

# Cyclotrons

## Chapter 3

- RF modelisation and computation
- B modelisation and computation
- Beam transport computation

# Modelisation and Computation

Putting dipoles and drift into a transport code is not going to work. We do not know *a priori* where the orbit is for any momentum neither the edge angles or the field index in that region.

The only realistic solution is to get the field map and the equation of motion.

# RF Modelisation and Computation

- Recently, computers became powerful enough to permit 3D electromagnetic field modeling of complex shapes (like Dees!) with large numbers of mesh points
- This way, parasitic cyclotron modes can be numerically confirmed.
- Beam-cavity interactions can also be investigated, the excitation of higher order modes in cavities and vacuum chambers can be analyzed and verified.
- Previously, integrated cavity design (vacuum chamber and cavity as one unit) was rather tedious, because the effects of mechanical forces and thermal effects on the RF geometry were very difficult to predict with reasonable accuracy.

# RF Modelisation and Computation

- Today, mechanical cavity design can be performed using FEA (finite element analysis) simulation.
- The RF geometry data is transferred into a thermal-and structural model (mech. simulation method), taking into account atmospheric pressure and thermal effects.

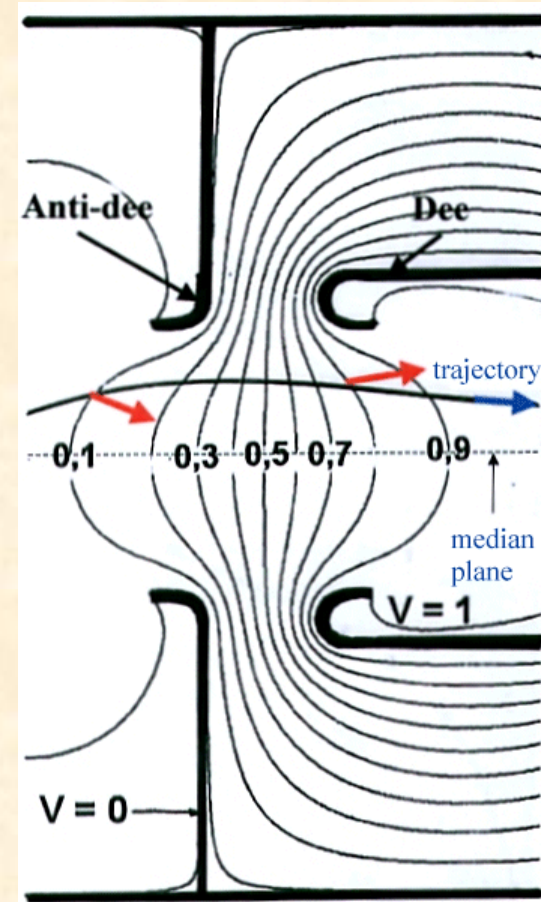
This procedure yields a **deformed RF geometry, which represents the operating geometry**

# Accelerating gaps

The transport of the particle through the accelerating gaps depends on its vertical Z-position. One has to take into account the real equipotential distribution.

**Especially in the central region when the energy is low.**

- The gap length has an equivalent length
- The transit time factor varies as a function of  $z$
- The vertical beam focusing is affected as well.



# Central region modelisation

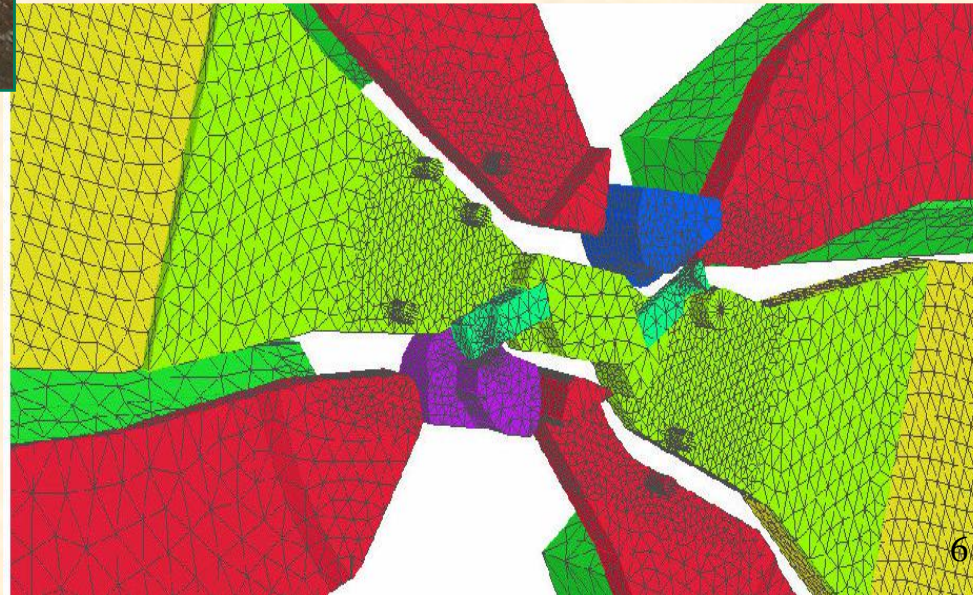


4 poles

2 dees (4 gaps)

