LECTURE 2

- modes in a resonant cavity
- TM vs TE modes
- types of structures
- from a cavity to an accelerator

wave equation -recap

- Maxwell equation for E and B field:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E}=0$$

- In free space the electromagnetic fields are of the *transverse electro magnetic*,TEM, type: the electric and magnetic field vectors are ⊥ to each other and to the direction of propagation.
- In a bounded medium (cavity) the solution of the equation must satisfy the boundary conditions :

$$\vec{E}_{//}=\vec{0}$$

$$\vec{B}_{\perp}=\vec{0}$$

TE or TM modes

- TE (=transverse electric) : the electric field is perpendicular to the direction of propagation. in a cylindrical cavity

$$TE_{nml}$$

n : azimuthal,

m : radial

l longitudinal component

- TM (=transverse magnetic) : the magnetic field is perpendicular to the direction of propagation

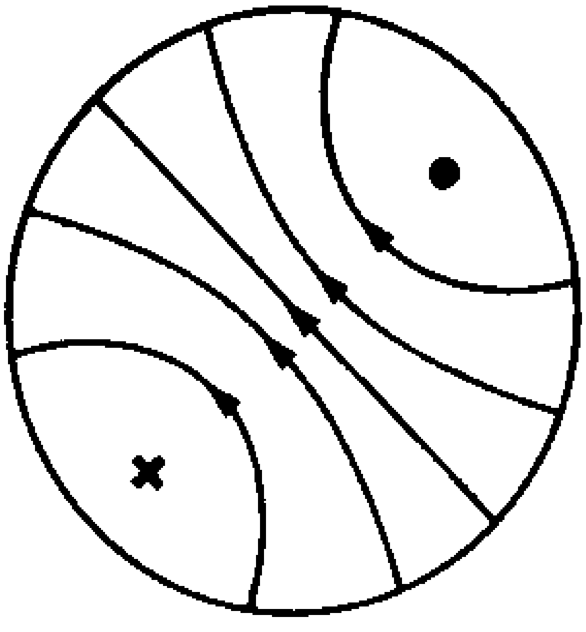
$$TM_{nml}$$

n : azimuthal,

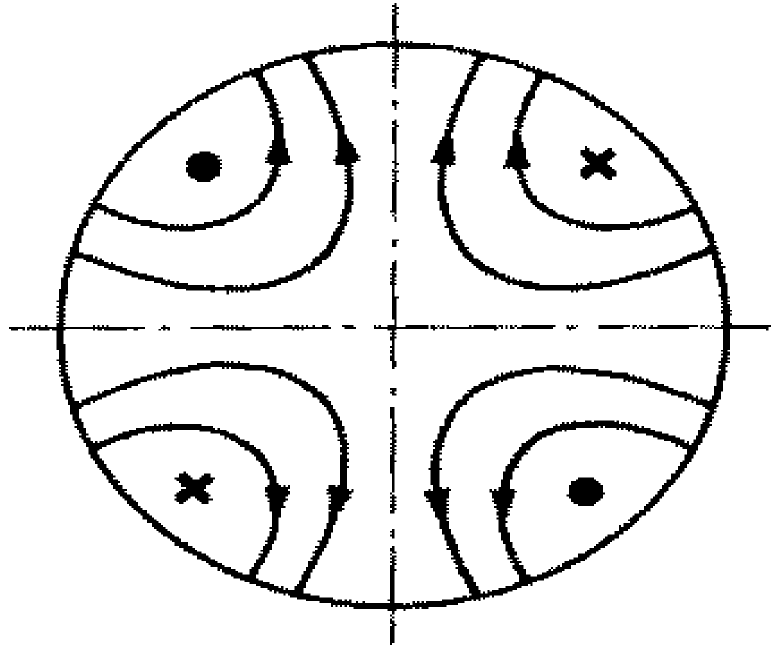
m : radial

l longitudinal component

TE modes



Empty cavity; mode TE_{11}

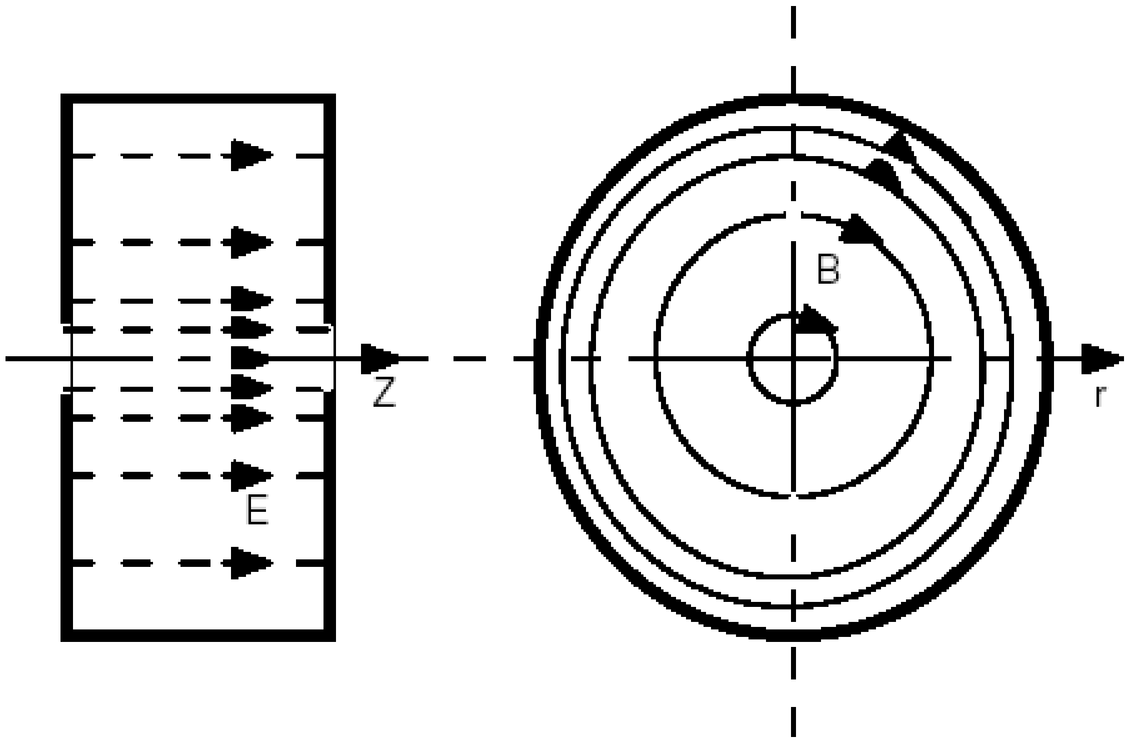


Empty cavity; mode TE_{21}

dipole mode

quadrupole mode used in Radio Frequency Quadrupole

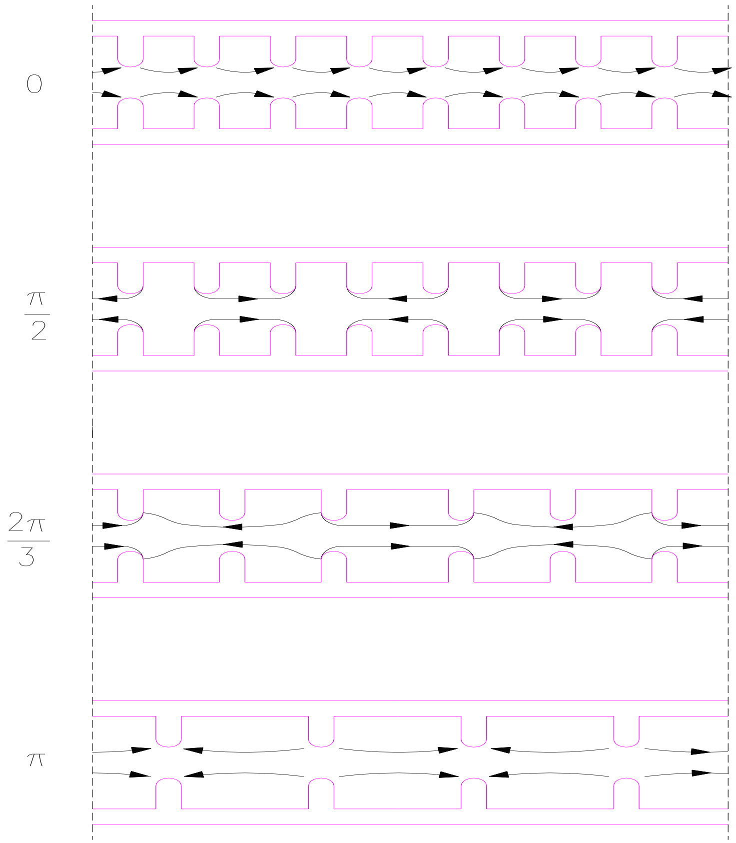
TM modes



TM010 mode , most commonly used accelerating mode

cavity modes

- • **0-mode** Zero-degree phase shift from cell to cell, so fields adjacent cells are in phase. Best example is DTL.
- • π**-mode** 180-degree phase shift from cell to cell, so fields in adjacent cells are out of phase. Best example is multicell superconducting cavities.
- • π**/2 mode** 90-degree phase shift from cell to cell. In practice these are biperiodic structures with two kinds of cells, accelerating cavities and coupling cavities. The CCL operates in a π/2structure mode. This is the preferred mode for very long multicell cavities, because of very good field stability.



basic accelerating structures

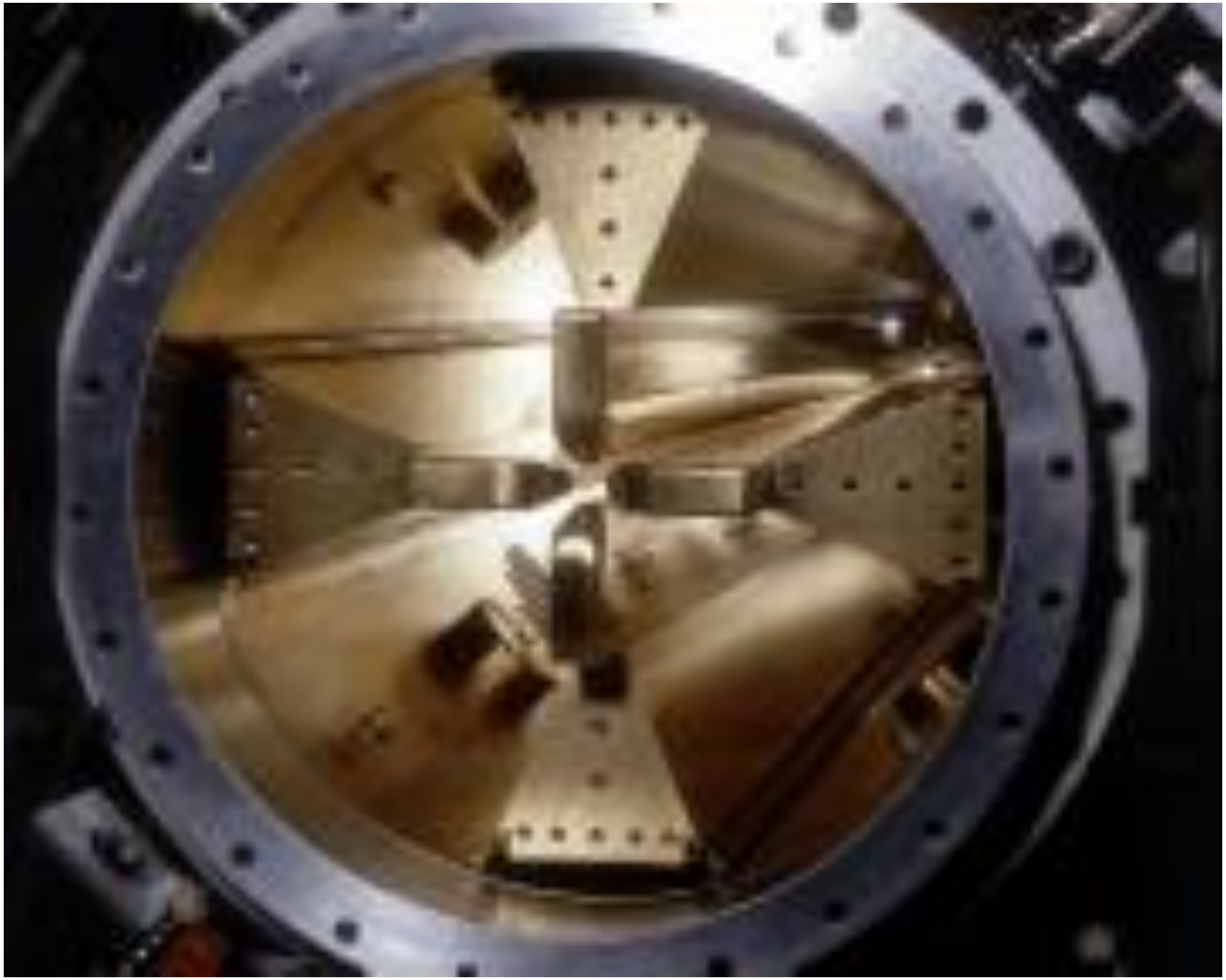
- Radio Frequency Quadrupole
- Interdigital-H structure

