Exercises for RF Tutorial — Solutions

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1 Cavities

Question 1

1. $\mathrm{c} = \lambda{\cdot}\mathrm{f} \rightarrow \lambda = \frac{\mathrm{c}}{\mathrm{f}} = \frac{3{\cdot}10^8\,\mathrm{ms}^{-1}}{150{\cdot}10^6\,\mathrm{s}^{-1}} = 2\,\mathrm{m}$

2. $\mathrm{a} = 0.383 \cdot \lambda = 0.383 \cdot 2\,\mathrm{m} = 0.766\ \mathrm{m} \approx 77\,\mathrm{cm} \rightarrow \mathrm{d} = 2{\cdot}\mathrm{a} = 1.54\,\mathrm{m}$

3. $\frac{\mathrm{R}}{\mathrm{Q}} = 185\frac{\mathrm{h}}{\mathrm{a}} \rightarrow \mathrm{h} = \frac{\mathrm{R}}{\mathrm{Q}} \cdot \frac{\mathrm{a}}{185} = 300\Omega \cdot \frac{0.77\,\mathrm{m}}{185\Omega} = 1.25\,\mathrm{m}$

4. $\mathrm{Q} = \frac{\mathrm{a}}{\delta}\left[1 + \frac{\mathrm{a}}{\mathrm{h}}\right]^{-1}$ with $\delta = \sqrt{\frac{2}{\omega\sigma\mu}} = \sqrt{\frac{1}{\pi\,\mathrm{f}\,\sigma\mu}}$

   Copper: $\mu = \mu_0 \cdot \mu_\mathrm{r}$; $\mu_0 = 4\pi \cdot 10^{-7}\,\frac{\mathrm{m\,kg}}{\mathrm{s^2\,A^2}}$, $\mu_\mathrm{r} = 1$; $\rightarrow \delta = \sqrt{\frac{1}{\pi 150{\cdot}10^6 58{\cdot}10^6 4\pi{\cdot}10^{-7}}} = 5.4 \cdot 10^{-6}\,\mathrm{m}$
   $\rightarrow \mathrm{Q} = \frac{0.77}{5.4{\cdot}10^{-6}} \cdot \left[1 + \frac{0.77}{1.25}\right]^{-1} = 88238$

   StSt: $\mu = \mu_0 \cdot \mu_\mathrm{r}$; $\mu_0 = 4\pi \cdot 10^{-7}\,\frac{\mathrm{m\,kg}}{\mathrm{s^2\,A^2}}$, $\mu_\mathrm{r} = 1$; $\rightarrow \delta = \sqrt{\frac{1}{\pi 150{\cdot}10^6 1.4{\cdot}10^6 4\pi{\cdot}10^{-7}}} = 3.5 \cdot 10^{-5}\,\mathrm{m}$
   $\rightarrow \mathrm{Q} = \frac{0.77}{3.5{\cdot}10^{-5}} \cdot \left[1 + \frac{0.77}{1.25}\right]^{-1} = 13614$

5. $\mathrm{R} = \frac{R}{Q} \cdot Q = 300 \cdot 88238 = 26.47 \cdot 10^6\,\Omega \approx 26\,\mathrm{M}\Omega$

   $\frac{R}{Q} = \omega L \rightarrow L = \frac{R}{Q}\frac{1}{\omega} = \frac{R}{Q}\frac{1}{2\pi f_\mathrm{res}} = 300\frac{1}{2\pi 150{\cdot}10^6} = 318.3\,\mathrm{nH}$

   $\frac{R}{Q} = \frac{1}{\omega C} \rightarrow C = \left[\frac{R}{Q}\omega\right]^{-1} = \left[\frac{R}{Q}2\pi f_\mathrm{res}\right]^{-1} = [300 \cdot 2\pi \cdot 150 \cdot 10^6]^{-1} = 3.5\,\mathrm{pF}$

Question 2

1. TM_{mnp}: $\lambda = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}} \rightarrow \mathrm{TM}_{101}, a = c$: $\lambda = \frac{2}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{a}\right)^2}} = \frac{2a}{\sqrt{2}} = \sqrt{2}a = \sqrt{2} \cdot 100 = 141.4\,\mathrm{mm}$

   $\mathrm{c} = \lambda \cdot f \rightarrow f_\mathrm{res} = \frac{\mathrm{c}}{\lambda} = \frac{3{\cdot}10^8}{141.4{\cdot}10^{-3}} = 2.12\,\mathrm{GHz}$

2. $\delta = \sqrt{\frac{2}{\omega\sigma\mu}} = \sqrt{\frac{2}{2\pi f_\mathrm{res}\sigma\mu}} = \sqrt{\frac{2}{2\pi 2.12{\cdot}10^9 58{\cdot}10^6 4\pi{\cdot}10^{-7}}} = 1.43\,\mu\mathrm{m}$

