

Aspects of QCD at the LHC

- ◆ Experiment and Theory
- ◆ QCD improved parton model
- ◆ Strong coupling constant
- ◆ Parton distribution function
- ◆ NLO, NNLO results
- ◆ Jet physics

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 - Supersymmetry
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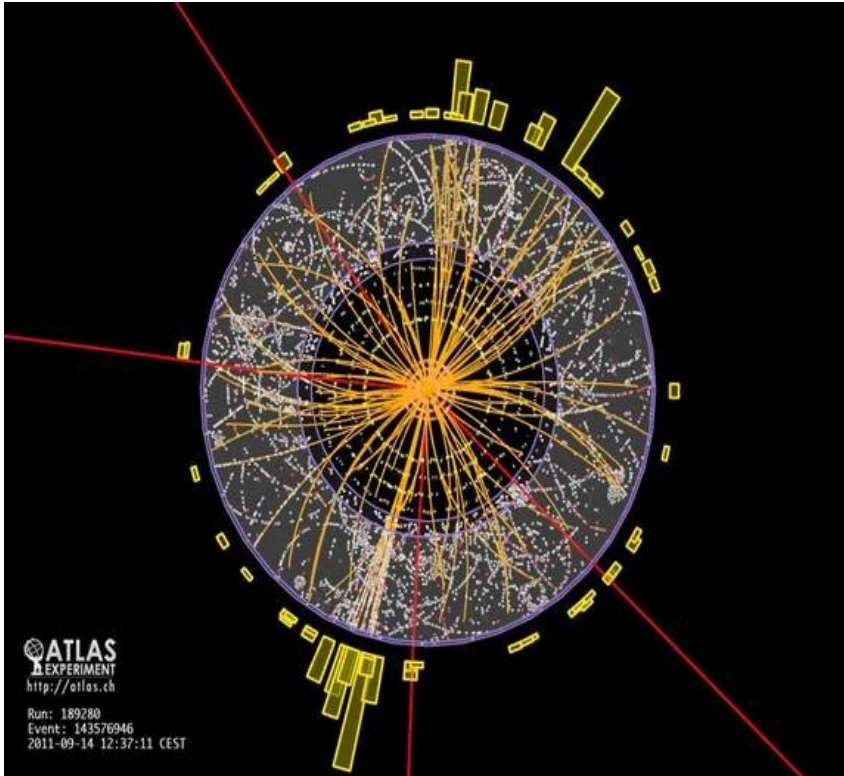
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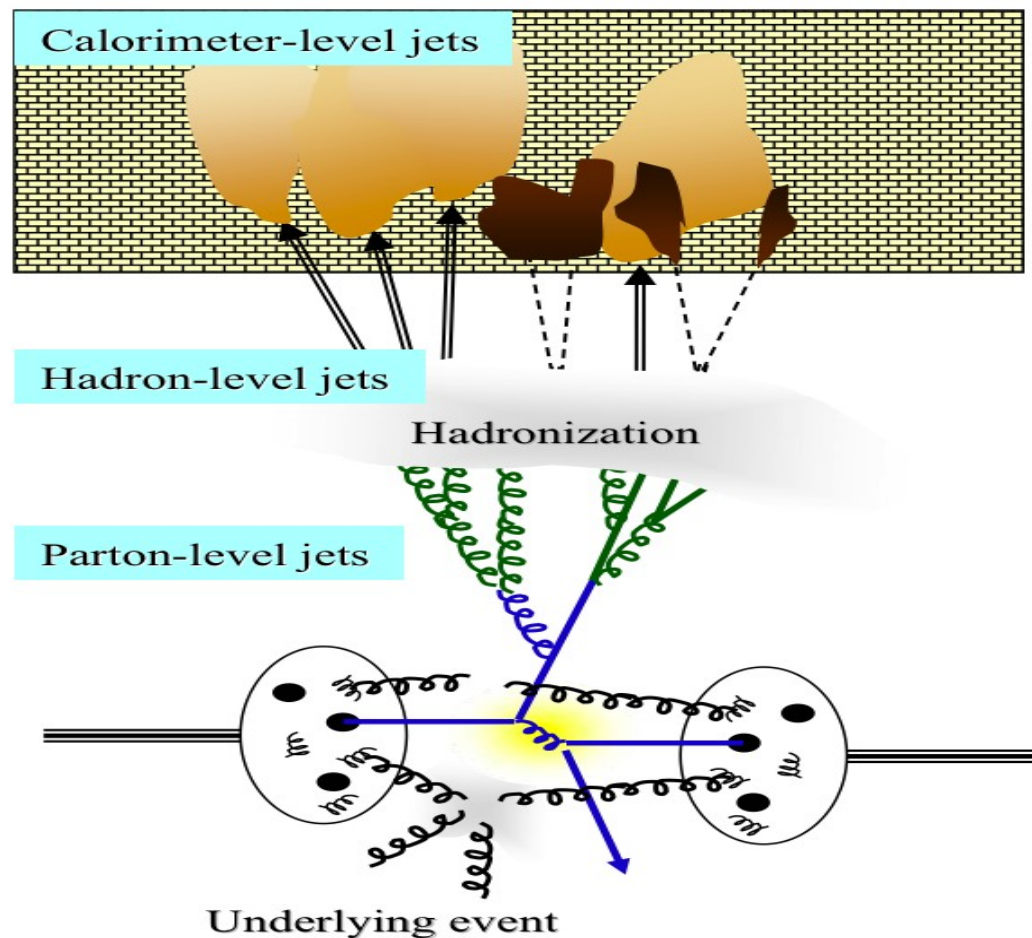
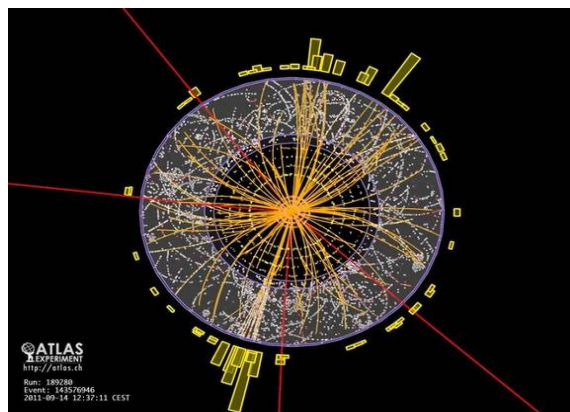
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- Theories:
 - Quantum Chromodynamics (QCD) effects
 - Electroweak (WE) effects
- Issues to be tackled:
 - Kinematics
 - Normalisation
 - Renormalisation and factorisation scale uncertainties
 - Parton Distribution Functions
 - Phase Space boundary effects and resummation of large logs

What we see in the experiment



- Large number of events of different kinds involving variety of particles at the detector level

What really happens



- Large number of events of different kinds involving variety of particles at the production and detector levels
- The underlying theory, Quantum Chromodynamics provides a physical picture.
- Exact computation of such an observable is unrealistic.

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- Parameter of QCD is strong coupling constant g_s or Λ_{QCD} .

QCD-a toolkit for discovering NEW PHYSICS at LHC

Infra-red safe observables in QCD

- We are interested processes involving hadrons
 - 1) Total cross sections: Higgs, slepton, gluino productions
 - 2) Differential cross sections: high p_T jets, rapidity of leptons
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- In perturbation theory, large logs originate from the infra-red sector of the theory:

$$\log\left(\frac{Q^2}{\Lambda^2}\right), \quad \log\left(\frac{P_T^2}{Q^2}\right), \quad \left(\frac{\log(1-x)}{(1-x)}\right)_+$$

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- We can compute them because they are "infra-red safe" due to their Factorisation properties

QCD improved Parton Model

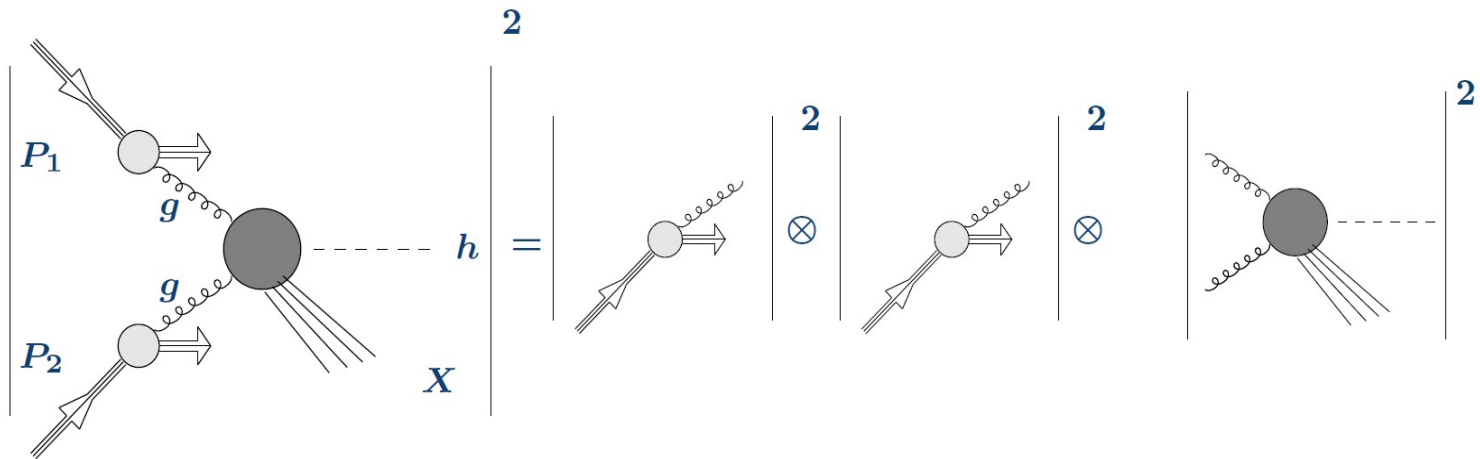
$$P_1 + P_2 \rightarrow \text{higgs} + X$$

$$d\sigma^{P_1 P_2} = \sum_{ab} \int dx_1 \int dx_2 f_{\frac{a}{P_1}}(x_1, \mu_F^2) f_{\frac{b}{P_2}}(x_2, \mu_F^2) d\hat{\sigma}^{ab}(x_1, x_2, \{p_i\}, \mu_F^2),$$

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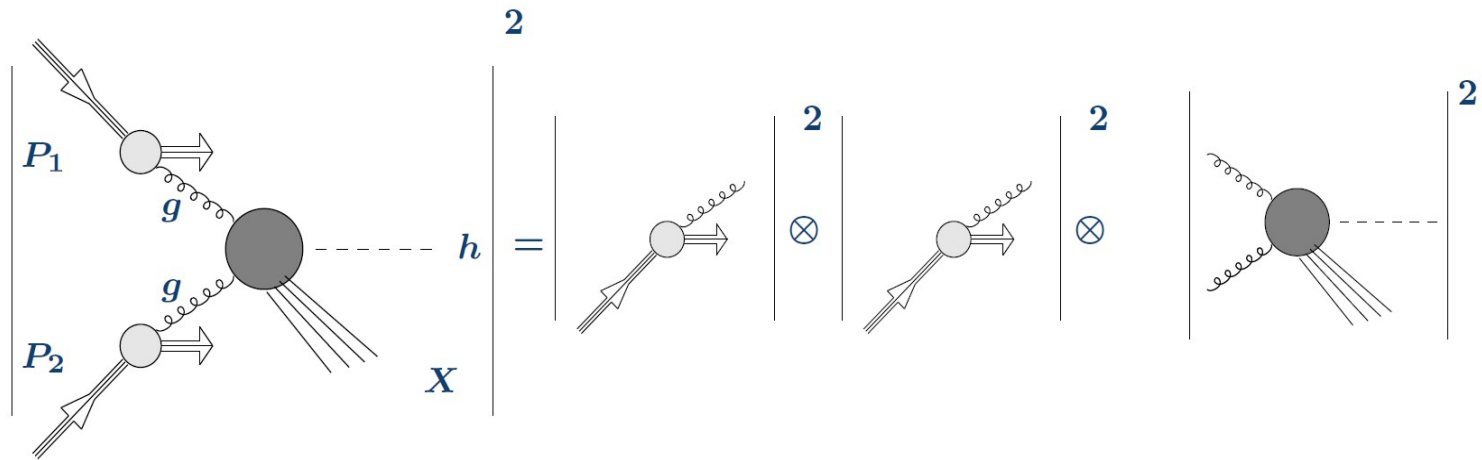
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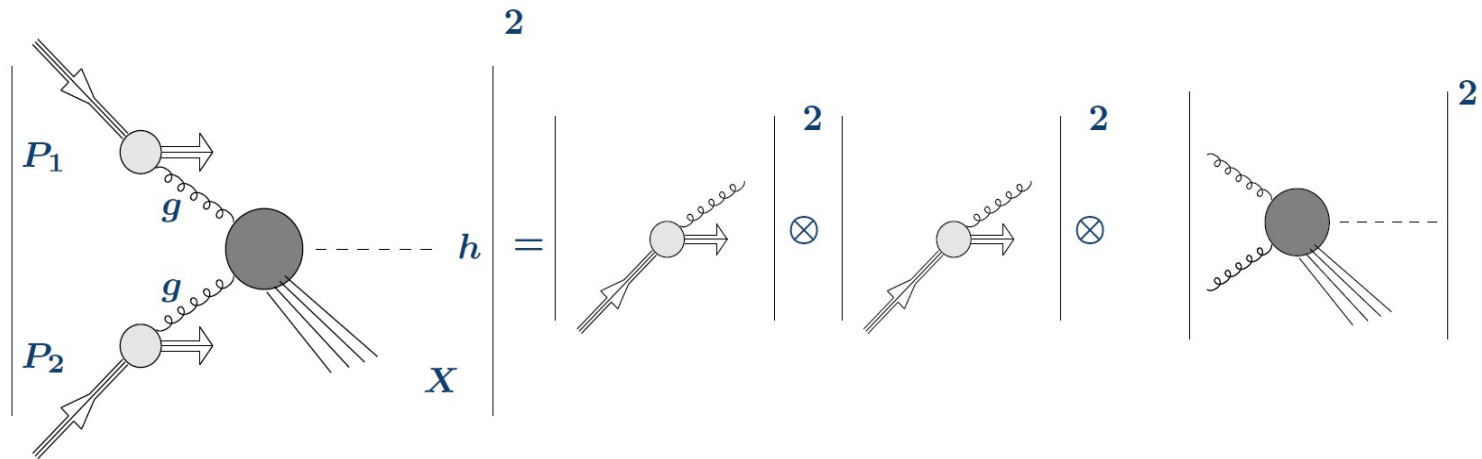


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- $\hat{\sigma}_{ab}(x_i, \{p_i\}, \mu_F^2)$ are the partonic cross sections.
- Perturbatively calculable.

Factorisation Theorem

Hadronic cross section in terms of partonic cross sections convoluted with appropriate PDF:

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- The Renormalisation group invariance:

$$\frac{d}{d\mu} \sigma^{P_1 P_2}(\tau, m_h^2) = 0, \quad \mu = \mu_F, \mu_R$$

LO is a crude approximation!

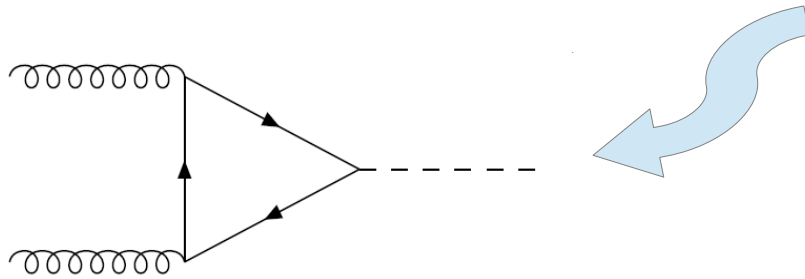
Higgs Production through gluon fusion:

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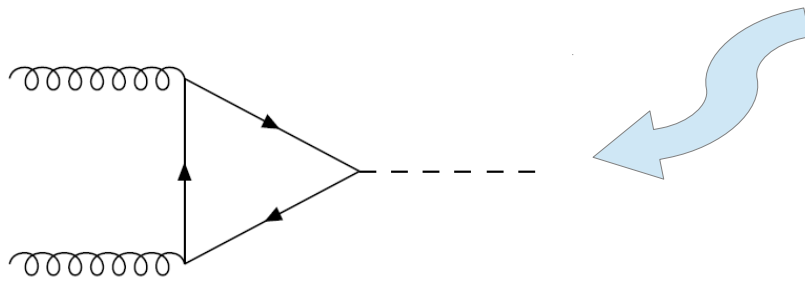
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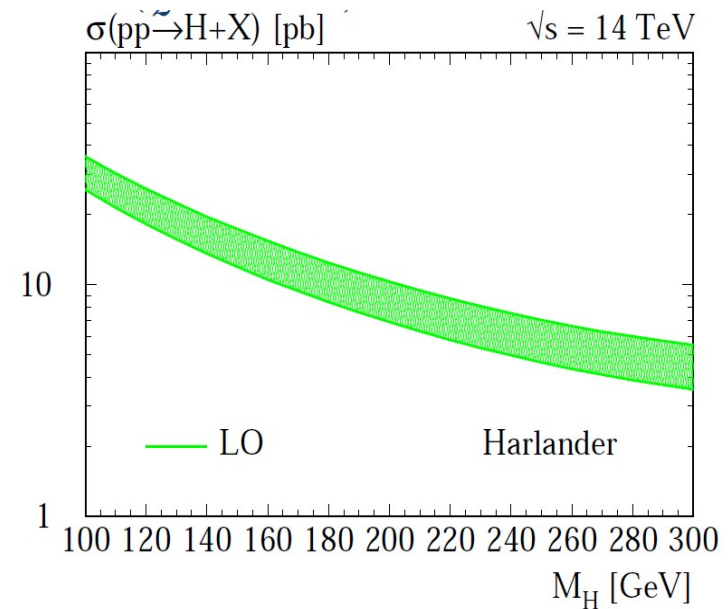
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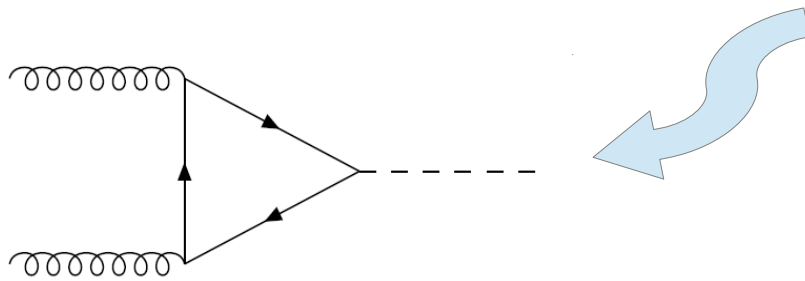
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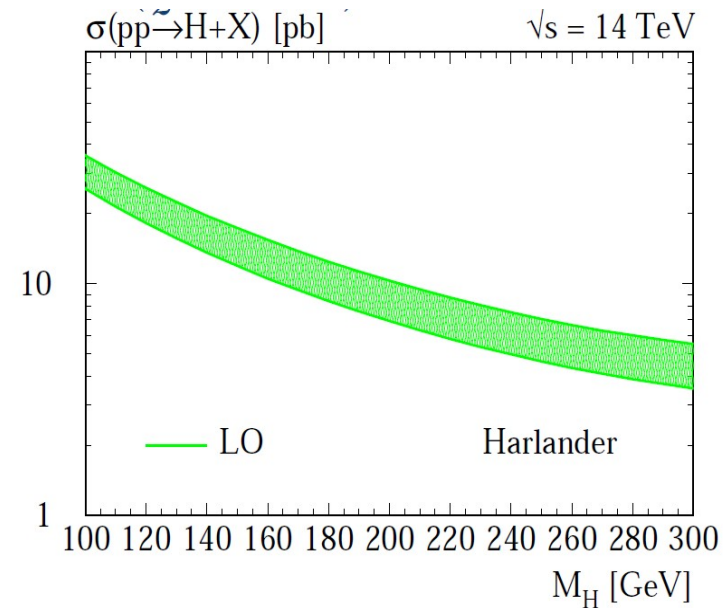
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$$2\hat{s} \hat{\sigma}_{gg}^{(0)}(\hat{s}, \mu_R) \sim \alpha_s^2(\mu_R) G_F \left[\frac{4m_t^2}{m_H^2} F\left(\frac{4m_t^2}{m_H^2}\right) \right], \quad \frac{m_H}{2} < \mu_R = \mu_F < 2m_H$$

LO prediction is Unreliable due 100 – 200% scale uncertainty

Inputs that can affect the predictions

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- Stability of perturbative result and missing higher order contributions.
- Any "Fixed order" perturbative result is bound to depend on μ_R and μ_F and type of PDF sets.
- Observables are "free" of μ_R and μ_F .

$$\mu \frac{d}{d\mu} \sigma^{P_1 P_2} = 0, \quad \mu = \mu_F, \mu_R, PDF$$

Strong Coupling Constant

$$\alpha_s(\mu_R)$$

Renormalisation Group Equation α_s

Renormalisation group equation for α_s :

