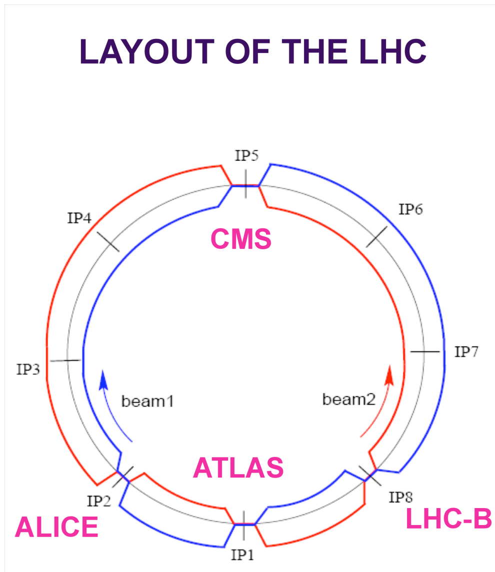
LONGITUDINAL BEAM DYNAMICS EXAMINATION

Consider the Large Hadron Collider (see Figure below) where the first beam (called beam 1 or blue beam) is circulating clockwise while the second beam (called beam 2 or red beam) is circulating anti-clockwise. The two beams circulate in two different vacuum chambers most of the time but they share the same beam pipe around the four experiments (ATLAS at the Interaction Point 1 (called IP1), ALICE at IP2, CMS at IP5 and LHC-B at IP8). Let's assume that each beam is composed of only one bunch such that the 2 counter-rotating bunches are colliding in IP1 and IP5 only.



The relevant characteristics of the LHC machine are summarized in the Table below.

Injection beam momentum	$p_i = 450$ GeV/c
Collision beam momentum	$p_c = 7000$ GeV/c
Ring circumference	Circ = 26658.883 m
Momentum compaction factor	$\alpha_p = 3.225 \times 10^{-4}$
Number of dipoles (for each beam)	$N_d = 1232$
Length of dipoles	$L_d = 14.3$ m
RF harmonic number	h = 35640
Total RF voltage	$V_{RF} = 16$ MV

1) What is the value of the relativistic gamma factor at transition γ_{tr}? What is the slip factor η at injection and collision energies? Does the LHC cross transition? What does this mean for the revolution period of a particle gaining energy (i.e. being accelerated)?

2) Compute the relativistic beta factor β at injection and collision energies. What can you conclude? What are the revolution frequency f_{rev} and revolution period T_{rev}? Deduce the RF frequency f_{RF}.

3) Once injected with the beam momentum $p_i = 450$ GeV/c, it takes 20 min to accelerate (constantly) the 2 bunches until the collision beam momentum $p_c = 7000$ GeV/c.

Compute the energy gain per turn. Deduce the RF synchronous phase during acceleration.

4) Compute the dipole bending angle θ_d, deduce the dipole bending radius ρ_d and then deduce the variation of the magnetic field dB / dt during acceleration. Compute the magnetic field at injection B_i (through the formula of the magnetic rigidity) and then deduce the magnetic field at collision energy B_c with the two available methods (checking that both give the same result).

5) What is the RF synchronous phase on the flat-top at 7000 GeV/c? Why?

6) Compute the synchrotron frequency, the synchrotron period and the synchrotron tune at top energy. How many LHC turns are necessary for a particle to perform one synchrotron oscillation in phase space?

7) In order to make the 2 bunches collide at other locations than IP1 and IP5, the following method is adopted. The bunch 2 is accelerated to an off-centered orbit corresponding to a momentum offset $\Delta p / p = 10^{-4}$ (we assume that the time needed to do this is infinitely small). After a certain time, the phase between the two beams has drifted by a certain amount. Once the desired crossing position is reached, the bunch 2 is decelerated back to the centered orbit (we assume that the time needed to do this is infinitely small).

   7.1) Will the bunch 2 move clockwise or anti-clockwise with respect to bunch 1 (looking for instance turn after turn at the time when bunch 1 is at IP1)?

   7.2) What are the shifts of the revolution frequency Δf_{rev} , RF frequency Δf_{RF} and LHC average radius ΔR for beam 2?

   7.3) Show that if we want the 2 bunches to collide in IP2 only, the bunch 2 needs to be shifted by a quarter of the LHC circumference compared to bunch 1.

   7.4) Show that the general formula to compute the time Δt_{IP2} needed to go from the case with the 2 bunches colliding in IP1 and IP5 only to the case with the 2 bunches colliding in IP2 only is given by

$$\Delta t_{IP2} = \frac{1}{4\, f_{rev} \left| \eta \dfrac{\Delta p}{p} \right|} .$$

   Deduce the time needed with the momentum offset $\Delta p / p = 10^{-4}$.

   7.5) What would have been the situation if the LHC were operated below transition (with the same absolute value of the slip factor)?

   7.6) What are the two methods to collide in IP8 only and how long would it take?

   7.7) What should we do if we would like to go faster? What could then be the problem?

N.B.: The proton rest energy is E_0 = 0.938 GeV, the elementary charge is $e = 1.6 \times 10^{-19}$ C and the velocity of light is $c = 2.997925 \times 10^8$ m/s.