   $\mathrm{Q} = \frac{\lambda}{\delta}\frac{b}{2}\frac{(a^s + c^2)^{\frac{3}{2}}}{c^3(a+2b) + a^3(c+2b)} \rightarrow a = c, \lambda = \sqrt{2}a$: $\mathrm{Q} = \frac{1}{\delta}\frac{ab}{a+2b} = \frac{1}{1.43{\cdot}10^{-2}}\frac{100{\cdot}50}{100+2{\cdot}50} = 17422$

3. $\frac{1}{Q_\mathrm{L}} = \frac{1}{Q_0} + \frac{1}{Q_\mathrm{ext}} \rightarrow Q_\mathrm{L} = Q_\mathrm{ext}$: $\frac{1}{Q_\mathrm{L}} = \frac{2}{Q_0} \rightarrow Q_\mathrm{L} = \frac{Q_0}{2} = \frac{17422}{2} = 8711$

4. $Q = \frac{f_\mathrm{res}}{\Delta f} \rightarrow \Delta f = \frac{f_\mathrm{res}}{Q} \rightarrow$ loaded cavity bandwidth: $\Delta f = \frac{f_\mathrm{res}}{Q_\mathrm{L}} = \frac{2.12{\cdot}10^9}{8711} = 243.6\,\mathrm{kHz}$

5. At critical coupling all power is going into the cavity and no power is reflected. Hence, all power is thermally dissipated: $\mathrm{P_{in}} = \mathrm{P_{TH}} = 50\,\mathrm{W}$

6. $Q = \frac{\omega W}{\mathrm{P}} \rightarrow W = \frac{Q\mathrm{P}}{\omega} = \frac{Q\mathrm{P}}{2\pi f_{\text{res}}} \rightarrow$ loaded cavity: $W = \frac{Q_{\mathrm{L}}\mathrm{P}_{\text{in}}}{2\pi f_{\text{res}}} = \frac{8711\cdot 50}{2\pi 2.12\cdot 10^{9}} = 32.7\,\mu\mathrm{J}$

Question 3

1. $\lambda = 2.61a = 417.6\,\mathrm{mm}$

   $\mathrm{c} = \lambda \cdot f \rightarrow f_{\text{res}} = \frac{\mathrm{c}}{\lambda} = \frac{3\cdot 10^{8}}{417.6\cdot 10^{-3}} = 718\,\mathrm{MHz}$

2. $\delta = \sqrt{\frac{2}{\omega\sigma\mu}} = \sqrt{\frac{2}{2\pi f_{\text{res}}\sigma\mu}} = \frac{2}{2\pi 718\cdot 10^{6} 1.4\cdot 10^{6} 4\pi\cdot 10^{-7}} = 15.9\,\mu\mathrm{m}$

   $Q = \frac{a}{\delta}\left[1 + \frac{a}{h}\right]^{-1} \rightarrow h = \frac{aQ\delta}{a - Q\delta} = \frac{160\cdot 10^{-3} 400015.9\cdot 10^{-6}}{160\cdot 10^{-3} - 400015.9\cdot 10^{-6}} = 105.2\,\mathrm{mm}$

3.
4. $Q = \frac{f_{\text{res}}}{\Delta f} \rightarrow \Delta f = \frac{f_{\text{res}}}{Q} = \frac{718\cdot 10^{6}}{4000} = 179.6\,\mathrm{kHz}$

5. $\frac{R}{Q} = 185\frac{h}{a} = 185\frac{105.2}{160} = 122\,\Omega$

   Since the $\frac{R}{Q}$ value is a geometrical factor only, it is independent of the material of the cavity.

6. $R = \frac{R}{Q}Q = 122 \cdot 4000 = 488\,\mathrm{k}\Omega$

   $\frac{R}{Q} = \omega L \rightarrow L = \frac{R}{Q}\frac{1}{\omega} = \frac{R}{Q}\frac{1}{2\pi f_{\text{res}}} = 122\frac{1}{2\pi 718\cdot 10^{6}} = 27\,\mathrm{nH}$

   $\frac{R}{Q} = \frac{1}{\omega C} \rightarrow C = \left[\frac{R}{Q}\omega\right]^{-1} = \left[\frac{R}{Q}2\pi f_{\text{res}}\right]^{-1} = [122 \cdot 2\pi \cdot 718 \cdot 10^{6}]^{-1} = 1.8\,\mathrm{pF}$

7. $R = \frac{V^2}{2\mathrm{P}} \rightarrow V = \sqrt{2\mathrm{P}R} = \sqrt{2 \cdot 10 \cdot 488 \cdot 10^{3}} = 3.1\,\mathrm{kV}$

   $k^2 = \frac{R}{R_{\text{in}}} \rightarrow k = \sqrt{\frac{R}{R_{\text{in}}}} = \sqrt{\frac{488\cdot 10^{3}}{50}} = 98.8$

8. $\delta = \sqrt{\frac{2}{\omega\sigma\mu}} = \sqrt{\frac{2}{2\pi f_{\text{res}}\sigma\mu}} = \frac{2}{2\pi 718\cdot 10^{6} 58\cdot 10^{6} 4\pi\cdot 10^{-7}} = 2.5\,\mu\mathrm{m}$