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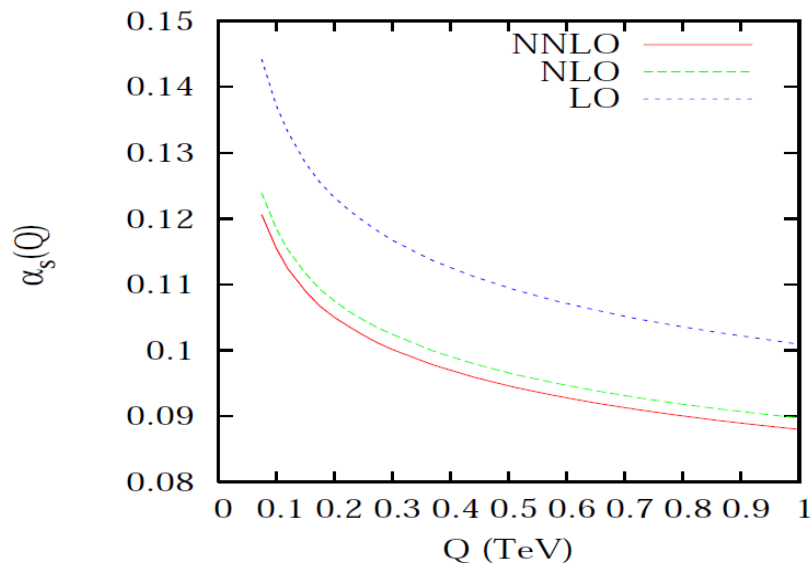
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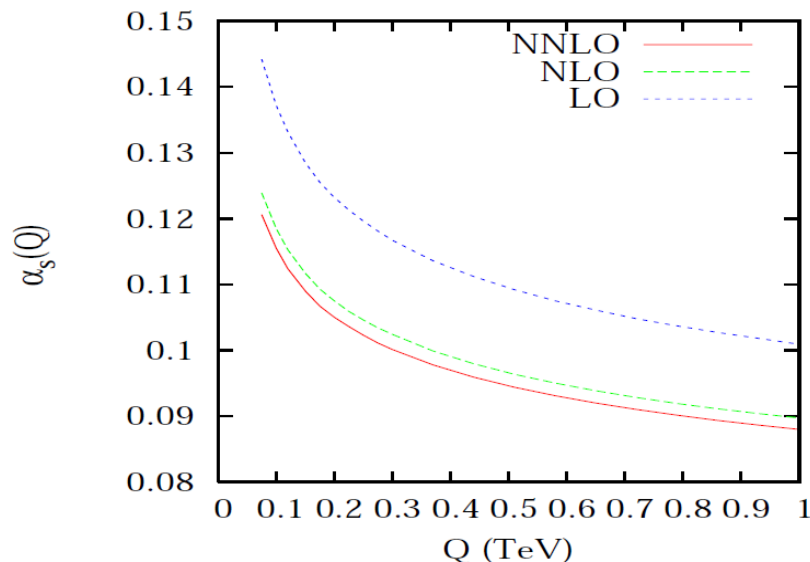


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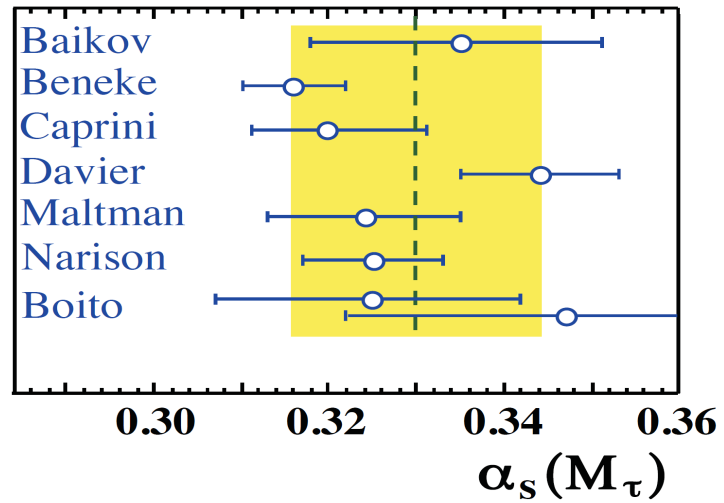
Measured from :

- Tau decays,
- lattice,
- heavy quarkonia decays,
- non-single structure functions,
- Jets from HERA,
- event shape variables from LEP

From various sources

S. Bethke

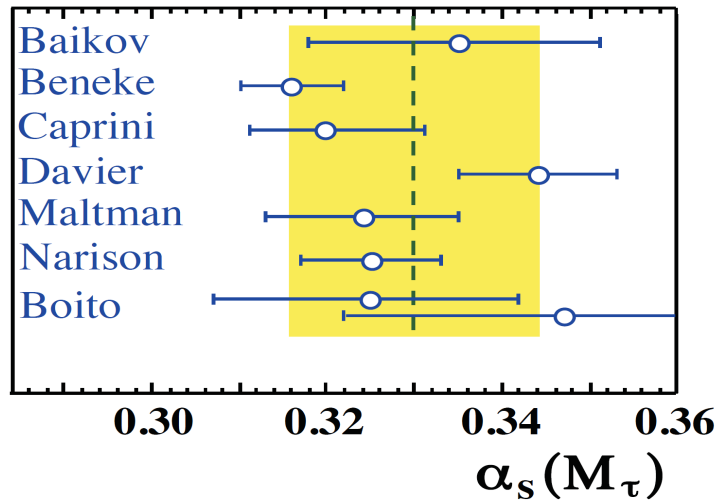
α_s from τ -decays



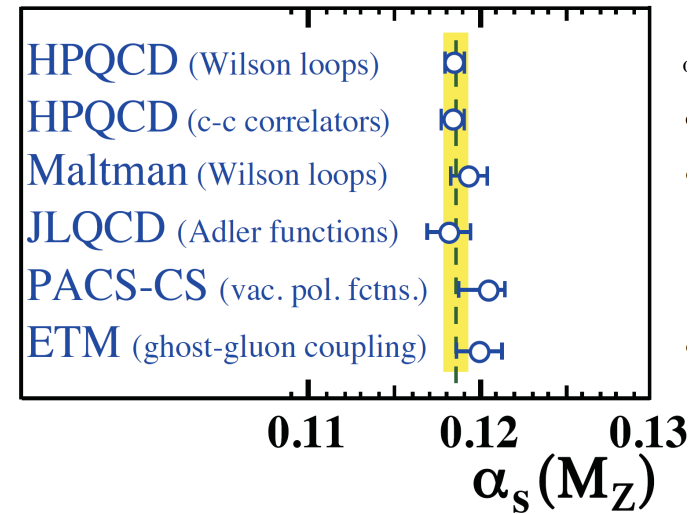
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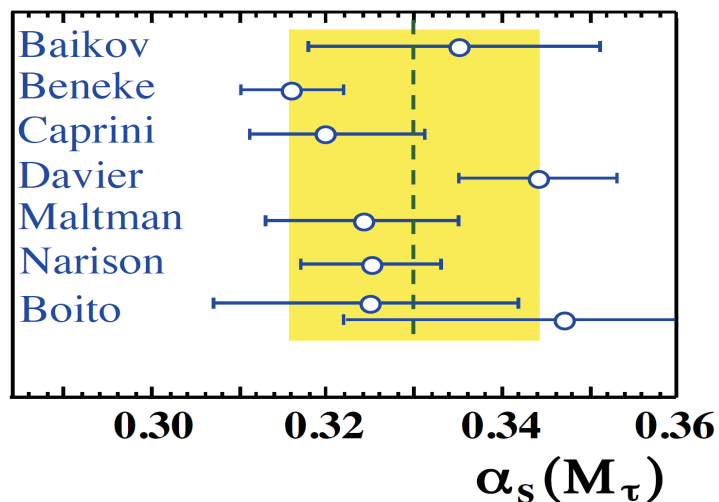
α_s from lattice QCD



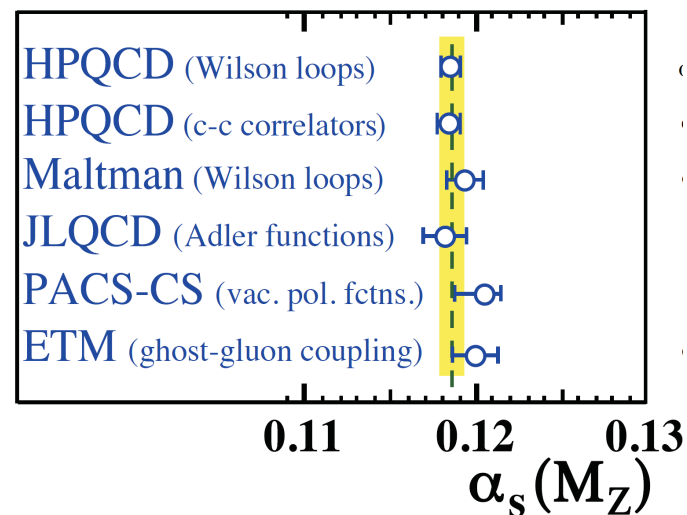
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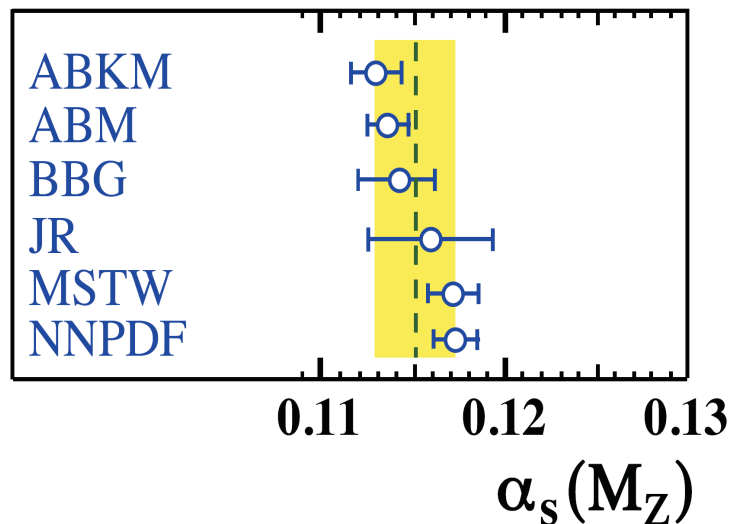
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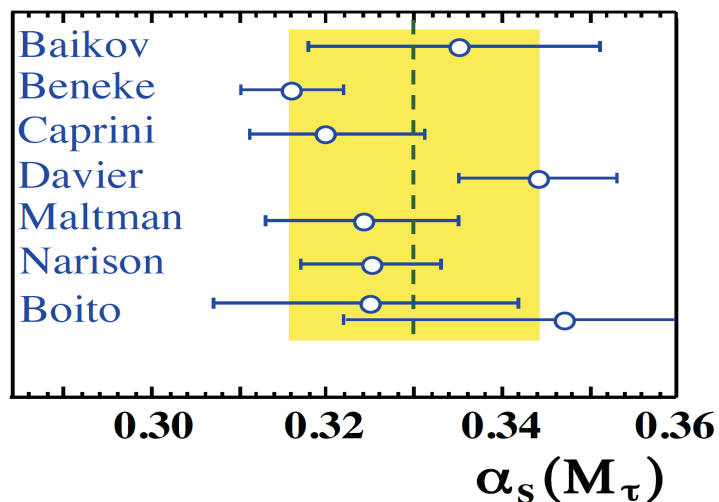
α_s from DIS structure functions



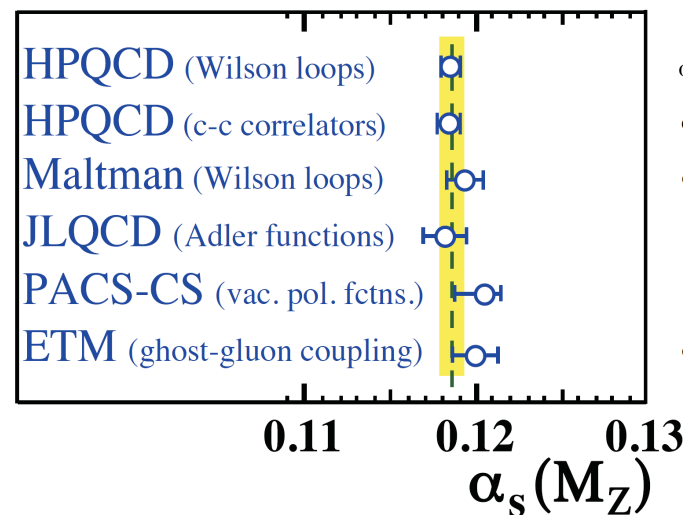
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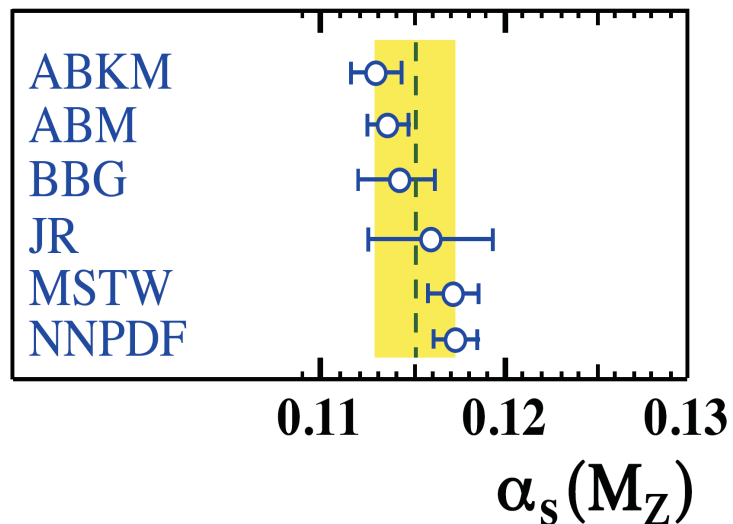
α_s from τ -decays



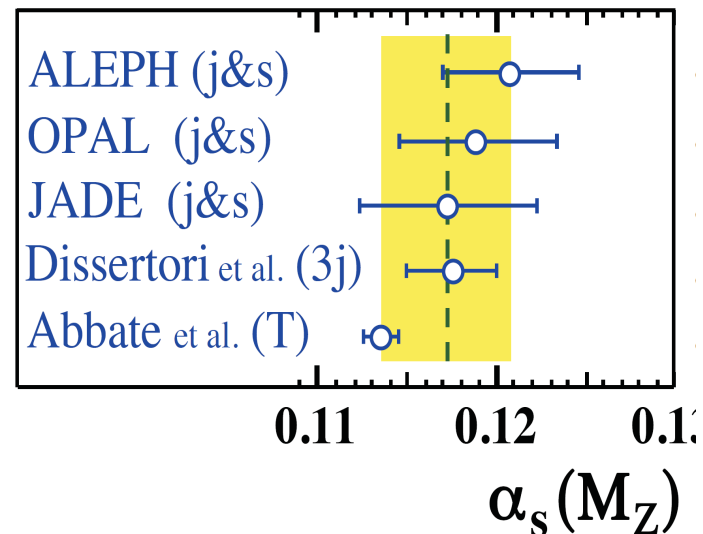
α_s from lattice QCD



α_s from DIS structure functions

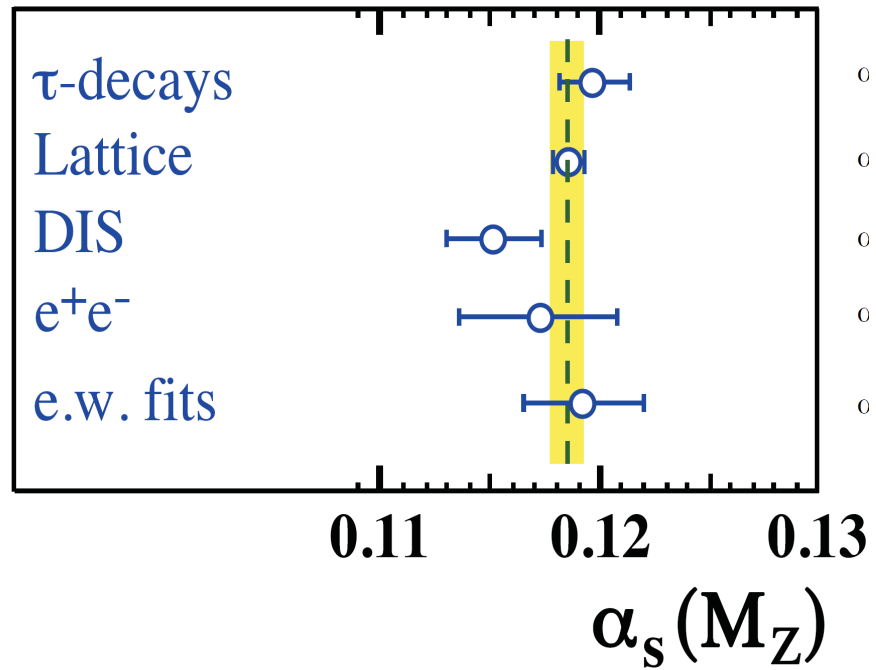


α_s from jets and event shapes in e^+e^- annihilation



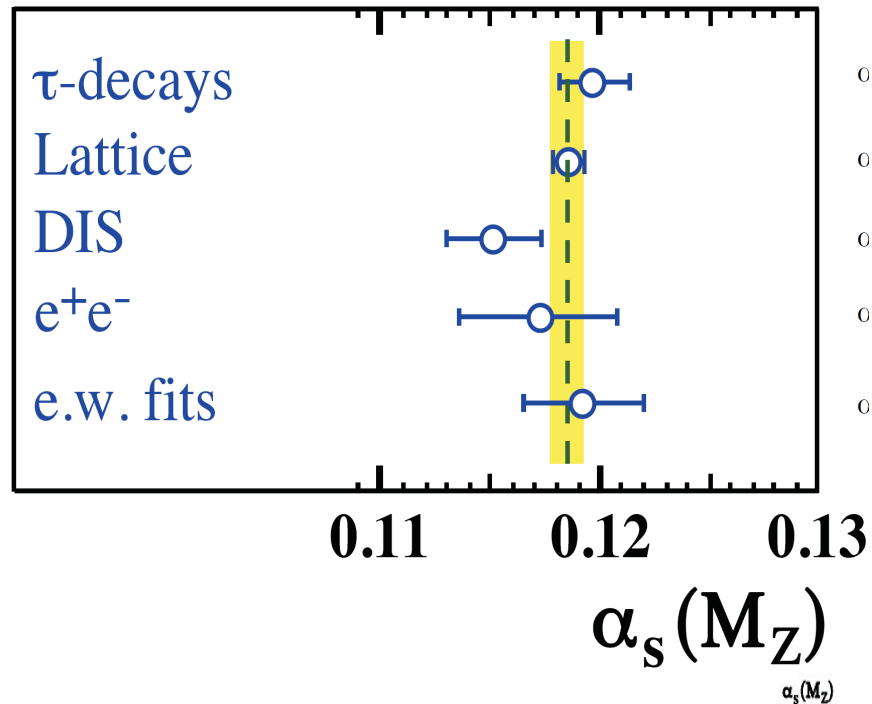
World Summary of α_s 2012:

S. Bethke



World Summary of α_s 2012:

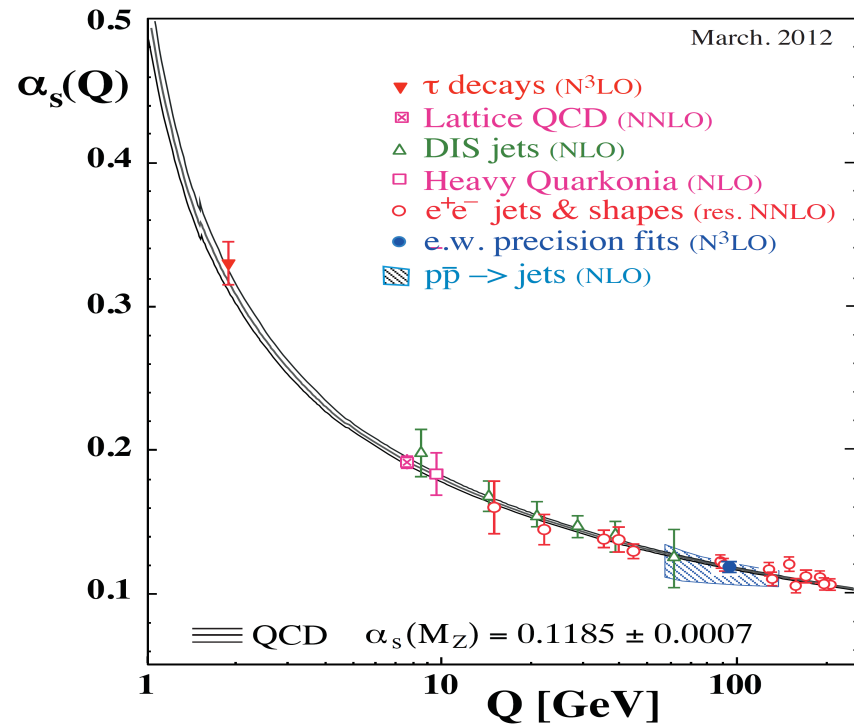
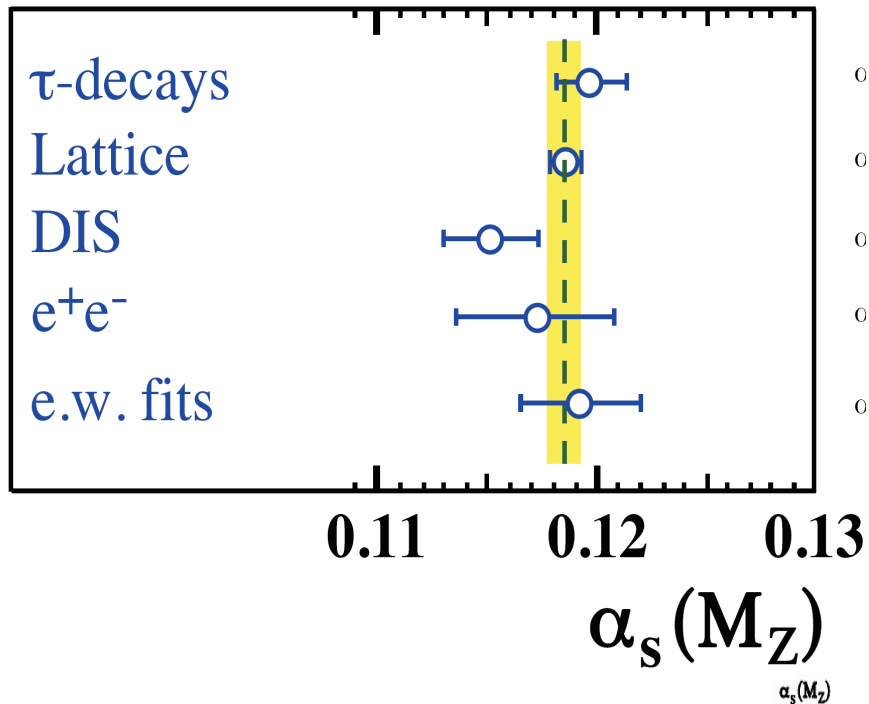
S. Bethke



