JUAS

Joint Universities Accelerator School

MINI-WORKSHOP DESIGNING A SYNCHROTRON

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Introduction

- **AIM:**

  To design a 20 GeV/c proton synchrotron cycling at 3.125 Hz with an intensity of 3×10^{13} protons per pulse [one pulse = all the particles in the ring] with dispersion-free regions for injection, extraction and RF.

- **ASSUME:**

  A fast cycling booster with a maximum momentum of 3.5 GeV/c. The booster can have several rings stacked on top of each other. The booster delivers 1-sigma, normalised emittances of 40π mm mrad horizontally and 20π vertically and 0.1 eV s longitudinally. Assume there are no losses and no emittance blowup.

Putting “it” together

The SPS Design Committee get down to business (1971).

Organisation

A. Project Leader :

Organises parameter list and forms working groups.

B. Lattice Working Group

1. **Choose a basic lattice cell and calculate its parameters.**
2. **Decide geometry and strategy for dispersion and find transition energy.**
3. **Calculate beam size, aperture, magnet design, power converter design etc.**

C. RF Working Group

4. **Type & number of cavities and parameters.**
5. **RF frequency, voltage and synchronous phase programmes.**

D. Collective Effects Working Group

6. **Position in tune diagram and self space-charge tune shifts.**
7. **Instabilities.**
8. **Calculate beam energy per pulse and power.**

Collect "given" information

- This forms the basis of the ***Parameter List***.
- The Parameter List can be an active spread sheet for calculating on-line the effect of changes in the design and the values of frequently-used parameters.

  - First part of Parameter List is for the booster synchrotron that will be injecting into the main synchrotron.

  - Second part of the Parameter List is for the main synchrotron.

Parameter list spreadsheet

Parameter List for Synchrotron and Booster

Bold entries can be edited and the spreadsheet will recalculate.

Constants:

Proton mass [GeV]		**0.938255**			
Velocity of light [m/s]		**2.9799E+08**			
Electronic charge [A s]		**1.6022E-19**			
Free space permeability [H/m]		**1.2566E-06**			
Free space permittivity [F/m]		**8.8542E-12**			
Free space impedance [Ω]		**3.7673E+02**			
Planck's constant/π [J s]		**1.0546E-34**			
Classical proton radius [m]		**1.53E-18**			
Booster:	Number of rings	**4**			
(fast cycling)	Average radius	**37.5**		Circumference [m]	235.619449
	Dipole occupation [fraction]	**0.28**		Length [m]	65.97344573
	Extraction momentum [GeV/c]	**3.5**		Top field [T]	1.111866667
				Gamma at extraction	3.862040361
				Beta at extraction	0.965895958
				Extraction kinetic energy [GeV]	2.685323679
				Rigidity at extraction [Tm]	11.674600
				Extraction revolution period [s]	8.186139E-07
	Harmonic number	**4**		RF frequency [Hz]	4.886309E+06
	Bunching factor (peak/average)	**5.66**		Bunch lgth/RF period	0.249861053
				Extracted bunch lgth [s]	5.12E-08
				Gap for kicker [s]	1.54E-07
	Kicker rise or fall time [s]	**1.00E-07**			
	Assume multi-turn injection with 20 turns with 50% efficiency after bunching giving 6.E12 particles/ring				
	Particles per ring	**6.00E+12**		Particles/bunch	1.50E+12
	Norm. H emit. [π mm mrad]	**40**		Geom. H. emit.[π mm mrad]	10.72291429
	Norm. V emit. [π mm mrad]	**20**		Geom. V. emit.[π mm mrad]	5.361457143
	Longitudinal emittance [eV s]	**0.1**			

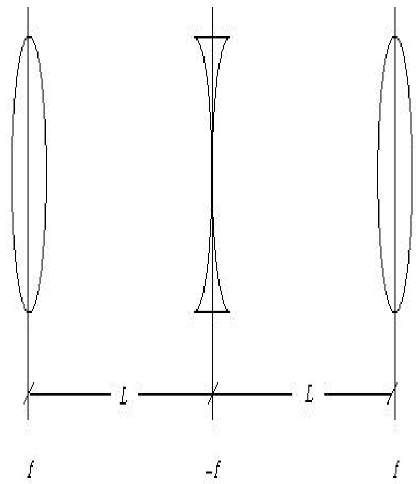
- **The left hand side is input data that can be edited and the right hand side is calculated.**
- **You will find this spreadsheet on your CD-ROM – except that some formulæ have been removed!**
- **Your first job is to enter as many of the formulæ as you can!**