- Drift Tube Linac
- Cell Coupled Linac
- Side Coupled Linac

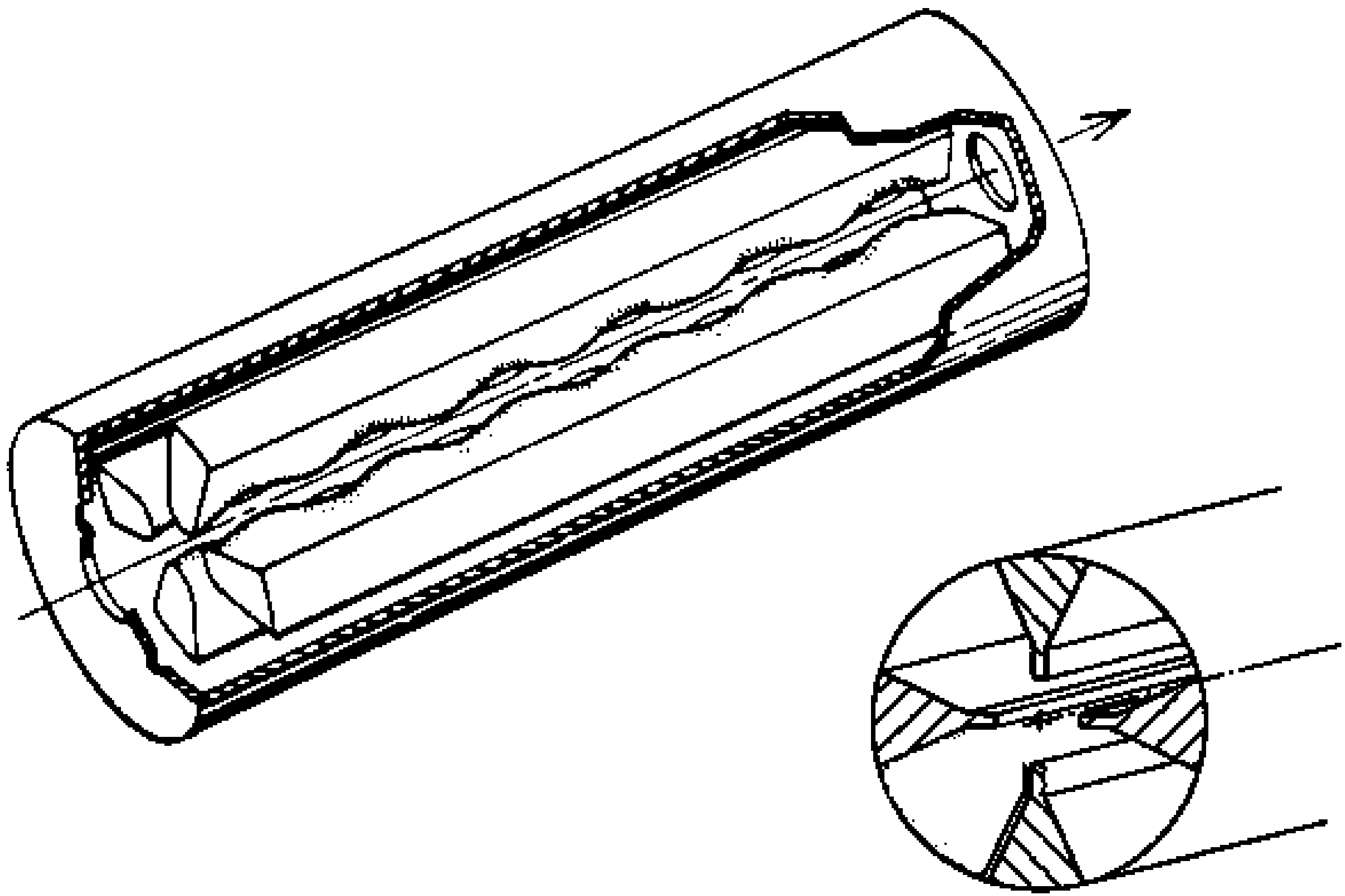
derived/mixed structure

- RFQ-DTL
- SC-DTL
- CH structure

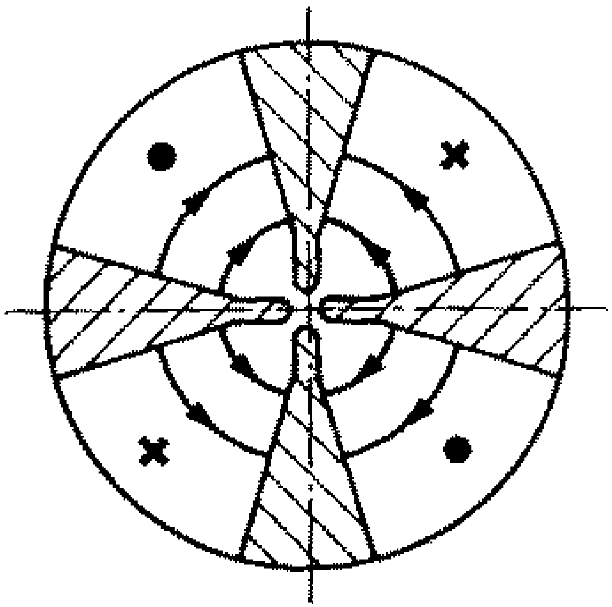
Radio Frequency Quadrupole



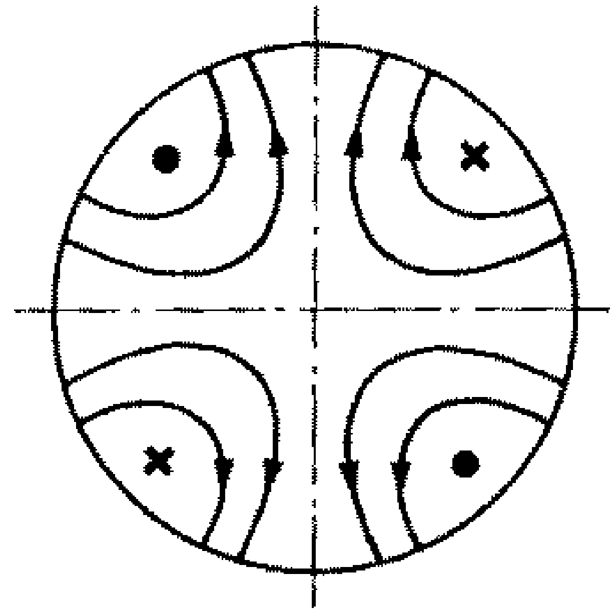
Radio Frequency Quadrupole



Radio Frequency Quadrupole



Cavity with vanes

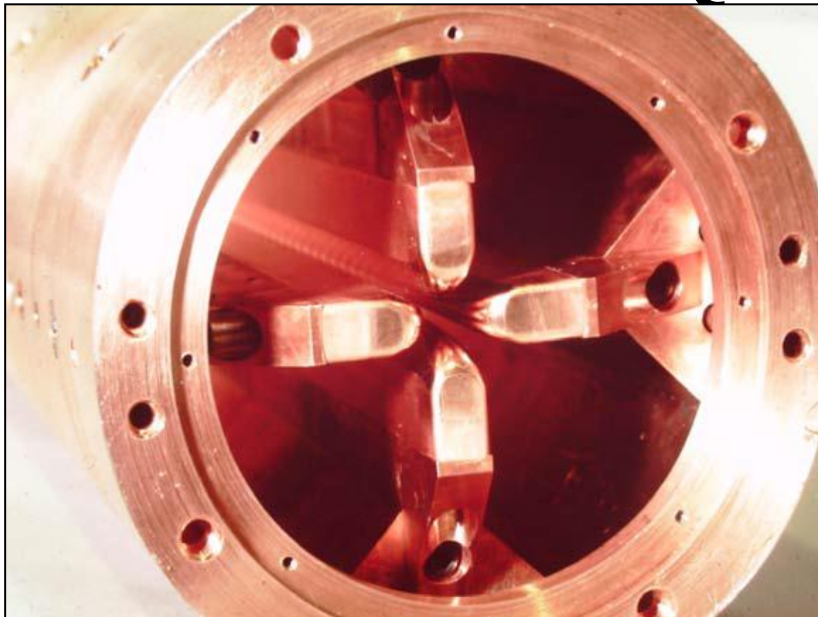


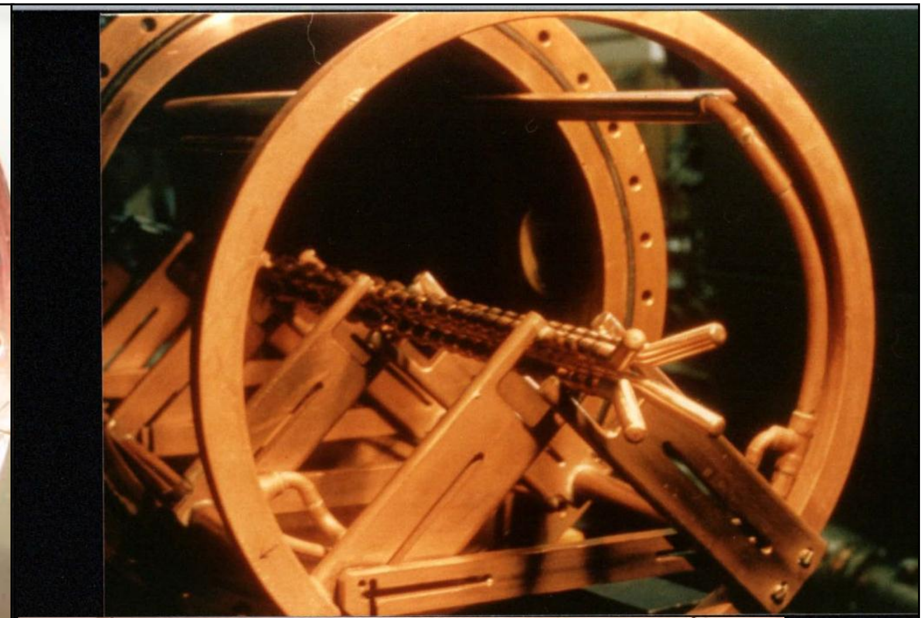
Empty cavity; mode TE_{21}

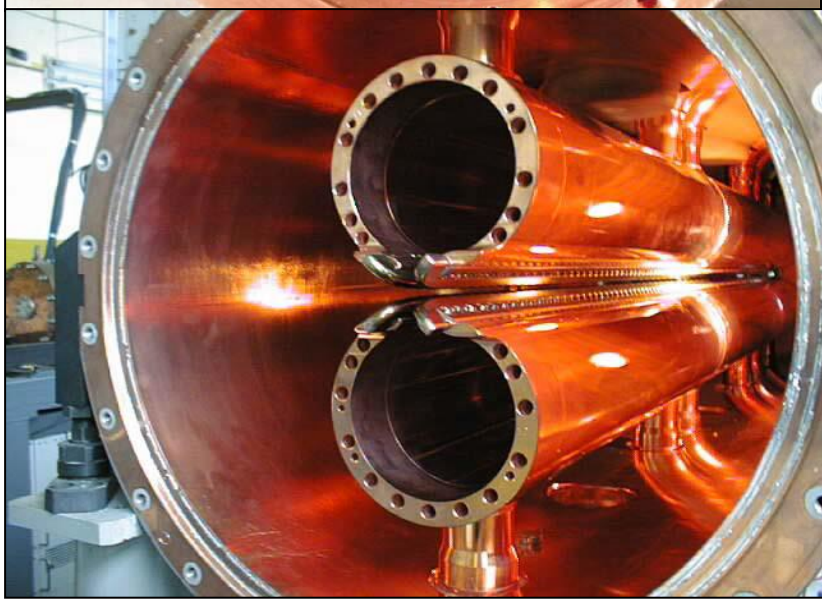
cavity loaded with 4 electrodes

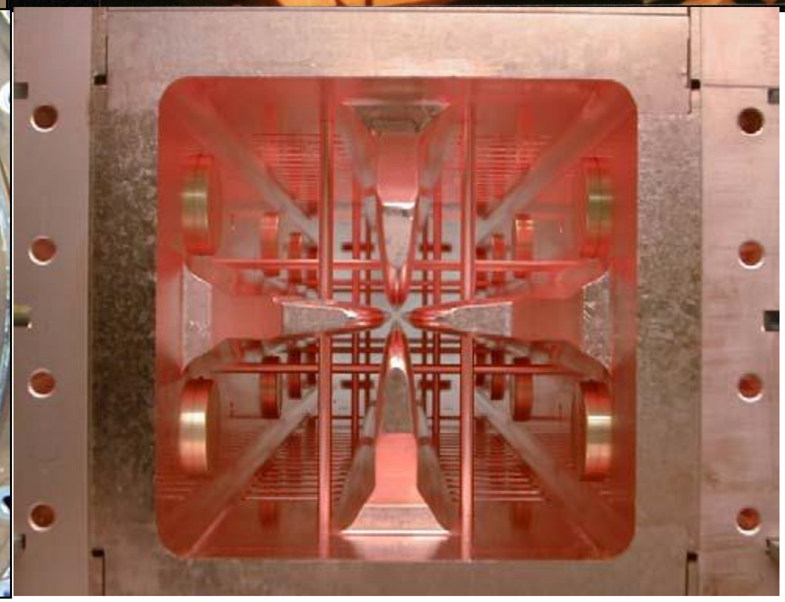
TE210 mode

RFQ Structures

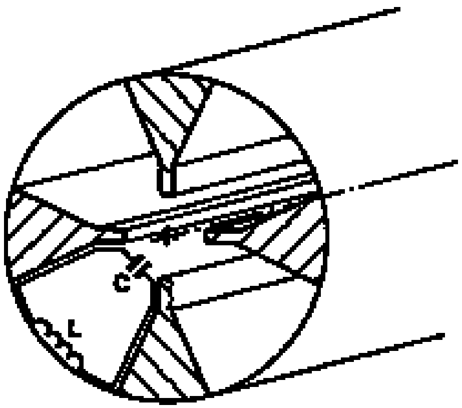


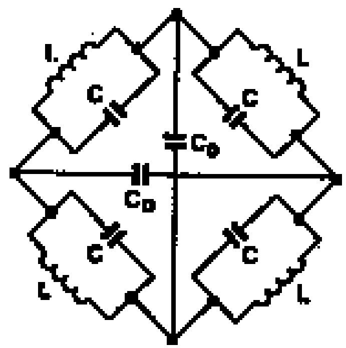


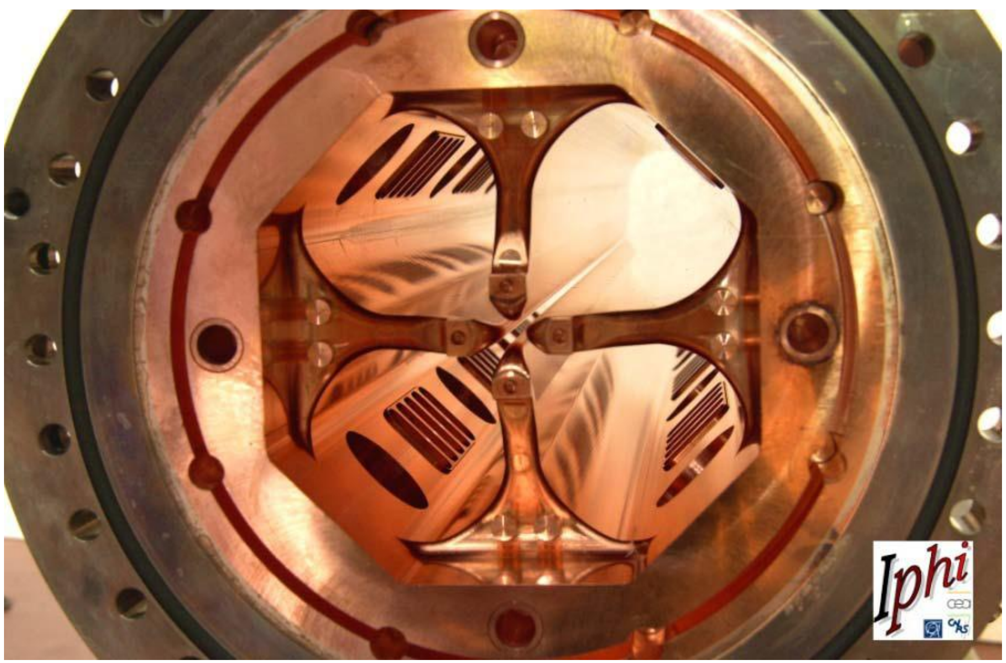




four vane-structure