Process	$\alpha_s(M_{Z^0})$	excl. mean $\alpha_s(M_{Z^0})$	std. dev.
τ -decays	0.1197 ± 0.0016	0.1183 ± 0.0007	0.8
Lattice QCD	0.1186 ± 0.0007	0.1182 ± 0.0011	0.3
DIS [F_2]	0.1151 ± 0.0022	0.1188 ± 0.0010	1.5
e^+e^- [jets & shps]	0.1172 ± 0.0037	0.1185 ± 0.0006	0.3
ew. prec. data]	0.1192 ± 0.0028	0.1185 ± 0.0006	0.2

World Summary of α_s 2012:

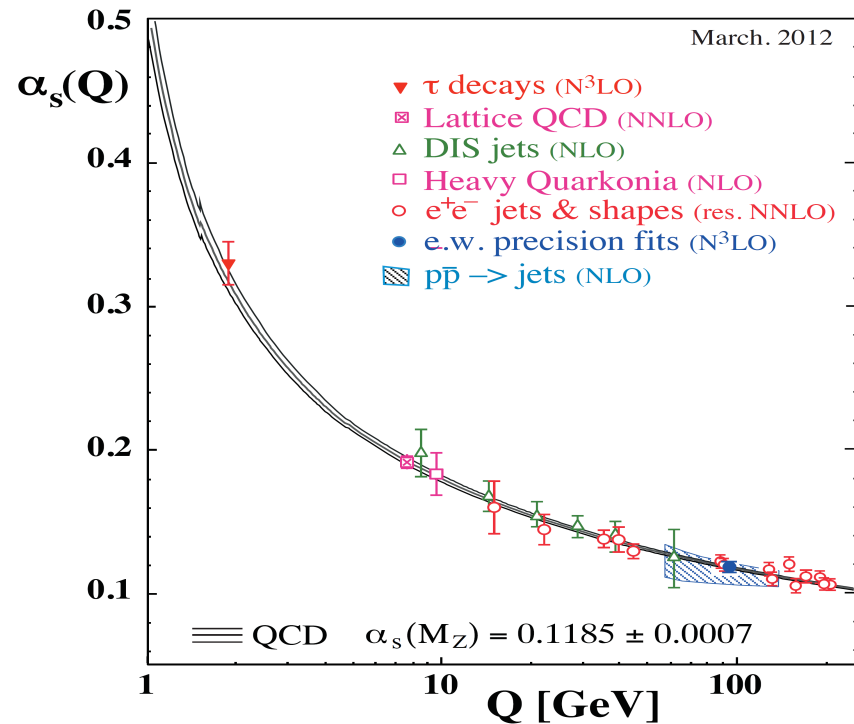
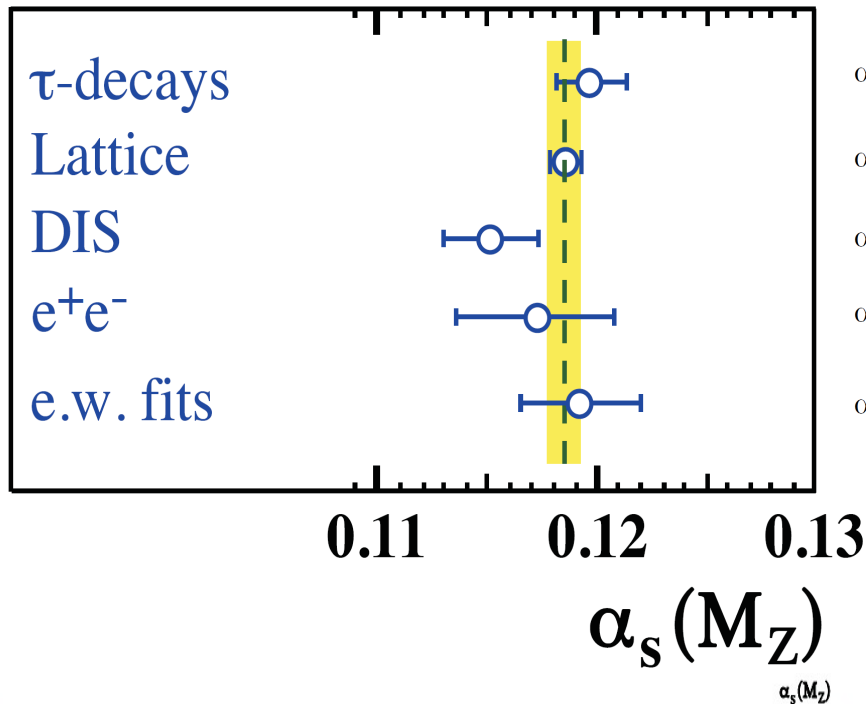
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World Summary of α_s 2012:

S. Bethke



$$\alpha_s(M_Z) = 0.1185 \pm 0.0007$$

$$\Lambda_{\overline{\text{MS}}}^{(5)} = (214 \pm 9) \text{ MeV}$$

$$\Lambda_{\overline{\text{MS}}}^{(4)} = (297 \pm 11) \text{ MeV}$$

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Parton Distribution Function

$$f_a(z, \mu_F)$$

PDF and DGLAP evolution equation

Renormalised parton density:

$$f_a(z, \mu_F) = \Gamma_{ab} \left(z, \mu_F, \frac{1}{\epsilon_{\text{IR}}} \right) \otimes f_a^B(z)$$

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Dakshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) Evolution equation:

$$\mu_F \frac{d}{d\mu_F} f_a(x, \mu_F) = \int_x^1 \frac{dz}{z} P_{ab}(z, \mu_F) f_b\left(\frac{x}{z}, \mu_F\right), \quad P \equiv \Gamma^{-1} \left(\mu_F \frac{d}{d\mu_F} \right) \Gamma$$

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Perturbatively Calculable:

$$\begin{aligned} P_{ab}(z, \mu_F) &= \left(\frac{\alpha_s(\mu_F)}{4\pi} \right) P^{(0)}(z) && \text{one loop (LO)} \\ &+ \left(\frac{\alpha_s(\mu_F)}{4\pi} \right)^2 P^{(1)}(z) && \text{two loop (NLO)} \\ &+ \left(\frac{\alpha_s(\mu_F)}{4\pi} \right)^3 P^{(2)}(z) && \text{three loop (NNLO)} \end{aligned}$$

NNLO is already known (summer 2004)

Scale Variation of Flux at the LHC

$$\Phi_{ab}^I(x, \mu_F) = \int_x^1 \frac{dz}{z} f_a^I(z, \mu_F) f_b^I\left(\frac{x}{z}, \mu_F\right) \quad I = LO, NLO, NNLO$$

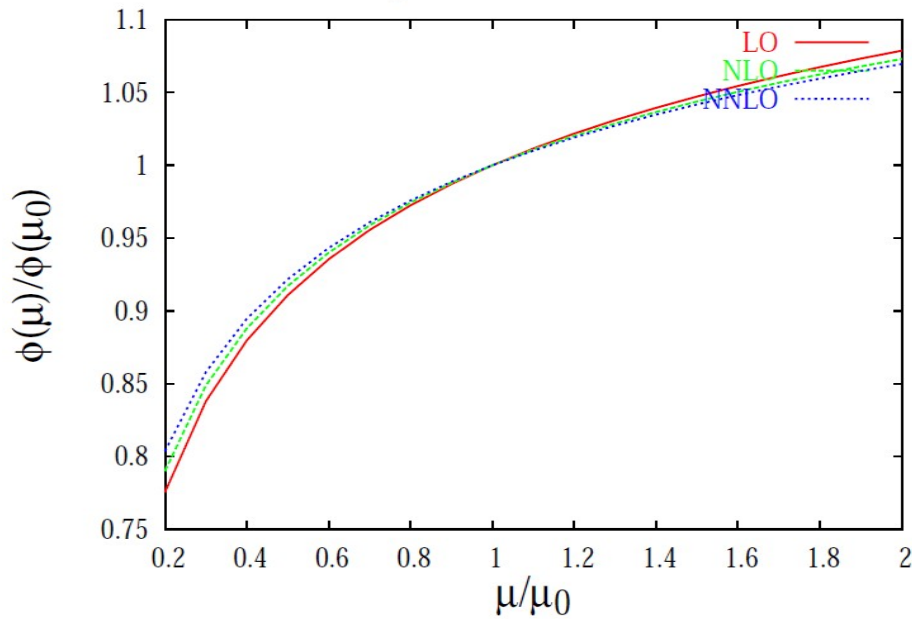
DGLAP evolution:

$$\mu_F \frac{d}{d\mu_F} f_a(x, \mu_F) = \int_x^1 \frac{dz}{z} P_{ab}(z, \mu_F) f_b\left(\frac{x}{z}, \mu_F\right) \quad \mu_F = \mu, \quad \mu_0 = 150 \text{ GeV}$$

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LHC(quark flux, Q=150 GeV)

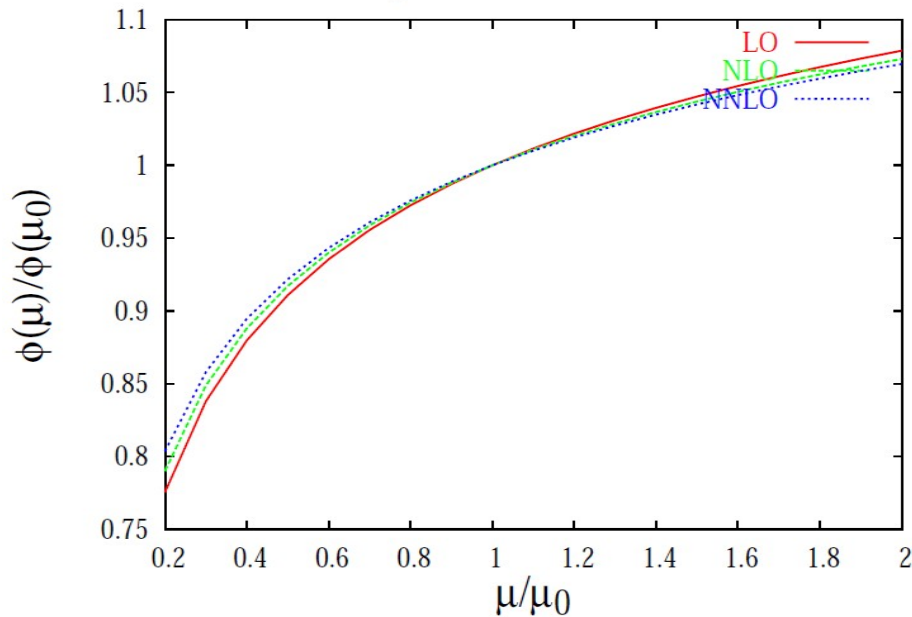


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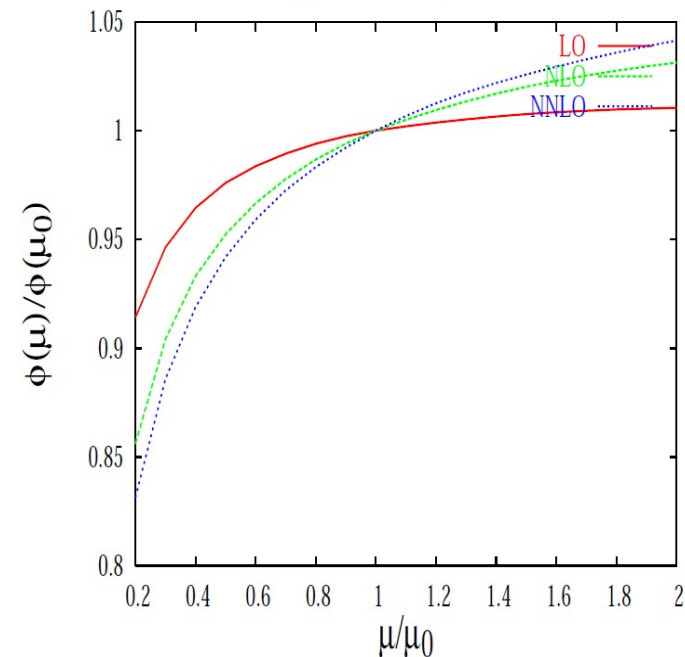
Scale Variation of Flux at the LHC

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LHC(quark flux, Q=150 GeV)



LHC(gluon flux, Q=150 GeV)



DGLAP evolution:

$$\mu_F \frac{d}{d\mu_F} f_a(x, \mu_F) = \int_x^1 \frac{dz}{z} P_{ab}(z, \mu_F) f_b\left(\frac{x}{z}, \mu_F\right) \quad \mu_F = \mu, \quad \mu_0 = 150 \text{ GeV}$$

PDF sets

Different Groups:

MSTW, CTEQ, ABKM, ABM, NNPD, HERAPDF, GJR,

PDF sets

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MSTW, CTEQ, ABKM, ABM, NNPD, HERAPDF, GJR,

Experimental inputs:

Deep Inelastic Scattering,
Drell-Yan,
Tevatron jets, Tevatron W,Z , ...

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PDF uncertainty:

Choice of data sets
Treatment of heavy quarks
Treatment of errors
Order of perturbation theory
Parametrisation of densities
Flavour symmetries
Asymptotic behaviour of pdfs

PDF sets

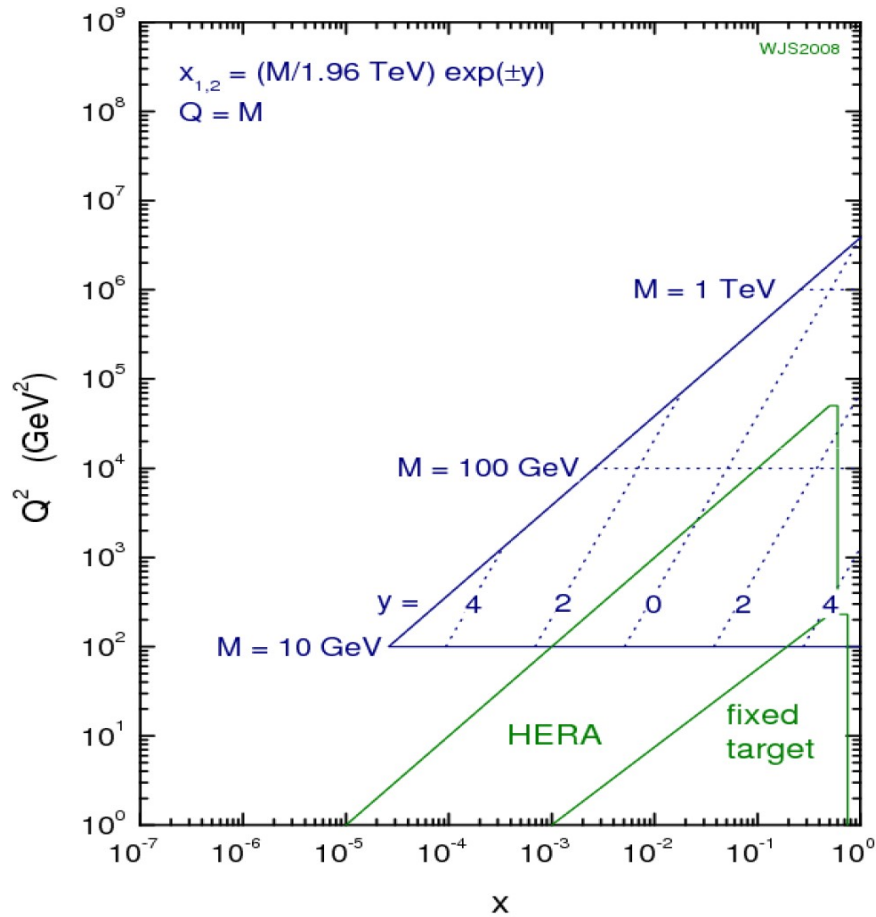
J. Stirling

	MSTW08	CTEQ6.6 ^x	NNPDF2.0	HERAPDF1.0	ABKM09	GJR08
HERA DIS	✓	✓	✓*	✓*	✓	✓
F-T DIS	✓	✓	✓	✗	✓	✓
F-T DY	✓	✓	✓	✗	✓	✓
TEV W,Z	✓	✓+	✓	✗	✗	✗
TEV jets	✓	✓+	✓	✗	✗	✓
GM-VFNS	✓	✓	✗	✓	✗	✗
NNLO	✓	✗	✗	✗	✓	✓

