Exercises on Wake Fields and Instabilities

Exercise 1:

Show that the impedance of an RLC parallel circuit is that of a resonant mode and relate R, L and C to Q, R_s *and* ω_r

Exercise 2:

Calculate the amplitude of the resonator wake field given $R_s = 1\ k\Omega$, $\omega_r = 5\ GHz$, $Q = 10^4$

Calculate the ratio $|Z(\omega_r)| / |Z(2\omega_r)|$ *for* $Q = 1, 10^3, 10^5$

Exercise 3: Beam Break Up

Consider a beam in a linac at 1 GeV without acceleration. Obtain the growth of the oscillation amplitude after 3 km if:

N = 5e10, $w_{\perp}$(-1 mm) = 63 V/(pC m), L_w = 3.5 cm, k_y = 0.06 1/m

Exercise 4: Beam Break Up (2)

Consider the same beam of the previous exercise being now accelerated from 1 GeV with a gradient g =16.7 MeV/m. Obtain the growth of the oscillation amplitude

$$E_f = E_0 + gL_L \approx gL_L = 50\,\text{GeV}$$

$$\left(\frac{\Delta\hat{y}_2}{\hat{y}_2}\right)_{\max} = \frac{cNew_\perp(z)L_L}{4\omega_y(E_f/e)L_w}\ln\frac{E_f}{E_0} = ?$$

Exercise 5: Haissinski equation with pure inductive impedance

Given the wake field in case of a pure inductive impedance, determine the longitudinal distribution

$$w_{\parallel}(z) = -c^2 L \delta'(z) \quad \Longrightarrow \quad \Psi(z) = ?$$

Exercise 6: Microwave instability threshold

Calculate the threshold average current of the microwave instability for an accelerator having the following parameters:

$|Z_{\|} / n| = .5\ \Omega$, $\sigma_z = 1$ cm, $\sigma_\varepsilon = 10^{-3}$, $\alpha_c = 0.027$,
$E_0 = 510$ MeV, $L_0 = 97.69$ m