LECTURE 3

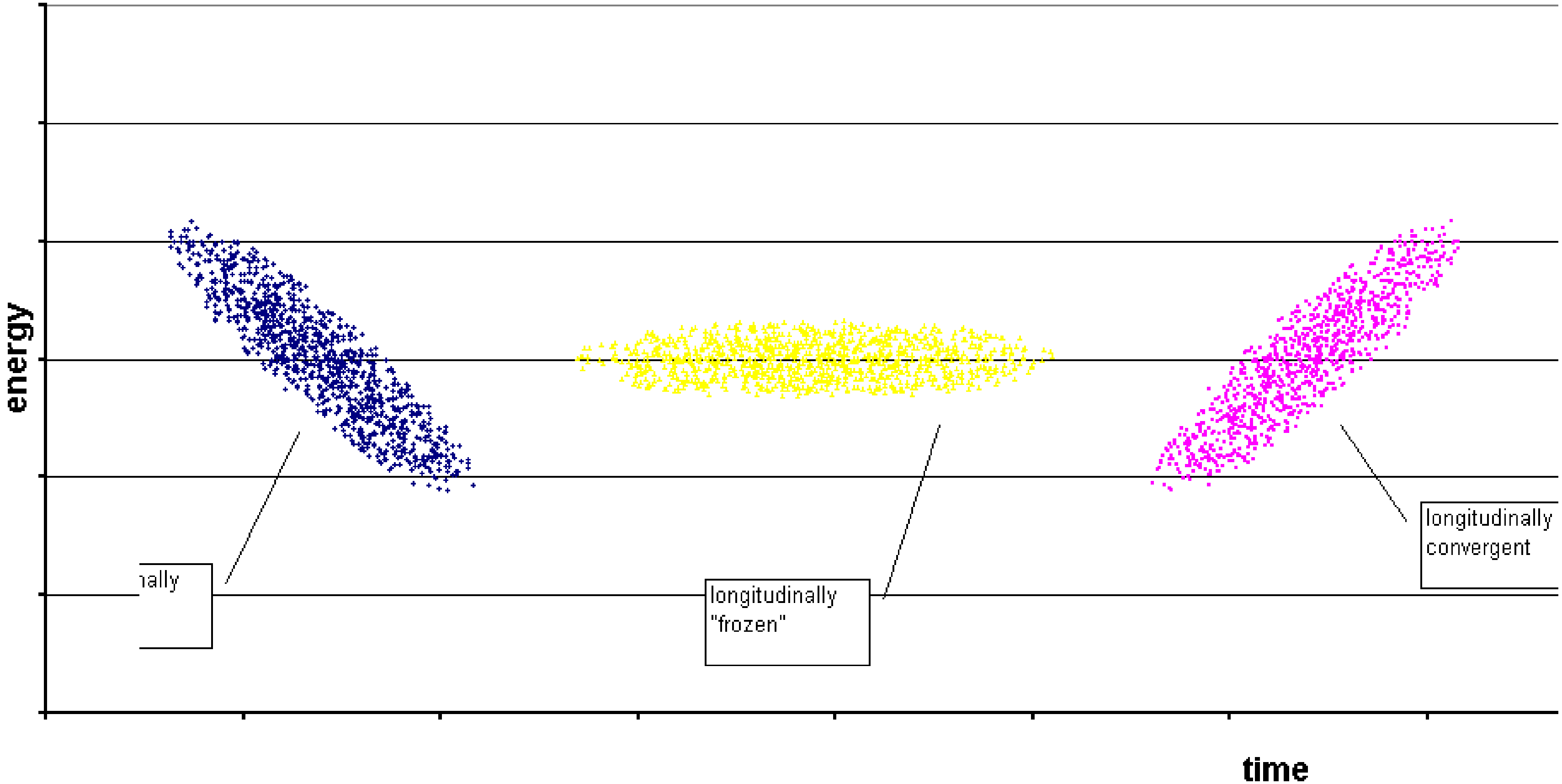
- single particles dynamics in a linear accelerator

- bunching, acceleration
- transverse and longitudinal focusing
- synchronous structures
- DTL drift-kick-drift dynamics
- slippage in a multicell cavity

Linac basics

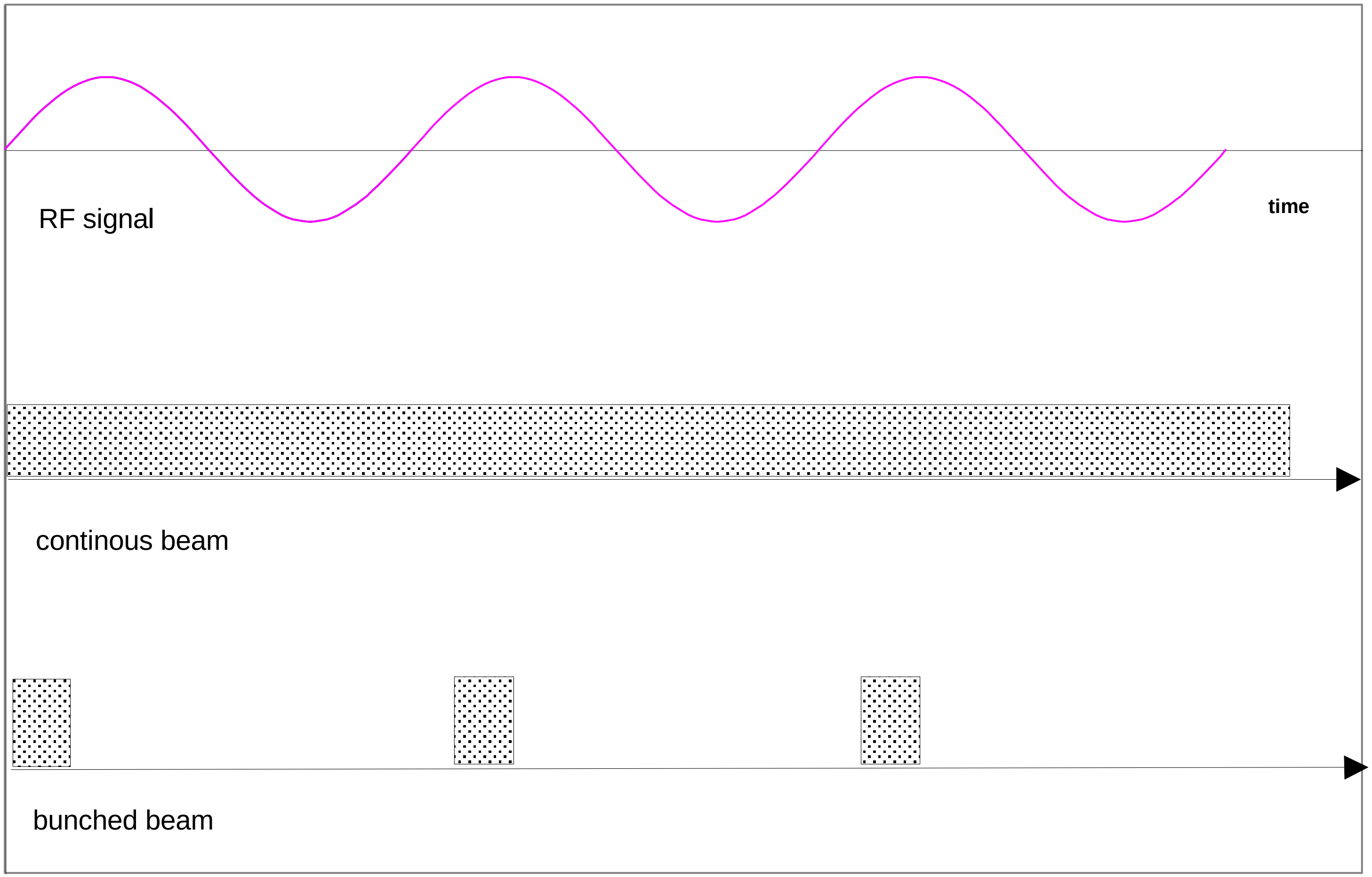
In order to increase the energy of a beam of particles while keeping them confined in space, we need to provide a longitudinal field for ACCELERATION and a transverse force for FOCUSING.

