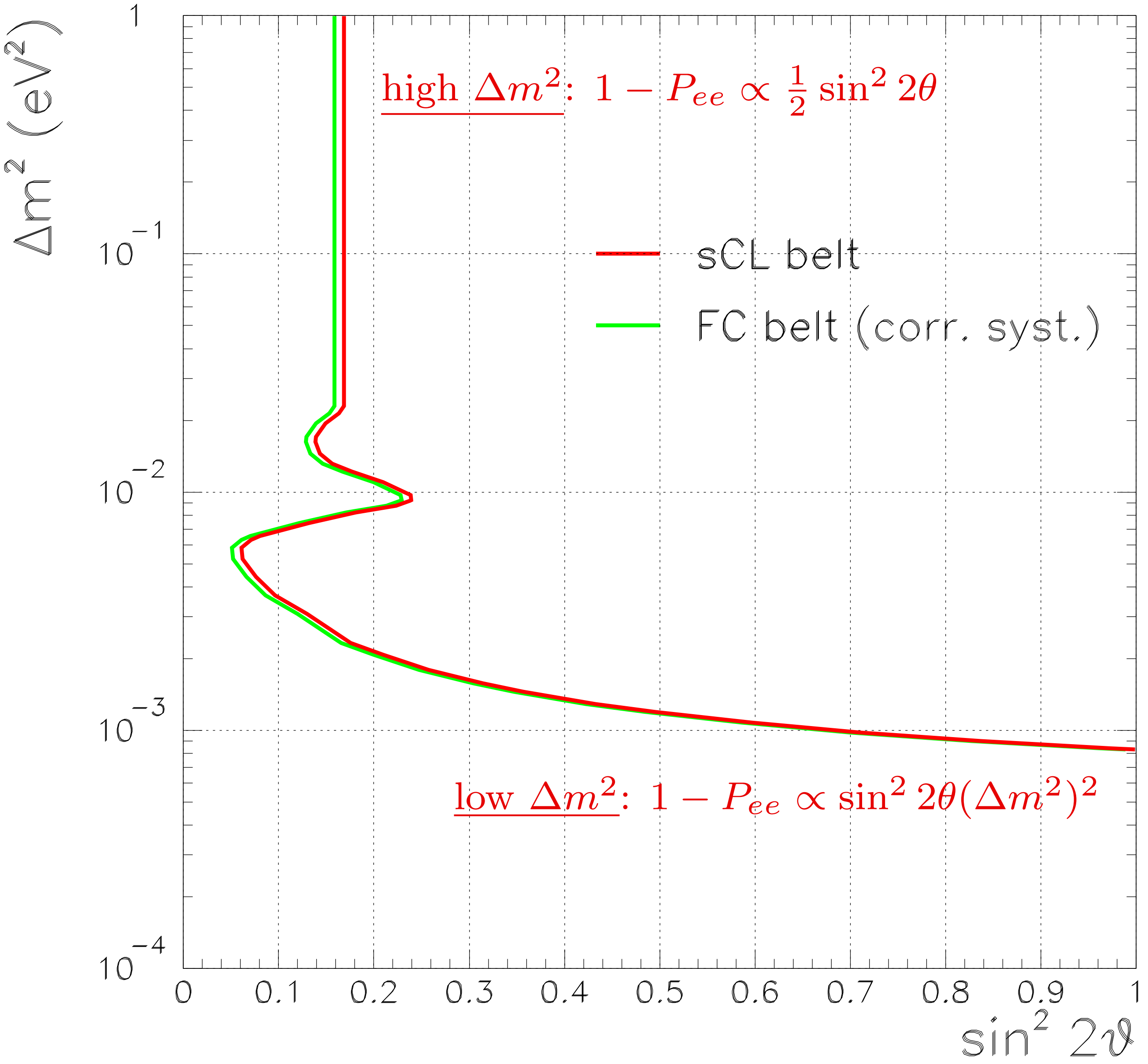
It is trivial to compute $P_{e\mu}(L) \equiv |\langle\nu_\mu|\nu(t, z=L)\rangle|^2$. It is just like a two-level system from basic undergraduate quantum mechanics! In the ultrarelativistic limit (always a good bet), $t \simeq L$, $E_i - p_{z,i} \simeq (m_i^2)/2E_i$, and

$$\boxed{P_{e\mu}(L) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)}$$

oscillation parameters: $\begin{cases} \pi \frac{L}{L_{\rm osc}} \equiv \frac{\Delta m^2 L}{4E} = 1.267 \left(\frac{L}{\rm km}\right) \left(\frac{\Delta m^2}{\rm eV^2}\right) \left(\frac{\rm GeV}{E}\right) \\ \text{amplitude } \sin^2 2\theta \end{cases}$



CHOOZ experiment



$$P_{ee} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

result: $1 - P_{ee} < 0.05$

There is a long (and oftentimes confused and confusing) history behind this derivation and several others. A comprehensive discussion can be found, for example, in

E.K. Akhmedov, A. Yu. Smirnov, 0905.1903 [hep-ph]

In a nutshell, neutrino oscillations as described above occur whenever

- Neutrino Production and Detection are Coherent $\rightarrow$ cannot "tell" ν_1 from ν_2 from ν_3 but "see" ν_e or ν_μ or ν_τ.
- Decoherence effects due to wave-packet separation are negligible $\rightarrow$ baseline not too long that different "velocity" components of the neutrino wave-packet have time to physically separate.
- The energy released in production and detection is large compared to the neutrino mass $\rightarrow$ so we can assign all of the effect to the neutrino propagation, independent from the production process. Also assures ultra-relativistic approximation good.

$P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$ Works great for $\sin^2 2\theta \sim 1$ and $\Delta m^2 \sim 10^{-3}$ eV^2

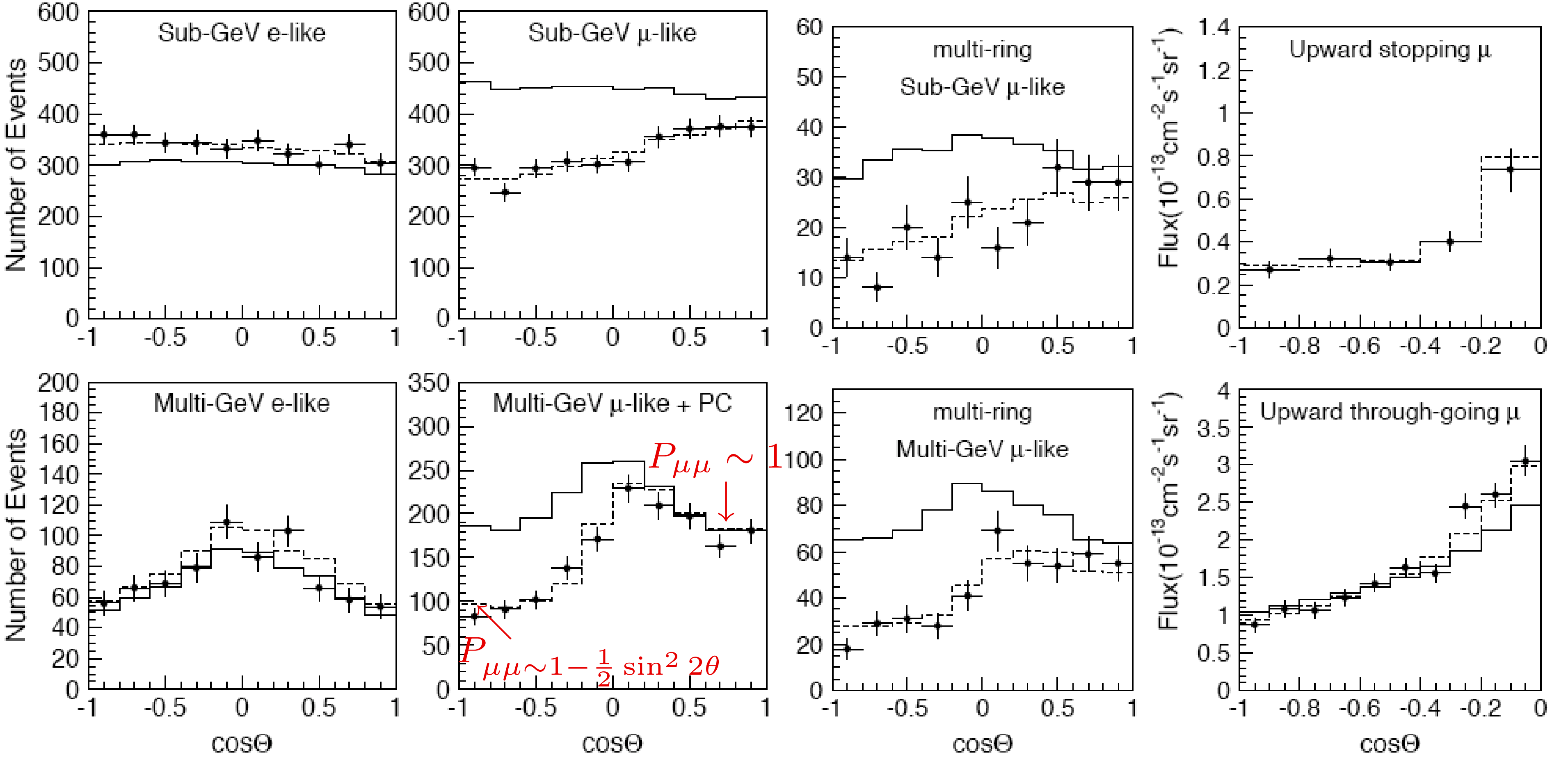
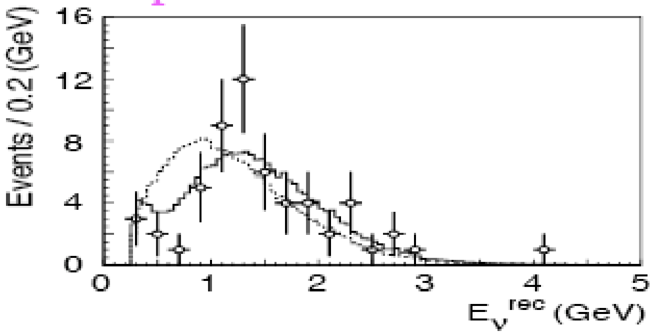


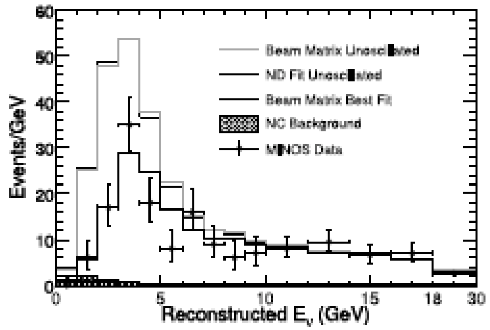
Figure 4. Zenith angle distribution for fully-contained single-ring e-like and μ-like events, multi-ring μ-like events, partially contained events and upward-going muons. The points show the data and the solid lines show the Monte Carlo events without neutrino oscillation. The dashed lines show the best-fit expectations for $\nu_\mu \leftrightarrow \nu_\tau$ oscillations. From M. Ishitsuka [Super-Kamiokande Collaboration], hep-ex/0406076.