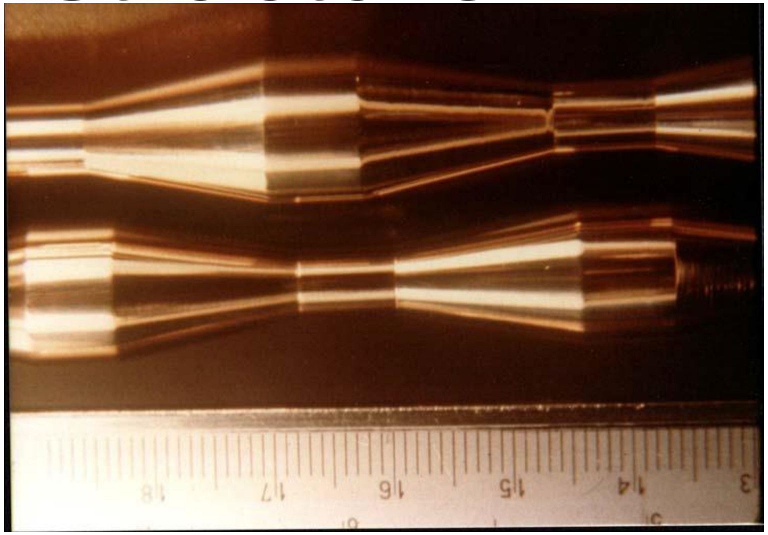






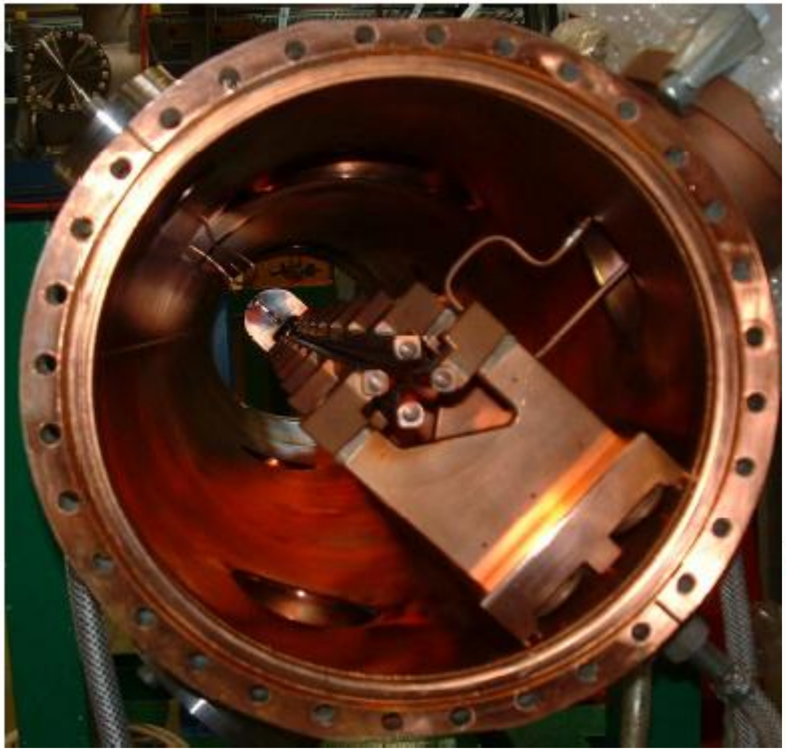
1. capacitance between vanetips, inductance in the intervane space
2. each vane is a resonator
3. frequency depends on cylinder dimensions (good at freq. of the order of 200MHz, at lower frequency the diameter of the tank becomes too big)
4. vane tip are machined by a computer controlled milling machine.
5. need stabilization (problem of mixing with dipole modeTE110)

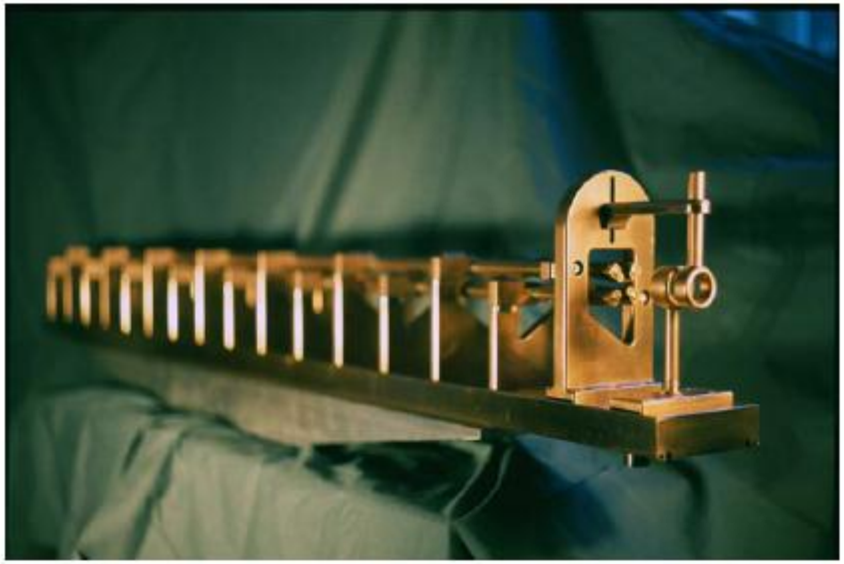
four rod-structure



- capacitance between rods, inductance with holding bars
- each cell is a resonator
- cavity dimensions are independent from the frequency,
- easy to machine (lathe)
- problems with end cells, less efficient than 4-vane due to strong current in the holding bars