Acceleration-basics



longitudinal phase plane
time-energy or time-momentum

Acceleration-basics



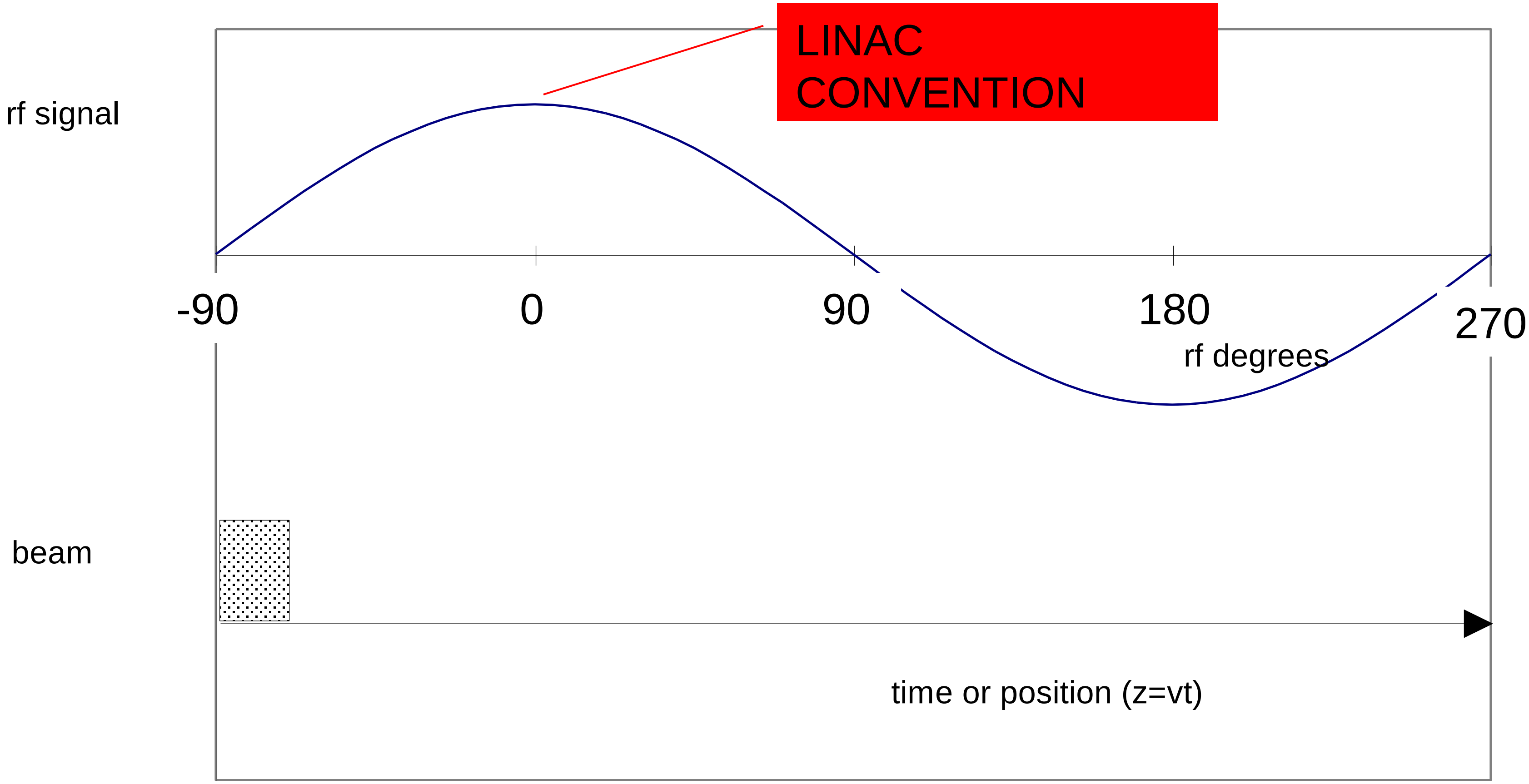
It is not possible to transfer energy to an un-bunched beam

BUNCHING

- need a structure on the scale of the wavelength to have a net transfer of energy to the beam

- need to bunch a beam and keep it bunched all the way through the acceleration : need to provide LONGITUDINAL FOCUSING

Acceleration



degrees of RF ⇔ length of the bunch(cm) ⇔ duration of the bunch (sec)

$$\Delta\Phi = \Delta z\ 360/\beta\lambda = \Delta t\ 360 * f$$

β is the relativistic parameter λ the RF wavelength, f the RF frequency

synchronous particle

- it’s the (possibly fictitious) particle that we use to calculate and determine the phase along the accelerator. It is the particle whose velocity is used to determine the synchronicity with the electric field.
- It is generally the particle in the centre (longitudinally) of the bunch of particles to be accelerated

Acceleration

- to describe the motion of a particle in the longitudinal phase space we want to establish a relation between the energy and the phase of the particle during acceleration
- energy gain of the synchronous particle $\Delta W_s = qE_0 LT \cos(\phi_s)$
- energy gain of a particle with phase Φ $\Delta W = qE_0 LT \cos(\phi)$
- assuming small phase difference ΔΦ =Φ-Φ_s

$$\frac{d}{ds}\Delta W = qE_0 T \cdot [\cos(\varphi_s + \Delta\varphi) - \cos\varphi_s]$$

- and for the phase

$$\frac{d}{ds}\Delta\varphi = \omega\left(\frac{dt}{ds} - \frac{dt_s}{ds}\right) = \frac{\omega}{c}\left(\frac{1}{\beta} - \frac{1}{\beta_s}\right) \cong -\frac{\omega}{\beta_s c}\frac{\Delta\beta}{\beta_s} = -\frac{\omega}{mc^3\beta_s^3\gamma_s^3}\Delta W$$

Acceleration-Separatrix

- Equation for the canonically conjugated variables phase and energy with Hamiltonian (total energy of oscillation):

$$\frac{\omega}{mc^3\beta_s^3\gamma_s^3}\left\{\frac{\omega}{2mc^3\beta_s^3\gamma_s^3}(\Delta W)^2 + qE_0T\left[\sin(\varphi_s+\Delta\varphi)-\Delta\varphi\cos\varphi_s-\sin\varphi_s\right]\right\}=H$$

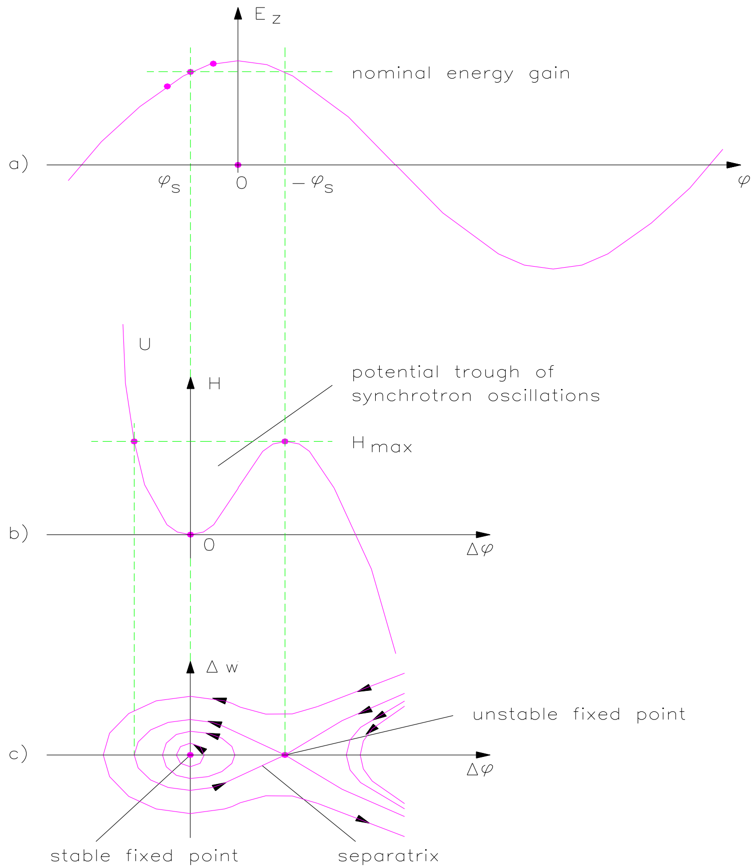
- For each H we have different trajectories in the longitudinal phase space .Equation of the separatrix (the line that separates stable from unstable motion)

$$\frac{\omega}{2mc^3\beta_s^3\gamma_s^3}(\Delta W)^2 + qE_0T\left[\sin(\varphi_s+\Delta\varphi)+\sin\varphi_s-(2\varphi_s+\Delta\varphi)\cos\varphi_s\right]=0$$

- Maximum energy excursion of a particle moving along the separatrix

$$\Delta\hat{W}_{\max} = \pm 2\left[\frac{qmc^3\beta_s^3\gamma_s^3E_0T(\varphi_s\cos\varphi_s-\sin\varphi_s)}{\omega}\right]^{\frac{1}{2}}$$

