Joint Universities Accelerator School

JUAS 2013

Archamps, France, 18th February 2013

# Normal-conducting accelerator magnets

Thomas Zickler,

CERN

# Scope of the lectures

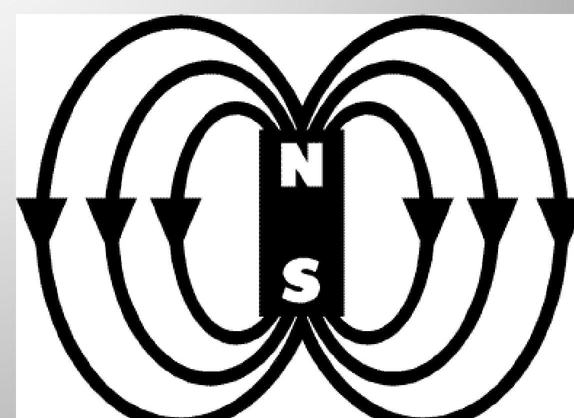
Overview of electro-magnetic technology as used in particle accelerators considering *normal-conducting, iron-dominated* electro-magnets (generally restricted to direct current situations)

## Main goal is to:

- create a fundmental understanding in accelerator magnet technology
- provide a guide book with practical instructions how to start with the design of a standard accelerator magnet
- focus on applied and practical design aspects using 'real' examples
- introduce finite element codes for practical magnet design

## Not covered:

– permanent magnet technology
– super-conducting technology



# Program (1)

## Session 1 (14:00 – 15:00)

### Lecture 1 – Introduction & Basic principles (60')

- A bit of history…
- Why do we need magnets?
- Basic principles and concepts
- Magnet types

## Session 2 (15:00 – 16:00)

### Lecture 2 - Analytical design (60')

- What do we need to know before starting?
- Yoke design
- Coil dimensioning
- Cooling layout
- Magnet manufacturing

## Coffee break (16:00 – 16:15)

## Session 3 (16:15 – 17:15)

### Lecture 3 – Applied numerical design (60')

- Creation of a basic 2D finite-element model
- Interpretation of results
- Outlook into 3D design

# Program (2)

## Session 4 Wednesday, 20.2. (15:00 – 17:15)

Case study (part 1) (120’)

Introduction

Students are invited to design and specify a ‚real‘ magnet

Analytical magnet design on paper

## Session 5 Thursday, 21.2. (9:00 – 12:15)

Case study (part 2) (180’)

Computer work

Numerical magnet design

## Session 6 Friday, 22.2. (9:00 – 12:00)

Practical works @ CERN ( 2 x 90’)

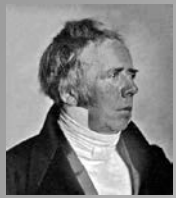
Manufacturing technologies, materials,

QA tests and magnetic measurements

# Lecture 1: Basic principles

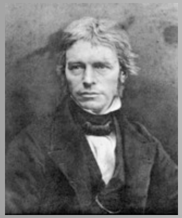
- A bit of history…
- Why do we need magnets?
- Magnet technologies
- Basic principles and concepts
- Magnet types and applications

# A bit of history…

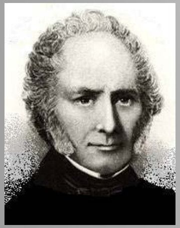


1820: Hans Christian Oersted (1777-1851) finds that electric current affects a compass needle

1820: Andre Marie Ampere (1775-1836) in Paris finds that wires carrying current produce forces on each other

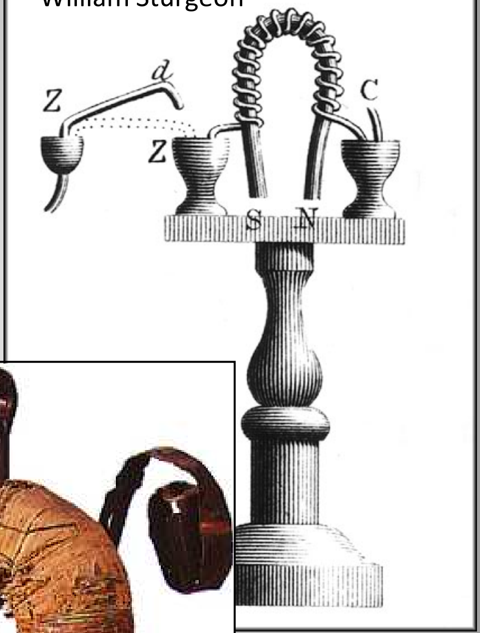


1820: Michael Faraday (1791-1867) at Royal Society in London develops the idea of electric fields and studies the effect of currents on magnets and magnets inducing electric currents



1825: British electrician, William Sturgeon (1783-1850) invented the first electromagnet

1860: James Clerk Maxwell (1831-1879), a Scottish physicist and mathematician, puts the theory of electromagnetism on mathematical basis

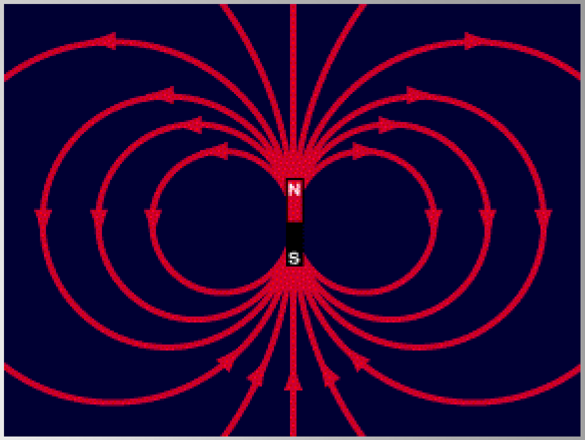


William Sturgeon


Joseph Henry

# Magnetic units

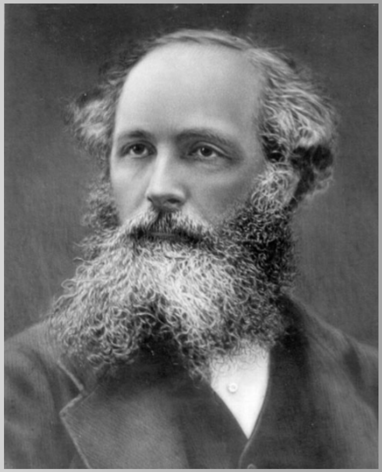
IEEE defines the following units:




- Magnetic field:
  - H (vector) [A/m]
  - the magnetizing force produced by electric currents
- Electromotive force:
  - e.m.f. or U [V or (kg $m^2$)/(A $s^3$)]
  - here: voltage generated by a time varying magnetic field
- Magnetic flux density or magnetic induction:
  - B (vector) [T or kg/(A $s^2$)]
  - the density of magnetic flux driven through a medium by the magnetic field
  - Note: induction is frequently referred to as "Magnetic Field“
  - $H$, $B$ and $\mu$ relates by: $B = \mu H$
- Permeability:
  - $\mu = \mu_0 \mu_r$
  - permeability of free space $\mu_0 = 4 \pi 10^{-7}$ [V s/A m]
  - relative permeability $\mu_r$ (dimensionless): $\mu_{air} = 1$; $\mu_{iron} > 1000$ (not saturated)
- Magnetic flux:
  - ϕ [Wb or (kg $m^2$)/(A $s^2$)]
  - surface integral of the flux density component perpendicular trough a surface

# Maxwell's equations

In 1873, Maxwell published "Treatise on Electricity and Magnetism" in which he summarized the discoveries of Coulomb, Oersted, Ampere, Faraday, et. al. in four mathematical equations:



## Gauss' law for electricity:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \oint_{\partial V} \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

## Gauss' law of flux conservation:

$$\nabla \cdot \vec{B} = 0 \qquad \oint_{\partial V} \vec{B} \cdot d\vec{A} = 0$$

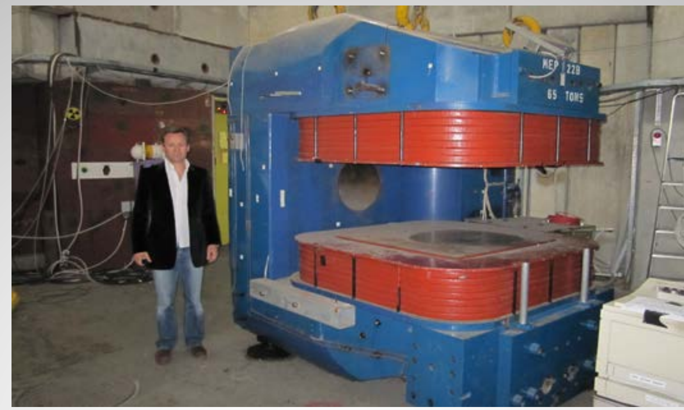
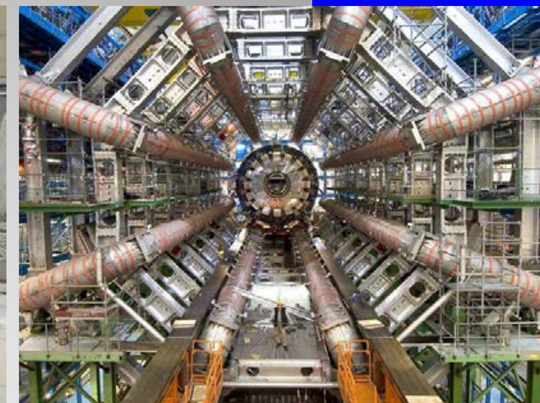
## Faraday's law of induction:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \oint_{\partial A} \vec{E} \cdot d\vec{s} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int_A \vec{B} \cdot d\vec{A}$$

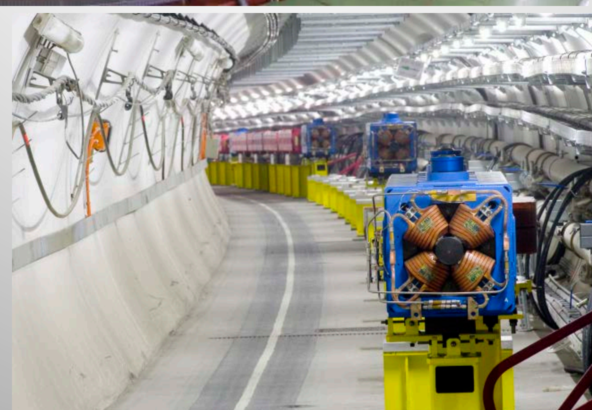
## Ampere's circuital law:

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \oint_{\partial A} \vec{B} \cdot d\vec{s} = \int_A \mu_0 \vec{J} \cdot d\vec{A} + \frac{d}{dt}\int_A \mu_0 \varepsilon_0 \vec{E} \cdot d\vec{A}$$

# Magnets everywhere...



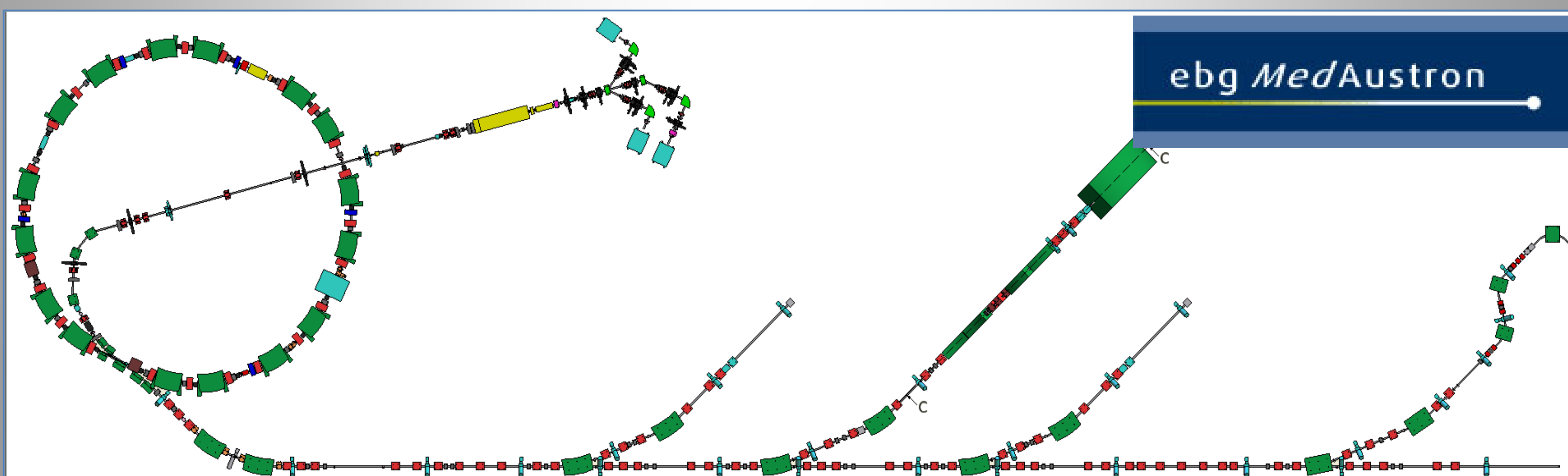



CERN

JUAS
Joint Universities Accelerator School

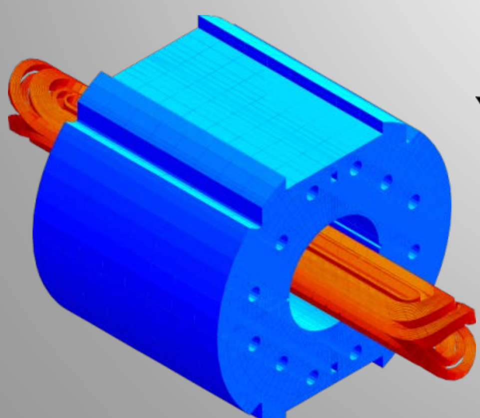
# Why do we need magnets?

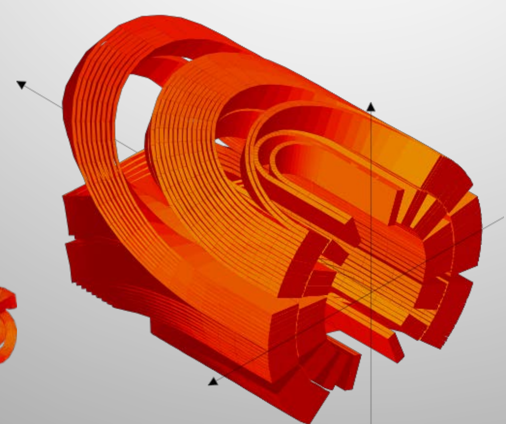
- Interaction with the beam
  - guide the beam to keep it on the orbit
  - focus and shape the beam
- Lorentz‘s force: $\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$
  - for relativistic particles this effect is equivalent if $\vec{E} = c\vec{B}$
  - if B = 1 T then E = $3 \cdot 10^8$ V/m

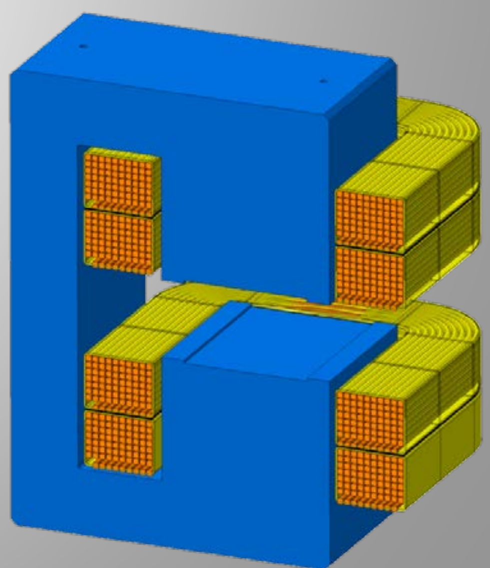


# Magnet technologies

- Magnets
  - Electro-magnets
    - Superconducting
      - **Coil dominated** B < 9 T
      - Iron dominated B < 2 T
    - Normal-conducting
      - Coil dominated B < 1 T
      - **Iron dominated** B < 2 T
  - Permanent magnets

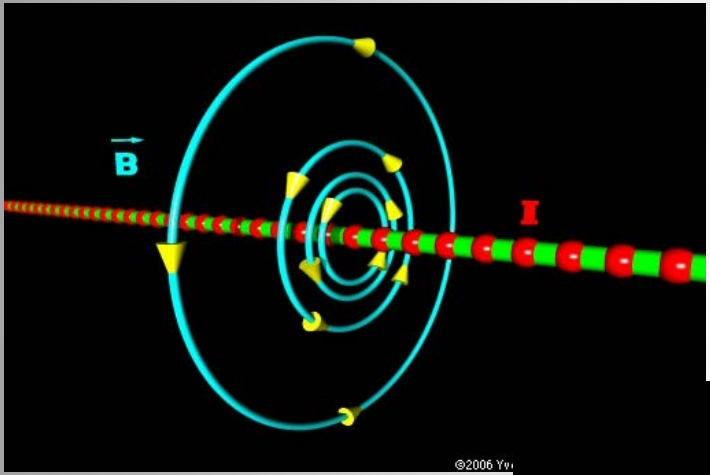






# How does a magnet work?

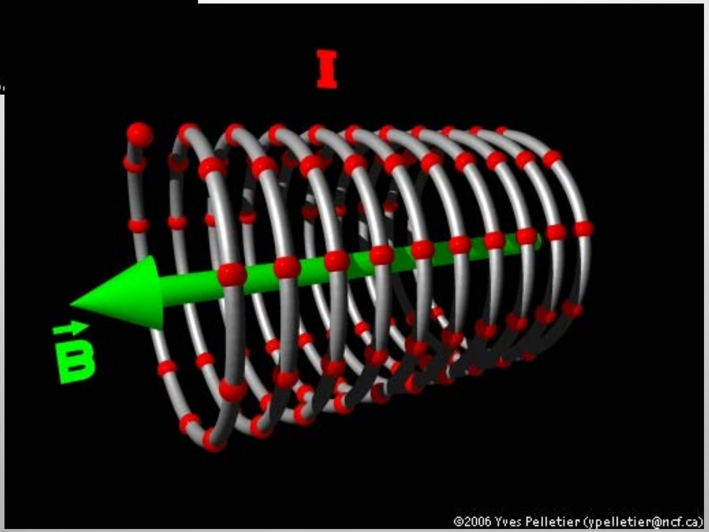
- Permanent magnets provide only constant magnetic fields
- Electro-magnets can provide adjustable magnetic fields



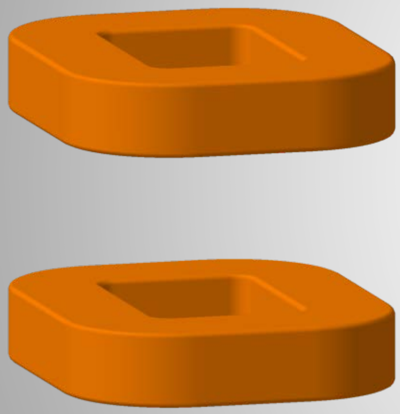

Maxwell & Ampere:

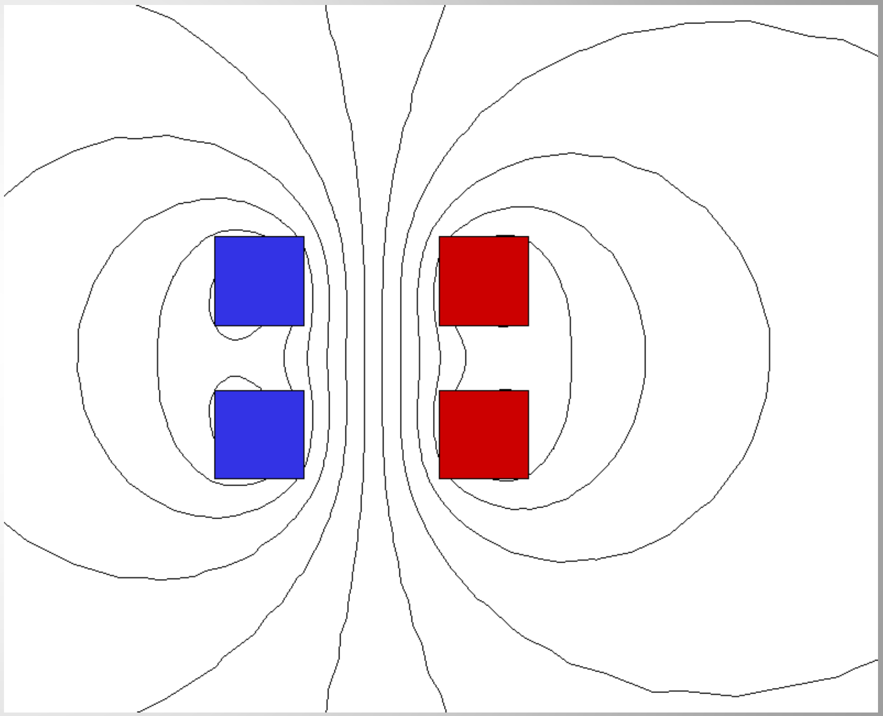
$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

„An electrical current is surrounded by a magnetic field“



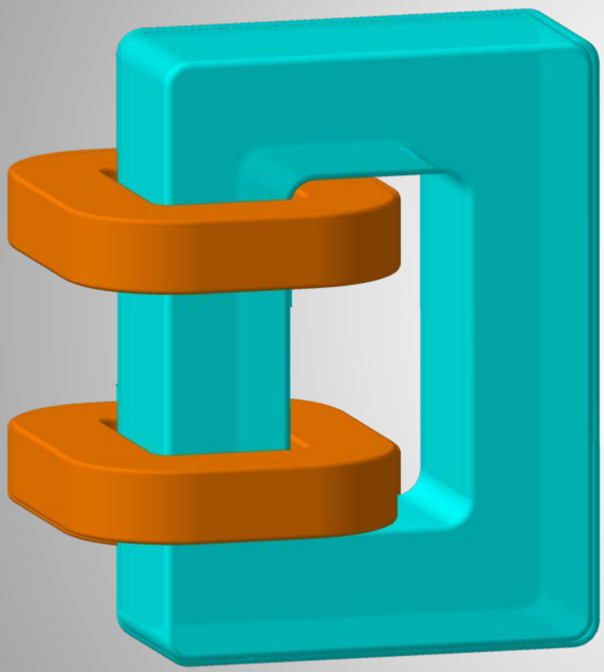
# Magnetic circuit

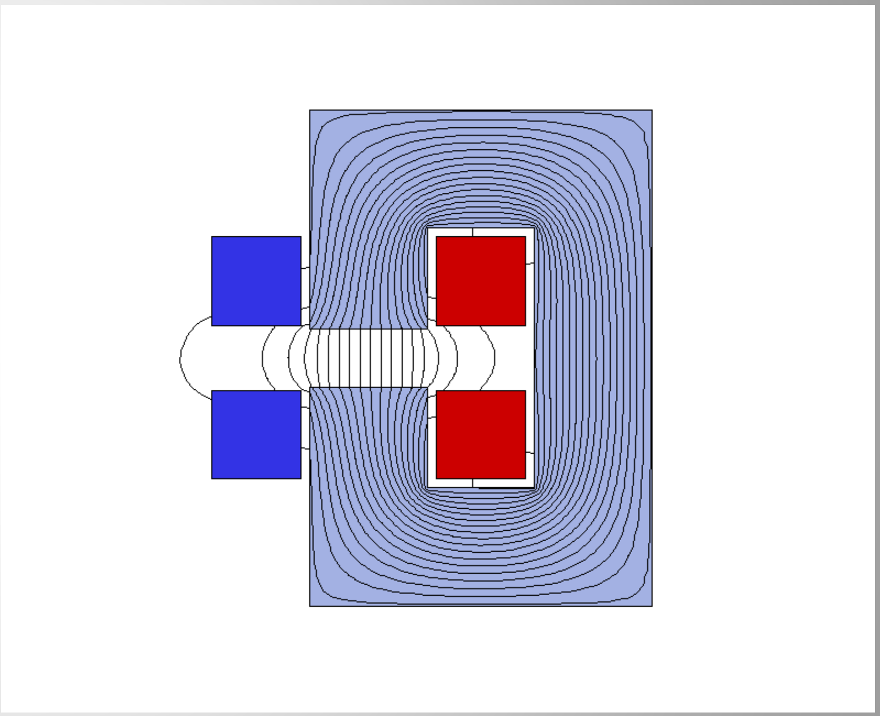




Flux lines represent the magnetic field
Coil colors indicate the current direction

# Magnetic circuit





Coils hold the electrical current
Iron holds the magnetic flux

# Excitation current in a dipole



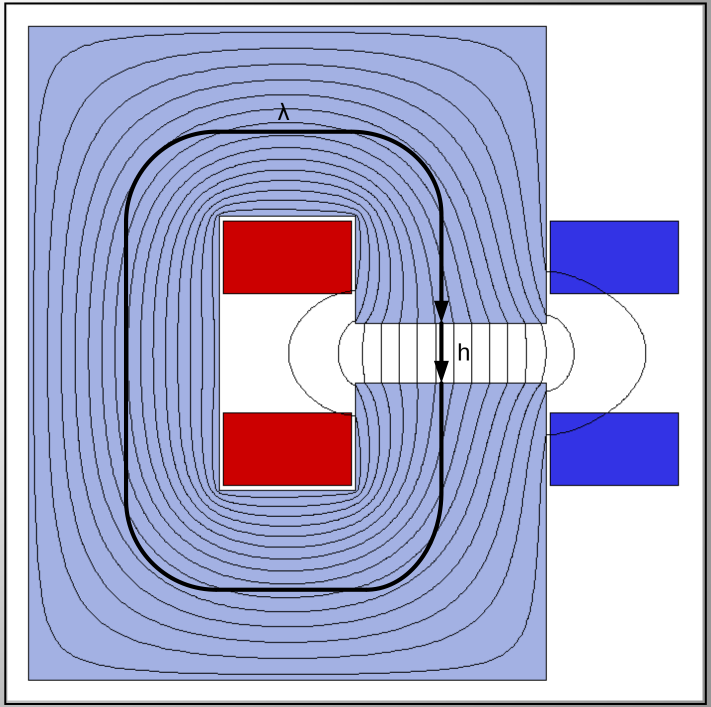
Ampere's law $\oint \vec{H} \cdot d\vec{l} = NI$ and $\vec{B} = \mu \vec{H}$ with $\mu = \mu_0 \mu_r$

leads to $$NI = \oint \frac{\vec{B}}{\mu} \cdot d\vec{l} = \int_{gap} \frac{\vec{B}}{\mu_{air}} \cdot d\vec{l} + \int_{yoke} \frac{\vec{B}}{\mu_{iron}} \cdot d\vec{l} = \frac{Bh}{\mu_{air}} + \frac{B\lambda}{\mu_{iron}}$$

assuming, that B is constant along the path

If the iron is not saturated: $$\frac{h}{\mu_{air}} >> \frac{\lambda}{\mu_{iron}}$$

then: $$NI_{(per\ pole)} \approx \frac{Bh}{2\mu_0}$$

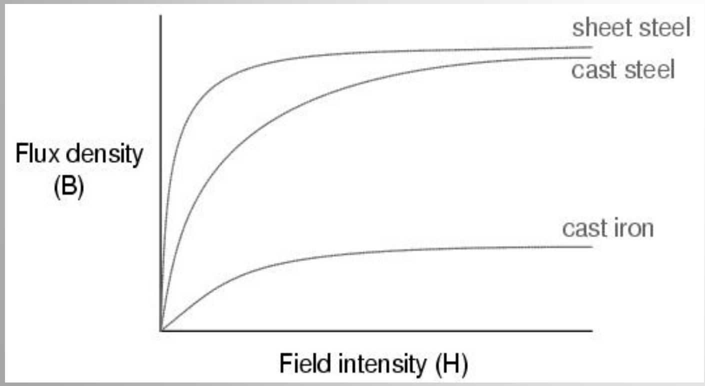


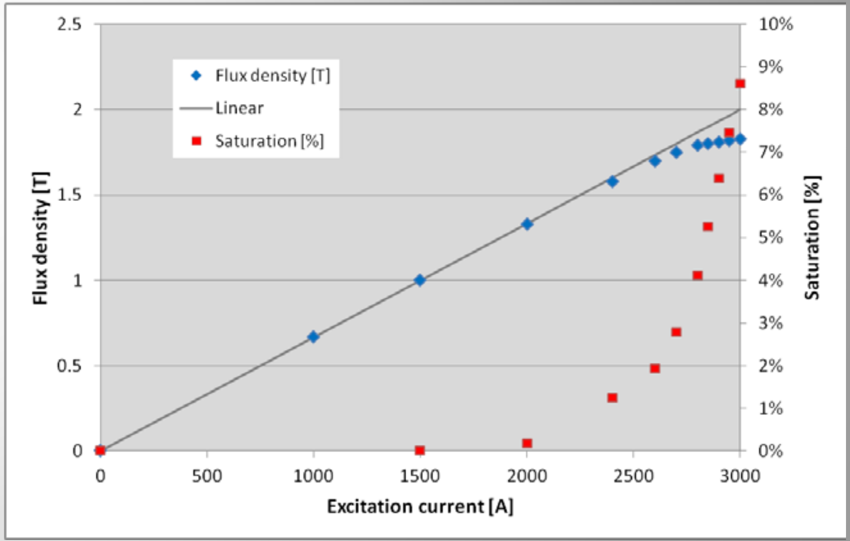
# Reluctance and efficiency

Similar to Ohm's law, one can define the 'resistance' of a magnetic circuit, called 'reluctance', as:

- *σ:* conductivity [S/m]
- *NI:* magneto-motive force [A]
- *Φ*: magnetic flux [Wb]
- $l_M$: flux path length in iron [m]
- $A_M$: iron cross section perpendicular to flux [m²]

$$R_E = \frac{U}{I} = \frac{l_E}{A_E \sigma} \Rightarrow R_M = \frac{NI}{\Phi} = \frac{l_M}{A_M \mu_r \mu_0}$$



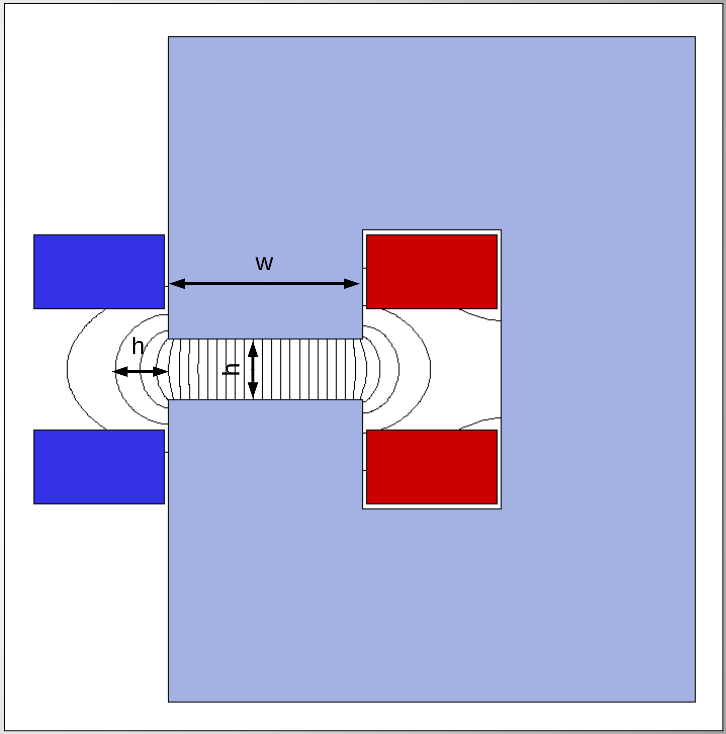

- Increase of B above 1.5 T in iron requires non-proportional increase of H
- Iron saturation (small $\mu_{iron}$) leads to inefficiencies

# Magnetic flux

Flux in the yoke includes the gap flux and stray flux

Total flux in the return yoke:

$$\Phi = \int_A B \cdot dA \approx B_{gap}(w + 2h) l_{mag}$$

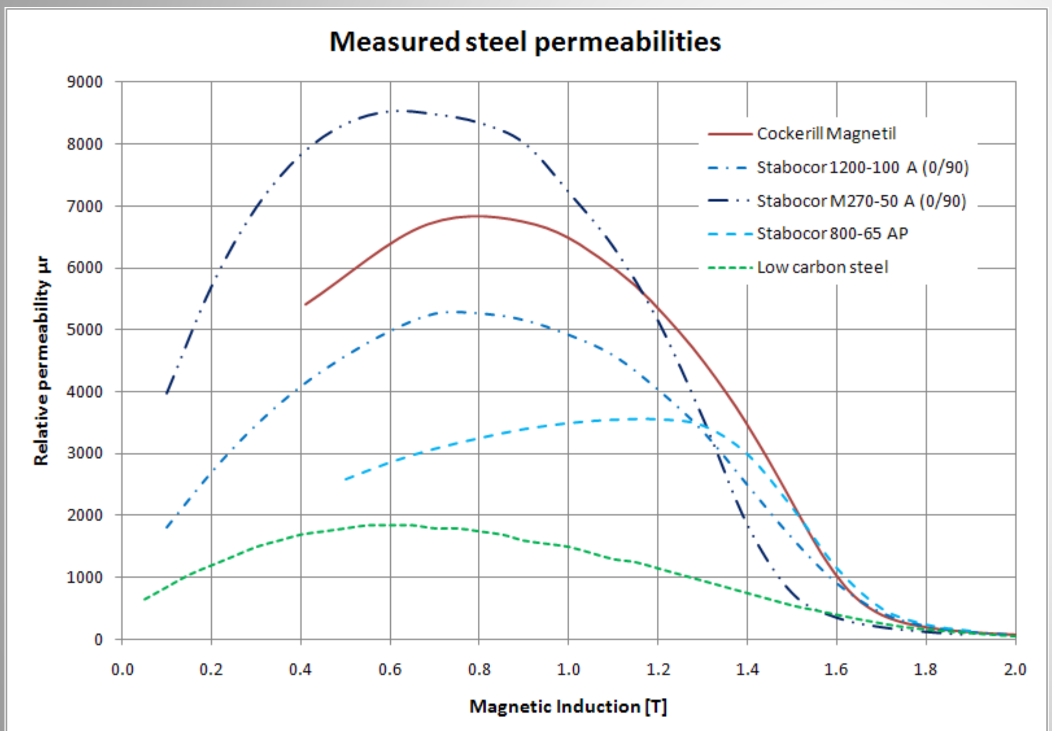


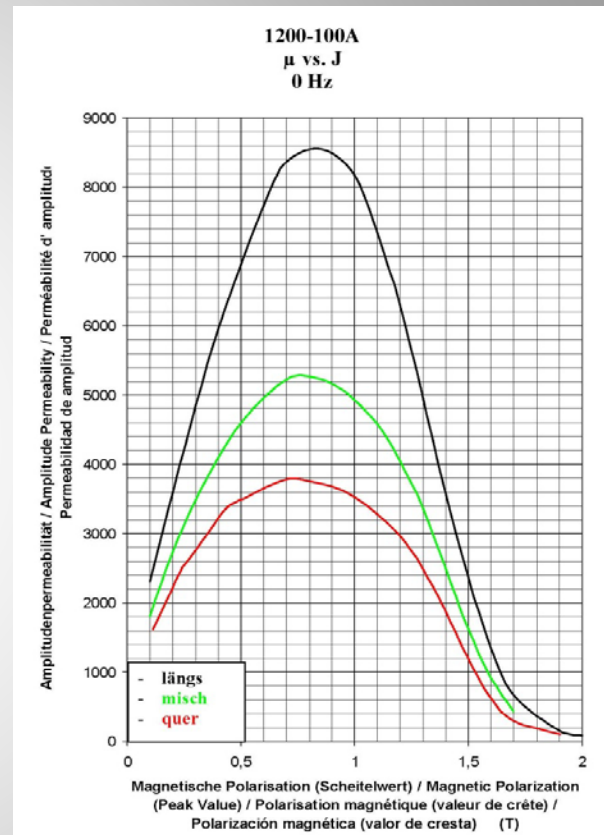

Keep yoke reluctance small by providing sufficient iron cross-section!

