



# Combining clustering & counts of galaxy clusters

## Cosmology and PNG with maxBCG sample

**Annalisa Mana\***, J.Weller, T.Giannantonio, B.Hoyle,  
G.Hütsi, B.Sartoris [Mana et al. 2013]

# Outline

---

- Introduction
- SDSS **maxBCG** catalogue and data
- **Theoretical modelling:**
  - Scaling relation
  - Counts and total masses
  - Power Spectrum
  - Primordial Non Gaussianity
- **Monte Carlo Markov Chain** analysis
- Cosmological **constraints**
- Conclusions

# Introduction

## Why/how cosmology with galaxy clusters?

### WHY?

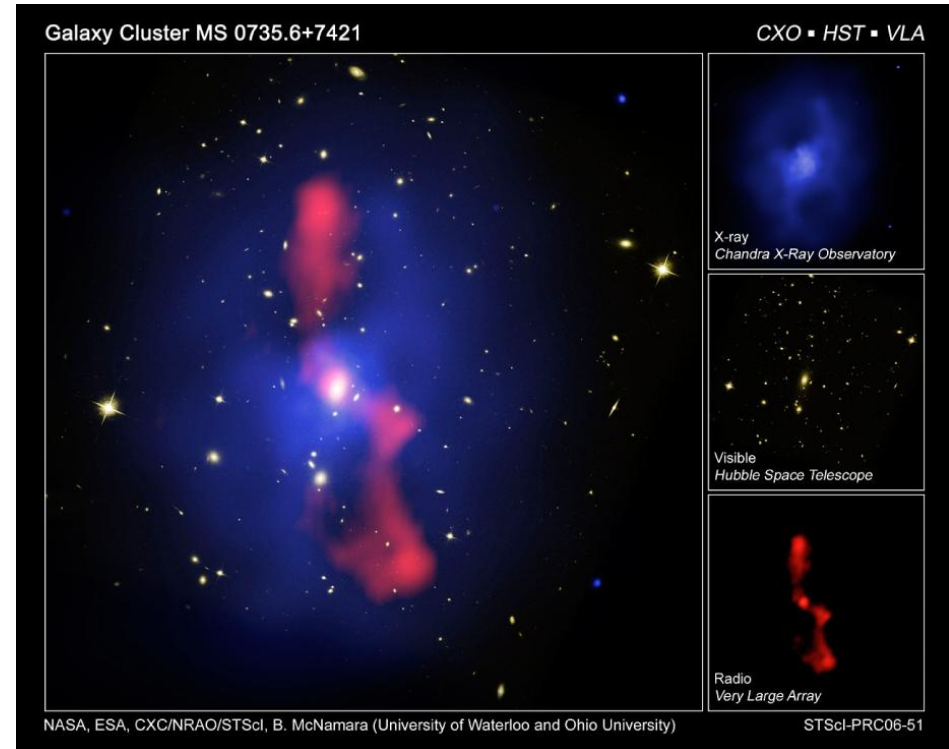
They probe the **expansion** of the Universe and the **growth rate** of cosmic structures.

### HOW?

Measuring cluster statistics as function of **mass proxy** (optical richness, X-ray properties, SZ signal, ...) and redshift.

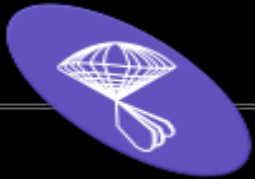


## CONSTRAINTS on COSMOLOGY and $M-M_{\text{obs}}$ RELATION



Composite image of galaxy cluster MS0735.6+7421. Optical: HST; X-ray: Chandra X-ray Observatory; Radio: Very Large Array telescope. Credit: NASA, ESA, CXC, STScI, B. McNamara; NRAO, L. Birzan and team.