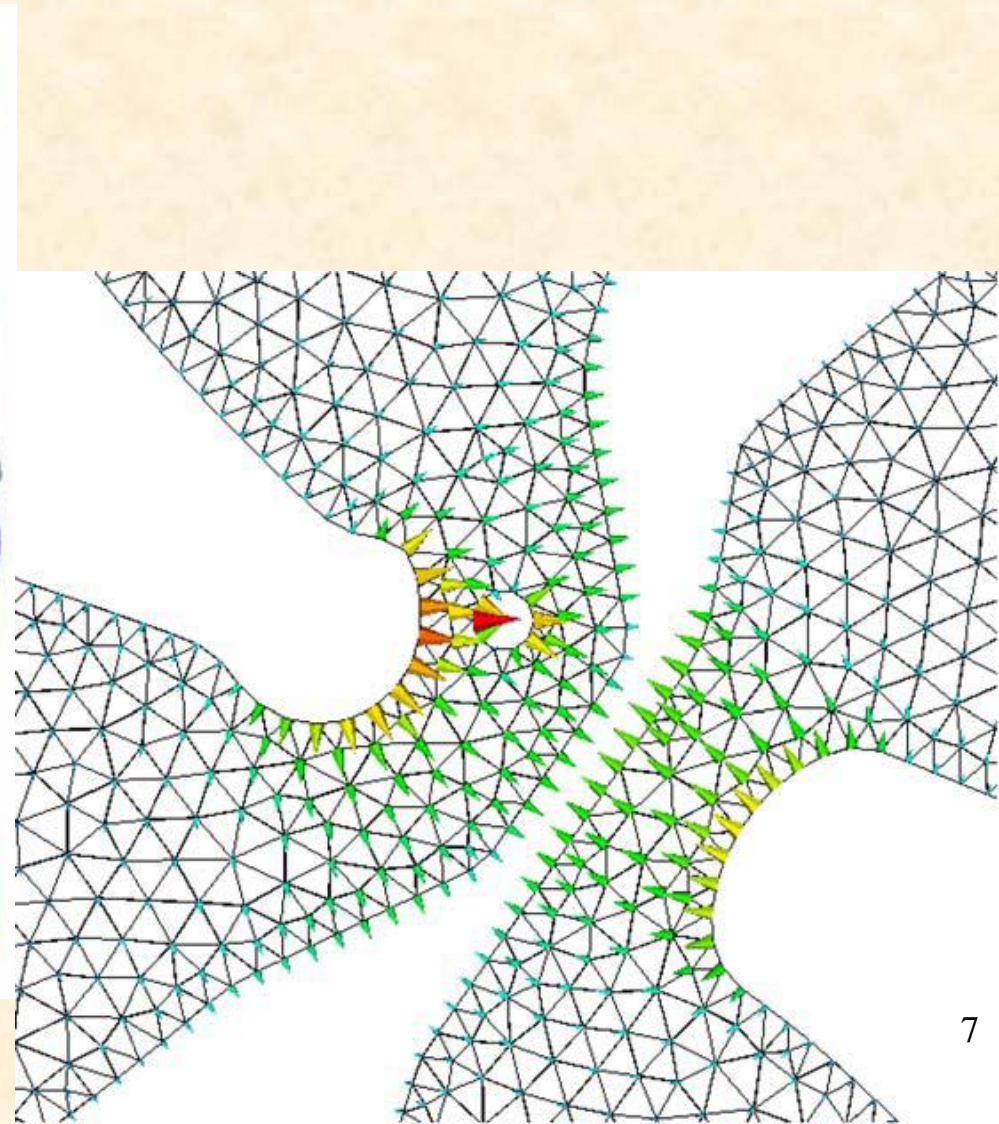
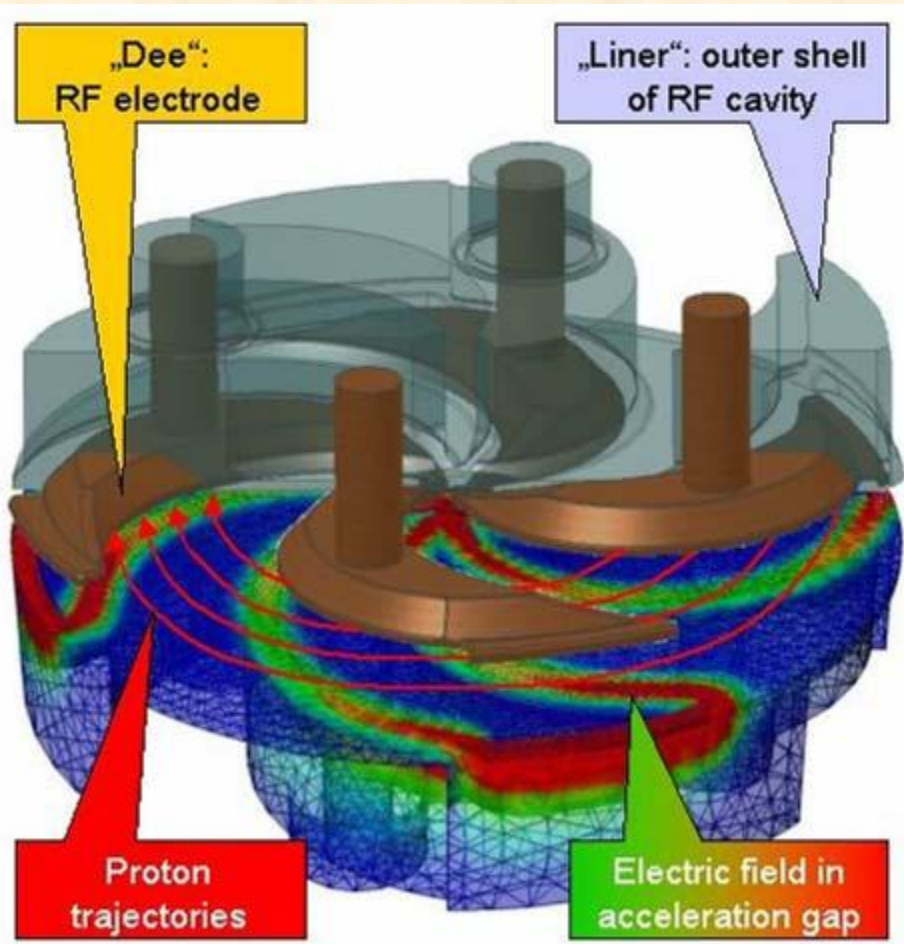
2 ion sources (H- and D-)