K2K	ν_μ at KEK	SK	L=250 km
MINOS	ν_μ at Fermilab	Soundan	L=735 km
Opera/Icarus	ν_μ at CERN	Gran Sasso	L=740 km

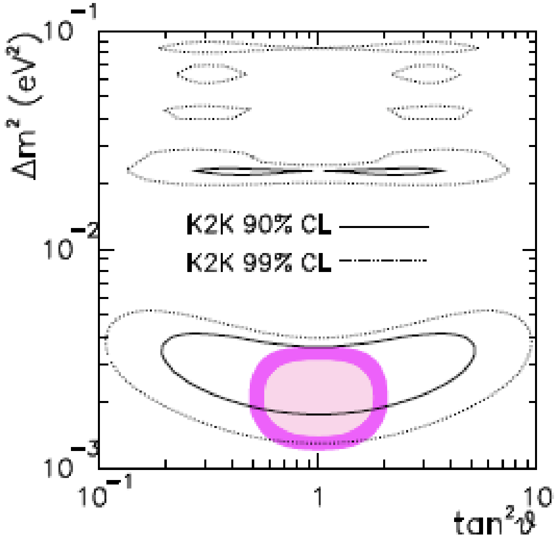
K2K 2004: spectral distortion



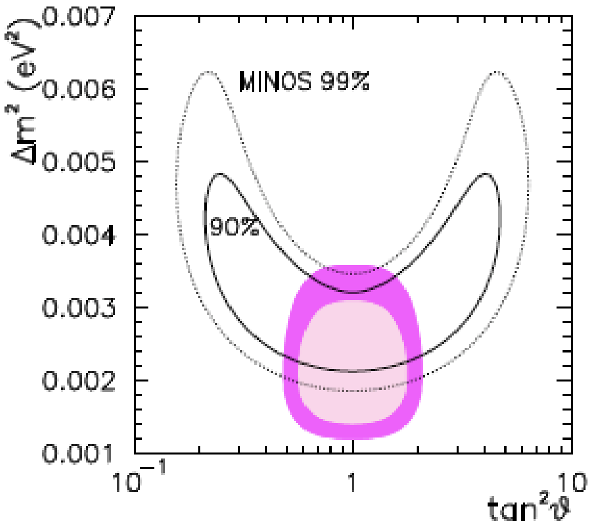
MINOS 2006: spectral distortion



Confirmation of ATM oscillations



Confirmation of ATM oscillations



[Gonzalez-Garcia, PASI 2006]

Matter Effects

The neutrino propagation equation, in the ultra-relativistic approximation, can be re-expressed in the form of a Shrödinger-like equation. In the mass basis:

$$i\frac{\mathrm{d}}{\mathrm{d}L}|\nu_i\rangle = \frac{m_i^2}{2E}|\nu_i\rangle,$$

up to a term proportional to the identity. In the weak/flavor basis

$$i\frac{\mathrm{d}}{\mathrm{d}L}|\nu_\beta\rangle = U_{\beta i}\frac{m_i^2}{2E}U^\dagger_{i\alpha}|\nu_\alpha\rangle.$$

In the 2×2 case,

$$i\frac{\mathrm{d}}{\mathrm{d}L}\left(\begin{array}{c} |\nu_e\rangle \\ |\nu_\mu\rangle \end{array}\right) = \frac{\Delta m^2}{2E}\left(\begin{array}{cc} \sin^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \cos^2\theta \end{array}\right)\left(\begin{array}{c} |\nu_e\rangle \\ |\nu_\mu\rangle \end{array}\right),$$

(again, up to additional terms proportional to the 2×2 identity matrix).

Fermi Lagrangian, after a Fiertz rearrangement of the charged-current terms:

$$\mathrm{L} \supset \bar{\nu}_{eL} i\partial_\mu \gamma^\mu \nu_{eL} - 2\sqrt{2}G_F \left(\bar{\nu}_{eL}\gamma^\mu \nu_{eL}\right)\left(\bar{e}_L \gamma_\mu e_L\right) + \ldots$$

Equation of motion for one electron neutrino state in the presence of a non-relativistic electron background, in the rest frame of the electrons:

$$\langle \bar{e}_L \gamma_\mu e_L \rangle = \delta_{\mu 0} \frac{N_e}{2}$$

where $N_e \equiv e^\dagger e$ is the average electron number density (at rest, hence $\delta_{\mu 0}$ term). Factor of 1/2 from the "left-handed" half.

Dirac equation for a one neutrino state inside a cold electron "gas" is (ignore mass)

$$(i\partial^\mu \gamma_\mu - \sqrt{2}G_F N_e \gamma_0)|\nu_e\rangle = 0.$$

In the ultrarelativistic limit, (plus $\sqrt{2}G_F N_e \ll E$), dispersion relation is

$$E \simeq |\vec{p}| \pm \sqrt{2}G_F N_e, \qquad + \text{ for } \nu, \quad - \text{ for } \bar{\nu}$$

$$i\frac{\mathrm{d}}{\mathrm{d}L}\left(\begin{array}{c}|\nu_e\rangle\\|\nu_\mu\rangle\end{array}\right)=\left[\frac{\Delta m^2}{2E}\left(\begin{array}{cc}\sin^2\theta & \cos\theta\sin\theta\\ \cos\theta\sin\theta & \cos^2\theta\end{array}\right)+\left(\begin{array}{cc}A & 0\\ 0 & 0\end{array}\right)\right]\left(\begin{array}{c}|\nu_e\rangle\\|\nu_\mu\rangle\end{array}\right),$$

$A=\pm\sqrt{2}G_FN_e$ (+ for neutrinos, − for antineutrinos).

Note: Similar effect from neutral current interactions common to all (active) neutrino species → proportional to the identity.

In general, this is hard to solve, as A is a function of L: two-level non-relativistc quantum mechanical system in the presence of time dependent potential.

In some cases, however, the solution is rather simple.

Constant A: good approximation for neutrinos propagating through matter inside the Earth [exception: neutrinos that see Earth's internal structure (the crust, the mantle, the outer core, the inner core)]

$$i\frac{\mathrm{d}}{\mathrm{d}L}\left(\begin{array}{c}|\nu_e\rangle \\ |\nu_\mu\rangle\end{array}\right) = \left(\begin{array}{cc} A & \Delta/2\sin 2\theta \\ \Delta/2\sin 2\theta & \Delta\cos 2\theta\end{array}\right)\left(\begin{array}{c}|\nu_e\rangle \\ |\nu_\mu\rangle\end{array}\right), \quad \Delta \equiv \Delta m^2/2E.$$

$$P_{e\mu} = \sin^2 2\theta_M \sin^2\left(\frac{\Delta_M L}{2}\right),$$

where

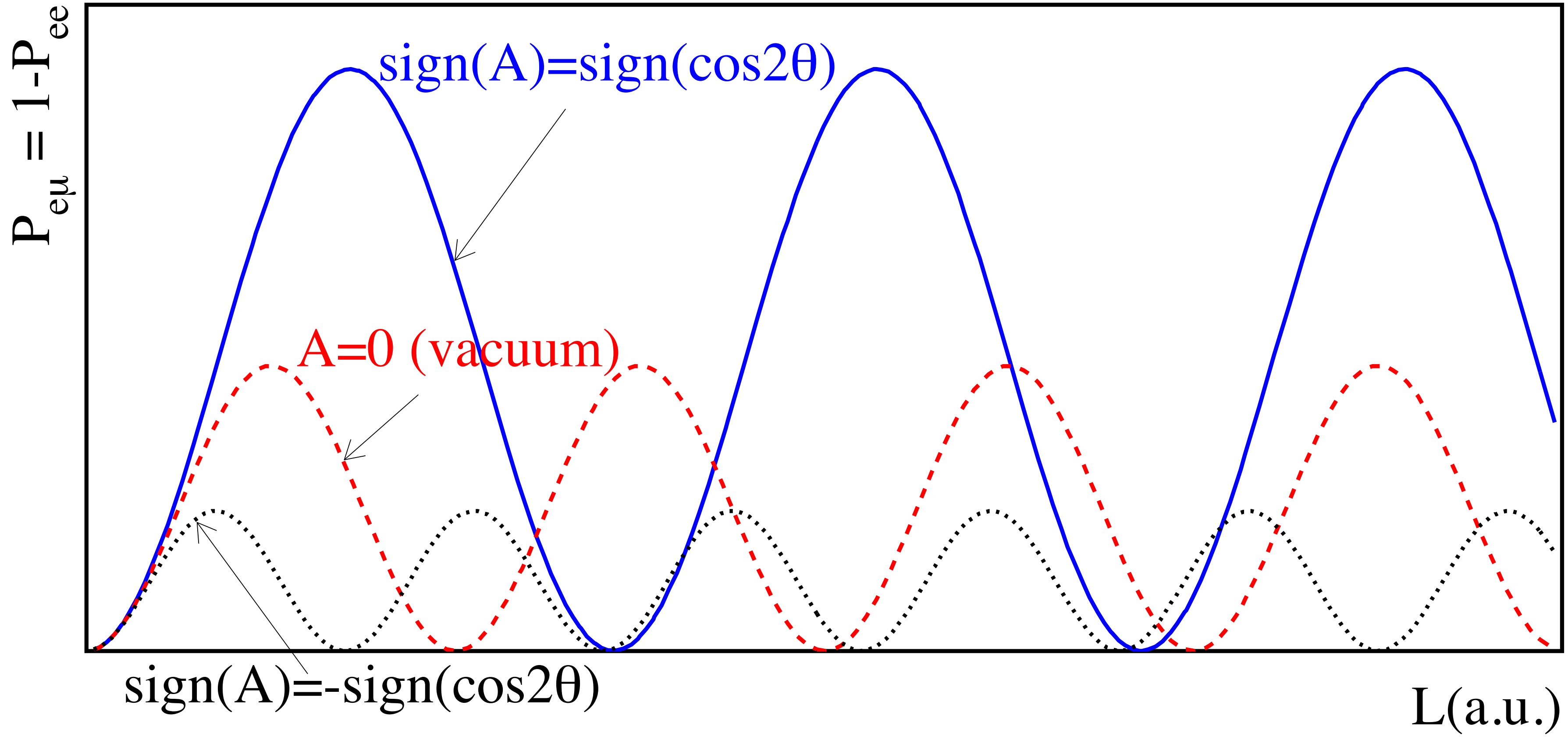
$$\begin{aligned}
\Delta_M &= \sqrt{(A-\Delta\cos 2\theta)^2 + \Delta^2\sin^2 2\theta}, \\
\Delta_M \sin 2\theta_M &= \Delta\sin 2\theta, \\
\Delta_M \cos 2\theta_M &= A - \Delta\cos 2\theta.
\end{aligned}$$

The presence of matter affects neutrino and antineutrino oscillation differently.