# Permeability



Ferro-magnetic materials: high permeability ($\mu_r$ >>1), but not constant



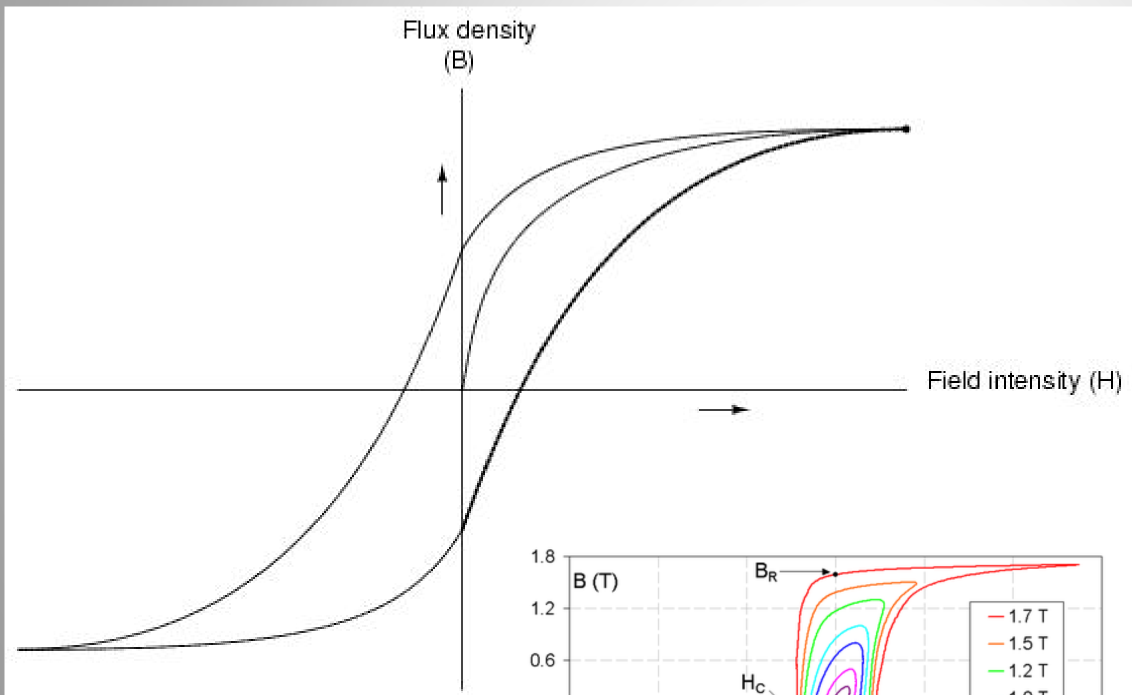



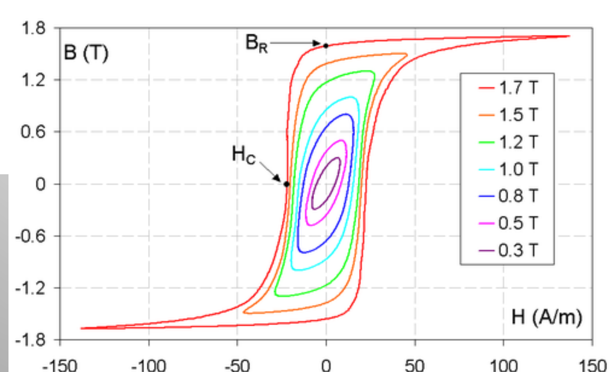

Data source: Thyssen/Germany

Anisotropy in sheet material can be partly cured by final annealing

# Steel hysteresis

Flux density *B(H)* as a function of the field strength is different, when increasing and decreasing excitation



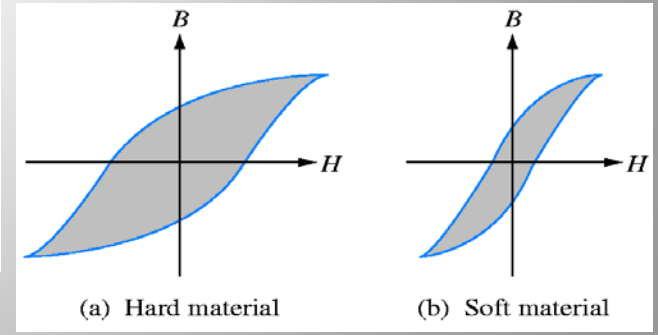




## Remanent field (Retentivity):

$H = 0 \rightarrow B = B_r > 0$

## Coercivity or coercive force:

$B = 0 \rightarrow H = H_c < 0$



# Residual field in a magnet

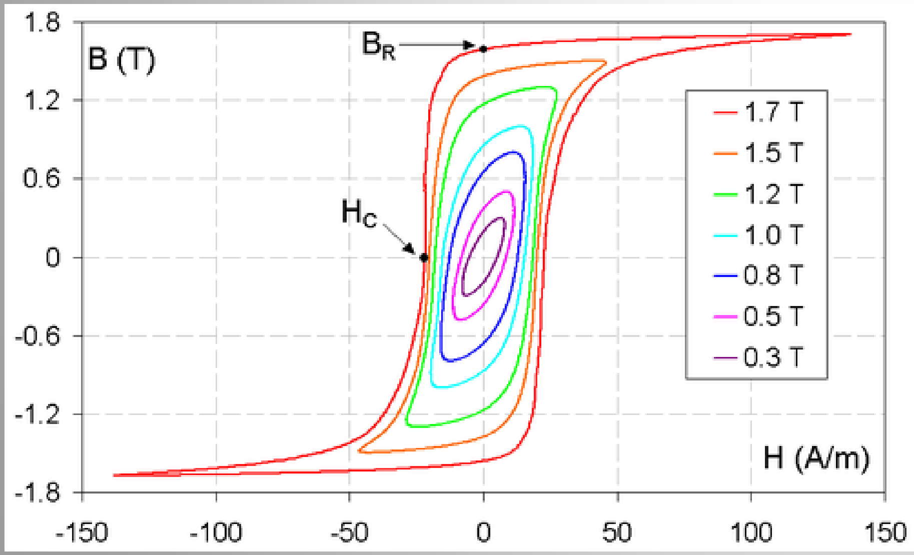
In a continuous ferro-magnetic core (transformer) the residual field is determined by the remanent field $B_r$

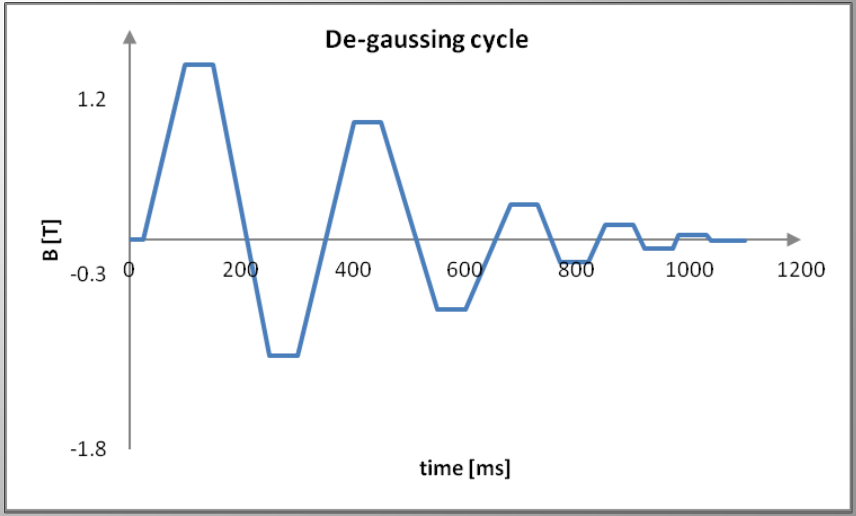
In a magnet core (gap), the residual field is determined by the coercivity $H_c$

Assuming the coil current *I=0*:

$$\oint \vec{H} \cdot \vec{dl} = \int_{gap} \vec{H}_{gap} \cdot \vec{dl} + \int_{yoke} \vec{H}_c \cdot \vec{dl} = 0$$

$$B_{residual} = -\mu_0 H_C \frac{l}{g}$$



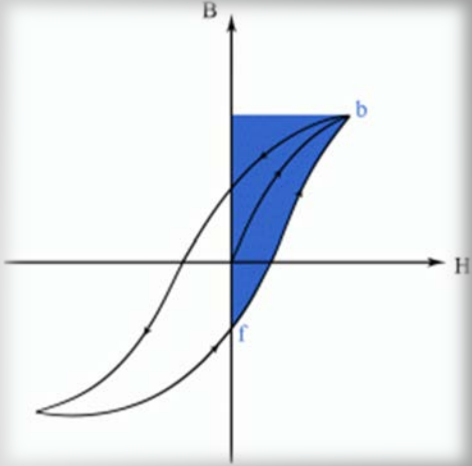




Demagnetization cycle!

# Stored energy & Inductance

Stored energy $E_S$ [J, joules] in a magnet depends on (non-uniform) field distribution in the gap, coils, and iron yoke:

$$E_S = \int_V \int_f^b H \cdot dB \cdot dv$$ and in case $\mu_r$ is linear: $$E_S = \frac{1}{2}\int_V H \cdot B \cdot dv$$




– difficult to calculate analytically
– usually done by numerical computations
– most of the energy is stored in the air gap

Inductance L [H] of a magnet is given by: $$L = \frac{2E_S}{I^2}$$

– total voltage on a pulsed magnet: $$V_{tot} = RI + L\frac{dI}{dt} = RI + \frac{2E_S}{I^2}\frac{dI}{dt}$$

– low inductance allows fast changes of magnetic field
– inductance depends on the magnetization in the iron

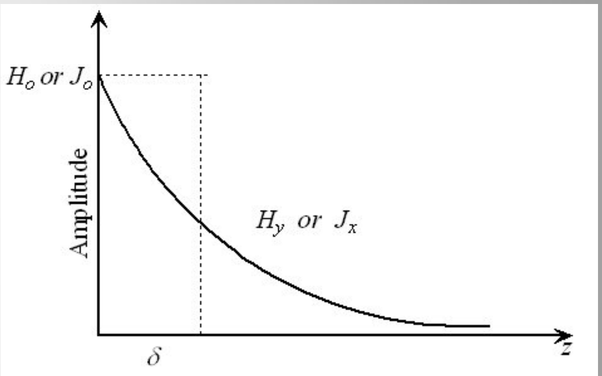
# Eddy currents

Faraday's law: varying magnetic field induces an e.m.f. (voltage) $U = -\frac{\partial \Phi}{\partial t}$

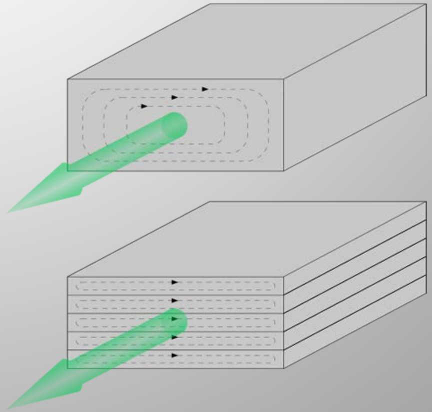
- Circulating (eddy) currents are generated in electrical conducting materials
  - creating a magnetic field opposing the original change in magnetic flux (Lenz's law)
  - opposing to the penetration of the magnetic field (skin effect)
  - producing losses (Joule heating)
  - causing delays to reach stable field value
  - damping high order modes (ripples)

$$H_y(z) = H_0 \cdot e^{-x/\delta} \qquad \delta = \frac{1}{\sqrt{\pi \cdot \mu_0 \cdot \mu_r \cdot f \cdot \sigma}}$$

  - $\delta$: skin depth [m]




- Magnetic circuits are made of insulated laminations to reduce eddy currents,
  - decrease lamination thickness (d < δ/2)
  - increase resistivity
  - decrease permeability
  - decrease frequency (∂Φ/∂t )



# Losses

## Losses in the coils:

Ohmic power loss $P_\Omega$ per length unit [W/m] in a coil conductor

$$\frac{P_\Omega}{l} = \frac{\rho}{a_{cond}} I^2$$

- $\rho$: resistivity [Ωm] (for copper: $1.86 \cdot 10^{-8}$ Ωm @ 40°C)
- $a_{cond}$ : conductor cross-section [m²]

## Losses in the iron yoke:

Hysteresis losses: Power loss $P_H$ per mass unit [W/kg] up to 1.5 T using Steinmetz's law

$$\frac{P_H}{m} = \eta \cdot f \cdot B^x$$

- $\eta$: material depending coefficient: $0.01 < \eta < 0.1$; $\eta \approx 0.02$ for silicon steel
- $x$: Steinmetz exponent: for iron $x = 1.6$
- $f$: operation frequency [Hz]

Eddy current losses: Power loss $P_E$ per volume unit [W/m³] if $\delta >> d_{lam}$

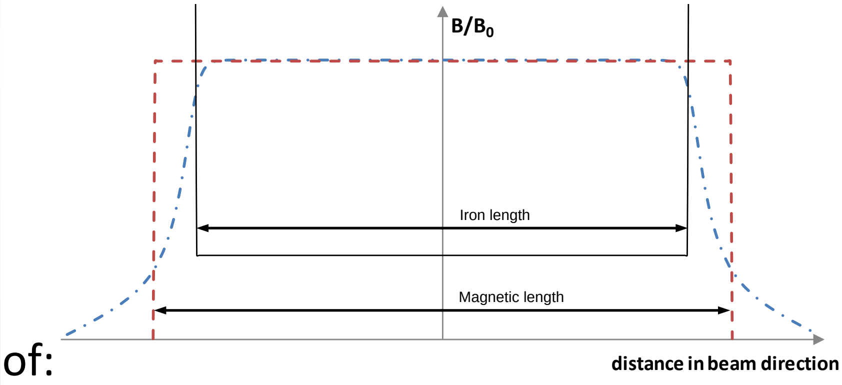
$$\frac{P_E}{V} = \frac{\pi^2 d_{lam}^2 f^2 B^2}{6\rho}$$

- $d_{lam}$: lamination thickness [m]

# Magnetic length

Coming from ∞, B increases towards the magnet center (stray flux)

Magnetic length: $$l_{mag} = \frac{\int_{-\infty}^{\infty} B(z) \cdot dz}{B_0}$$

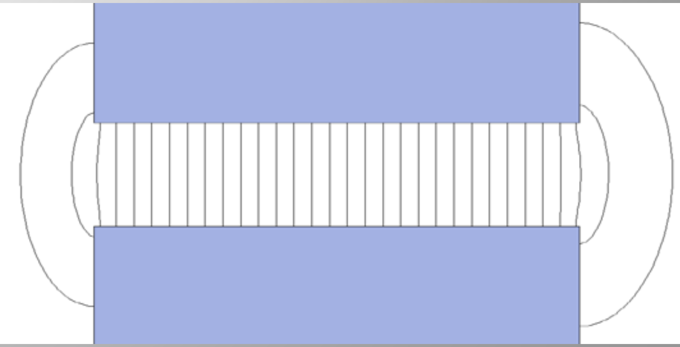



'Magnetic' length > iron length

Approximation for a dipole: $$l_{mag} = l_{iron} + 2hk$$

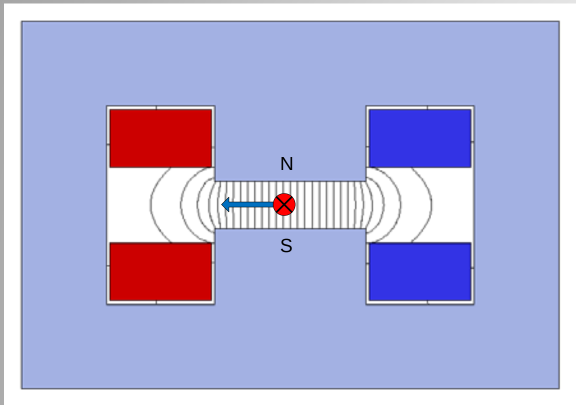
Geometry specific constant $k$ gets smaller in case of:

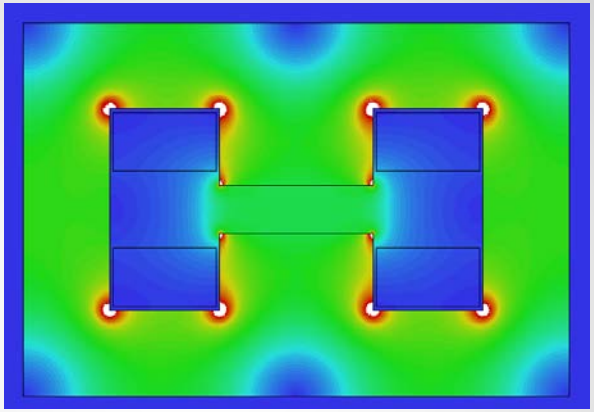
- pole length < gap height
- saturation
- precise determination only by measurements or numerical calculations

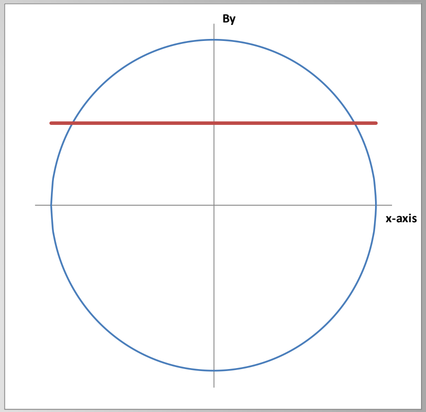


# Dipoles

- Purpose: bend or steer the particle beam



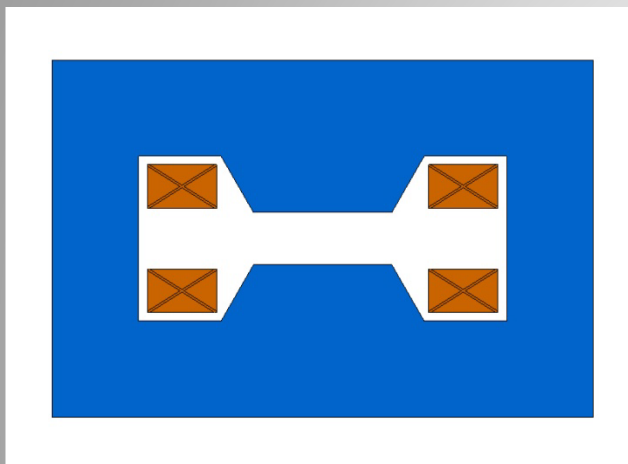


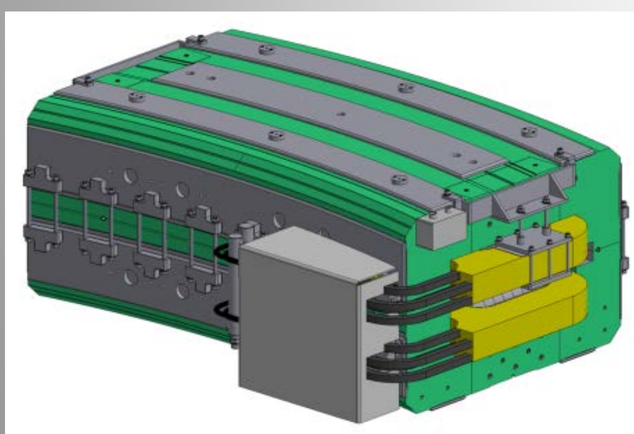

- Equation for normal (non-skew) ideal (infinite) poles: $y = \pm r$
  (r = half gap height)
- Magnetic flux density: $B_x = 0$; $B_y = b_1 = \text{const.}$
- Applications: synchrotrons, transfer lines, spectrometry, beam scanning

JUAS
Joint Universities Accelerator School

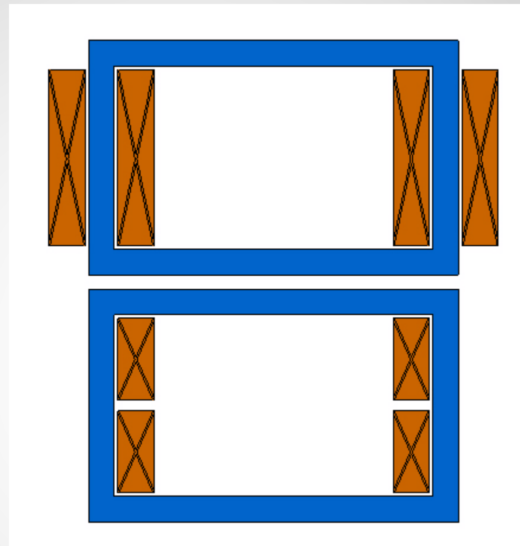
# Dipole types

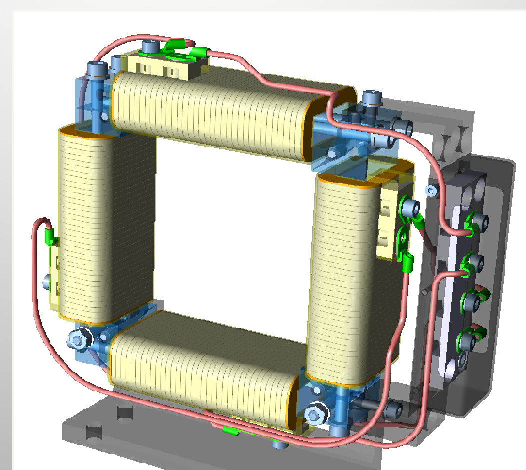
## H-Shape



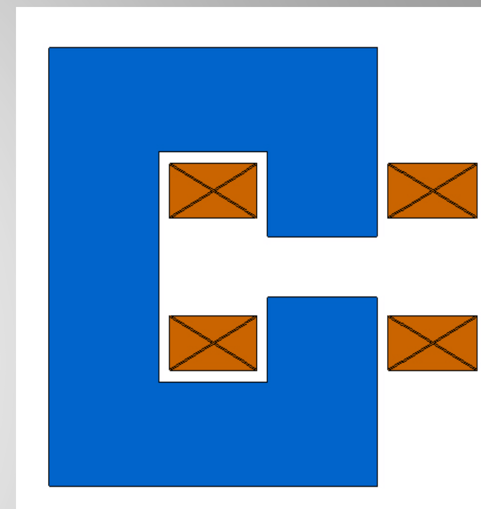


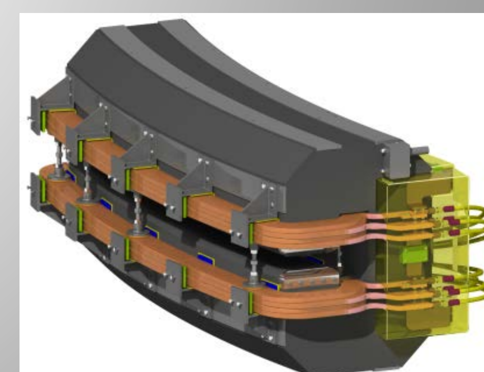
## O-Shape





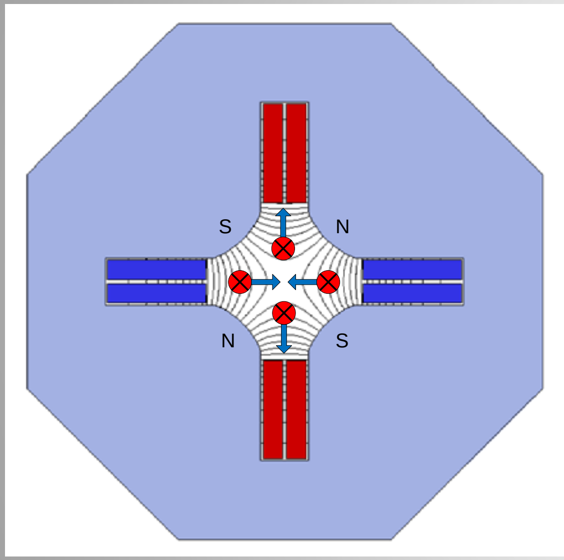
## C-Shape

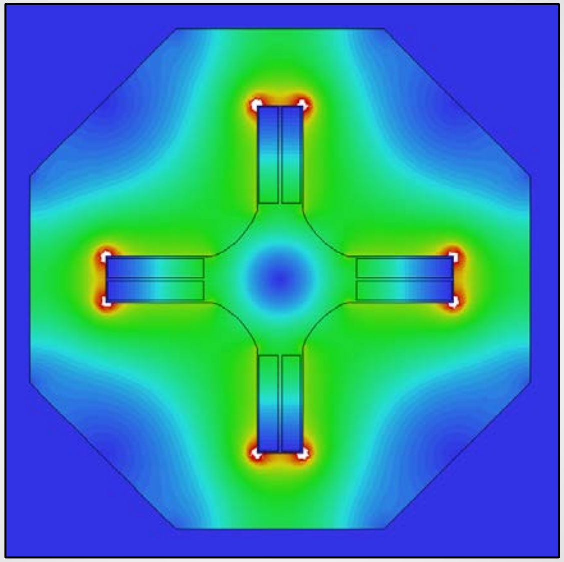


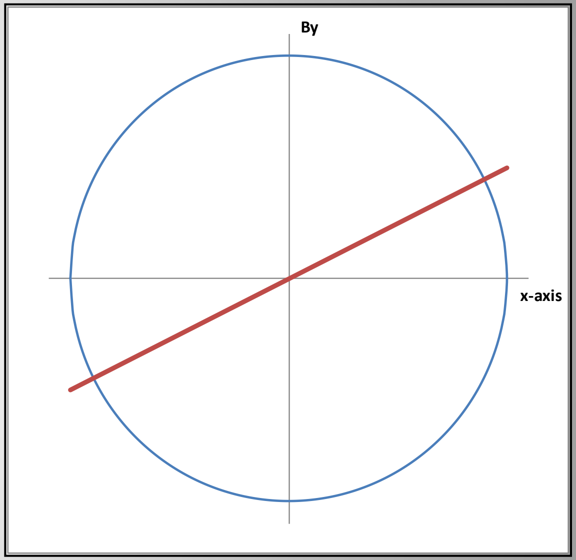


# Quadrupoles

- Purpose: focusing the beam (horizontally focused beam is vertically defocused)



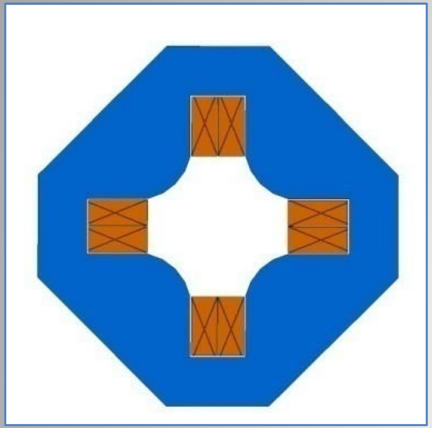





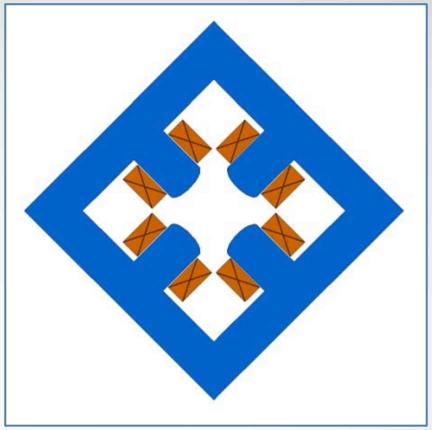

- Equation for normal (non-skew) ideal (infinite) poles: $2xy = \pm r^2$ (r = aperture radius)
- Magnetic flux density: $B_x = b_2 y$; $B_y = b_2 x$