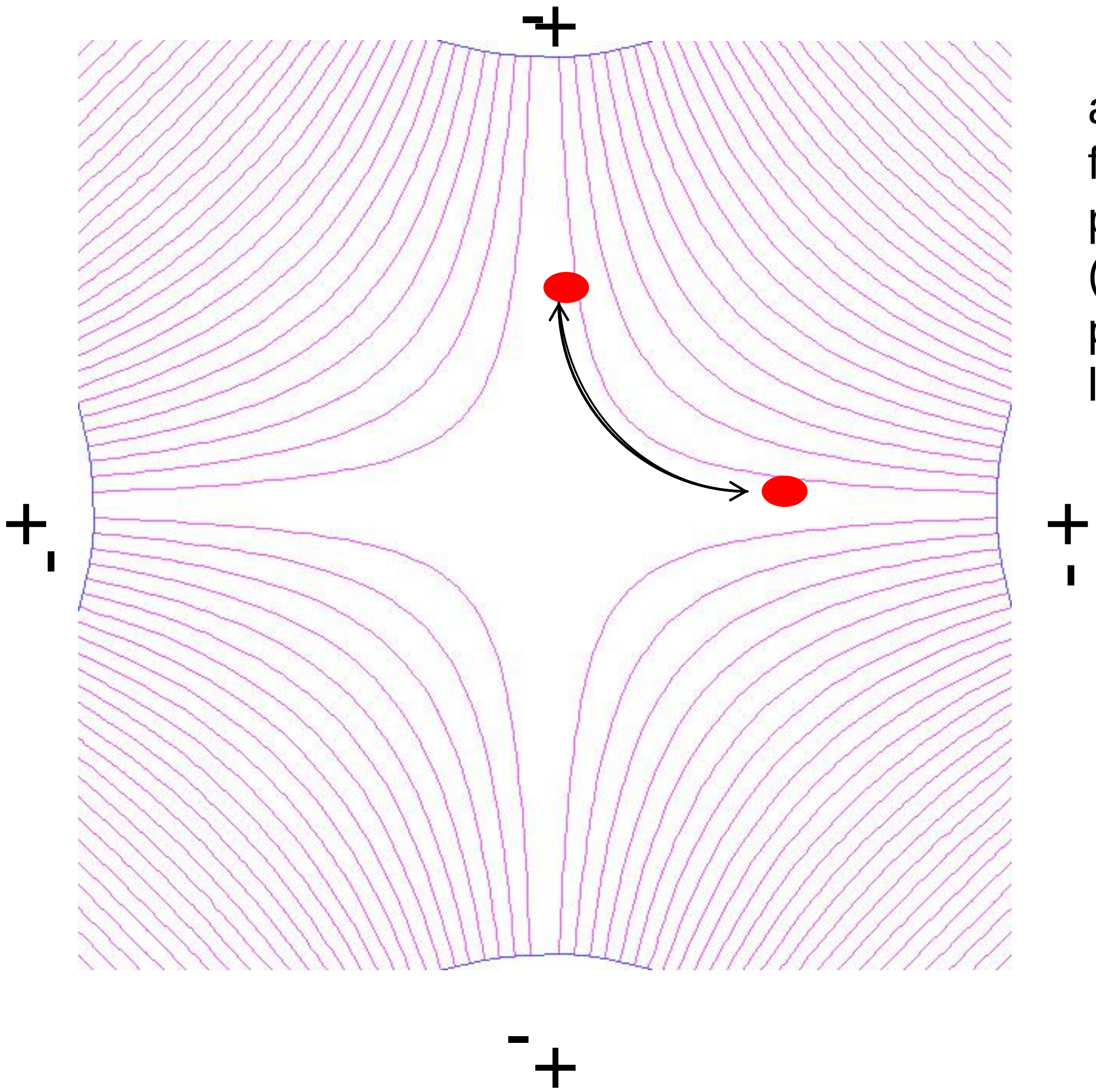
CNAO RFQ





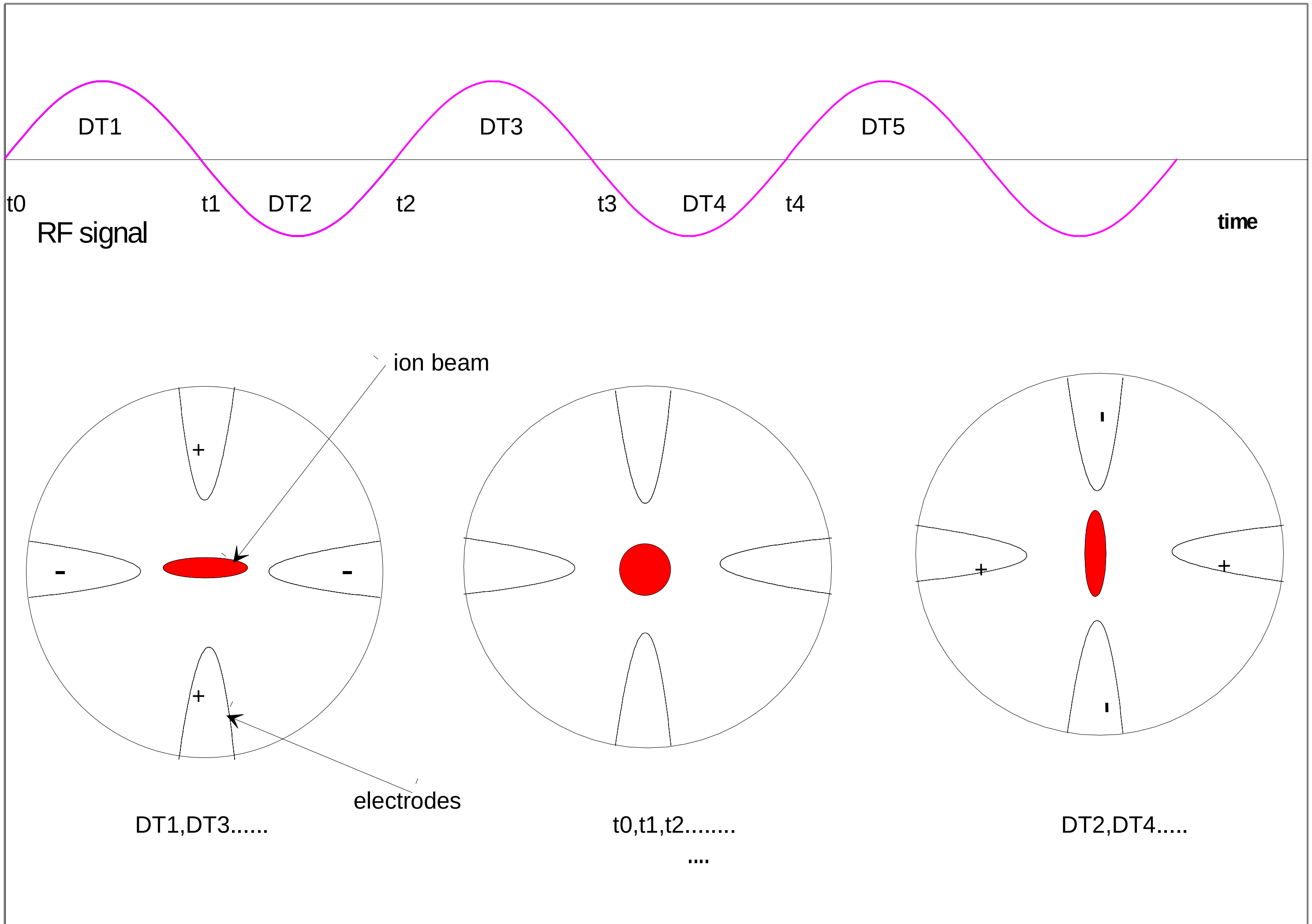
Beam energy in – out	8 – 400 keV/u
Electrode length	1.28 m
Tank diameter	0.25 m
Tank length	1.44 m
Electrode voltage	70 kV
RF power loss (pulse)	190 – 200 kW

transverse field in an RFQ

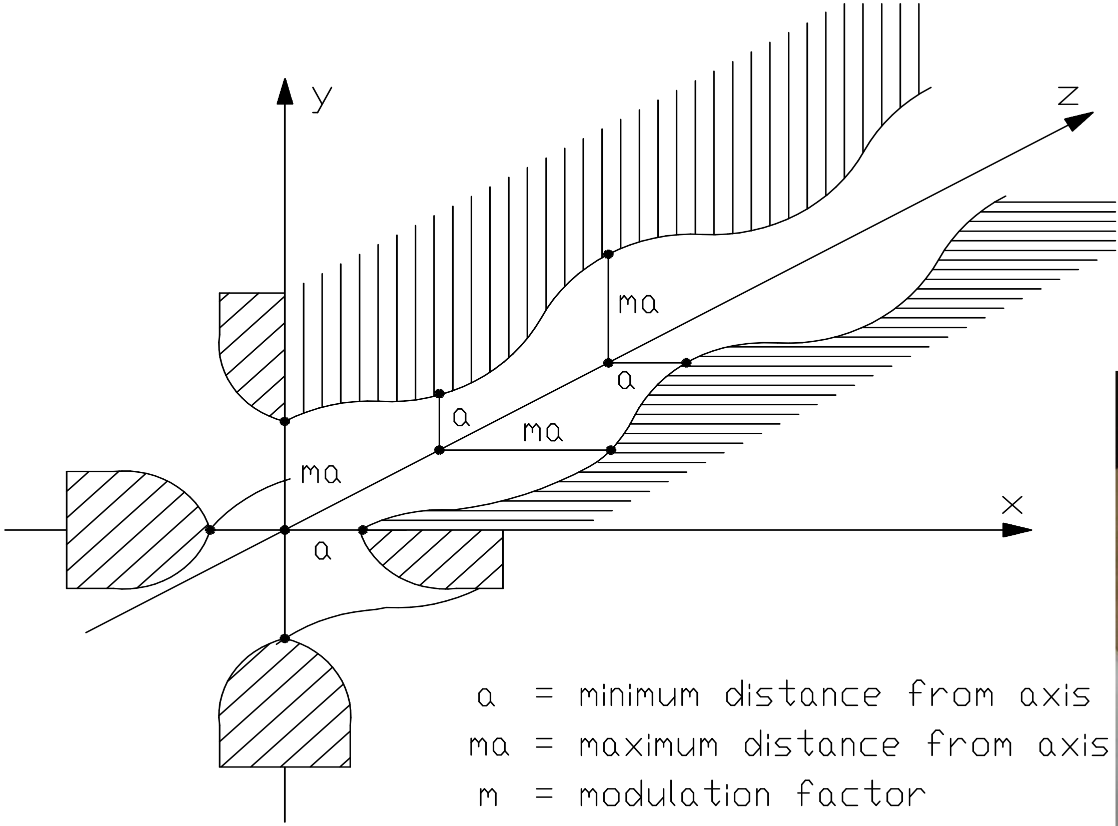


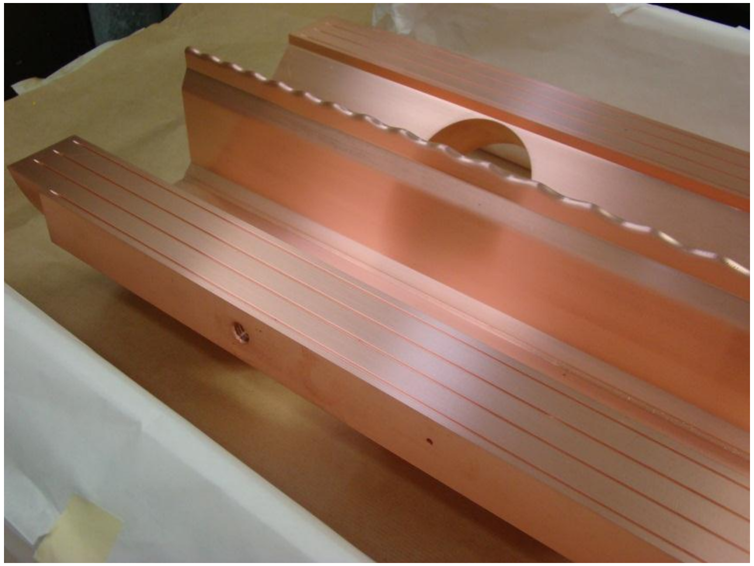
alternating gradient focussing structure with period length $\beta\lambda$
(in half RF period the particles have travelled a length $\beta\lambda/2$)

transverse field in an RFQ



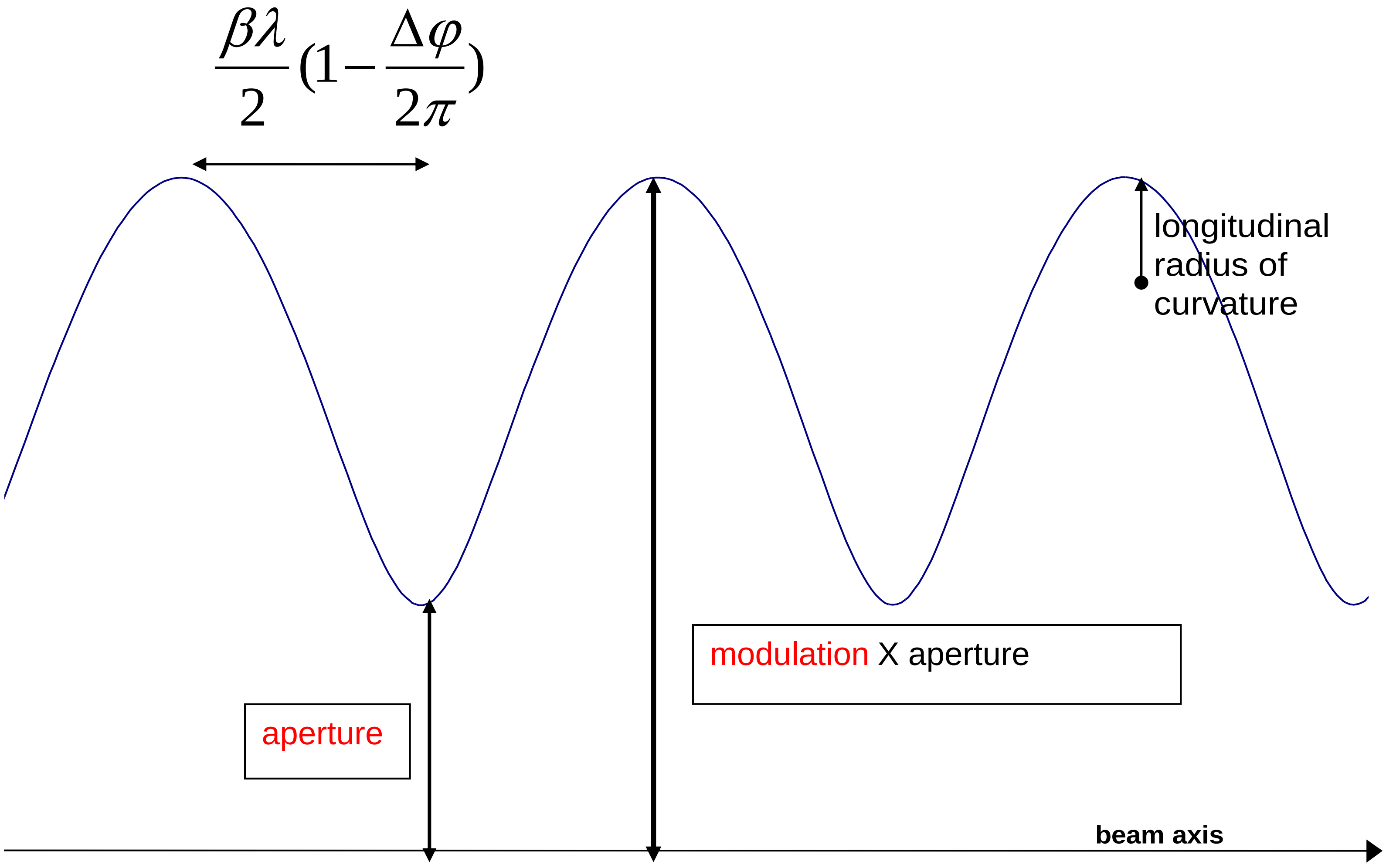
acceleration in RFQ





longitudinal modulation on the electrodes creates a longitudinal component in the TE mode

acceleration in an RFQ



important parameters of the RFQ

$$B = \left(\frac{q}{m_0}\right)\left(\frac{V}{a}\right)\left(\frac{1}{f^2}\right)\frac{1}{a}\left(\frac{I_o(ka)+I_o(mka)}{m^2 I_o(ka)+I_o(mka)}\right)$$

type of particle

limited by sparking

Transverse field distortion due to modulation (=1 for un-modulated electrodes)

$$E_0 T = \frac{m^2 - 1}{m^2 I_o(ka)+I_o(mka)} \cdot V \frac{2}{\beta \cdot \lambda} \frac{\pi}{4}$$