Acceleration

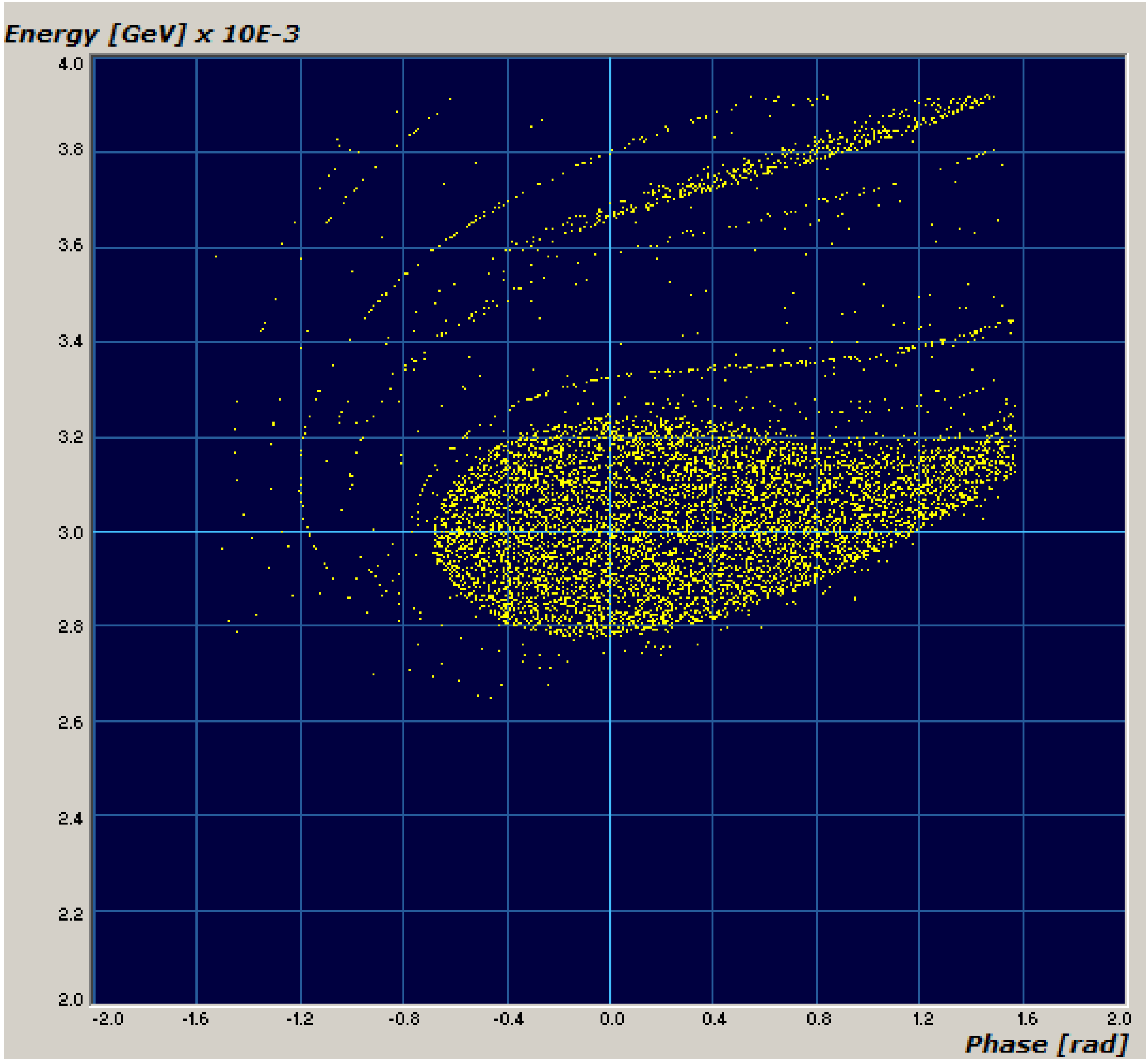


RF electric field as function of phase.

Potential of synchrotron oscillations

Trajectories in the longitudinal phase space each corresponding to a given value of the total energy (stationary bucket)

Longitudinal acceptance



IH beam dynamics-KONUS

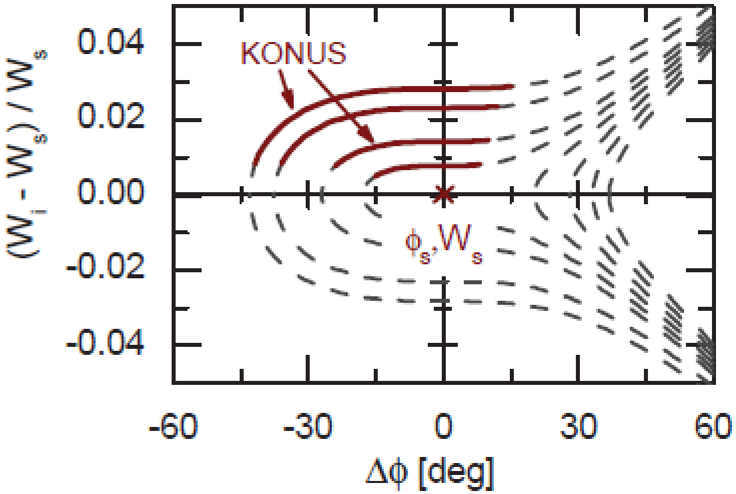


Figure 2: Single particle orbits in $\Delta W/W_s - \Delta\phi$ phase space at $\phi_s = 0°$ with color marking of the area used by KONUS.

Higher accelerating efficiency

Less RF defocusing (see later) – allow for longer accelerating sections w/o transverse focusing

Need re-bunching sections

Acceleration

- definition of the acceptance : the maximum extension in phase and energy that we can accept in an accelerator :

$$\Delta\varphi \cong 3\varphi_s$$

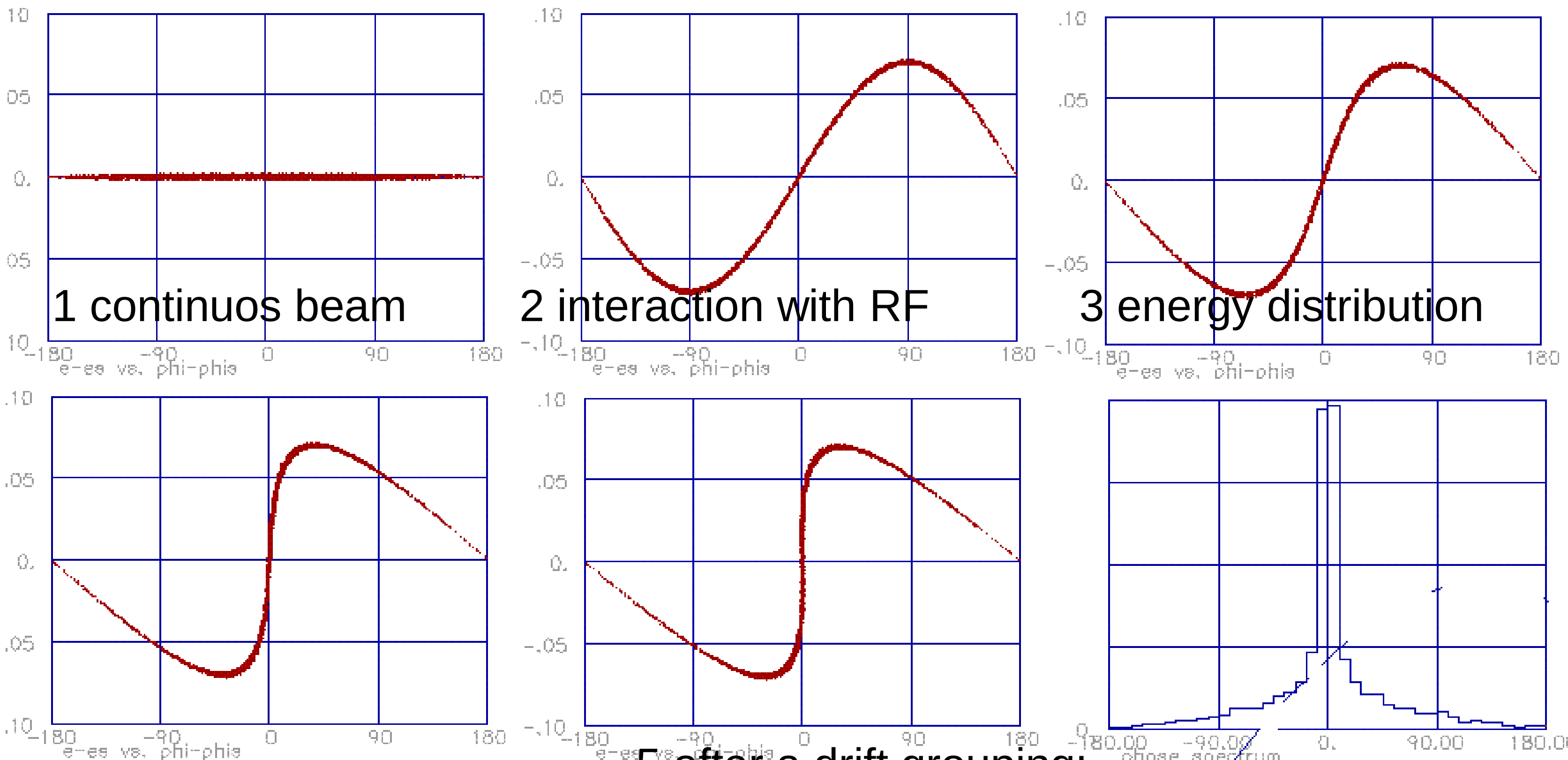
$$\Delta\hat{W}_{\max} = \pm 2\left[\frac{qmc^3\beta_s^3\gamma_s^3 E_0 T(\varphi_s \cos\varphi_s - \sin\varphi_s)}{\omega}\right]^{\frac{1}{2}}$$