Nothing wrong with this: CPT-theorem relates the propagation of neutrinos in an electron background to the propagation of antineutrinos in a positron background.

Enlarged parameter space in the presence of matter effects.

For example, can tell whether $\cos 2\theta$ is positive or negative.



The MSW Effect

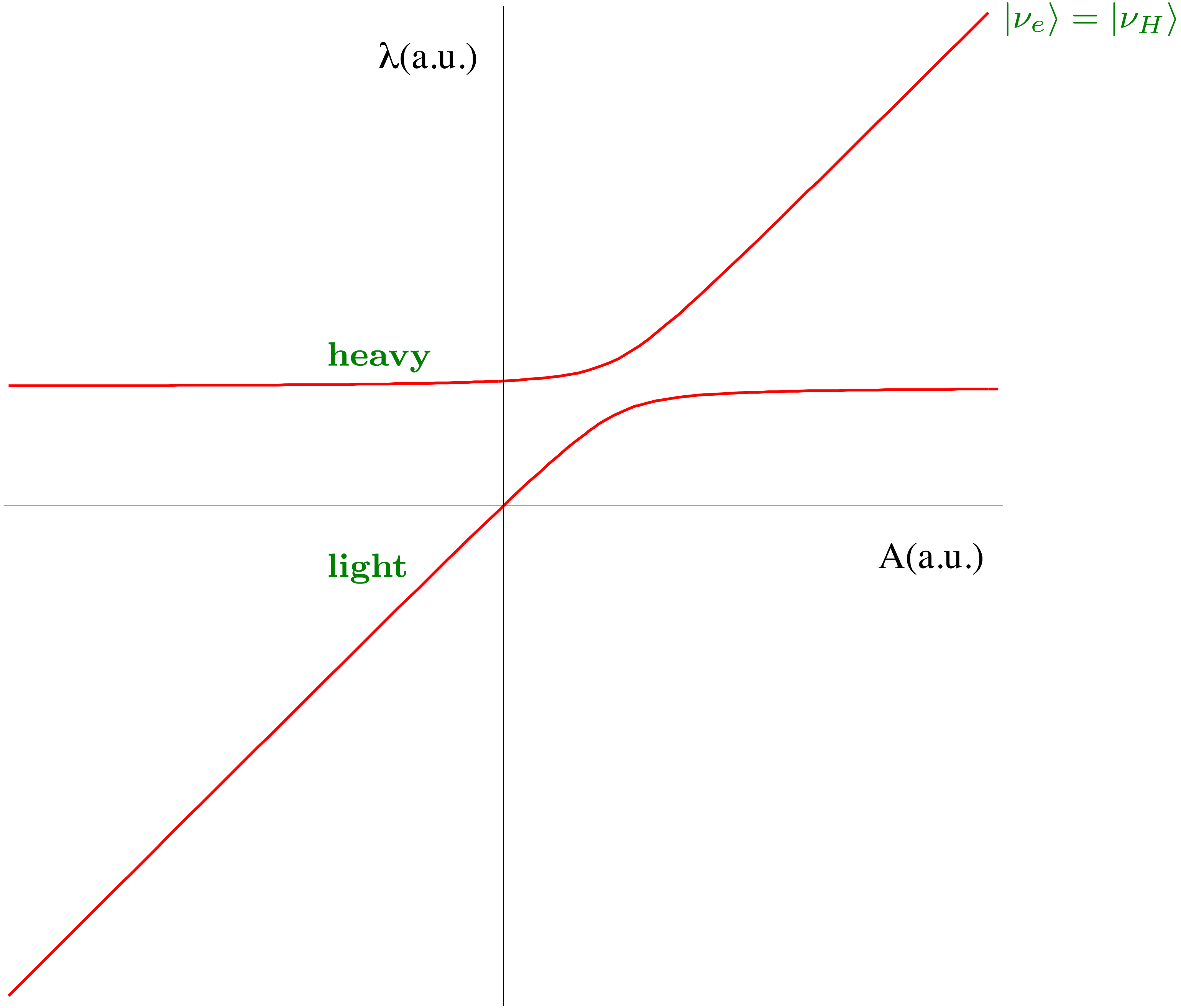
Curiously enough, the oldest neutrino puzzle is the one that is most subtle to explain. This is because solar neutrinos traverse a strongly varying matter density on their way from the center of the Sun to the surface of the Earth.

For the Hamiltonian

$$\left[\Delta\begin{pmatrix}\sin^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \cos^2\theta\end{pmatrix} + A\begin{pmatrix}1 & 0 \\ 0 & 0\end{pmatrix}\right],$$

it is easy to compute the eigenvalues as a function of A:

(remember, $\Delta = \Delta m^2/2E$)



$|\nu_e\rangle = |\nu_H\rangle$
λ(a.u.)
heavy
light
A(a.u.)

A decreases "slowly" as a function of L $\Rightarrow$ system evolves adiabatically.

$$|\nu_e\rangle = |\nu_{2M}\rangle \text{ at the core } \rightarrow |\nu_2\rangle \text{ in vacuum,}$$
$$P_{ee}^{\rm Earth} = |\langle\nu_e|\nu_2\rangle|^2 = \sin^2\theta.$$

Note that $P_{ee} \simeq \sin^2\theta$ applies in a wide range of energies and baselines, as long as the approximations mentioned above apply —ideal to explain the energy independent suppression of the ^{8}B solar neutrino flux!

Furthermore, large average suppressions of the neutrino flux are allowed if $\sin^2\theta \ll 1$. Compare with $\bar{P}_{ee}^{\rm vac} = 1 - 1/2\sin^2 2\theta > 1/2$.

One can expand on the result above by loosening some of the assumptions. $|\nu_e\rangle$ state is produced in the Sun's core as an *incoherent* mixture of $|\nu_{1M}\rangle$ and $|\nu_{2M}\rangle$. Introduce adiabaticity parameter P_c, which measures the probability that a $|\nu_{iM}\rangle$ matter Hamiltonian state will *not* exit the Sun as a $|\nu_i\rangle$ mass-eigenstate.

$$
\begin{aligned}
|\nu_e\rangle \quad &\rightarrow \quad |\nu_{1M}\rangle, \text{ with probability } \cos^2\theta_M, \\
&\rightarrow \quad |\nu_{2M}\rangle, \text{ with probability } \sin^2\theta_M,
\end{aligned}
$$

where θ_M is the matter angle at the neutrino **production point**.

$$
\begin{aligned}
|\nu_{1M}\rangle \quad &\rightarrow \quad |\nu_1\rangle, \text{ with probability } (1-P_c), \\
&\rightarrow \quad |\nu_2\rangle, \text{ with probability } P_c, \\
|\nu_{2M}\rangle \quad &\rightarrow \quad |\nu_1\rangle \text{ with probability } P_c, \\
&\rightarrow \quad |\nu_2\rangle \text{ with probability } (1-P_c).
\end{aligned}
$$

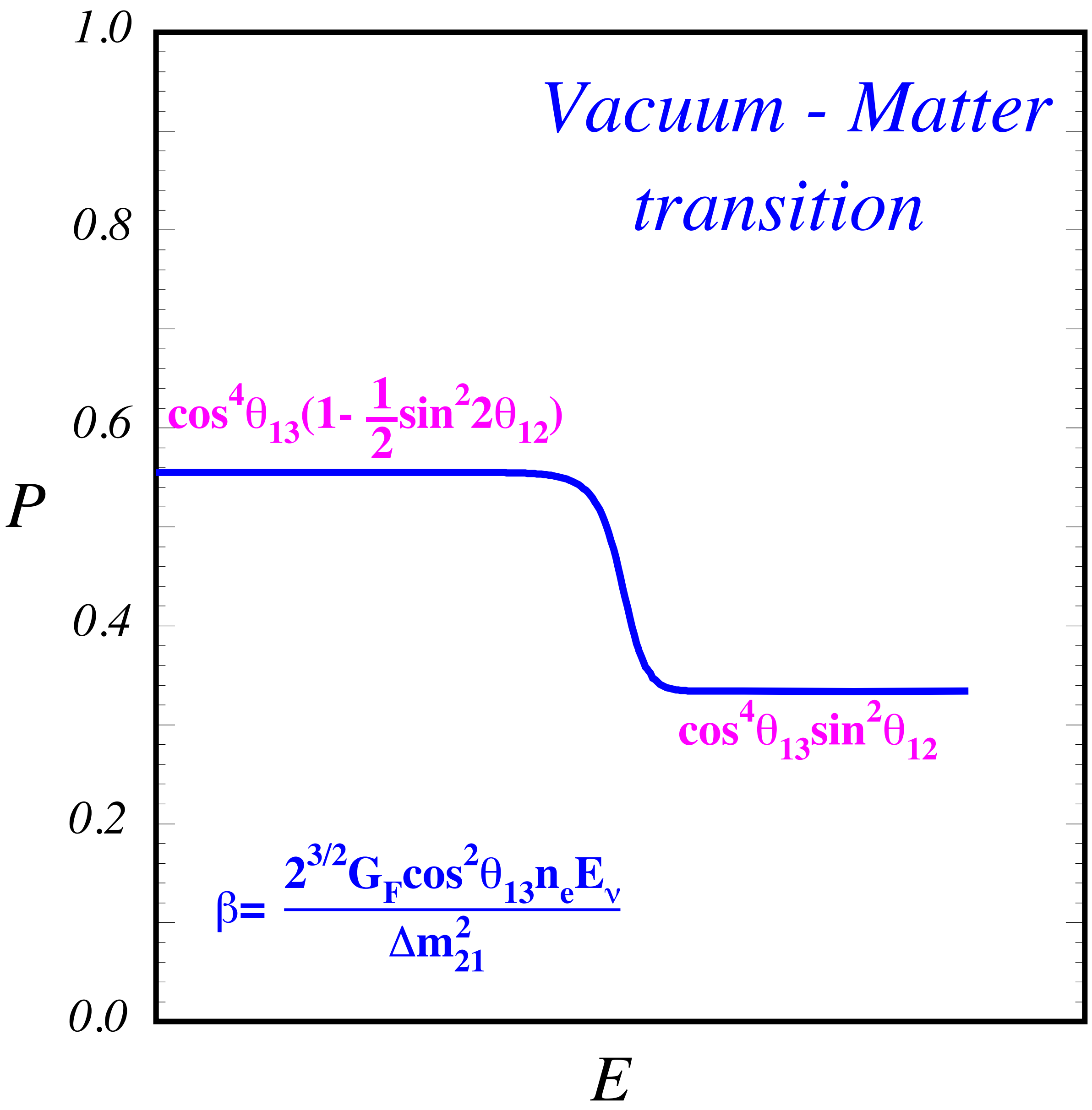
$P_{1e} = \cos^2\theta$ and $P_{2e} = \sin^2\theta$ so

$$
\begin{aligned}
P_{ee}^{\rm Sun} = \quad & \cos^2\theta_M\left[(1-P_c)\cos^2\theta + P_c\sin^2\theta\right] \\
& + \sin^2\theta_M\left[P_c\cos^2\theta + (1-P_c)\sin^2\theta\right].
\end{aligned}
$$

For $N_e = N_{e0}e^{-L/r_0}$, P_c, (crossing probability), is exactly calculable

$$
P_c = \frac{e^{-\gamma\sin^2\theta} - e^{-\gamma}}{1-e^{-\gamma}}, \quad \gamma = 2\pi r_0\Delta. \tag{1}
$$

Adiabatic condition: $\gamma \gg 1$, when $P_c \rightarrow 0$.