   $Q = \frac{a}{\delta}\left[1 + \frac{a}{h}\right]^{-1} = \frac{160\cdot 10^{-3}}{2.5\cdot 10^{-6}}\left[1 + \frac{160}{105.2}\right]^{-1} = 25746$

   $R = \frac{R}{Q}Q = 122 \cdot 25746 = 3.1\,\mathrm{M}\Omega$

   $R = \frac{V^2}{2\mathrm{P}} \rightarrow V = \sqrt{2\mathrm{P}R} = \sqrt{2 \cdot 10 \cdot 3.1 \cdot 10^{6}} = 7.9\,\mathrm{kV}$

2 Decibel

Question 4

The solutions are printed in bold font:

Voltage ratio	Power ratio	dB
3.1623	**10**	**10**
10	100	**20**
100	**10000**	40

Question 5

The solutions are printed in bold font:

dBm (50 Ω)	RMS Voltage (50 Ω)	milli Watt
0	**0.224 V**	**1**
+ 30	**7.1 V**	**1000**
– 60	**0.224 mV**	$\mathbf{10^{-6}}$
+ 20	**2.23 V**	100

To determine the RMS Voltage, the following relations can be used:
P = U · I, I = U/R → P = U^2/R → U = $\sqrt{P \cdot R}$

3 Multiple choice

Question 6

1. How will the resonant frequency f_{res} of the E_{010} (TM_{010}) mode of a pill box cavity change if height of the cavity is doubled? (check 1)
   ○ The f_{res} decreases by a factor 2
   ○ The f_{res} decreases by a factor $\sqrt{2}$
   ○ The f_{res} increases by a factor 2
   ○ The f_{res} increases by a factor $\sqrt{2}$
   ○ **The f_{res} will not change** The resonance frequency is only dependent on the radius for this mode.
2. A critically coupled aluminum pill-box cavity is driven by an RF generator with an output power of 100 kW. How much power would be dissipated by the cavity if it were made of silver? $\sigma_{\text{Aluminium}} = 38 \cdot 10^6$ S/m, $\sigma_{\text{Silver}} = 63 \cdot 10^6$ S/m. Note: the silver cavity would also be critically coupled (check 1)
   ○ The power dissipation decreases by a factor $\sqrt{\frac{\sigma_{\text{Aluminium}}}{\sigma_{\text{Silver}}}}$
   ○ The power dissipation increases by a factor $\sqrt{\frac{\sigma_{\text{Aluminium}}}{\sigma_{\text{Silver}}}}$
   ○ **The power dissipation will not change** Critical coupling is independent of the material.
3. Calculate the minimal thickness of a copper shielding box if we want to allow less than 1% of 50 Hz currents flowing in the internal side of the box walls. $\sigma_{\text{Copper}} = 58 * 106$ S/m, $\mu = \mu_0 \mu_r$, $= 4\pi \cdot 10^{-7}$ Vs/Am, $< 1\% \approx 5$ sigma (check 1)
   ○ 46.7 **mm** δ = 9.3 mm at 50 Hz which corresponds to $1\sigma \rightarrow 5\sigma = 5 \cdot 9.3 \cdot 10^{-6}$ = 46.7 mm
   ○ 4.67 mm
   ○ 0.46 mm
   ○ 0.046 mm
4. A rectangular waveguide has a width of a = 10 cm. (check 2)
   ○ The mode TE_{10} or H_{10} has a cutoff frequency of 3 GHz
   ○ **The mode TE_{10} or H_{10} has a cutoff frequency of 1.5 GHz** 10 cm = $\frac{\lambda}{2} \rightarrow f_{\text{res}} = \frac{c}{\lambda} = \frac{3 \cdot 10^8}{20 \cdot 10^{-2}}$ = 1.5 GHz
   ○ The electric field is parallel to the side with the larger dimensions
   ○ **The electric field is orthogonal to the side with the larger dimensions** see cross section pictures
5. Which mode is the fundamental mode (lowest cut-off frequency) in a cylindrical waveguide of circular cross-section without inner conductor? (check 1)

   ○ **TE** separation condition/mode chart
   ○ TEM
   ○ TM

6. Which mode is the fundamental mode in a cylindrical waveguide with inner conductor (coaxial line)? (check 1)

   ○ TE
   ○ **TEM**   not simply connected cross section, thus electrostatic potential between inner and outer conductor possible
   ○ TM

7. Adding capacitive loading to a cavity (check 1)

   ○ **lowers the resonance frequency** $f_{res} \propto \frac{1}{\sqrt{C}} \to C \uparrow \Leftrightarrow f_{res} \downarrow$
   ○ does not affect the resonance frequency
   ○ increases the resonance frequency

