

Exercises on Space Charge

Exercise 1

Compute the transverse space charge forces and the tune shifts for a cylindrical beam in a circular beam pipe, having the following longitudinal distributions: parabolic, sinusoidal modulation, Gaussian

<i>parabolic</i>	$\lambda(z) = \frac{3Ne}{2l_o} \left[1 - \left(\frac{2z}{l_o} \right)^2 \right]$
<i>sinusoidal modulation</i>	$\lambda(z) = \lambda_o + \Delta\lambda \cos(k_z z) \quad ; \quad k_z = 2\pi / \lambda_w$
<i>Gaussian</i>	$\lambda(z) = \frac{Ne}{\sqrt{2\pi}\sigma_z} \exp\left(-\frac{z^2}{2\sigma_z^2}\right)$

Exercise 2

Compute the transverse space charge force and the tune shift for a cylindrical beam in a circular beam pipe, having a bi-Gaussian longitudinal and transverse distribution.

bi – Gaussian

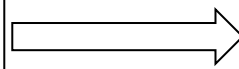
$$\lambda(z) = \frac{Ne}{\sqrt{2\pi}\sigma_z} \exp\left(-\frac{z^2}{2\sigma_z^2}\right)$$

$$\rho(r, z) = \frac{\lambda(z)}{2\pi\sigma_r^2} \exp\left(\frac{-r^2}{2\sigma_r^2}\right)$$

Exercise 3

Compute the longitudinal space charge force of a transverse uniform cylindrical beam in a circular perfectly conducting beam pipe

$$E_z(r,z) = -\frac{1}{\gamma^2} \frac{\partial}{\partial z} \int_r^b E_r(r,z) dr$$



$$F_z(r,z) = -\frac{e}{\gamma^2} \frac{\partial}{\partial z} \int_r^b E_r(r',z) dr'$$

Exercise 4

Compute the longitudinal space charge forces for a cylindrical beam in a circular beam pipe, having the following longitudinal distributions: parabolic, sinusoidal modulation, Gaussian

parabolic
$$\lambda(z) = \frac{3Ne}{2l_o} \left[1 - \left(\frac{2z}{l_o} \right)^2 \right]$$

sinusoidal modulation
$$\lambda(z) = \lambda_o + \Delta\lambda \cos(k_z z) \quad ; \quad k_z = 2\pi / l_w$$

Gaussian
$$\lambda(z) = \frac{Ne}{\sqrt{2\pi}\sigma_z} \exp\left(-\frac{z^2}{2\sigma_z^2}\right)$$

$$F_z(r, z) = -\frac{e}{\gamma^2} \frac{\partial}{\partial z} \int_r^b E_r(r', z) dr'$$

$$F_z(r, z) = -\frac{e}{4\pi\epsilon_0\gamma^2} \left(1 - \frac{r^2}{a^2} + 2 \ln \frac{b}{a}\right) \frac{\partial\lambda(z)}{\partial z}$$

Exercise 5

Compute the incoherent betatron tune shift of a uniform proton beam inside two perfectly conducting parallel plates