bunching

Preparation to acceleration :

- generate a velocity spread inside the beam
- let the beam distribute itself around the particle with the average velocity

Discrete Bunching



1 continuos beam

2 interaction with RF

3 energy distribution

4 earlier particles are slowed down, later particles are accelerated

5.after a drift grouping: around the “central” particle

6 distribution of the particles

Adiabatic bunching

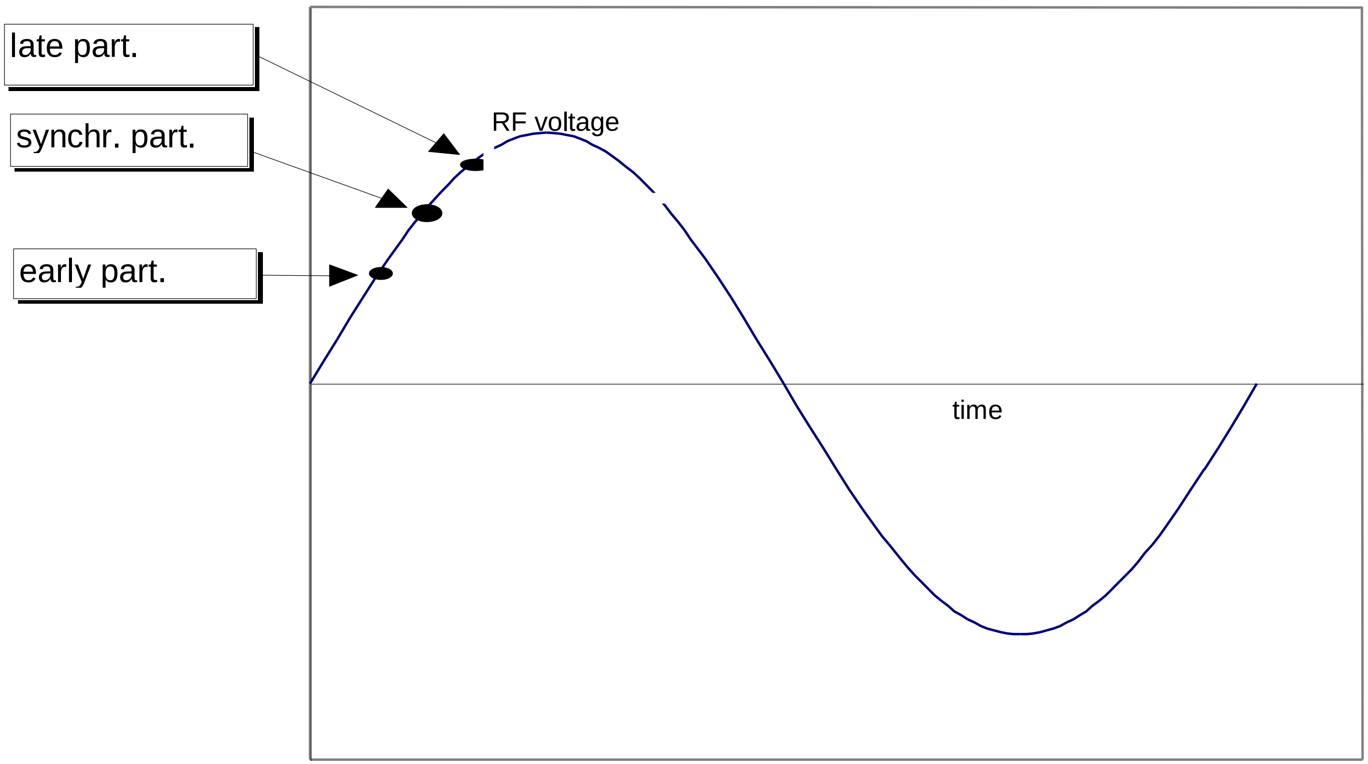
- generate the velocity spread continuously with small longitudinal field : bunching over several oscillation in the phase space (up to 100!) allows a better capture around the stable phase : 95% capture vs 50 %
- in an RFQ by slowly increasing the depth of the modulation along the structure it is possible to smoothly bunch the beam and prepare it for acceleration.

movie of the RFQ rfq2.plt

Adiabatic bunching

- Get pictures from RFQ

Keep bunching during acceleration



for phase stability we need to accelerate when dEz/dz >0 i.e. on the rising part of the RF wave

Longitudinal phase advance

- if we accelerate on the rising part of the positive RF wave we have a LONGITUDINAL FORCE keeping the beam bunched. The force (harmonic oscillator type) is characterized by the LONGITUDINAL PHASE ADVANCE

$$k_{0l}^{2} = \frac{2\pi q E_0 T \sin(-\varphi_s)}{mc^2 \beta_s^3 \gamma^3 \lambda} \left[\frac{1}{m^2} \right]$$

- long equation

$$\frac{d^2 \Delta\varphi}{ds^2} + k_{0l}^{2} \left(\Delta\varphi - \frac{\Delta\varphi^2}{2\tan(-\varphi_s)} \right) = 0$$

Longitudinal phase advance

- Per meter

$$k_{0l} = \sqrt{\frac{2\pi q E_0 T \sin(-\varphi_s)}{mc^2 \beta_s^3 \gamma^3 \lambda}} \left[\frac{1}{m}\right]$$