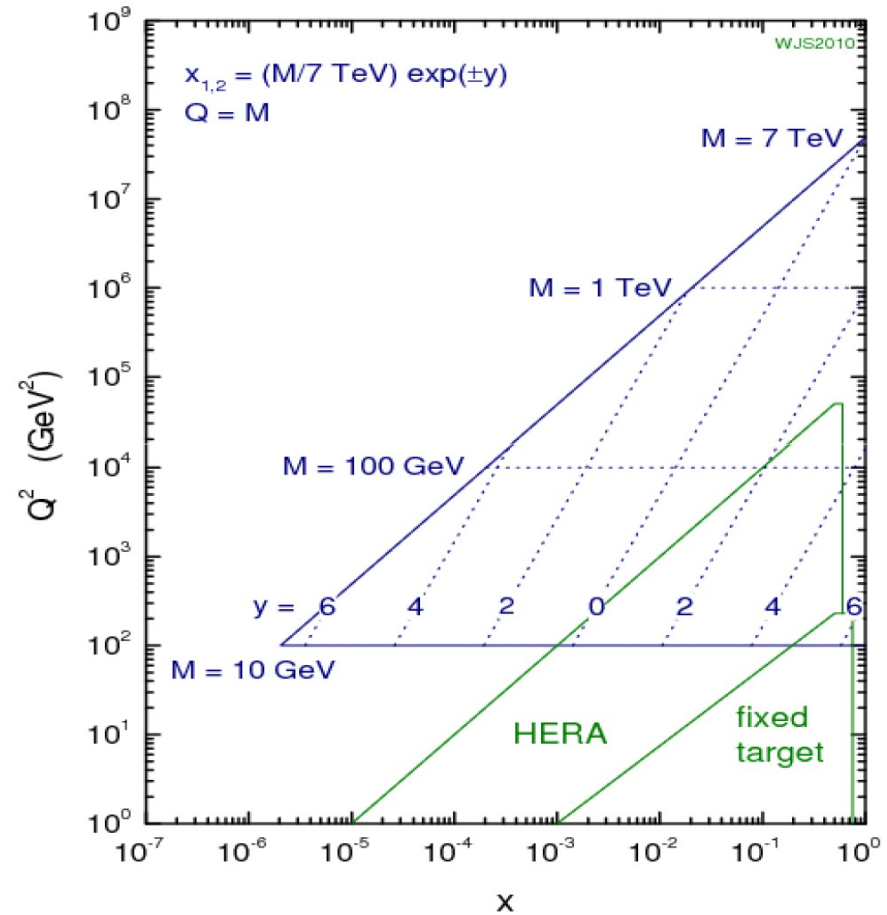
LHC-testing ground

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Tevatron parton kinematics

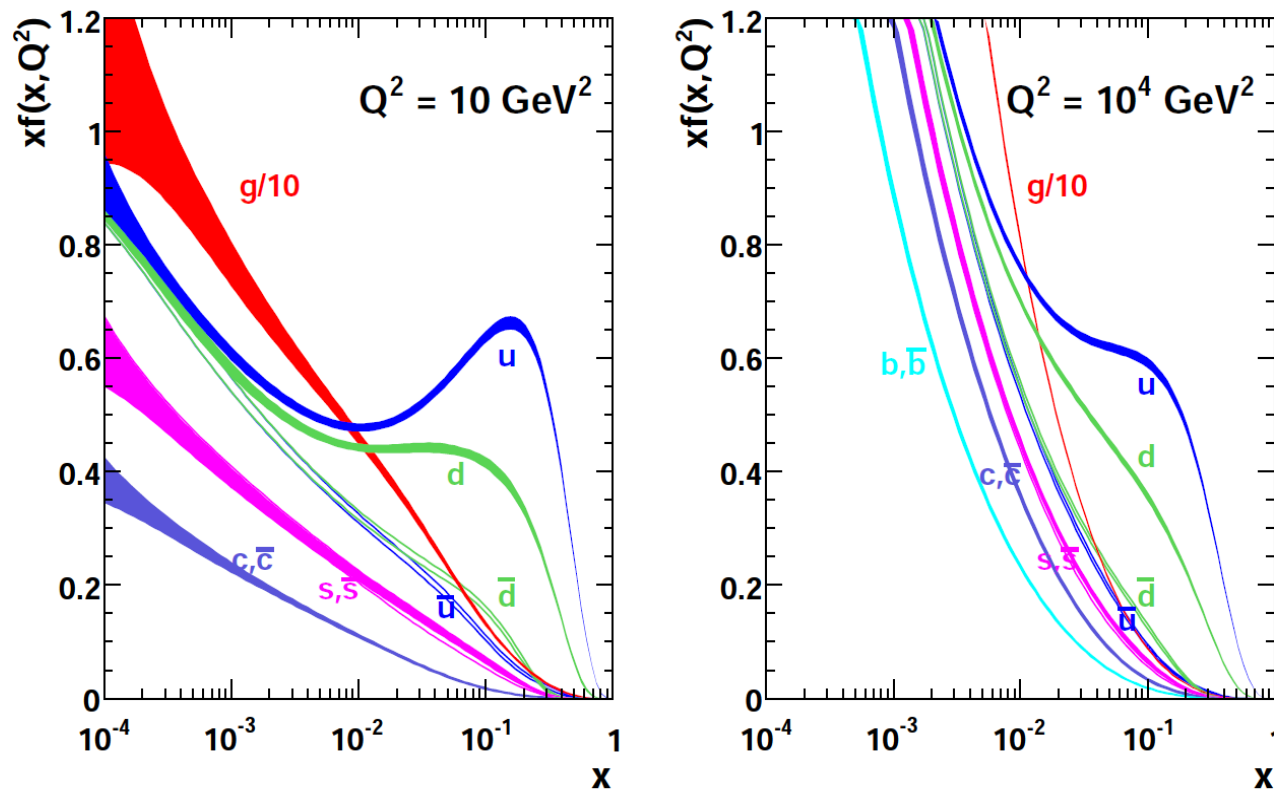


7 TeV LHC parton kinematics



Uncertainty from Parton Distribution Functions

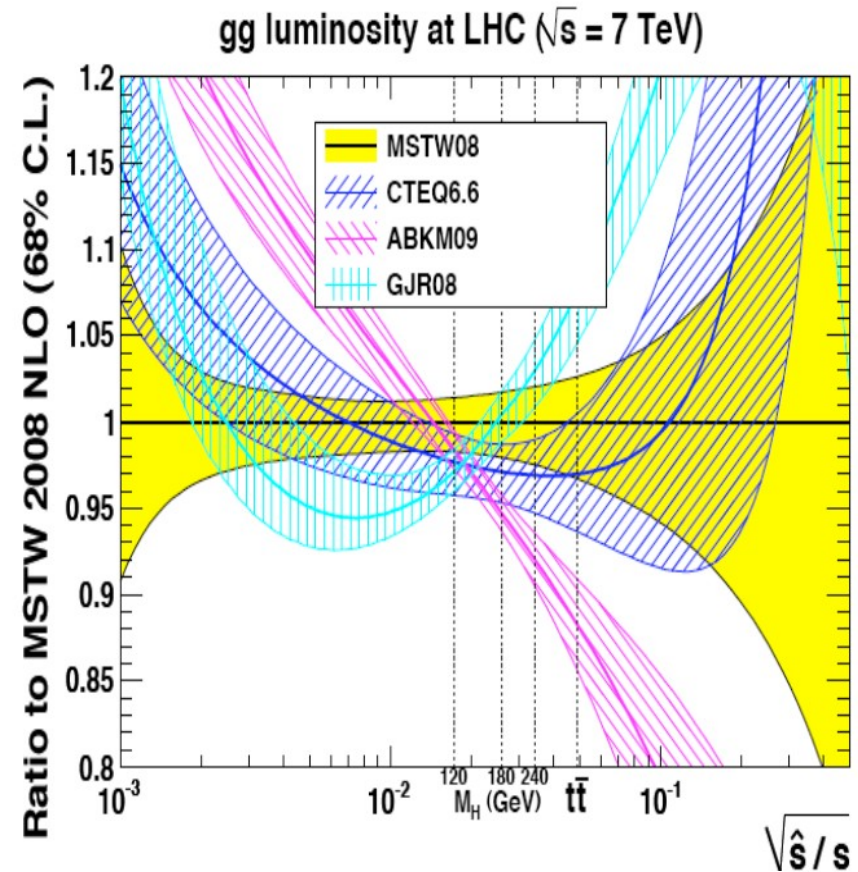
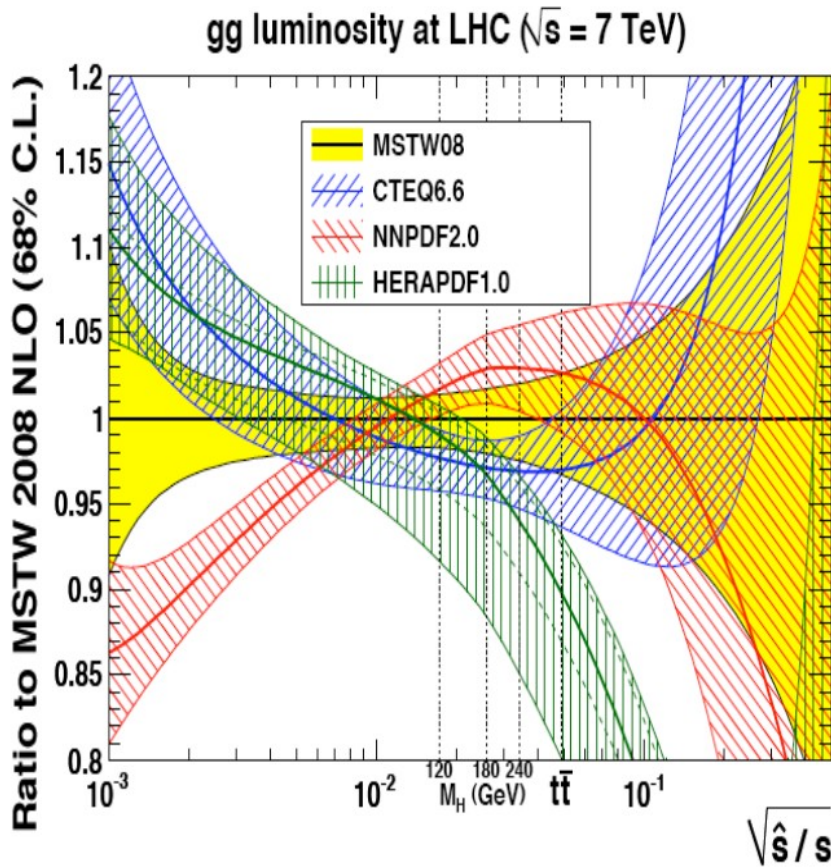
MSTW 2008 NLO PDFs (68% C.L.)



MSTW 2008 NLO PDFs at $Q^2 = 10 \text{ GeV}^2$ and $Q^2 = 10^4 \text{ GeV}^2$.

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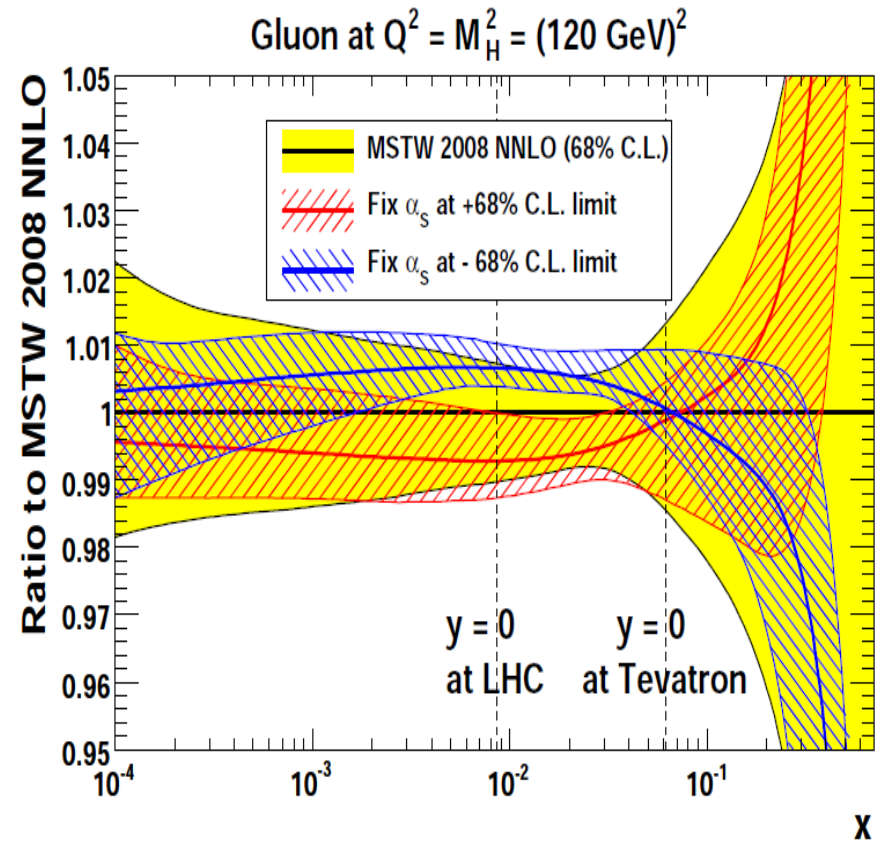
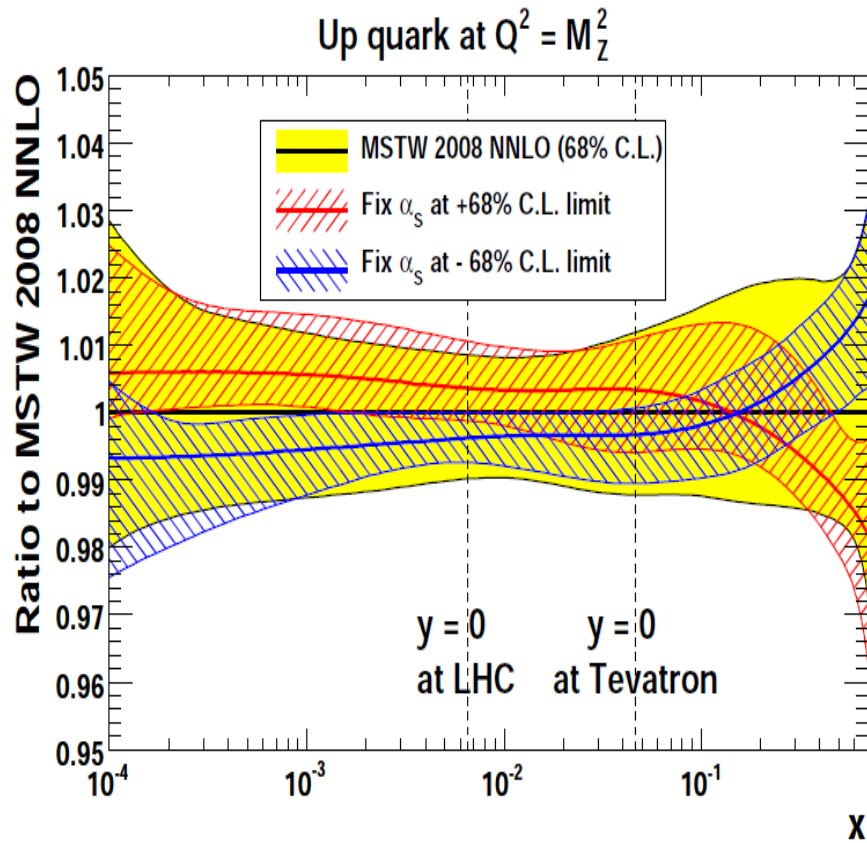
Gluon Luminosity



- Data sets: Electroproduction, hadron production (fixed target and collider)
- Fits procedure: Hessian and Monte Carlo
- Treatment: α_s , m_b and m_c

J. Stirling

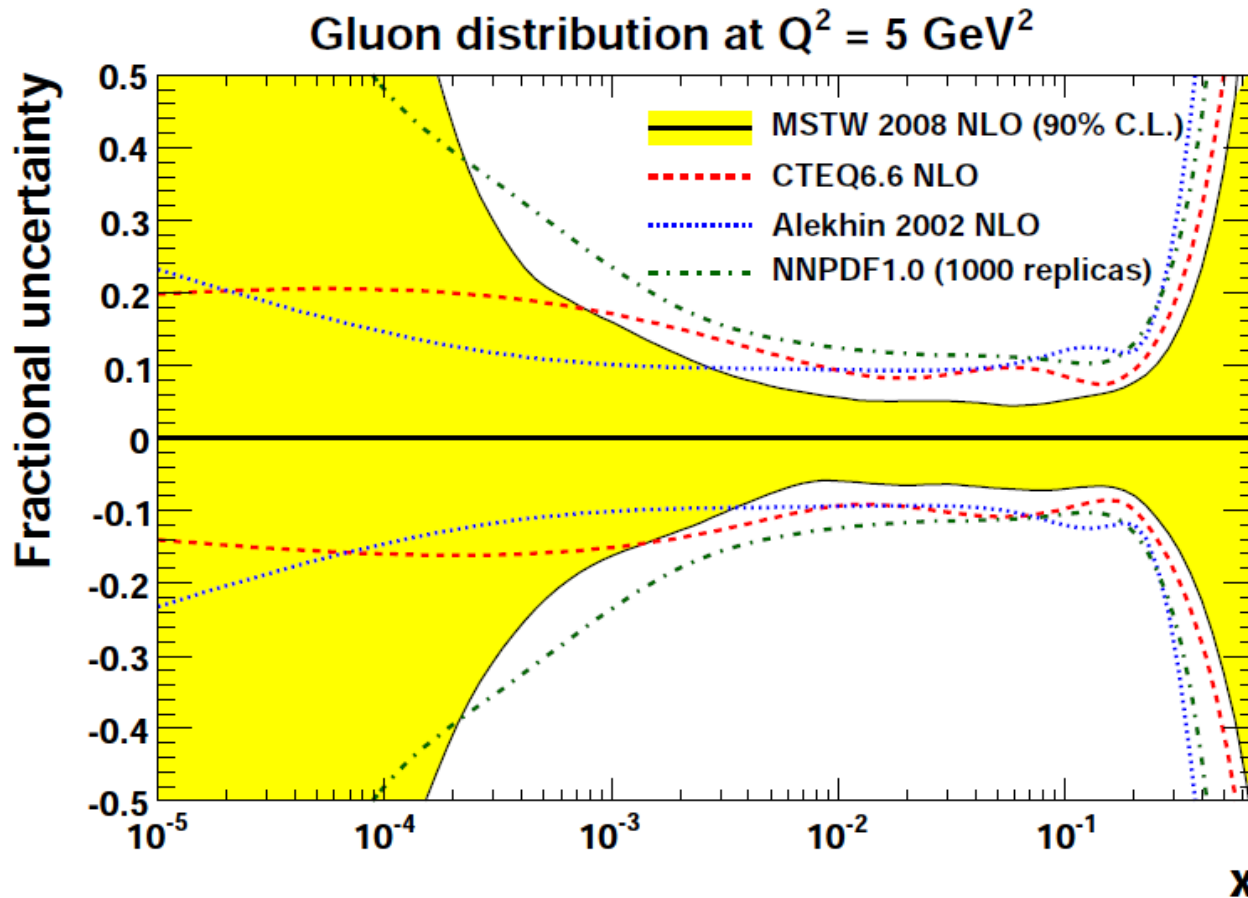
PDF Uncertainty



J. Stirling

Gluon PDF

J. Stirling

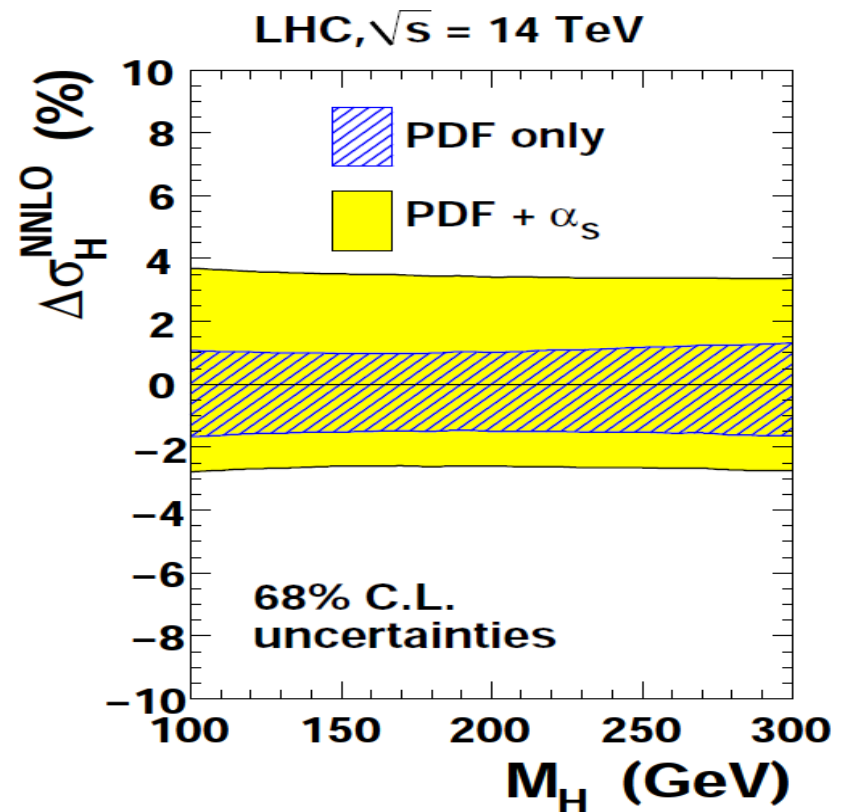
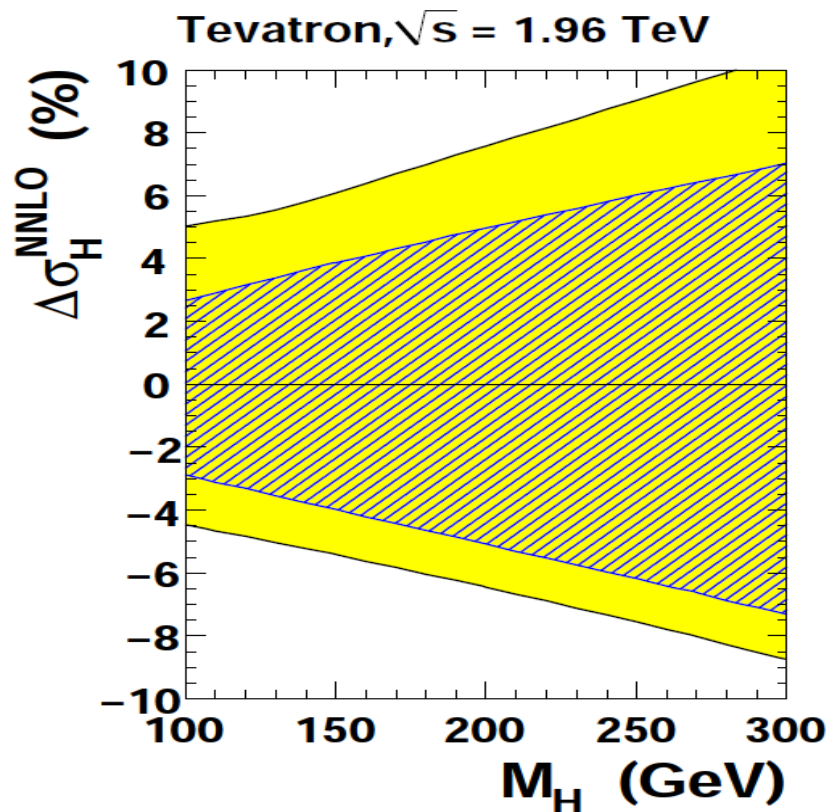


A comparison of the fractional uncertainty for the present MSTW, CTEQ6.6, Alekhin and NNPDF1.0 NLO gluon distributions at $Q^2 = 5 \text{ GeV}^2$. All uncertainty bands represent a 90% C.L. limit.

PDF and Higgs cross section

J. Stirling

Higgs cross sections with MSTW 2008 NNLO PDFs



PDF and α_s

- For consistent prediction, PDFs along with appropriate α_s have to be used.
- MSTW does global fits for both PDFs and α_s using DIS data and other hadronic data.
- Also it uses LO, NLO and NNLO corrected cross sections computed in \overline{MS} scheme.

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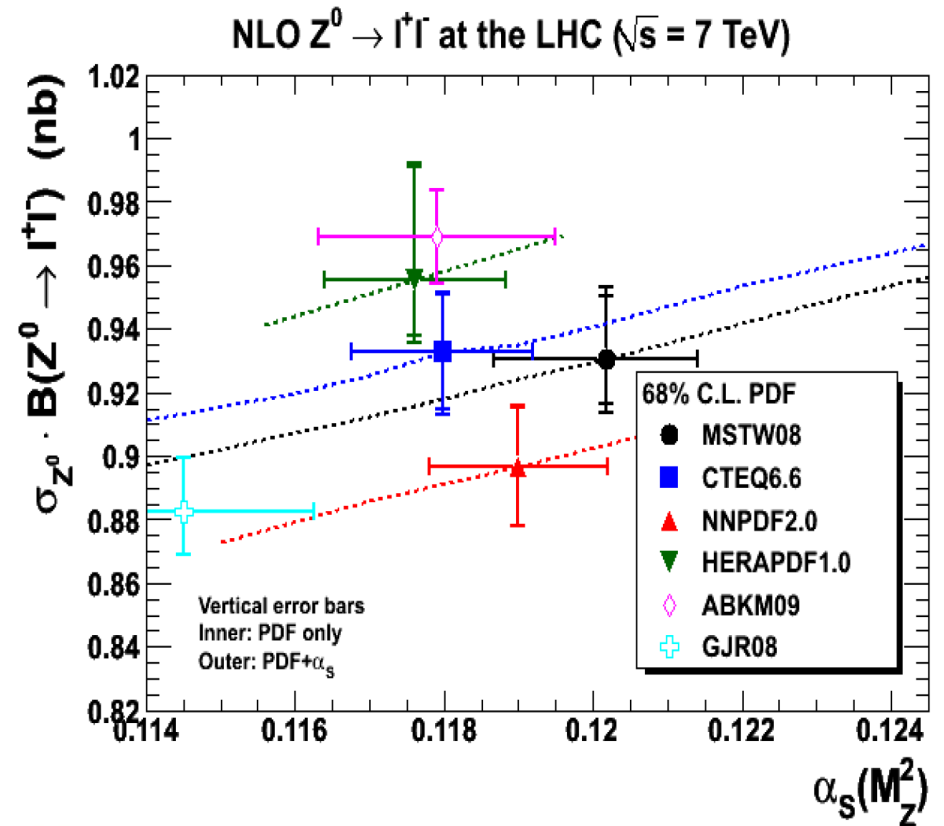
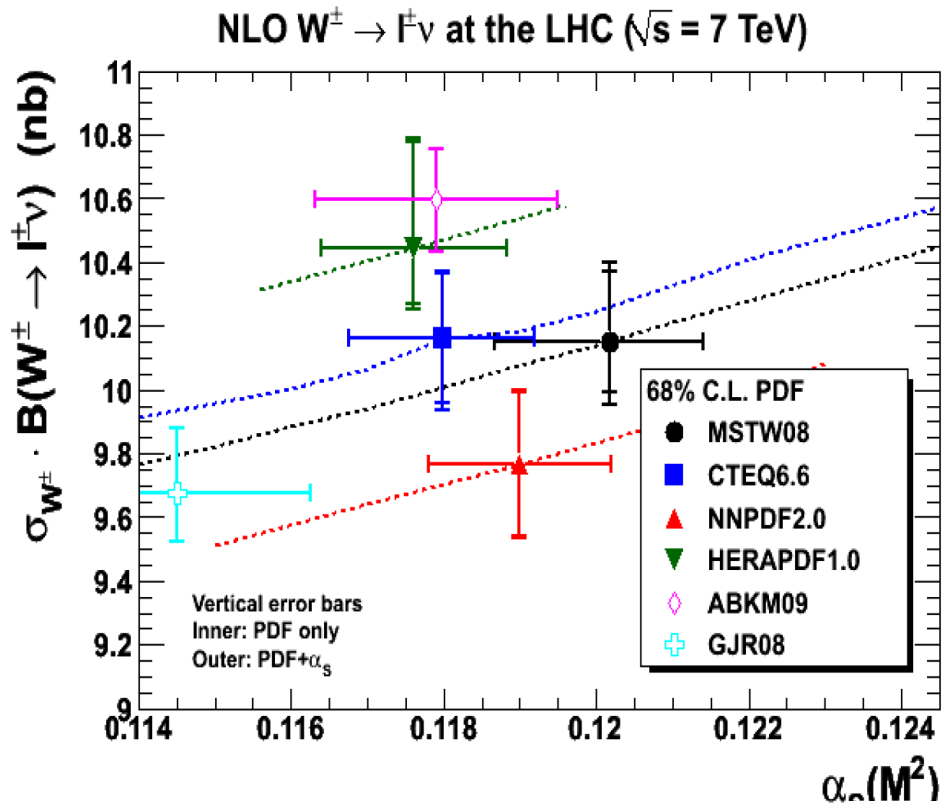
Such a fit gives:

$$NLO : \alpha_s(M_Z^2) = 0.1202_{-0.0015}^{+0.0012} (68\%C.L.)_{-0038}^{+0032} (90\%C.L.)$$

$$NLO : \alpha_s(M_Z^2) = 0.1171_{-0.0014}^{+0.0014} (68\%C.L.)_{-0034}^{+0034} (90\%C.L.)$$