We need:

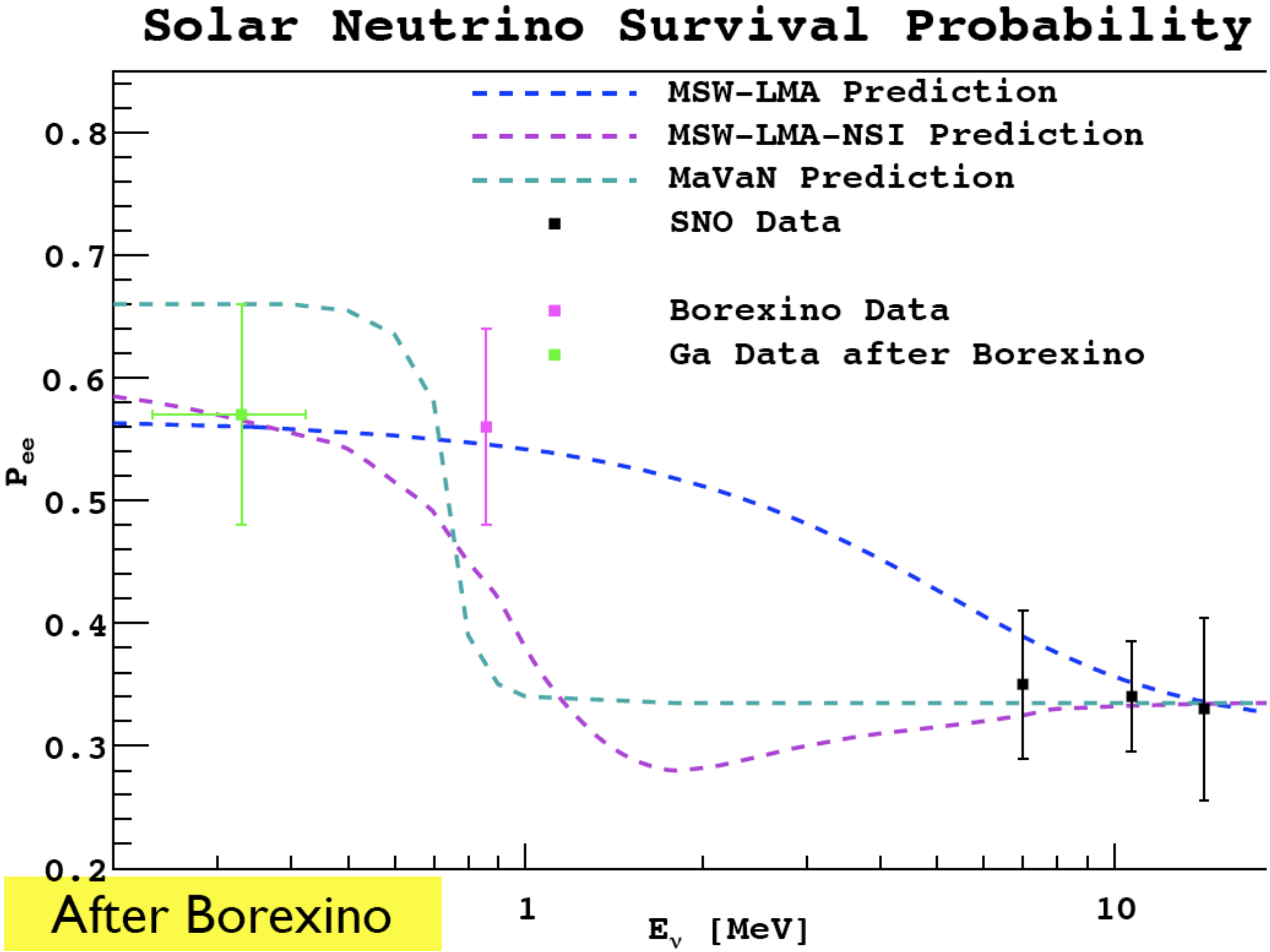
- $P_{ee} \sim 0.3$ (^{8}B neutrinos)
- $P_{ee} \sim 0.6$ (^{7}Be, pp neutrinos)

$\Rightarrow \sin^2\theta \sim 0.3$

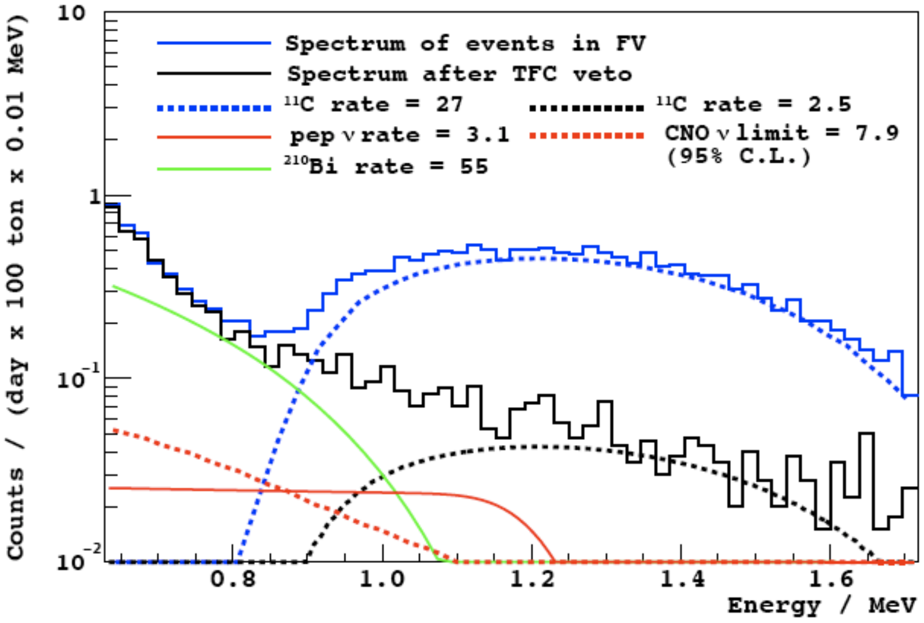
$\Rightarrow \Delta m^2 \sim 10^{-(5\text{ to }4)}\ \mathrm{eV}^2$

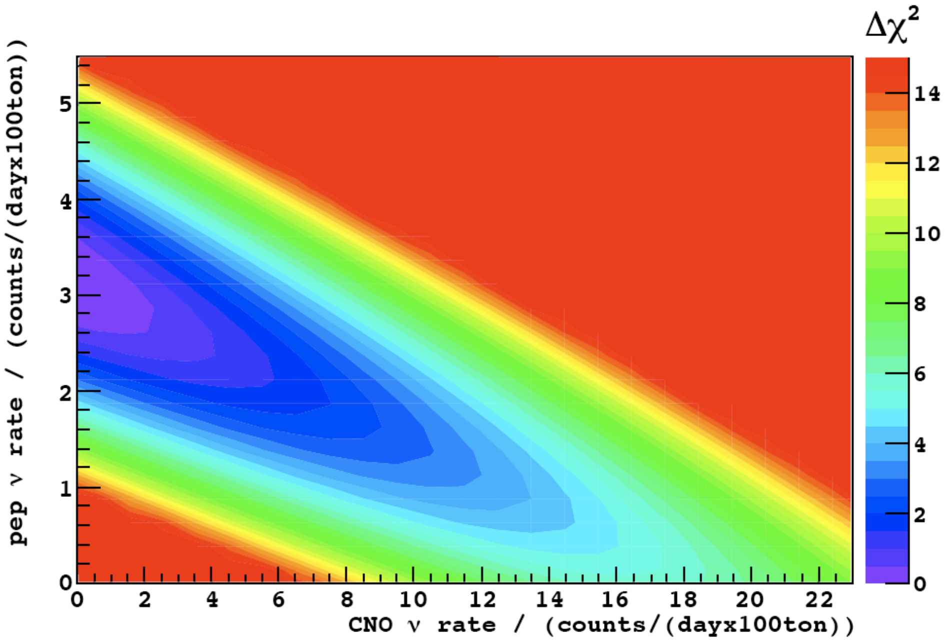
for a long time, there were many other options!

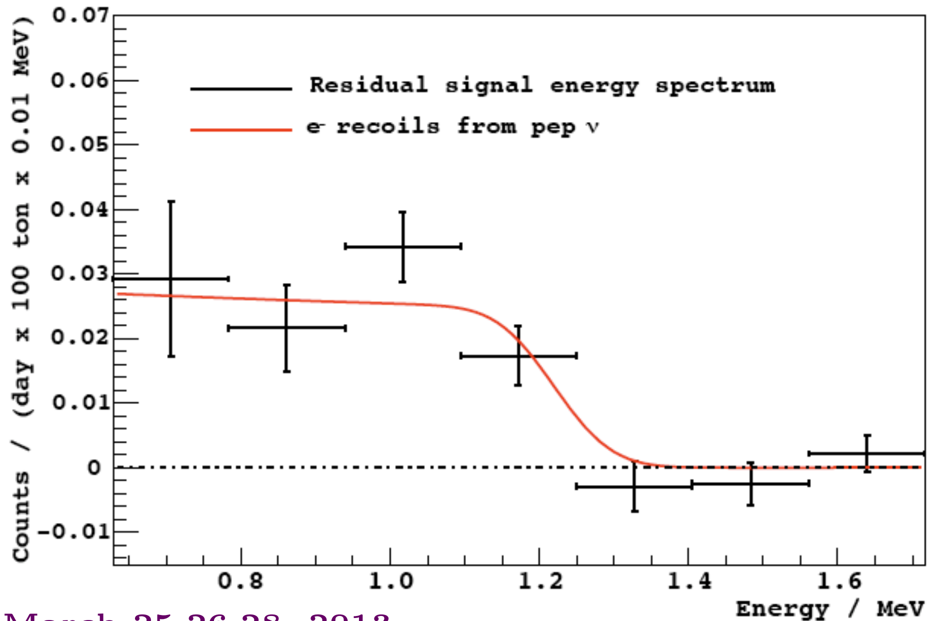
(LMA, LOW, SMA, VAC)