Central plug to adjust field  
in the center



# RF Modelisation and Computation

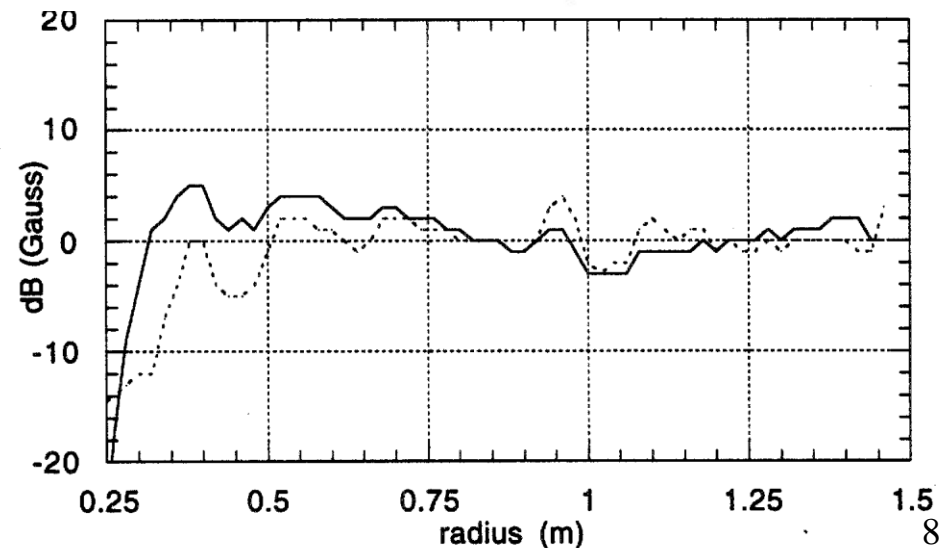
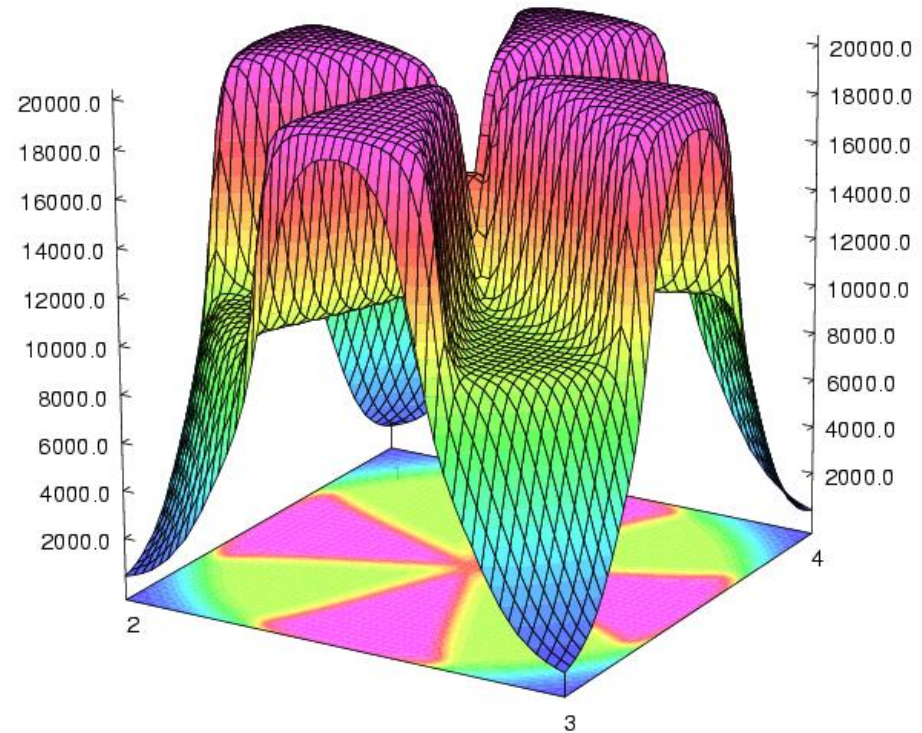
The PSI 4 Sector 250 MeV-P-Medical Cyclotron built by ACCEL GmbH



P. Sigg: RF for Cyclotrons Tutorial at CAS 05

# Field map Modelisation and Computation

- The use of codes such as TOSCA allows the determination of a magnet field map in 3D finite elements.
- The computation figures are remarkably close to the measurements.
- The transport of particles through the 3D field map will predict the behaviour of the beam during the acceleration.
- One can rely on modelisation even for large machine.
- Magnetic field measurements not needed

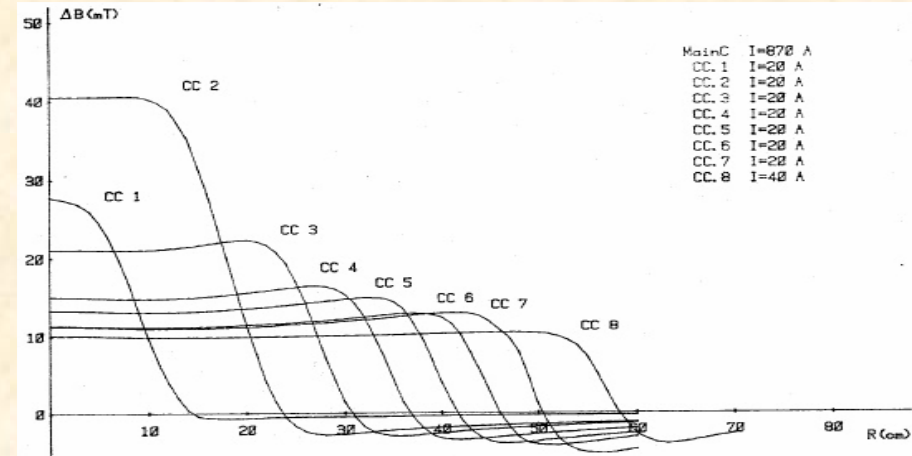
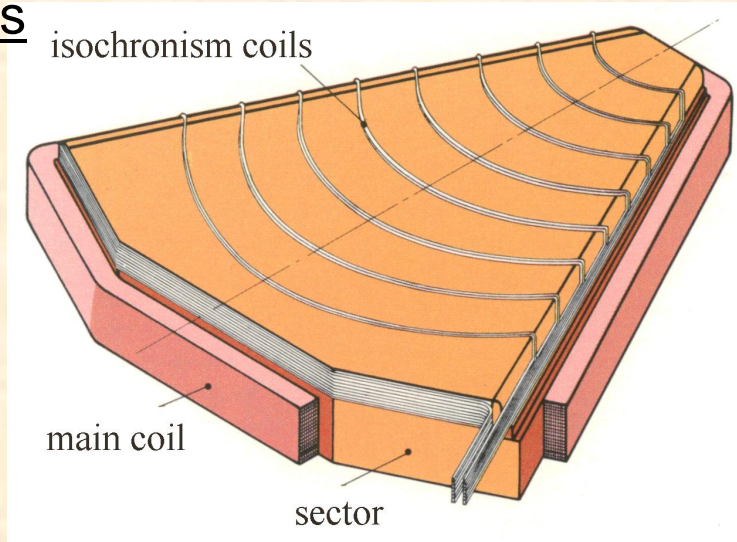