NNLO	$\alpha_s(M_Z^2)$ (expt. unc. only)	
MSTW	0.1171	$+0.0014$ -0.0014
AMP	0.1128	± 0.0015
BBG	0.1134	$+0.0019$ -0.0021
ABKM	0.1129	± 0.0014
JR	0.1158	± 0.0035

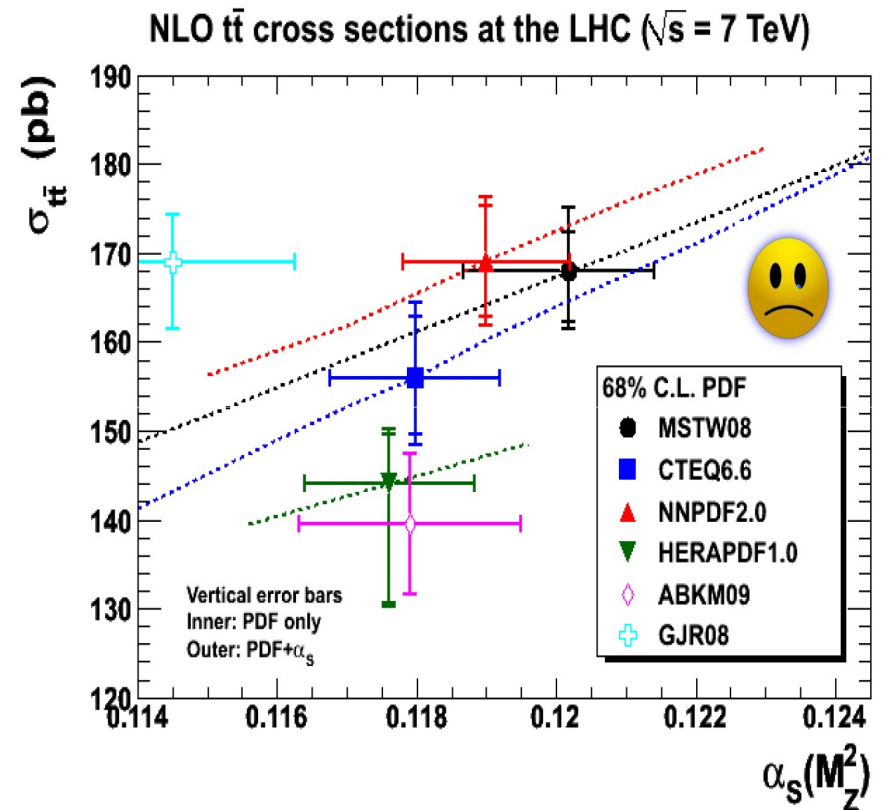
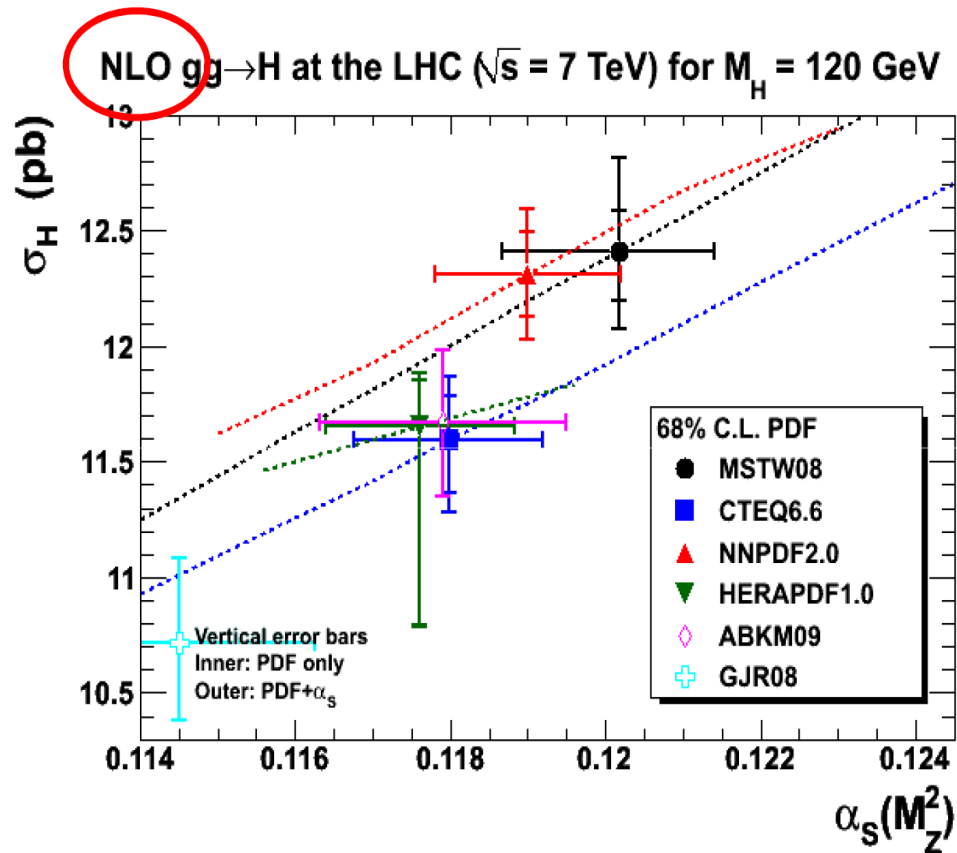
Benchmark cross sections for W and Z



J. Stirling

Benchmark cross sections for Higgs and Top

J. Stirling

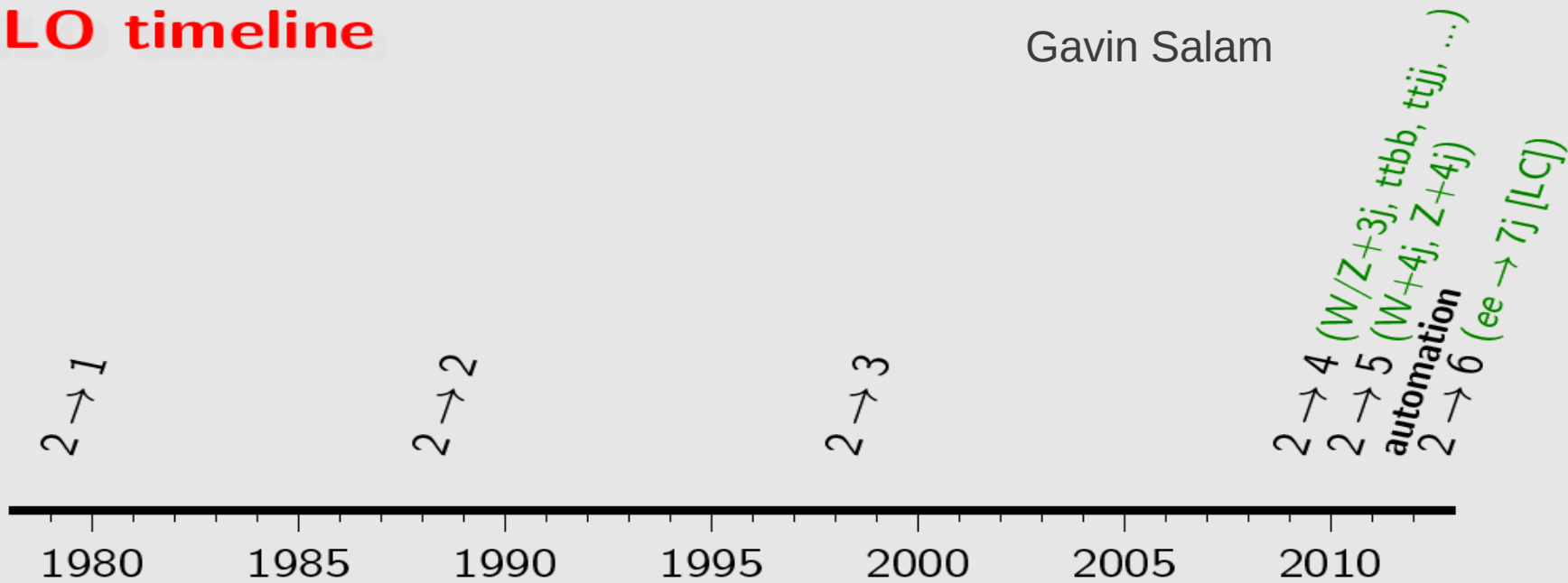


NLO

NLO revolution

NLO timeline

Gavin Salam



1979: NLO Drell-Yan [Altarelli, Ellis & Martinelli]

1991: NLO $gg \rightarrow$ Higgs [Dawson; Djouadi, Spira & Zerwas]

1987: NLO high- p_t photoproduction [Aurenche et al]

1988: NLO $b\bar{b}$, $t\bar{t}$ [Nason et al]

1988: NLO dijets [Aversa et al]

1993: Vj [JETRAD, Giele, Glover & Kosower]

1998: NLO $Wb\bar{b}$ [MCFM: Ellis & Veseli]

2000: NLO $Zb\bar{b}$ [MCFM: Campbell & Ellis]

2001: NLO $3j$ [NLOJet++: Nagy]

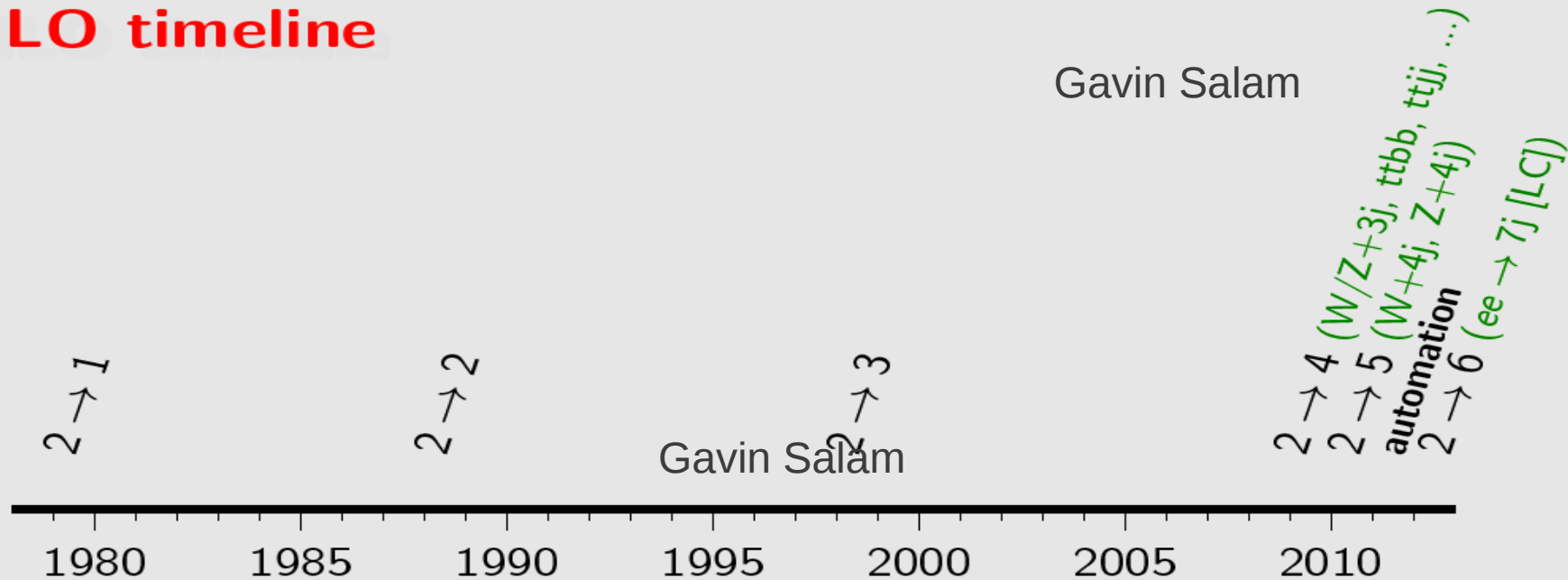
...

2007: NLO $t\bar{t}j$ [Dittmaier, Uwer & Weinzierl '07]

...

NLO revolution

NLO timeline



- 2009: NLO $W+3j$ [Rocket: Ellis, Melnikov & Zanderighi]
- 2009: NLO $W+3j$ [BlackHat+Sherpa: Berger et al]
- 2009: NLO $t\bar{t}b\bar{b}$ [Bredenstein et al]
- 2009: NLO $t\bar{t}b\bar{b}$ [HELAC-NLO: Bevilacqua et al]
- 2009: NLO $q\bar{q} \rightarrow b\bar{b}b\bar{b}$ [Golem: Binoth et al]
- 2010: NLO $t\bar{t}jj$ [HELAC-NLO: Bevilacqua et al]
- 2010: NLO $Z+3j$ [BlackHat+Sherpa: Berger et al]
- ...

Advances at NLO

Analytical Methods

- Faster way of generating Feynman diagrams:

QGRAF

- Sympolic manipulation:

FORM, Mathematica

- On-shell methods
- Recursion techniques

Merging NLO with Parton Showers:

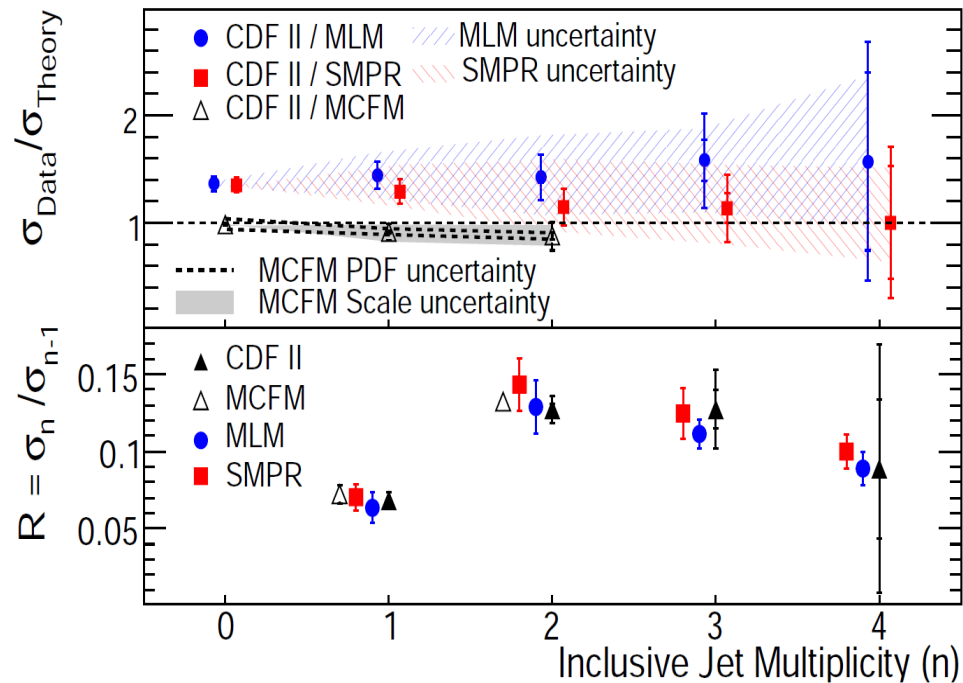
- MC@NLO
- POWEG
- SHERPA
- VINCIA
- GENeVa
- aMC@NLO
- KRKMC

Semi-numerical methods

- Helac-NLO
- CutTools
- BlackHat
- Rocket
- SAMURAI
- MadLoop
- GoSam
- Ngluon

Role of NLO corrections

W + n-jet cross section

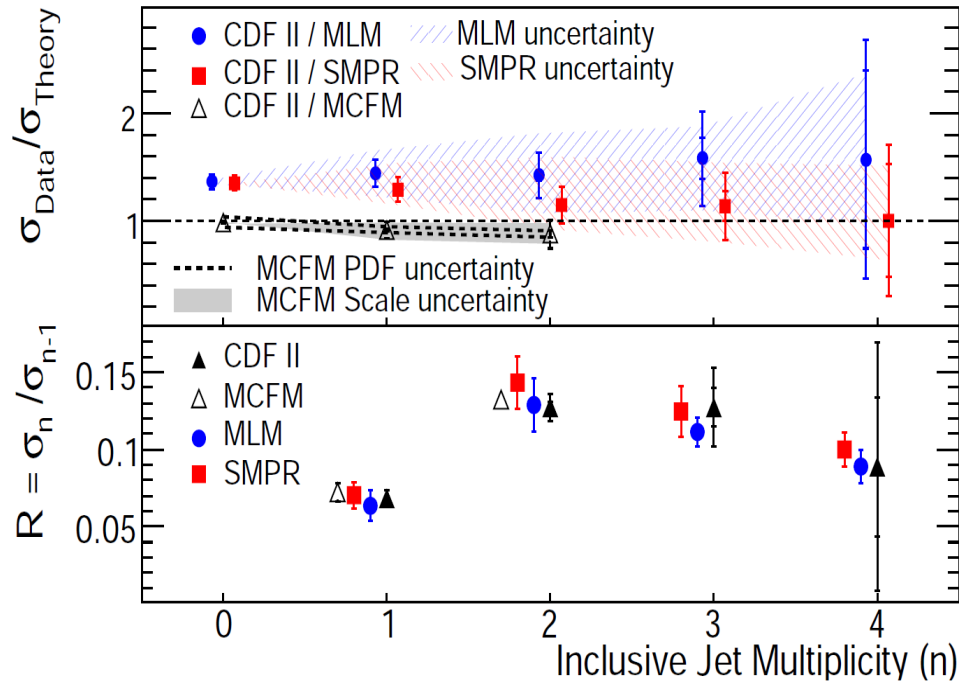


- SMPR: Madgraph, Pythia, Jetglu
- MLM: ALPGEN, Herwig, Jetglu
- MCFM: NLO QCD + showering

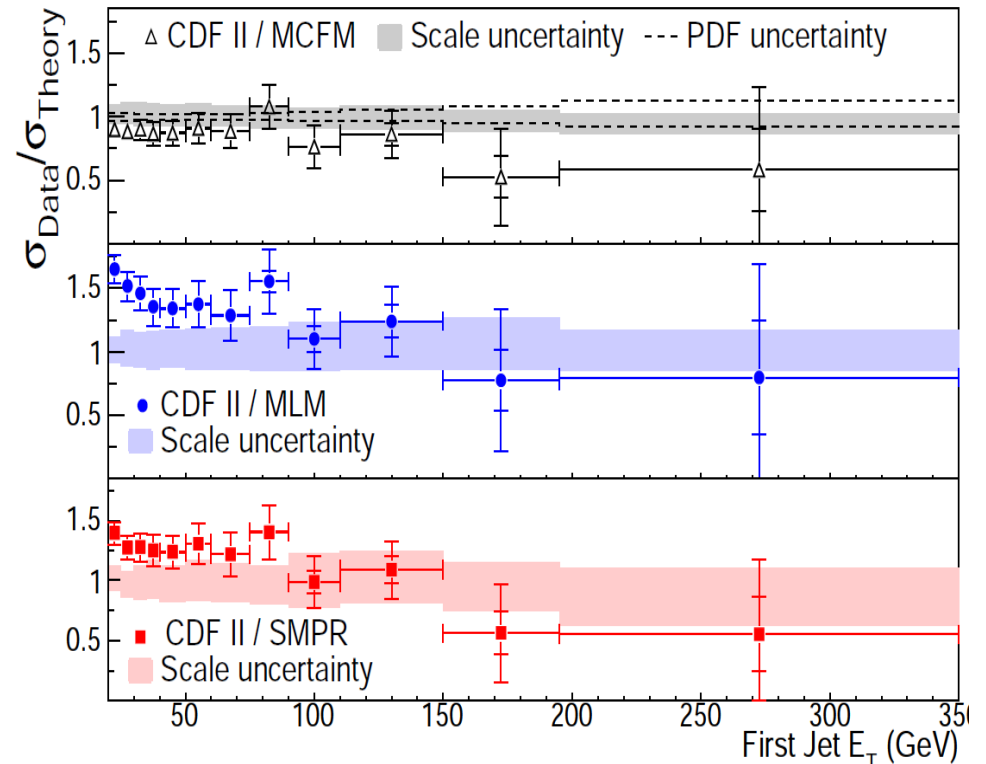
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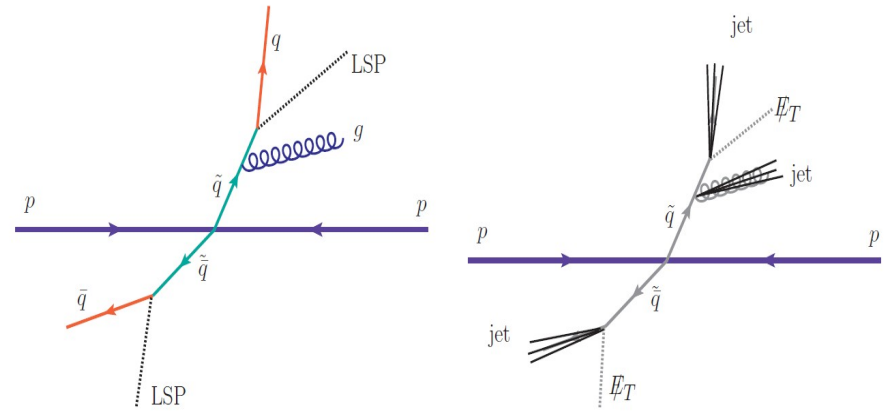