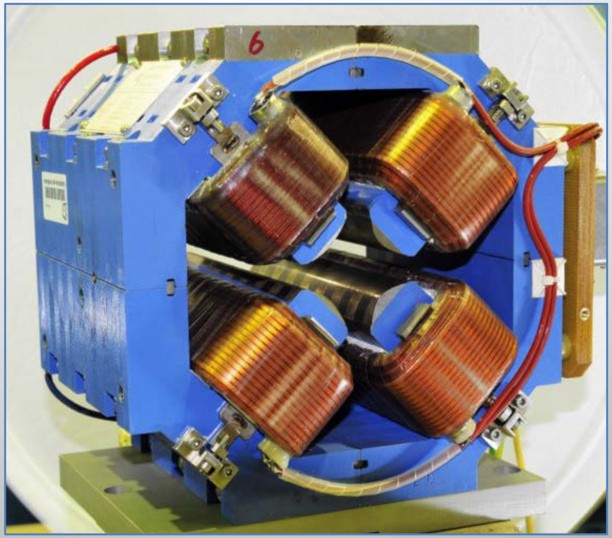
# Quadrupole types

Standard quadrupole

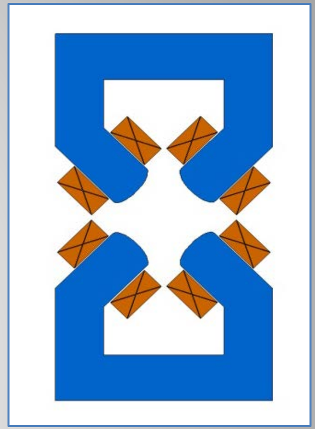


Standard quadrupole



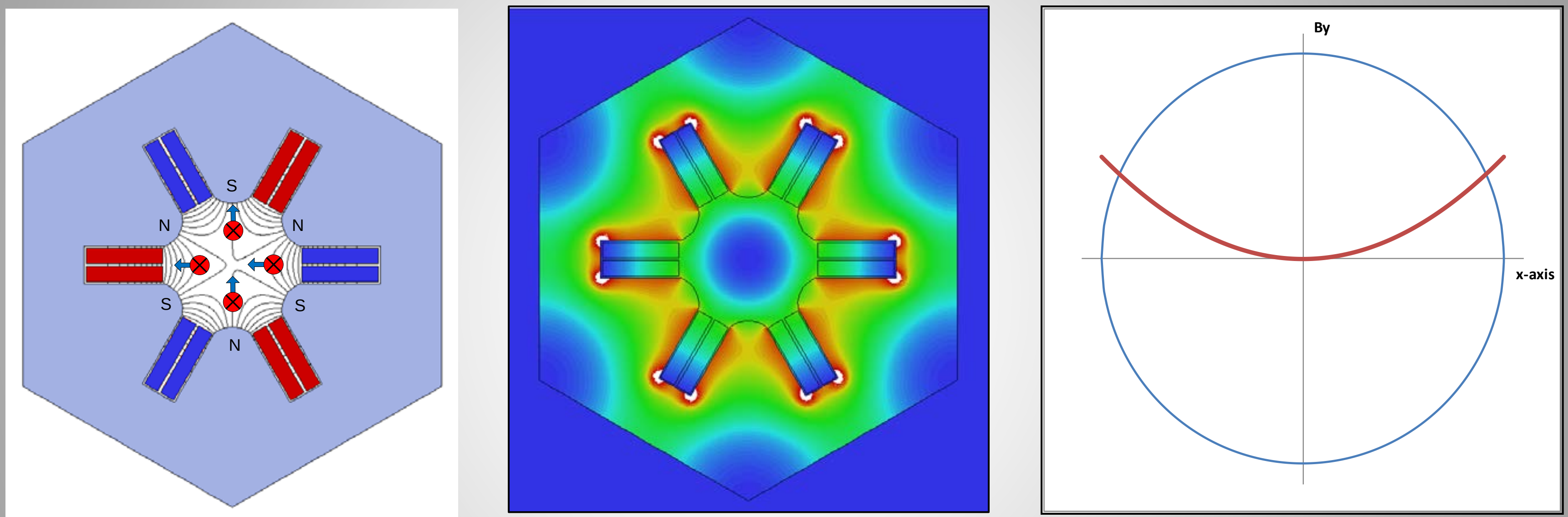


Collins or Figure-of-Eight



# Sextupoles

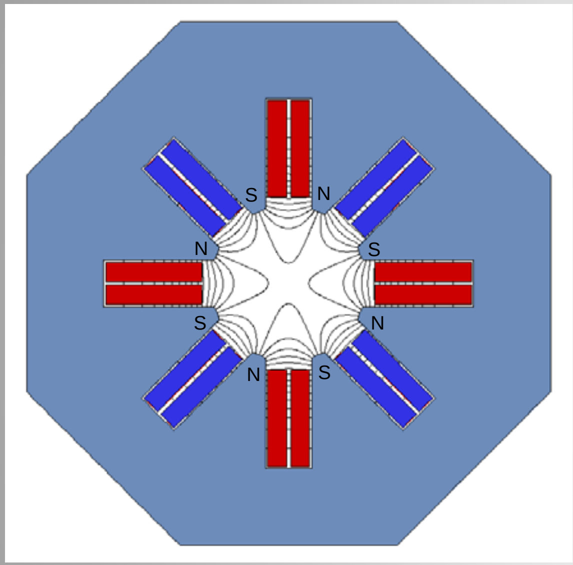
- Purpose: correct chromatic aberrations of 'off-momentum' particles

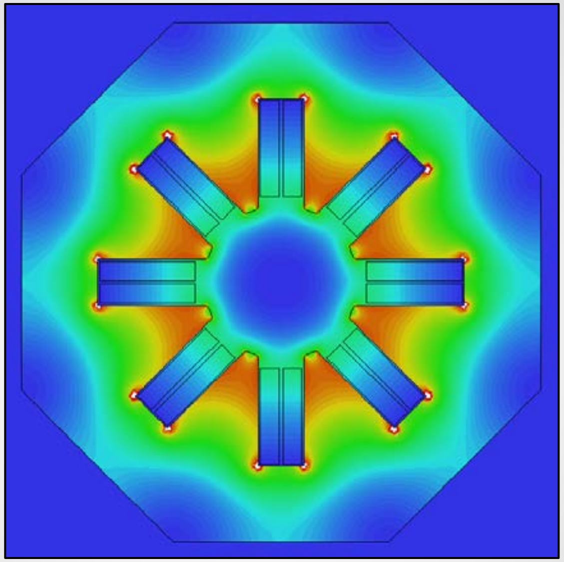


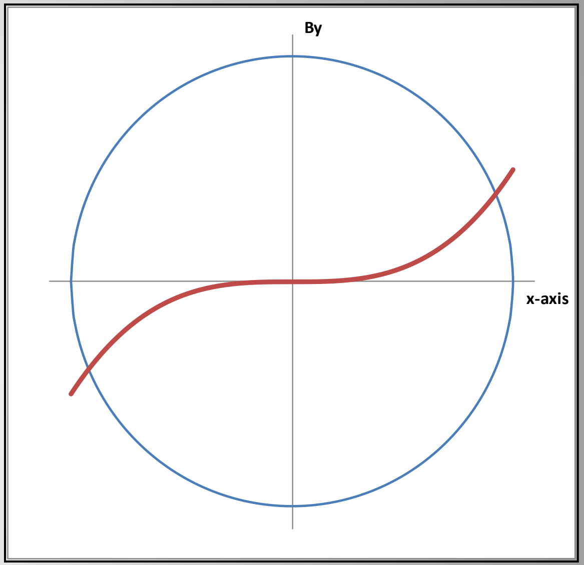

- Equation for normal (non-skew) ideal (infinite) poles: $3x^2y - y^3 = \pm r^3$ (r = aperture radius)
- Magnetic flux density: $B_x = b_3xy$; $B_y = b_3(x^2 - y^2)/3$

# Octupoles

- Purpose: 'Landau' damping

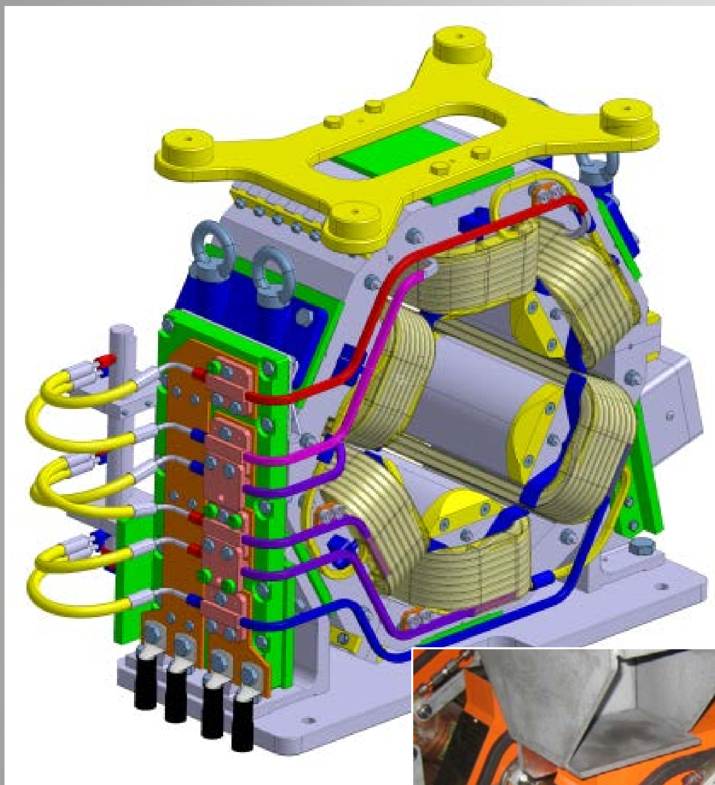


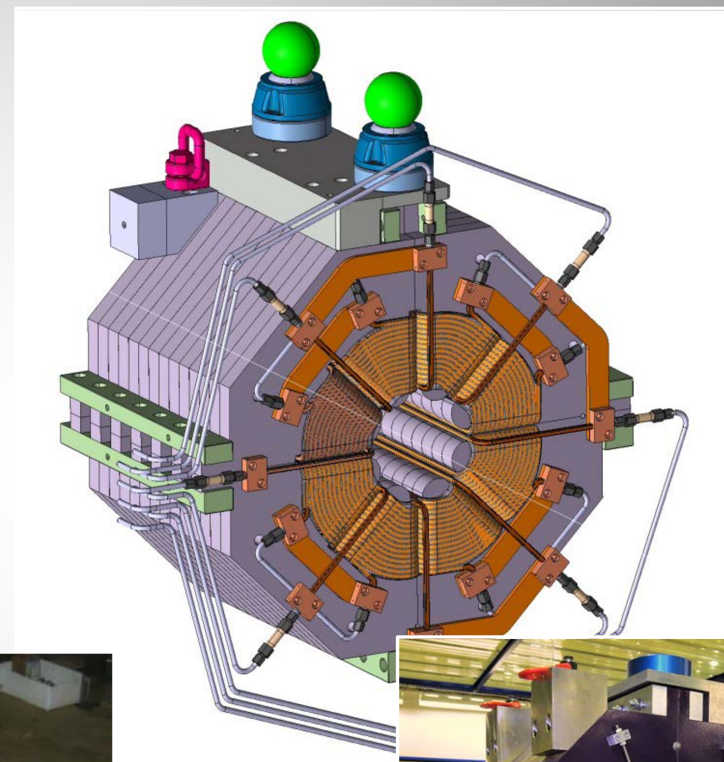



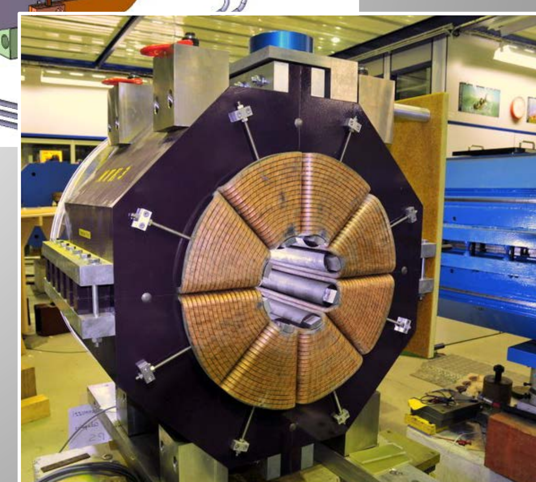



- Equation for normal (non-skew) ideal poles: $4(x^3y - xy^3) = \pm r^4$ (r = aperture radius)
- Magnetic flux density: $B_x = b_4(3x^2y - y^3)/6$; $B_y = b_4(x^3 - 3xy^2)/6$

# Sextupoles & Octupoles

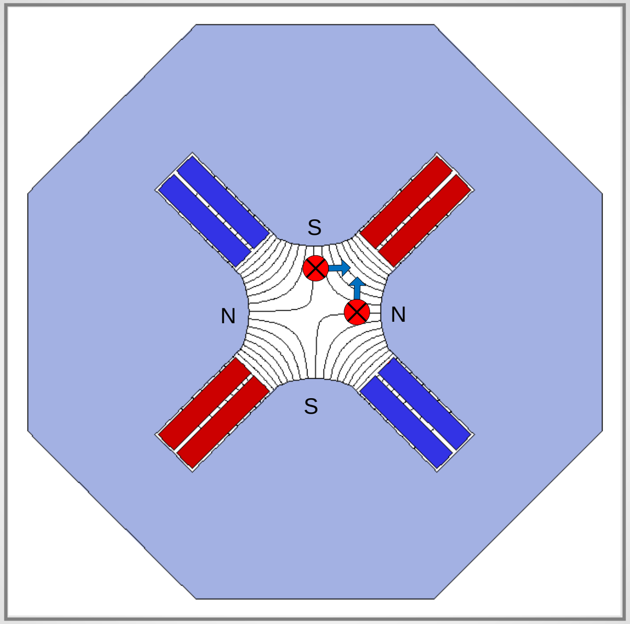






# Skew quadrupole

- Purpose: coupling horizontal and vertical betatron oscillations



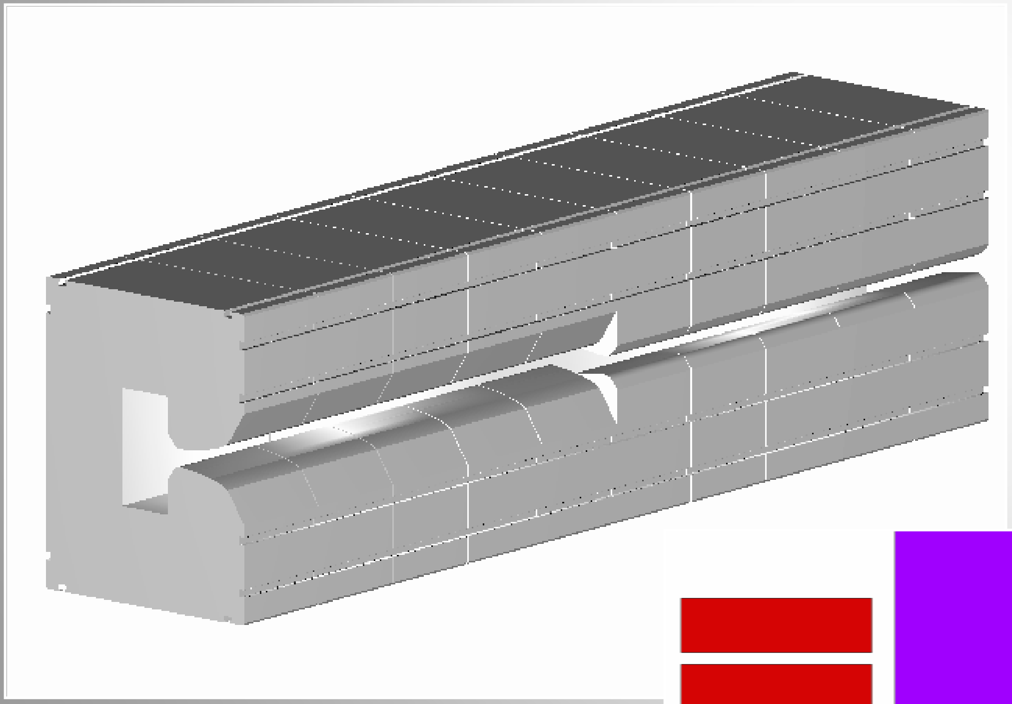

Rotation by π/2n

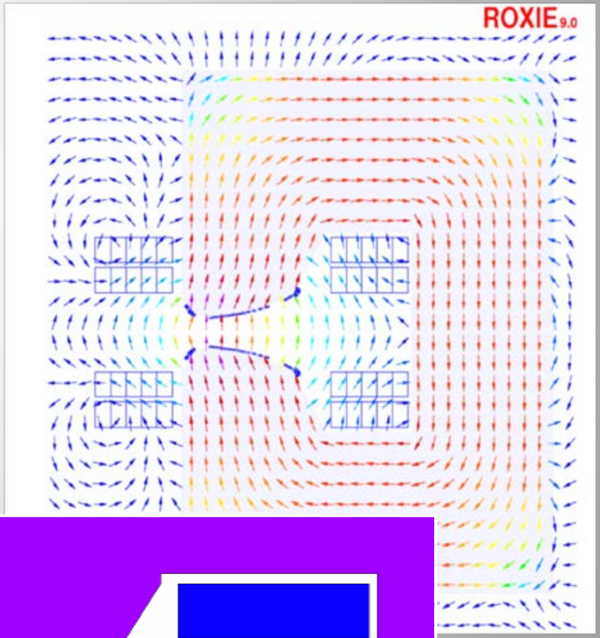
- Beam that has horizontal displacement (but no vertical) is deflected vertically
- Beam that has vertical displacement (but no horizontal) is deflected horizontally

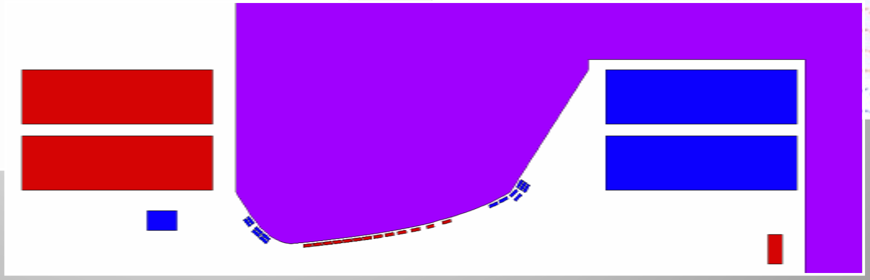
# Combined function magnets

Functions generated by pole shape (sum a scalar potentials):

- Amplitudes cannot be varied independently
- Dipole and quadrupole: PS main magnet (PFW, Fo8…)





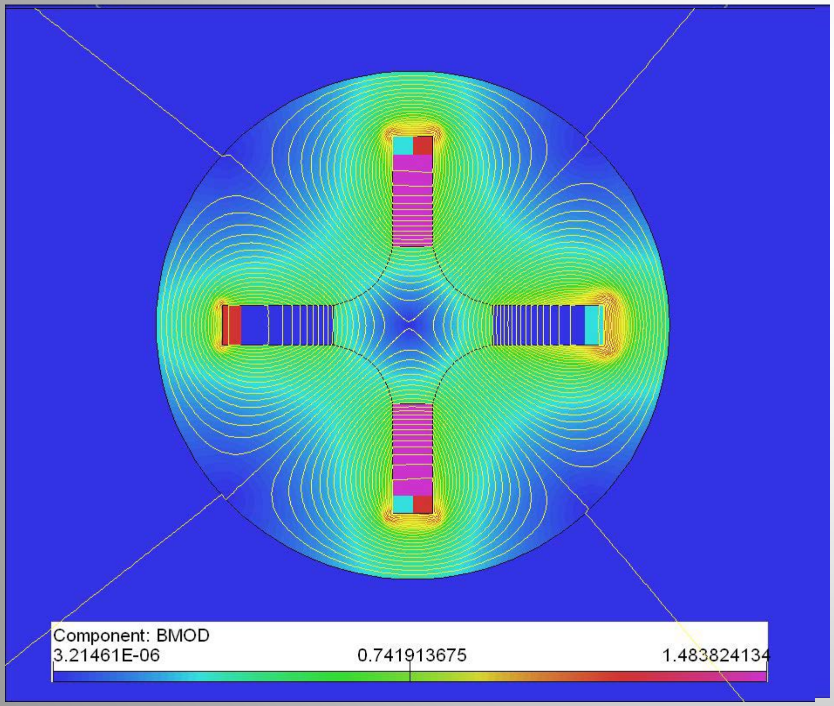



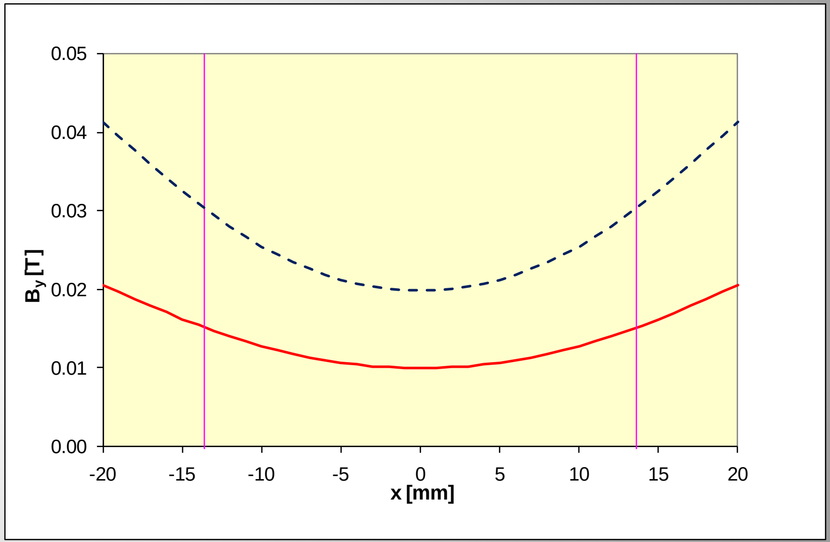
# Combined function magnets



## Functions generated by individual coils:

Amplitudes can be varied independently

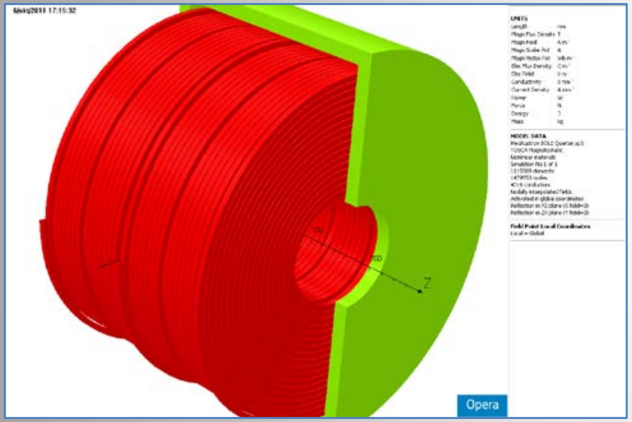


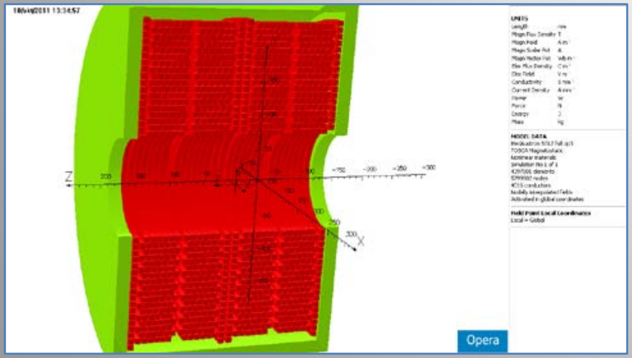



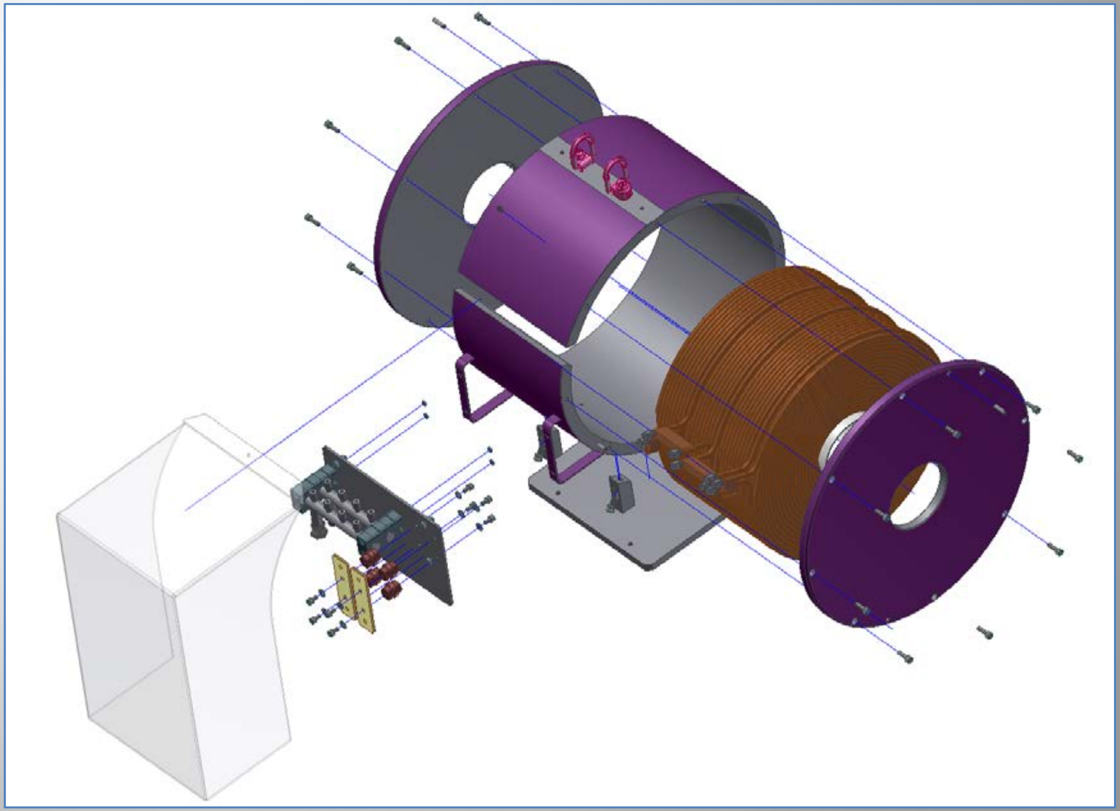

Quadrupole and corrector dipole (strong sextupole component in dipole field)

# Solenoids

- Weak focusing, non-linear elements
- Main field component in z-direction, focusing by end fields
- Usually only used in experiments or low-energy beam lines



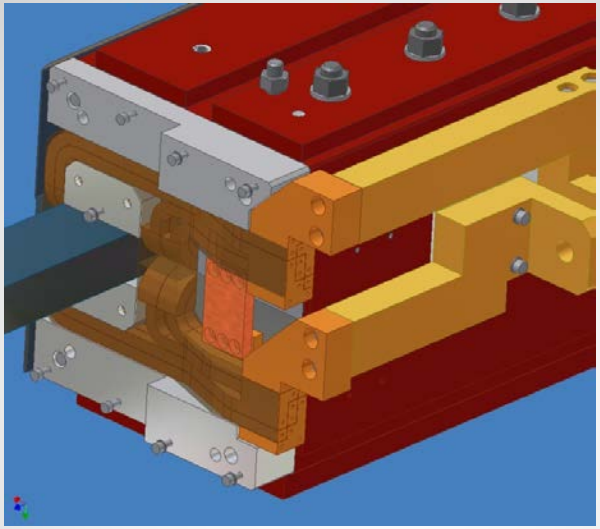



# Special magnets

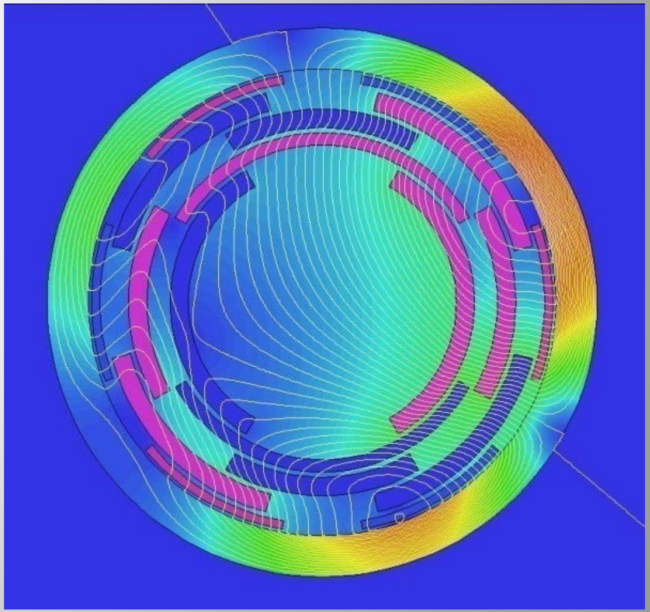


For beam injection and extraction

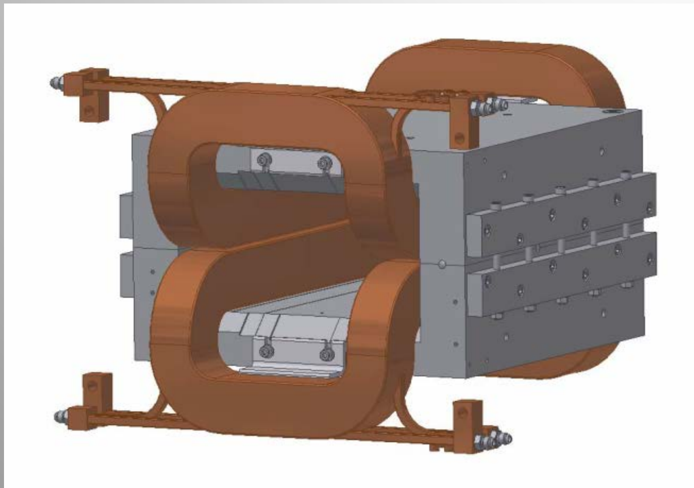
- Septa
- Kicker magnets
- Bumper magnets



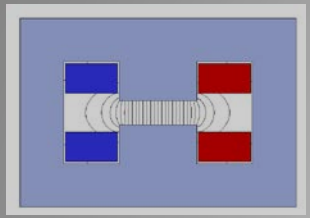
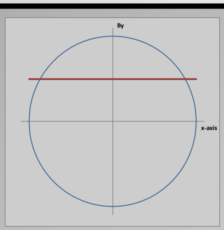
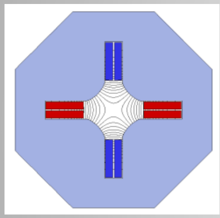
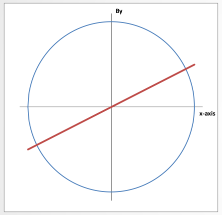
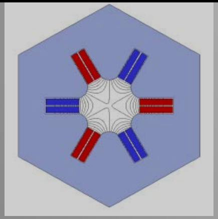
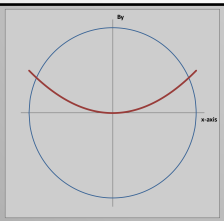
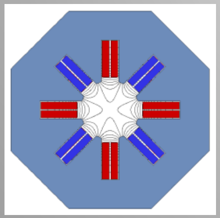
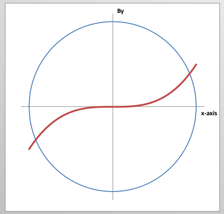
Coil-dominated magnets



Scanning magnets



# Overview

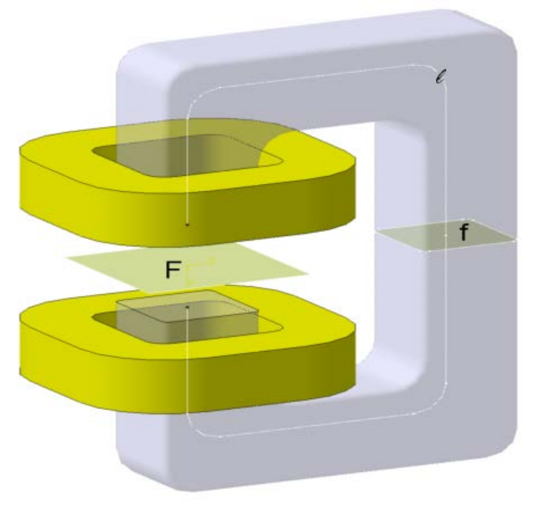
| Pole shape | Field distribution | Pole equation | $B_x$, $B_y$ |
|---|---|---|---|
|  |  | $y = \pm r$ | $B_x = 0$<br>$B_y = b_1 = B_0 = \text{const.}$ |
|  |  | $2xy = \pm r^2$ | $B_x = b_2 y$<br>$B_y = b_2 x$ |
|  |  | $3x^2y - y^3 = \pm r^3$ | $B_x = b_3 xy$<br>$B_y = b_3(x^2 - y^2)/2$ |
|  |  | $4(x^3y - xy^3) = \pm r^4$ | $B_x = b_4(3x^2y - y^3)/6$<br>$B_y = b_4(x^3 - 3xy^2)/6$ |

# Summary

- Magnets are needed to guide and shape particle beams
- Coils carry the electrical current, the iron yoke carries the magnets flux
- Steel properties have a significant influence on the magnet performance
- In case of time-varying fields, eddy currents can appear
- Different magnet types providing different functions

# Lecture 2: Analytical design

- What do we need to know before starting?
- Deriving the main parameters
- Coil design and cooling
- Cost estimate & optimization
- Magnet manufacturing & testing

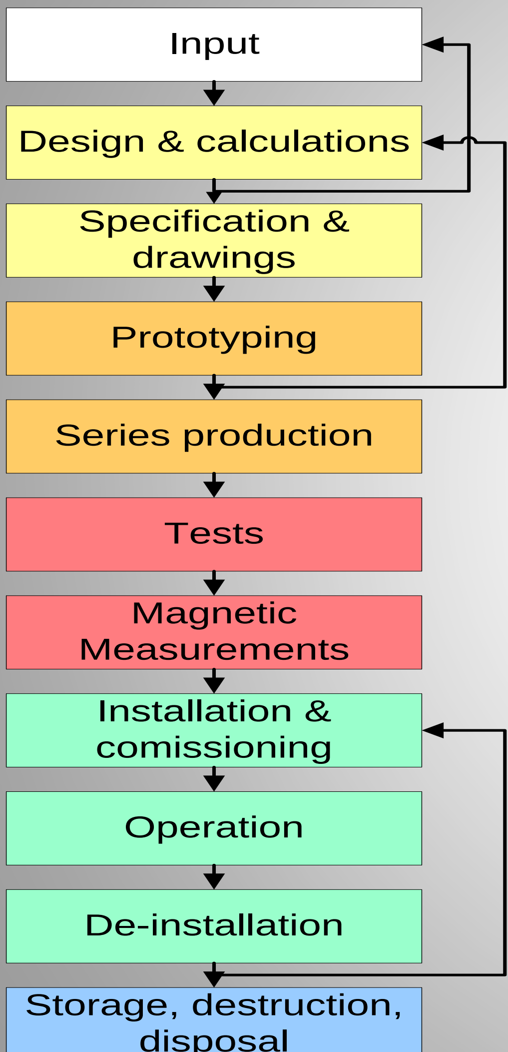


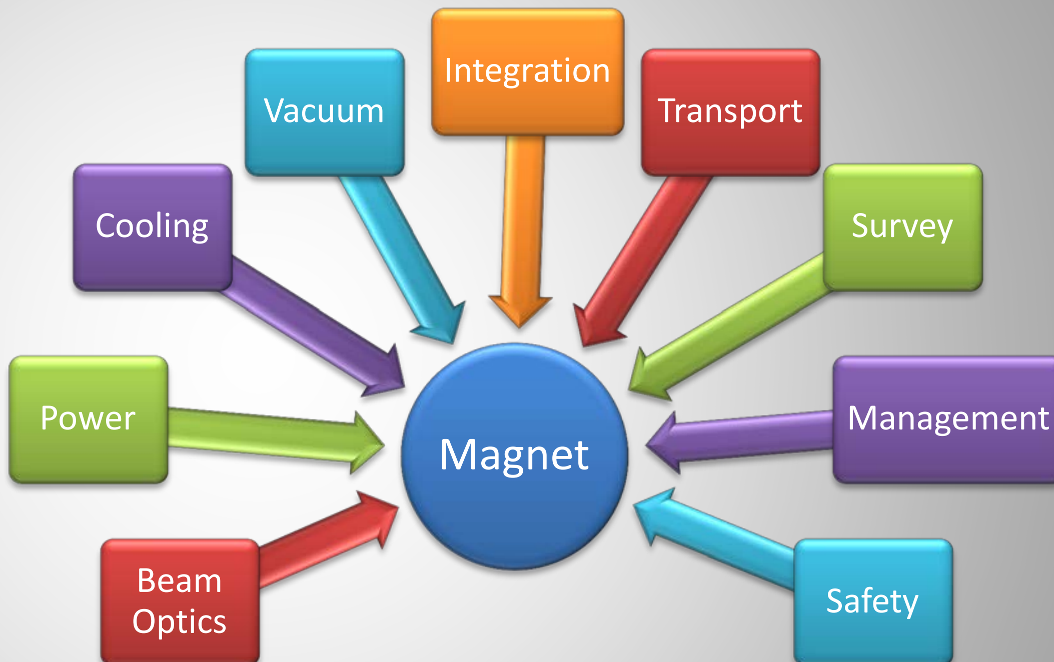
# Goals in magnet design

The goal is to produce a product just good enough to perform reliably with a sufficient safety factor at the lowest cost and on time.