8. Advantages of a nose cone cavity compared to an ordinary pill box cavity of same dimension (check 1)

   ○ Smaller skin depth
   ○ **Higher R/Q**   field concentration near nose cone increases R/Q
   ○ Higher Q

9. Superconducting cavities usually do not have nose cones because (check 2)

   ○ Superconductors are expensive, so don't waste them for nose cones
   ○ **Nose cones are sensitive to multipactoring, which causes excessive heating and must therefore be avoided**
   ○ The shunt impedance is so high that it can't be increased any more by changing the geometry
   ○ **Superconductors are sensitive to high electric field around the nose cones**

10. When doing numerical simulations, geometrical symmetries are exploited in order to (check 2)

    ○ ensure convergence of the simulation algorithms for resonant structures
    ○ **reduce calculation time**
    ○ account for the transit time factor
    ○ **rule out certain higher order modes**

11. The GSM standard specifies a minimum sensitivity requirement of about -100 dBm, while the maximum output power is in the order of 1 W. This corresponds to how many orders of magnitude in power? (Exact values: -102 dBm minimum sensitivity, 1 to 5 W maximum output power) (check 1)

    ○ 5
    ○ 8
    ○ **13** -100 dBm = 10^{-10} for mW, 1 mW = 10^{-3} W $\to$ 13 orders of magnitude

12. When you cover then antenna of your mobile with your hand while using it, the attenuation caused is in the order of 20 dB. Human tissue is a rather good absorber, so you can neglect reflections for this calculation. How many percent of the mobile's output power stay in the head and hand? (check 1)

    ○ 9
    ○ **99** -20 dB = 10^{-2} = 0.01 = 1% goes through $\to$ 99% are absorbed
    ○ 99.99

4 Resonant circuits and impedance plane

Question 7

1. $\omega_{\text{res}} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-7}\,\text{m}^2\,\text{kg}\,\text{s}^{-2}\,\text{A}^{-2}\cdot 10^{-11}\,\text{s}^4\,\text{A}^2\,\text{m}^2\text{kg}^{-1}}} = \frac{1}{\sqrt{10^{-18}}}\,\text{s}^{-1} = 1\,\text{GHz} \rightarrow \text{f} = \frac{1}{2\pi}\cdot\omega$

   $\frac{R}{Q} = \omega\cdot L \rightarrow Q = \frac{R}{\omega L} = \frac{10^3\,\text{kg}\,\text{m}^2\,\text{s}^{-3}\,\text{A}^{-2}}{10^9\,\text{s}^{-1}\cdot 10^{-7}\,\text{m}^2\,\text{kg}\,\text{s}^{-2}\,\text{A}^{-2}} = 10$

   $Q = \frac{f_{\text{res}}}{\Delta f} \rightarrow \Delta f = \frac{f_{\text{res}}}{Q} = \frac{159\cdot 10^6}{10} = 15.9\,\text{MHz}$

   $Q = \frac{\omega_{\text{res}}}{\Delta\omega} \rightarrow \Delta\omega = \frac{\omega_{\text{res}}}{Q} = 100\,\text{MHz}$

2. To sketch the circuits in either plane, we calculate the **admittances** for certain 'strategic' frequencies:

   $\omega = 0$:

   $Z_C = -j\,\frac{1}{\omega C} = -j\,\infty \qquad Y_C = j\,\frac{1}{\infty} = j\,0$
   $Z_L = j\,\omega\,L = j\,0 \qquad Y_L = -j\,\frac{1}{0} = -j\,\infty$
   $Z_R = 10^3 \qquad Y_R = 10^{-3}$

   this leads to:

   $Y_{\text{tot}} = Y_C + Y_L + Y_R = j\,0 - j\,infty + 10^{-3} = 10^{-3} - j\,infty$

   $Z_{\text{tot}} = \frac{1}{Y_{\text{tot}}} = \frac{1}{10^{-3} - j\,\infty} = \frac{10^{-3} + j\,\infty}{10^{-6} + \infty^2} = \frac{10^3}{\infty^s} + j\,\frac{\infty}{\infty^2} = 0$

   $\omega = \infty$:

   after applying the same principle as above, we get:

   $Y_{\text{tot}} = 10^{-3} + j\,\infty$

   $Z_{\text{tot}} = 0$

   $\omega = \omega_{\text{res}}$:

   $Z_C = -j\,\frac{1}{\omega C} = -j\,\frac{1}{10^9\cdot 10^{-11}} = -j\,100 \qquad Y_C = j\,\frac{1}{100} = j\,0.01$
   $Z_L = j\,\omega\,L = j\,10^9\cdot 10^{-7} = j\,100 \qquad Y_L = -j\,0.01$
   $Z_R = 10^3 \qquad Y_R = 10^{-3}$

   this leads to:

