Joint University Accelerator School

Examination of Transverse Beam Dynamics

JUAS 2013

5th February 2013

1 Exercise

A transport lattice with no acceleration consists of FODO cells with quadrupole spacing $L = 10$ m and focal length $f = 7$ m.

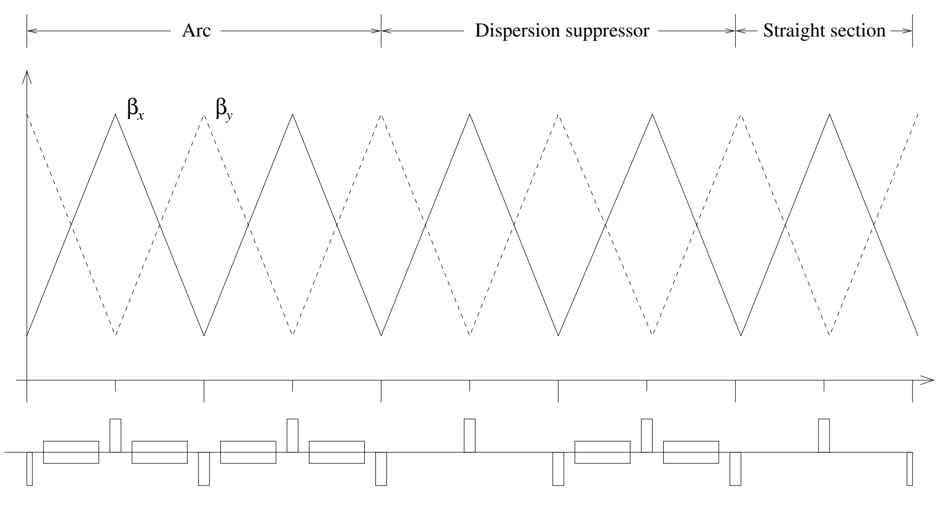
- How large is the phase advance?
- Compute the beta maximum $\hat{\beta}$ and the beta minimum $\check{\beta}$ for this FODO cell
- Does this cell provide a stable periodic solution?

Now, put bending magnets in the space between the quadrupoles (consider thin bending magnets in the middle of the drifts between quadrupoles), in order to make a storage ring consisting of 72 of these FODO cells.

- What is the necessary bending angle for the dipole magnets?
- What is the maximum dispersion of this ring?
- Compute the ring tunes.

2 Exercise

The picture shows a "missing-magnet" dispersion suppressor embedded in a FODO lattice, whose β-functions are also shown. The beam propagates from left to right and the dispersion-free region is located to the right, i.e. the last FODO cell.



Questions:

1. draw in the plot, qualitatively, the evolution of the dispersion function from left to right (use arbitrary units)
2. imagining that the phase advance in the horizontal axis is $\mu = 90°$ per cell, draw the trajectory of two on-energy particles that propagate through the cells:
   (a) one starting with $x = 0$, and $x' > 0$
   (b) one starting with $x > 0$, and $x' = 0$

3 Exercise

The RHIC collider collides fully stripped gold ions ($A = 197$, $Z = 79$) at a total energy of 100 GeV per nucleon per beam. The circumference of the ring is 3834 m. Assume the rest mass of a gold ion is 197×0.93113 GeV/c^2.
Questions:

1. If the injection energy is 10.5 GeV/nucleon, what is the revolution frequency at the injection energy and at full energy, respectively?
2. If we assume there are 192 identical dipoles per ring, what is the field at full energy? Assume each dipole is 10 m long.

4 Exercise

During the lectures we have seen how the beam ellipse $\begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$ is deformed when the beam travels through a drift space, a focussing quadrupole, and a bending magnet.

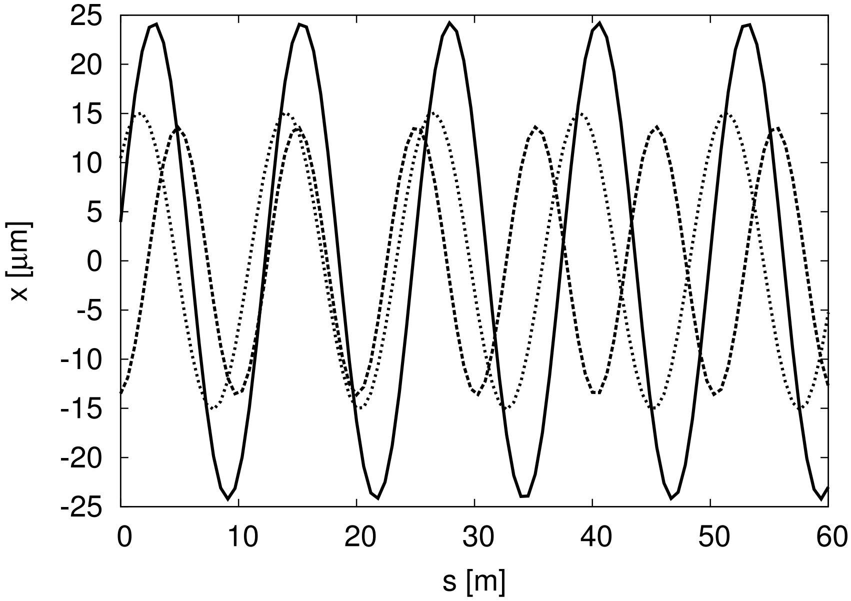
1. Using only geometric arguments (and not Liouville's theorem) show that the area of the beam ellipse remains the same when the beam propagate through a drift space and through a focussing quadrupole.
2. How is the beam ellipse deformed when the beam traverses a defocusing quadrupole?
3. How is the ellipse deformed in a thin bending magnet?

5 Exercise

Consider the parameters $\{\beta(s), \alpha(s), \mu(s)\}$, where β and α are the Twiss parameters and μ is the phase advance. Now imagine a beamline composed by two sections where the first transports the parameters $\{\beta_0, \alpha_0, 0\}$ into $\{\beta_1, \alpha_1, \mu_1\}$, and the second transports $\{\beta_1, \alpha_1, 0\}$ into $\{\beta_2, \alpha_2, \mu_2\}$. Show that the total beamline transports $\{\beta_0, \alpha_0, 0\}$ into $\{\beta_2, \alpha_2, \mu_1 + \mu_2\}$.

6 Exercise

The following plot represents the trajectories of three particles traveling in a non-dispersive transfer line with constant focusing strength.



Among the three particles, one is significantly off-momentum. Which one is it (full, small-dot or large-dot line)? Is its rigidity higher or lower than the on-momentum particles? Justify the answer.

7 Exercise

The main linac of CLIC consists of 12 sectors filled with FODO cells with increasing length and fixed phase advance, $\mu = 72^\circ$ per cell, in both axes. In each sector the cell length scales approximatively with the square root of the energy: $L_{\text{cell}} \propto \sqrt{E}$. If one wants to keep fixed the quadrupole gradient G in all sectors, how must the length of the quadrupole magnets scale? Justify your answer.

8 Exercise

Define what is the momentum compaction factor.