MCFM: NLO QCD works better



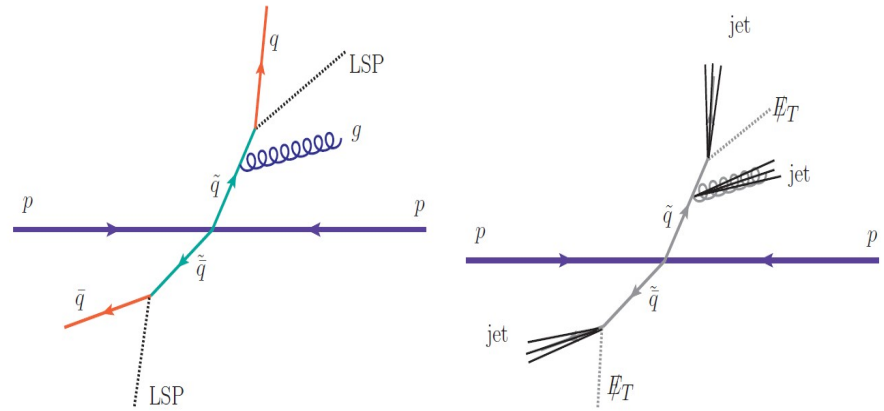
Z background to SUSY searches

- Susy searches require estimate on the Z background
- Hard to measure Z background
- Photon rates are 6 times larger easy to measure.
- Use theory to get the ratio $R_{Z/\gamma}$



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$$\sigma(pp \rightarrow Z(\rightarrow \nu\bar{\nu}) + \text{jets}) = \sigma(pp \rightarrow \gamma + \text{jets}) \times R_{Z/\gamma}$$



Background



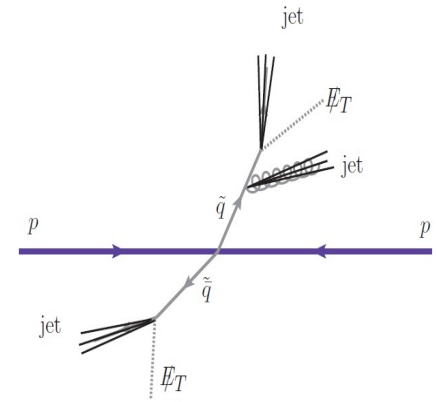
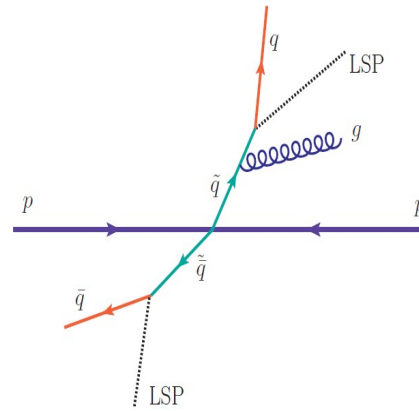
measured



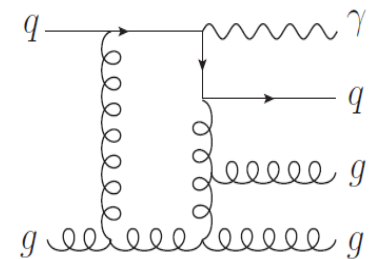
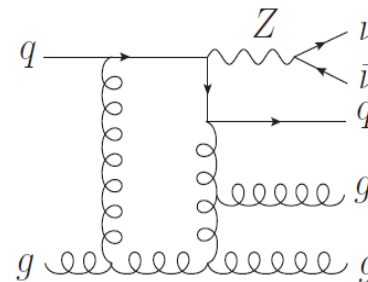
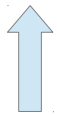
theory

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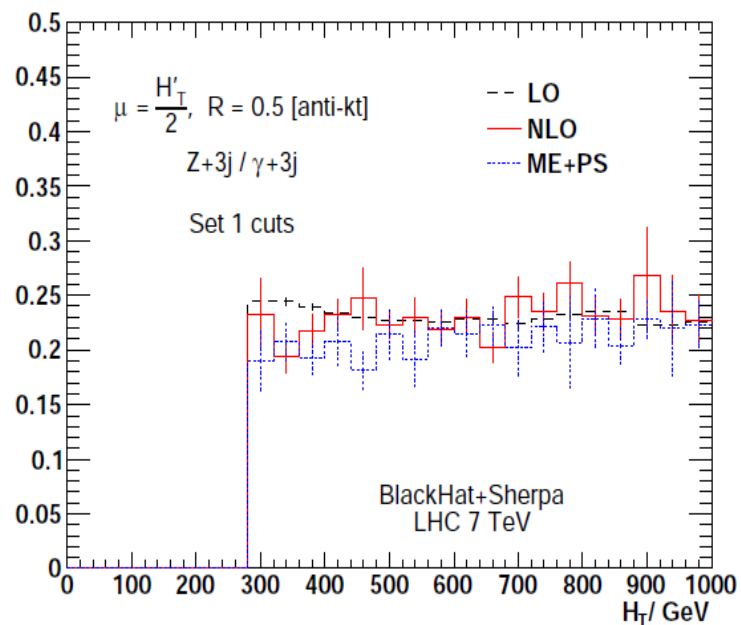


Background

measured

theory

Theory predictions

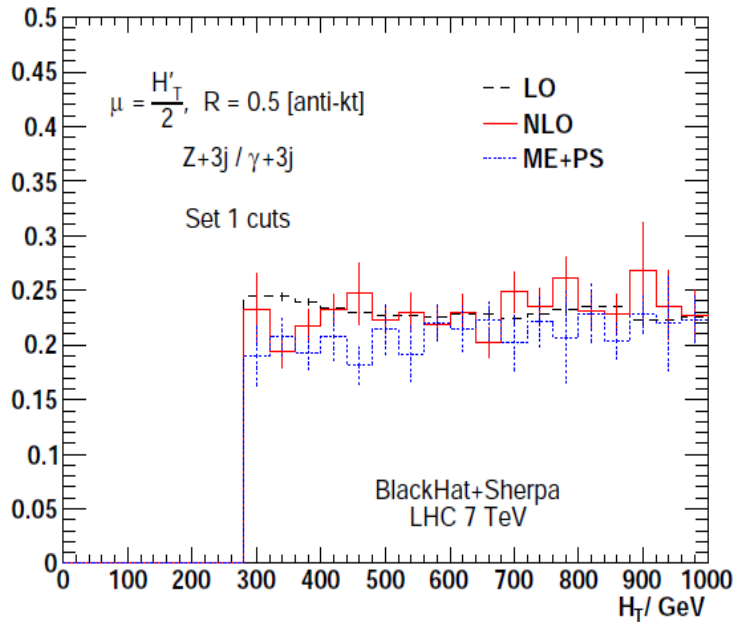


BlackHat

Virtual: On-shell and Unitarity cut techniques
Real : SHERPA

CMS and ATLAS use this
to estimate METZJ background
for SUSY searches

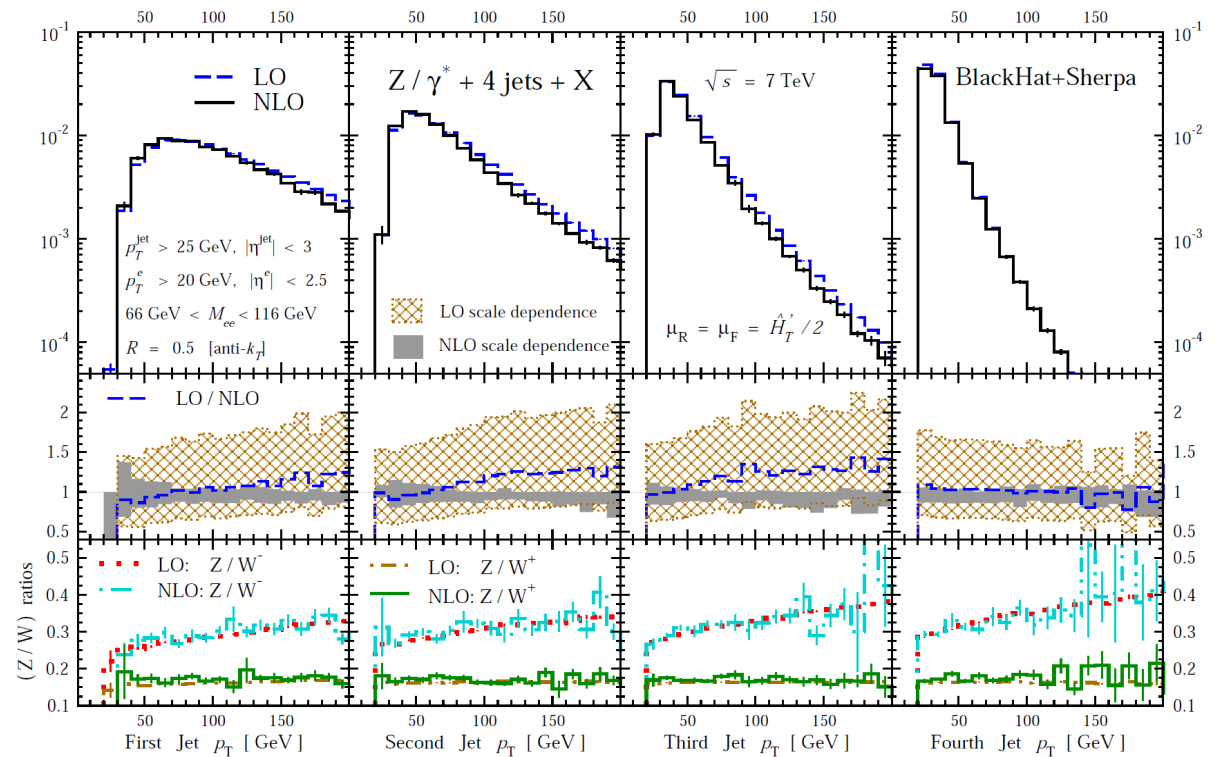
Theory predictions



CMS and ATLAS use this to estimate METZJ background for SUSY searches

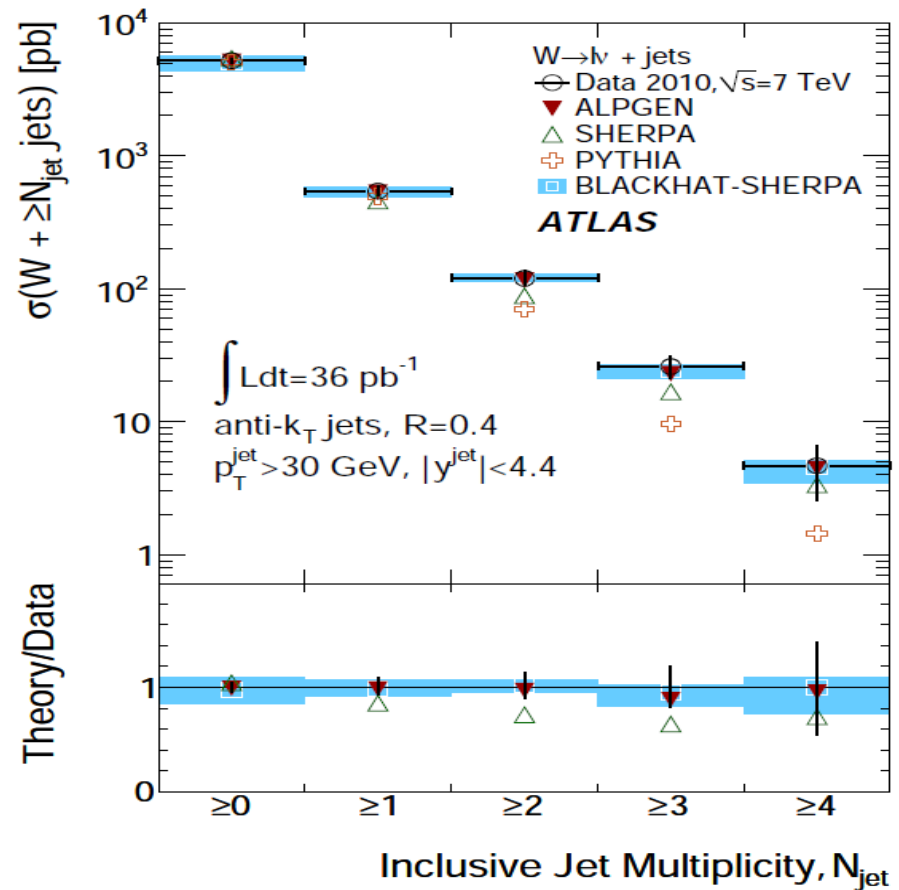
BlackHat

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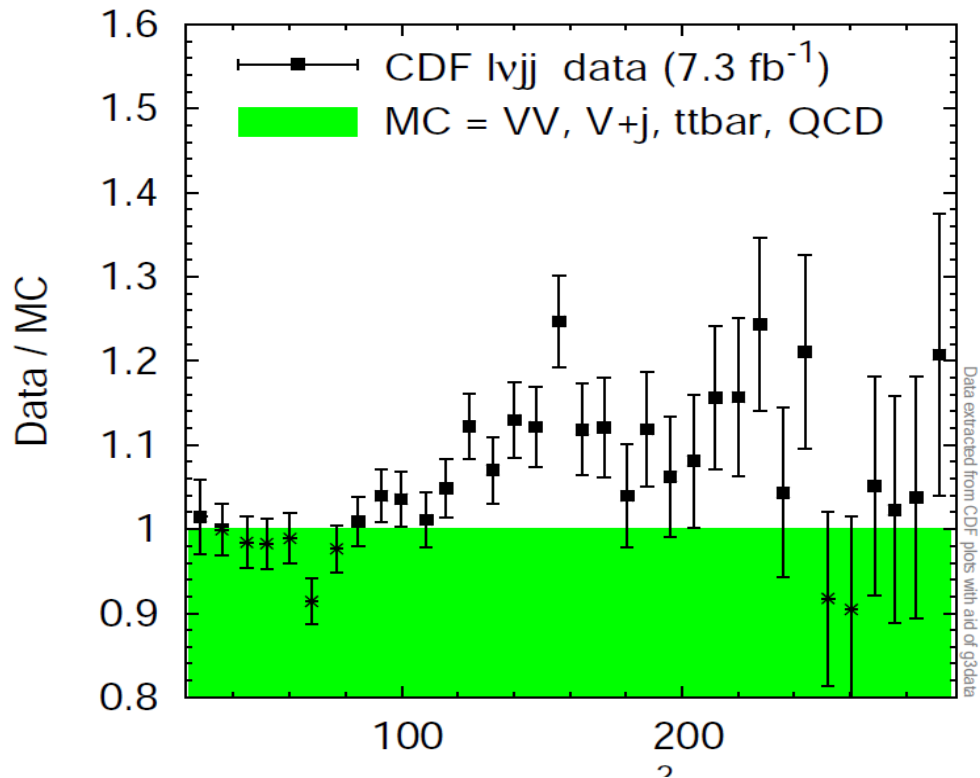


NLO predictions

- NLO tree level matrix elements beyond leading order
- Virtual corrections
- Parton showering
- Good agreement with data

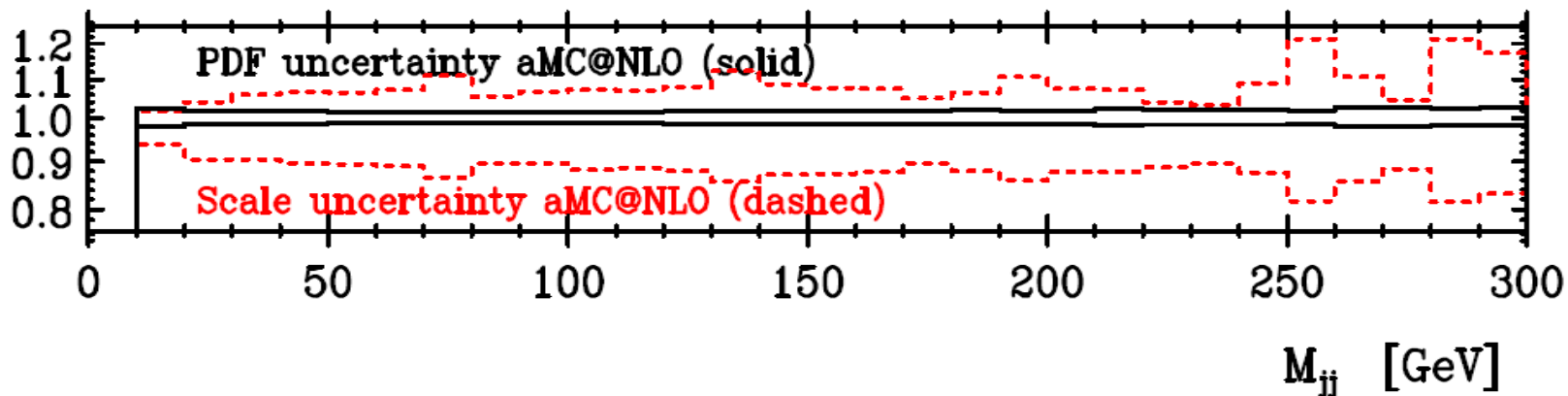


W+2 jet anomaly at CDF – NLO effect?



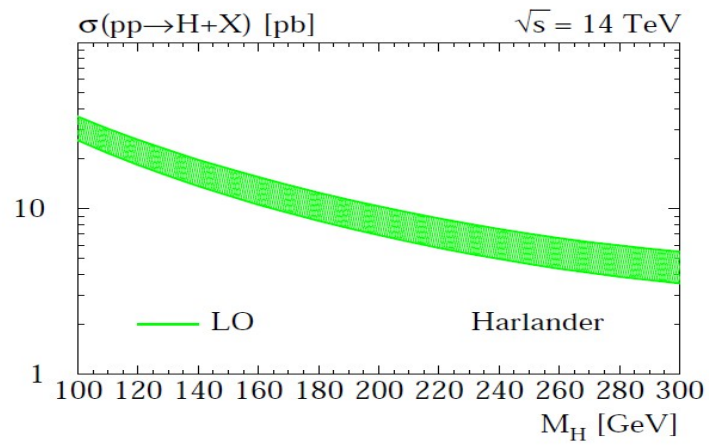
- CDF and D0 use ALPGEN and Parton showering for their analysis
- **AMC@NLO** Gives the similar prediction
- Theory uncertainty is comparable to Anomaly!