- Good enough:
  - Obvious parameters clearly specified, but tolerance difficult to define
  - Tight tolerances lead to increased costs
- Reliability:
  - Get MTBF and MTTR reasonably low
  - Reliability is usually unknown for new design
  - Requires experience  to search for a compromise between extreme caution and extreme risk (expert review)
- Safety factor:
  - Allows operating a device under more demanding condition as initially foreseen
  - To be negotiated between the project engineer and the management
  - Avoid inserting safety factors a multiple levels (costs!)

# Magnet life cycle



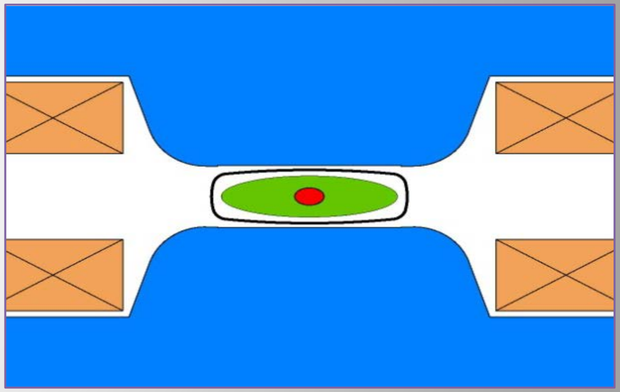

A magnet is not a stand-alone device!

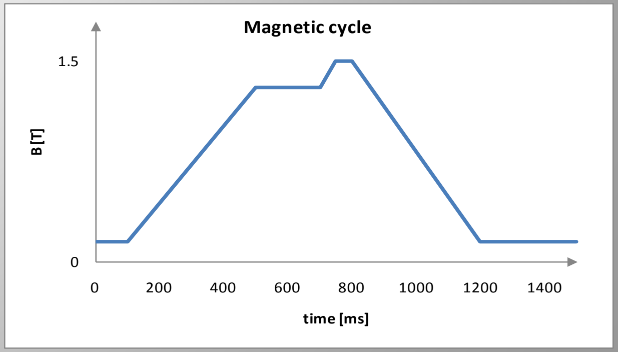
# Design process

Electro-magnetic design is an iterative process:

Collect input data → Analytical design → Numerical 2D/3D simulations → Mechanical design → Drawings & specifications

- Field strength (gradient) and magnetic length
- Integrated field strength (gradient)
- Aperture and ‚good field region‘
- Field quality:
  - field homogeneity
  - maximum allowed multipole errors
  - settling time (time constant)
- Operation mode: continous, cycled
- Electrical parameters
- Mechanical dimensions
- Cooling
- Environmental aspects





# General requirements

| | |
|---|---|
| Magnet type and purpose | • Dipole: bending, steering, extraction<br>• Quadrupole, sextupole, octupole<br>• Combined function, solenoid, special magnet |
| Installation | • Storage ring, synchrotron light source, collider<br>• Accelerator<br>• Beam transport lines |
| Quantity | • Installed units<br>• Spare units (~10 %) |

# Performance requirements

**Beam parameters**

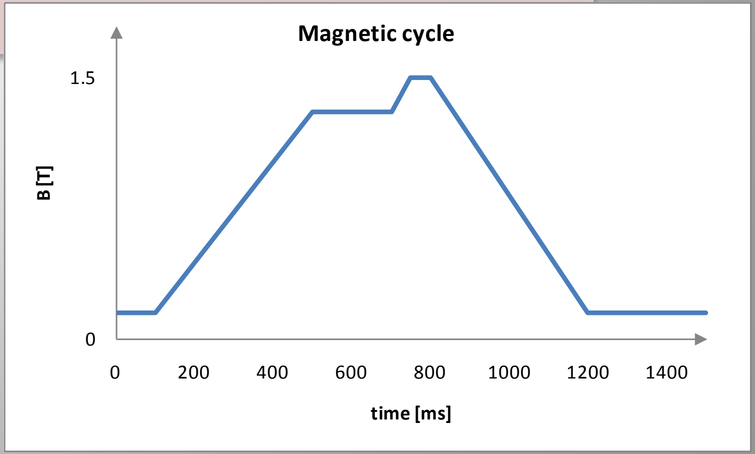
- Type of beam, energy range, deflection angle
- Integrated field (gradient)
- Local field (gradient) and magnetic length

**Aperture**

- Physical aperture
- 'Good field region'

**Operation mode**

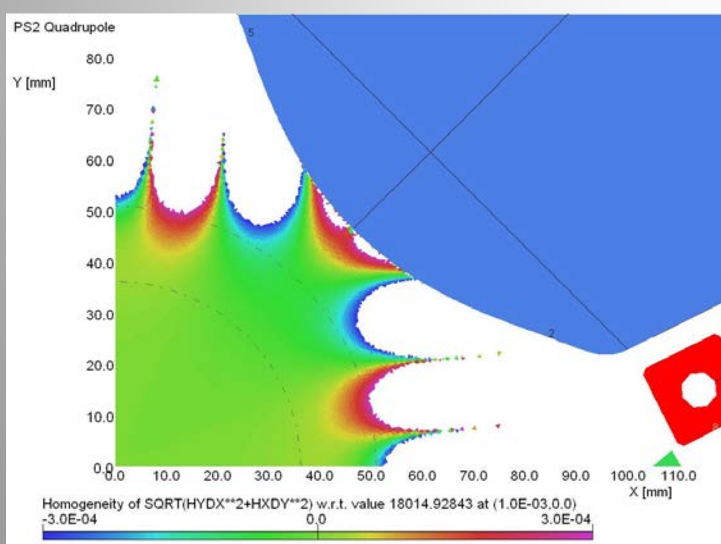
- Continuous
- Pulse-to-pulse modulation (ppm)
- Ramp rate (T/s)

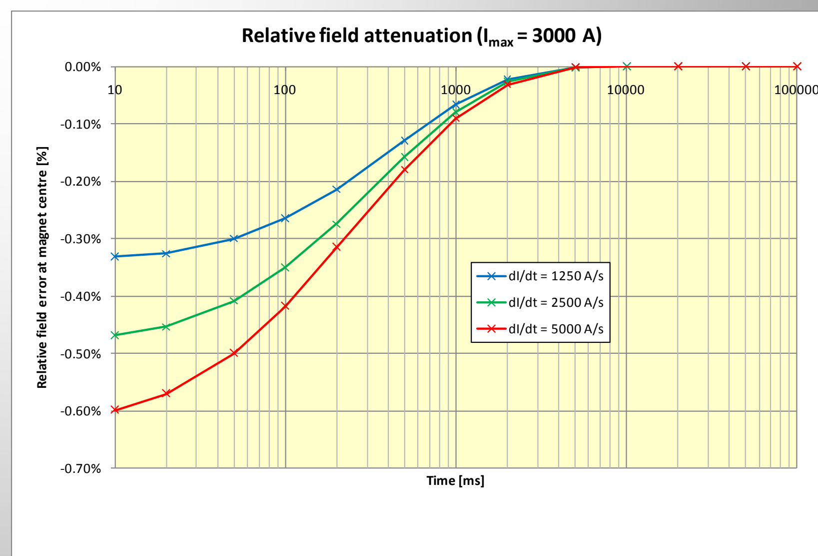


# Performance requirements

## Field quality

- Homogeneity (uniformity)
- Maximum allowed multipole errors
- Stability & reproducibility
- Settling time (time constant)
- Allowed residual field)



# Physical requirements

**Geometric boundaries**

- Available space
- Transport limitations
- Weight limitations

**Accessibility**

- Crane
- Connections (electrical, hydraulic)
- Alignment targets

# Interfaces

Equipment linked to the magnet is defining the boundaries and constraints

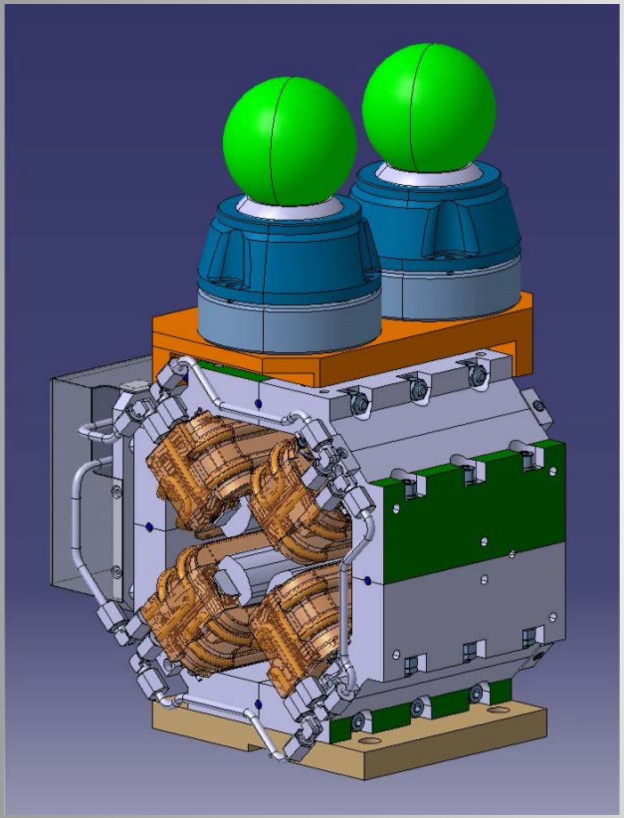
| | |
|---|---|
| Power converter | • Max. current (peak, RMS)<br>• Max. voltage<br>• Pulsed/dc |
| Cooling | • Max. flow rate and pressure drop<br>• Water quality (aluminium/copper circuit)<br>• Inlet temperature<br>• Available cooling power |
| Vacuum | • Size and material of vacuum chamber<br>• Space for pumping ports, bake out<br>• Captive vacuum chamber |

# Environmental aspects

Other aspects, which can have an influence on the magnet design

| | |
|---|---|
| Environment temperature | • Risk of condensation<br>• Heat dissipation into the tunnel |
| Ionizing radiation | • High radiation levels require radiation hard materials<br>• Special design to allow fast repair/replacement |
| Electro-magnetic compatibility | • Magnetic fringe fields disturbing other equipment (beam diagnostics)<br>• Surrounding equipment perturbing field quality |
| Safety | • Electrical safety<br>• Interlocks |

# Magnet Components



Alignment targets

Yoke

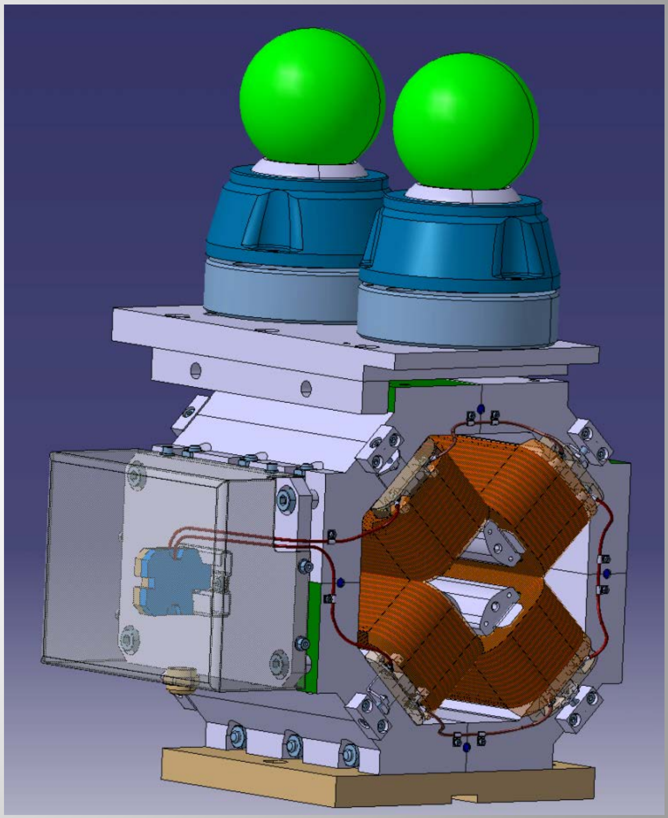
Coils

Sensors

Cooling circuit

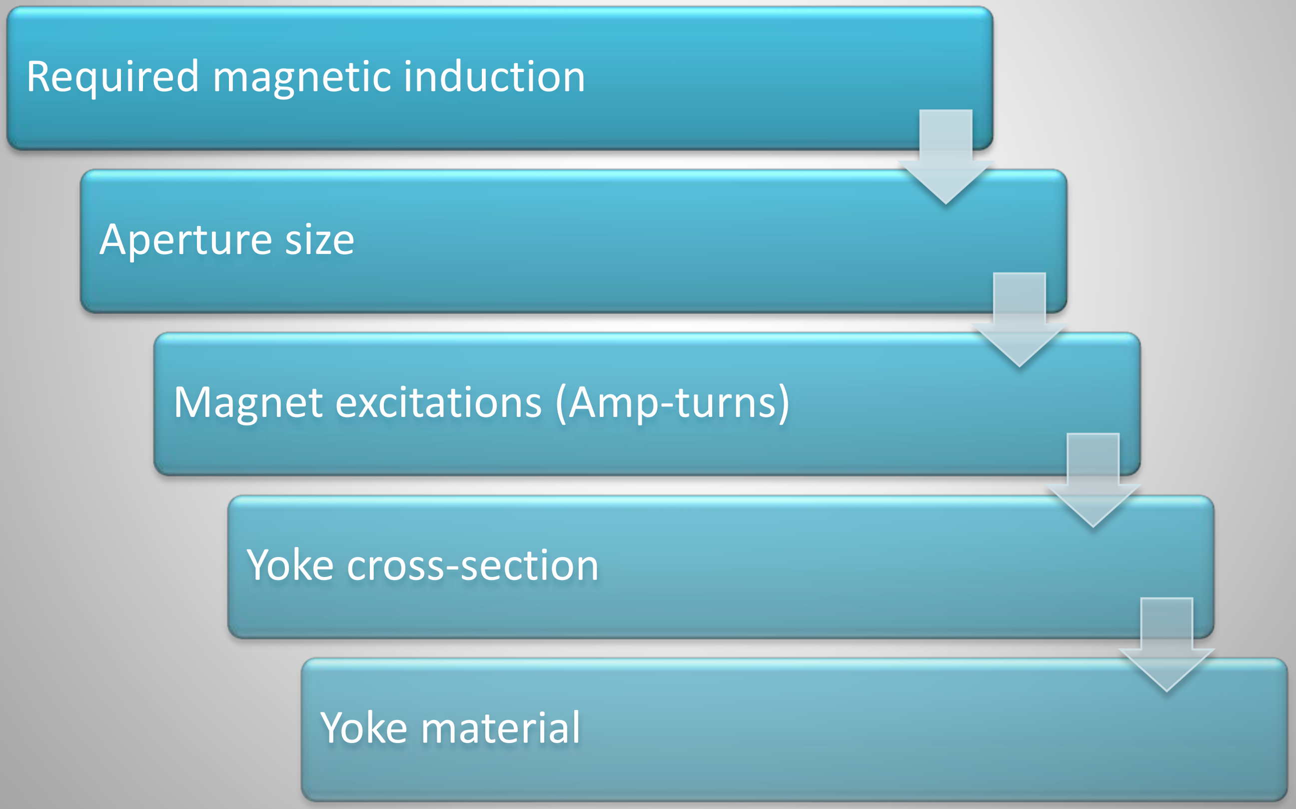
Connections

Support



# Magnetic design

Translate the beam optic requirements into a magnetic design



# Beam rigidity

Beam rigidity $B\rho$ [Tm]:

$$B\rho = \frac{1}{qc}\sqrt{T^2 + 2T\,E_0}$$

- $q$: particle charge number [Coulombs]
- $c$: speed of light [m/s]
- $T$: kinetic beam energy [eV]
- $E_0$: particle rest mass energy [eV]
  ( 0.51 MeV for electrons, 938 MeV for protons)

# Magnetic induction

Dipole bending field *B* [T]:

$$B = \frac{B\rho}{r_M}$$

- *B*: Flux density or magnetic induction (vector) [T]
- $r_M$: magnet bending radius [m]

Quadrupole field gradient *B'* [T/m]:

$$B' = B\rho k$$

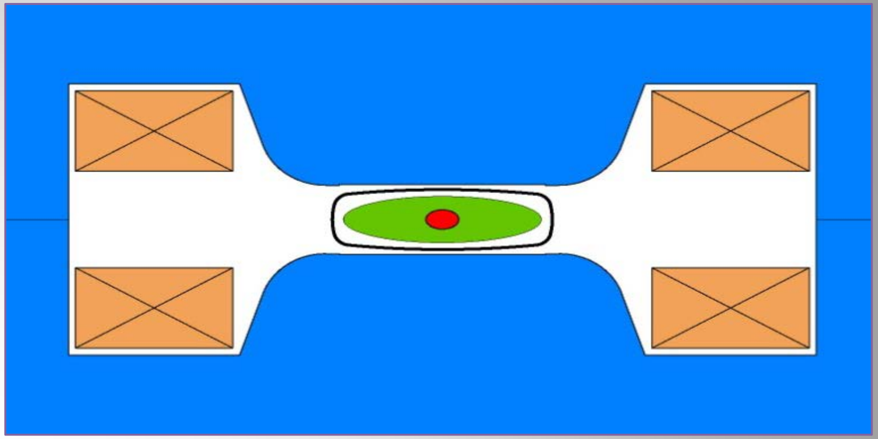
- *k*: quadrupole strength [$m^{-2}$]

Sextupole differential gradient *B''* [$T/m^2$]:

$$B'' = B\rho m$$

- *m*: sextupole strength [$m^{-3}$]

# Aperture size



Aperture =

Good field region

Maximum beam size

- Lattice functions: beta functions and dispersion
- Geometrical transverse emittancies (energy depended)
- Momentum spread
- Envelope (typical 3-sigma)
- Largest beam size usually at injection

$$\sigma = \sqrt{\varepsilon\,\beta + \left(D\frac{\Delta p}{p}\right)^2}$$

+ Closed orbit distortions (few mm)

+ Vacuum chamber thickness (0.5 – 5 mm)

+ Installation and alignment margin (0 – 10 mm)

# Excitation current in a dipole

Ampere's law $\oint \vec{H} \cdot d\vec{l} = NI$ and $\vec{B} = \mu\vec{H}$ with $\mu = \mu_0 \mu_r$

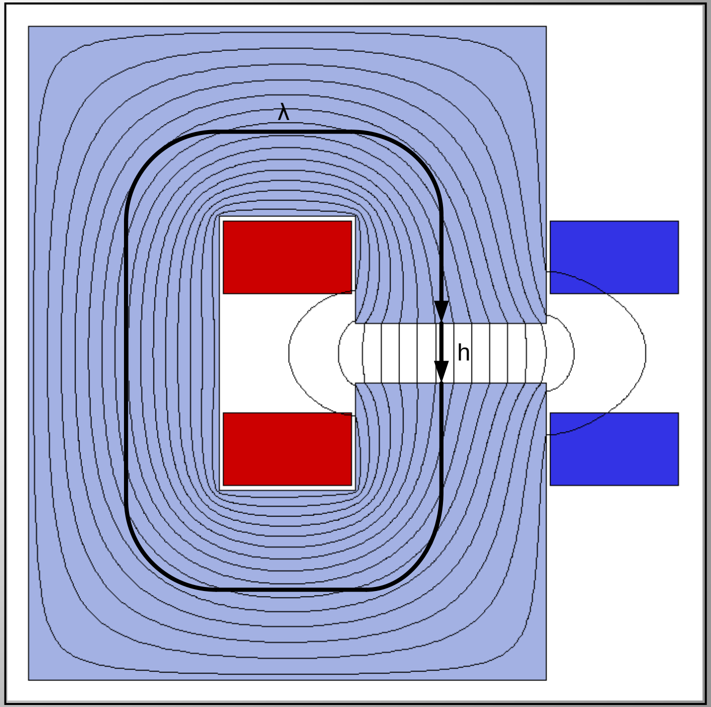
leads to $NI = \oint \frac{\vec{B}}{\mu} \cdot d\vec{l} = \int_{gap} \frac{\vec{B}}{\mu_{air}} \cdot d\vec{l} + \int_{yoke} \frac{\vec{B}}{\mu_{iron}} \cdot d\vec{l} = \frac{Bh}{\mu_{air}} + \frac{B\lambda}{\mu_{iron}}$

assuming, that B is constant along the path

If the iron is not saturated: $\frac{h}{\mu_{air}} >> \frac{\lambda}{\mu_{iron}}$

then: $NI_{(per\ pole)} \approx \frac{Bh}{2\eta\mu_0}$

- *h*: gap height [m]
- *η*: efficiency (typically 95% - 99 %)



# Reluctance and efficiency

Reluctance: $$R_M = \frac{NI}{\Phi} = \frac{l_M}{A_M \mu_r \mu_0}$$

- $\Phi$: magnetic flux [Wb]
- $l_M$: flux path length in iron [m]
- $A_M$: iron cross section perpendicular to flux [$m^2$]

Term ( $\frac{\lambda}{\mu_{iron}}$ ) in previous slide is called 'normalized reluctance' of the yoke

Keep iron yoke reluctance less than a few % of air reluctance ( $\frac{h}{\mu_0}$ ) by providing sufficient iron cross section ($B_{iron}$ < 1.5 T)

Efficiency: $$\eta = \frac{R_{M,gap}}{R_{M,gap} + R_{M,yoke}} \approx 99\%$$

# Excitation current in a Quadrupole



Choosing the shown integration path gives:

$$NI = \oint \vec{H} \cdot \vec{dl} = \int_{s1} \vec{H}_1 \cdot \vec{dl} + \int_{s2} \vec{H}_2 \cdot \vec{dl} + \int_{s3} \vec{H}_3 \cdot \vec{dl}$$

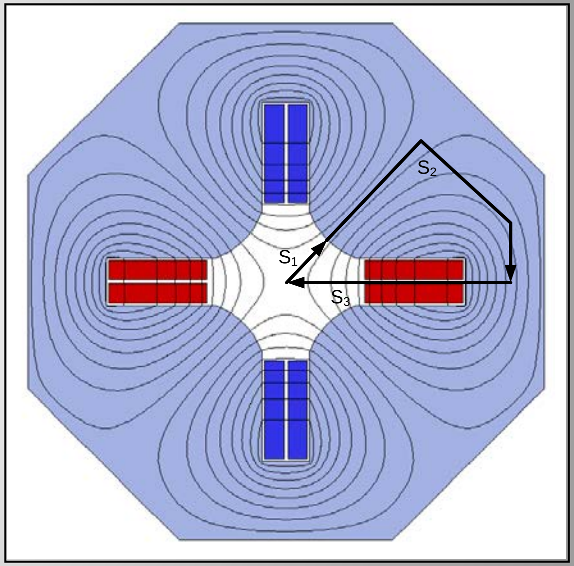
For a quadrupole, the gradient $B' = \frac{dB}{dr}$ is constant

and $B_y = B'x \qquad B_x = B'y$

Field modulus along $s_1$: $H(r) = \frac{B'}{\mu_0}\sqrt{x^2 + y^2} = \frac{B'}{\mu_0} r$

Neglecting H in $s_2$ because: $R_{M,s2} = \frac{s_2}{\mu_{iron}} << \frac{s_1}{\mu_{air}}$

and along $s_3$: $\int_{s3} \vec{H}_3 \cdot \vec{dl} = 0$




Leads to: $NI \approx \int_0^R H(r)\, dr = \frac{B'}{\mu_0}\int_0^R r\, dr$

$$NI_{(per\ pole)} = \frac{B' r^2}{2\eta\mu_0}$$

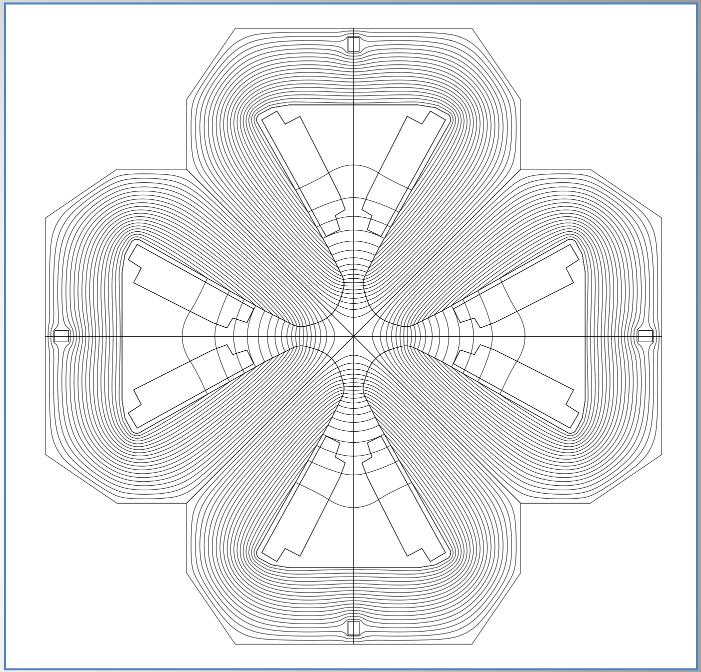
# Magnetic length



Magnetic length for a quadrupole:

$$l_{mag} = l_{iron} + 2r\,k$$

- *k: geometry specific constant (≈ 0.45)*



NI increases with the square of the quadrupole aperture:

$$NI \propto r^2 \qquad P \propto r^4$$

More difficult to accommodate the necessary Ampere-turns (= coil cross section)

→ truncating the hyperbola leads to a decrease in field quality

# Excitation current in a Sextupole

Same approach as for quadrupole:

For a sextupole, the field is parabolic and $B'' = \frac{d^2B}{dr^2}$ is constant

so $H(r) = \frac{B''}{2\mu_0} r^2$

leads to: $NI = \oint \vec{H} \cdot \vec{dl} \approx \int_0^R H(r)\,dr = \frac{B''}{2\mu_0} \int_0^R r^2\,dr$

$$NI_{(per\ pole)} = \frac{B'' r^3}{6\eta\mu_0}$$

NI increases with the 3rd power of the aperture:

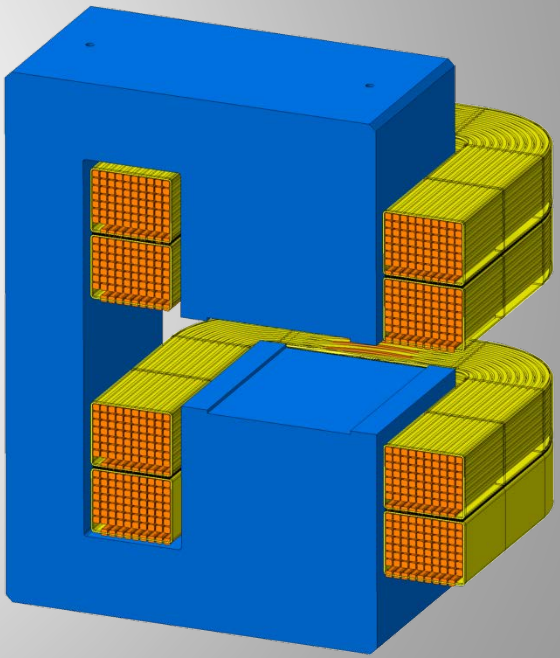
$$NI \propto r^3 \qquad P \propto r^6$$

Fortunately, sextupole fields are usually much smaller than quadrupole fields

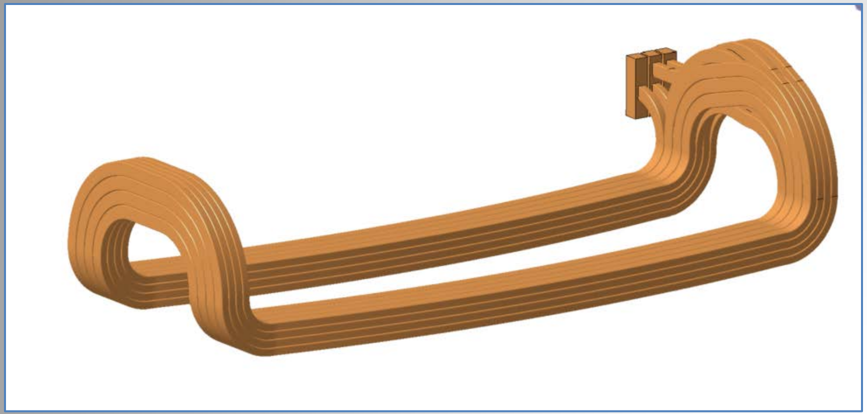
# Coil design

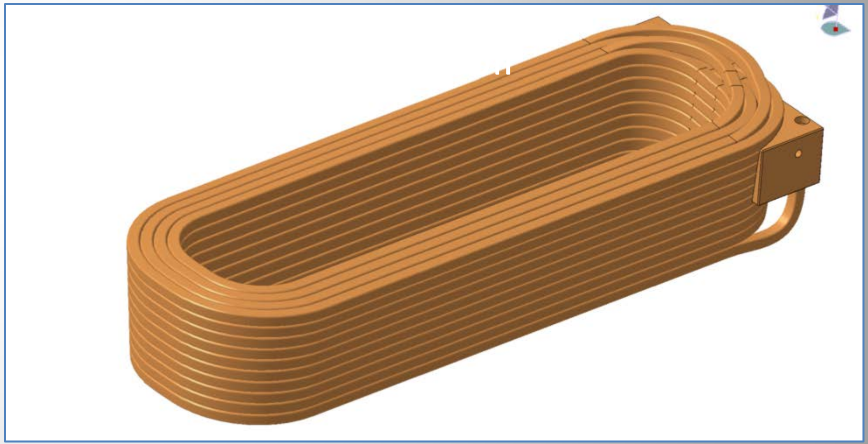
Ampere-turns NI are determined, but the current density j, the number of turns N and the coil cross section need to be decided