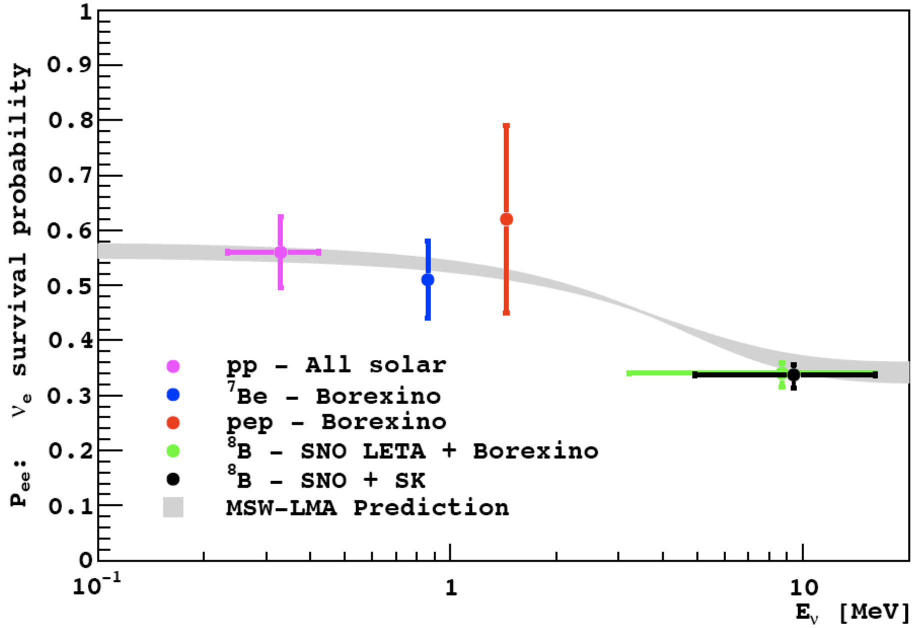

Solar Neutrino Survival Probability
MSW-LMA Prediction
MSW-LMA-NSI Prediction
MaVaN Prediction
SNO Data
Borexino Data
Ga Data after Borexino
P_{ee}
0.8
0.7
0.6
0.5
0.4
0.3
0.2
1
10
E_ν [MeV]
After Borexino

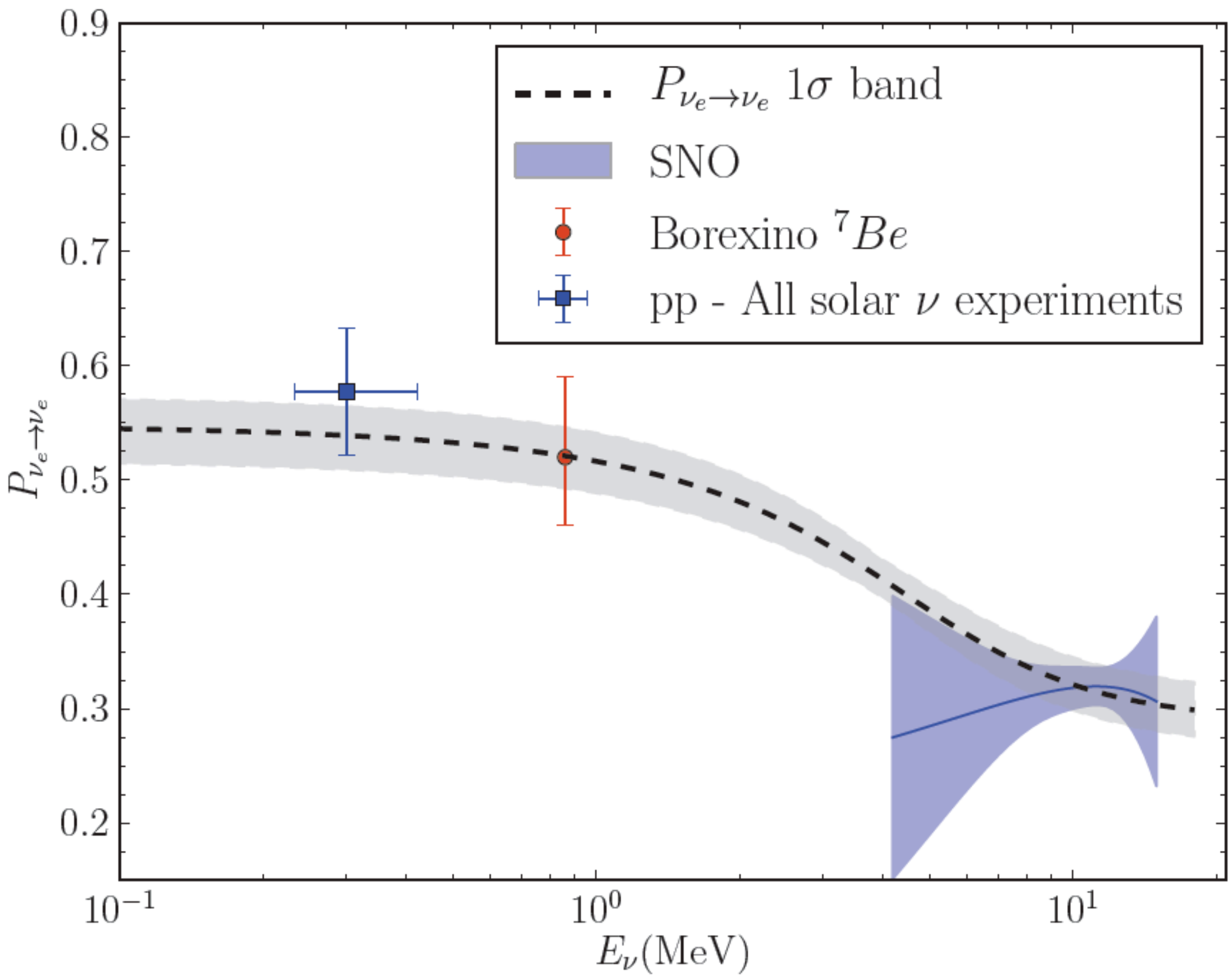
Borexino, 1110.3230

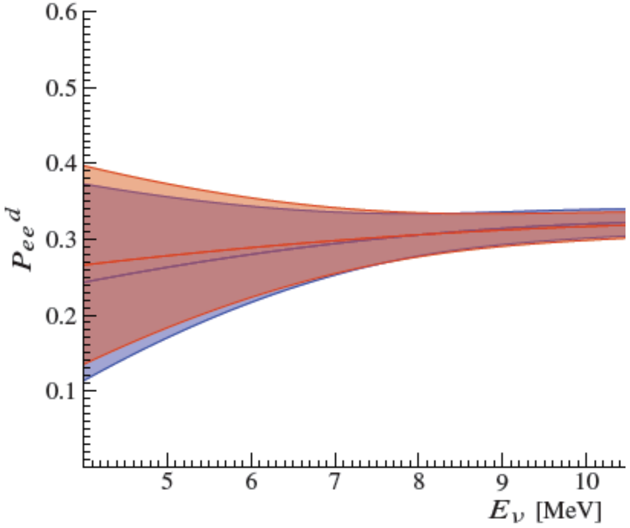


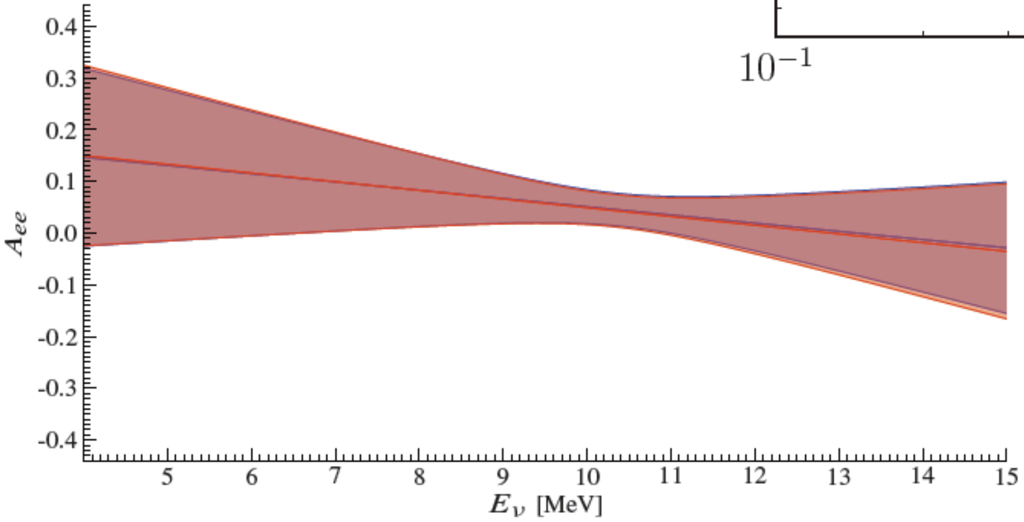










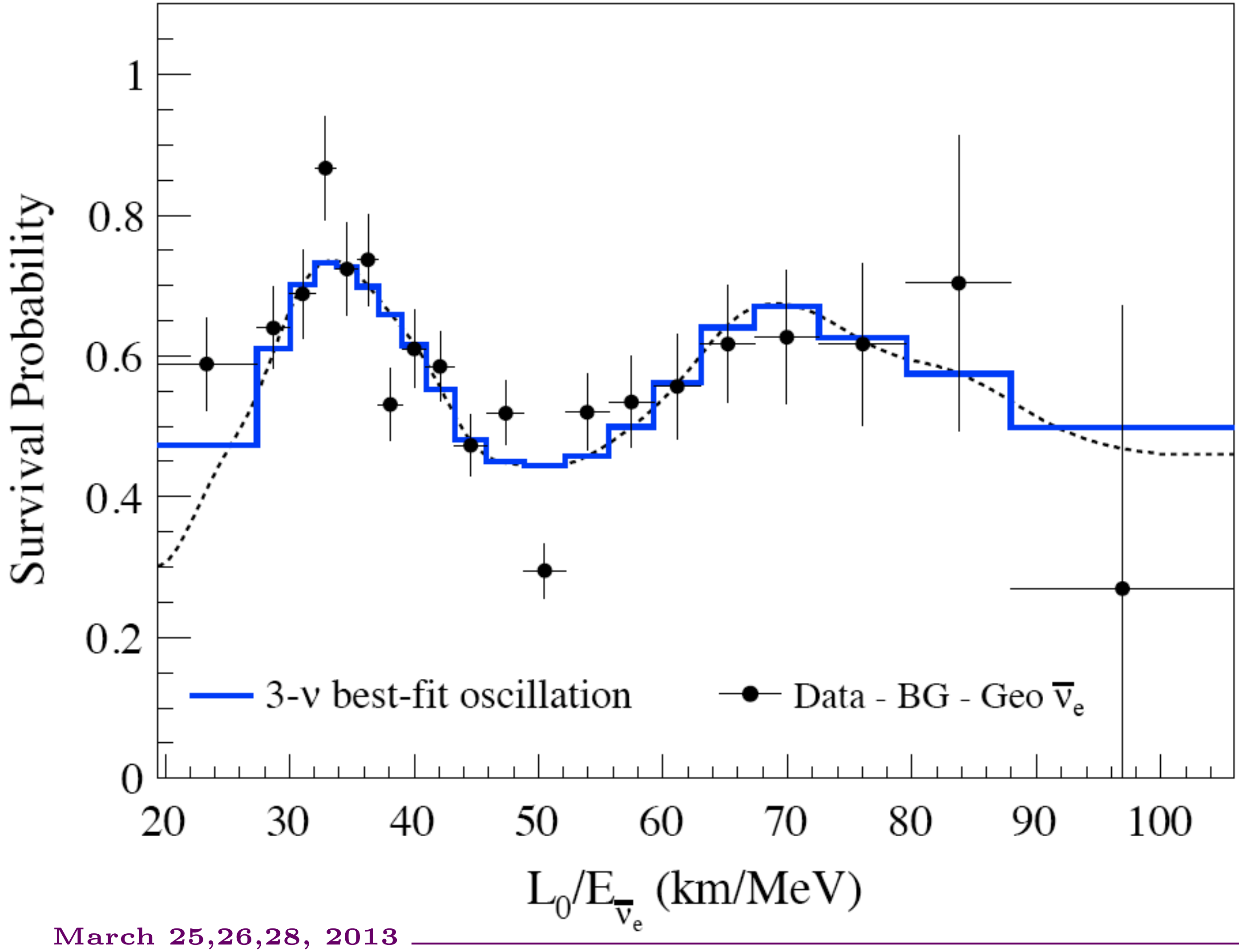


"Final" SNO results, 1109.0763

Solar oscillations confirmed by Reactor experiment: KamLAND

[arXiv:1303.4667]