Length of focusing period

L=(Number of RF gaps) βλ

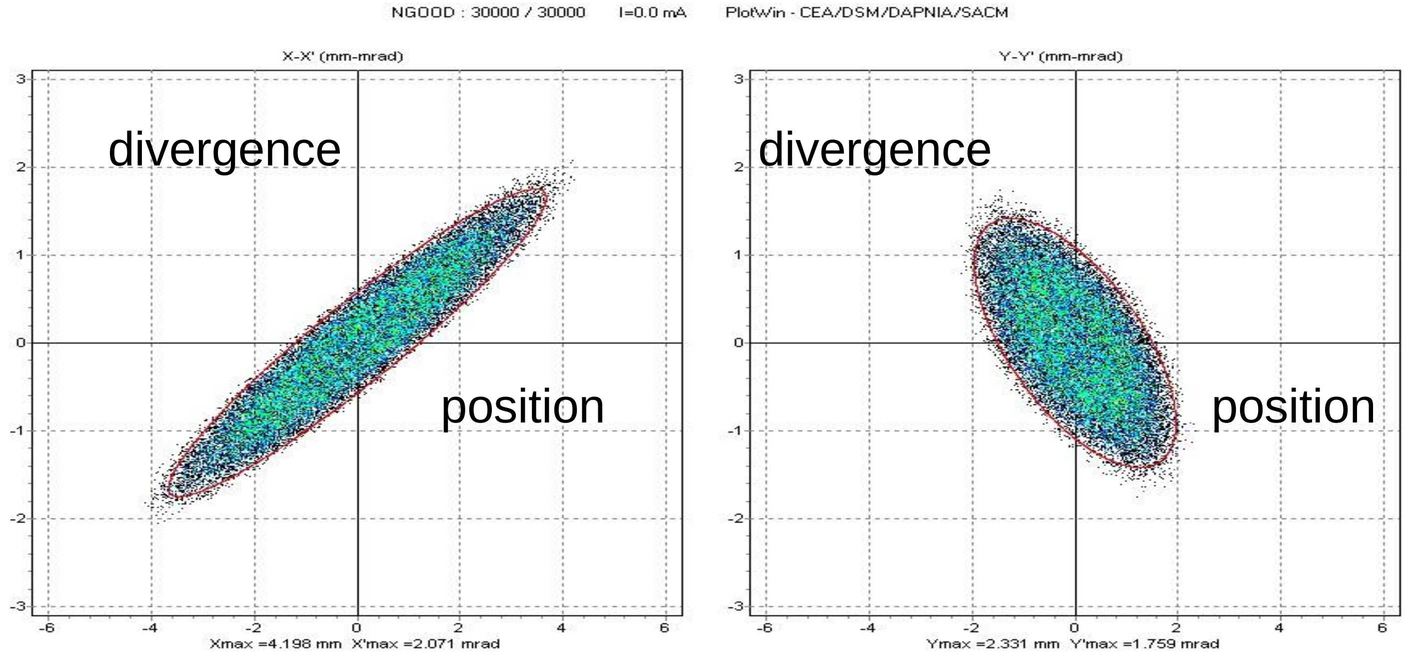
- Per focusing period

$$\sigma_{0l} = \sqrt{\frac{2\pi q E_0 T N^2 \lambda \sin(-\varphi_s)}{mc^2 \beta_s \gamma^3}}$$

- Per RF period

$$\sigma_{0l} = \sqrt{\frac{2\pi q E_0 T \sin(-\varphi_s)}{m \beta_s \gamma^3 \lambda}} \left[\frac{1}{s}\right]$$

Transverse phase space and focusing



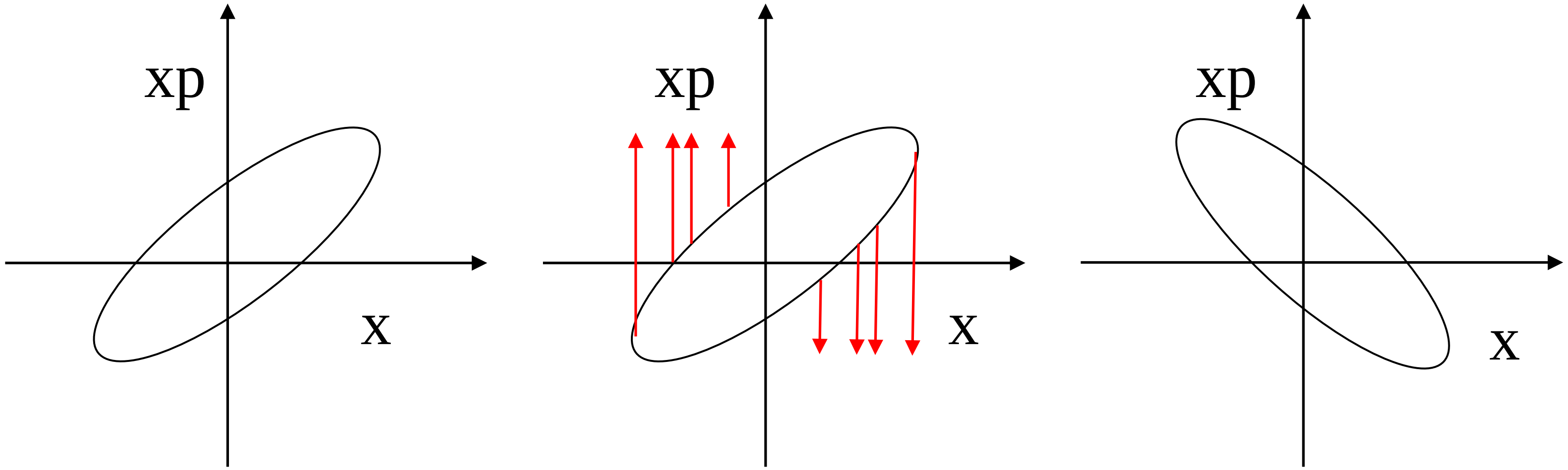
Bet = 6.3660 mm/Pi.mrad
Alp = -2.8807

DEFOCUSED

Bet = 1.7915 mm/Pi.mrad
Alp = 0.8318

FOCUSED

Focusing force



defocused beam

apply force towards the axis proportional to the distance from the axis

focused beam

F(x)=-K x

Focusing

- MAGNETIC FOCUSING

(dependent on particle velocity)

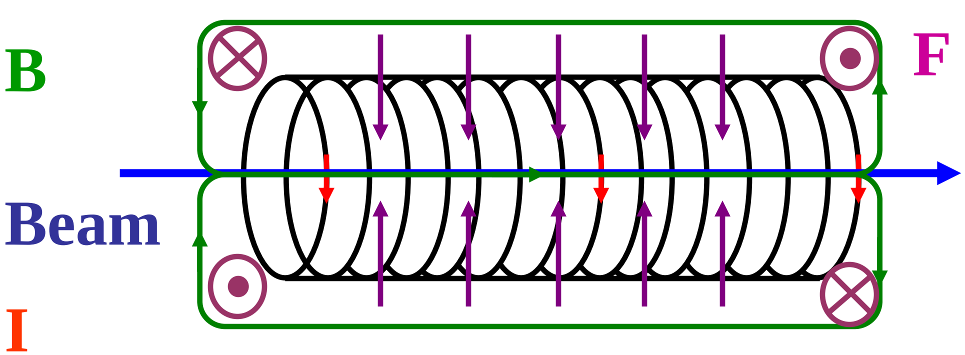
$$\vec{F} = q\vec{v} \times \vec{B}$$

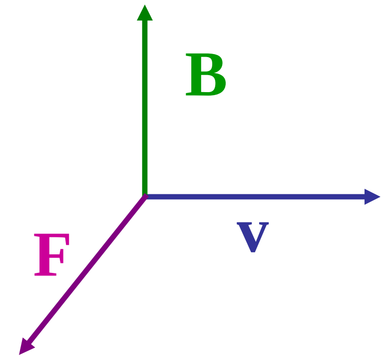
- ELECTRIC FOCUSING

(independent of particle velocity)

$$\vec{F} = q \cdot \vec{E}$$

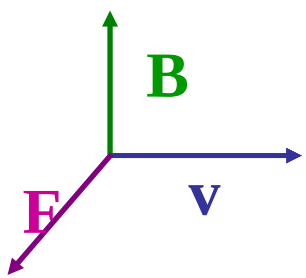
Solenoid





Input : $B = B_{\perp}$

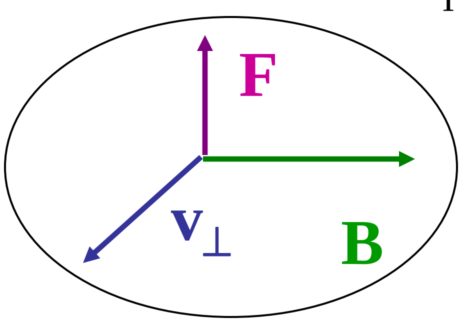
$F \propto v \cdot B$



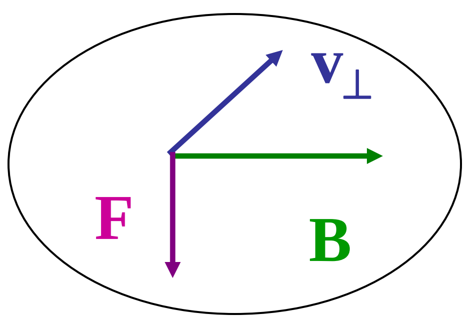
Beam transverse rotation :