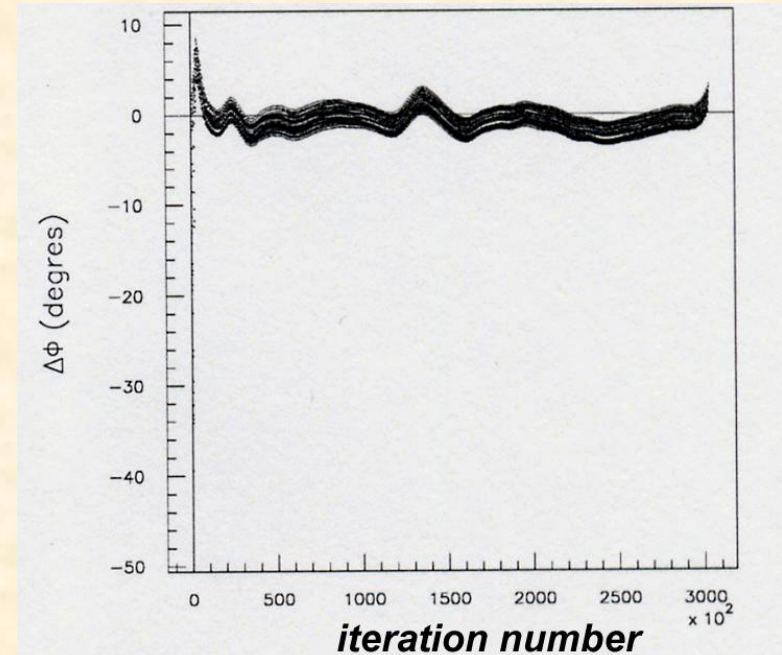
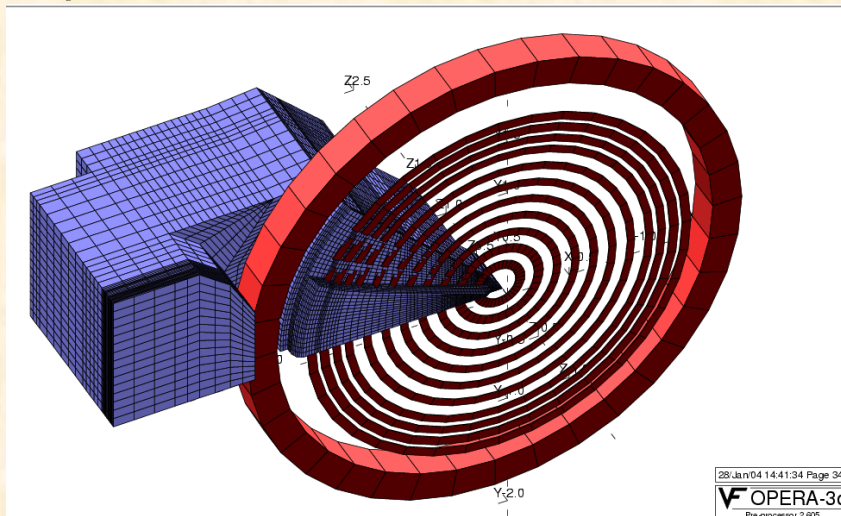
# Field map Modelisation and Computation