Lattice working group

- **The spreadsheet is fine, but can we find a lattice that fits the spreadsheet data?**
- **If we base ourselves on the JUAS course, then a FODO is the natural choice, but anyone who has other ideas is welcome to propose an alternative.**
- **On the CD-ROM you will find a FODO cell with 60 degree phase advances and some intermediate steps towards making a ring. The ring lattice obtained provides dispersion-free straight sections for the cavities and for injection/extraction.**
- **Try analysing the FODO with the thin-lens formulæ.**

Analytic thin-lens formulæ

- **A FODO is an example of an alternating-gradient lattice. By alternating focusing & defocusing lenses, an overall focusing situation can be created. Remember quadrupoles focus in one plane and defocus in the other.**



- **Multiply the thin-lens matrices**

$$M=\begin{pmatrix}1 & 0\\ \mp 1/2f & 1\end{pmatrix}\begin{pmatrix}1 & L\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ \pm 1/f & 1\end{pmatrix}\begin{pmatrix}1 & L\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ \mp 1/2f & 1\end{pmatrix}$$

$$=\begin{pmatrix}1-L^2/2f^2 & , & 2L(1\pm L/2f)\\ -L/2f^2(1\mp L/2f)\,, & & 1-L^2/2f^2\end{pmatrix}=\begin{pmatrix}\cos\mu+\alpha\,\sin\mu\,, & \beta\,\sin\mu\\ -\gamma\,\sin\mu & ,\ \cos\mu-\alpha\,\sin\mu\end{pmatrix}$$

- **Comparing the terms to the general Twiss transfer matrix gives:**

$$\left.\begin{aligned}\cos\mu &= 1-L^2/2f^2\\ \sin(\mu/2) &= L/2f\end{aligned}\right\}$$

$$\left.\begin{aligned}\beta &= 2L\frac{[1\pm\sin(\mu/2)]}{\sin\mu}\\ \alpha_{x,z} &= 0\,.\end{aligned}\right\}$$

$$\frac{\hat{\beta}}{\breve{\beta}}=\frac{1+\sin(\mu/2)}{1-\sin(\mu/2)}\,,$$

Some approximate values

- You can get approximate values for the transition energy, the average betatron amplitude function and dispersion as follows:
- The tune of the machine comes from the number of cells,

