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Guided study and tutorial on Cyclotrons

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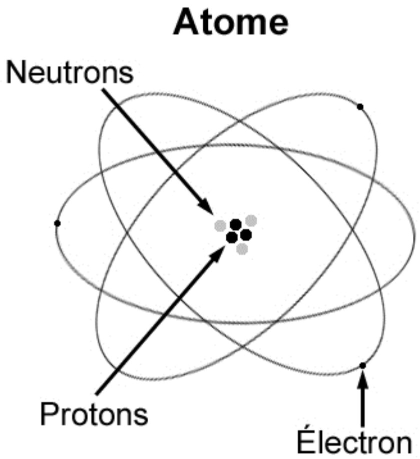
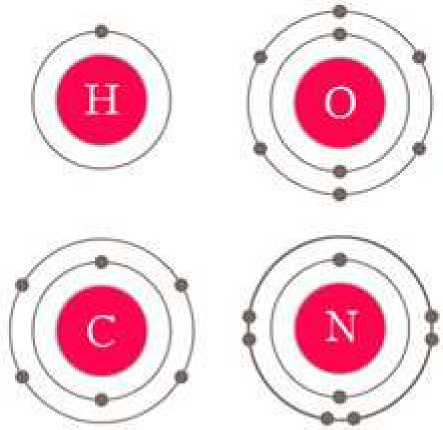




Recall :
Number of proton = Z
Number of mass = protons + neutrons = A
Number of electron=Q

Useful expressions

CONSTANTS

c = 2,997925x10^8 m/s
1 u.m.a. = 931,478 MeV= 1,67.10^{-27} kg
e = 1,602x10^{-19} C

W in MeV/nucleon, R en mètres et F_{RF} en MHz

1. $\beta = \dfrac{\sqrt{W(W+1862,956)}}{W+931,478} = 2,0958.10^{-2}\dfrac{F_{RF}R}{h}$

2. $\gamma = \dfrac{931,478+W}{931,478}$

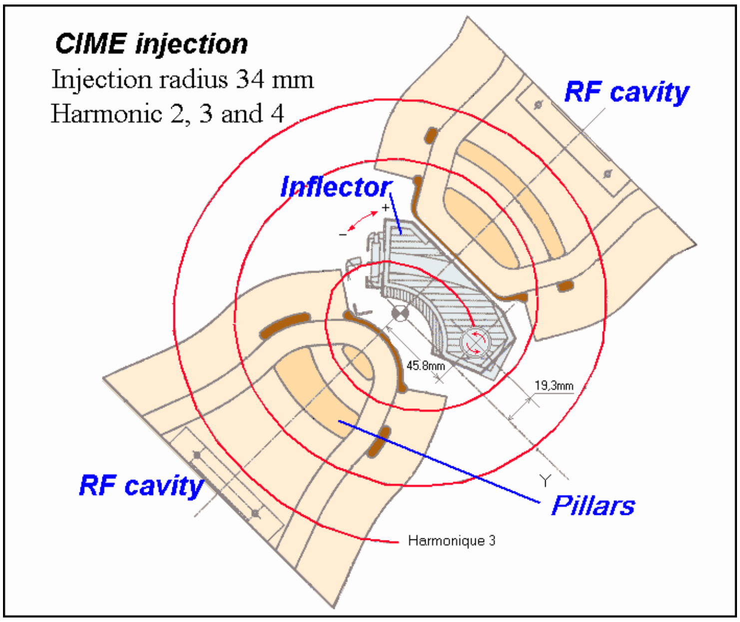
3. B = 6.5121.10^{-2} F_{rev} (A/Q)

4. $W = \sqrt{931.478^2 + \left(\dfrac{299,7925\ BRQ}{A}\right)^2} - 931,478 = 96,488\left(\dfrac{QBR}{A}\right)^2 \dfrac{1}{\gamma+1}$

PROBLEM 1

In a variable energy isochronous cyclotron with an axial injection system and a fixed geometry of the central region (posts, pillars, etc…) all beams must follow the same path at least on the first turns.

1. It is also convenient to carry out this condition up to extraction, so that the position of the electrostatic deflector remains constant.



If r_0 is the first and unique radius of curvature of the orbit at the exit of the inflector, find a relation linking the mean magnetic field B_0, the RF frequency F_{RF}, h the harmonic number and the ion source plateform voltage V_{inj}: $\frac{1}{\pi \cdot r^2{}_0} = \frac{F_{RF} B_0}{h V_{inj}}$

2. if $R_{injection} = 0.04$ m, $R_{extraction} = 2$ m and $W_{extraction} = 10$ MeV/A
   a. Find the revolution frequency
   b. Express the formula of the magnetic field at the injection
   c. For an ion of carbon 12C6+ (A=12 and Q=6) find the voltage, V_{inj}, to inject the ion on the good radius

PROBLEM 2

From the lecture, it has been stated that to reach high energy with cyclotron, the magnetic field requires satisfying the relation below:

$$B_z(r) = \gamma(r) B_z(0)$$

1. What sign should be the field index $n = -\frac{r}{B(r)}\frac{dB(r)}{dr}$ in order to satisfy the isochronism condition to reach high energies:

PROBLEM 3

Stripping Problem

To extract protons out of a cyclotron it is easier to accelerate H^- (constituted by a proton and 2 electrons) and in the cyclotron to insert a thin carbon foil to strip the ion the last two electrons.

1. Draw the particle trajectory before and after stripping
2. What is the minimum energy of the proton beam as a function of R_{max} and W_{max} that can be produced?

PROBLEM 4

The kinetic energy W (in MeV/nucleon) reached by an ion of mass A and charge Q accelerated to a mean radius R in a mean magnetic field $B(r) = \gamma(r)B_0$ can be expressed by:

$$W[\text{MeV/A}] = 96{,}488\left(\frac{QBR}{A}\right)^2 \frac{1}{\gamma+1}$$

1. Show that this energy W can also be simply expressed in terms of γ, R, the RF frequency FRF and the harmonic number h. The parameters B, Q and A should not appear in the formula.
2. Check that the two formulae give the same numerical result in the following case:

A=12, Q=6, R=1 m, B0=1.5 Tesla, Frf=34.56 MHz and H=3