Corpuscular optics (JM De Conto)

Exercises + solutions:

1 Consider $^{12}C^{6+}$ ions, with an energy of 400 MeV/amu. Calculate the Lorentz factors and the magnetic rigidity. What is the diameter of a storage ring for such particles (CNAO synchrotron, normal conducting)? Take: m_p~0.94 GeV

$$\gamma = \frac{400+940}{940} = 1.4 \rightarrow \beta = 0.7$$

$$B\rho = \frac{\beta\gamma m_0 c}{ne} = \frac{\beta\gamma m_0 c^2}{nec} = \frac{\beta\gamma E_0}{nc}$$

With E_0 : energy at rest (12*0.94 MeV)

$$B\rho = \frac{\beta\gamma E_0}{nc} = 6.22\ \mathrm{Tm}$$

For B~1T, if half ring is done by bending magnets, the radius is~12 m (the exact size!)

2 Consider a beam whose rigidity is 5 Tm. Consider a magnetic quadrupole with focal length equal to 1.5 m. What is the minimum length of the quadrupole? If the quadrupole length is 0.5m and the inner radius 40mm, give the magnetic field on the poles.

The maximum gradient is supposed to be g=20 T/m (for normal conducting magnets). This limitation is NOT explicit in the quadrupole definition (it comes from some saturation that occur somewhere in the yoke). $\frac{1}{f} = \frac{gL}{B\rho}$ So the minimum length is $\frac{B\rho}{f g_{max}} = 0.17m$. For L=0.5, g=6.7T/m so B=0.27T on the pole.

3 Compute the transfer matrix of the quadrupole in the focusing plane and calculate the position of the principal planes. Conclusion?

$$\theta = KL = \sqrt{\frac{g}{B\rho}} \cdot L = 0.58$$

$$M = \begin{bmatrix} 0.837 & 0.473 \\ -0.634 & 0.837 \end{bmatrix}$$

$h_1 = h_2 = \frac{1-M_{11}}{M_{21}} = -0.26$ (symetrical system) ~ in the middle of the quadrupole ~ thin lens

4 Consider 2 thin lenses. The first one is focusing, the next is defocusing. The focal lengths are respectively f_1 and f_2. They are separated by a distance L. Compute the transfer matrix. Give a condition on L and f_1 to have a focusing system like on transparency 31 (2 arguments).

$$M = \begin{bmatrix} 1 & 0 \\ 1/f_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f_1 & 1 \end{bmatrix} = \begin{bmatrix} 1 - L/f_1 & L \\ \frac{1}{f_2} - \frac{1}{f_1} - \frac{L}{f_1 f_2} & 1 + L/f_2 \end{bmatrix}$$

To fulfill transp 31, we must have x>0 at exit for x>0 at entrance and x' at entrance equal to 0. So the condition is $1-\frac{L}{f_1}>0$. The system is converging if $\frac{1}{f_2}-\frac{1}{f_1}-\frac{L}{f_1 f_2}<0$

5 Compute the transfer matrix of a FODO cell. Express cosμ versus L and f and express the quantity $\psi = sin(\mu/2)$ versus L and f.

$$M=\begin{bmatrix}1 & L\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\1/f & 1\end{bmatrix}\begin{bmatrix}1 & L\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\-1/f & 1\end{bmatrix}=\begin{bmatrix}1-\frac{L}{f}-\frac{L^2}{2f^2} & 2L+\frac{L^2}{f}\\-\frac{L}{f^2} & 1+L/f\end{bmatrix}$$

$$cos\mu=\frac{1}{2}Tr(M)=1-\frac{L^2}{2f^2}=1-2sin^2\left(\frac{\mu}{2}\right)=1-2\psi^2$$

$$\psi=\frac{L}{2f}$$

Deduce (some calculations required) the following relation for the TWISS β on the converging lens:

$$\beta_c=\frac{L}{\psi}\sqrt{\frac{1+\psi}{1-\psi}}$$

$$\beta_c sin\mu=2\beta_c\psi\sqrt{1-\psi^2}=L\left(2+\frac{L}{f}\right)=2L(1+\psi)$$

$$\beta_c=L\frac{(1+\psi)}{\psi\sqrt{1-\psi^2}}=\frac{L}{\psi}\sqrt{\frac{1+\psi}{1-\psi}}$$

For which phase advance angle is the angular acceptance maximum? It is the phase such the amplitude is minimum for any angle. So is is when $\beta_c$is minimum with μ

$\frac{d\beta_c}{d\psi}=\frac{\psi^2+\psi-1}{D}$ where D is a (complicated) denominator.

$$\psi^2+\psi-1=0\rightarrow\psi=\frac{-1\pm\sqrt{5}}{2}$$

Only the positive value is acceptable

$$sin\left(\frac{\mu}{2}\right)=\frac{-1+\sqrt{5}}{2}\rightarrow\mu=76.34\;deg$$

Note that $\frac{-1\pm\sqrt{5}}{2}$ is the inverse of the golden ratio!

6 Consider a 45 degree bending magnet (hard edge, uniform field, no edge focusing) with a bending radius of 2m. This dipole is followed, 3 m later, by the same dipole rotating in the same direction. A thin lens is put between the two dipoles. What is its focal length to assume the system to be achromatic?

See slide 43 (detailed calculation) with L=3/2

$$f = \frac{\rho(1 - cos\theta) + Lsin\theta}{sin\theta} = 2.3m$$

7 Is waist to waist imaging (WTW) equivalent to point to point (PTP) imaging? If yes demonstrate it. If no, give the conditions to have the 2 simultaneously.

No: WTW imaging is a property of the beam. PTP imaging is the property of the system:

WTW: $M_{11}M_{21}\beta_0 + M_{22}M_{12}\gamma_0 = 0$ (slide 54)

PTP: M_{21}=0

WTW+PTP if and only if $\alpha_0 = \alpha_1 = 0\ and\ M_{11}M_{21} = 0$. As M_{21}=0, the only solution is M_{21}=0. (M_{11} cannot be 0, because the determinant of the matrix is not zero). So PTP+WTW needs

$$\alpha_0 = \alpha_1 = 0\ and\ M = \begin{bmatrix} G & 0 \\ 0 & 1/G \end{bmatrix}$$

8 Is an ideal quadrupole purely linear (consider an electrostatic one)?

No. A particule off axis is on a different equipotential line. So its kinetic energy is not the nominal one, and the focal length varies with position. A pure, “perfect” quadrupole has always third order abberations.