$$N\mu = 2\pi Q$$

- The tune is also given by the integral of 1/β, so that

$$\int \frac{ds}{\beta} = \int d\mu$$

$$\frac{2\pi R}{\overline{\beta}} = 2\pi Q$$

$$\therefore \quad \overline{\beta} = \frac{R}{Q}$$

and

$$\gamma_{tr} \approx Q_x$$

$$\frac{1}{\gamma_{tr}^{\,2}} = \frac{\overline{D}_x}{R}$$

$$\therefore \overline{D} = \frac{R}{Q_x^{\,2}}$$

Accurate values

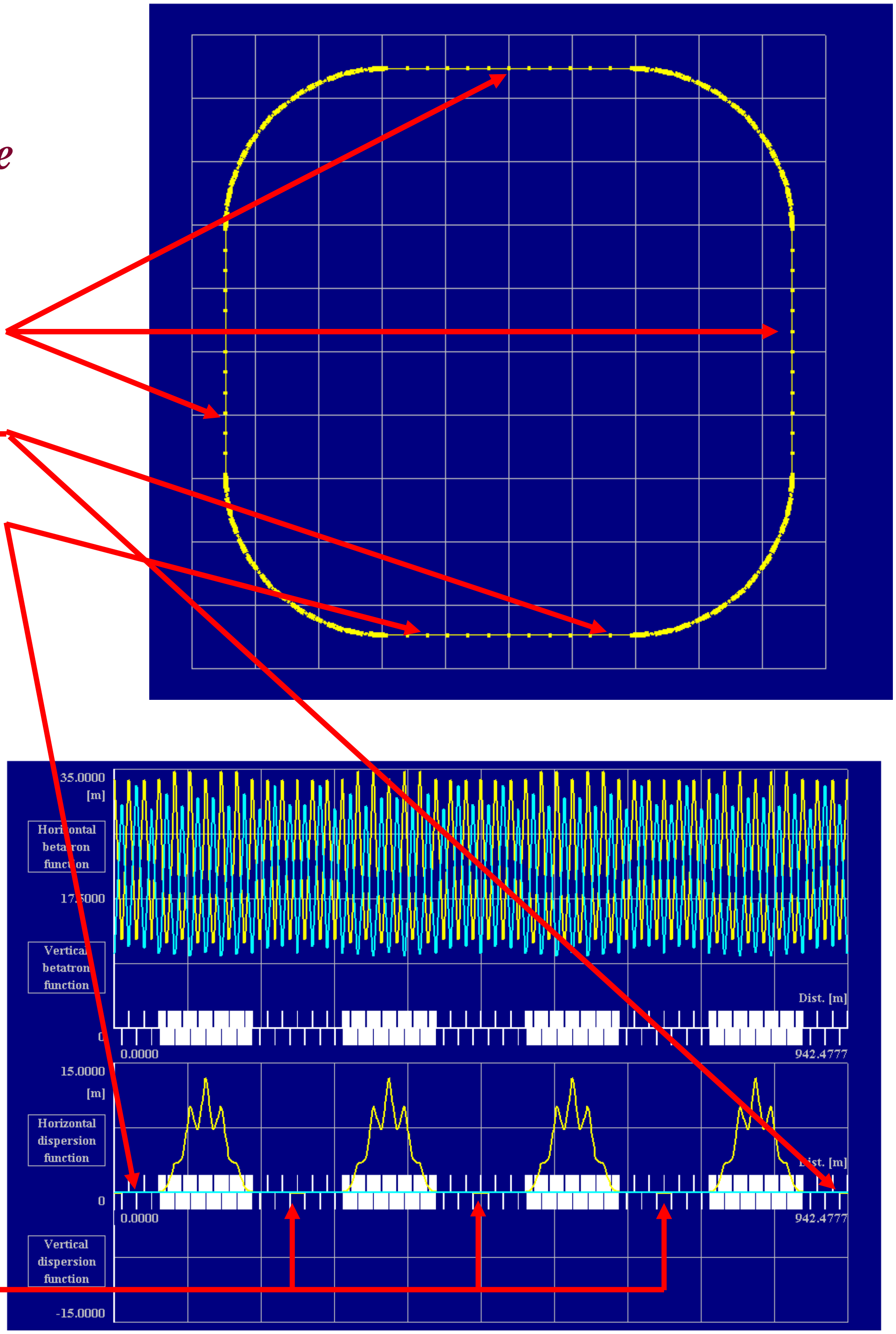
An accurate analysis of the basic cell and lattice can be obtained by running WinAGILE on your CD-ROM.