AMC@NLO

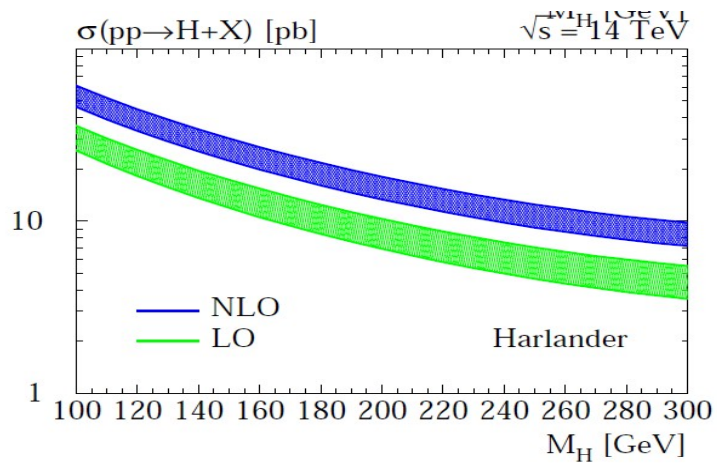
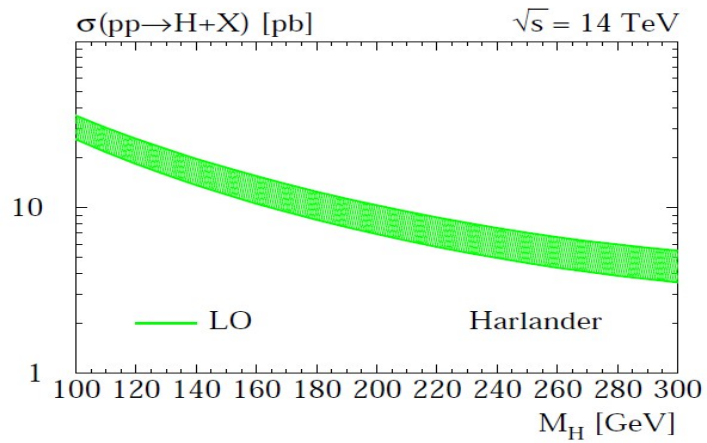


NNLO

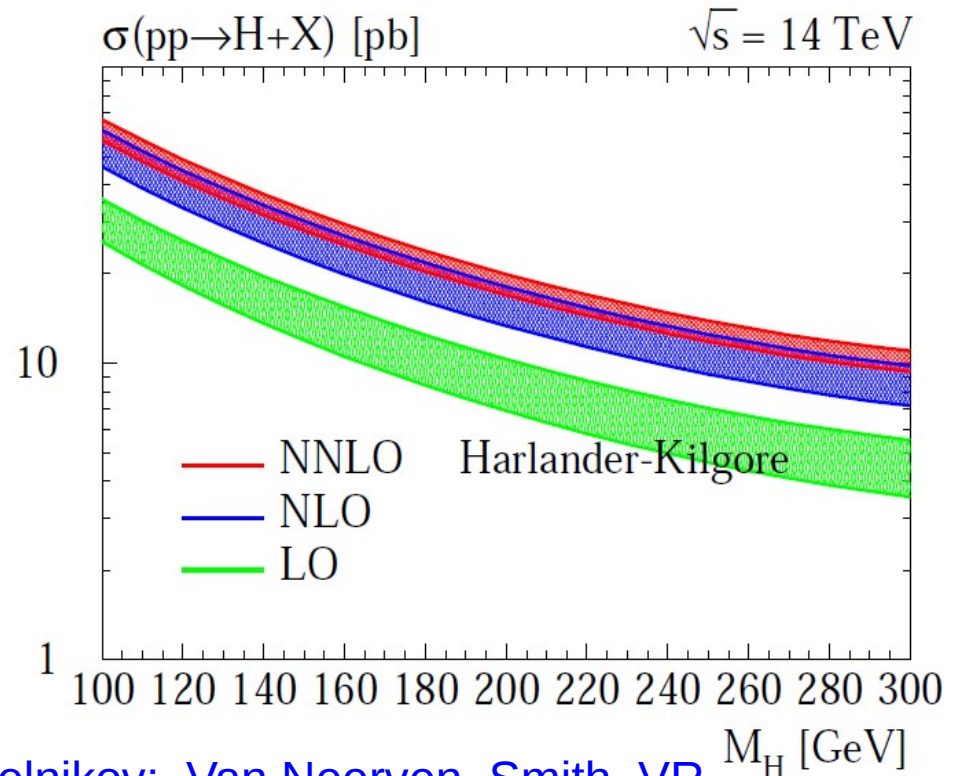
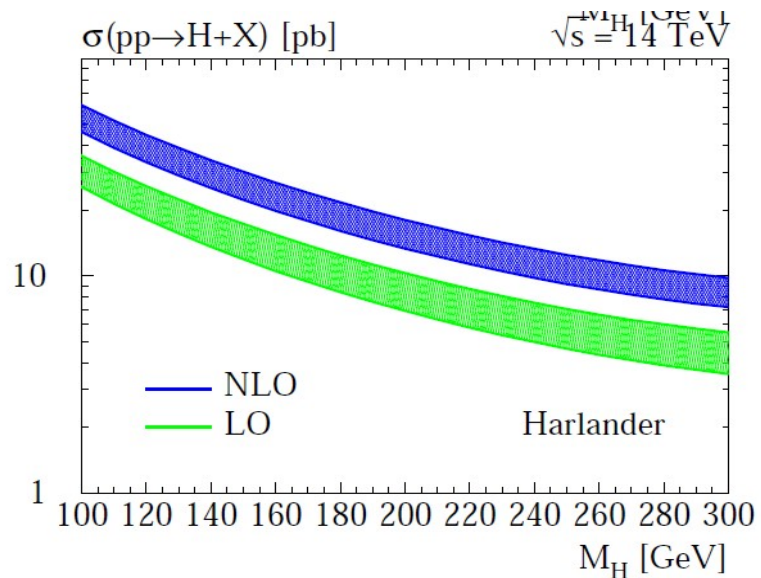
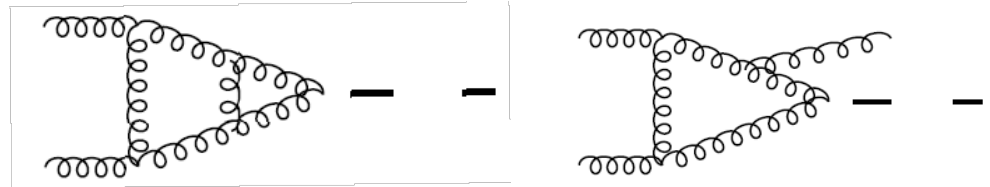
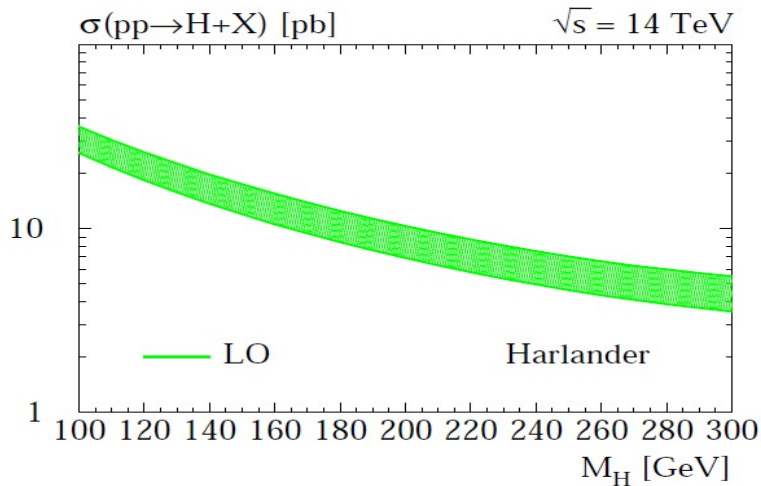
Higgs cross section at NNLO



Higgs cross section at NNLO



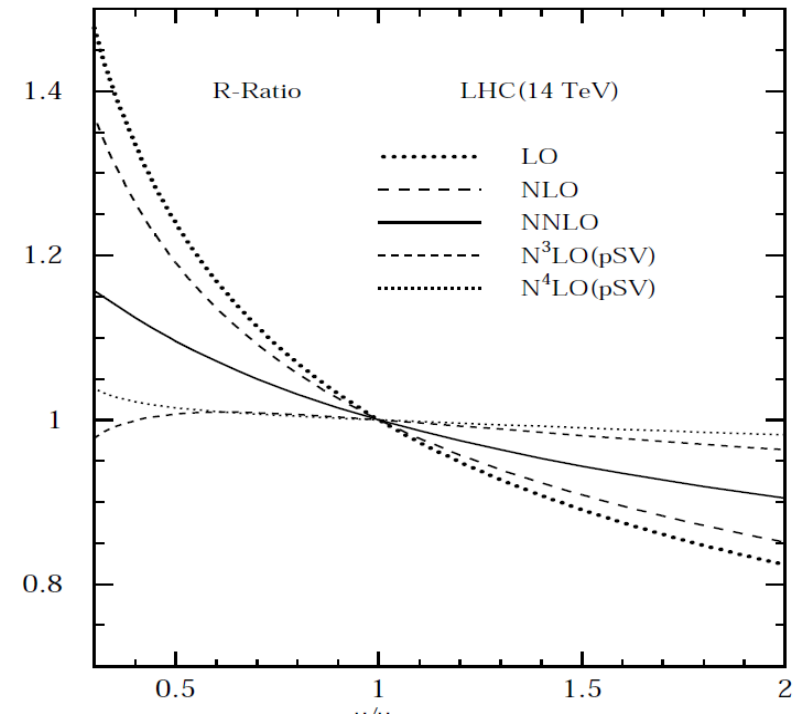
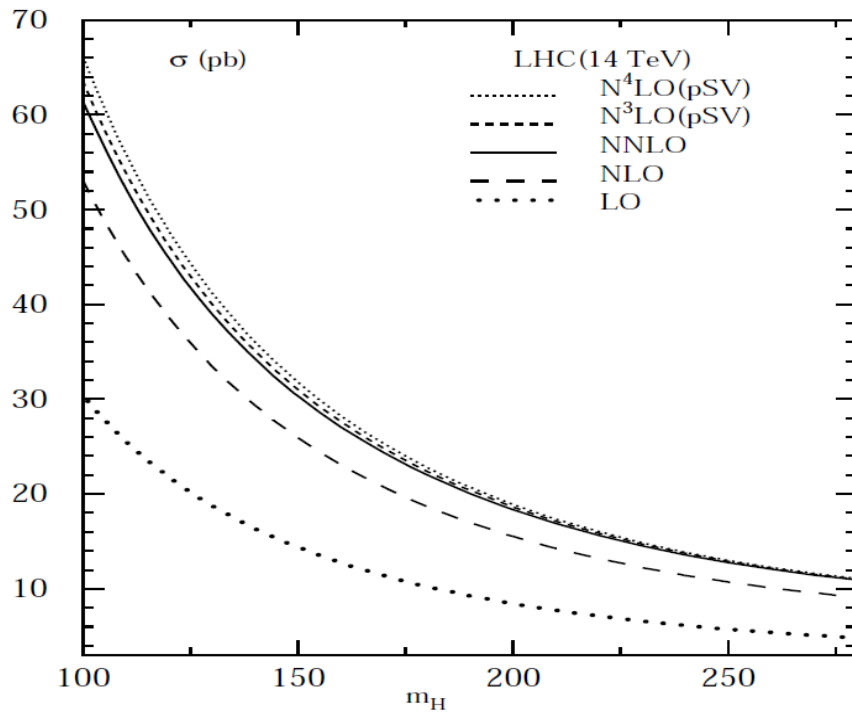
Higgs cross section at NNLO



Harlander, Kilgore; Anastasiou, Melnikov; Van Neerven, Smith, VR

Higgs Cross section at NNNLO (approx.)

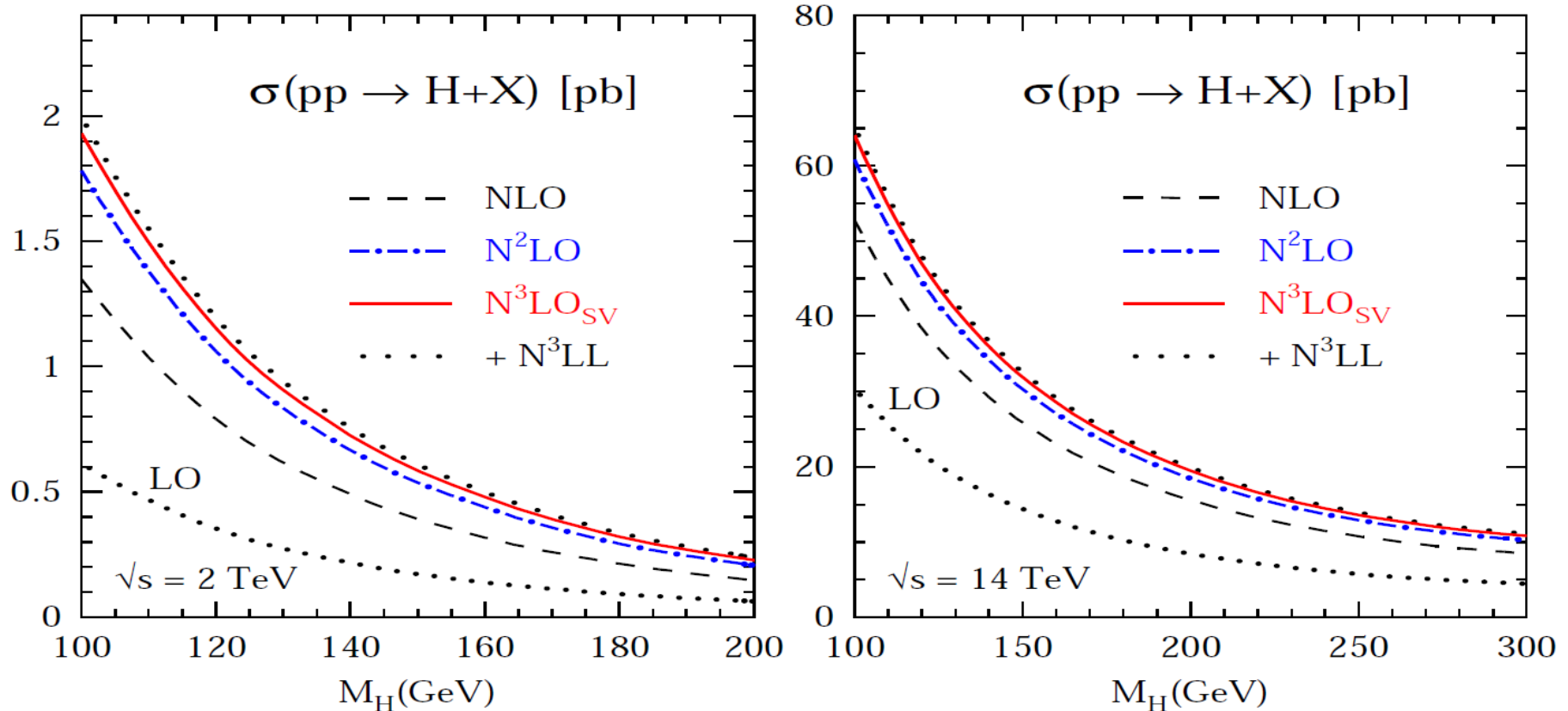
$$R = \frac{\sigma_{N^i LO}(\mu)}{\sigma_{N^i LO}(\mu_0)}$$



- Scale uncertainty improves a lot
- Additional 7 – 9% increase in cross section due to $N^3 LO$ soft gluons.

Resummed Higgs cross section

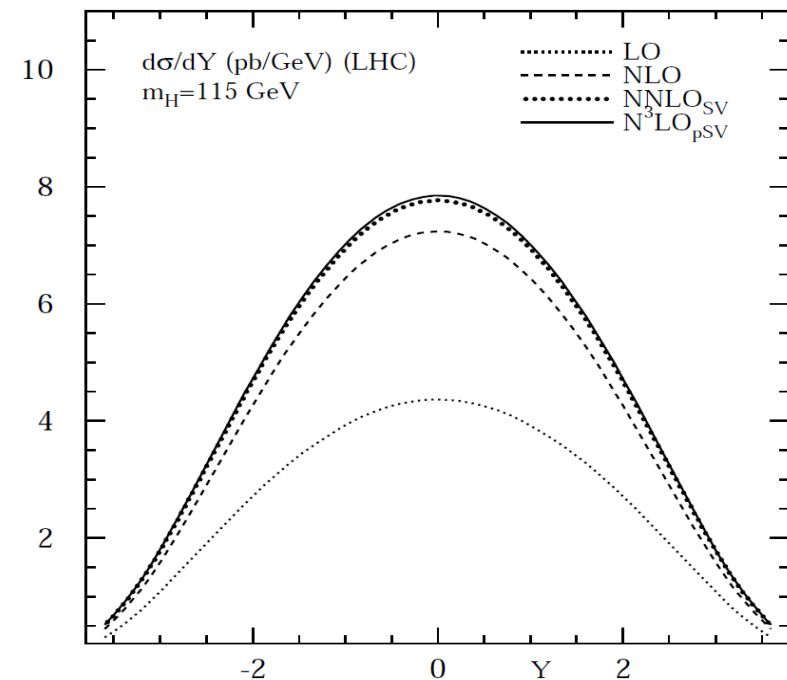
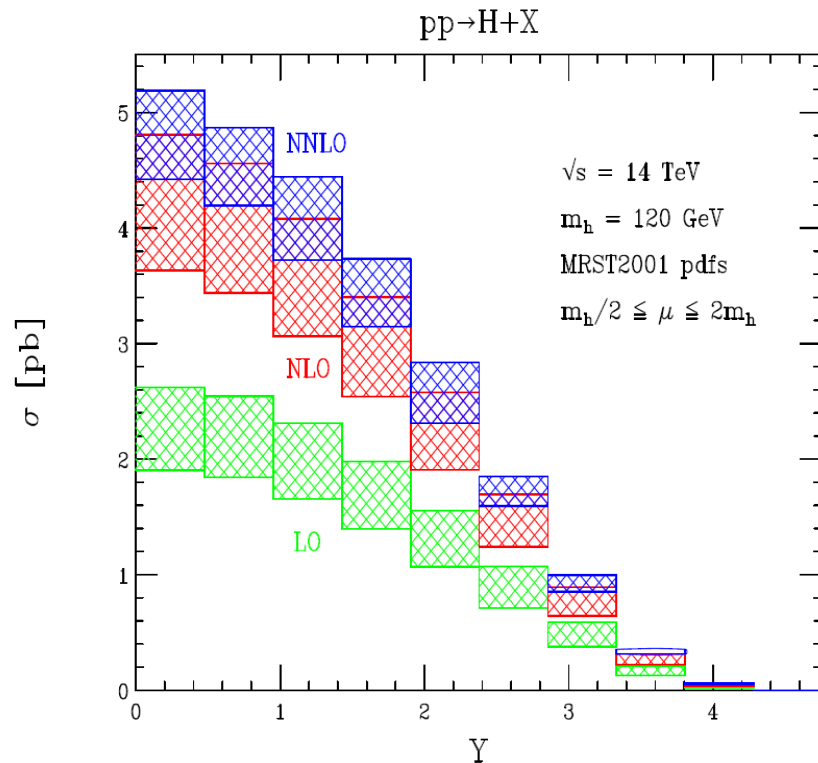
Catani and Grazzini; Vogt and Moch



- N^3LL resummation exponents are available now.
- N^3LL resummation does not change the picture much. Fixed order N^3LO_{pSV} is very close to the N^3LL resummed result.

Rapidity of Higgs and its scale dependence at $NNLO, N^3LO$

Anastaiou et al



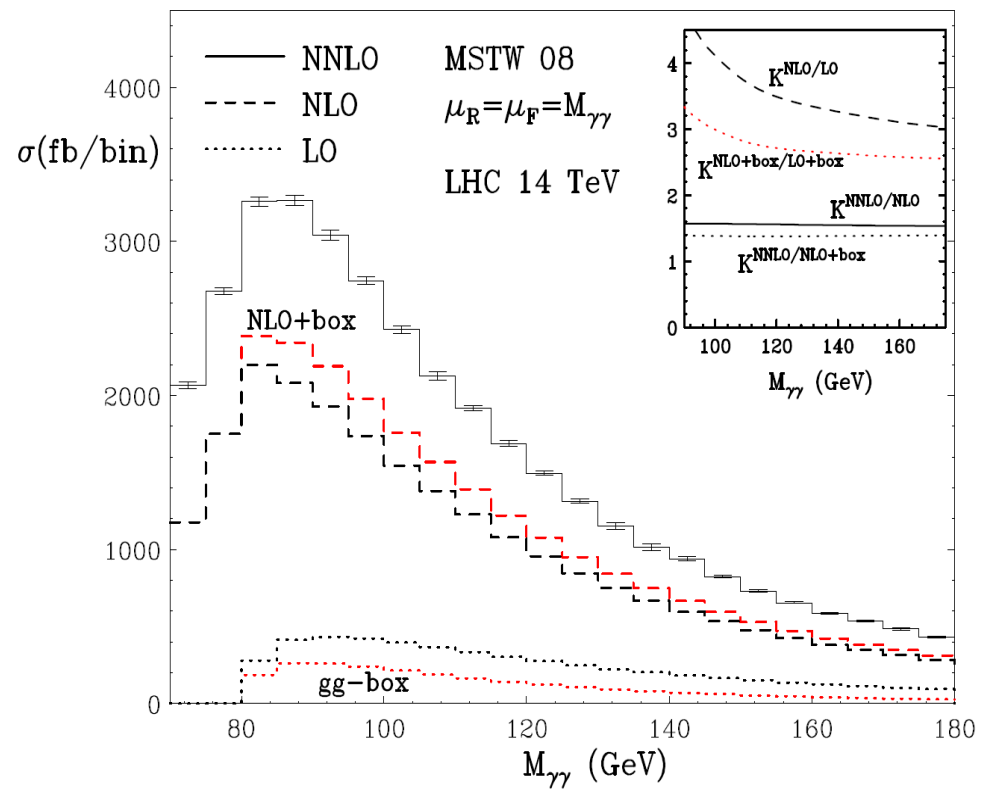
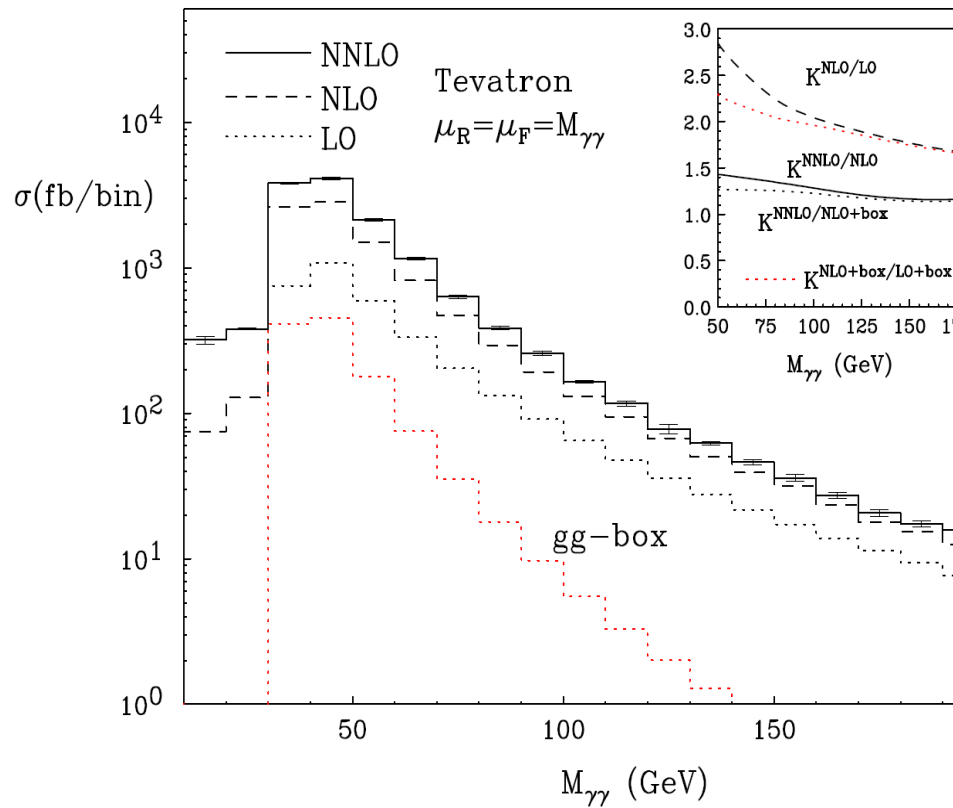
- NNLO exact in the large top limit reduces the scale uncertainty significantly
- One of the most difficult computations in QCD. Is it the end?