   $Y_{\text{tot}} = Y_C + Y_L + Y_R = j\,100 - j\,100 + 10^{-3} = 10^{-3}$

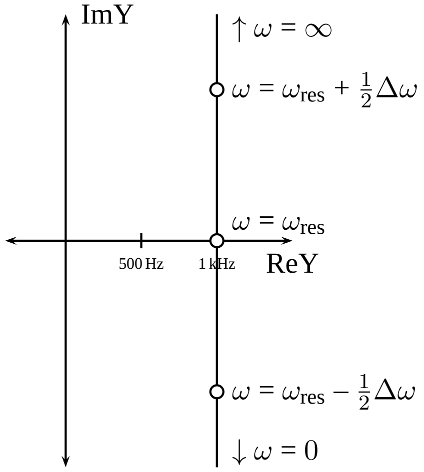
   $Z_{\text{tot}} = \frac{1}{Y_{\text{tot}}} = 10^3$

   $\omega = \omega_{\text{res}} \pm \frac{\Delta f}{2}$:

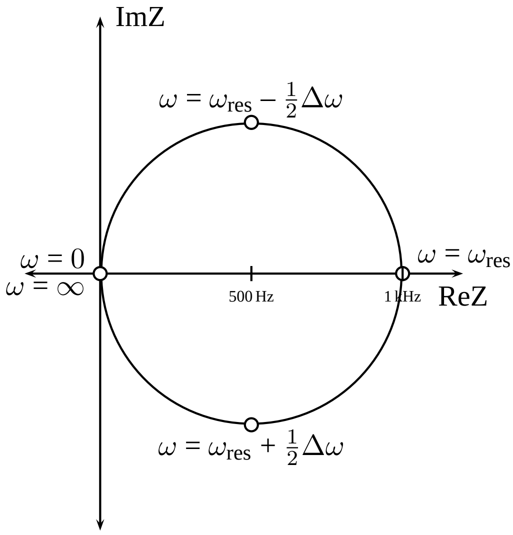
$Y_{tot} = 10^{-3} \pm j\,0.01$

$Z_{tot} = 0.5 \cdot 10^3 \mp j\,0.5 \cdot 10^3$

plotting the circuit in the admittance plane looks like:



3. in the impedance plane:



1. $\omega_{res} = 2\pi\, f_{res} = 100\,\text{MHz}$

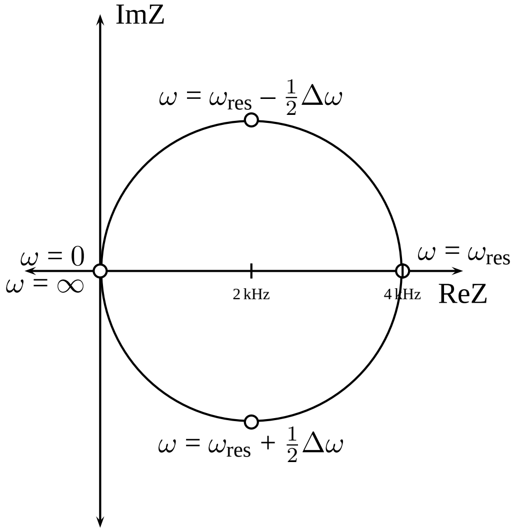
   $\frac{R}{Q} = \omega_{res}\, L \rightarrow L = \frac{R}{Q} \cdot \frac{1}{\omega_{res}} = 2\,\mu\text{H}$

   $\omega_{res} = \frac{1}{\sqrt{L\,C}} \rightarrow C = \frac{1}{\omega^2\, L} = 50\,\text{pF}$

2. $Q = \frac{\omega_{res}}{\Delta\omega} \rightarrow \Delta\omega = \frac{\omega_{res}}{Q} = 5\,\text{MHz}$

Frequency	Admittance	Impedance
0	$-\mathrm{j}\,\infty$	0
$\omega_{\mathrm{res}} - \frac{\Delta\omega}{2}$	$0.25 \cdot 10^{-3} - \mathrm{j}\,0.25 \cdot 10^{-3}$	$2 \cdot 10^{3} + \mathrm{j}\,2 \cdot 10^{3}$
ω_{res}	$0.25 \cdot 10^{3-}$	$4 \cdot 10^{3}$
$\omega_{\mathrm{res}} + \frac{\Delta\omega}{2}$	$0.25 \cdot 10^{-3} + \mathrm{j}\,0.25 \cdot 10^{-3}$	$2 \cdot 10^{3} - \mathrm{j}\,2 \cdot 10^{3}$
∞	$\mathrm{j}\,\infty$	0

Sketching this in the impedance plane, we get:



5 Transmission lines and striplines

Question 9

1. $\mathrm{Z} = \sqrt{\frac{L'}{C'}} \rightarrow L' = \mathrm{Z}^2 C'$

   $\mathrm{v} = \frac{1}{\sqrt{L'C'}} \rightarrow L' = \frac{1}{\mathrm{v}^2 C'}$

   $\rightarrow \frac{1}{\mathrm{v}^2 C'} = \mathrm{Z}^2 C' \rightarrow C' = \frac{1}{\mathrm{vZ}} = \frac{1}{0.5 \cdot 3 \cdot 10^8 \cdot 75} = 88.9\,\mathrm{pF}$

