Transverse Beam Dynamics

JUAS tutorial 4

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1 Exercise: chromaticity in a FODO cell

Consider a ring made of N identical FODO cells with equally spaced quadrupoles. Assume that the two quadrupoles are both of length l_q, but their strengths may differ.

1.1 Calculate the maximum and the minimum betatron function in the FODO cell as a function of the length l_q, the focal lengths f_F, f_D and the phase advance μ. (*Use the thin lens approximations*)

1.2 Calculate the natural chromaticities for this machine.

1.3 Show that for short quadrupoles, if $f_F \simeq f_D$,

$$\xi_N \simeq -\frac{2\tan\frac{\mu}{2}}{\mu}$$

2 Exercise: measurement of Twiss parameters

One of the possible ways to determine experimentally the Twiss parameters at a given point makes use of a so-called quadrupole scan. One can measure the transverse size of the beam in a profile monitor, called Wire Beam Scanner (WBS), located at a distance L downstream a focusing quadrupole, as a function of the normalized gradient in this quadrupole. This allows to compute the emittance of the beam, as well as the β- and the α- functions at the entrance of the quadrupole.

Let's consider a quadrupole Q with a length of $l = 20$ cm. This quadrupole is installed in an electron transport line where the particle momentum is 300 MeV/c. At a distance $L = 10$ m from the quadrupole the transverse beam size is measured with a WBS, for various values of the current I_Q. The maximum value of the quadrupole gradient G is obtained for a current of 100 A, and is $G = 1$ T/m. G is proportional to the current.

Advice: use thin-lens approximation.

2.1 How does the normalized focusing strength K vary with I_Q?

2.2 Let Σ_1 and Σ_2 be the 2×2 matrices with the twiss parameters, $\Sigma = \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$, at the quadrupole entrance and at the wire scanner, respectively.

- Give the expression Σ_2 as function of α_1, β_1, and γ_1
- Show that β_2 can be written in the form: $\beta_2 = A_2\,(Kl)^2 + A_1\,(Kl) + A_0$
- Express A_0, A_1, and A_2 as a function of L, α_1,β_1, and γ_1

Hint for the next questions: show that if you express β_2 as

$$\beta_2 = B_0 + B_1\,(Kl - B_2)^2$$

you have:

$B_0 = A_0 - A_1^2/4A_2^2 = L^2/\beta_1$

$B_1 = A_2 = L^2\beta_1$

$B_2 = -A_1/A_2 = 1/L - \alpha_1/\beta_1$

2.3 **The transverse beam r.m.s. beam size is $\sigma = \sqrt{\epsilon\beta}$, where ϵ is the transverse emittance. Express σ_2 as a function of Kl and find its minimum, $(Kl)_{\mathbf{min}}$. Give the expression for $\frac{\mathrm{d}\sigma_2}{\mathrm{d}(Kl)}$.**

2.4 **How does σ_2 vary with Kl when $|Kl - (Kl)_{\mathbf{min}}| \gg 1/\beta_1$?**

2.5 **Deduce the values of α_1,β_1, and γ_1 from the measurement σ_2, as a function of the quadrupole current I_Q.**