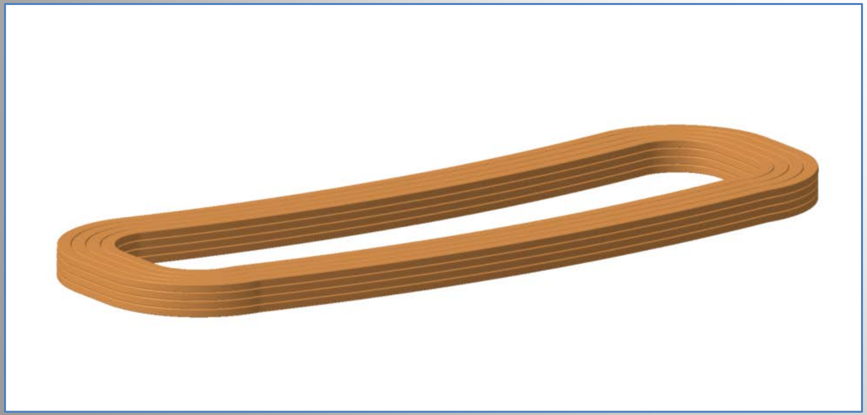
- Coil type selection
- Power requirements
- Cooling circuit computation
- Conductor selection
- Optimization



# Standard coil types







# Power requirements

Assuming the magnet cross-section and the yoke length are known, one can calculate the total dissipated power per magnet:

$$P_{dipole} = \rho \frac{Bh}{\eta \mu_0} j l_{avg}$$

$$P_{qudrupole} = 2\rho \frac{B' r^2}{\eta \mu_0} j l_{avg}$$

$$P_{sextupole} = \rho \frac{B'' r^3}{\eta \mu_0} j l_{avg}$$

- *j:* current density [A/m²]: $j = \frac{NI}{f_c A} = \frac{I}{a_{cond}}$
- *ρ*: resistivity [Ωm]
- $l_{avg}$: average turn length [m]; approximation: 2.5 $l_{iron}$ < $l_{avg}$ < 3 $l_{iron}$ for racetrack coils
- $a_{cond}$: conductor cross section [m²]
- *A*: coil cross section [m²]
- $f_c$: filling factor = $\frac{\text{net conductor area}}{\text{coil cross section}}$ (includes geometric filling factor, insulation, cooling duct, edge rounding)

Note: for a constant geometry, the power loss *P* is proportional to the current density *j*

# Number of turns

The determined power can be divided into voltage and current: $P = UI$

Basic relations: $R_{magnet} \propto N^2 j$ $V_{magnet} \propto Nj$ $P_{magnet} \propto j$

The number of turns N are chosen to match the impedances of the power converter and connections:

**Large N = low current = high voltage**

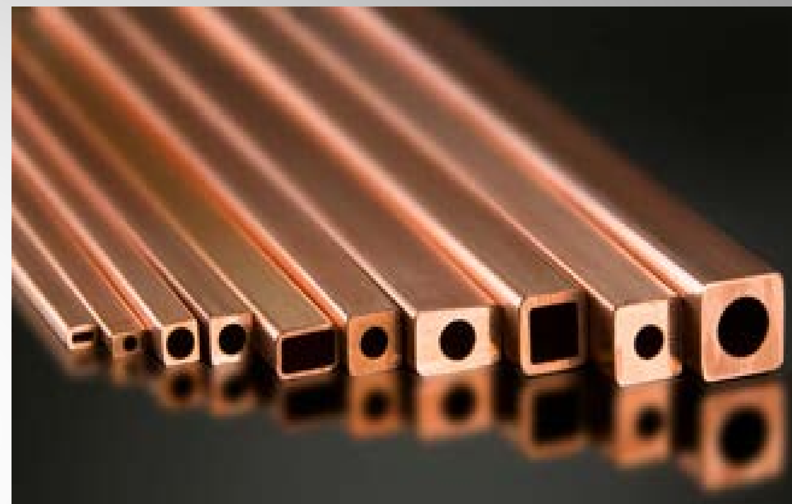
- Small terminals
- Small conductor cross-section
- Thick insulation for coils and cables
- Less good filling factor in the coils
- Large coil volume
- Low power transmission loss

**Small N = high current = low voltage**

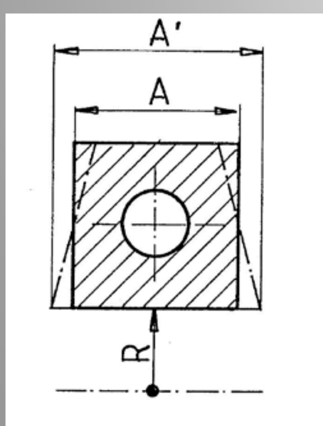
- Large terminals
- Large conductor cross-section
- Thin insulation in coils and cables
- Good filling factor in the coils
- Small coil volume
- High power transmission loss

# Conductor materials

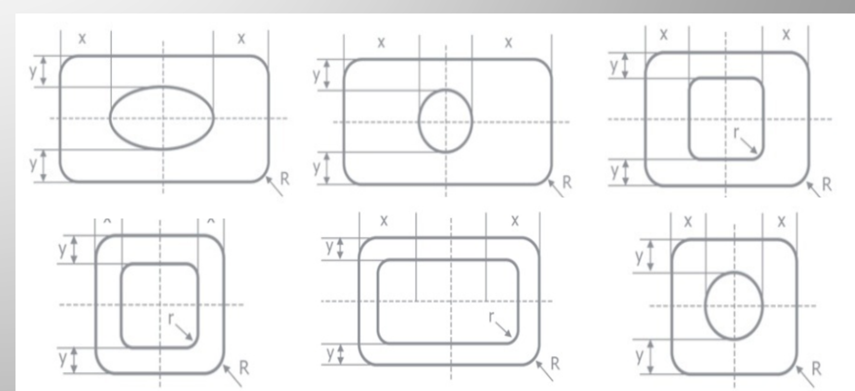
| | Al | Cu (OF) |
|---|---|---|
| Purity | 99.7 % | 99.95 % |
| Resistivity @ 20°C | 2.83 µΩ cm | 1.72 µΩ cm |
| Thermal resistivity coeff. | 0.004 $K^{-1}$ | 0.004 $K^{-1}$ |
| Specific weight | 2.70 $g/cm^3$ | 8.94 $g/cm^3$ |
| Thermal conductivity | 2.37 W/cm K | 3.91 W/cm K |



Key-stoning: risk of insulation damage & decrease of cooling duct cross-section




$$R = 3 \cdot A \Rightarrow \frac{\Delta A}{A} = 3.6\%$$



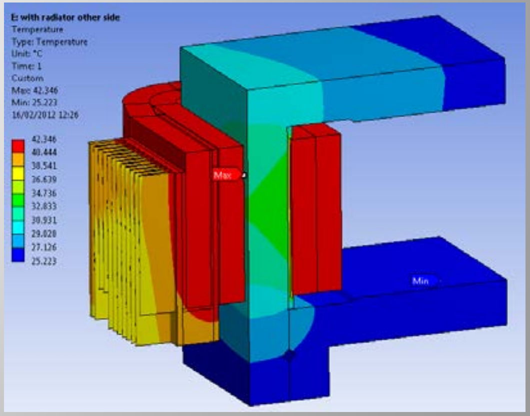
# Coil cooling

## Air cooling by natural convection:

- Current density
  - $j \leq 2$ A/mm$^2$ for small, thin coils
- Cooling enhancement:
  - Heat sink with enlarged radiation surface
  - Forced air flow (cooling fan)
- Only for magnets with limited strength (e.g. correctors)

## Direct water cooling:

- Typical current density $j \leq 10$ A/mm$^2$
- Requires demineralized water (low conductivity) and hollow conductor profiles




## Indirect water cooling:

- Current density $j \leq 3$ A/mm$^2$
- Tap water can be used

# Direct water cooling

Practical recommendations and canonical values:

- Water cooling: 2 A/mm$^2$ ≤ j ≤ 10 A/mm$^2$
- Pressure drop: 0.1 ≤ Δp ≤ 1.0 MPa (possible up to 2.0 MPa)
- Low pressure drop might lead to more complex and expensive coil design
- Flow velocity should be high enough so flow is turbulent
- Flow velocity $u_{avg}$ ≤ 5 m/s to avoid erosion and vibrations
- Acceptable temperature rise: ΔT ≤ 30°C
- For advanced stability: ΔT ≤ 15°C

Assuming:

- Long, straight and smooth pipes without perturbations
- Turbulent flow = high Reynolds number (*Re* > 4000)
- Good heat transfer from conductor to cooling medium
- Temperature of inner conductor surface equal to coolant temperature
- Isothermal conductor cross section

# Direct water cooling

Useful simplified formulas using water as cooling fluid:

Reynolds number *Re* []: $\mathrm{Re} = \frac{u_{avg} d}{\nu}$

Average coolant velocity $u_{avg}$ [m/s]: $u_{avg} \approx 0.3926 \cdot d^{0.714} \left( \frac{\Delta p}{l} \right)^{0.571}$

Water flow *Q* [litre/s] necessary to remove heat *P*: $Q_{water} = 0.2388 \frac{P}{\Delta T}$

Water flow *Q* [litre/s] inside a round tube : $Q = u_{avg} \frac{\pi d^2}{4} 10^3$

Temperature increase *ΔT* [°C] : $\Delta T = 304 \frac{P}{u_{avg} d^2}$

- *P:* power [W]
- *l,d*: cooling circuit length [m] and diameter [m]
- *ν:* kinematic viscosity of coolant is temperature depending, for simplification it is assumed to be constant ($9.85 \cdot 10^{-7}$ $m^2/s$ @ 21°C for water)

# Direct water cooling

Number of cooling circuits per coil: $\Delta p \propto \frac{1}{K_W^3}$

→ Doubling the number of cooling circuits reduces the pressure drop by a factor of eight for a constant flow

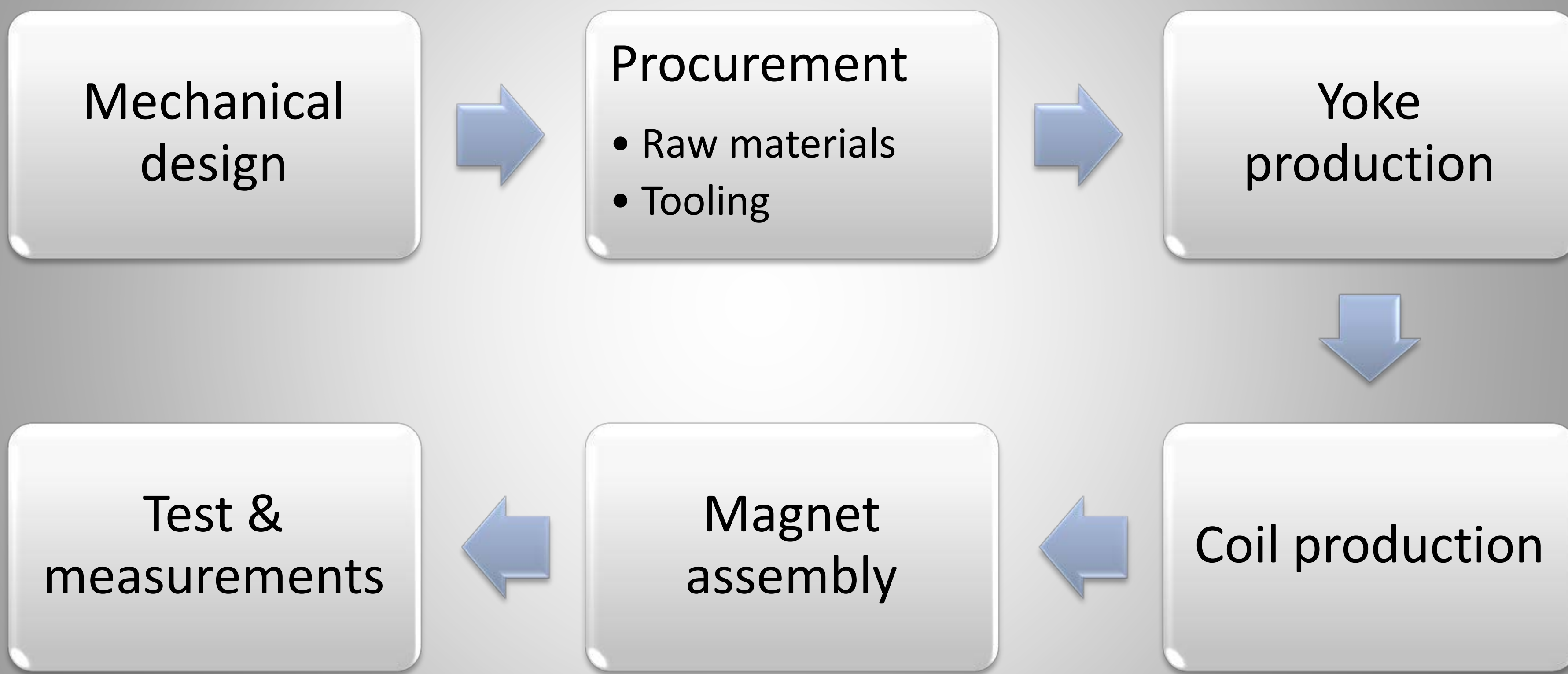
Diameter of cooling channel: $\Delta p \propto \frac{1}{d^5}$

→ Increasing the cooling channel by a small factor can reduce the required pressure drop significantly

# Cooling water properties

- For the cooling of hollow conductor coils demineralised water is used (exception: indirect cooled coils)
- Water quality essential for the performance and the reliability of the coil (corrosion, erosion, short circuits)
- Resistivity > 0.1 x $10^6$ Ωm
- pH between 6 and 6.5 (= neutral)
- Dissolved oxygen below 0.1 ppm
- Filters to remove particles and loose deposits to avoid cooling duct obstruction

# Magnet manufacturing



# Magnetic steel

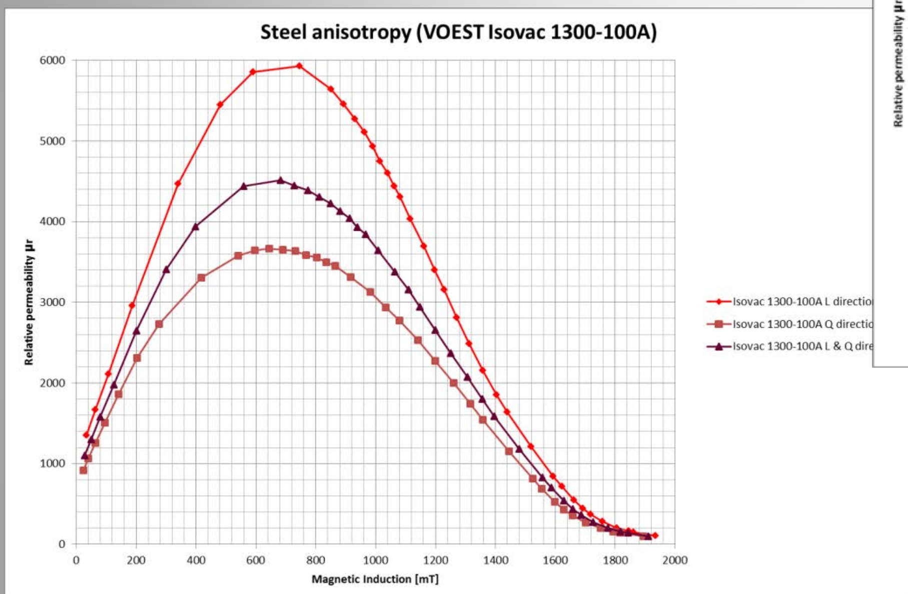
Massive (cast ingot) iron only for dc magnets

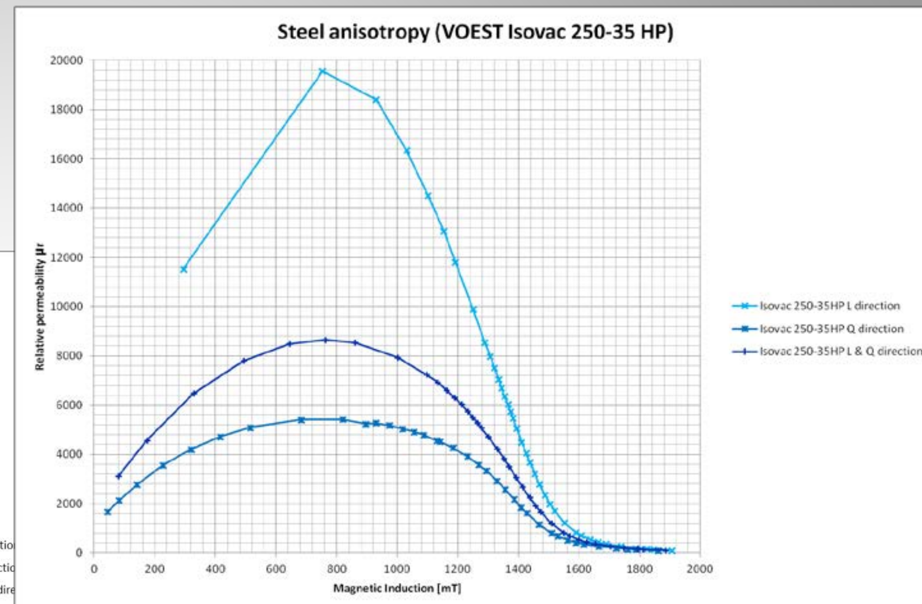
Today's standard: cold rolled, non-oriented electro-steel sheets (EN 10106)

- Magnetic and mechanical properties can be adjusted by final annealing
- Reproducible steel quality even over large productions
- Magnetic properties (permeability, coercivity) within small tolerances
- Homogeneity and reproducibility among the magnets of a series can be enhanced by selection, sorting or shuffling
- Organic or inorganic coating  for insulation and bonding
- Material is usually cheaper, but laminated yokes are labour intensive and require more expensive tooling (fine blanking, stacking)


# NGO steel

ISOVAC 1300-100A: $H_c$ = 65 A/m




ISOVAC 250-35HP: $H_c$ = 30 A/m

**Sheet thickness:**
$0.3 \leq t \leq 1.5$ mm

**Specific weight:**
$7.60 \leq \delta \leq 7.85$ g/cm$^3$

**Electr. resistivity** @20°C:
0.16 (low Si) $\leq \rho$
$\leq$ 0.61 µΩm (high Si)

# Yoke manufacturing

Stamping laminations

↓

Stacking laminations into yokes

↓

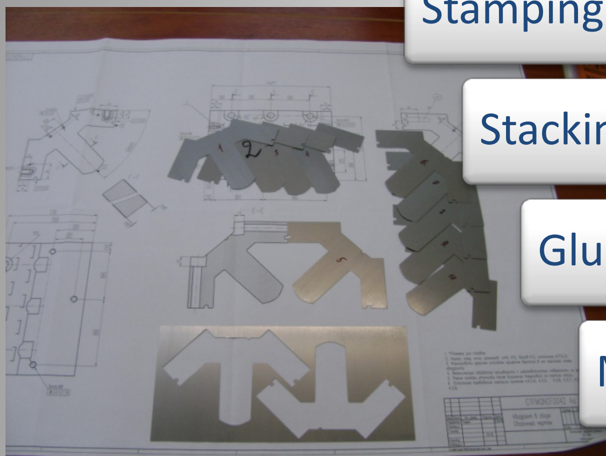
Gluing and/or welding

↓

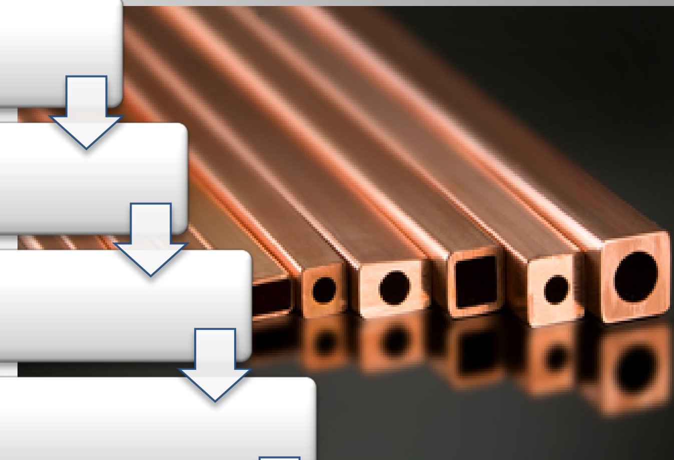
Machining

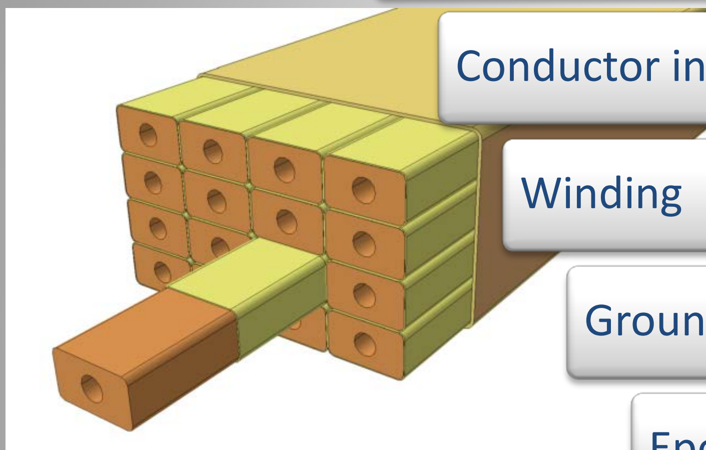
↓

Assembly (preliminary)

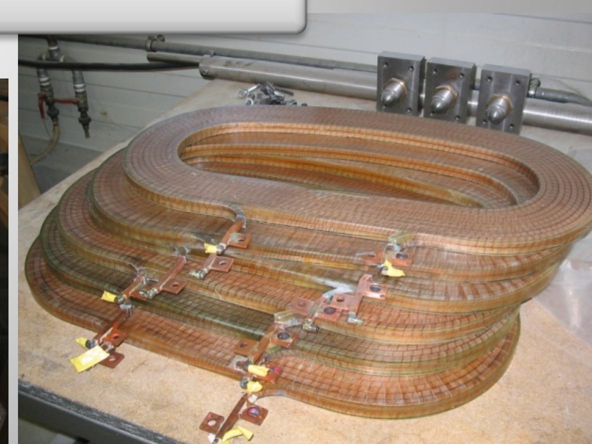


# Coil manufacturing





- Define conductor type and material
- Conductor insulation
- Winding
- Ground insulation
- Epoxy impregnation



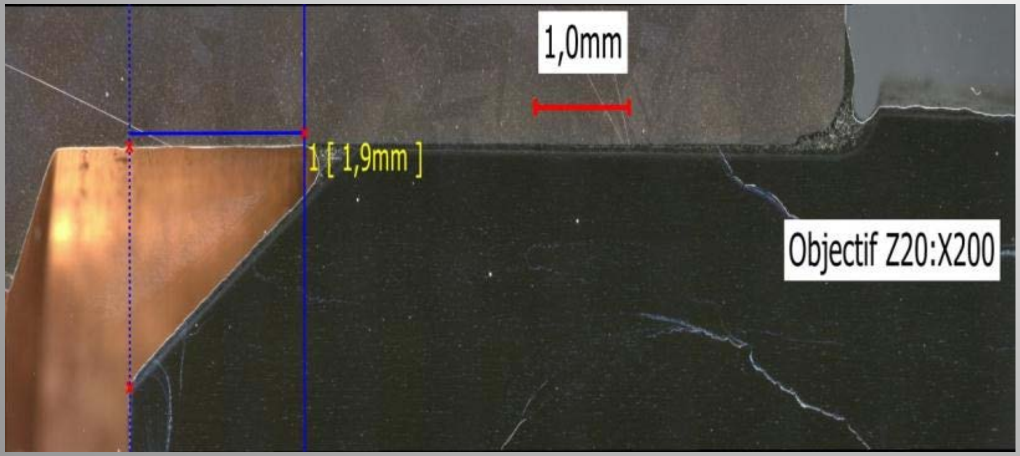
# QA & Acceptance tests

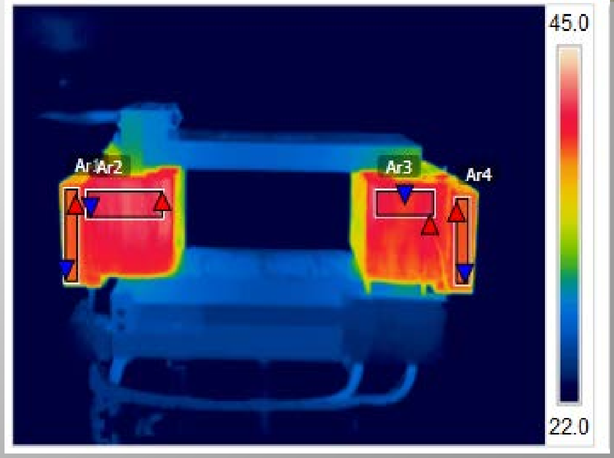
QA is important at each production stage

Constant monitoring of critical items

Acceptance test include electrical, hydraulic, mechanical measurements

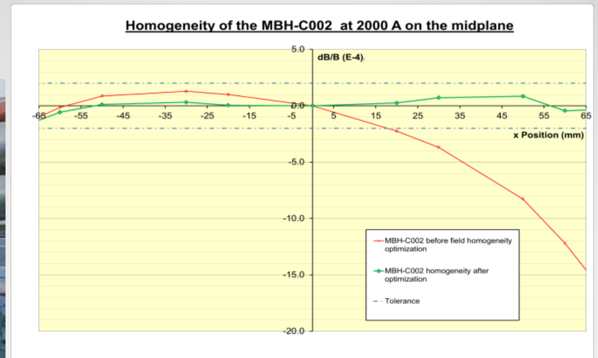


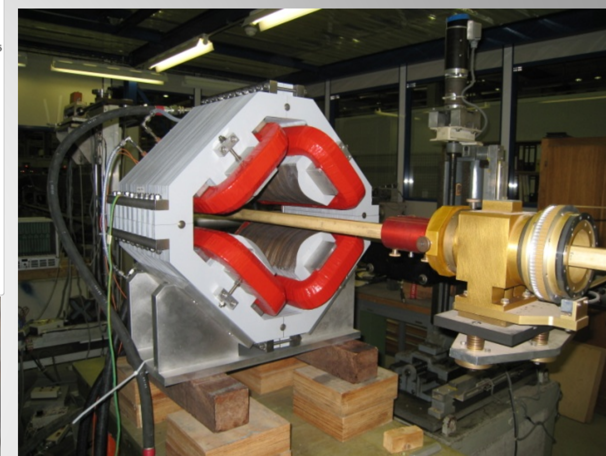



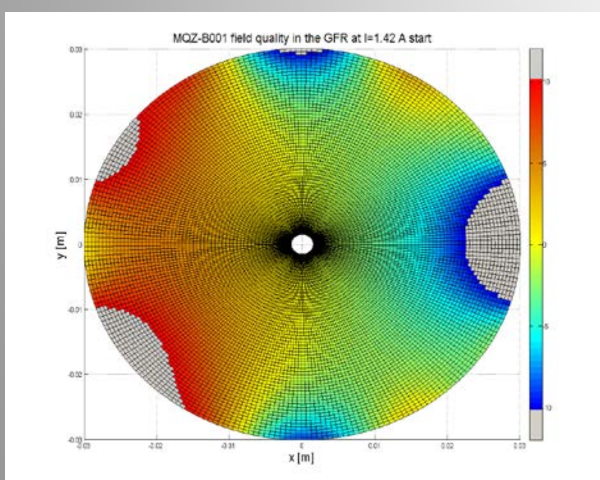


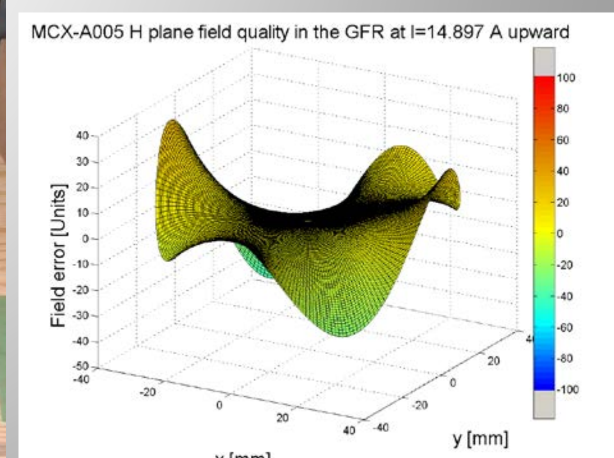
# Magnetic measurements

Magnetic measurements as final proof if design and manufacturing is correct





# Cost estimate

## Production specific tooling:

5 to 15 k€/tooling

## Material:

Steel sheets: 1.0 - 1.5 € /kg

Copper conductor: 10 to 20 € /kg

## Yoke manufacturing:

Dipoles: 6 to 10 € /kg (> 1000 kg)

Quads/Sextupoles: 50 to 80 € /kg (> 200 kg)

Small magnets: up to 300 € /kg

## Coil manufacturing:

Dipoles: 30 to 50 € /kg (> 200 kg)

Quads/Sextupoles: 65 to 80 € /kg (> 30 kg)

Small magnets: up to 300 € /kg

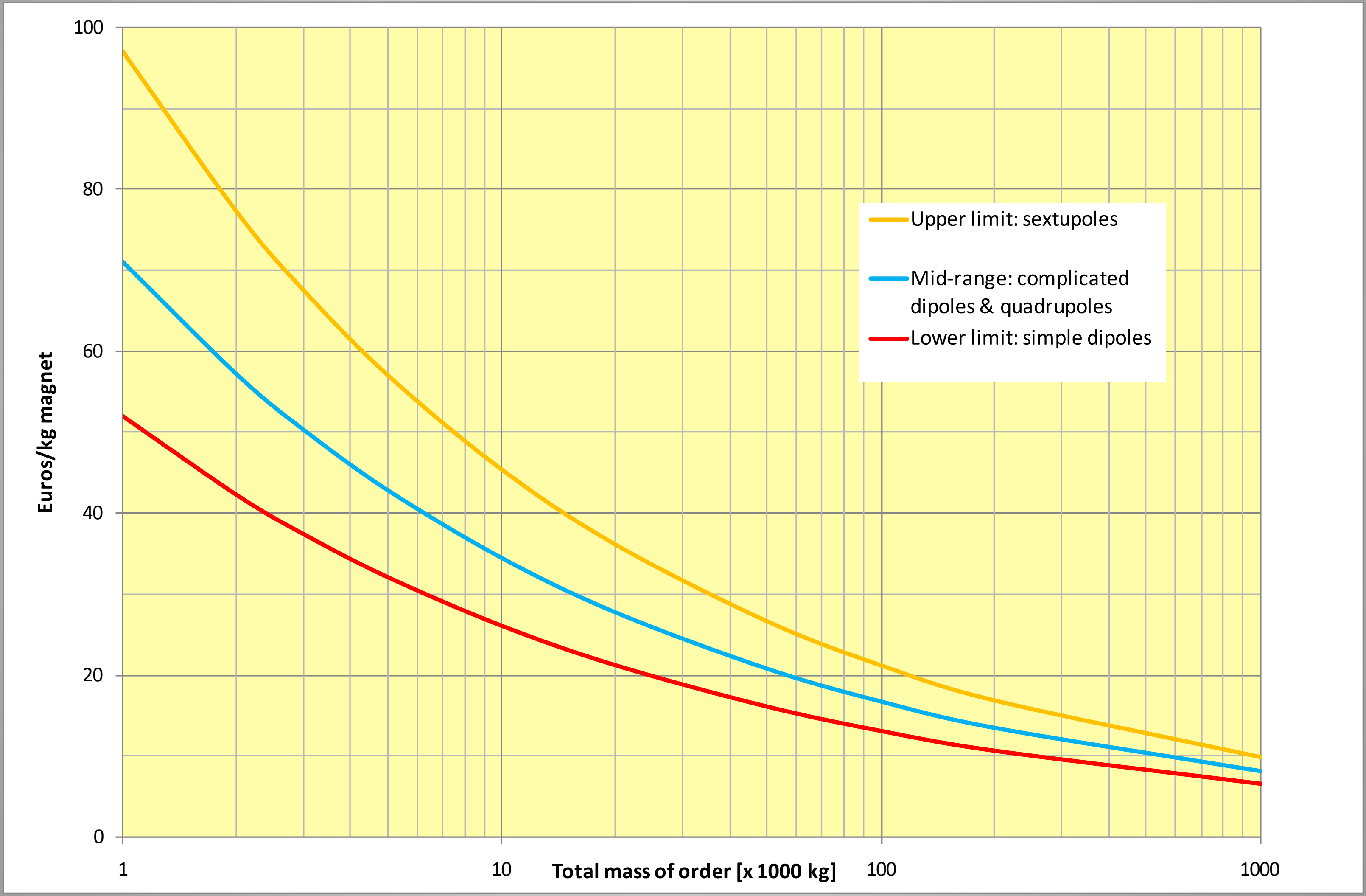
## Contingency:

10 to 20 %

| | | |
|---|---|---|
| Magnet | ***Magnet type*** | ***Dipole*** |
| | Number of magnets (incl. spares) | 18 |
| | Total mass/magnet | 8330 kg |
| Fixed costs | Design | 14 kEuros |
| | Punching die | 12 kEuros |
| | Stacking tool | 15 kEuros |
| | Winding/molding tool | 30 kEuros |
| Yoke | Yoke mass/magnet | 7600 kg |
| | Used steel (incl. blends)/magnet | 10000 kg |
| | Yoke manufacturing costs | 8 Euros/kg |
| | Steel costs | 1.5 Euros/kg |
| Coil | Coil mass/magnet | 730 kg |
| | Coil manufacturing costs | 50 Euros/kg |
| | Cooper costs (incl. insulation) | 12 Euros/kg |
| Total costs | Total order mass | 150 Tonnes |
| | Total fixed costs | 71 kEuros |
| | Total Material costs | 428 kEuros |
| | Total manufacturing costs | 1751 kEuros |
| | Total magnet costs | 2250 kEuros |
| | Contingency | 20 % |
| | **Total overall costs** | **2700 kEuros** |

NOT included: magnetic design, supports, cables, water connections, alignment equipment, magnetic measurements, transport, installation
Prices for 2011

# Cost estimate

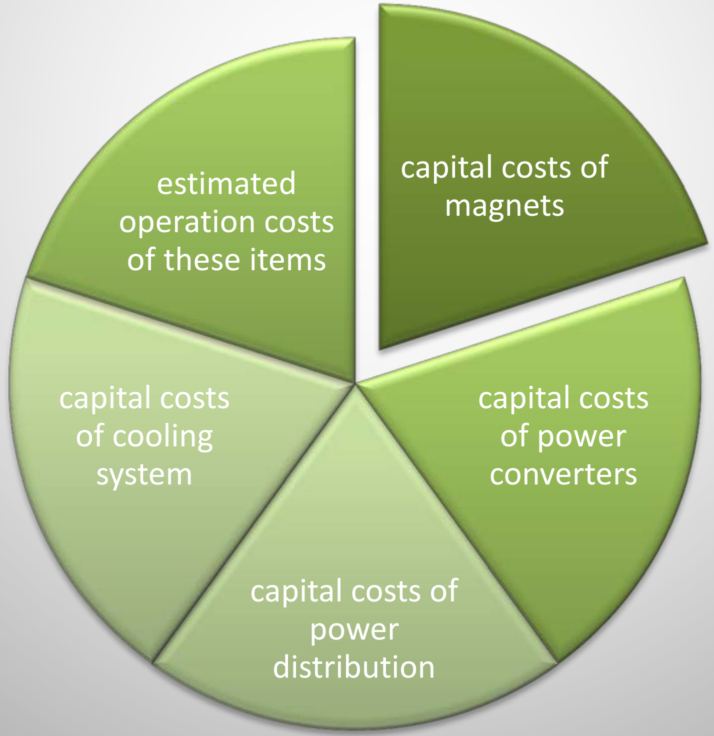


# Cost optimization

*Focus on economic design!*

Design goal: Minimum total costs over projected magnet life time by optimization of capital (investment) costs against running costs (power consumption)

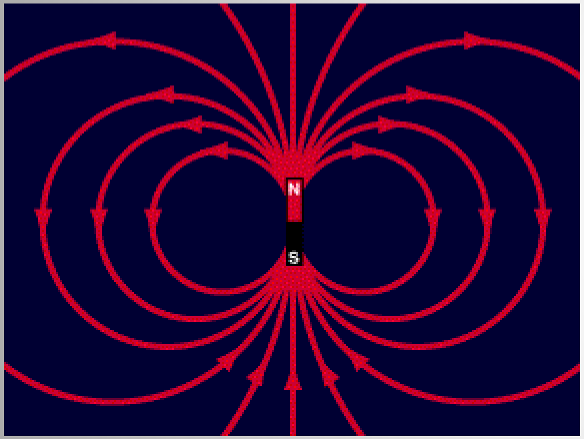
Total costs include:



# Summary

- Magnetic desing means translating beam optic requirements
- Before starting the design, all input parameters, requirements, contraints and interfaces have to be known and well understood
- Analytical design is neccessary to derive the main parameters of the future magnet before entering into a detailed design using numerical methods
- Magnet design is an iterative process often requiring a high level of experience
- Cost optimization is an important design aspect, in particular in view of future energy costs

# Lecture 3: Numerical design




Which code shall I use?