## Isochronism $B(r) = \gamma(r)B_0$

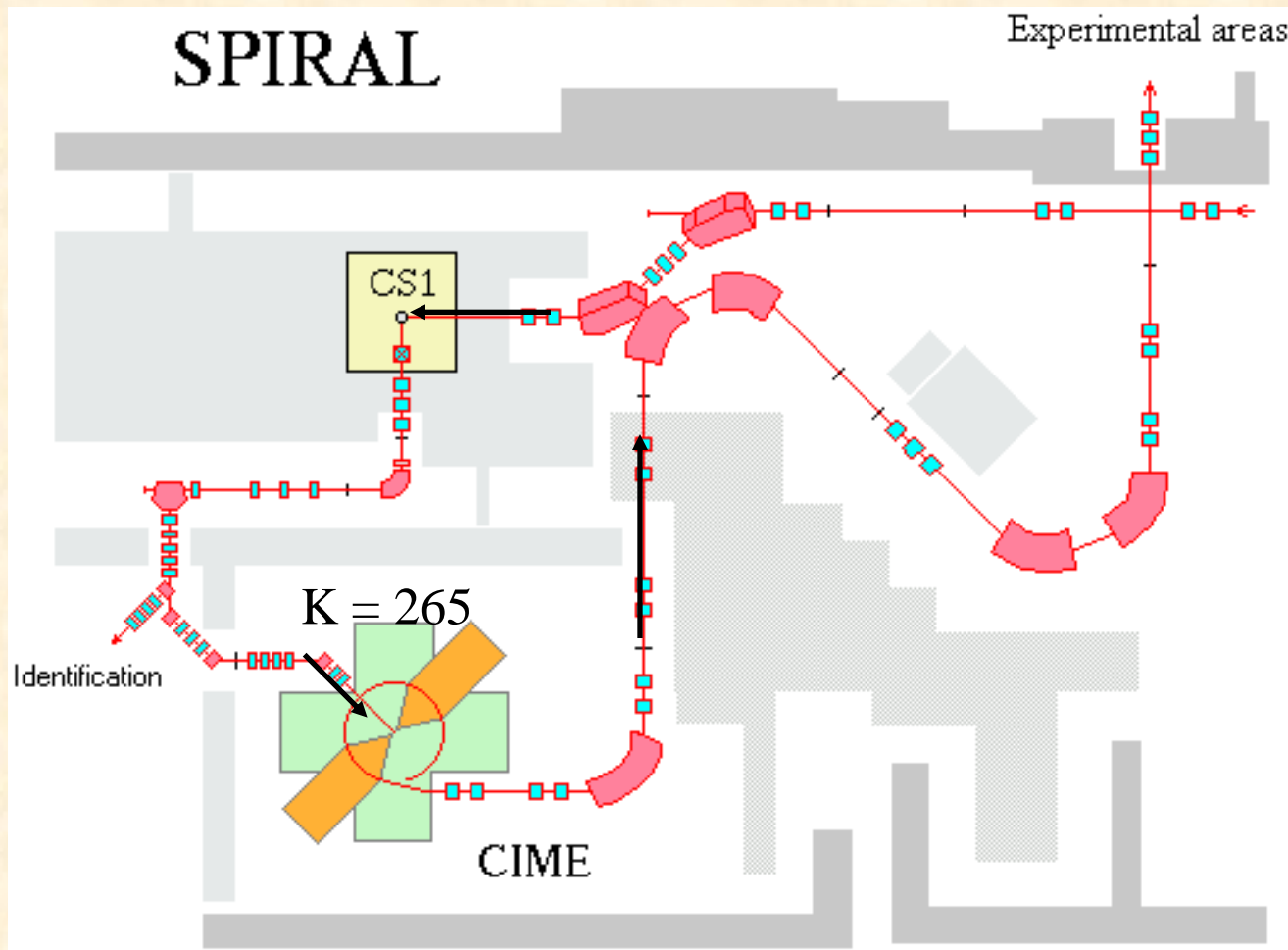
### Sectors



### Compact



# Beam transport SPIRAL cyclotron example



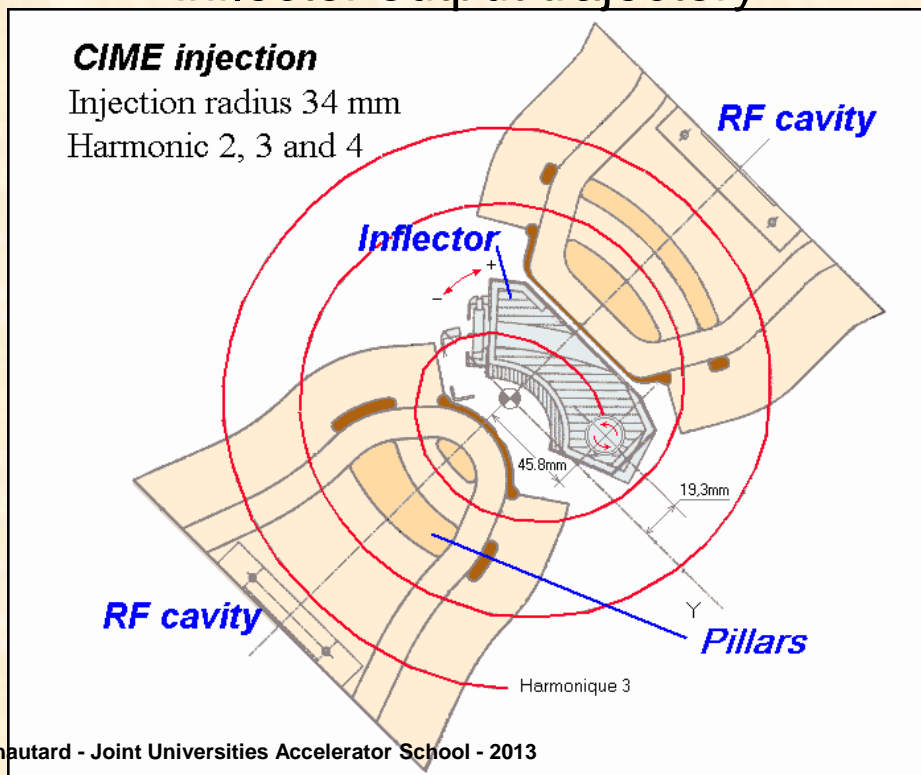
# SPIRAL cyclotron example

## Cyclotron modelisation

- Magnetic configuration: Computed field maps (Tosca ...) or measured field maps at various field level (10 field levels)
- RF cavity field models (for 6 harmonics)
- Multiparticle computation codes
  - ⇒ find a tuning for the whole working diagram

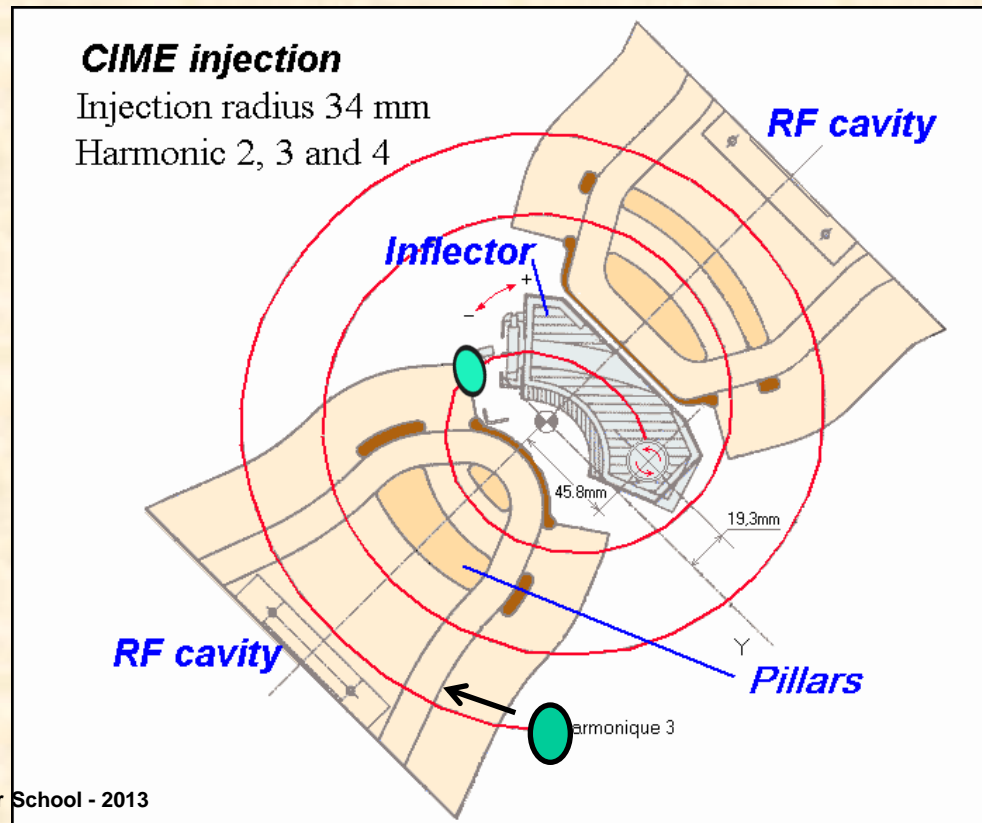
# Trajectories and matching recipes (1/4)