$v_{\perp} \propto v \cdot B \cdot r$

Middle : $B = B_{l}$



$x < 0$

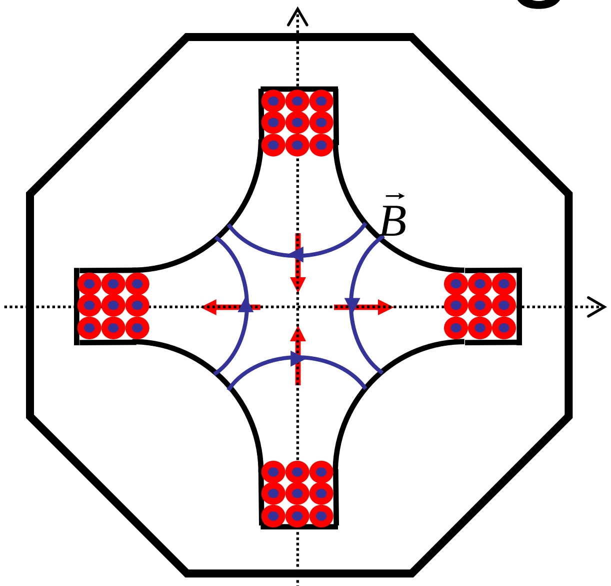


$x > 0$

$F \propto v_{\perp} \cdot B \propto v \cdot B^2 \cdot r$

Beam linear focusing

Magnetic quadrupole



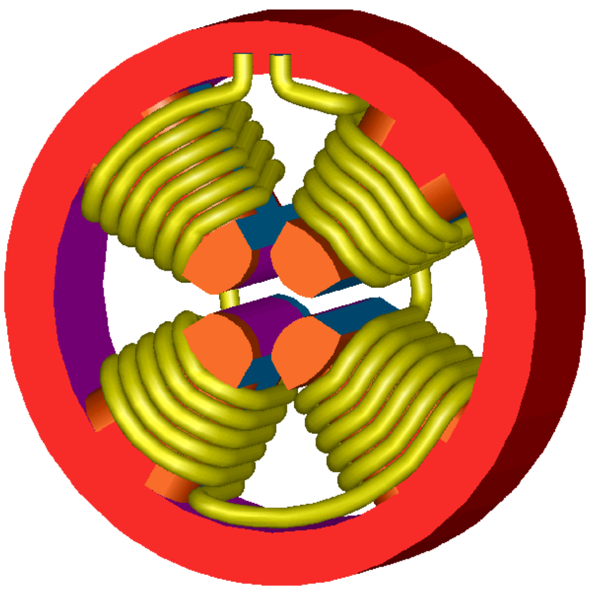
Magnetic field

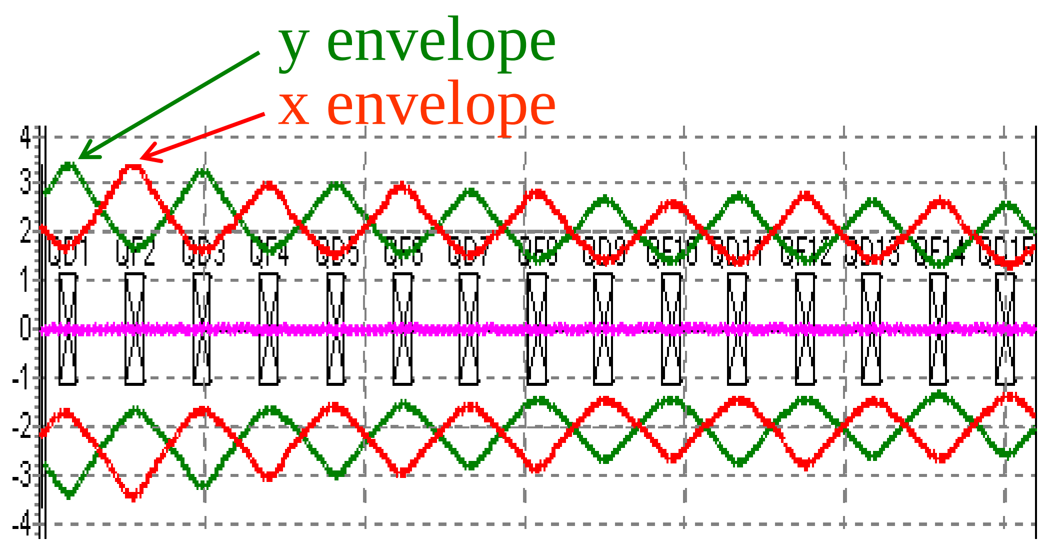
$$\begin{cases} B_x = G \cdot y \\ B_y = G \cdot x \end{cases}$$

Magnetic force

$$\begin{cases} F_x = -q \cdot v \cdot G \cdot x \\ F_y = q \cdot v \cdot G \cdot y \end{cases}$$

Focusing in one plan, defocusing in the other





sequence of focusing and defocusing quadrupoles

FODO

- periodic focusing channel : the beam 4D phase space is identical after each period
- Equation of motion in a periodic channel (Hill's equation) has periodic solution :

$$x(z) = \sqrt{\varepsilon_0 \beta(z)} \cdot \cos(\sigma(z))$$

emittance

beta function , has the periodicity of the focusing period

transverse phase advance

$$\beta(z+l) = \beta(z)$$

$$\sigma(z) = \int_0^z \frac{dz}{\beta(z)}$$

review N. Pichoff course

quadrupole focusing

$$\sigma_{0t} = \sqrt{\frac{\theta_0^4}{8\pi^2} + \Delta_{rf}}$$

zero current phase advance per period in a LINAC

G magnetic quadrupole gradient, [T/m]
N= number of magnets in a period

$$\theta_0^2 = \frac{qG\lambda^2 N^2 \beta\chi}{m_0 c\gamma}$$

for $+-$ (*N=2*)

$$\chi = \frac{4}{\pi}\sin(\frac{\pi}{2}\Gamma)$$

for $++--$ (*N=4*)

$$\chi = \frac{8}{\sqrt{2}\pi}\sin(\frac{\pi}{4}\Gamma)$$

Γ is the quadrupole filling factor (quadrupole length relative to period length).

RF defocusing

Maxwell equations

$$\nabla \cdot E = 0 \quad \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

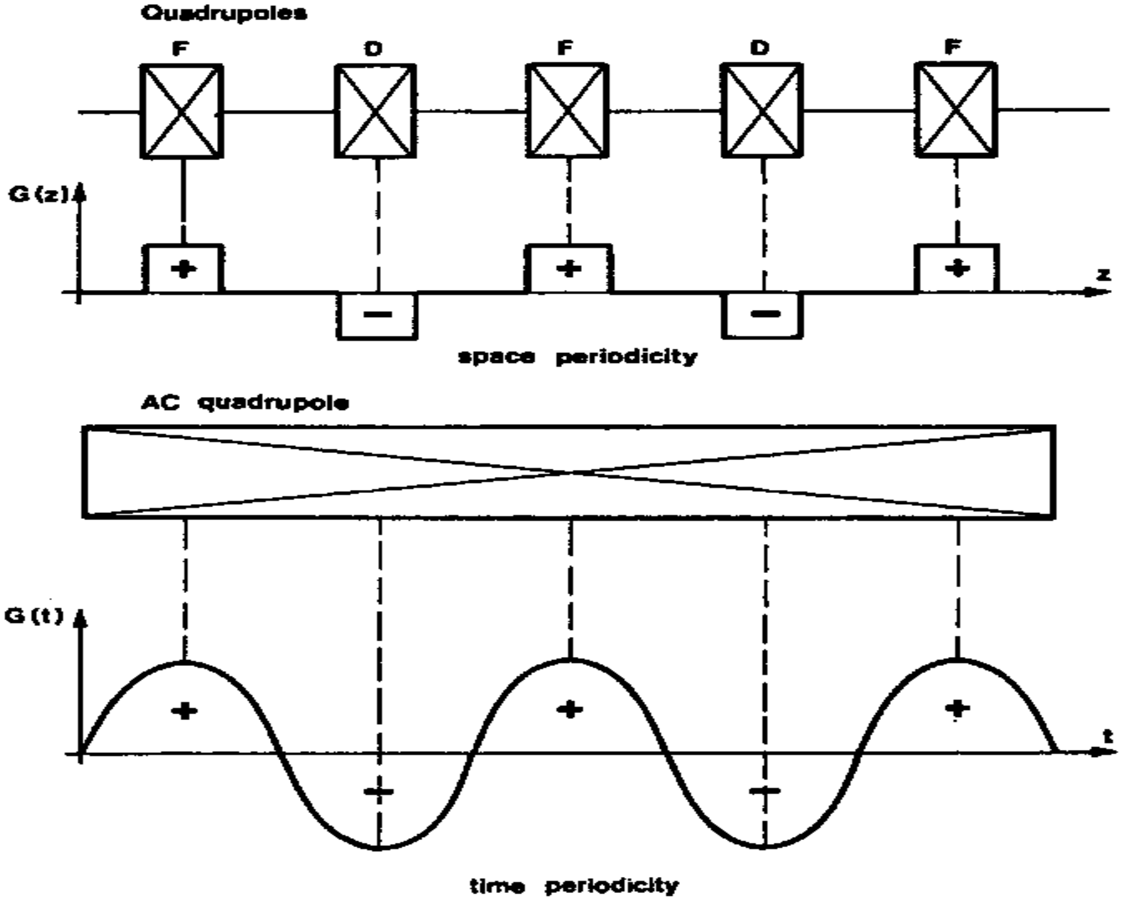
when longitudinal focusing (phase stability) , there is defocusing (depending on the phase) in the transverse planes

$$\Delta_{rf} = \frac{1}{2}\sigma_{0l}^2 = \frac{\pi q \lambda N^2 E_0 T \sin\phi_s}{m_0 c^2 \beta\gamma^3}$$

Number of RF gap in a transverse focusing period

IH transverse focusing

FODO in RFQ vs FODO in DTL



sequence of magnetic quadrupoles

focusing field

RF quadrupole

focusing field

Other

- FOFODODO

- sequence of triplets

- solenoidal channel

matching

- matched condition to a static focusing channel
- matching to a time varying focusing channel

First order rules for designing an accelerator

- Acceleration : choose the correct phase, maintain such a phase thru the process of acceleration

- Focusing : choose the appropriate focusing scheme and make sure it is matched

Synchronous particle and geometrical beta β_g.