Introduction to 2D numerical design

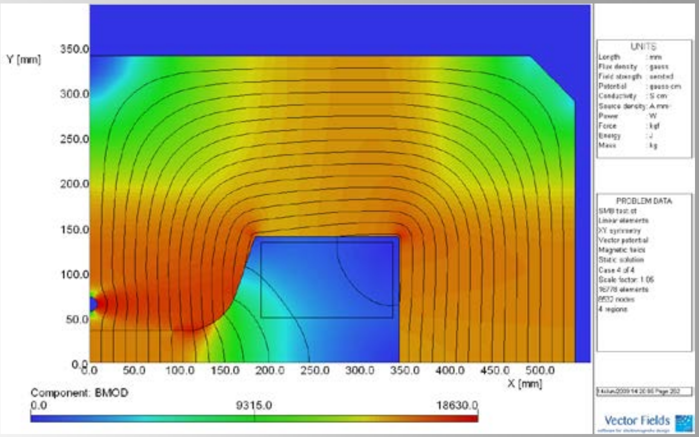
How to evaluate the results

Typical application examples

A brief look into 3D...

# Numerical design

Common computer codes: Opera (2D) or Tosca (3D), Poisson, ANSYS, Roxie, Magnus, Magnet, Mermaid, Radia, FEMM, etc…

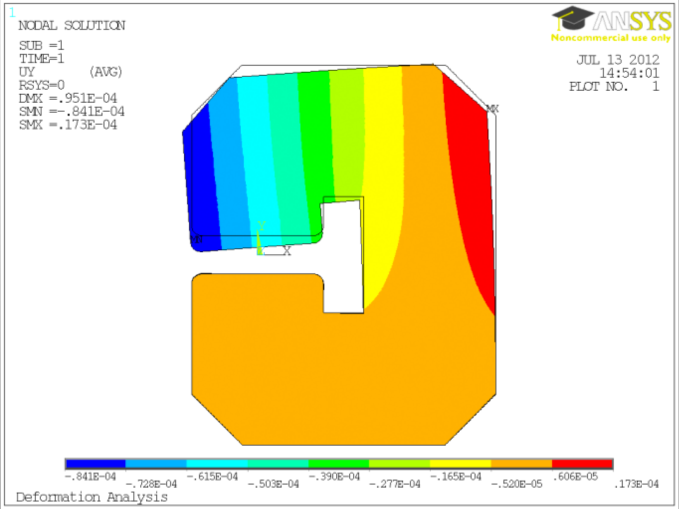


Technique is iterative

- calculate field generated by a defined geometry
- adjust geometry until desired distribution is achieved

Advanced codes offer:

- modeller, solver and post-processors
- mesh generator with elements of various shapes
- multiple solver iterations for non-linear material properties
- anisotropic material characterisation
- optimization routines
- combination with structural and thermal analysis
- time depended analysis (steady state, transient)




FEM codes are powerful tools, but be cautious:

- Always check results if they are 'physical reasonable'
- Use FEM for quantifying, not to qualify

# Which code shall I use ?

Selection criteria:

- The more powerful, the harder to learn
- Powerful codes require powerful CPU and large memory
- More or less user-friendly input (text and/or GUI, scripts)
- OS compatibility and lincense costs

Computing time increases for high accuracy solutions, non-linear problems and time dependent analysis

- Compromise between accuracy and computing time
- Smart modelling can help to minimize number of elements

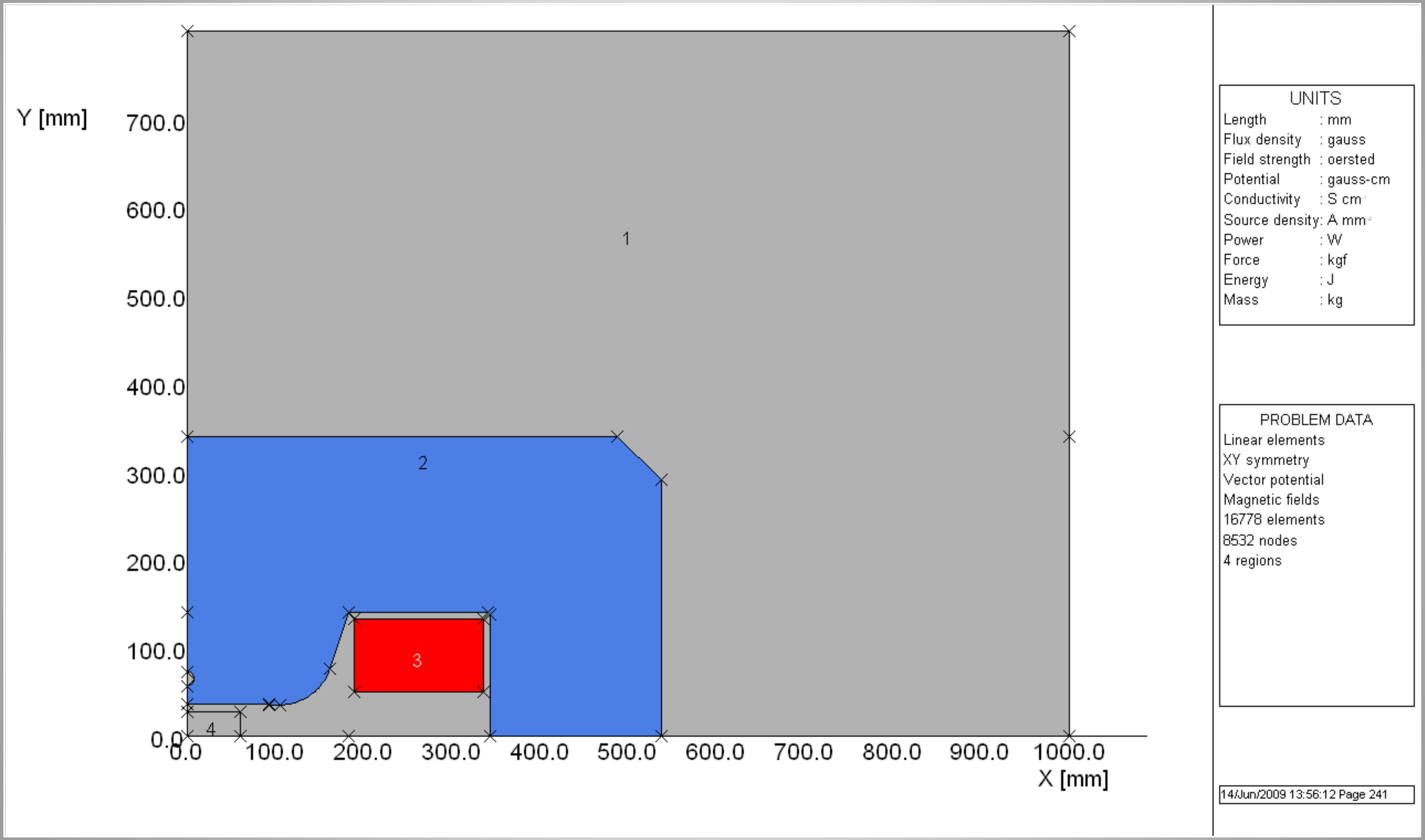
| 2D | 3D |
| --- | --- |
| • 2D analysis is often sufficient<br>• magnetic solvers allow currents only perpendicular to the plane<br>• fast | • produces large amount of elements<br>• mesh generation and computation takes significantly longer<br>• end effects included<br>• powerful modeller |

# Numerical design process

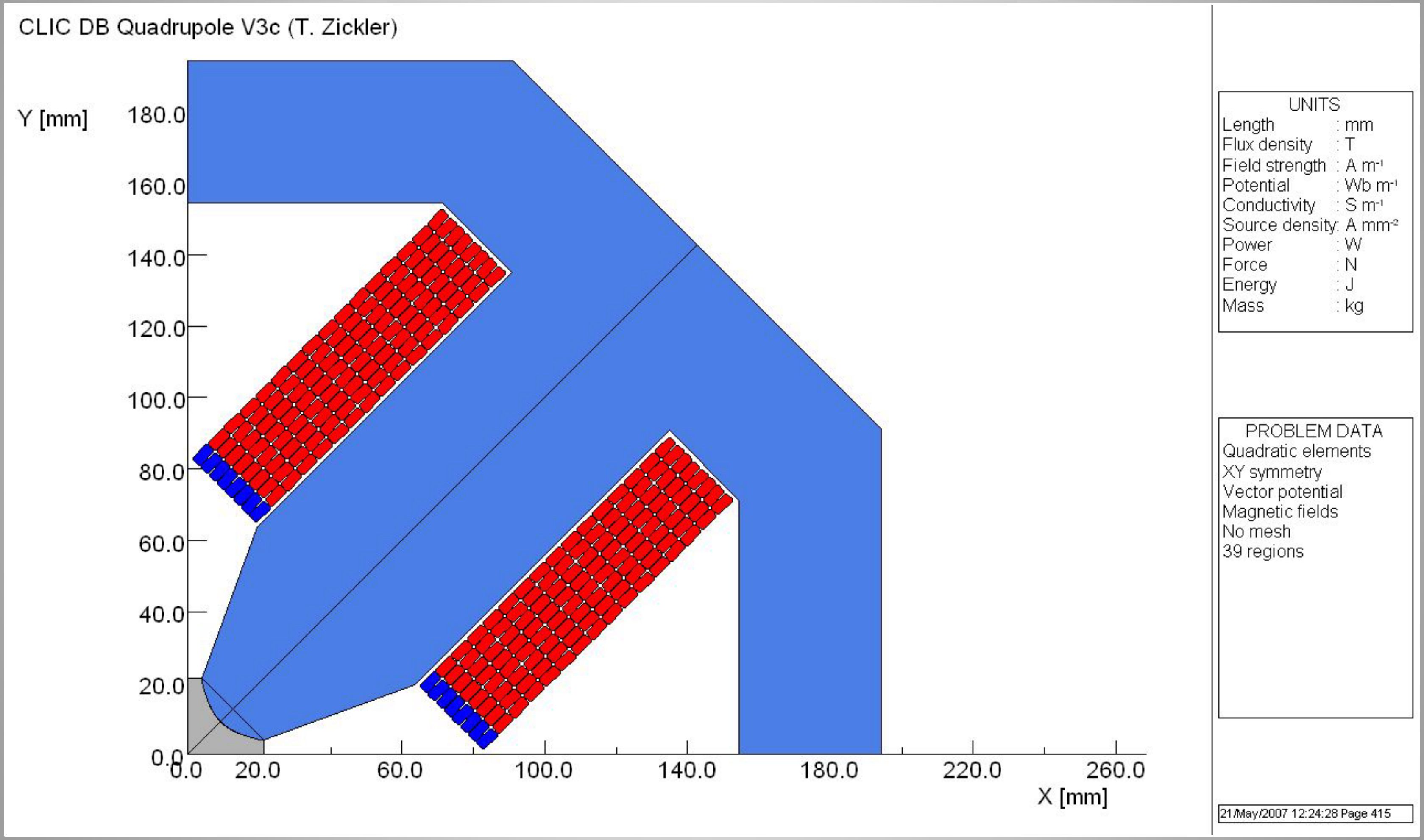
Design process in 2D (similar in 3D):

1. Create the model (pre-processor or modeller)
2. Define boundary conditions, set material properties
3. Calculations (solver)
4. Visualize and asses the results (post-processor)
5. Optimization by adjusting the geometry (manually or optimization code)

# Creating the model

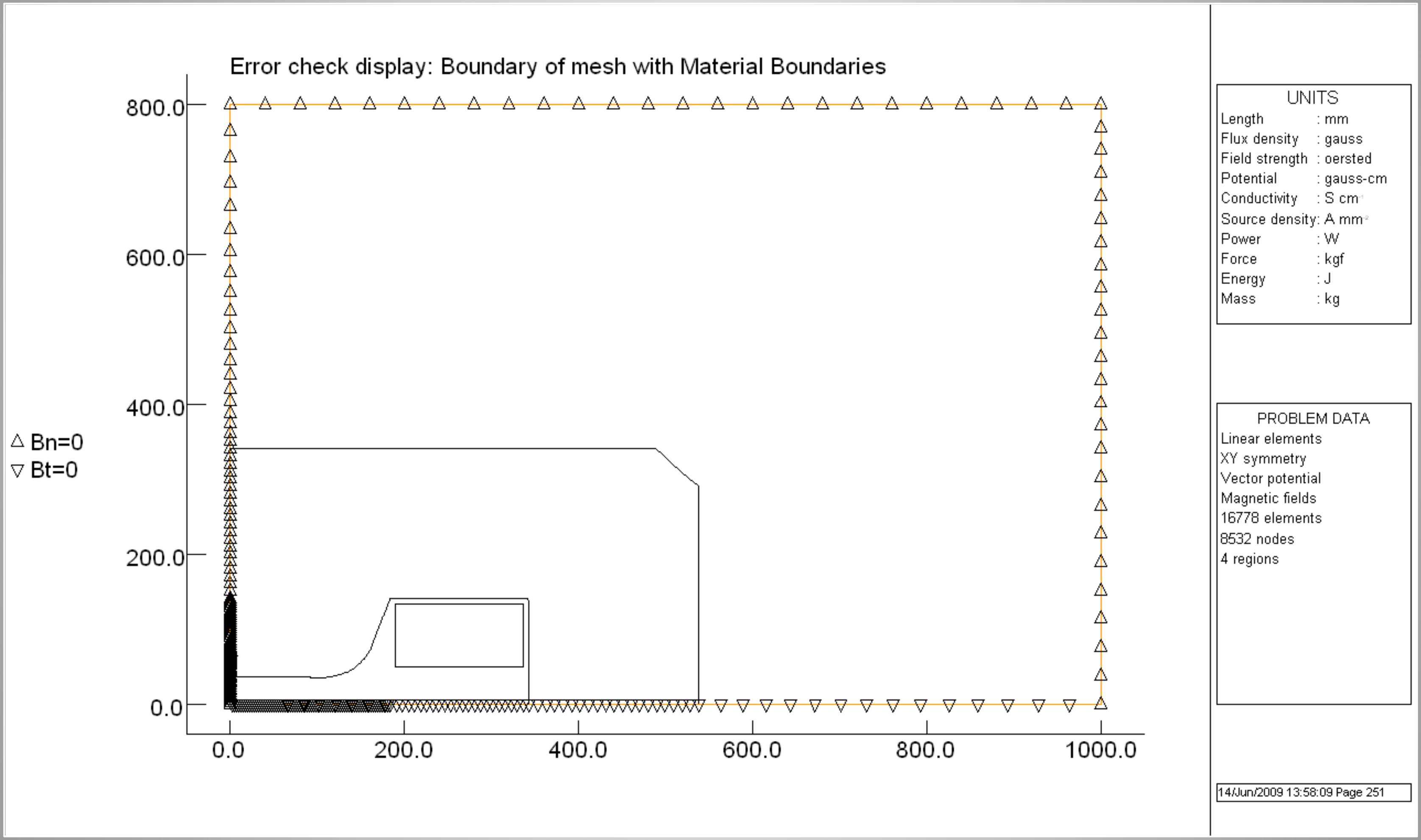


# Model symmetries

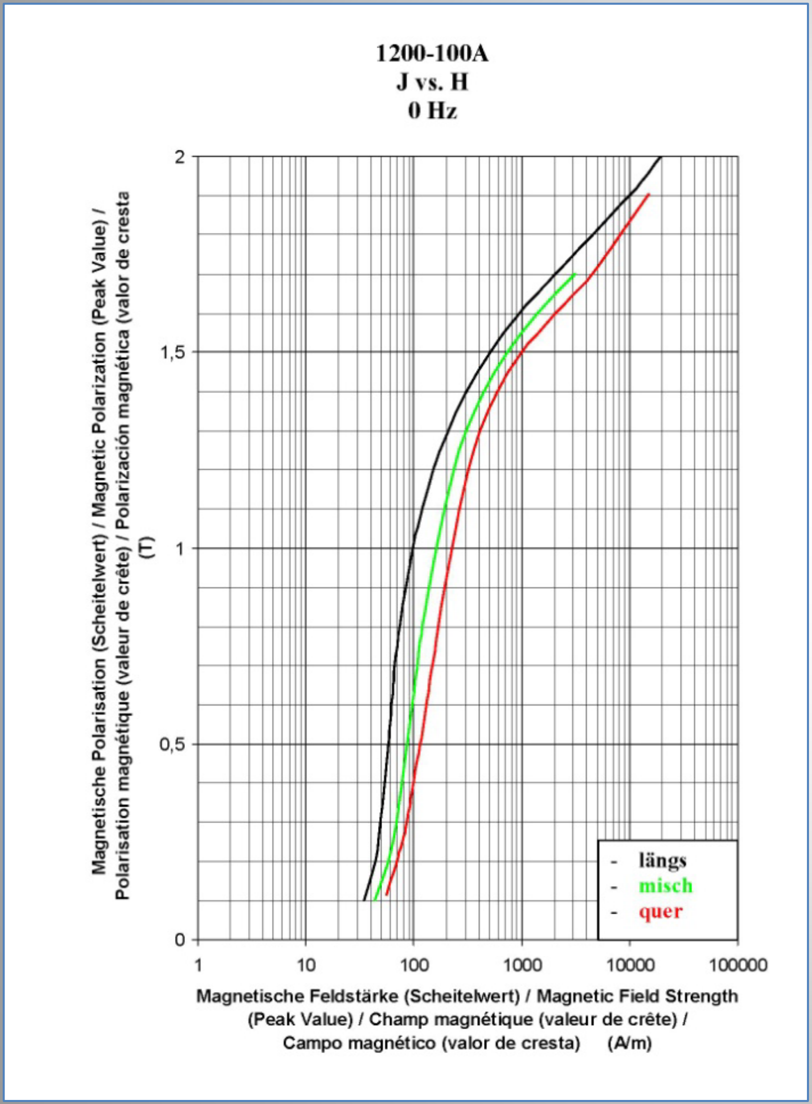



Note: one eighth of quadrupole could be used with opposite symmetries defined on horizontal and y = x axis

# Boundary conditions



# Material properties




Data source: Thyssen/Germany

## Permeability:

- either fixed for linear solution
- or permeability curve for non-linear solution
- can be anisotropic
- apply correction for steel packing factor
- pre-defined curves available

## Conductivity:

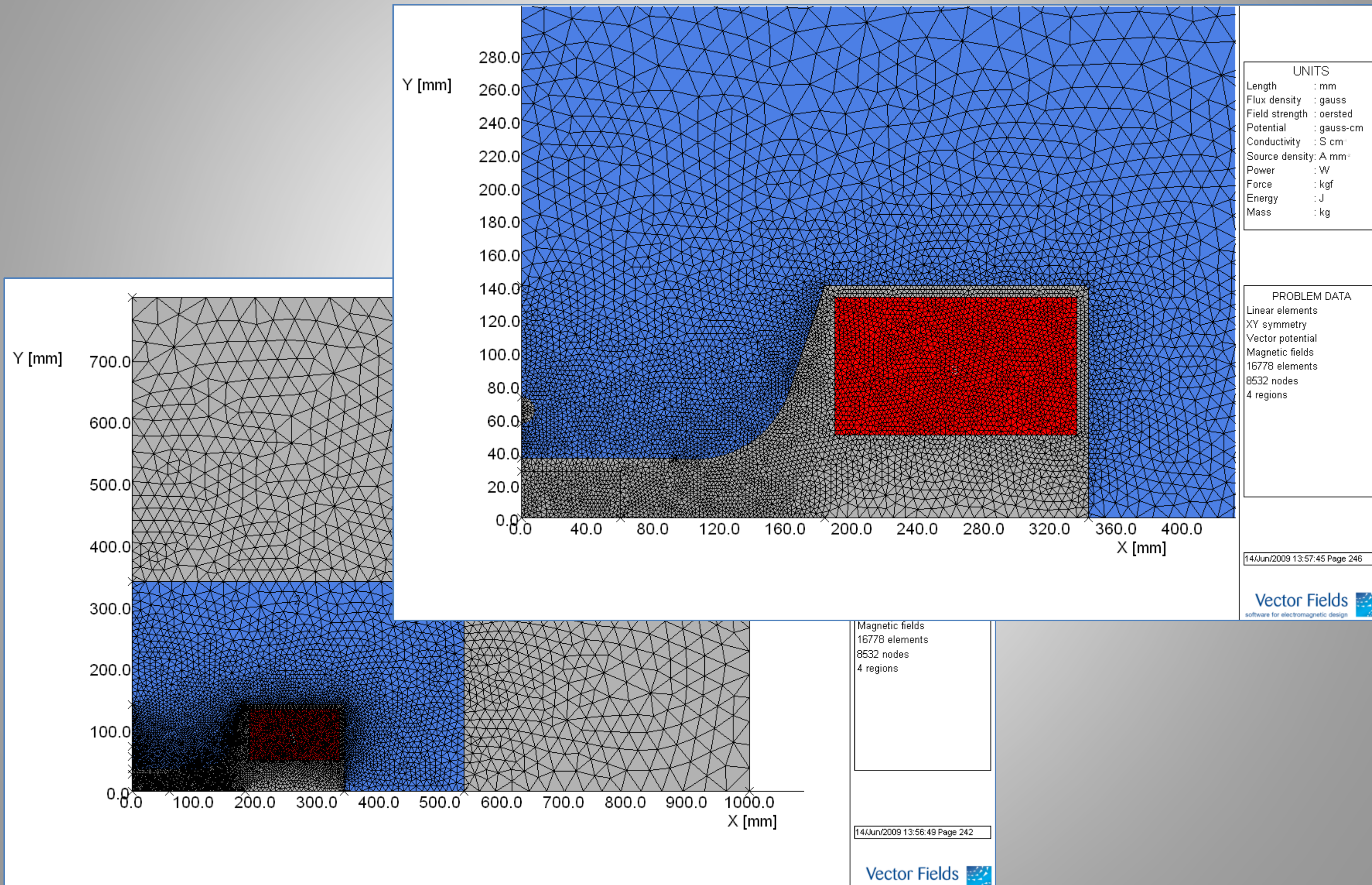
- for coil and yoke material
- required for transient eddy current calculations

## Mechanical and thermal properties:

- in case of combined structural or thermal analysis

## Current density in the coils

# Mesh generation



# Data processing

**Solution**

- linear: predefined constant permeability for a single calculation
- non-linear: permeability table for iterative calculations

**Solver types**

- static
- steady state (sine function)
- transient (ramp, step, arbitrary function, ...)

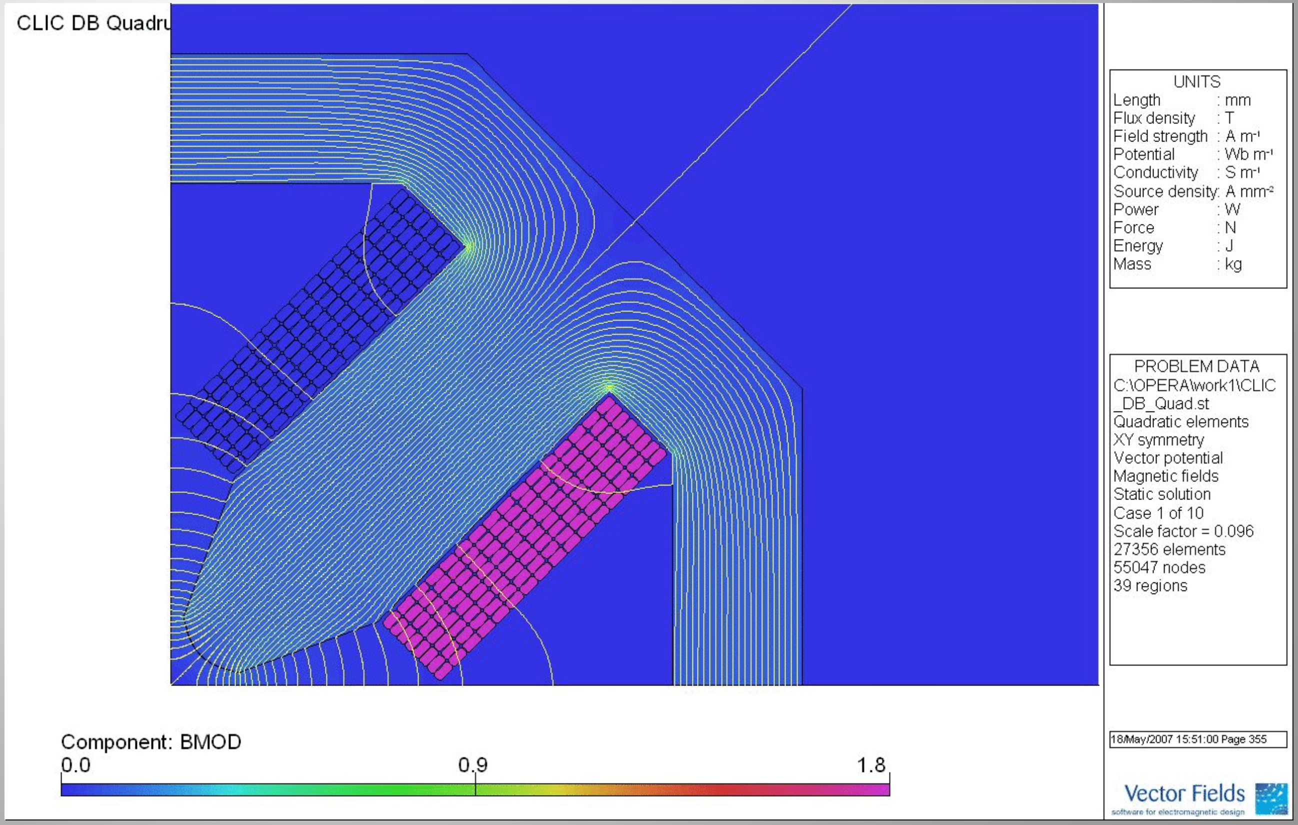
**Solver settings**

- number of iterations,
- convergence criteria
- precision to be achieved, etc...