   $\rightarrow L' = \mathrm{Z}^2 C' = 75^2 \cdot 88.9 \cdot 10^{-12} = 500\,\mathrm{nH}$

2. $\mathrm{v} = \frac{\mathrm{c}}{\mu_{\mathrm{r}}\epsilon_{\mathrm{r}}} \rightarrow \epsilon_{\mathrm{r}} = \left[\frac{\mathrm{v}^2}{\mathrm{c}^2}\mu_{\mathrm{r}}\right]^{-1} = \frac{0.5 \cdot \mathrm{c}^s}{\mathrm{c}^2} = 4$
3. $\mathrm{Z} = \sqrt{\frac{\mu_{\mathrm{r}}}{\epsilon_{\mathrm{r}}}} 60 \ln\left(\frac{R}{r}\right) \rightarrow r = R \cdot \mathrm{e}^{-\frac{\mathrm{Z}}{60}\sqrt{\frac{\epsilon_{\mathrm{r}}}{\mu_{\mathrm{r}}}}} = 10 \cdot \mathrm{e}^{\frac{75}{60}\frac{1}{\sqrt{4}}} = 0.82\,\mathrm{mm}$

Question 10

1. $\mathrm{Z} = \frac{60\Omega}{\sqrt{\epsilon_r}} \cdot \ln\left[\frac{1.9b}{0.8w+t}\right] = \frac{60\Omega}{\sqrt{4}} \cdot \ln\left[\frac{1.9 \cdot 15}{0.8 \cdot 3.1 + 0.02}\right] = 73\,\Omega$

2. $\mathrm{Z} = \sqrt{\epsilon_\mathrm{r}}\frac{1}{C'\mathrm{c}} \to C' = \sqrt{\epsilon_\mathrm{r}}\frac{1}{\mathrm{Zc}} = \sqrt{4}\frac{1}{73\cdot 3\cdot 10^8} = 91.3\,\mathrm{pF/ul}$

   $\mathrm{Z} = \sqrt{\frac{L'}{C'}} \to L' = \mathrm{Z}^2 C' = 73^2 \cdot 91.3 \cdot 10^{-12} = 486.7\,\mathrm{nH/ul}$

3. $\mathrm{v} = \frac{\mathrm{c}}{\sqrt{\epsilon_\mathrm{r}}} = \frac{\mathrm{c}}{\sqrt{4}} = 0.5\,\mathrm{c}$

Question 11

1. $\mathrm{Z} = \sqrt{\epsilon_\mathrm{r}}\frac{1}{C'\mathrm{c}} \to C' = \sqrt{\epsilon_\mathrm{r}}\frac{1}{\mathrm{Zc}} = \sqrt{2.1}\frac{1}{50\cdot 3\cdot 10^8} = 96.6\,\mathrm{pF/ul}$

   $\mathrm{Z} = \sqrt{\frac{L'}{C'}} \to L' = \mathrm{Z}^2 C' = 50^2 \cdot 96.6 \cdot 10^{-12} = 241.5\,\mathrm{nH/ul}$

2. $\mathrm{v} = \frac{\mathrm{c}}{\sqrt{\epsilon_\mathrm{r}}} = \frac{\mathrm{c}}{\sqrt{2.1}} = 0.69\,\mathrm{c}$

6 S-parameters

Question 12

Component	Isolator	Circulator	Transmission line, length $\lambda/2$	3 dB attenuator
S-matrix	S_2	S_4	S_3	S_1

Question 13

Transmission line:

$$S_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & \mathrm{e}^{-\mathrm{j}\beta l} \\ \mathrm{e}^{-\mathrm{j}\beta l} & 0 \end{bmatrix} = \begin{bmatrix} 0 & \mathrm{e}^{-\mathrm{j}\frac{2\pi}{\lambda}l} \\ \mathrm{e}^{-\mathrm{j}\frac{2\pi}{\lambda}l} & 0 \end{bmatrix} = \begin{bmatrix} 0 & \mathrm{e}^{-\mathrm{j}\frac{2\pi}{\lambda}\frac{\lambda}{4}} \\ \mathrm{e}^{-\mathrm{j}\frac{2\pi}{\lambda}\frac{\lambda}{4}} & 0 \end{bmatrix} = \begin{bmatrix} 0 & \mathrm{e}^{-\mathrm{j}\frac{\pi}{2}} \\ \mathrm{e}^{-\mathrm{j}\frac{\pi}{2}} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\mathrm{j} \\ -\mathrm{j} & 0 \end{bmatrix}$$