- design a linac for one “test” particle. This is called the “synchronous” particle.

- the length of each accelerating element determines the time at which the synchronous particles enters/exits a cavity.
- For a given cavity length there is an optimum velocity (or beta) such that a particle traveling at this velocity goes through the cavity in half an RF period.
- The difference in time of arrival between the synchronous particles and the particle traveling with speed corresponding to the geometrical beta determines the phase difference between two adjacent cavities
- in a synchronous machine the geometrical beta is always equal to the synchronous particle beta and EACH cell is different

Adapting the structure to the velocity of the particle

- Case1 : the geometry of the cavity/structure is continuously changing to adapt to the change of velocity of the “synchronous particle”

- Case2 : the geometry of the cavity/structure is adapted in step to the velocity of the particle. Loss of perfect synchronicity, phase slippage.

- Case3 : the particle velocity is beta=1 and there is no problem of adapting the structure to the speed.

Case1 : $\beta_s = \beta_g$

- The absolute phase φ_i and the velocity β_{i-1} of this particle being known at the entrance of cavity i, its RF phase ϕ_i is calculated to get the wanted synchronous phase ϕ_{si}, $\phi_i = \varphi_i - \phi_{si}$
- the new velocity β_i of the particle can be calculated from, $\Delta W_i = qV_0 T \cdot \cos\phi_{si}$

  ① if the phase difference between cavities i and $i+1$ is given, the distance D_i between them is adjusted to get the wanted synchronous phase ϕ_{si+1} in cavity $i+1$.

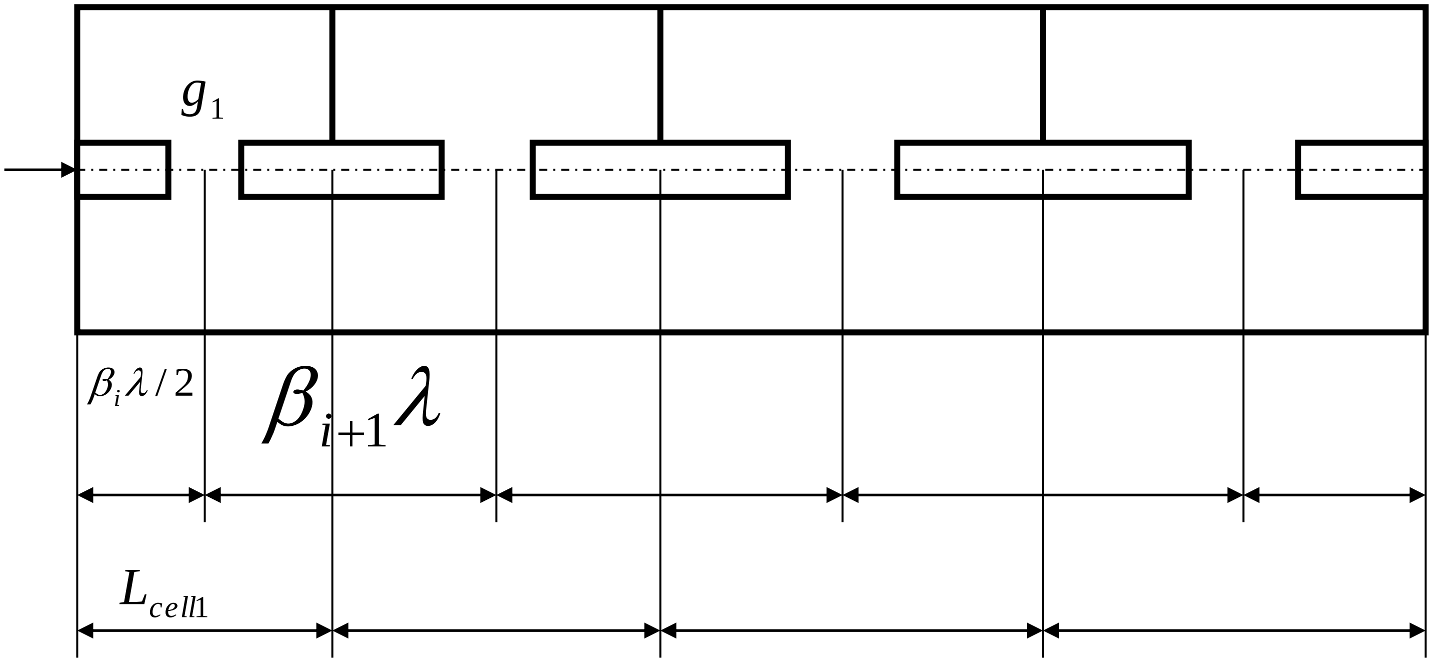
  ②if the distance D_i between cavities i and $i+1$ is set, the RF phase ϕ_i of cavity $i+1$ is calculated to get the wanted synchronous phase ϕ_{si+1} in it.

RF phase	ϕ_{i-1}		ϕ_i		ϕ_{i+1}
Particle velocity		β_{si-1}		β_{si}	
Distances		D_{i-1}		D_i	
Synchronous phase	ϕ_{si-1}		ϕ_{si}		ϕ_{si+1}
Cavity number	i-1		i		$i+1$

Synchronism condition :

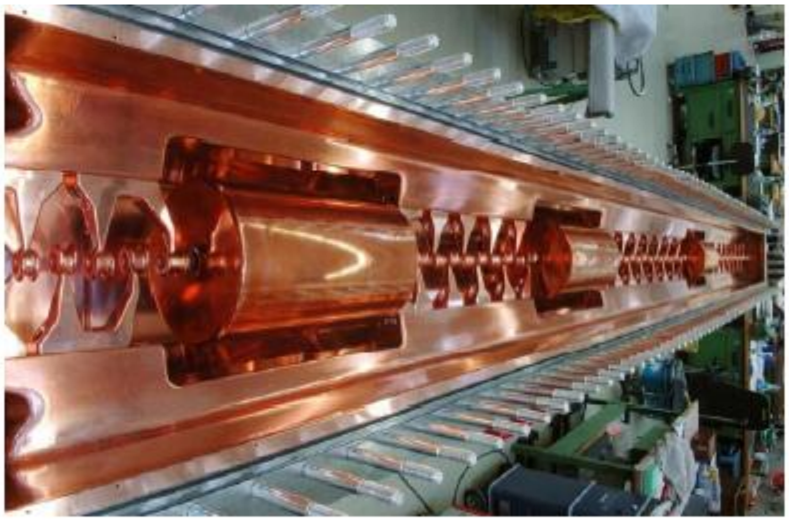
$$\phi_{si+1} - \phi_{si} = \omega \cdot \frac{D_i}{\beta_{si}\, c} + \phi_{i+1} - \phi_i + 2\pi n$$

$\beta_s = \beta_g$ DTL drift-kick-drift design



- fix phase RF and gap lenght
- the syncr. particle should travel a distance L in one RF period
- assume that the particle goes to mid gap with the initial velocity.
- at mid gap the particle velocity is increased by an amount corresponding to the effect of the integrated electric field times the transit time factor. $\Delta W = qE_oTL \cos(\Phi)$
- the particle drifts with this velocity till the end of the cell
- and so on and so on..........