An example solution

Cavities

Injection

Extraction



Calculation with WinAGILE

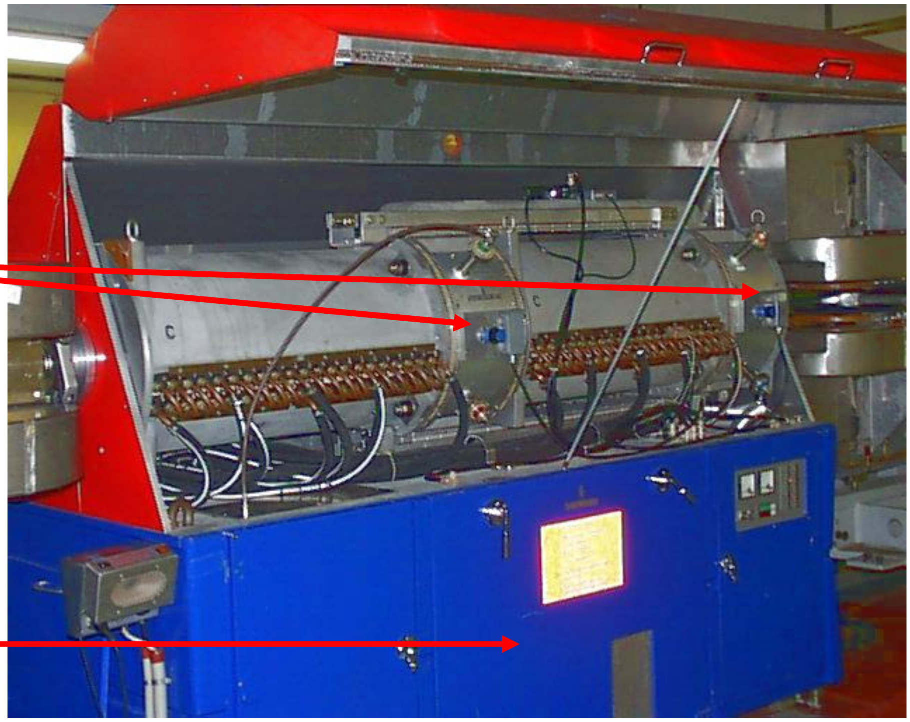
- There is no single right solution to this problem. The ring could be built in many ways.
- On the CD-ROM in the folder entitled “RingDesign” in the folder “MiniWorkshop” there are step-by-step instructions for building a possible solution.
- The instructions start with a basic FODO cell with 60 degree phase advance in both planes (arcfodo1.lat). A ring comprising 48 of these cells approximates to what is needed, but the circumference is a little too large. This can be adjusted by changing the lengths of the quadupoles and rematching the phase advance before saving as arcfodo2.lat.
- The next stage is to build a half-quadrant with a dispersion suppressor. Since we have chosen a 60 degree phase advance, we can use the well-known missing-magnet dispersion suppressor in the lecture notes. The result is in octant1.lat.
- The suppressor now has to be refined to match the dispersion exactly to zero. The result is in octant3.lat.
- The ring can now be assembled. First build a quadrant with the reflection of the octant and then concatenate four quadrants (ring1.lat).
- Finally, the ring can be modified to have better tunes etc.

RF working group

- A relatively low-frequency cavity is required.
- What is the max. d*B*/d*t* for a sinusoidal excitation of the dipoles?
- What is the maximum voltage needed?
- Perhaps sin ϕ_s can be increased once the bunch has adiabatically damped?
- Try to get some idea of the voltage, frequency and synchronous-phase programs.

CERN PS cavity

- 3-10 MHz
- Gaps,
  10 kV/gap
- About 2 m long
- Tetrode



Collective effects working group

- Self-field, space-charge tune shift for a coasting beam,

$$\Delta Q_{\mathrm{x,self-field}} = -\frac{1}{2}\frac{N r_0}{\beta^2\gamma^3(2\varepsilon_{\mathrm{x}})}\frac{1}{(1+b/a)}$$

$$\Delta Q_{\mathrm{y,self-field}} = -\frac{1}{2}\frac{N r_0}{\beta^2\gamma^3(2\varepsilon_{\mathrm{y}})}\frac{1}{(1+a/b)}$$

where N= number of particles in ring, $a = \sqrt{2} \times$ 1-σ beam width, $b = \sqrt{2} \times$ 1-σ beam height, r_0 is the classical proton radius = 1.5347×10^{-18} m and ε_x and ε_z are the geometric 1-σ emittances. You may have a slightly different formula from this course.

- For the bunched beam, multiply the tune shifts by the bunching factor.
- Due to the γ^3, the tune shift at injection will dominate.
- What effect does this tune shift have in the tune diagram?