- Find a central trajectory (1 particle)
  - For a isochronous field level and a given frequency
    - ⇒ Start from a closed orbit at large radius (no RF field)
    - ⇒ Then turn on RF field to decelerate the central particle to the injection.
    - ⇒ Tune the RF and the magnetic field at the injection to join the inflector output trajectory.



# Trajectories and matching recipes (2/4)

- Find a central trajectory (1 particle)
- Find a matched beam in the cyclotron (**multiparticles**)
  - ⇒ Start with **a matched beam** at large radius around the central trajectory (6D matching)
  - ⇒ Again **in backward** tracking through the field maps determine the 6D phase-space at the injection



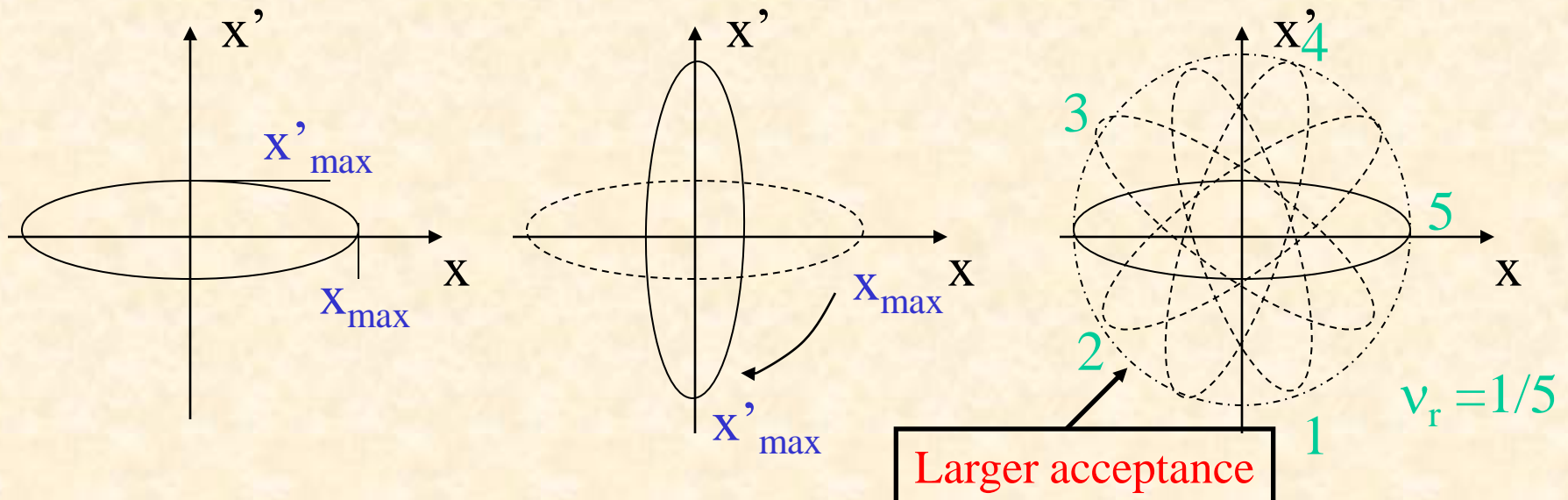
# Mismatched beam recall

We define a closed orbit  $\Rightarrow$  without acceleration

$$\begin{cases} x(t) = x_{\max} \cos(v_r \omega_0 t) \\ x'(t) = x'_{\max} \sin(v_r \omega_0 t) \end{cases}$$

Emittance area :  $\varepsilon = \pi x_{\max} \cdot x'_{\max}$  (and  $\varepsilon = \pi z_{\max} \cdot z'_{\max}$ )

Betatron oscillation with mismatched beam



# Matched beam recall

$$\begin{cases} x(t) = x_{\max} \cos(\nu_r \omega_0 t) \\ x'(t) = dx/ds = dx/R \omega_0 dt = -(x_{\max} \nu_r / R) \sin(\nu_r \omega_0 t) \end{cases}$$

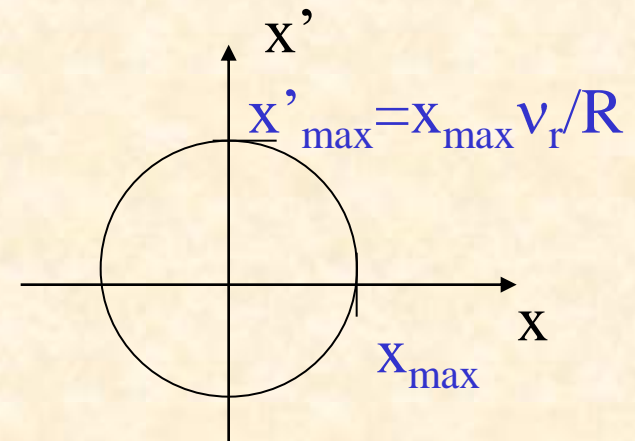
$$|x'_{\max}| = |x_{\max} \nu_r / R| \text{ and } \varepsilon = \pi x_{\max} \cdot x'_{\max} = \pi \cdot x_0^2 \nu_r / R$$

⇒ Initial beam conditions depend of the tune ( $\nu_r$ ) of the cyclotron at the matching point.