# Analyzing the results

With the help of the post-processor, field distribution and field quality and be visualized in various forms on the pre-processor model:

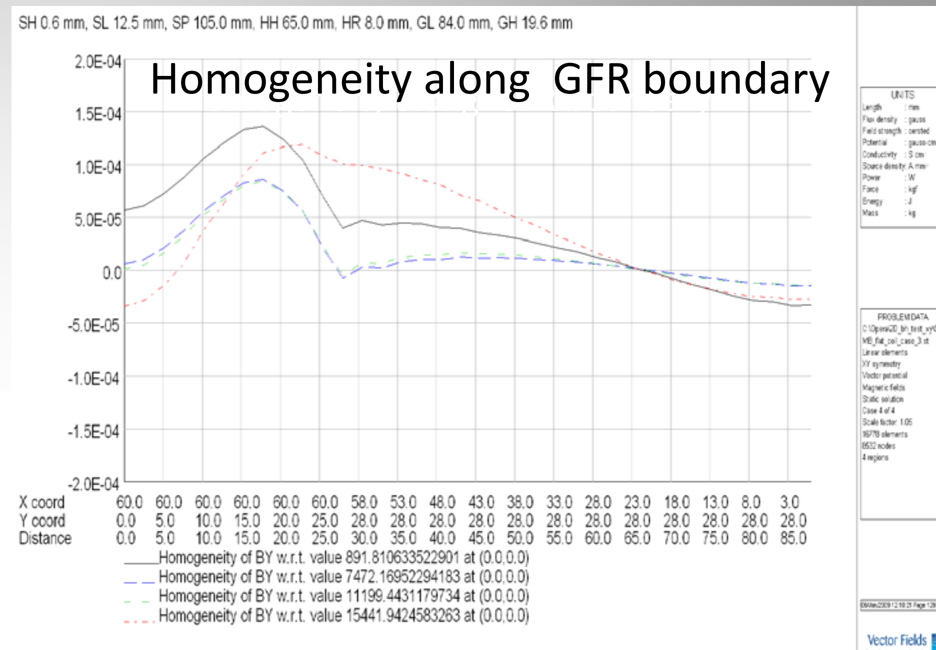
- Field lines and colour contours plots of flux, field, and current density
- Graphs showing absolute or relative field distribution
- Homogeneity plots

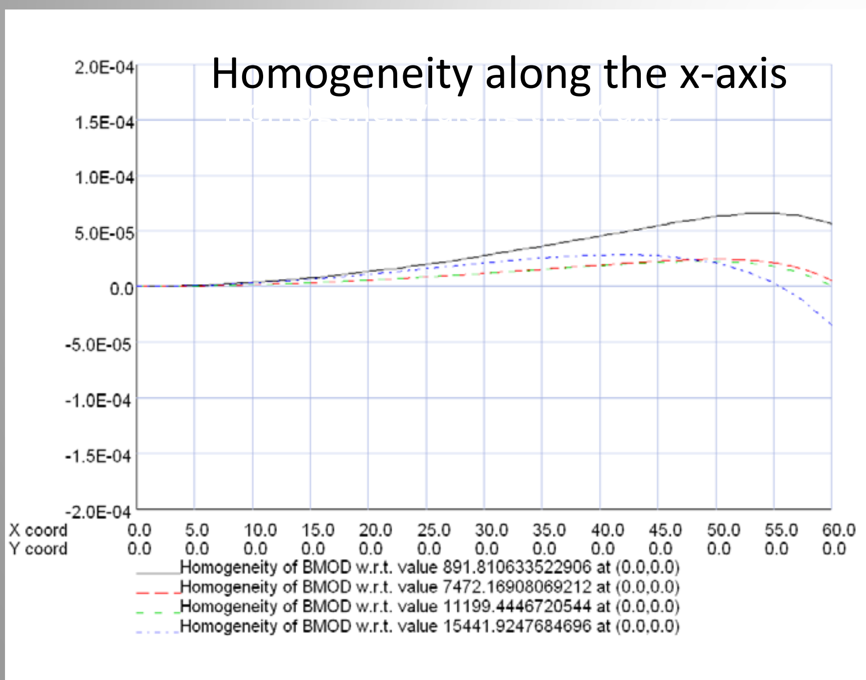


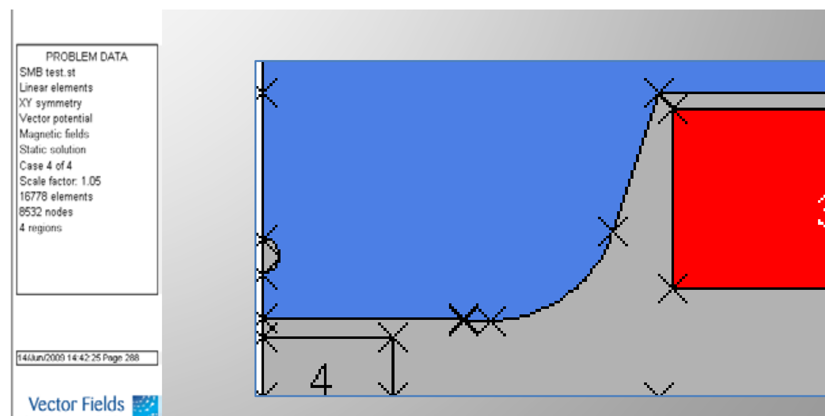
# Field homogeneity in a dipole

A simple judgment of the field quality can be done by plotting the field homogeneity

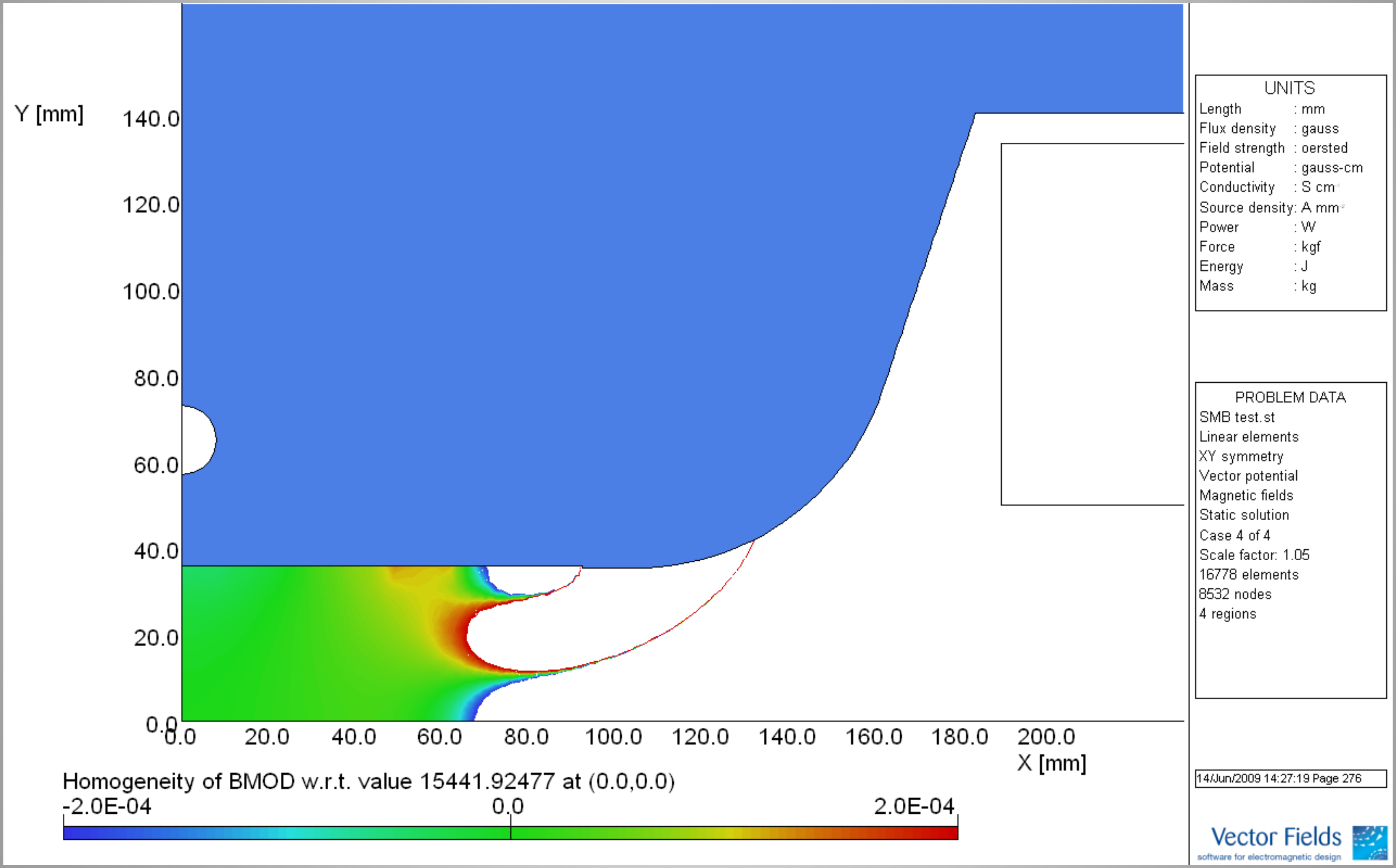
$$\frac{\Delta B}{B_0} = \frac{B_y(x,y)}{B_y(0,0)} - 1 \qquad \frac{\Delta B}{B_0} \leq 0.01\%$$



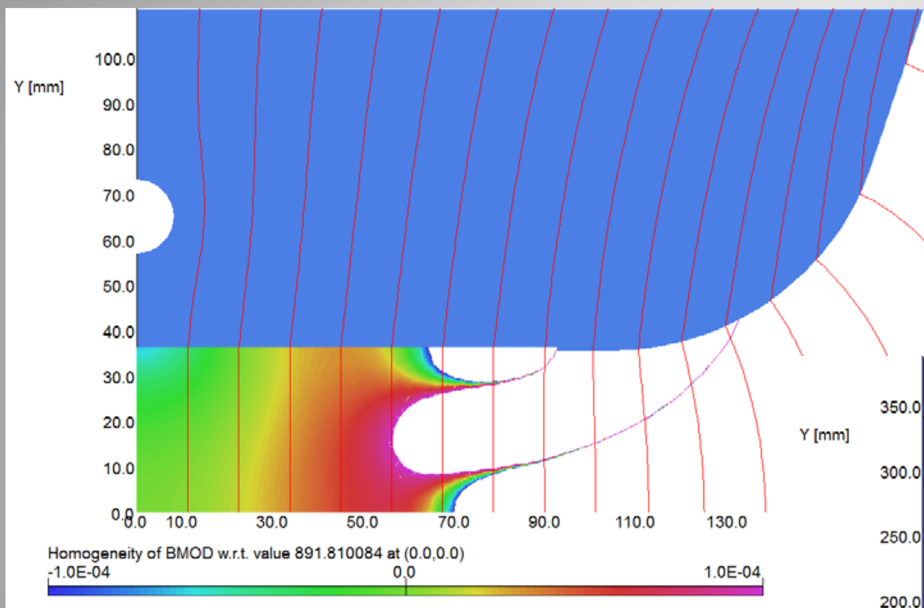



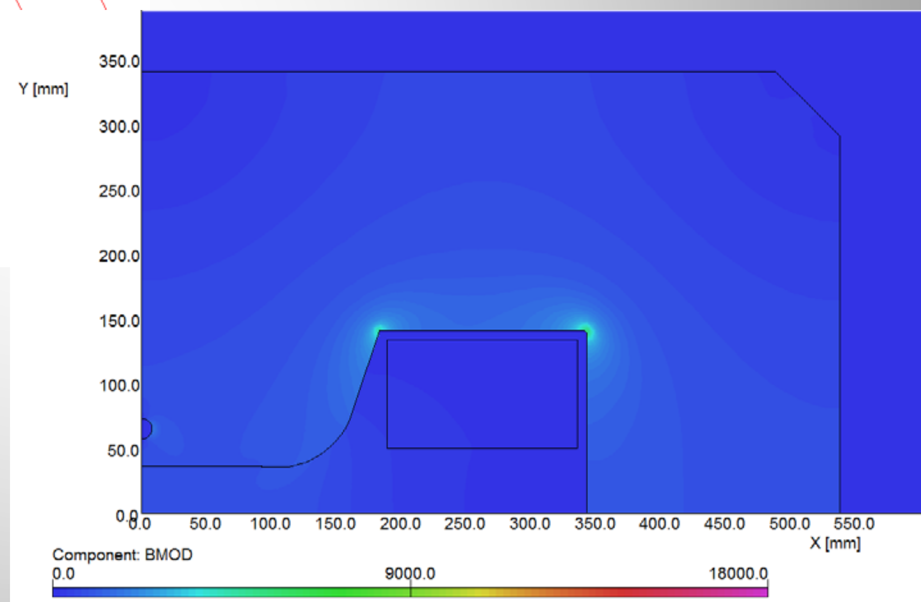
# Field homogeneity in a dipole



# Saturation and field quality




Also very low fields can disturb the field quality significantly



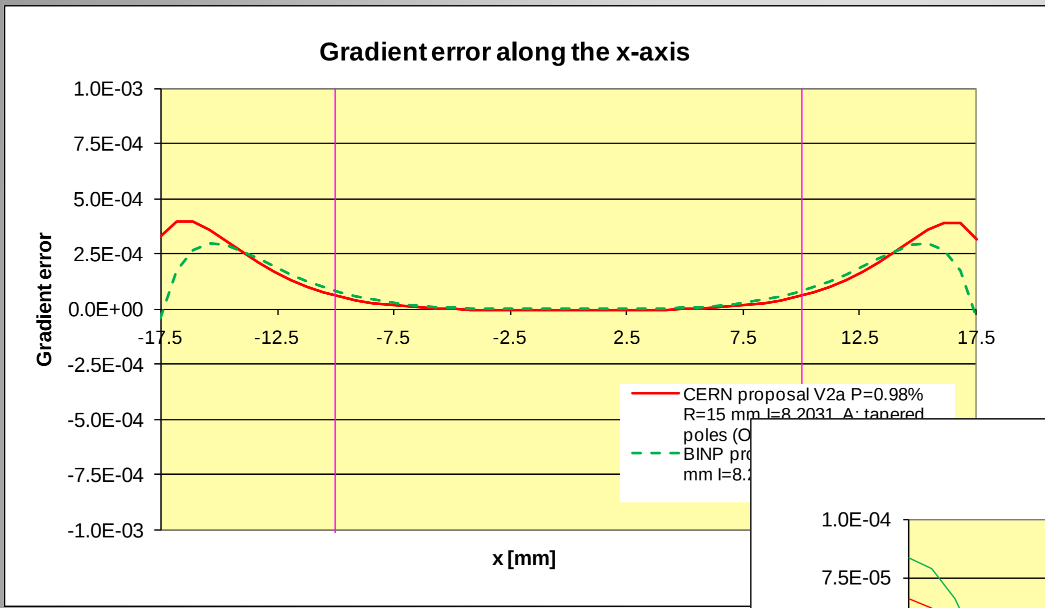

Field quality can vary with field strength due to saturation

# Field homogeneity in a quadrupole

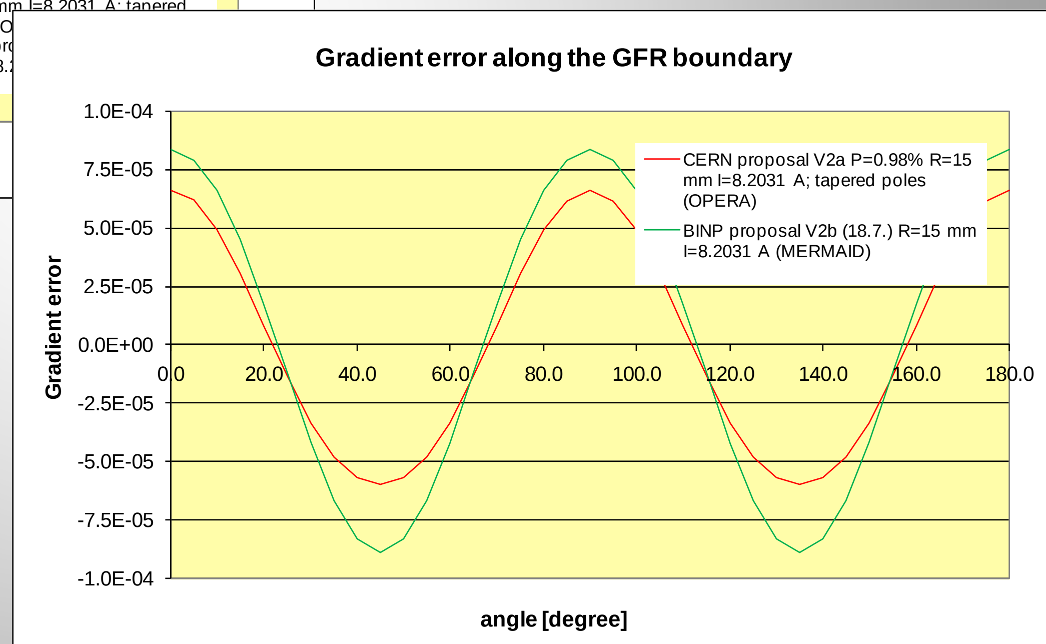


Field homogeneity in a quadrupole

$$\frac{\Delta B'}{B'_0} = \frac{B_r(x,y)}{B'(0,0)\sqrt{x^2+y^2}} - 1$$



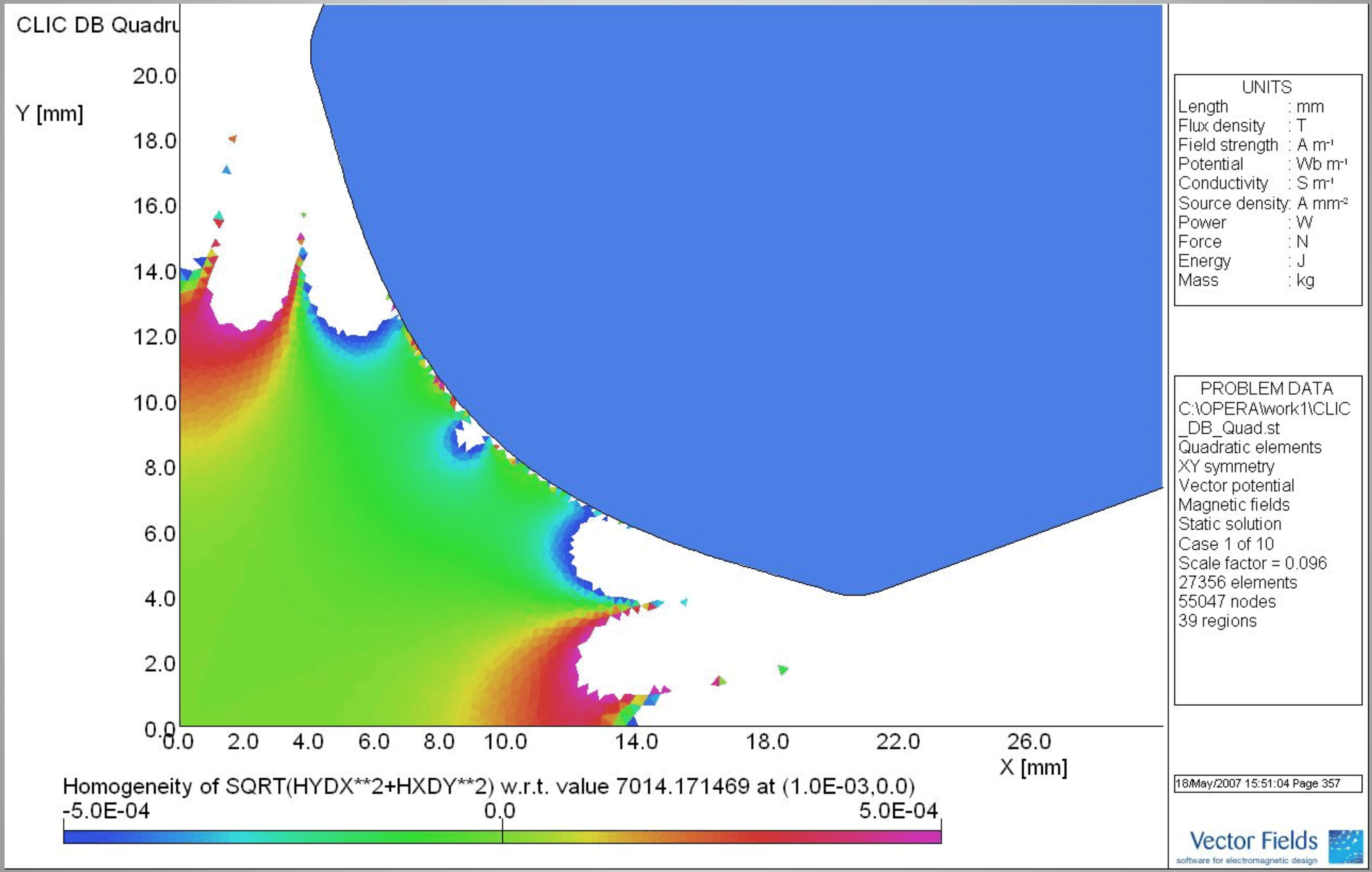

Gradient homogeneity along the x-axis




$$\frac{\Delta B'}{B'_0} \leq 0.1\%$$

Gradient homogeneity along circular GFR

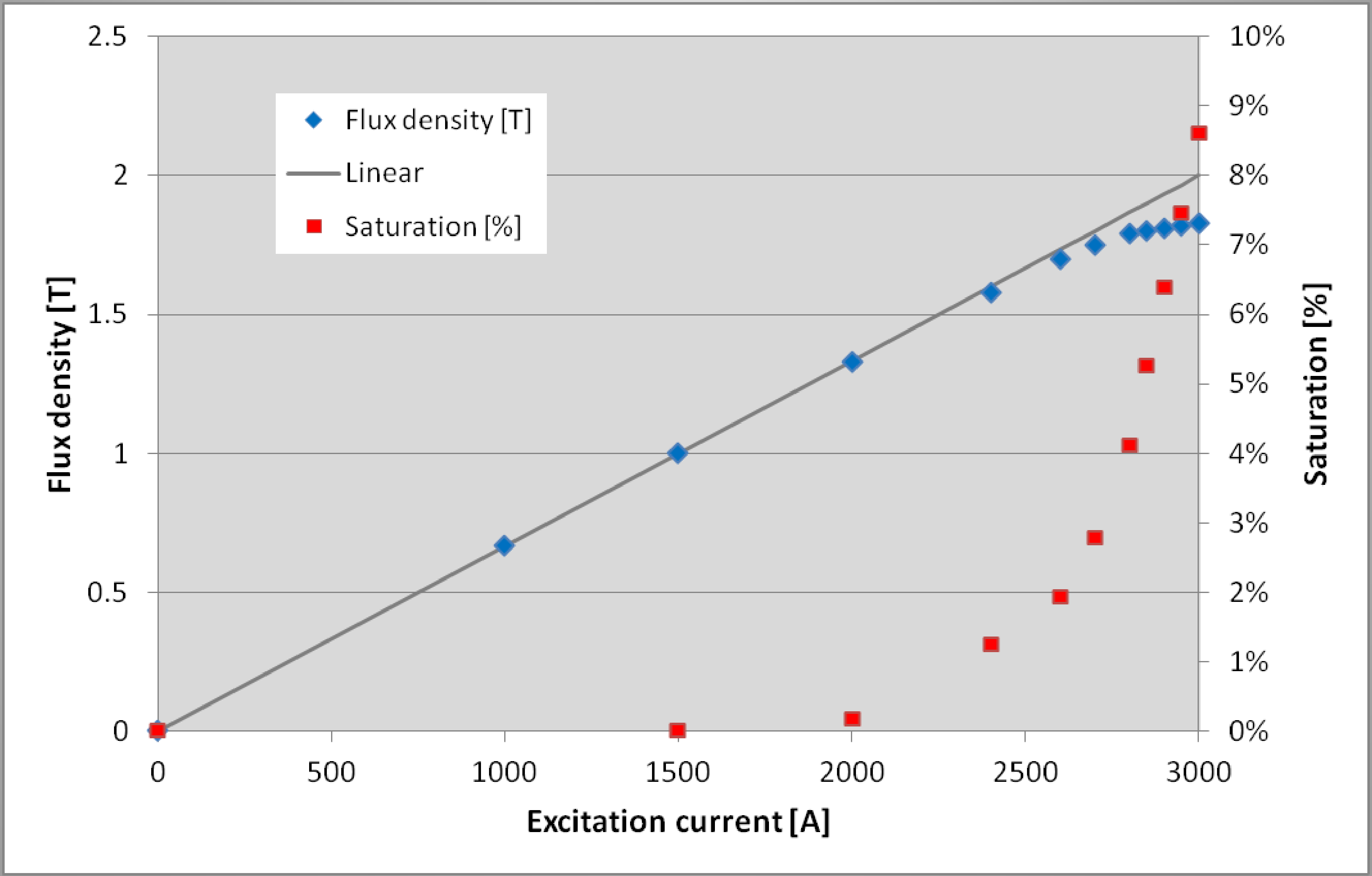
# Saturation and field quality




Field quality varies with field strength due to saturation

# Saturation




Flux density [T]
Linear
Saturation [%]
Flux density [T]
Saturation [%]
Excitation current [A]
0
0.5
1
1.5
2
2.5
0%
1%
2%
3%
4%
5%
6%
7%
8%
9%
10%
0
500
1000
1500
2000
2500
3000

# Multipole expansion

The amplitude and phase of the harmonic components in a magnet are good 'figures of merit' to asses the field quality of a magnet

$$B_y + iB_x = B_{ref} \sum_{n=1}^{\infty} (b_n + ia_n) \cdot \left( \frac{x + iy}{R_{ref}} \right)^{n-1}$$

- The normal ($b_n$) and the skew ($a_n$) multipole coefficients are useful:
  - to describe the field errors and their impact on the beam in the lattice, so the magnetic design can be evaluated
  - in comparison with the coefficients resulting from magnetic measurements to judge acceptability of a manufactured magnet
- Due to the finite size of the poles, higher order multipole components appear
- They are intrinsic to the design and called ‚allowed' multipoles

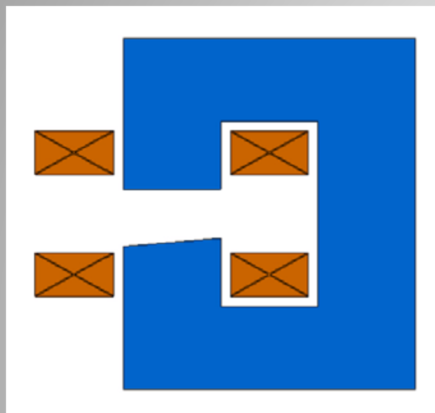
$$n = N(2m+1)$$

- *n*: order of multipole component
- *N*: order of the fundamental field
- *m*: integer number ($m \geq 1$)

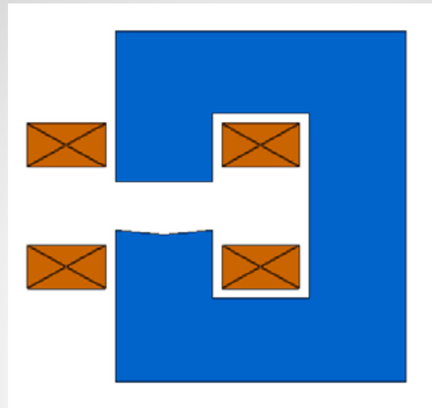
- ‚Non-allowed' multipoles result from a violation of symmetry and indicate a fabrication or assembly error

# Asymmetries

Asymmetries generating 'non-allowed' harmonics

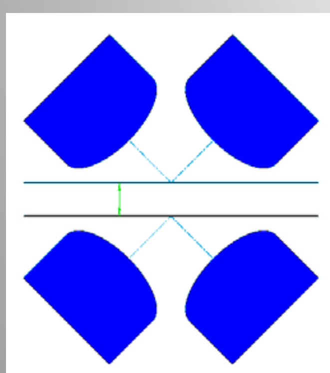


n = 2, 4, 6, ...

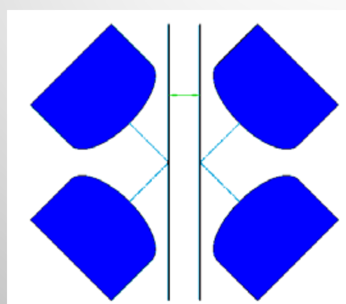


n = 3, 6, 9, ...

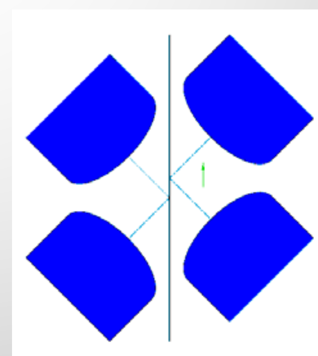
Comprehensive studies about the influence of manufacturing errors on the field quality have been done by K. Halbach.



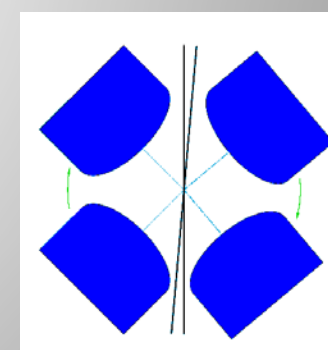
n = 4 (neg.)



n = 4 (pos.)



n = 3

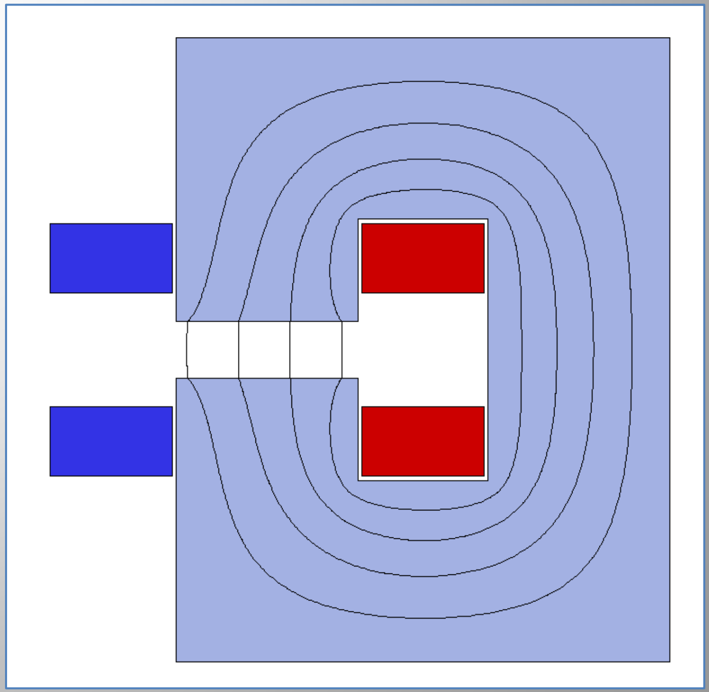


n = 2, 3

These errors can seriously affect machine behaviour and must be controlled!

# Asymmetry in a C-magnet

- C-magnet: one-fold symmetry
- Since $NI = \oint \vec{H} \cdot \vec{dl} = const.$ the contribution to the integral in the iron has different path lengths
- Finite (low) permeability will create lower B on the outside of the gap than on the inside
- Generates ‘forbidden’ harmonics with n = 2, 4, 6, ... changing with saturation
- Quadrupole term resulting in a gradient around 0.1% across the pole



# Pole tip design

It is easy to derive perfect mathematical pole configurations for a specific field configuration

In practice poles are not ideal: finite width and end effects result in multi-pole errors disturbing the main field

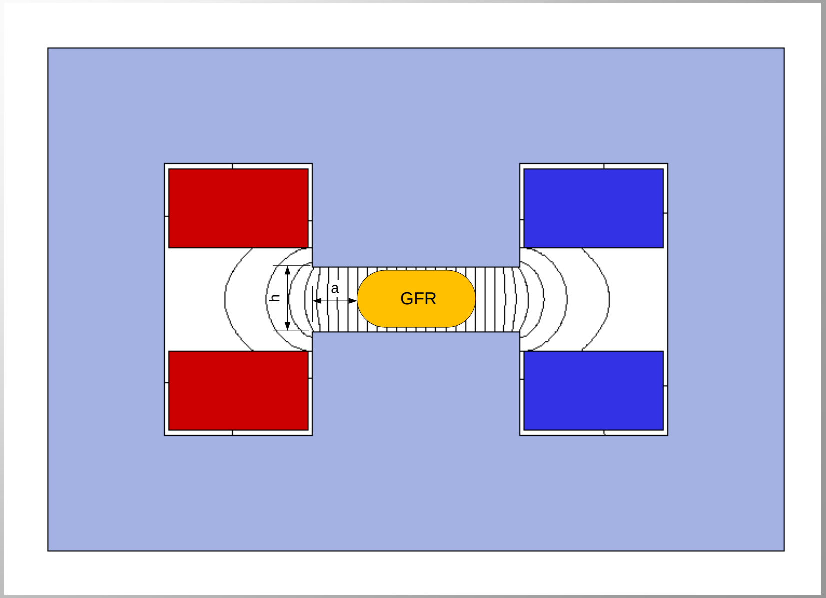
The uniform field region is limited to a small fraction of the pole width

Estimate the size of the poles and calculate the resulting fields

Better approach: calculate the necessary pole overhang using:

$$x_{unoptimized} = 2\frac{a}{h} = -0.36\ln\frac{\Delta B}{B_0} - 0.90$$

- *x*: pole overhang normalized to the gap
- *a*: pole overhang: excess pole beyond the edge of the good field region to reach the required field uniformity
- *h*: magnet gap

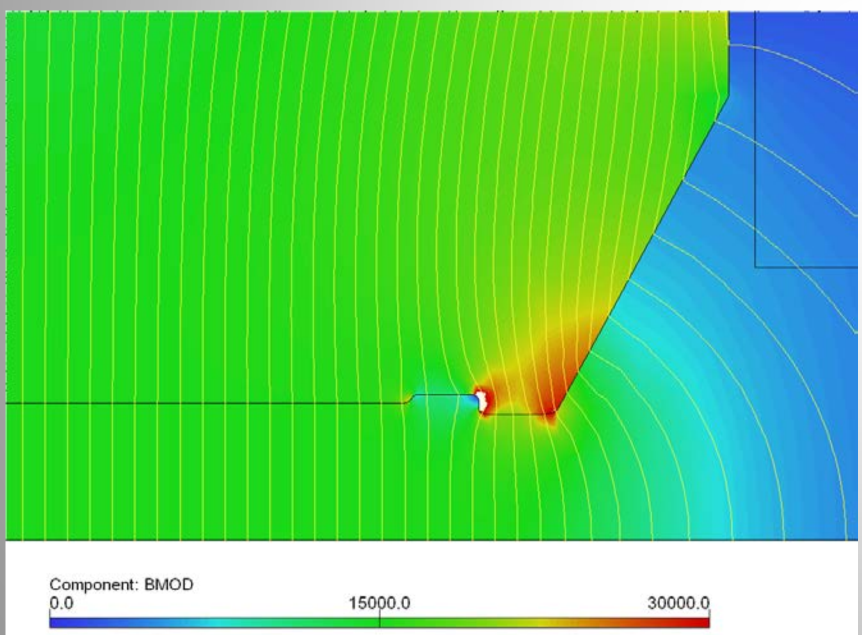


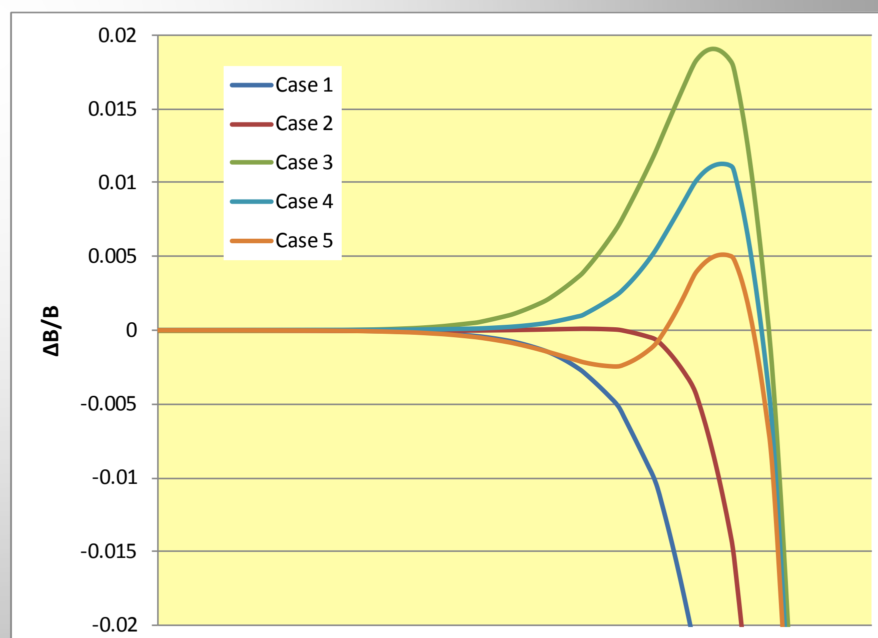
# Pole optimization



'Shimming' (often done by 'try-and-error') can improve the field homogeneity

1. Add material on the pole edges: field will rise and then fall
2. Remove some material: curve will flatten
3. Round off corners: takes away saturation peak on edges
4. Pole tapering: reduces pole root saturation

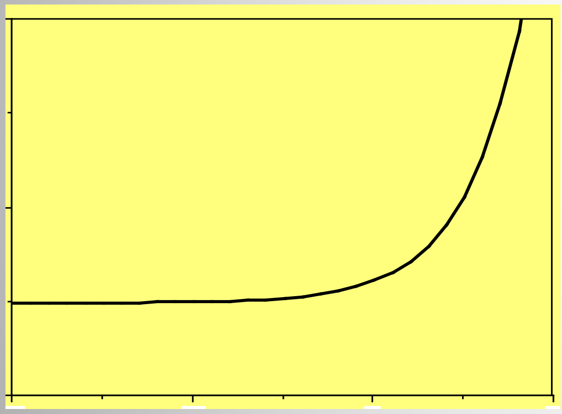


# Rogowsky roll-off

The 'Rogowsky' profile provides the maximum rate of increase in gap with a monotonic decrease in flux density at the surface, i.e. no saturation at the pole edges!