Vertically-stacked rings

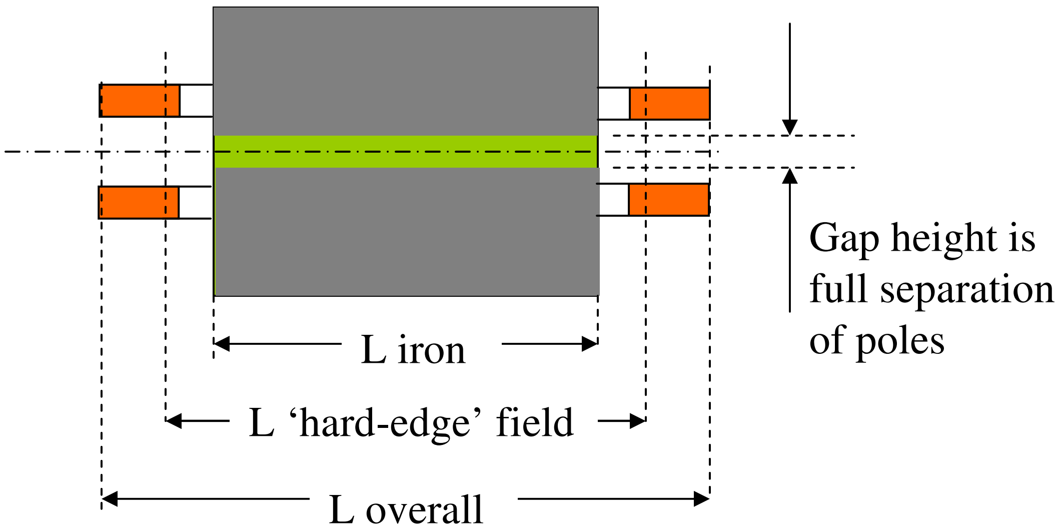
- Yes vertically-stacked rings exist.
- This is the CERN booster that accelerates simultaneously in 4 rings to 1.4 GeV/c.

Free space

- Remember, there is never enough free space in a lattice.

- Remember lattice programs normally show the boundaries corresponding to the ‘hard-edge’ model. This representation may cause you to over-estimate the free space between magnetic elements. See next slides.

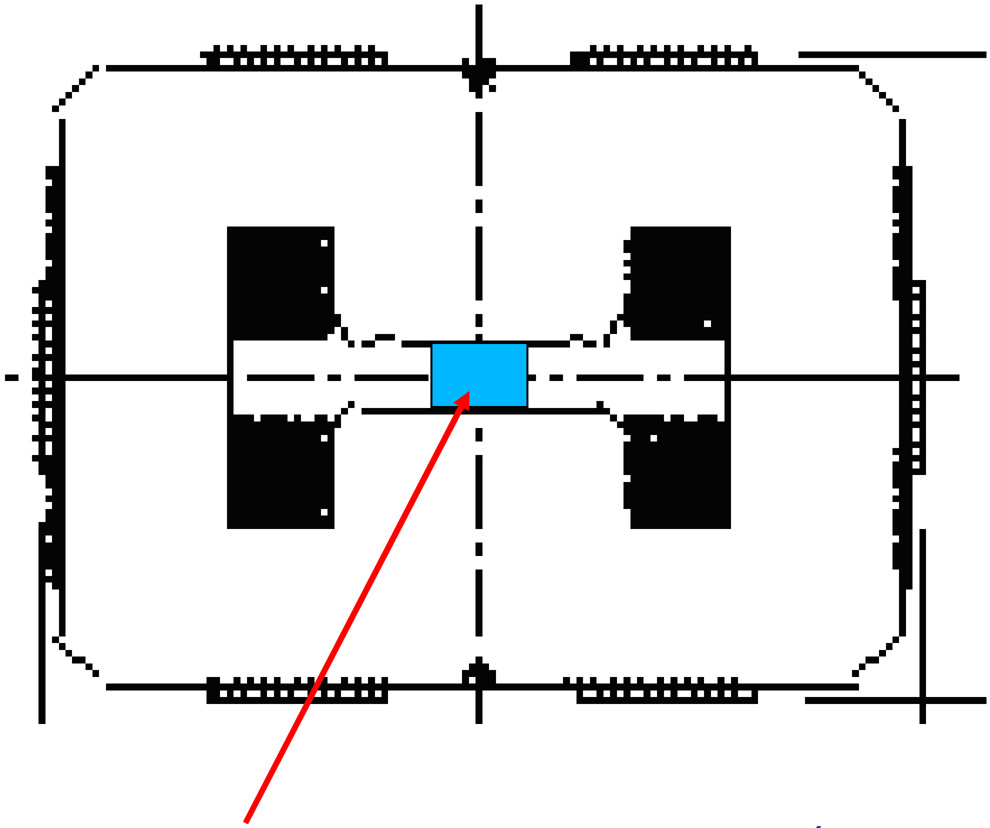
Estimating a dipole (1)



- **Effective field length of 'hard-edge' model**
  $= L_{\text{iron}} + 1.3 \times$ gap height (unsaturated)
  $= L_{\text{iron}} + 0.7 \times$ gap height (saturated).