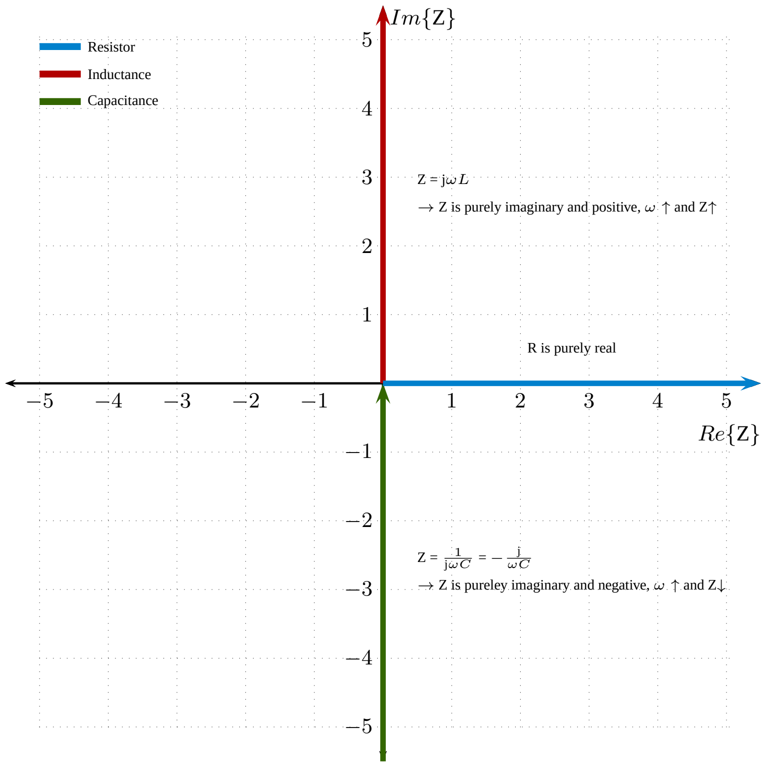
Amplifier:

$$S_2 = \begin{bmatrix} 0 & 0 \\ \sqrt{10} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 3.16 & 0 \end{bmatrix} \tag{1}$$