The edge profile is shaped according to:



$$y = \frac{h}{2} + \left(\frac{h}{\pi}\right)\exp\left(\left(\frac{x\pi}{h}\right) - 1\right)$$

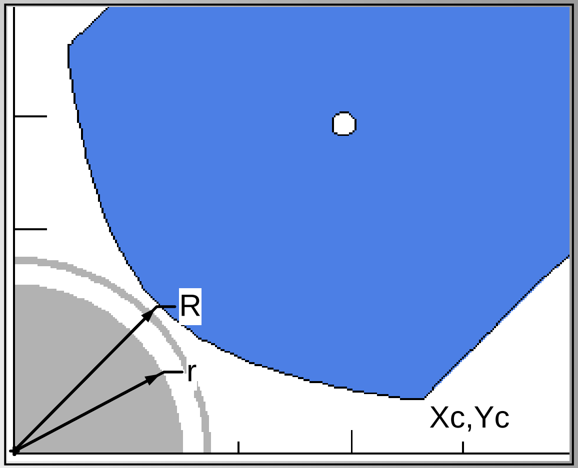
For an optimized pole: $$x_{optimized} = 2\frac{a}{h} = -0.14\ln\frac{\Delta B}{B_0} - 0.25$$

# Pole optimization

Similar technique can be applied for quadrupoles:

$$\frac{x_c}{R} = \sqrt{\frac{1}{2}\left(\sqrt{\left(\rho^2 + x_d\right)^2 + 1} + \rho^2 + x_d\right)}$$

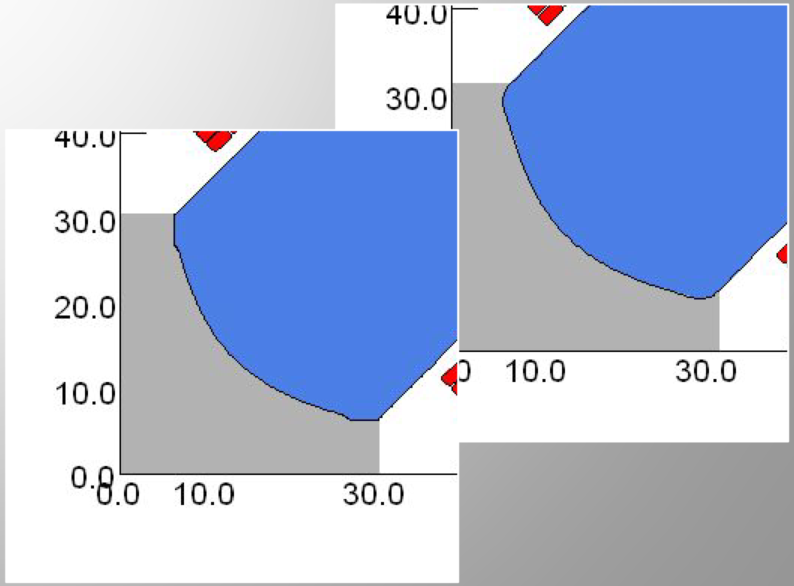
$$\frac{y_c}{R} = \sqrt{\frac{1}{2}\left(\sqrt{\left(\rho^2 + x_d\right)^2 + 1} - \rho^2 - x_d\right)}$$




- $x_c$: un-optimized resp. optimized pole overhang from dipole
- $\rho$: normalized good field radius r/R

Pole optimization:

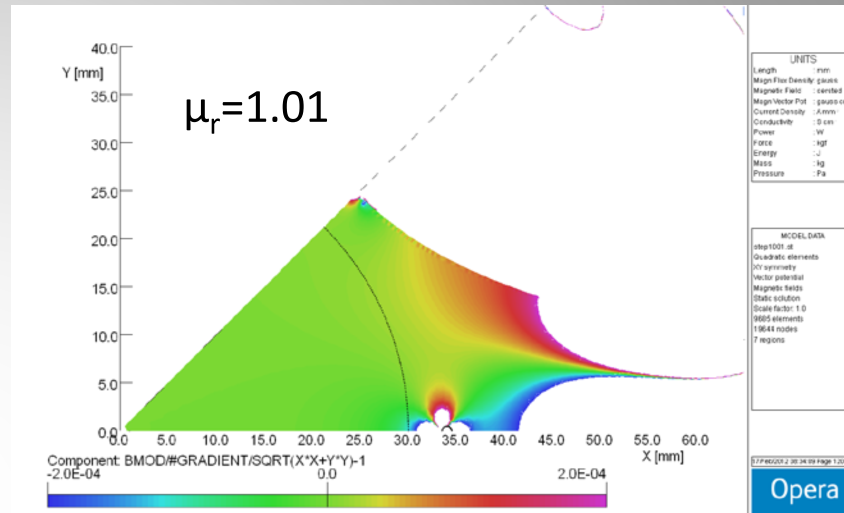
- Tangential extension of the hyperbola
- Additional bump = shim
- Round off sharp edge
- Tapered pole

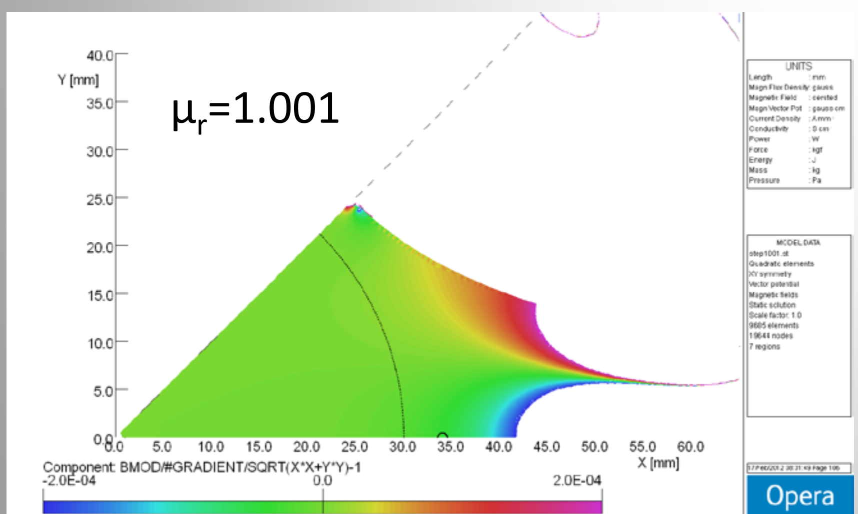


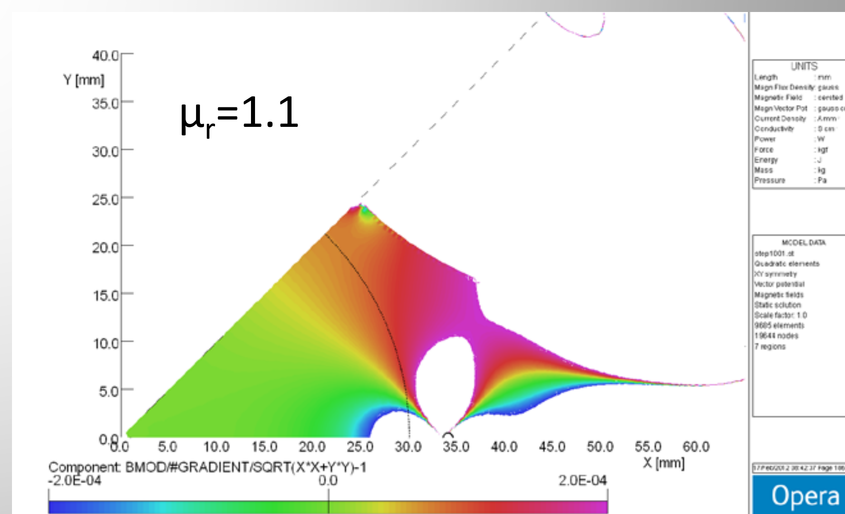
# A material problem...

Welding seam on stainless-steel vacuum chamber:

- GFR radius: 30 mm
- Chamber radius: 35 mm
- Welding seam diameter: 1 mm
- Rel. permeability of 316 LN: < 1.001

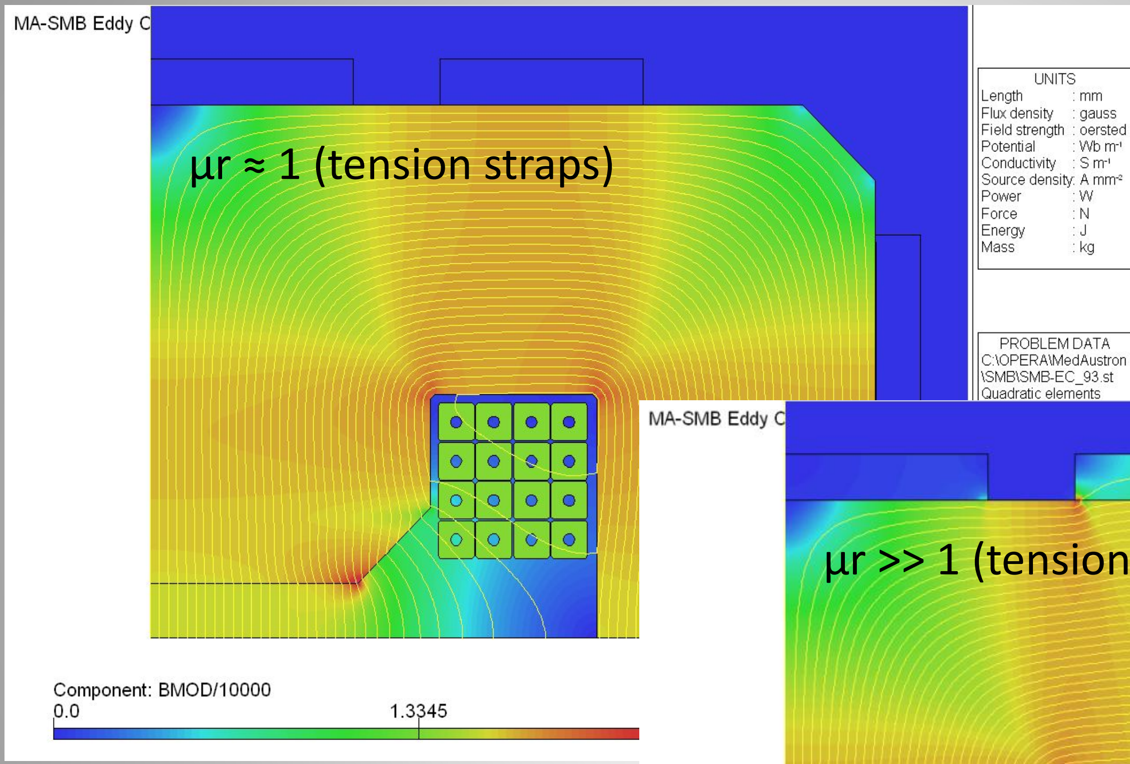


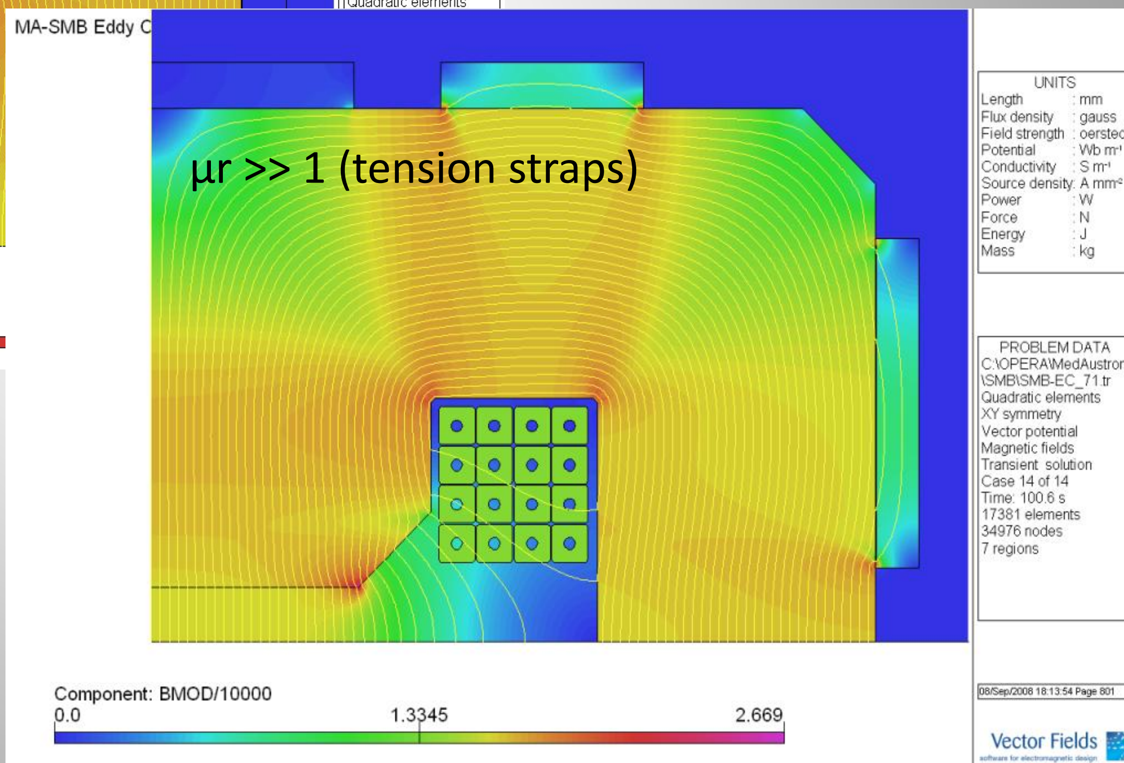



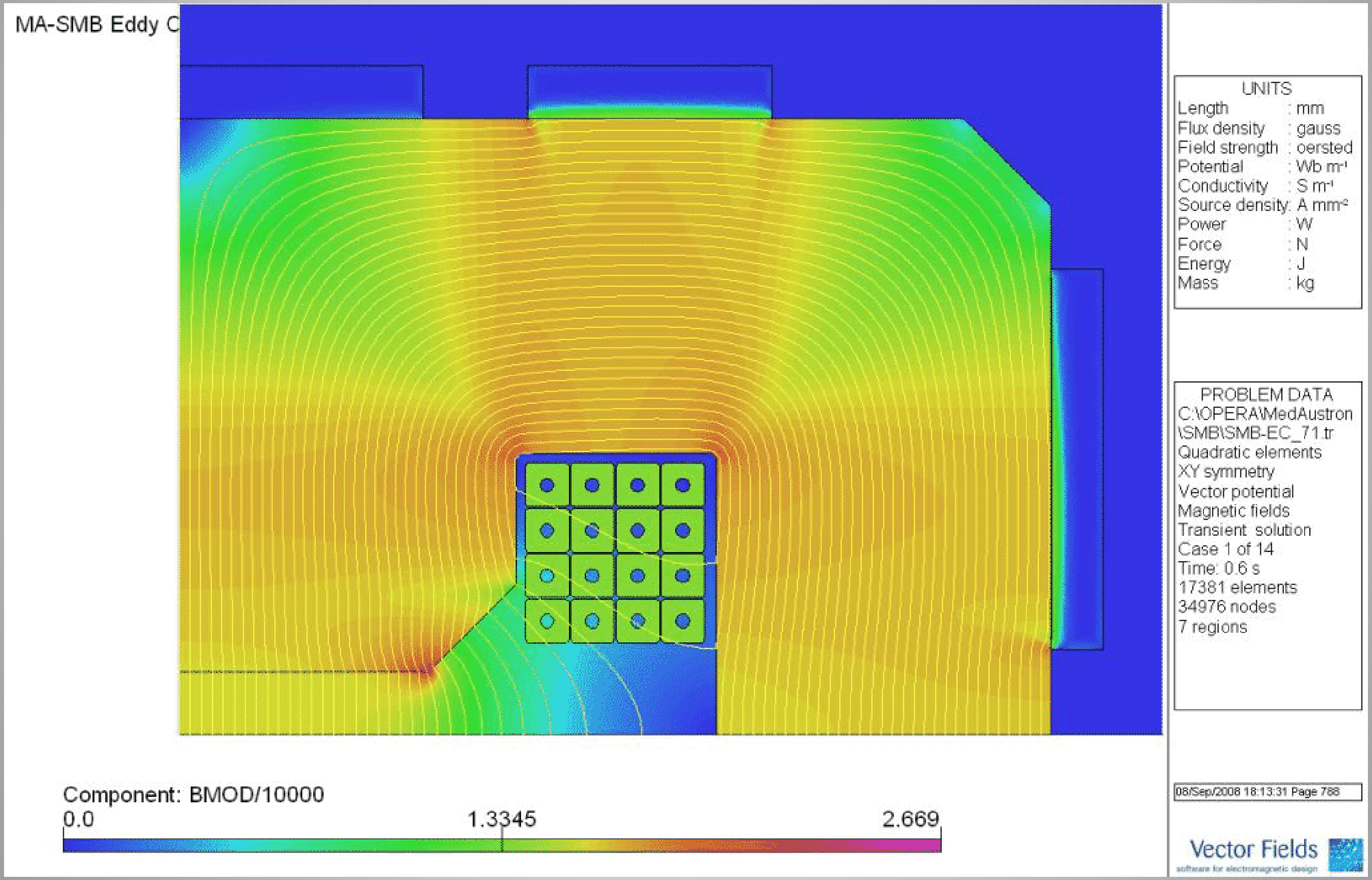

A small distortion in the GFR can significantly influence the field quality!

# Eddy currents - static case

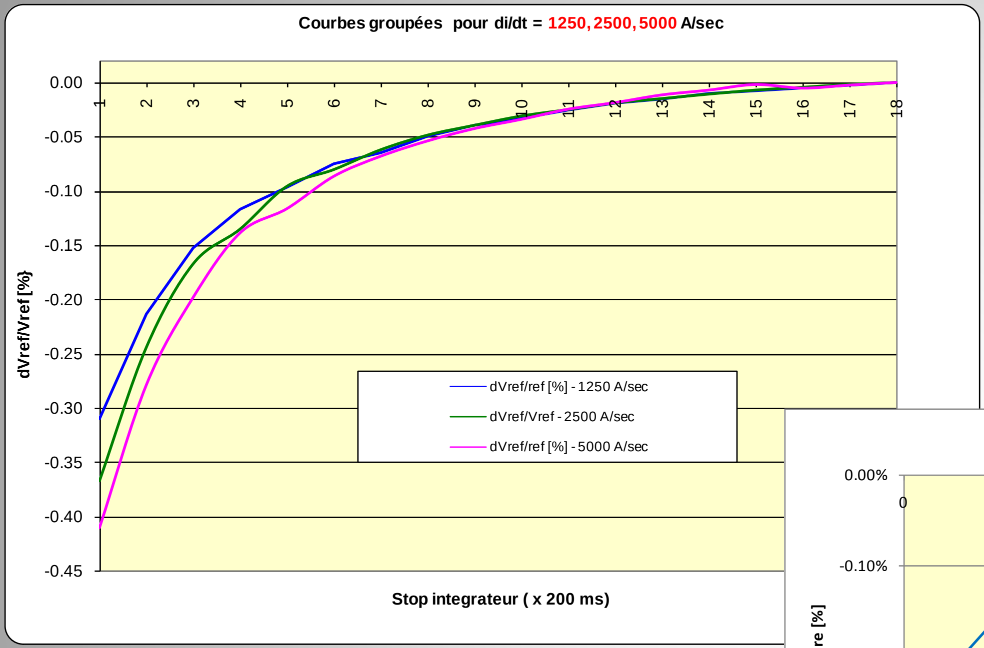


# Eddy currents - dynamic behavior

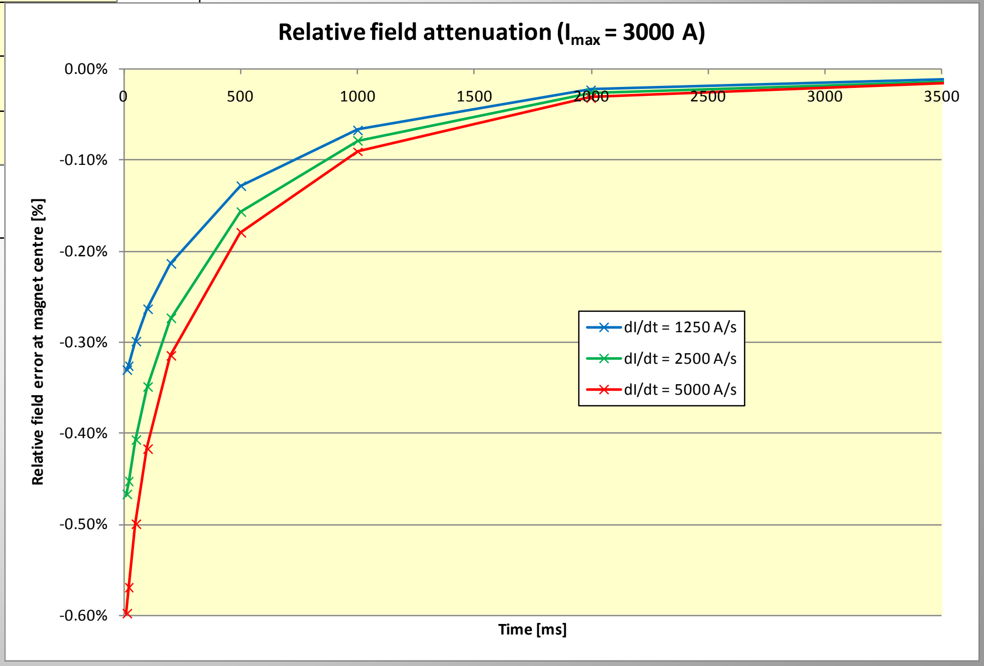


# Eddy currents – field lag




Measured curves (D. Cornuet, R. Chritin)

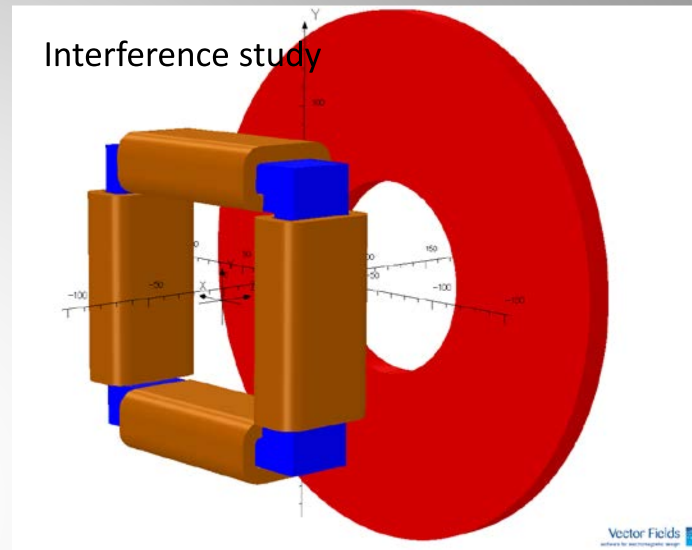
Calculated curves (OPERA 2D Transient)

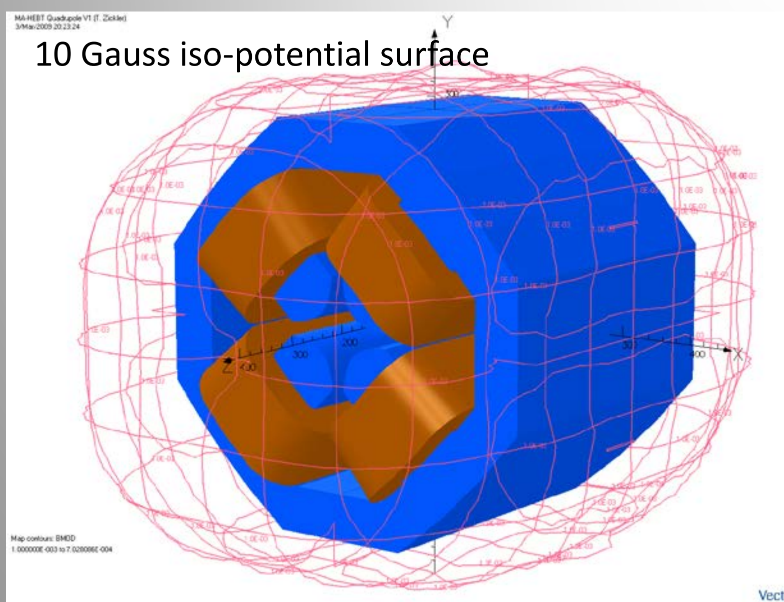


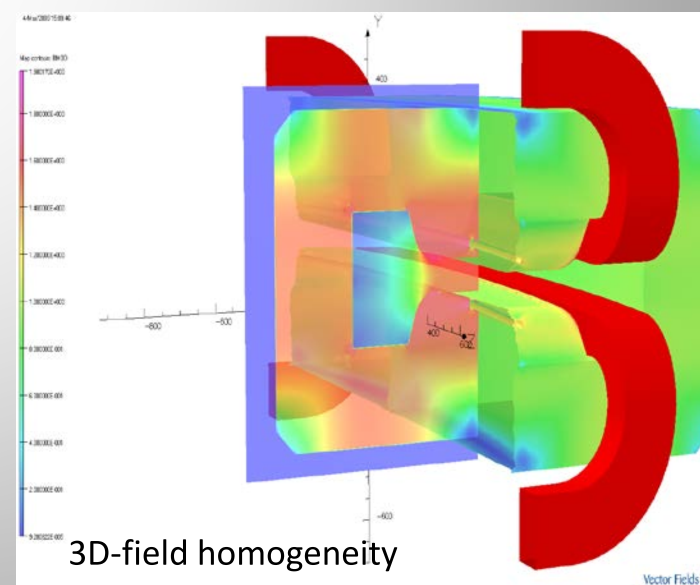
# 3D Design

Becomes necessary to study:

- the longitudinal field distribution
- end effects in the yoke
- end effects from coils
- magnets where the aperture is large compared to the length



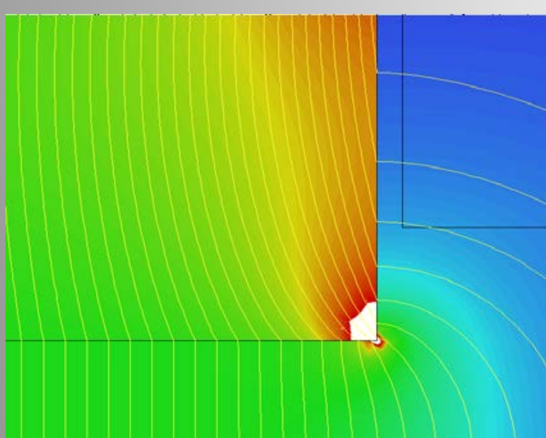
# Magnet ends

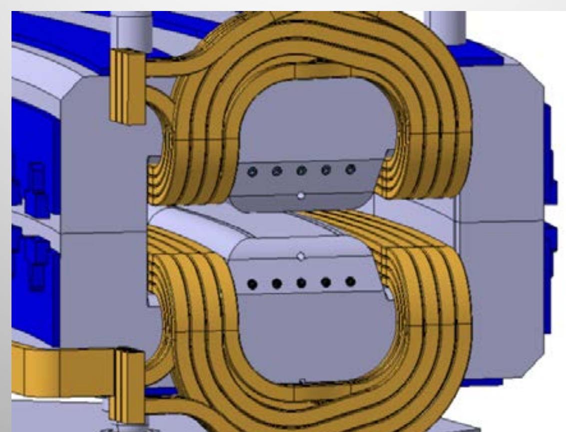
Special attention has to be paid to the magnet ends:

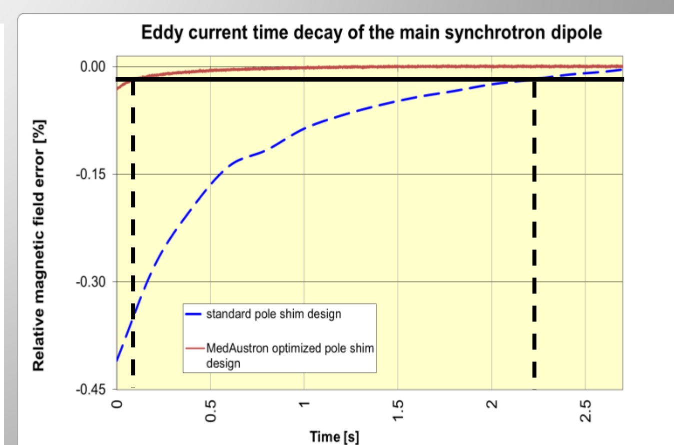
- A square end will introduce significant higher order multi-poles
- Therefore, it is necessary to terminate the magnet in a controlled way by shaping the end either by cutting away or adding material → longitudinal shimming

The goal of successful shimming is to:

- adjust the magnetic length
- prevent saturation in a sharp corner
- maintain magnetic length constant across the good field region
- prevent flux entering perpendicular to the laminations inducing eddy currents







# Summary

- A large varity of FE-codes with different features exist – the right choice depends of the complexity of the problem
- The FE-models shall be as simple as possible and adapted to the problem to reduce computing time
- Numeric computations should be used to quantify, not to qualify
- Benchmarking the results with measurements is a good practice

# Literature

JUAS
Joint Universities Accelerator School


- Fifth General Accelerator Physics Course, CAS proceedings, University of Jyväskylä, Finland, September 1992, CERN Yellow Report 94-01
- International Conference on Magnet Technology, Conference proceedings
- Iron Dominated Electromagnets, J. T. Tanabe, World Scientific Publishing, 2005
- Magnetic Field for Transporting Charged Beams, G. Parzen, BNL publication, 1976
- Magnete, G Schnell, Thiemig Verlag, 1973 (German)
- Field Computation for Accelerator Magnets: Analytical and Numerical Methods for Electromagnetic Design and Optimization, S. Russenschuck, Wiley-VCH, 2010
- CAS proceedings, Magnetic measurements and alignment, Montreux, Switzerland, March 1992, CERN Yellow Report 92-05
- CAS proceedings, Measurement and alignment of accelerator and detector magnets, Anacapri, Italy, April 1997, CERN Yellow Report 98-05
- Physik der Teilchenbeschleuniger und Synchrotronstrahlungsquellen, K. Wille, Teubner Verlag, 1996
- CAS proceedings, Magnets, Bruges, Belgium, June 2009, CERN Yellow Report 2010-004

# Thanks for your attention…