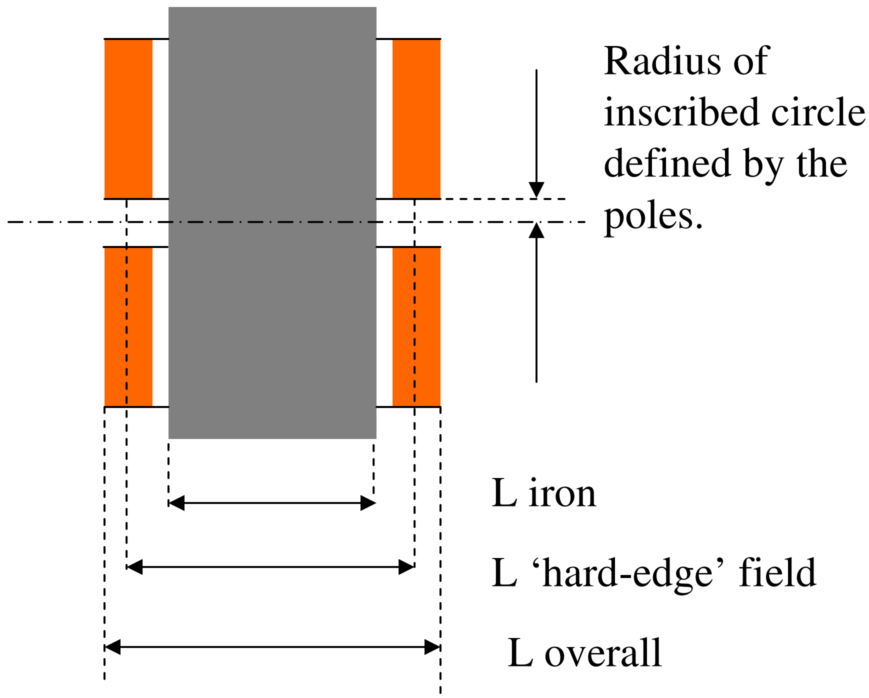
- **Overall length is more variable, but until a design is made, assume**
  $= L_{\text{iron}} + 4 \times$ gap height.

Estimating a dipole (2)



- **Good field : $\Delta B/B = \pm 2\times 10^{-4}$; B = 1.5 T; gap height (vert.) × 2 gap height (horiz.).**
- **Pole width ~ 5 × gap height.**
- **Coil X-section ~ [1.5-2 ×gap height]^2.**
- **Overall width ~ 13-15 ×gap height.**
- **Overall height ~ 10-11 × gap height.**
- **Side, top and bottom yokes = half of pole width +15%.**

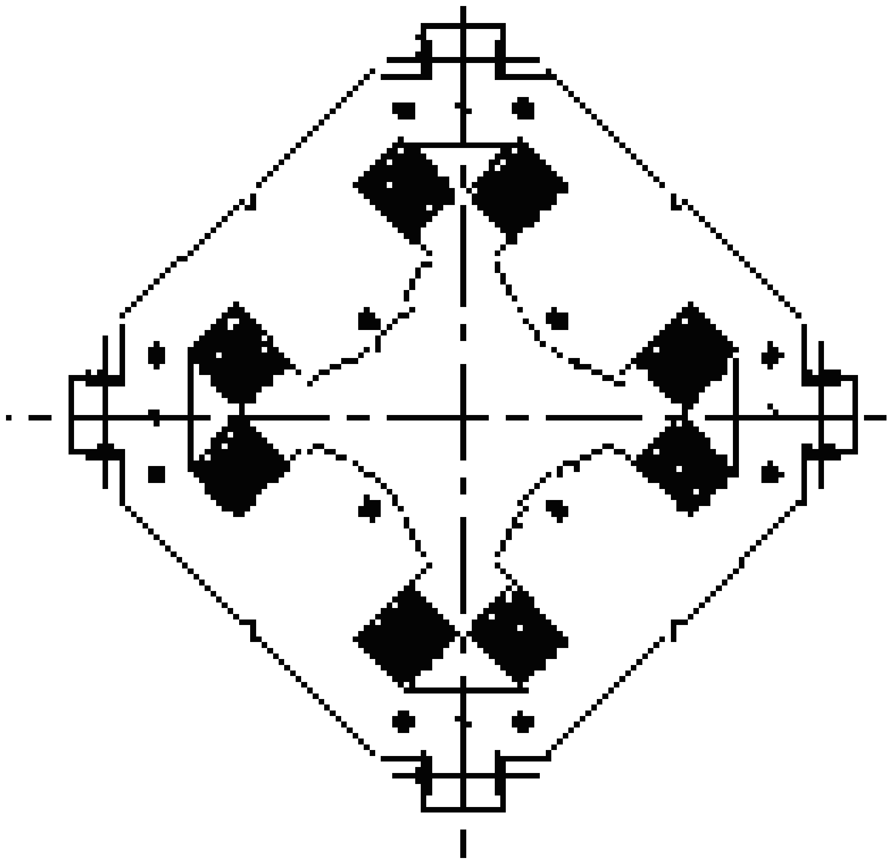
Estimating a quadrupole (1)