# SDSS\* maxBCG catalogue



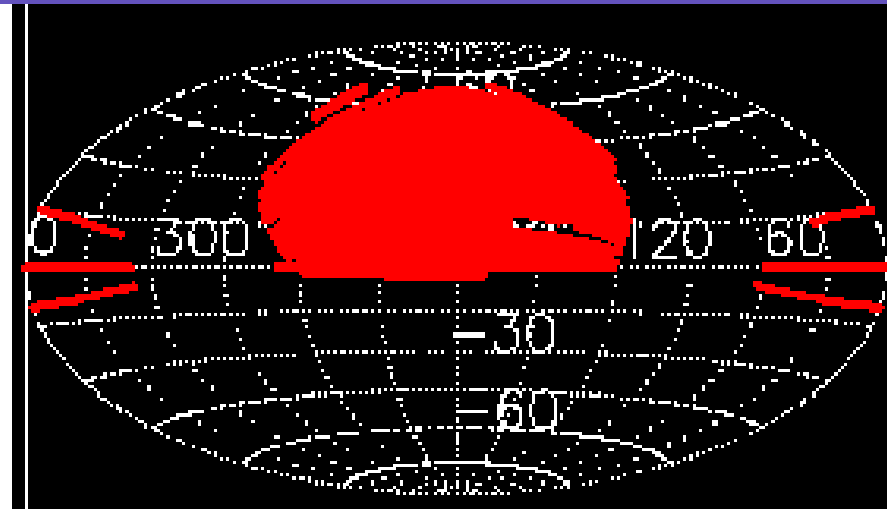
## SDSS Data Release 5

Sloan Digital Sky Survey

### maxBCG sample:

[Koester et al. 2007]

- Optically selected clusters by the red-sequence detection method
- **13,823** clusters  
500 Mpc<sup>3</sup> covering 7,398 deg<sup>2</sup> of sky  
 $z_{\text{photo}} \in [0.1, 0.3]$   
 $10 < N_{200} < 190$   
 $M_{\text{lim}} \sim 7 \times 10^{13} h^{-1} M_{\odot}$



SDSS DR5 Imaging Sky Coverage (Aitoff projection of Equatorial coordinates). Credit: SDSS webpage.

\*[www.sdss.org](http://www.sdss.org)

# MaxBCG Counts and WL masses

- Observable: **optical richness**  $N_{200}$  = nr. of red galaxies within  $R_{200}$

- Number counts

[Rozo et al. 2010]

$$11 < N_{200} < 120$$

$$7 \times 10^{13} h^{-1} M_{\odot} < M < 1.2 \times 10^{15} h^{-1} M_{\odot}$$

Poisson errors + sample variance +  
uncertainty on purity/completeness

Richness	No. of Clusters
11-14	5167
14-18	2387
19-23	1504
24-29	765
30-38	533
39-48	230
49-61	134
62-78	59
79-120	31

