Solutions of the problems on linear imperfections

1. The rms orbit distortion is given by:

$$u_{rms}(s) = \frac{\sqrt{\beta_u(s)\,\beta_u^{rms}}\,\sqrt{N}}{2\sqrt{2}\,|\sin(\pi Q_u)|}\,\Theta^u_{rms} \quad (1)$$

The kick for a quadrupole displacement is:

$$\Theta^u_{rms} = \frac{G}{B\rho}\cdot \ell \cdot (\delta u)_{rms} \quad (2)$$

The magnetic rigidity is

$$B\rho\,[\mathrm{T.m}] = \frac{1}{0.2998}\,\beta E\,[\mathrm{GeV}] \quad (3)$$

The total energy is $E = T + E_0 = \underline{\underline{1.938\ \mathrm{GeV}}}$ (4)

The relativistic γ is $\gamma = \frac{E}{E_0} = 2.07$ (5)

The relativistic β is

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.875 \quad (6)$$

$$(3) \xrightarrow{(4),(6)} B\rho = \underline{\underline{5.657\ \mathrm{Tm}}} \quad (7)$$

$$(2) \xrightarrow{(7)} \Theta^u_{rms} = \frac{5}{5.657}\cdot 1 \cdot 10^{-3}\ \mathrm{rad} = \underline{8.8\cdot 10^{-4}\ \mathrm{rad}} \quad (8)$$

$$(1) \xrightarrow{(8)} \left\{ x_{rms} = \frac{\sqrt{12.8}\cdot\sqrt{18}}{2\sqrt{2}\,|\sin(\pi\, 6.23)|}\cdot 8.8\cdot 10^{-4}\ \mathrm{m} = \underline{\underline{13.6\ \mathrm{mm}}} \right.$$

(1) $\overset{(2)}{\rightsquigarrow}$ $$y_{rms} = \frac{\sqrt{2\cdot 8}\ \sqrt{18}}{2\sqrt{2}\ |\sin(\pi \cdot 6.20)|} \cdot 8.8\cdot 10^{-4}\,m$$

$$= \underline{\underline{9\ mm}}$$

For $Q_x = 6.1 \overset{(1)}{\rightarrow}$ $x_{rms} = \underline{\underline{41.9\ mm}}$

For $Q_x = 6.01 \overset{(1)}{\rightsquigarrow}$ $x_{rms} = \underline{\underline{0.41\ m}}$

The vertical rms orbit distortion remains unchanged.

2. The equations for the 3-bump are:

$$\frac{\Theta_1 \sqrt{\beta_1}}{\sin \psi_{23}} = \frac{\Theta_2 \sqrt{\beta_2}}{\sin \psi_{31}} = \frac{\Theta_3 \sqrt{\beta_3}}{\sin \psi_{12}}$$

$$\psi_{12} = \psi_{23} = \pi/4$$

$$\psi_{13} = \psi_{12} + \psi_{23} = \pi/2 \rightsquigarrow \underline{\underline{\psi_{31} = -\pi/2}}$$

So $\Theta_1 = \Theta_3$ and $\underline{\underline{\Theta_2 = -\Theta_1 \sqrt{12}}}$

3. The magnetic rigidity is

$$\left.\begin{array}{l} B\rho = \dfrac{1}{0.2998}\,\beta E \quad (1) \\ \beta \simeq 1 \end{array}\right\} B\rho = 1334\ \mathrm{Tm} \quad (2)$$

The focusing normalized gradient is

$$k_F = \frac{G_F}{B\rho} \overset{(2)}{=} \frac{19.4}{1334}\ m^{-2} = \underline{\underline{0.015\ m^{-2}}} \quad (3)$$

The defocusing is just the opposite sign.

$$k_D = -0.015\ m^{-2} \quad (4).$$

$$\delta Q_u = \frac{1}{4\pi} \sum_i B_u^i\, k_i \left(\frac{\delta k}{k}\right)_i \ell_i \overset{\text{Split F/D quads.}}{=}$$

$$= \frac{1}{4\pi}\left(N_F\, B_u^F\, K_F \left(\frac{\delta K}{K}\right)_F \cdot \ell_F + N_D\, B_u^D\, K_D \left(\frac{\delta K}{K}\right)_D \cdot \ell_D\right) \quad (5)$$

As $N_F = N_D = N$, $\ell_F = \ell_D = \ell$ and $K_F = -K_D = K$

we have from (5)

$$\delta Q_{x,y} = \frac{1}{4\pi} N \cdot \ell \cdot K \left[\pm B_{x,y}^F \left(\frac{\delta K}{K}\right)_F \mp B_{x,y}^D \left(\frac{\delta K}{K}\right)_D\right] \quad (6)$$

From (6):

$$\delta Q_x = \frac{108 \cdot 3.22 \cdot 0.015}{4\pi}(108 \cdot 0.01 - 18 \cdot 0.005)$$

$$= \underline{\underline{0.398}}$$

$$\delta Q_y = \frac{108 \cdot 3.22 \cdot 0.015}{4\pi}(-18 \cdot 0.01 + 108 \cdot 0.005)$$

$$= \underline{\underline{0.145}}$$

The chromaticity of the machine is

$$\xi_{x,y} = -\frac{1}{4\pi} \sum_i B^i_{x,y} K^i_{x,y} l_i$$ and splitting

again F/D quads' contribution, we have:

$$\xi_{x,y} = -\frac{1}{4\pi} N \cdot l \cdot k \left(\pm B^F_{x,y} \mp B^D_{x,y}\right) \rightarrow$$

$$\xi_{x,y} = -\frac{108 \cdot 3.22 \cdot 0.015}{4\pi}(108 - 18) =$$

$= -36.2$ for both planes!

4. The vertical field for a sextupole is:

$$B_y = \frac{S}{2} x^2$$ and considering that

x is replaced by $x + \delta x$ we have:

$$B_y = \frac{S}{2}\left[\underbrace{x^2}_{\text{sextupole}} + \underbrace{2\delta x\, x}_{\text{Quadrupole}} + \underbrace{(\delta x)^2}_{\text{dipole}}\right]$$

For an octupole:

$$B_y = \frac{O}{3} x^3$$ and considering $x \longrightarrow x + \delta x$

$$B_y = \frac{O}{3}\left[\underbrace{x^3}_{\text{Octupole}} + \underbrace{3\delta x\, x^2}_{\text{Sextupole}} + \underbrace{3(\delta x)^2 x}_{\text{Quadrupole}} + \underbrace{(\delta x)^3}_{\text{Dipole}}\right]$$

The vertical field of a $2n$-pole is written as:

$$B_y = \frac{b_n}{n-1} x^{n-1}$$. Letting $x \rightarrow x + \delta_x$

we have

$$B_y = \frac{b_n}{n-1} \left[x^{n-1} + (n-1)\, \delta_x\, x^{n-2} + \ldots (\delta_x)^{n-1} \right]$$

The leading order feed-down is a $\underline{\underline{2(n-1)\text{-pole}}}$.