- **Effective field length of 'hard-edge' model**
  = L_{iron} + 1.0 × inscribed radius (unsaturated)
  = L_{iron} + 0.6 × inscribed radius (saturated).

- **Overall length is more variable, but until a design is made, assume**
  =L_{iron} + 2 × inscribed radius.

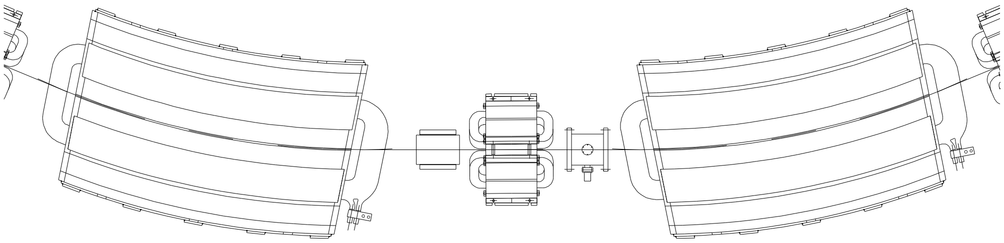
Estimating a quadrupole (2)



- **Good field : $\Delta G/G = \pm 5\times 10^{-4}$;**
  G = 6 T/m; 0.5 T on pole; over region of radius 0.6-0.7 × inscribed radius of poles.

- **Overall width ~ 7-8 × inscribed radius of poles.**

Vacuum system



- Once you have the magnet designs, make a layout and check if there is enough room for the flanges of the vacuum system and to access the bolts on the flanges. Check if there is enough room for bellows for alignment and temperature variations.

- Do not be surprised if you need to enlarge the lattice, because there is never enough room !!