Accelerating efficiency : fraction of the field deviated in the longitudinal direction (=0 for un-modulated electrodes)

cell length

transit time factor

.....and their relation

$$\left(\frac{I_o(ka)+I_o(mka)}{m^2 I_o(ka)+I_o(mka)}\right)+\frac{m^2-1}{m^2 I_o(ka)+I_o(mka)}\cdot I_0(ka)=1$$

focusing efficiency

accelerating efficiency

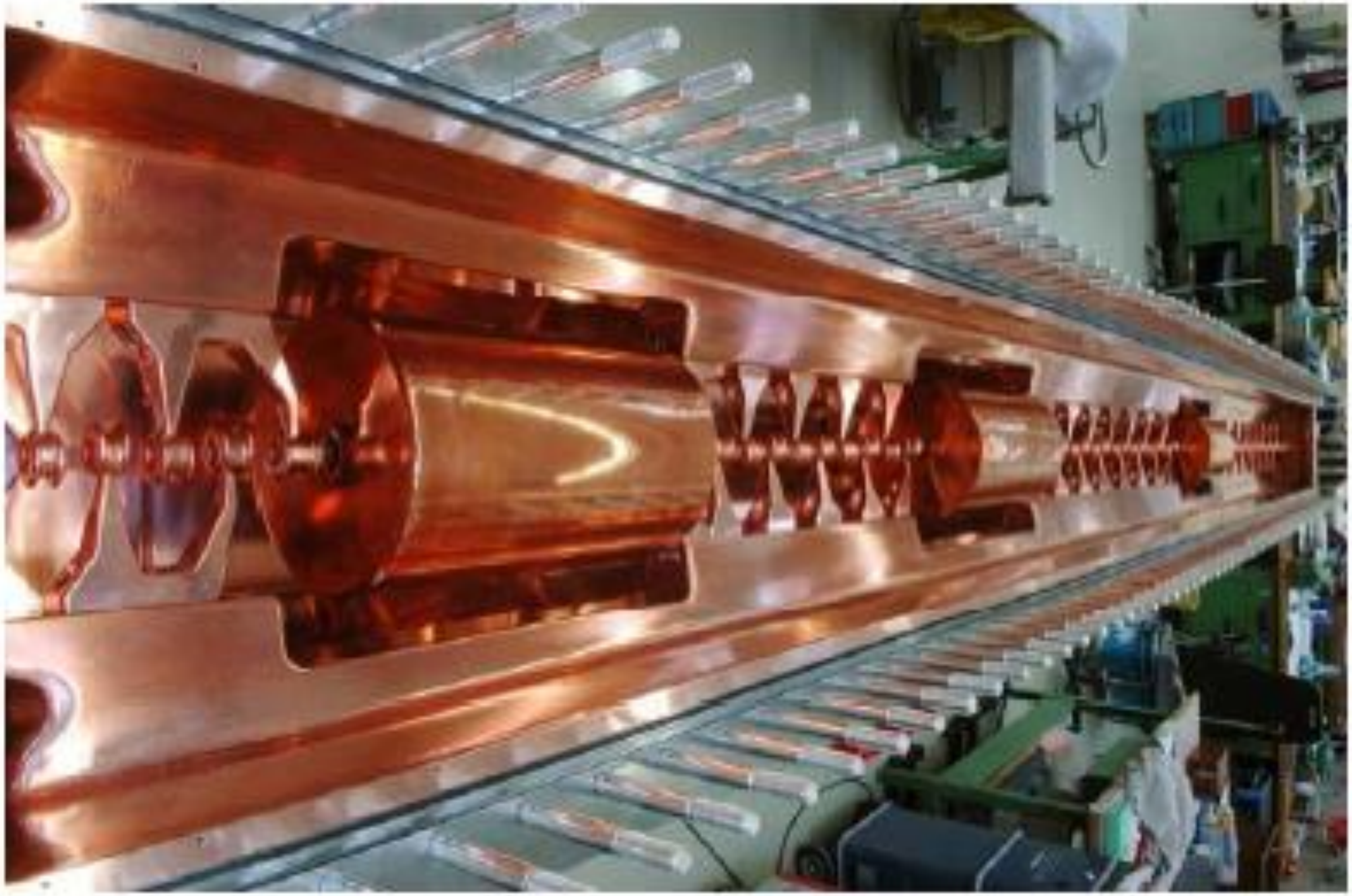
a=bore radius, β,γ=relativistic parameters, *c*=speed of light, *f*= rf frequency, *I0,1*=zero,first order Bessel function, *k*=wave number, λ=wavelength, *m*=electrode modulation, *m0*=rest *q*=charge, *r*= average transverse beam dimension, *r0*=average bore, *V*=vane voltage

RFQ

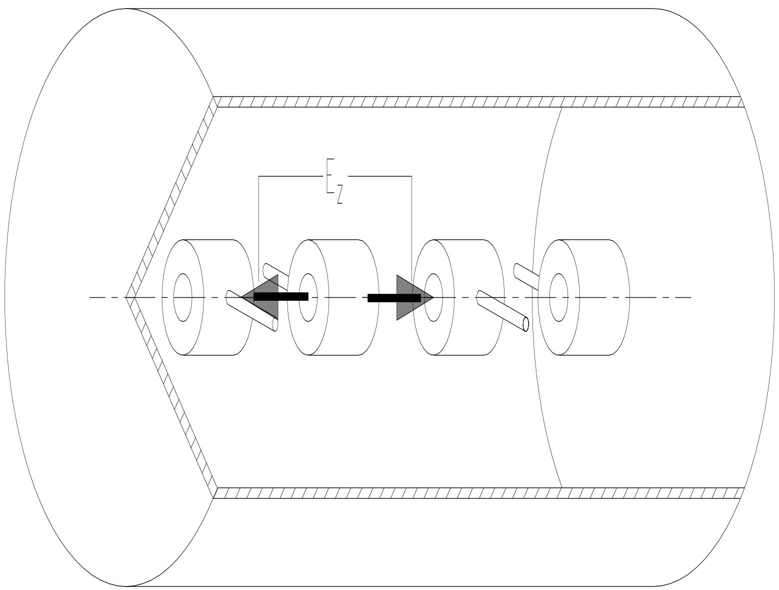
- The resonating mode of the cavity is a focusing mode
- Alternating the voltage on the electrodes produces an alternating focusing channel
- A longitudinal modulation of the electrodes produces a field in the direction of propagation of the beam which bunches and accelerates the beam
- Both the focusing as well as the bunching and acceleration are performed by the RF field
- The RFQ is the only linear accelerator that can accept a low energy CONTINOUS beam of particles
- 1970 Kapchinskij and Teplyakov propose the idea of the radiofrequency quadrupole (I. M. Kapchinskii and V. A. Teplvakov, Prib.Tekh. Eksp. No. 2, 19 (1970))

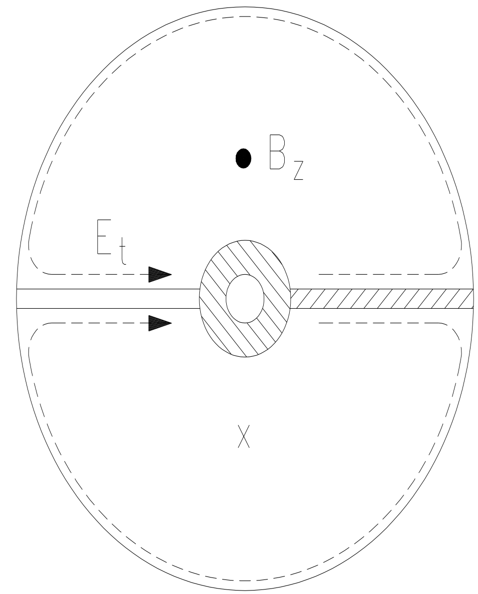
Interdigital H structure

CNAO IH



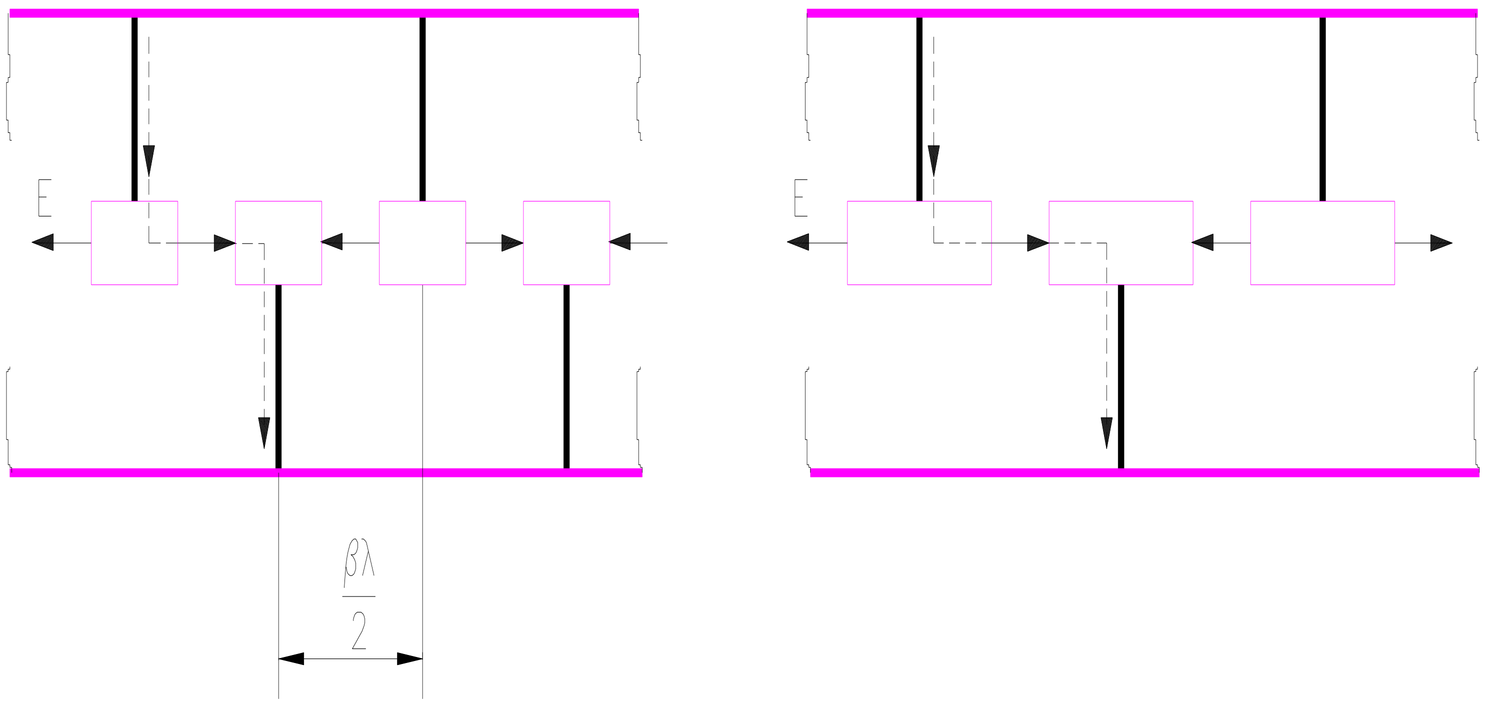
Interdigital H structure





the mode is the TE110

Interdigital H structure



•stem on alternating side of the drift tube force a longitudinal field between the drift tubes

•focalisation is provided by quadrupole triplets places OUTSIDE the drift tubes or OUTSIDE the tank