$$\text{phase} = 1.27\left(\frac{\Delta m^2}{5\times10^{-5}\ \text{eV}^2}\right)\left(\frac{5\ \text{MeV}}{E}\right)\left(\frac{L}{100\ \text{km}}\right)$$

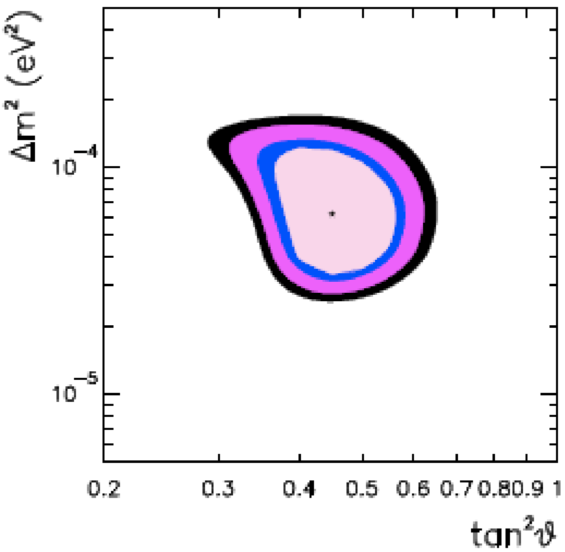


$$P_{ee} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

oscillatory behavior!

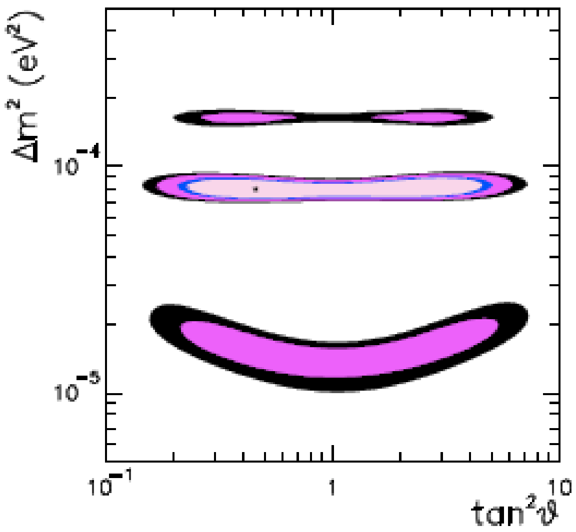
Solar

$\nu_e \to \nu_{\rm active}$

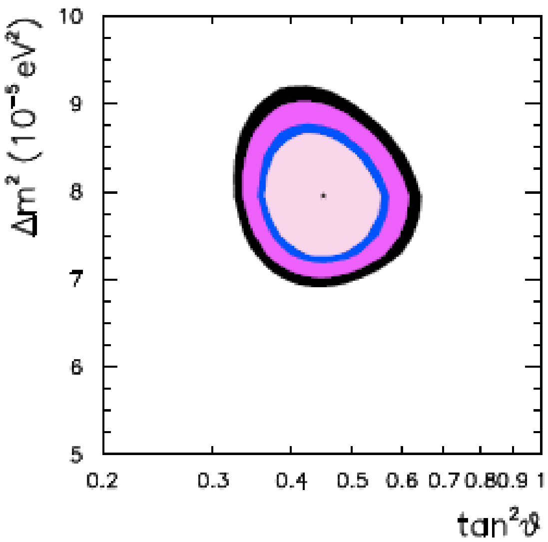


+ KamLAND

$\bar{\nu}_e \not\to \bar{\nu}_e$



ν_e oscillation parameters compatible with $\overline{\nu}_e$: *Sensible* to assume CPT: $P_{ee} = P_{\bar{e}\bar{e}}$

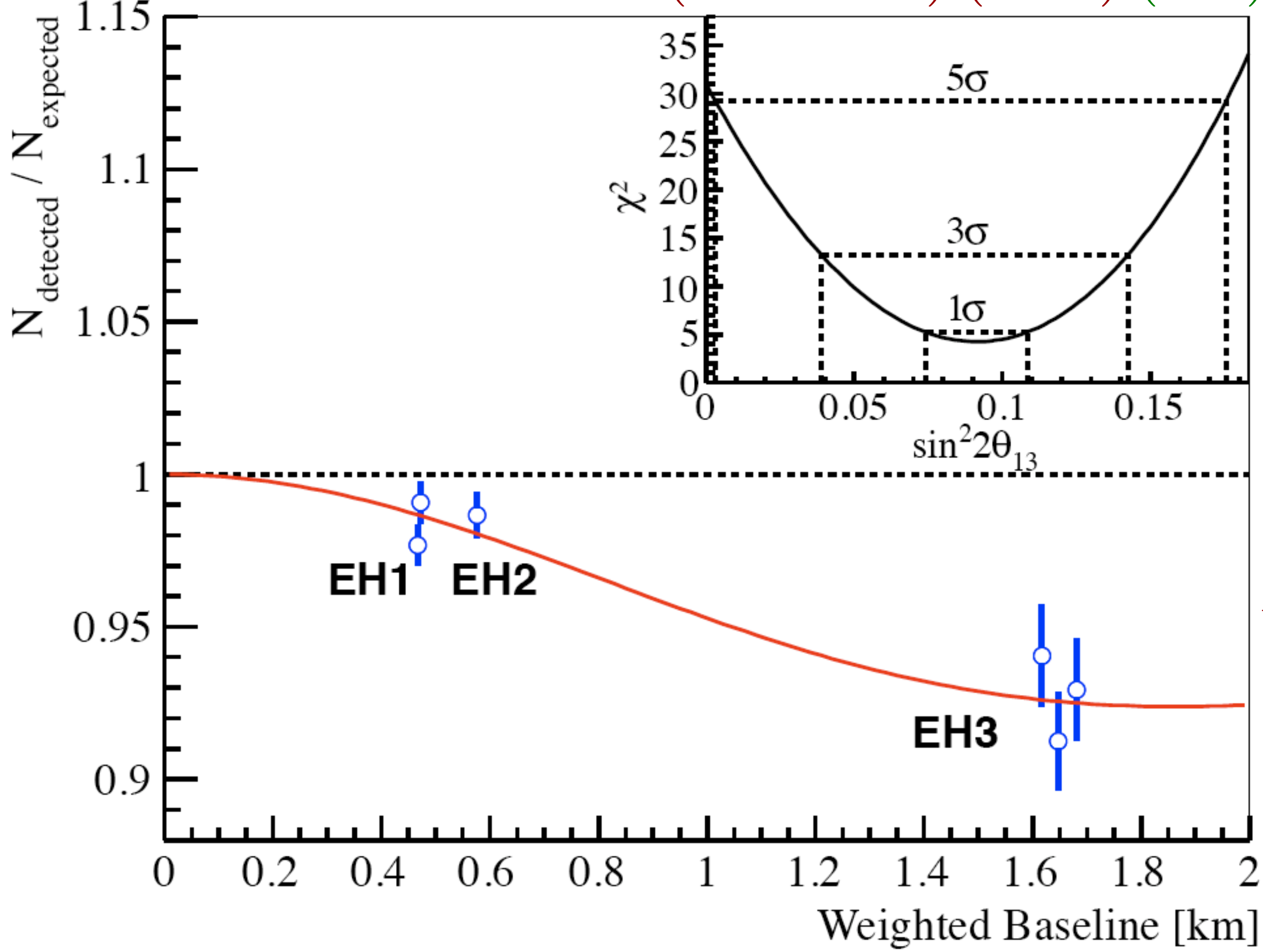


$$\Delta m^2_\odot = (8^{+0.4}_{-0.5}) \times 10^{-5}\ \text{eV}^2 \quad (1\sigma)$$
$$\tan^2\theta_\odot = 0.45^{+0.05}_{-0.05}$$

[Gonzalez-Garcia, PASI 2006]

Atmospheric Oscillations in the Electron Sector: Daya Bay, RENO, Double Chooz

$$\text{phase} = 0.64\left(\frac{\Delta m^2}{2.5\times10^{-3}\ \text{eV}^2}\right)\left(\frac{5\ \text{MeV}}{E}\right)\left(\frac{L}{1\ \text{km}}\right)$$



$$P_{ee} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Summarizing:

Both the solar and atmospheric puzzles can be properly explained in terms of two-flavor neutrino oscillations:

- **solar**: $\nu_e \leftrightarrow \nu_a$ (linear combination of ν_μ and ν_τ): $\Delta m^2 \sim 10^{-4}$ eV^2, $\sin^2\theta \sim 0.3$.
- **atmospheric**: $\nu_\mu \leftrightarrow \nu_\tau$: $\Delta m^2 \sim 10^{-3}$ eV^2, $\sin^2\theta \sim 0.5$ ("maximal mixing").
- **short-baseline reactors:** $\nu_e \leftrightarrow \nu_a$ (linear combination of ν_μ and ν_τ): $\Delta m^2 \sim 10^{-3}$ eV^2, $\sin^2\theta \sim 0.02$.