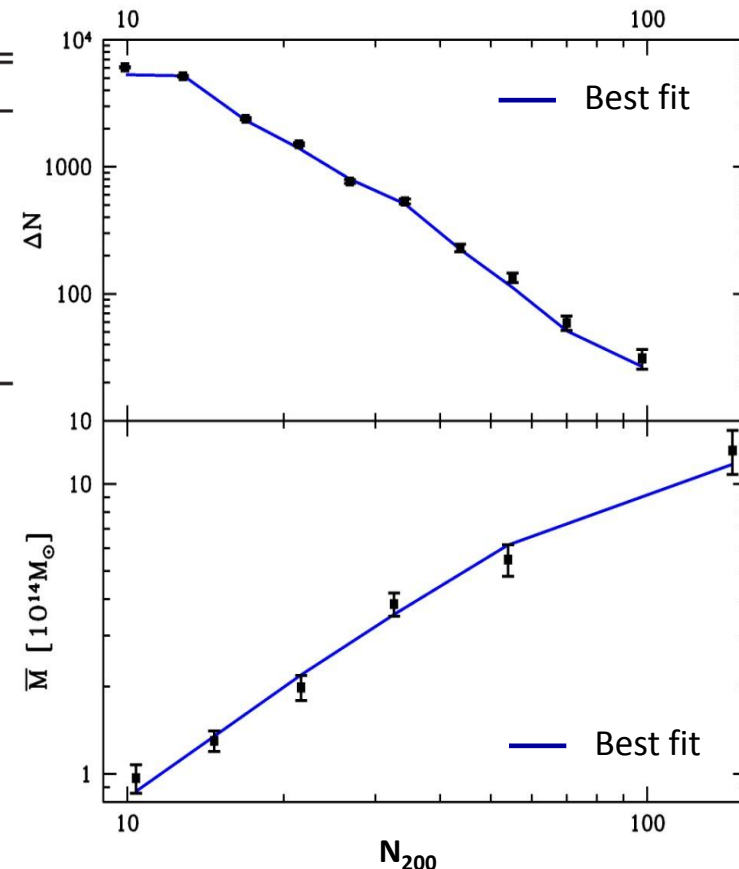
- Weak Lensing masses

[Sheldon et al. 2009,  
Johnston et al. 2007]

$$12 < N_{200} < 300$$

Richness	$\langle M_{200b} \rangle [10^{14} M_{\odot}]$
12-17	1.298
18-25	1.983
26-40	3.846
41-70	5.475
71+	13.03

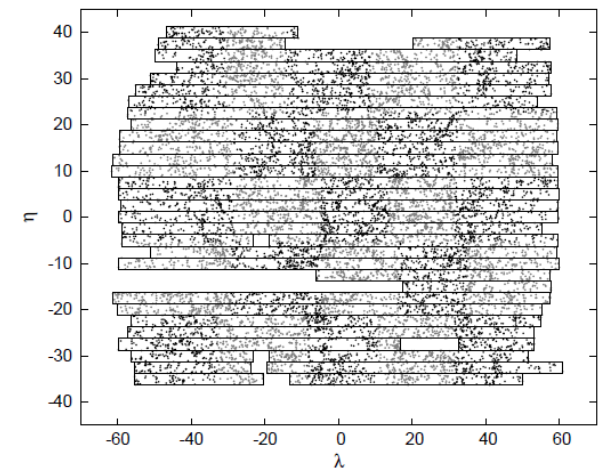
$\sim 10\%$  errors + offset factor  $\beta$   
with a prior width of 6%