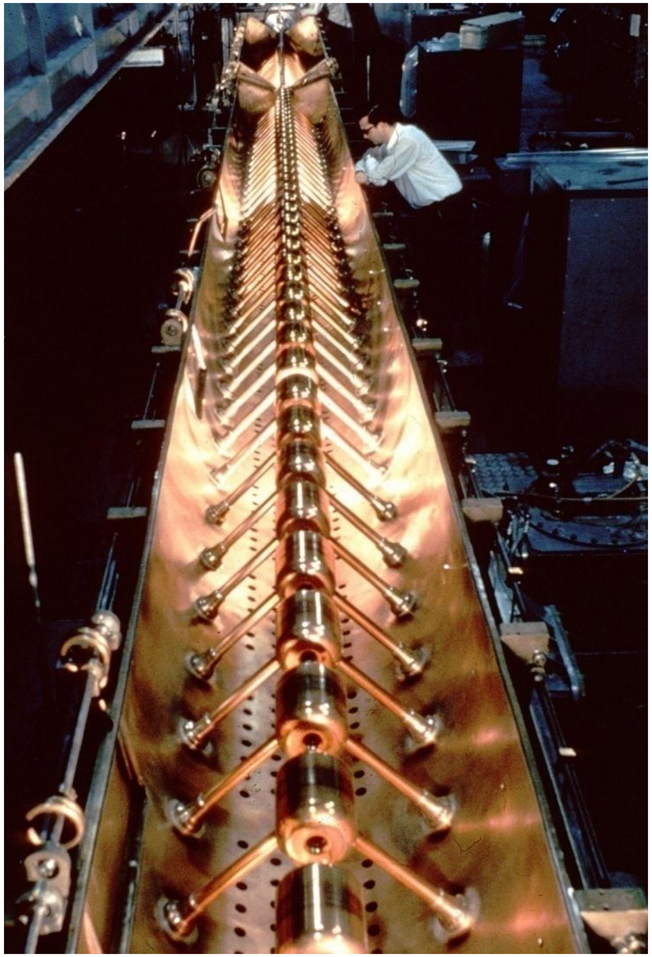
IH use

- very good shunt impedance in the low beta region (($\beta \cong 0.02$ to 0.08) and low frequency (up to 200MHz)

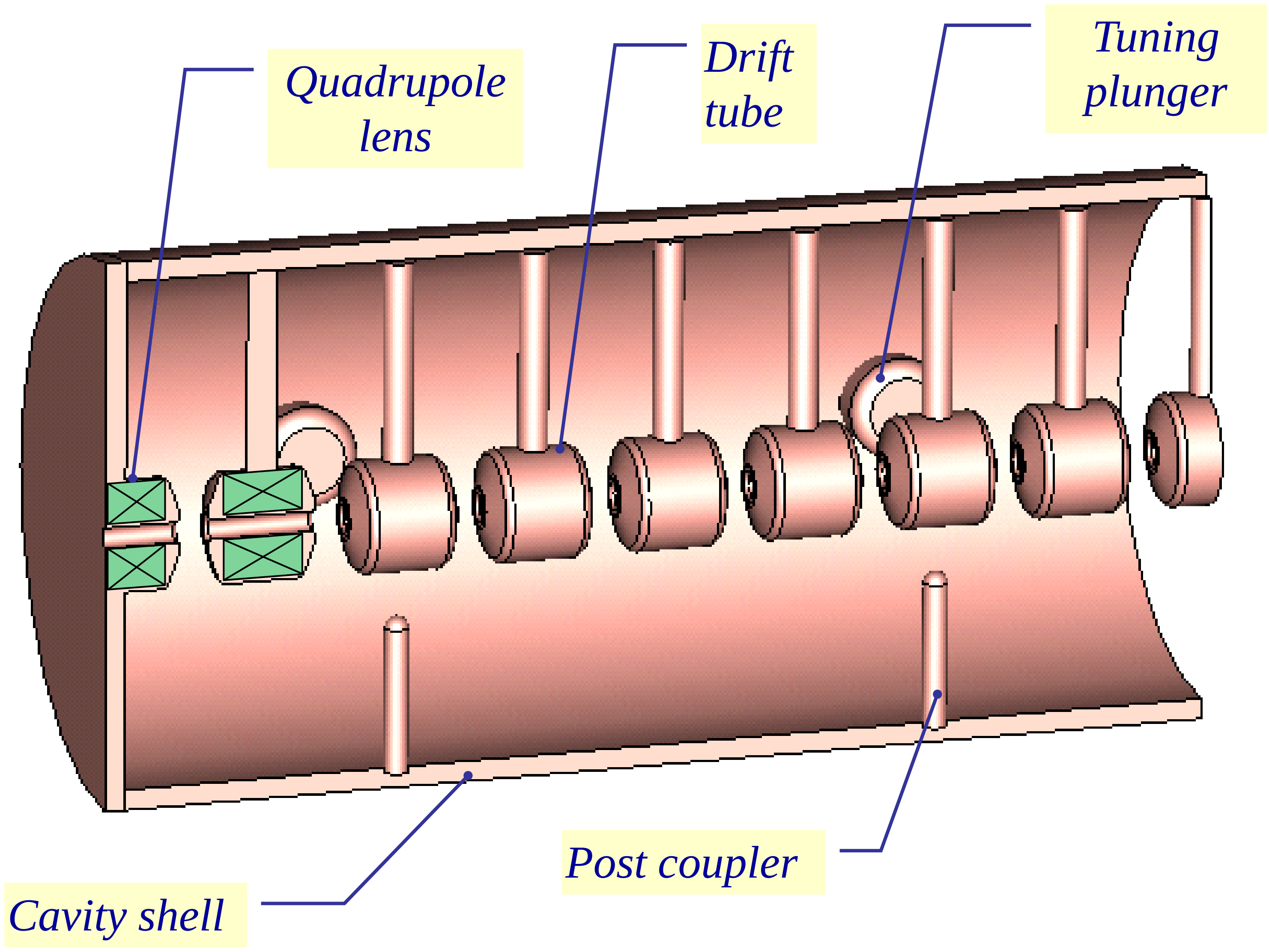
- not for high intensity beam due to long focusing period

- ideal for low beta heavy ion acceleration

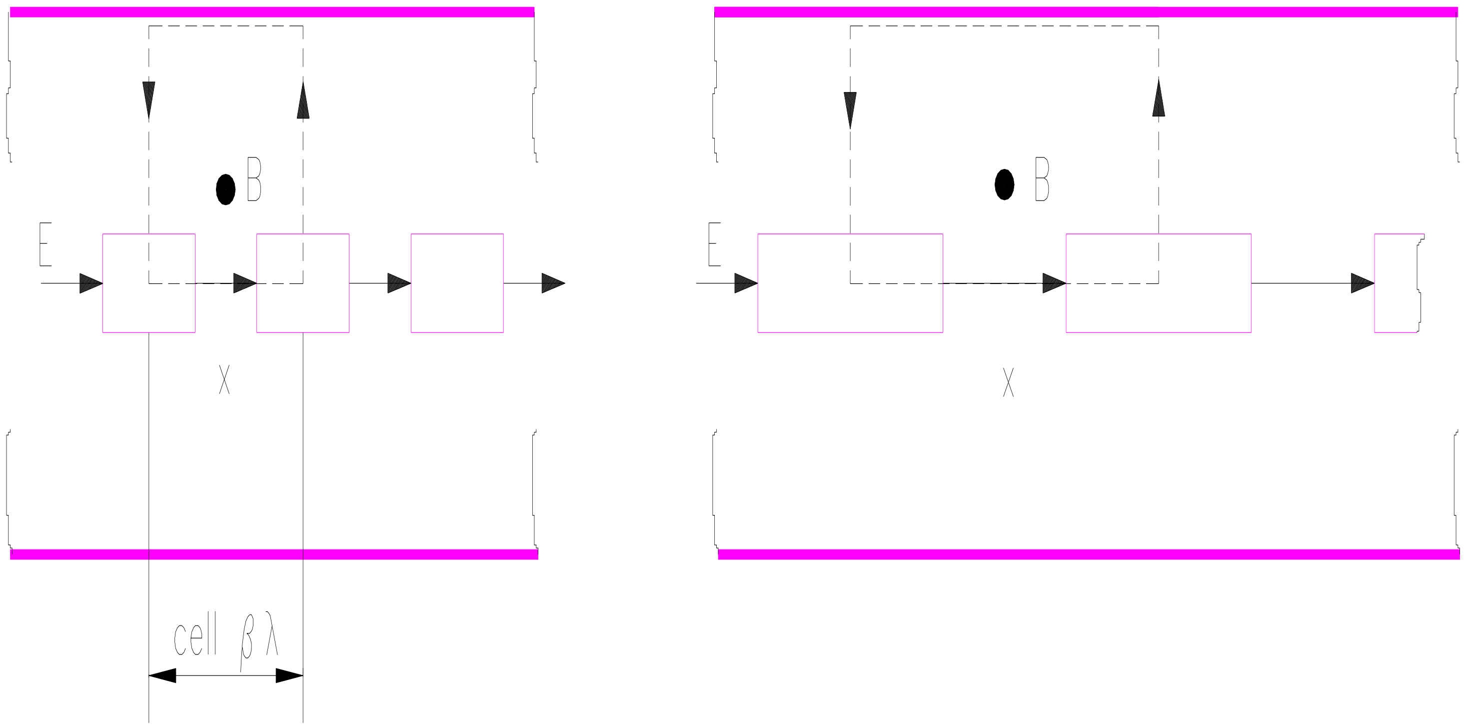
Drift Tube Linac



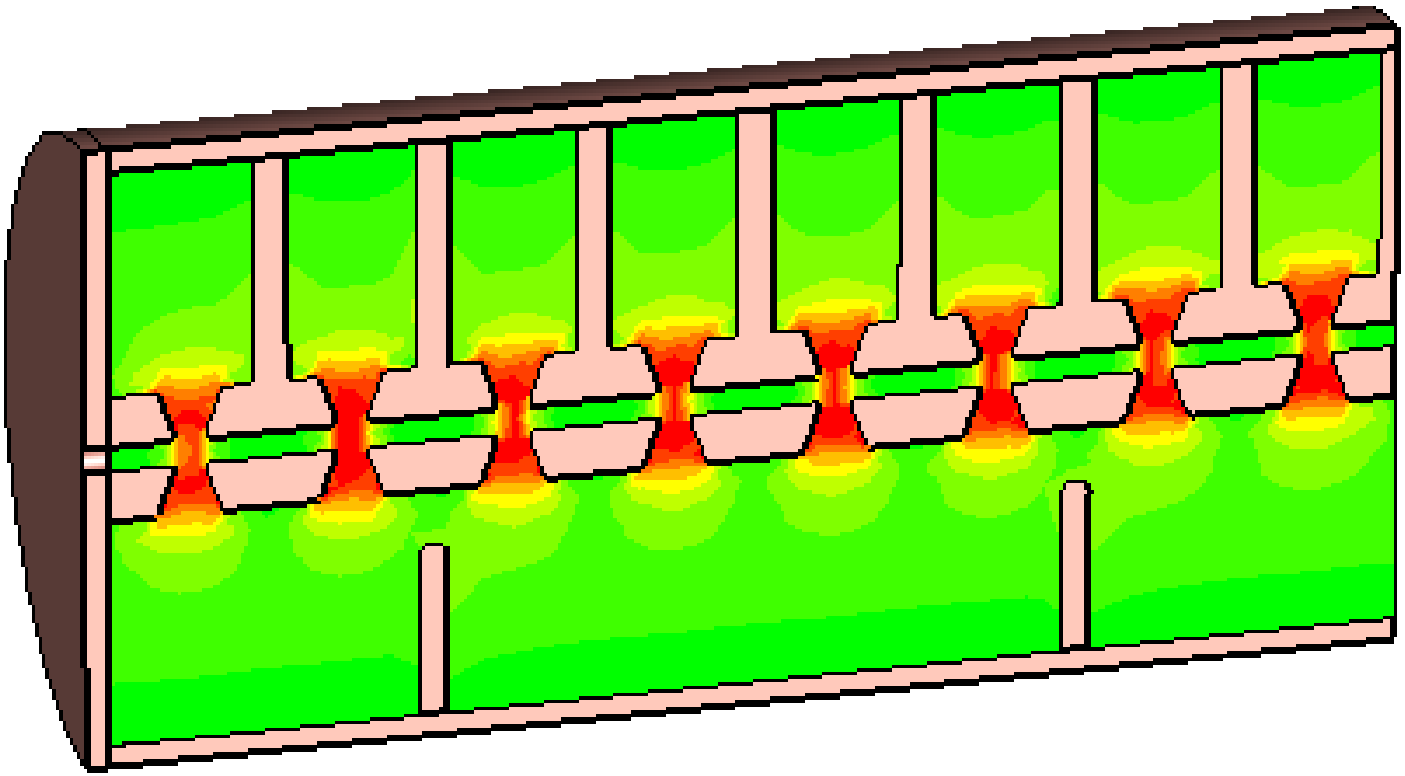
DTL – drift tubes



Drift Tube Linac

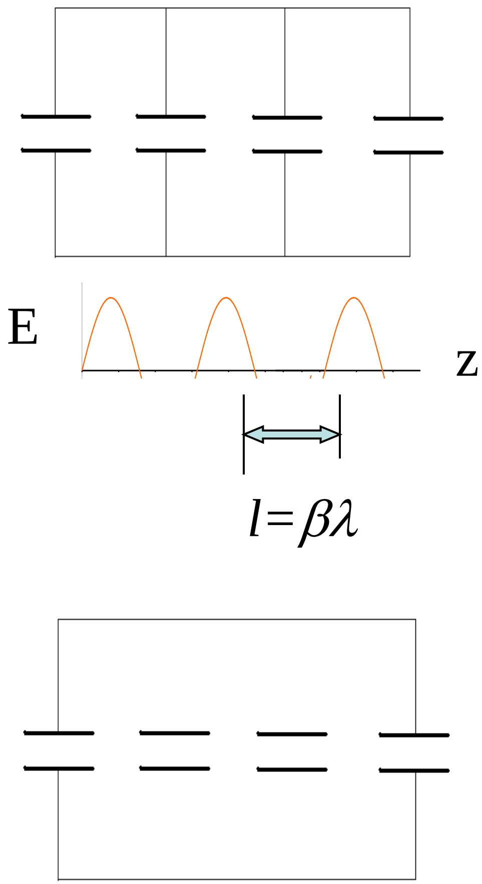


DTL : electric field



Mode is TM010

DTL



The DTL operates in **0 mode**
for protons and heavy ions in the
range β=0.04-0.5 (750 keV - 150 MeV)