⇒ Betatron oscillation disappears

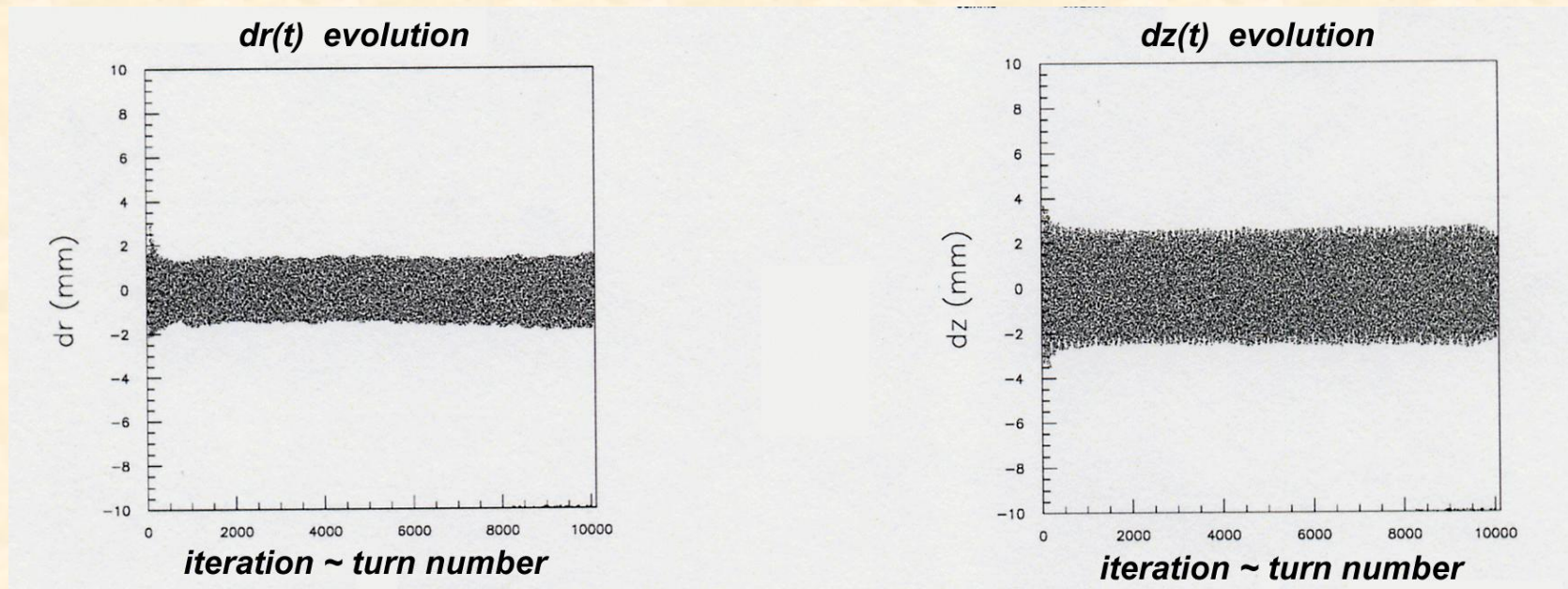
⇒ Matched beam

⇒ Minimum of acceptance



# Beam matching

A matched beam, remains matched as long as  $v_r$  and  $v_z$  change slowly under acceleration. Under acceleration and taking into account relativistic mass increase, the normalized emittances  $\varepsilon_x$  and  $\varepsilon_z$  remain constant



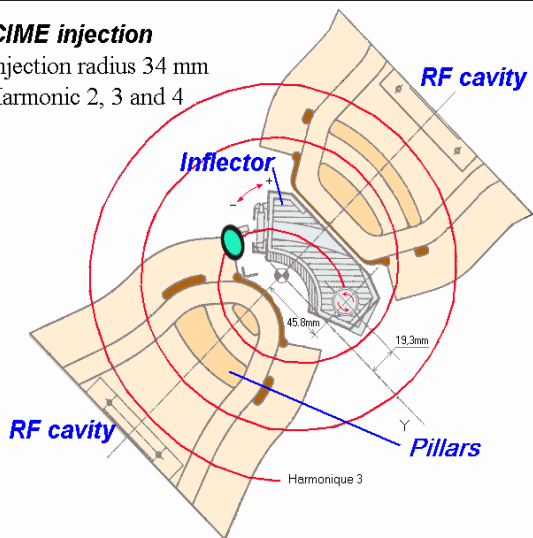
Geometric beam size



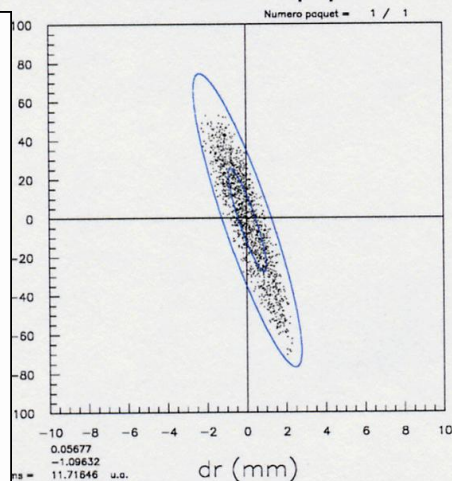
# Final backward 6D matching @ injection

## CIME injection

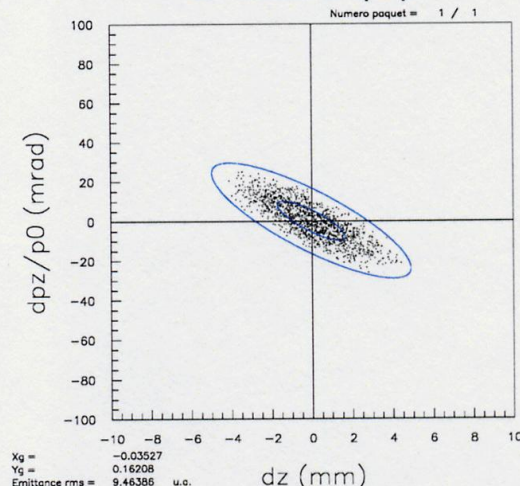
Injection radius 34 mm  
Harmonic 2, 3 and 4