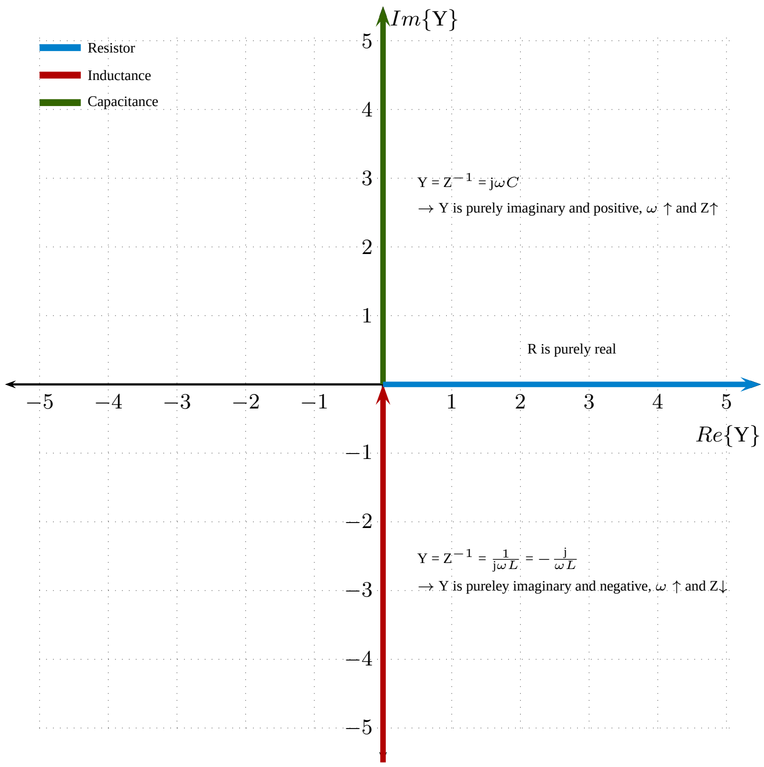
7 Impedances and Smith chart

Question 14

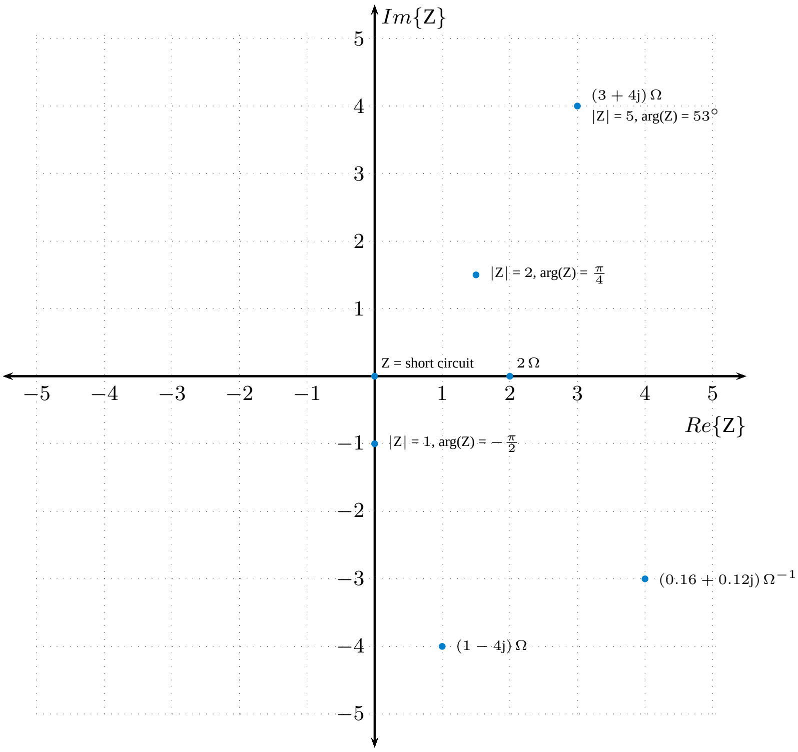
1. Impedance plane



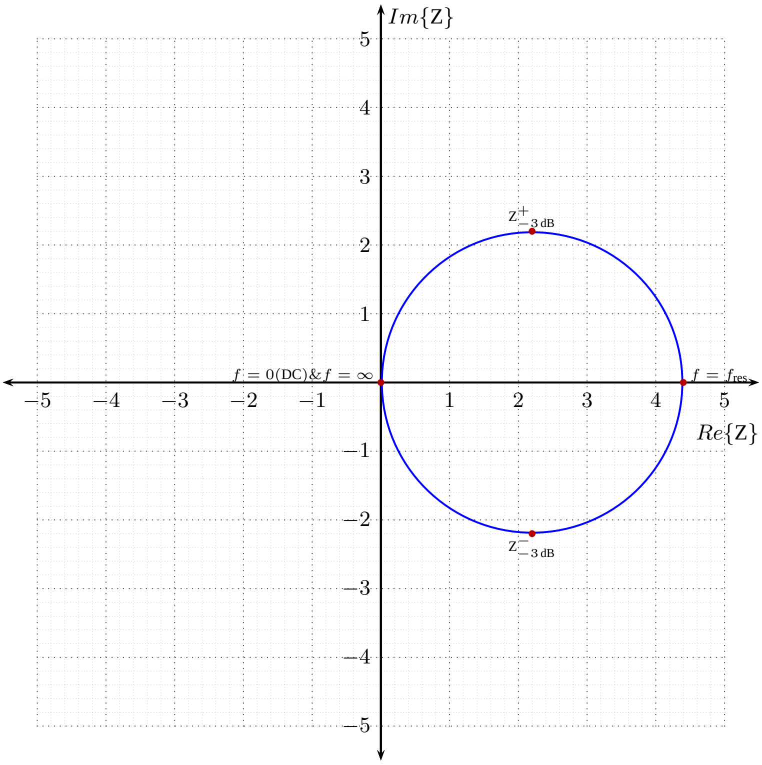
2. Admittance plane



Question 15



Question 16



$$f_{\text{res}} = \frac{1}{2\pi}\frac{1}{\sqrt{LC}} = \frac{1}{2\pi}\frac{1}{\sqrt{150\cdot 10^{-9}\cdot 80\cdot 10^{-12}}} = 45.9\,\text{MHz}$$

Question 17

1. At resonance $Im\{\mathrm{Z}\} = 0 \rightarrow f_{\mathrm{res}} = f_7 = 105.2\,\mathrm{MHz}$

2. Lower $3\,\mathrm{dB}$ point: $\arg\{\mathrm{Z}\} = 45° \rightarrow f^{-}_{-3\mathrm{dB}} = f_2$

   Upper $3\,\mathrm{dB}$ point: $\arg\{\mathrm{Z}\} = -45° \rightarrow f^{+}_{-3\mathrm{dB}} = f_5$

   $\mathrm{BW} = f_5 - f_2 = 300\,\mathrm{kHz}$

3. The resonant circuit is a parallel resonator with RLC

4. $R = \mathrm{Z}(f_7) = 230\,\mathrm{k\Omega}$

5. The locus of impedance is a vertical line in the Y-plane with $\mathrm{Re}\{\mathrm{Y}\} = 4.35\,\mu\mathrm{S}$ and $|\mathrm{Y}_{-3\mathrm{dB}}| = 6.15\,\mu\mathrm{S}$

6. $Q = \frac{f_{\mathrm{res}}}{\Delta f_{\mathrm{res}}} = \frac{105.2 \cdot 10^6}{300 \cdot 10^3} = 350$

   $\frac{R}{Q} = \omega L \rightarrow L = \frac{R}{Q}\frac{1}{\omega} = \frac{230 \cdot 10^3}{350}\frac{1}{2\pi 105.2 \cdot 10^6} = 994\,\mathrm{nH}$

   $\frac{R}{Q} = \frac{1}{\omega C} \rightarrow C = \left[\frac{R}{Q}\omega\right]^{-1} = 2.3\,\mathrm{pF}$

Question 18 - Question 20

Check your results with the online tool (Dellsperger)!