Synchronism condition (0 mode):

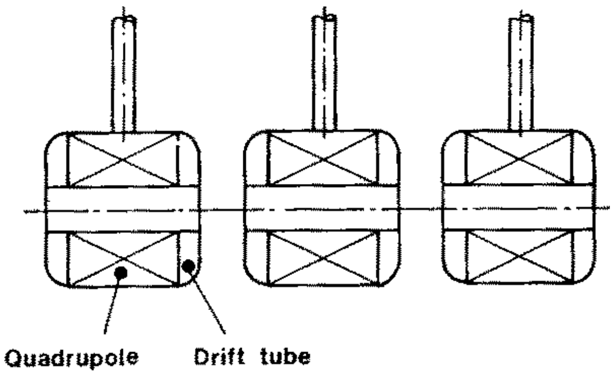
$$l = \frac{\beta c}{f} = \beta\lambda$$

The beam is inside the "drift tubes" when the electric field is decelerating

The fields of the 0-mode are such that if we eliminate the walls between cells the fields are not affected, but we have less RF currents and higher shunt impedance

Drift Tube Linac

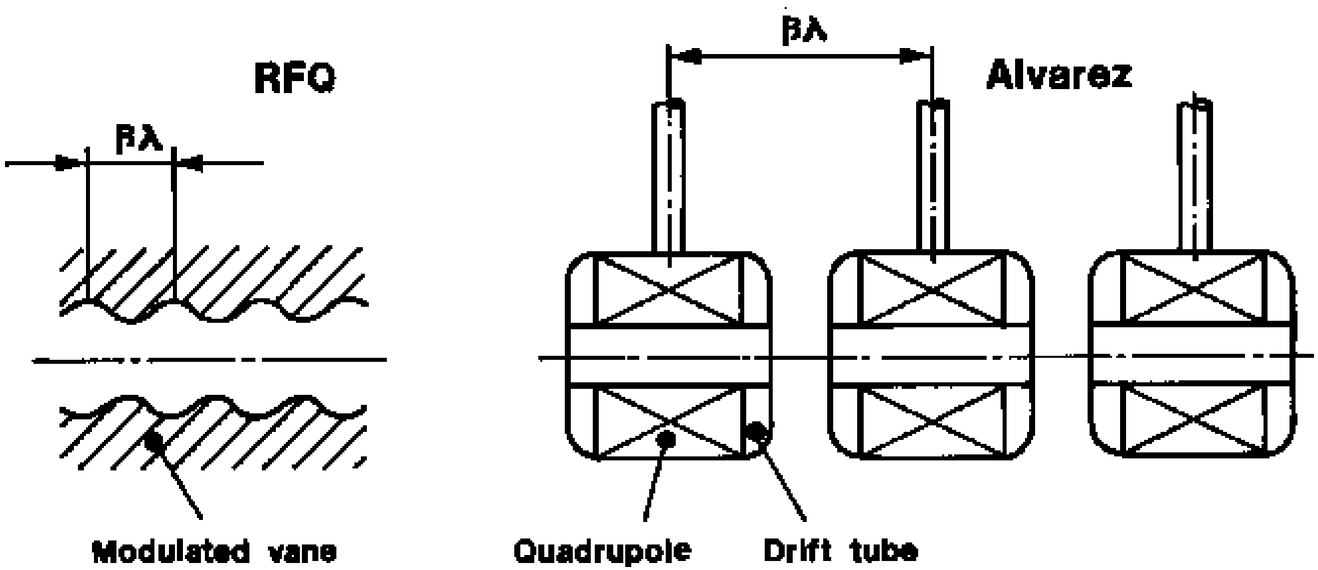
1. There is space to insert **quadrupoles** in the drift tubes to provide the strong transverse focusing needed at low energy or high intensity



2. The **cell length** ($\beta\lambda$) can **increase** to account for the increase in beta

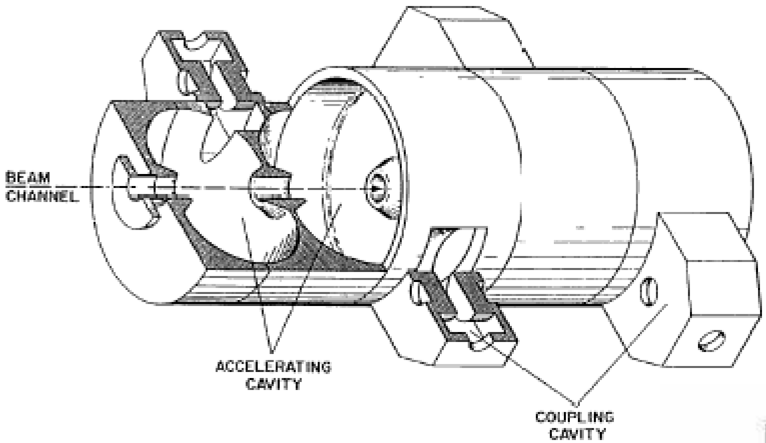
$\Rightarrow$ **the DTL is the ideal structure for the low β - low W range**

RFQ vs. DTL



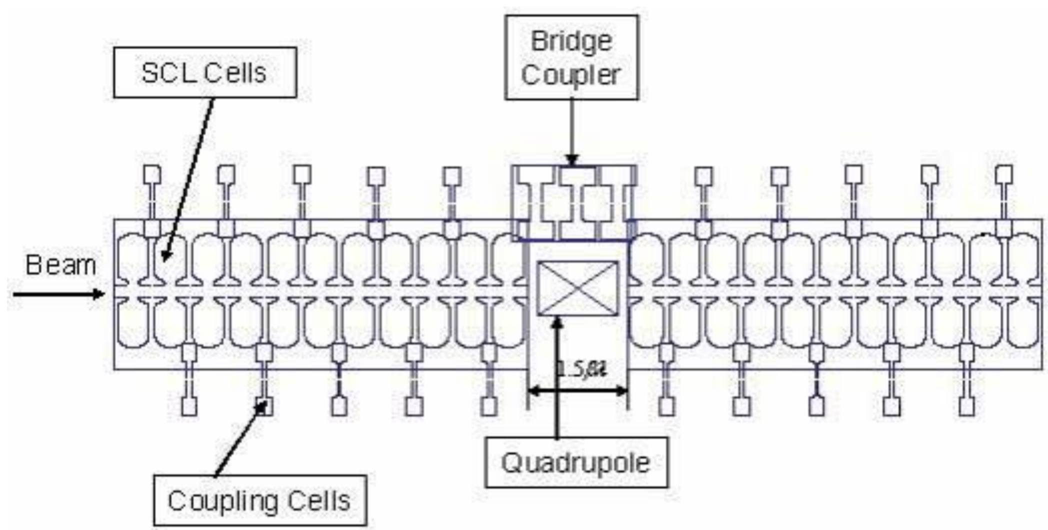
DTL can't accept low velocity particles, there is a minimum injection energy in a DTL due to mechanical constraints

Side Coupled Linac

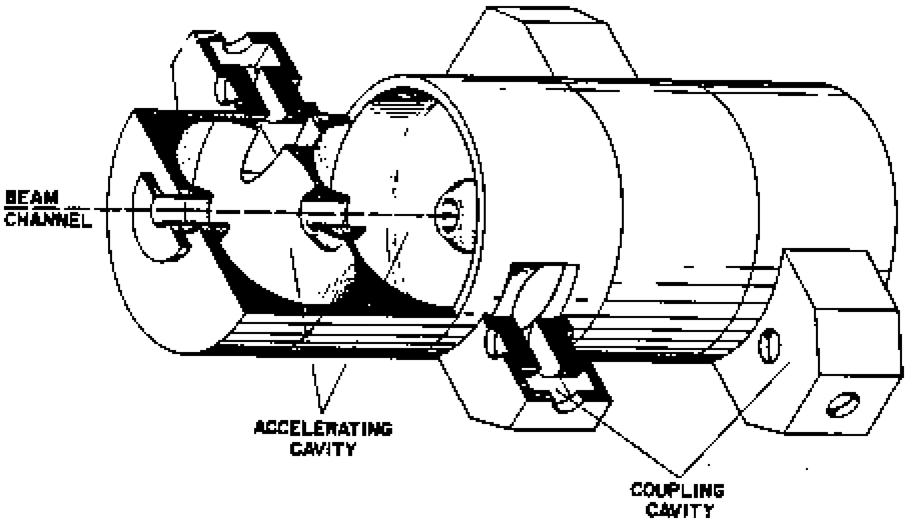


Chain of cells, coupled via slots and off-axis coupling cells.
Invented at Los Alamos in the 60's.
Operates in the $\pi/2$ mode (stability).

CERN SCL design:
Each klystron feeds 5 tanks of 11 accelerating cells each, connected by 3-cell bridge couplers.
Quadrupoles are placed between tanks.



The Side Coupled Linac

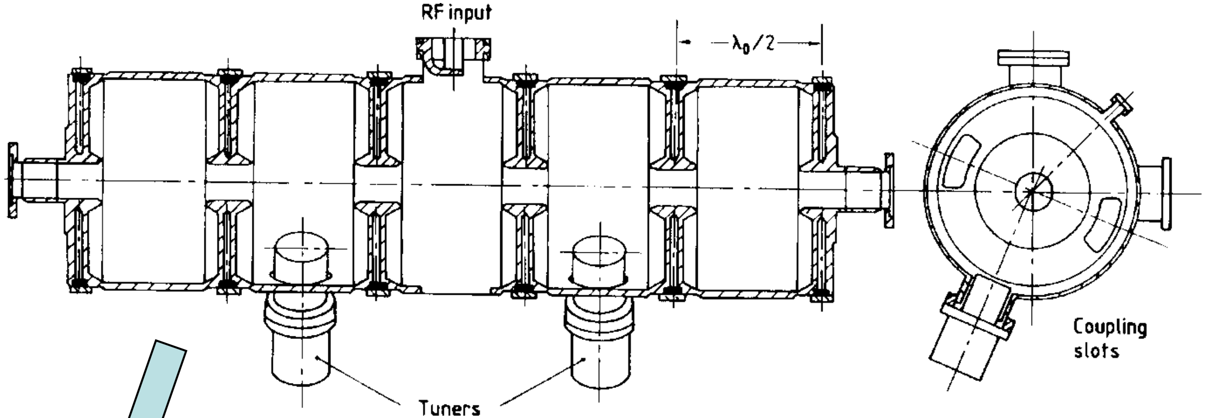


multi-cell Standing Wave structure in **$\pi/2$ mode**
frequency 800 - 3000 MHz
for protons (β=0.5 - 1)

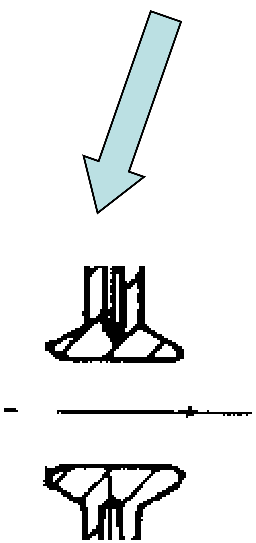
Rationale: high beta ⇒ cells are longer ⇒ advantage for high frequencies
• at high *f*, high power (> 1 MW) klystrons available ⇒ long chains (many cells)
• long chains ⇒ high sensitivity to perturbations ⇒ operation in $\pi/2$ mode

Side Coupled Structure:
- from the wave point of view, $\pi/2$ mode
- from the beam point of view, π mode

Room Temperature SW structure:
The LEP1 cavity



5-cell Standing Wave structure in **π mode** frequency 352 MHz for electrons (β=1)

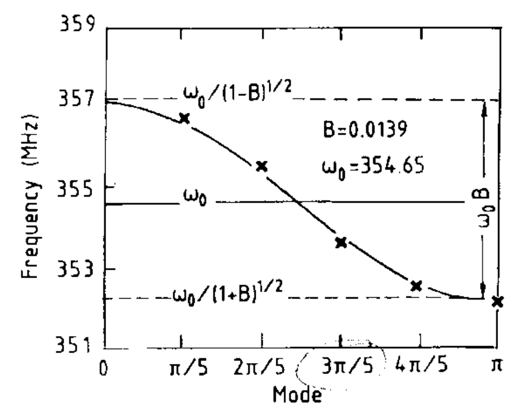


“noses”