Putting it all together – 3 flavor mixing:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{e\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Definition of neutrino mass eigenstates (who are ν_1, ν_2, ν_3?):

- $m_1^2 < m_2^2$ $\Delta m_{13}^2 < 0$ – Inverted Mass Hierarchy
- $m_2^2 - m_1^2 \ll |m_3^2 - m_{1,2}^2|$ $\Delta m_{13}^2 > 0$ – Normal Mass Hierarchy

$$\tan^2\theta_{12} \equiv \frac{|U_{e2}|^2}{|U_{e1}|^2}; \quad \tan^2\theta_{23} \equiv \frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2}; \quad U_{e3} \equiv \sin\theta_{13} e^{-i\delta}$$

[For a detailed discussion see AdG, Jenkins, PRD78, 053003 (2008)]

It Turns Out That ...

- Two Mass-Squared Differences Are Hierarchical, $\Delta m^2_{12} \ll |\Delta m^2_{13}|$;
- One of the Mixing Angles Is Small, $\sin^2\theta_{13} \sim 0.02$.

$\Rightarrow$ Two Puzzles Decouple, and Two-Flavor Interpretation Captures Almost All the Physics:

- Atmospheric Neutrinos Determine $|\Delta m^2_{13}|$ and θ_{23};
- Solar Neutrinos Determine Δm^2_{12} and θ_{12}.

(small θ_{13} guarantees that $|\Delta m^2_{13}|$ effects governing electron neutrinos are small, while $\Delta m^2_{12} \ll |\Delta m^2_{13}|$ guarantees that Δm^2_{12} effects are small at atmospheric and accelerator experiments).

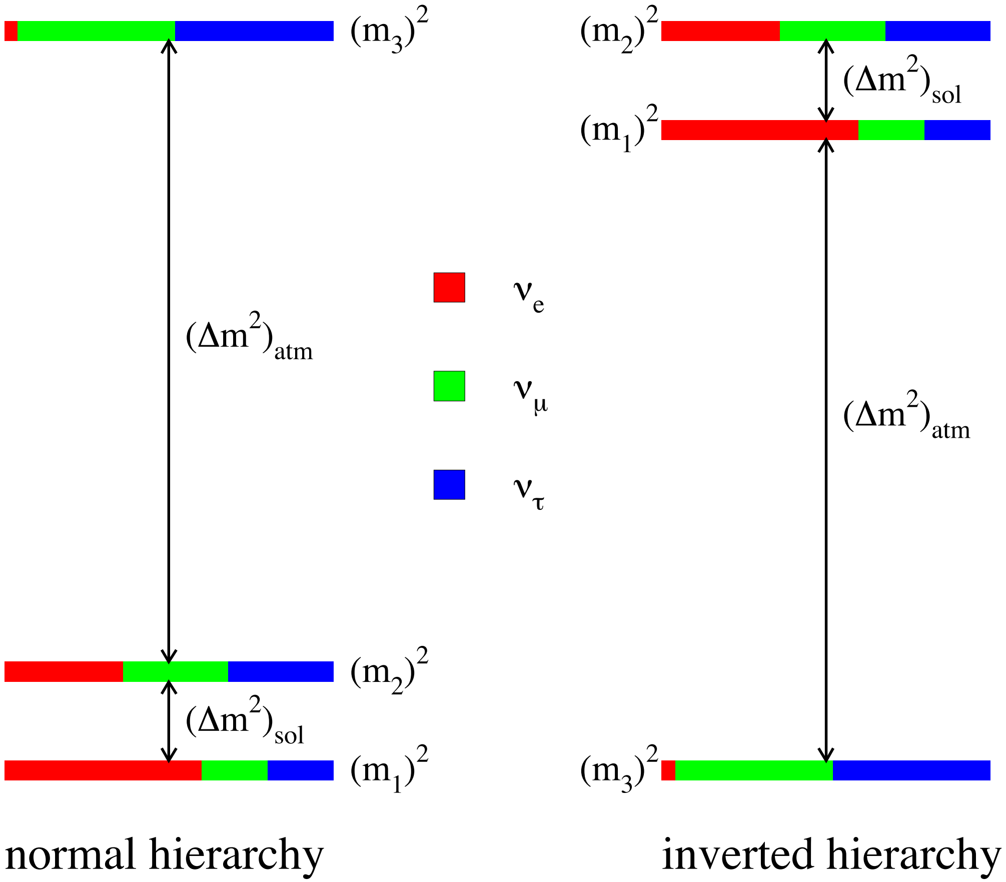
Three Flavor Mixing Hypothesis Fits All* Data Really Well.

parameter	best fit $\pm 1\sigma$	2σ	3σ
Δm^2_{21} $[10^{-5}\text{eV}^2]$	7.62 ± 0.19	7.27–8.01	7.12–8.20
Δm^2_{31} $[10^{-3}\text{eV}^2]$	$2.53^{+0.08}_{-0.10}$ $-(2.40^{+0.10}_{-0.07})$	$2.34 - 2.69$ $-(2.25 - 2.59)$	$2.26 - 2.77$ $-(2.15 - 2.68)$
$\sin^2\theta_{12}$	$0.320^{+0.015}_{-0.017}$	0.29–0.35	0.27–0.37
$\sin^2\theta_{23}$	$0.49^{+0.08}_{-0.05}$ $0.53^{+0.05}_{-0.07}$	0.41–0.62 0.42–0.62	0.39–0.64
$\sin^2\theta_{13}$	$0.026^{+0.003}_{-0.004}$ $0.027^{+0.003}_{-0.004}$	0.019–0.033 0.020–0.034	0.015–0.036 0.016–0.037
δ	$(0.83^{+0.54}_{-0.64})\,\pi$ 0.07π [a]	$0 - 2\pi$	$0 - 2\pi$

* Modulo short-baseline anomalies.

[Forero, Tórtola, Valle, 1205.4018]

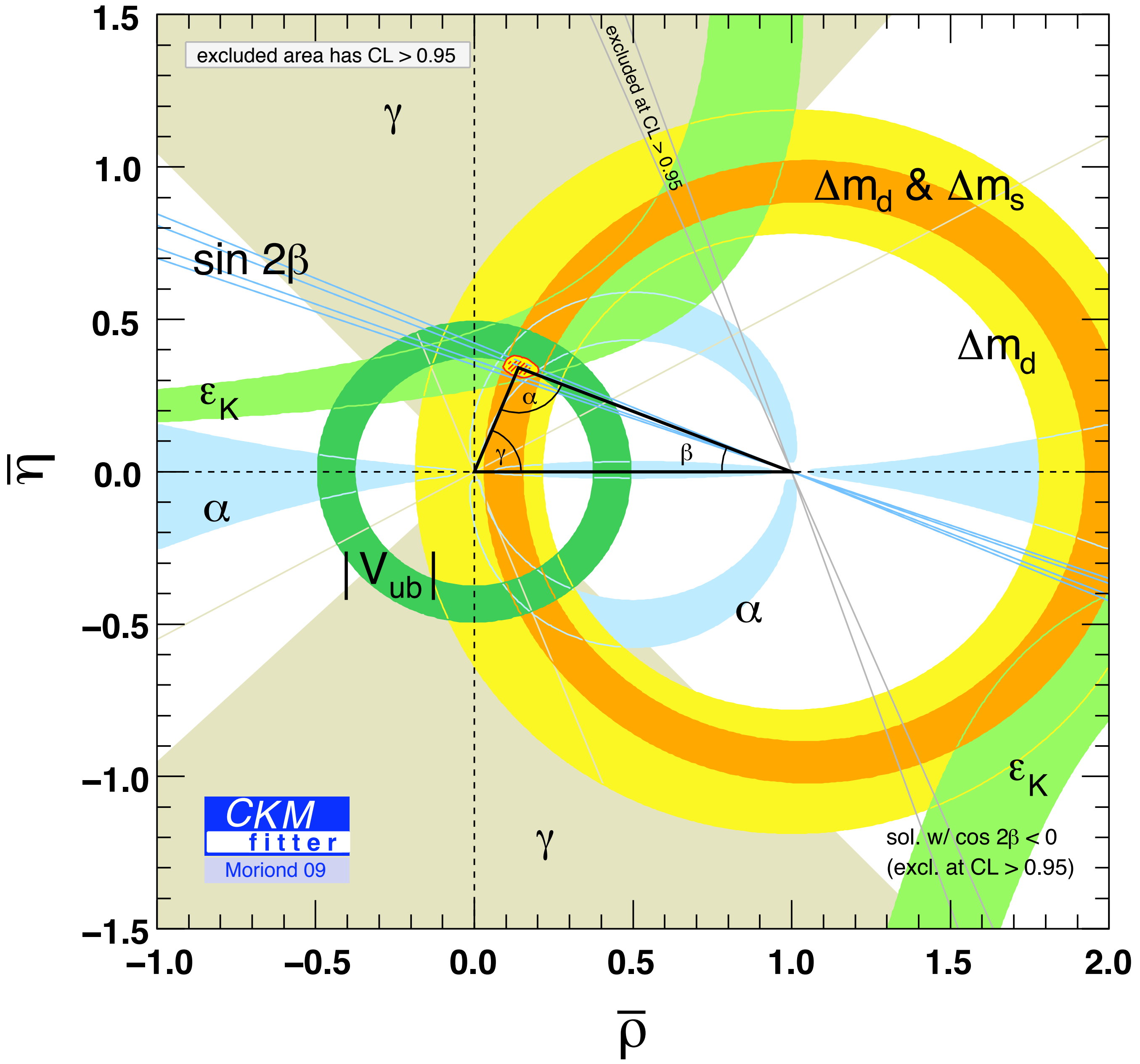
4– What We Know We Don't Know: Missing Oscillation Parameters



- ~~What is the ν_e component of ν_3?~~ ($\theta_{13} \neq 0$!)
- Is CP-invariance violated in neutrino oscillations? ($\delta \neq 0, \pi$?)
- Is ν_3 mostly ν_μ or ν_τ? ($\theta_{23} > \pi/4$, $\theta_{23} < \pi/4$, or $\theta_{23} = \pi/4$?)
- What is the neutrino mass hierarchy? ($\Delta m^2_{13} > 0$?)