Powering magnets

- **Dipole:**

  NI = (gap height) B/μ_0

- **Quadrupole:**

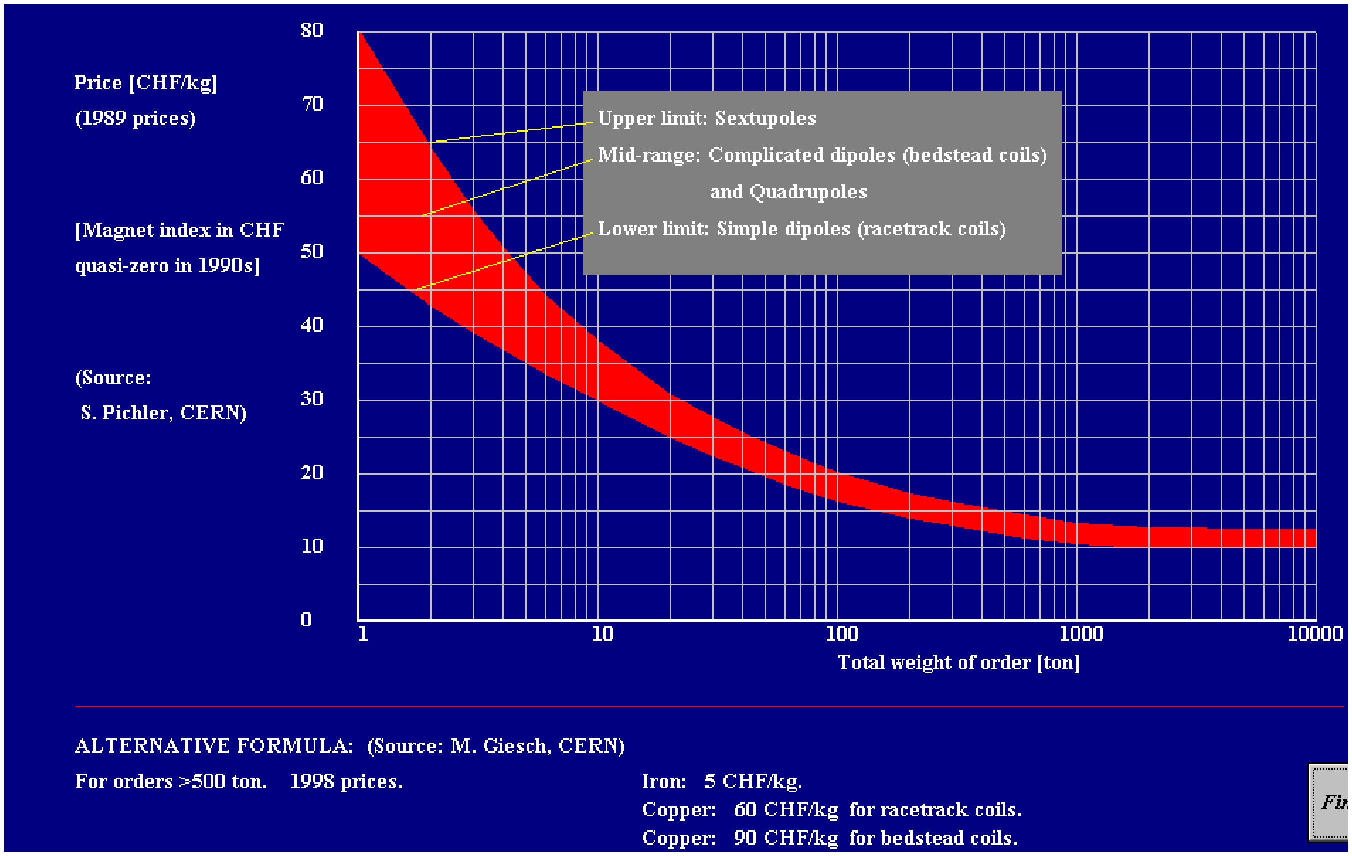
  $NI_{\text{per pole}} = (1/2)\, G\, (\text{inscribed radius})^2 / \mu_0$

- **More ampere-turns will be needed if iron is saturated.**

- **More voltage will be needed to pulse the magnets.**

- **For water-cooled coils see design aid in WinAGILE.**

Magnet cost



Graph can be found in WinAGILE

Eddy currents

- In general magnets are laminated to stop eddy currents.
- Large solid yokes can take up to tens of seconds to stabilise and often show poor reproducibility. This is probably due to the eddy current paths being unstable due to temperature and the recent history of powering which leaves the magnet with different hysteresis patterns in the yoke between runs.
- However, it is sometimes useful to accept some power loss in eddy currents during ramping to benefit from the eddy current smoothing on the flat top. This can stop high frequencies being passed from the power converters via the magnet to the beam.
- A simple expression for the time constant of a lamination is,

$$\tau = \frac{\sigma\mu_0}{\pi^2} a^2$$

  where *a* is the lamination thickness and σ the conductivity (strictly this applies for no air gaps) .
- For a more detailed analysis of the time constants in a magnet, use the 'Eddy current design aid' in WinAGILE. This includes the air gap.

Conclusion of Mini-Workshop

- **Please use the lectures as much as possible to fill out the design study of the synchrotron.**
- **Try to suggest how the intensity and repetition rate could be increased.**
- **This will act as a sort of revision.**
- **The use of WinAGILE will be further explored during the Computer Workshop.**
- **The Project Leader and the various Working Groups should report in the Concluding Meeting (see lecture programme).**