Di-photon at NNLO

Tevatron

LHC

Catani et al.



Cross section increases by 30-40%

Top quark production at NNLO

**Percent level precision physics at the Tevatron:
first genuine NNLO QCD corrections to $q\bar{q} \rightarrow t\bar{t} + X$**

Peter Bärnreuther and Michał Czakon
*Institut für Theoretische Teilchenphysik und Kosmologie,
RWTH Aachen University, D-52056 Aachen, Germany*

Alexander Mitov
Theory Division, CERN, CH-1211 Geneva 23, Switzerland
(Dated: April 25, 2012)

The total top quark pair production cross-section at hadron colliders through $\mathcal{O}(\alpha_S^4)$

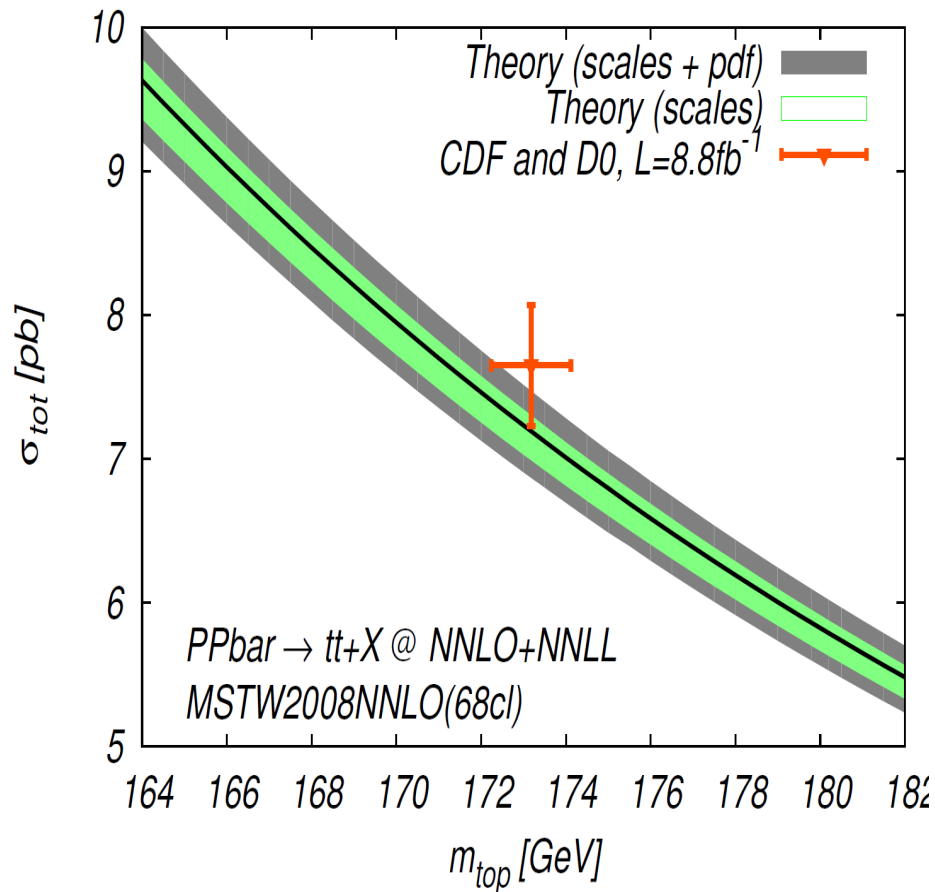
Michał Czakon and Paul Fiedler
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RWTH Aachen University, D-52056 Aachen, Germany*

Alexander Mitov
Theory Division, CERN, CH-1211 Geneva 23, Switzerland
(Dated: March 26, 2013)

Top@Tevatron and LHC

Czakon et al.

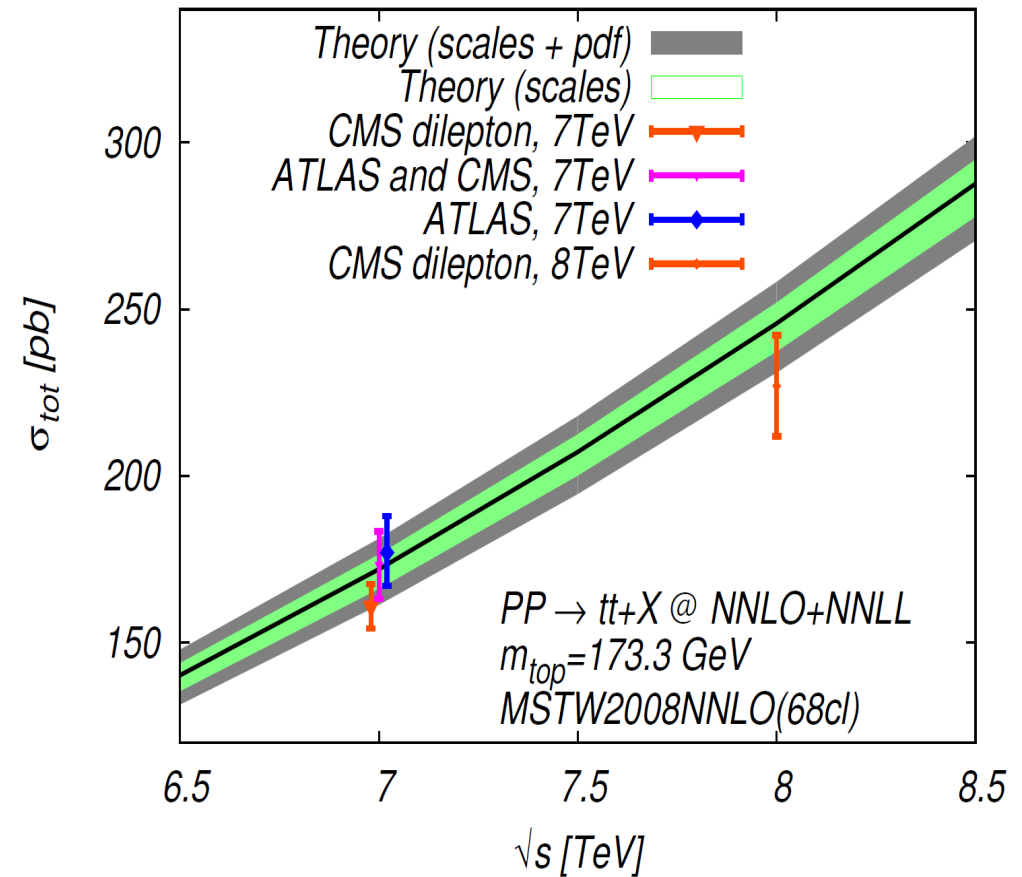
Tevatron



Scale dependence decreases 3-4 times
Compared NLO+ results

Scale uncertainty is 2.2%

LHC

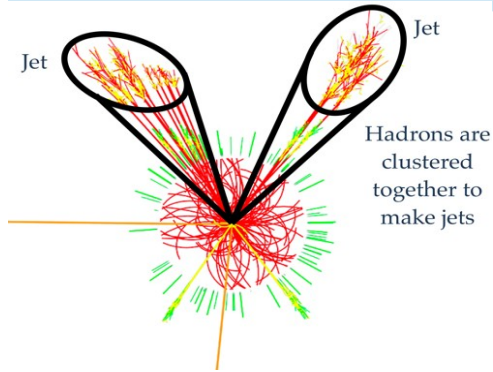


Scale dependence decreases 2-4 times
Compared to NLO+ results

Scale uncertainty is 3%

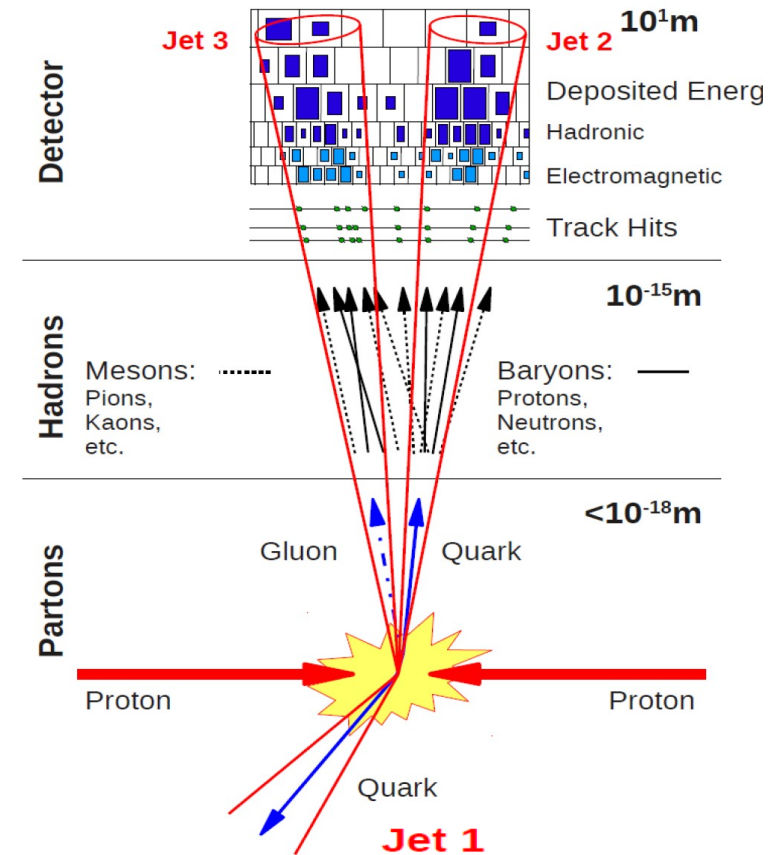
Jets

Infra-red safe observables

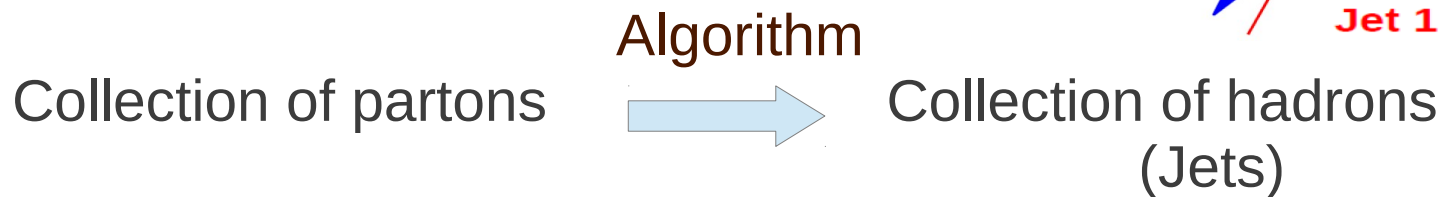


- We do not see quarks and gluons, we see only hadrons/bunch of hadrons (jets), leptons, photons, weak b

- Infra-red Safe observables are the only measurable
- How to construct infra-red safe quantities in QCD?
- Example: What is a Jet



Infra-red safe definition of a Jet



Jet Algorithms

- k_t Algorithm
- Cambridge/Aachen algorithm
- Anti- k_t algorithm

SIS Cone
ATLAS Cone
CMS Iterative Cone
GetJet
....
....

Successively Recombine the nearby partons

$$d_{ij} = \min(k_{t,i}^{2p}, k_{t,j}^{2p}) (\Delta y_{ij}^2 + \Delta\phi_{ij}^2)$$

$p = 1$: k_t algorithm

[Catani, Dokshitzer, Seymour, Webber, 93]

$p = 0$: Cambridge/Aachen (C/A) algorithm

[Dokshitzer, Leder, Moretti, Webber, 93]

$p = -1$: anti- k_t algorithm

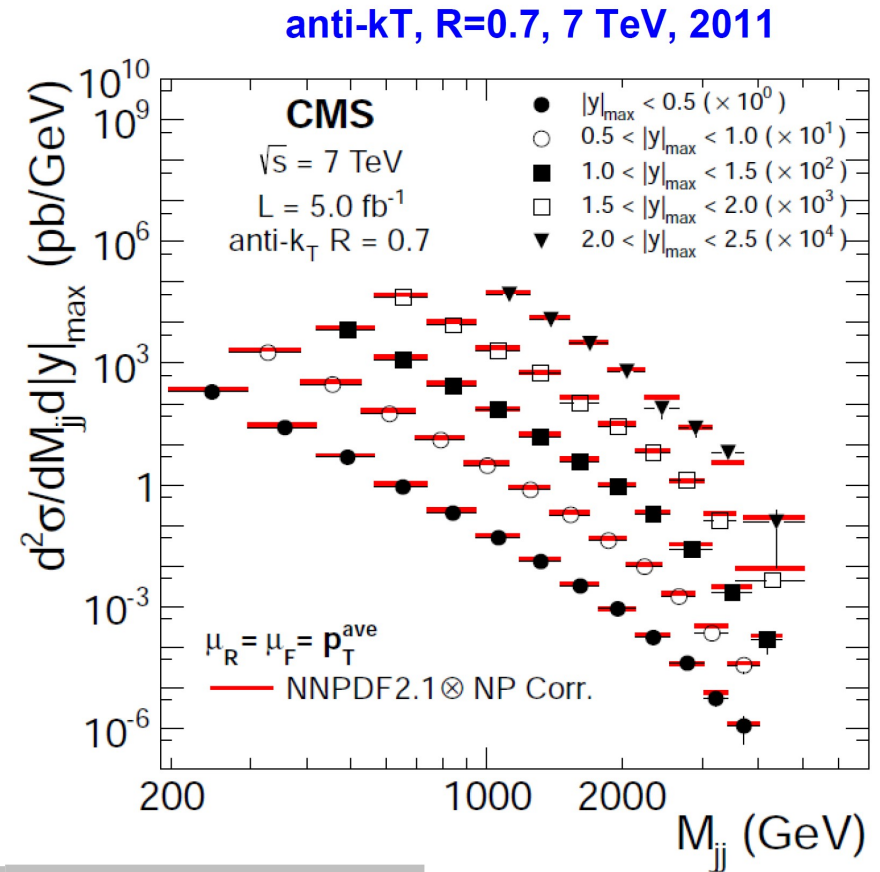
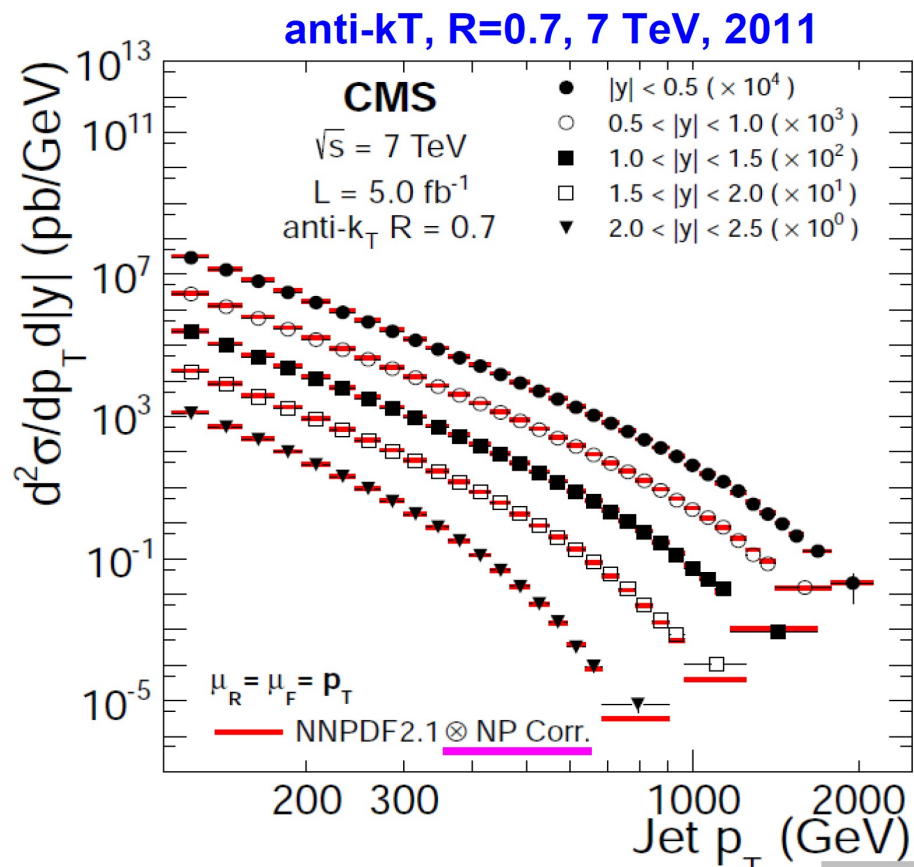
[Cacciari, Salam, GS, 08]

Cone: \approx flow of energy in a cone (of fixed R) centred on the cone

centre: **SISCone**

[Salam, GS, 07]

High Pt and invariant mass distributions of jets



Excellent agreement with NLO QCD predictions

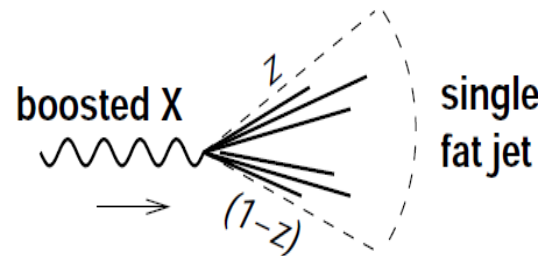
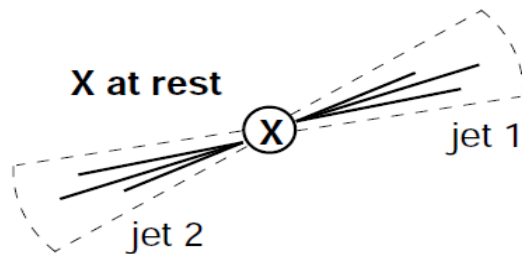
Fine Jets and Boosted Jets

Filtering: undo the last recombination, keep the subjets

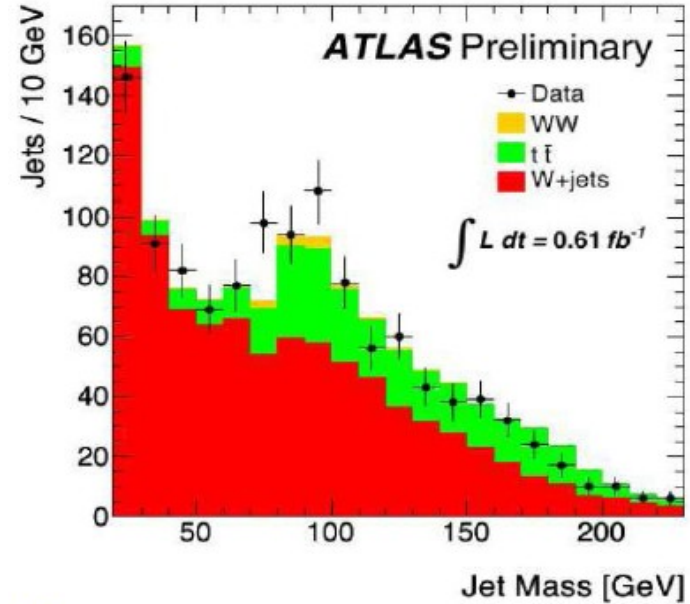
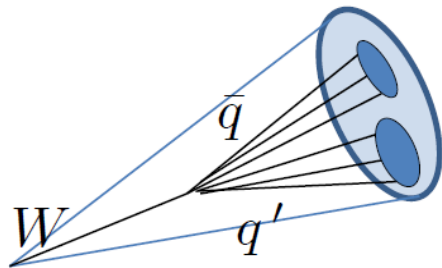
Trimming: remove low energetic deposits near a jet

Pruning: recluster each jet in way wide angle recombination are absent

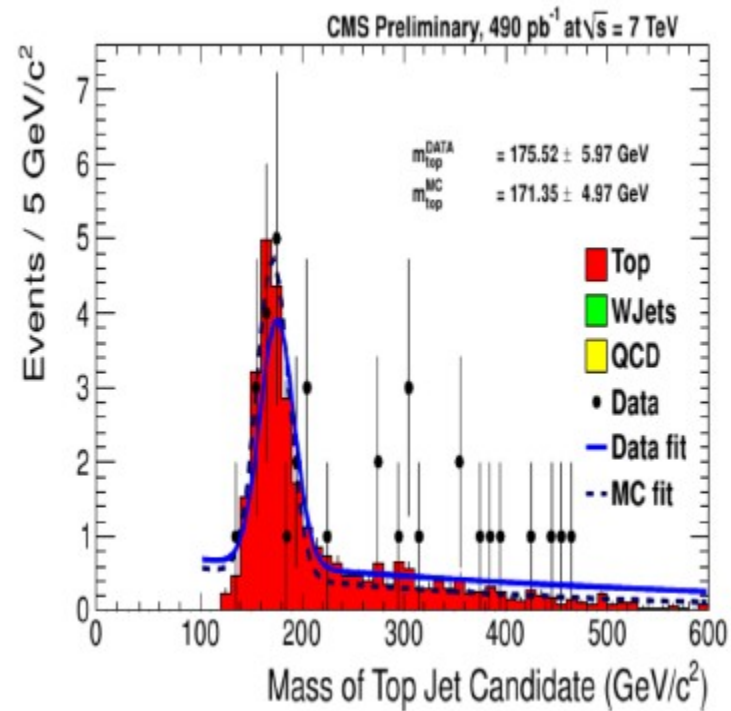
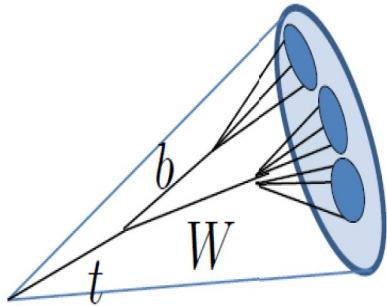
Boosted jets can probe Heavy states: **new physics**



Boosted Jet from W Boson



Boosted Jet from top quark



Conclusions

- QCD is a tool kit at Hadron Colliders
- Factorisation plays an important role for predictions
- Strong coupling constant and PDFs are under control
- Many NLO and few NNLO results are available to test SM and new physics
- Jet physics provides alternate ground for probing new physics.