$\Rightarrow$ All of the above can "only" be addressed with new neutrino oscillation experiments

Ultimate Goal: Not Measure Parameters but Test the Formalism (Over-Constrain Parameter Space)



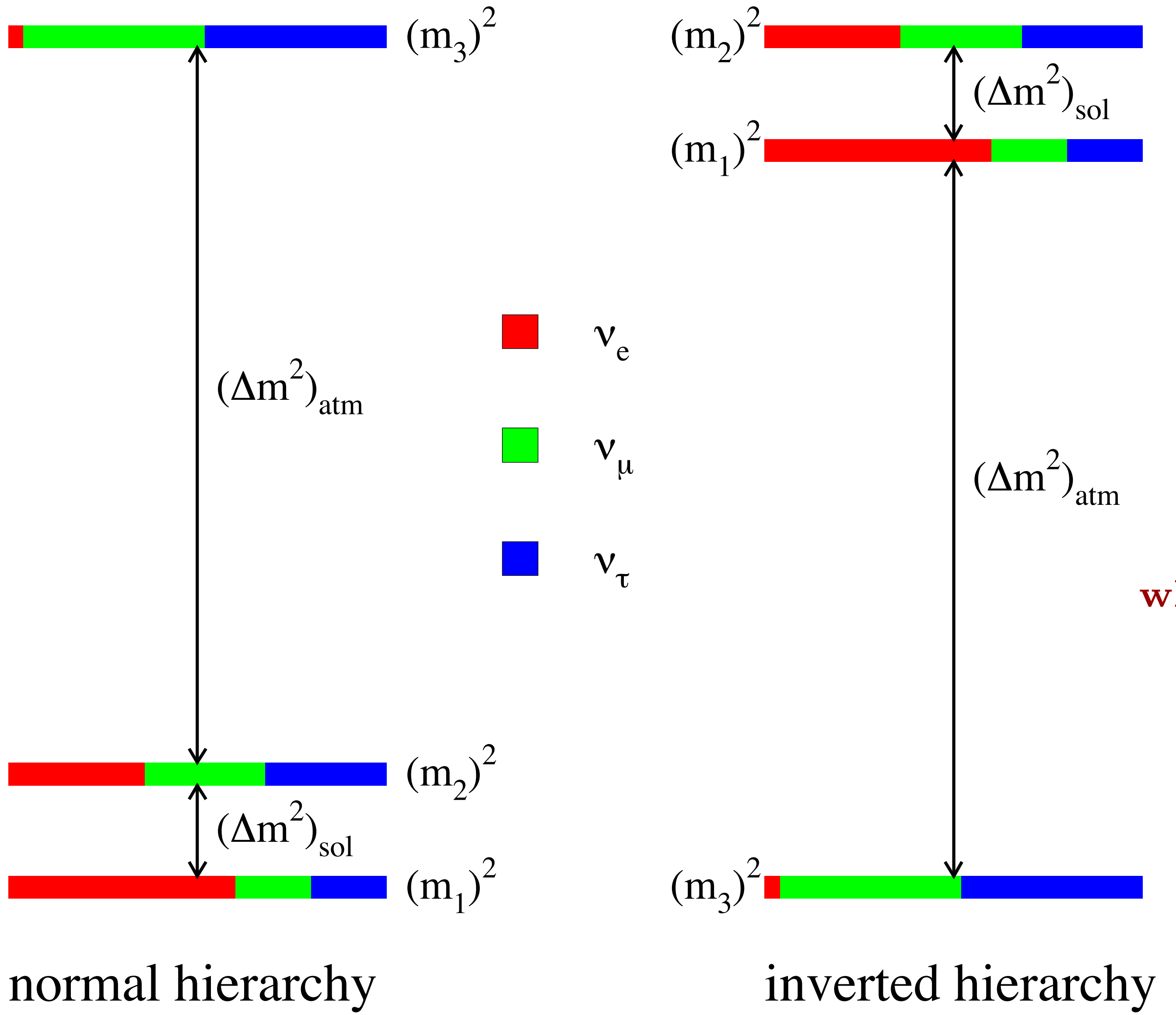
We need to do <u>this</u> in the lepton sector!

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

What we have **really measured** (very roughly):

- Two mass-squared differences, at several percent level – many probes;
- $|U_{e2}|^2$ – solar data;
- $|U_{\mu2}|^2 + |U_{\tau2}|^2$ – solar data;
- $|U_{e2}|^2|U_{e1}|^2$ – KamLAND;
- $|U_{\mu3}|^2(1-|U_{\mu3}|^2)$ – atmospheric data, K2K, MINOS;
- $|U_{e3}|^2(1-|U_{e3}|^2)$ – Double Chooz, Daya Bay, RENO;
- $|U_{e3}|^2|U_{\mu3}|^2$ (upper bound $\rightarrow$ hint) – MINOS, T2K.

We still have a ways to go!



The Neutrino Mass Hierarchy

which is the right picture?

Why Don't We Know the Neutrino Mass Hierarchy?

Most of the information we have regarding θ_{23} and Δm^2_{13} comes from atmospheric neutrino experiments (SuperK). Roughly speaking, they measure

$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m^2_{13} L}{4E}\right) + \text{ subleading.}$$

It is easy to see from the expression above that the leading term is simply not sensitive to the sign of Δm^2_{13}.

On the other hand, because $|U_{e3}|^2 < 0.05$ and $\frac{\Delta m^2_{12}}{\Delta m^2_{13}} < 0.06$ are both small, we are yet to observe the subleading effects.

Determining the Mass Hierarchy via Oscillations – the large U_{e3} route

Again, necessary to probe $\nu_\mu \to \nu_e$ oscillations (or vice-versa) governed by Δm^2_{13}. This is the oscillation channel that (almost) all next-generation, accelerator-based experiments are concentrating on, including the next generation experiments T2K and NOνA.

In vaccum

$$P_{\mu e} = \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\left(\frac{\Delta m^2_{13}L}{4E}\right) + \text{ “subleading”},$$

so that, again, this is insensitive to the sign of Δm^2_{13} at leading order. However, in this case, matter effects may come to the rescue.

As I discussed already, neutrino oscillations get modified when these propagate in the presence of matter. Matter effects are sensitive to the neutrino mass ordering (in a way that I will describe shortly) and different for neutrinos and antineutrinos.

If $\Delta_{12} \equiv \frac{\Delta m^2_{12}}{2E}$ terms are ignored, the $\nu_\mu \to \nu_e$ oscillation probability is described, in constant matter density, by

$$P_{\mu e} \simeq P_{e\mu} \simeq \sin^2\theta_{23}\sin^2 2\theta^{\rm eff}_{13}\sin^2\left(\frac{\Delta^{\rm eff}_{13}L}{2}\right),$$

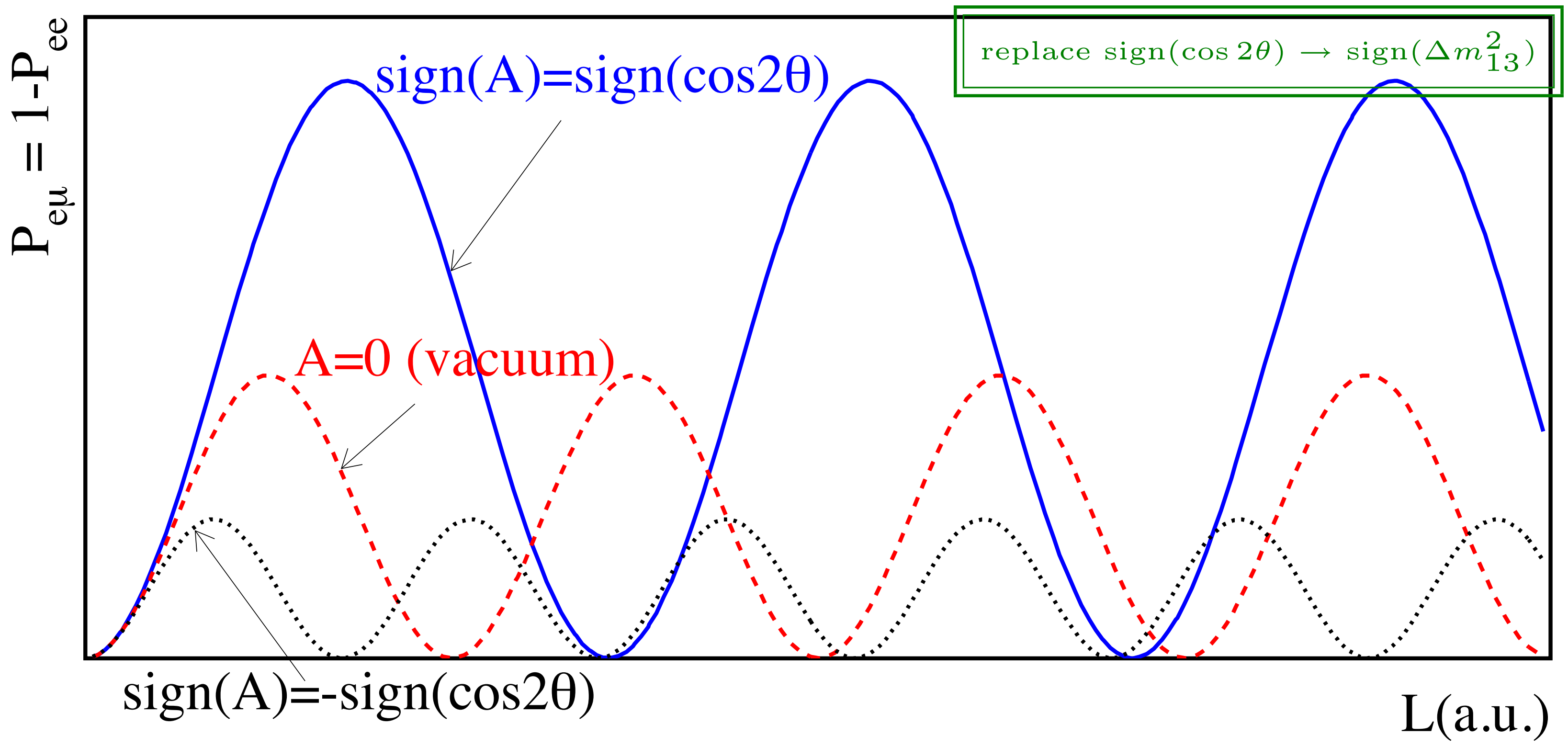
$$\sin^2 2\theta^{\rm eff}_{13} = \frac{\Delta^2_{13}\sin^2 2\theta_{13}}{(\Delta^{\rm eff}_{13})^2},$$

$$\Delta^{\rm eff}_{13} = \sqrt{(\Delta_{13}\cos 2\theta_{13} - A)^2 + \Delta^2_{13}\sin^2 2\theta_{13}},$$

$$\Delta_{13} = \frac{\Delta m^2_{13}}{2E},$$

$A \equiv \pm\sqrt{2}G_F N_e$ is the matter potential. It is positive for neutrinos and negative for antineutrinos.

$P_{\mu e}$ depends on the relative sign between Δ_{13} and A. It is different for the two different mass hierarchies, and different for neutrinos and antineutrinos.



Requirements:

- $\sin^2 2\theta_{13}$ large enough – otherwise there is nothing to see!
- $|\Delta_{13}| \sim |A|$ – matter potential must be significant but not overwhelming.
- $\Delta_{13}^{\rm eff} L$ large enough – matter effects are absent near the origin.

The "Holy Graill" of Neutrino Oscillations – CP Violation

In the old Standard Model, there is only one[a] source of CP-invariance violation:

$\Rightarrow$ The complex phase in V_{CKM}, the quark mixing matrix.

Indeed, as far as we have been able to test, all CP-invariance violating phenomena agree with the CKM paradigm:

- ϵ_K;
- ϵ'_K;
- $\sin 2\beta$;
- etc.

Recent experimental developments, however, provide strong reason to believe that this is not the case: neutrinos have mass, and leptons mix!

[a] modulo the QCD θ-parameter, which will be "willed away" henceforth.