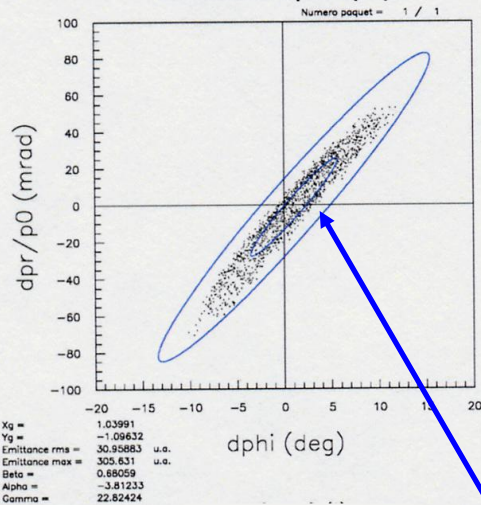
correlation  $r\text{-}dpr/p_0$



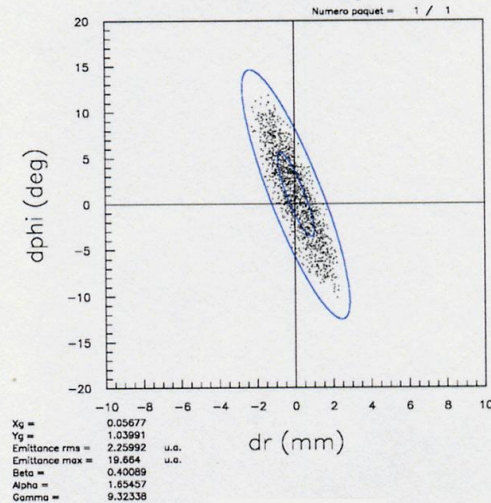
correlation  $z\text{-}dpz/p_0$



correlation  $dphi\text{-}dpr/p_0$



correlation  $r\text{-}dphi$



Not well represented by a gaussian beam  $\Rightarrow$  mismatch in forward

# Trajectories and matching recipes (3-4/4)

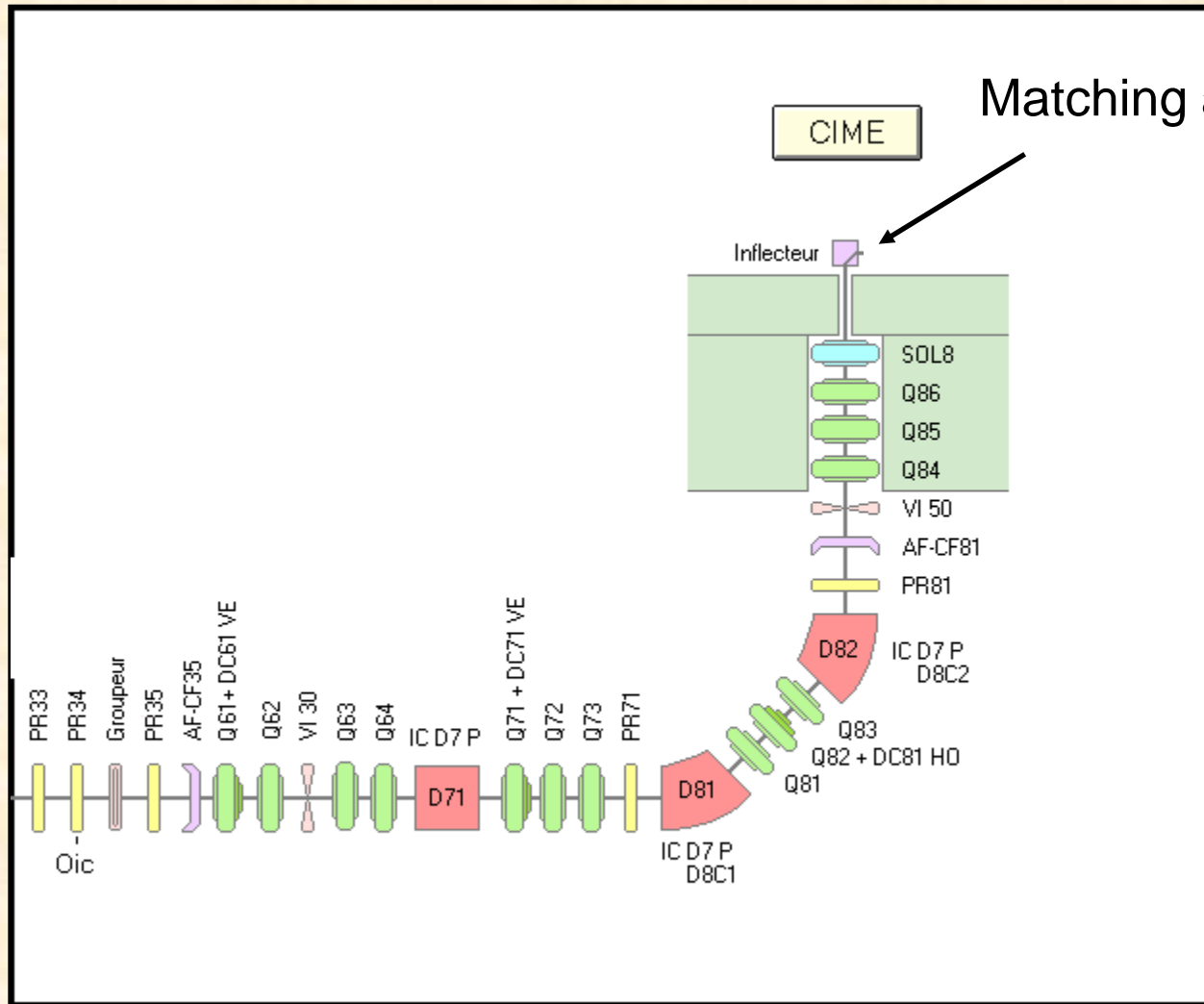
- Find a central trajectory (1 particle)
- Find a matched beam in the cyclotron (multiparticles)
- Forward tracking (**multiparticles**)
  - ⇒ confirm the matching to the extraction
  - ⇒ tune the isochronism
  - ⇒ and if the matching at the injection is not feasible by the injection line predict the new beam envelope and extraction
- Extraction (**multiparticles**)

# Trajectories and matching recipes

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**Iterative process**

# Backward 6D matching



Classical transport line problems