Synchronous structures

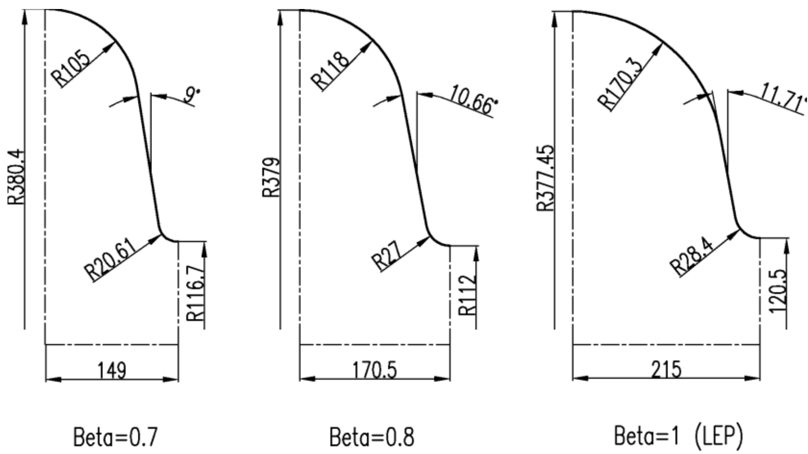


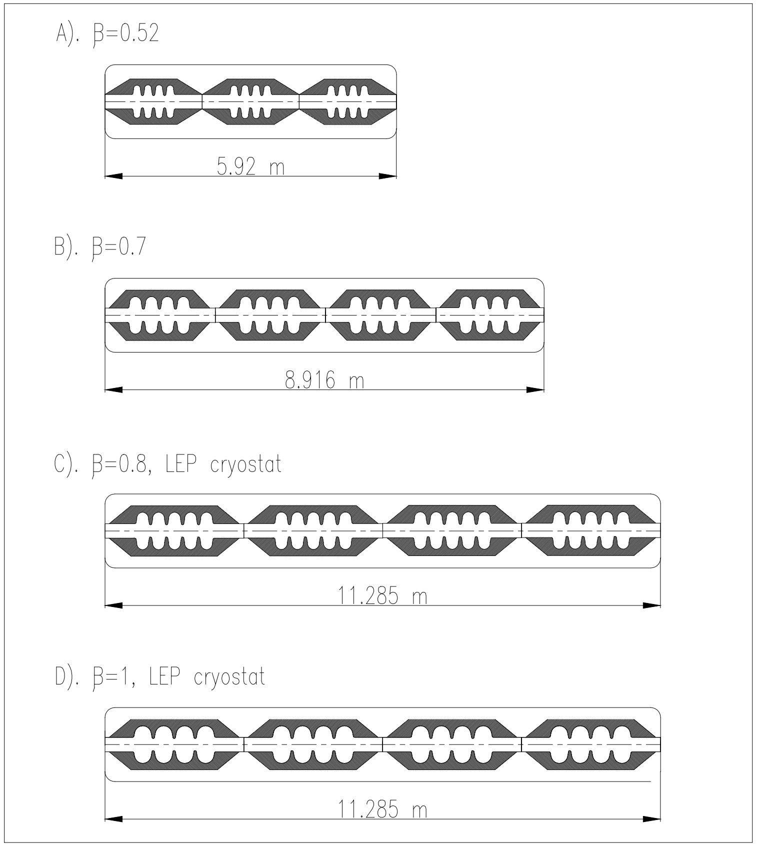
Case 2 : $\beta_s \sim \beta_g$

- for simplifying construction and therefore keeping down the cost, cavities are not individually tailored to the evolution of the beam velocity but they are constructed in blocks of identical cavities (tanks). several tanks are fed by the same RF source.
- This simplification implies a “phase slippage” i.e. a motion of the centre of the beam . The phase slippage is proportional to the number of cavities in a tank and it should be carefully controlled for successful acceleration.

Linacs made of superconducting cavities

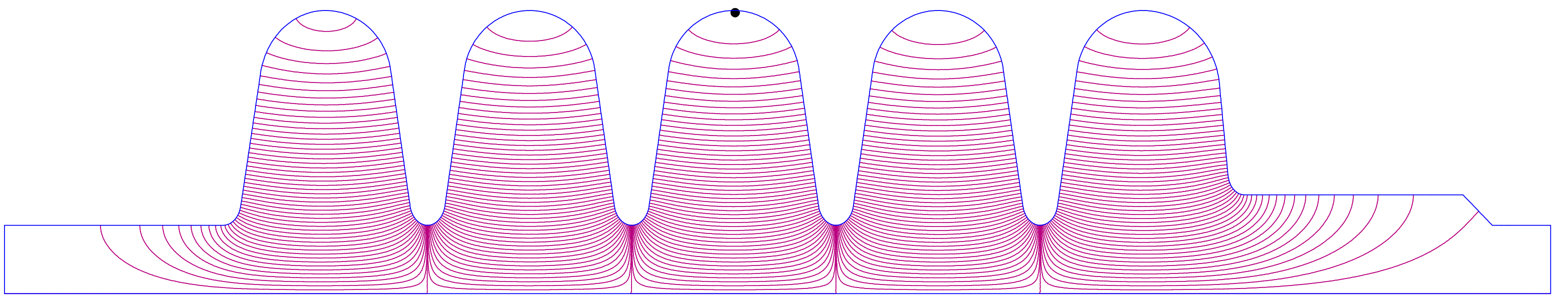
Need to standardise construction of cavities:
only few different types of cavities are made for some β's
more cavities are grouped in cryostats





Example:
CERN design, SC linac 120 - 2200 MeV

phase slippage



Lcavity = $\beta_g\lambda/2$

particle enters the cavity with $\beta_s < \beta_g$. It is accelerated

the particle has not left the cavity when the field has changed sign : it is also a bit decelerated

the particle arrives at the second cavity with a “delay”

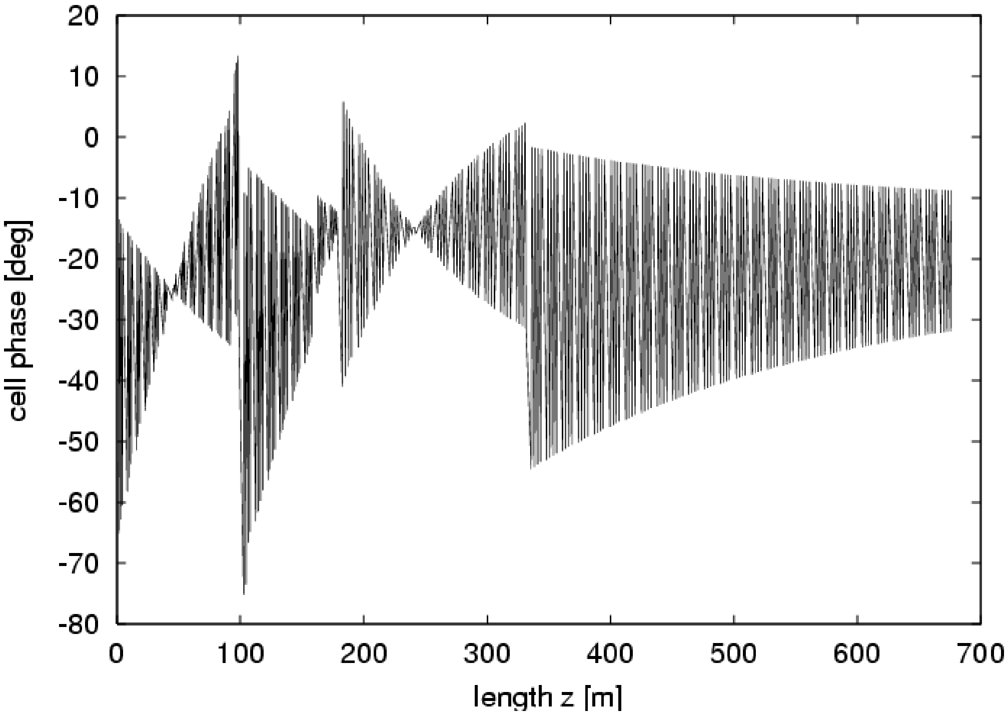
........and so on and so on

we have to optimize the initial phase for minimum phase slippage

for a given velocity there is a maximum number of cavity we can accept in a tank

Phase slippage

In each section, the cell length (βλ/2, π mode!) is correct only for one beta (energy): at all other betas the phase of the beam will differ from the design phase



Example of phase slippage:
CERN design for a 352 MHz SC linac
Four sections:
β = 0.52 (120 - 240 MeV)
β = 0.7 (240 - 400 MeV)
β = 0.8 (400 MeV - 1 GeV)
β = 1 (1 - 2.2 GeV)

← Phase at the first and last cell of each 4-cell cavity (5-cell at β=0.8)

lecture 3 summary

- in a linac we need to provide transverse and longitudinal focusing during acceleration.
- we need to bunch the beam to prepare it for acceleration : electron sources (gun) can provide a bunched beam, proton and ion source provide a continuous beam.