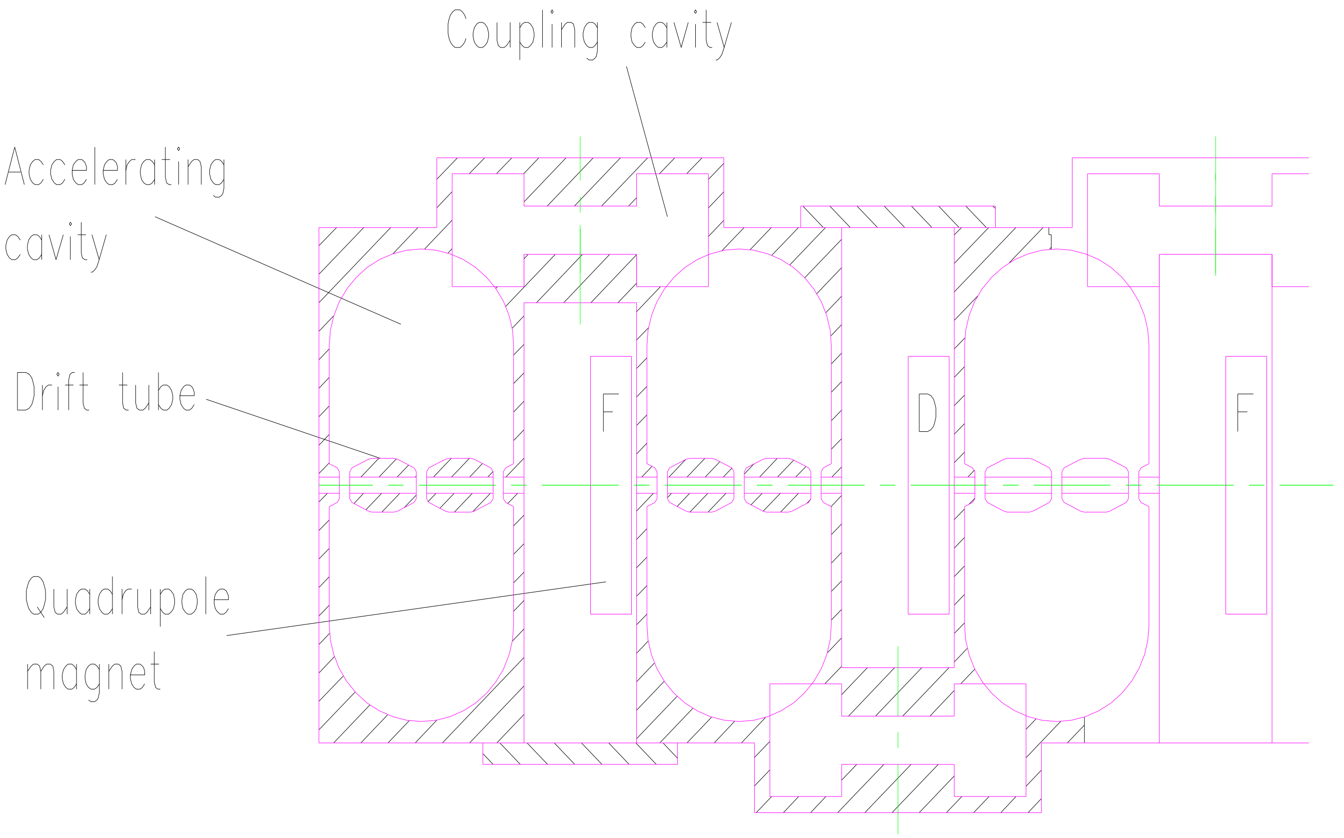
To increase shunt impedance :

1. “noses” concentrate E-field in “gaps”
2. curved walls reduce the path for RF currents

BUT: to close the hole between cells would “flatten” the dispersion curve ⇒ introduce coupling slots to provide magnetic coupling

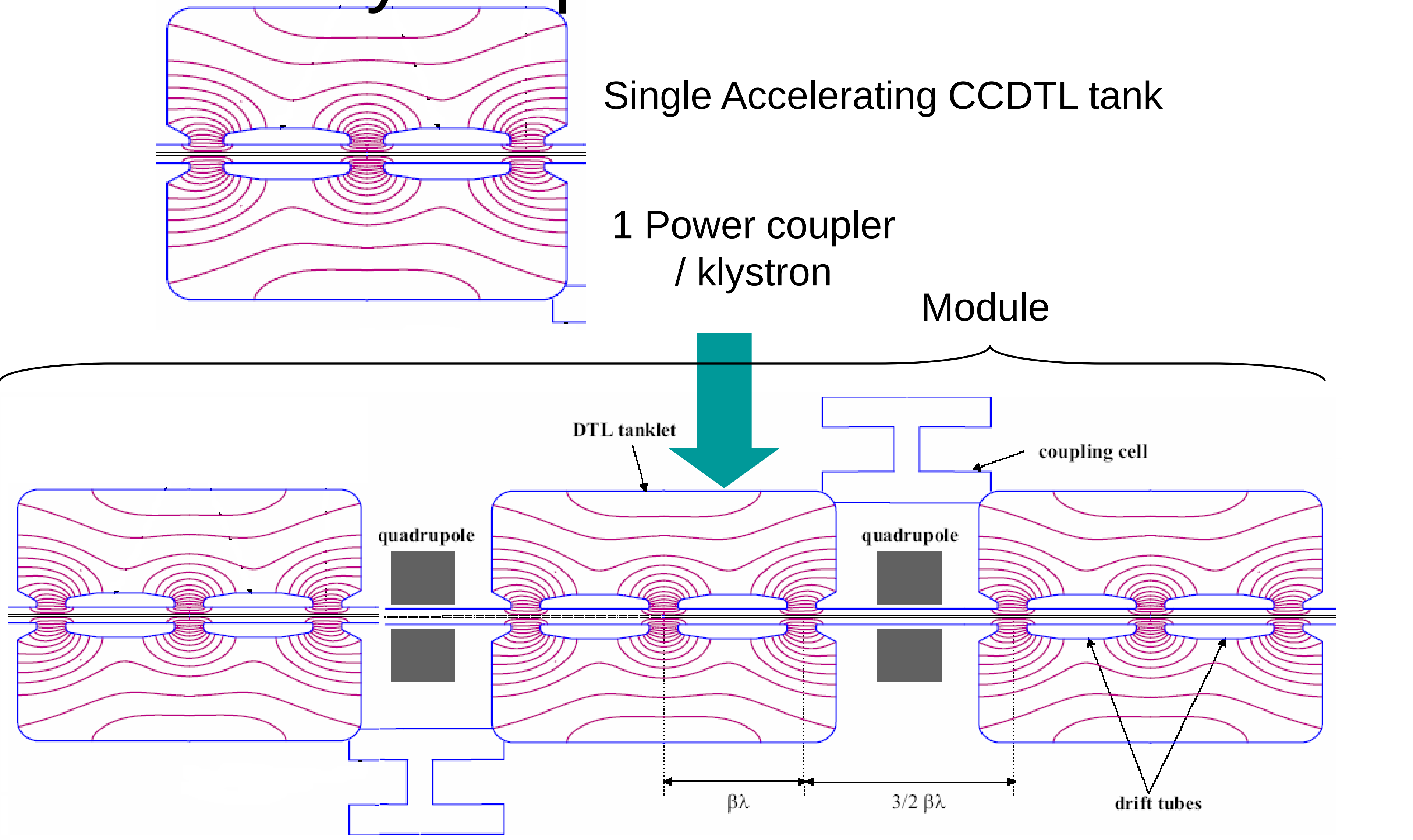


example of a mixed structure : the cell coupled drift tube linac



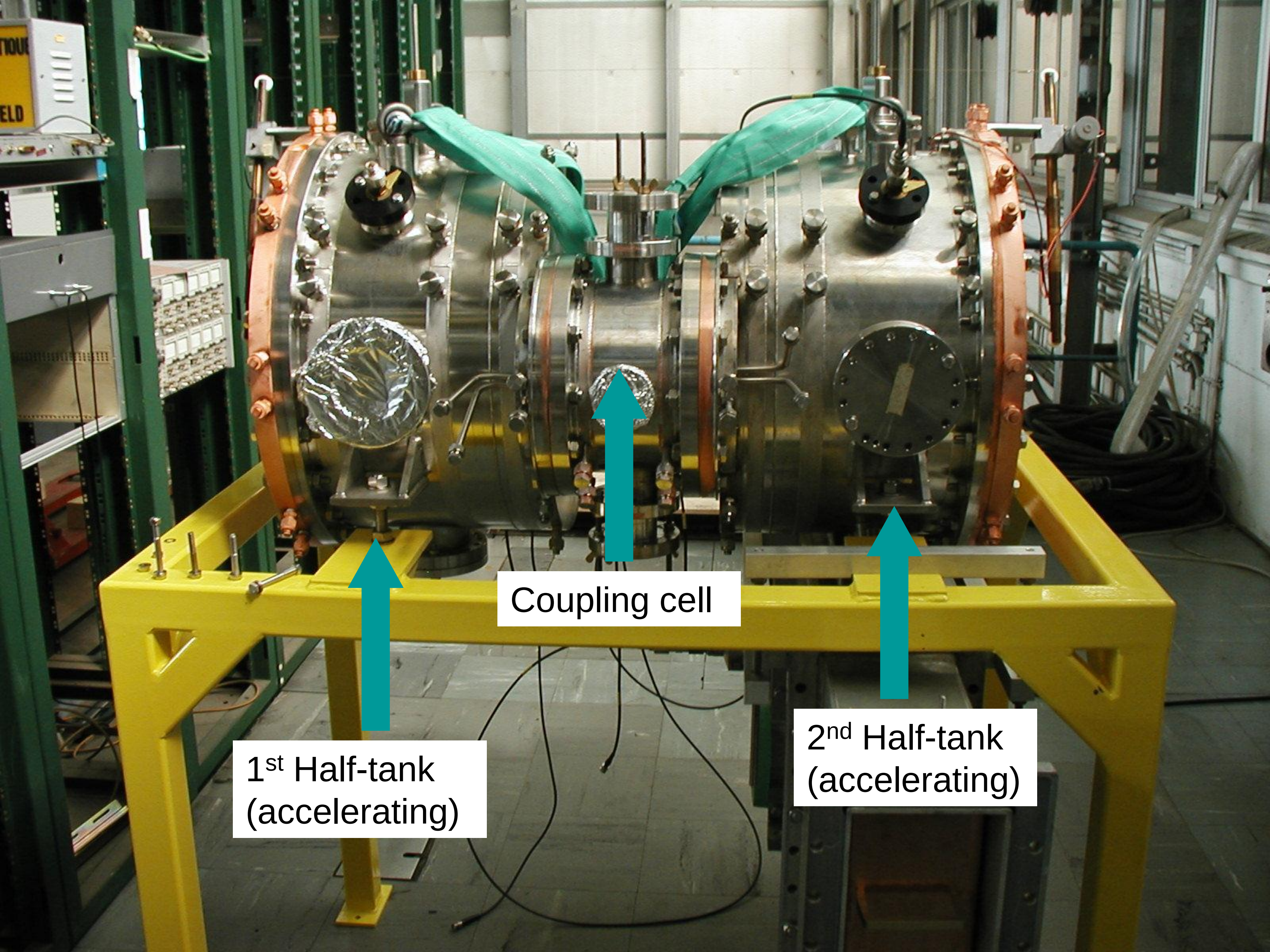
linac with a reasonable shunt impedance in the range of $0.2 < \beta < 0.5$, i. e. at energies which are between an optimum use of a DTL and an SCL accelerator

example of a mixed structure : the cavity coupled drift tube linac



CCDTL – cont’ed

- In the energy range 40-90 MeV the velocity of the particle is high enough to allow long drifts between focusing elements so that…
- …we can put the quadrupoles lenses outside the drift tubes with some advantage for the shunt impedance but with great advantage for the installation and the alignment of the quadrupoles…
- the final structure becomes easier to build and hence cheaper than a DTL.
- The resonating mode is the p/2 which is intrinsically stable


Coupling cell
1st Half-tank
(accelerating)
2nd Half-tank
(accelerating)

overview

take with CAUTION!

	Ideal range of beta	frequency	Particles
RFQ	Low!!! - 0.05	40-400 MHz	Ions / protons
IH	0.02 to 0.08	40-100 MHz	Ions and also protons
DTL	0.04-0.5	100-400 MHz	Ions / protons
SCL	Ideal Beta=1 But as low as beta 0.5	800 - 3000 MHz	protons / electrons

Summary of lesson 2

- wave equation in a cavity
- loaded cavity
- TM and TE mode
- some example of accelerating structures ad their range of use