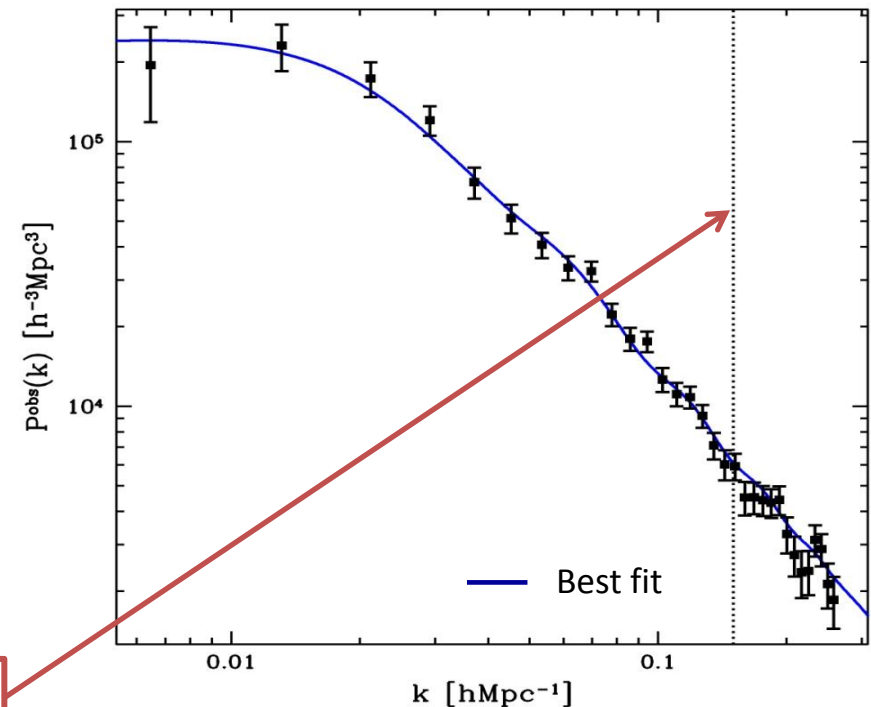
# MaxBCG Power Spectrum

## ● Power spectrum [Hütsi 2010]

1. Determination of **survey selection function** by a random catalog with 1,261,600 objects.
2. Calculation of **overdensity field** on a grid using TSC [Hockney & Eastwood 1988] mass assignment scheme.
3. **FFT** of the gridded overdensity field.
4. The raw 3D power spectrum is the **modulus squared of the FFT**.
5. Subtraction of the **shot noise**.
6. Recovery of the angle averaged spectrum using a modified version of iterative scheme of Jing (2005).
7. Error estimates: FKP [Feldman et al. 1994], **Monte Carlo** and 'jackknife'.
8. Negligible off-diagonal covariance between clustering and counts.



Top: Ang. distribution of maxBCG clusters and angular mask. Credit: Hütsi (2010).  
Bottom:  $P(k)$  data and our best fit model.



Quasi-linear regime only:  $k_{\max} = 0.15 \text{ hMpc}^{-1}$

# Theoretical modelling: mass-richness relation

Probability of observing  $N_{\text{gal}}^{\text{obs}}$  member galaxies given the true mass  $M$

$$p(N_{\text{gal}}^{\text{obs}} | M) = \int p(N_{\text{gal}}^{\text{obs}} | N_{\text{gal}}) p(N_{\text{gal}} | M) dN_{\text{gal}}$$

$$p(N_{\text{gal}}^{\text{obs}} | N_{\text{gal}}) = \frac{1}{\sqrt{2\pi\sigma^2_{\ln N_{\text{gal}}^{\text{obs}} | M}}} \exp[-x^2(N_{\text{gal}}^{\text{obs}})]$$

$$x(N_{\text{gal}}^{\text{obs}}) = \frac{\ln N_{\text{gal}}^{\text{obs}} - \ln N_{\text{gal}}(M)}{\sqrt{2\sigma^2_{\ln N_{\text{gal}}^{\text{obs}} | M}}}$$

[Lima, Hu 2005]

Delta function

# Theoretical modelling: mass-richness relation

Probability of observing  $N_{\text{gal}}^{\text{obs}}$  member galaxies given the true mass  $M$

$$p(N_{\text{gal}}^{\text{obs}} | M) = \int p(N_{\text{gal}}^{\text{obs}} | N_{\text{gal}}) p(N_{\text{gal}} | M) dN_{\text{gal}}$$

$$p(N_{\text{gal}}^{\text{obs}} | N_{\text{gal}}) = \frac{1}{\sqrt{2\pi\sigma^2_{\ln N_{\text{gal}}^{\text{obs}} | M}}} \exp[-x^2(N_{\text{gal}}^{\text{obs}})]$$

$$x(N_{\text{gal}}^{\text{obs}}) = \frac{\ln N_{\text{gal}}^{\text{obs}} - \ln N_{\text{gal}}(M)}{\sqrt{2\sigma^2_{\ln N_{\text{gal}}^{\text{obs}} | M}}}$$

[Lima, Hu 2005]

Delta function

- **Power law in mass** [Rozo et al. 2010, Johnston et al. 2007], fitted by fixing 2 pivot points in mass ( $M_1=1.3 \times 10^{14} M_{\odot}$ ,  $M_2=1.3 \times 10^{15} M_{\odot}$ )

$$\ln M = \ln M_{200|20} + \alpha_N \ln(N_{\text{gal}}/20)$$

with scatter  $\sigma_{\ln M | N_{\text{gal}}^{\text{obs}}} = 0.45^{+0.20}_{-0.18}$   
[Rozo 2009]

Intercept (mass of a cluster with 20 member galaxies)

slope



# Theoretical modelling: Number Counts

- Tinker's universal mass function [Tinker et al. 2010]

Expected nr. density of virialized DM haloes

$$\frac{dn(M, z)}{d \ln M} = \bar{\rho}_m \left| \frac{d \ln \sigma^{-1}}{dM} \right| f(\nu)$$

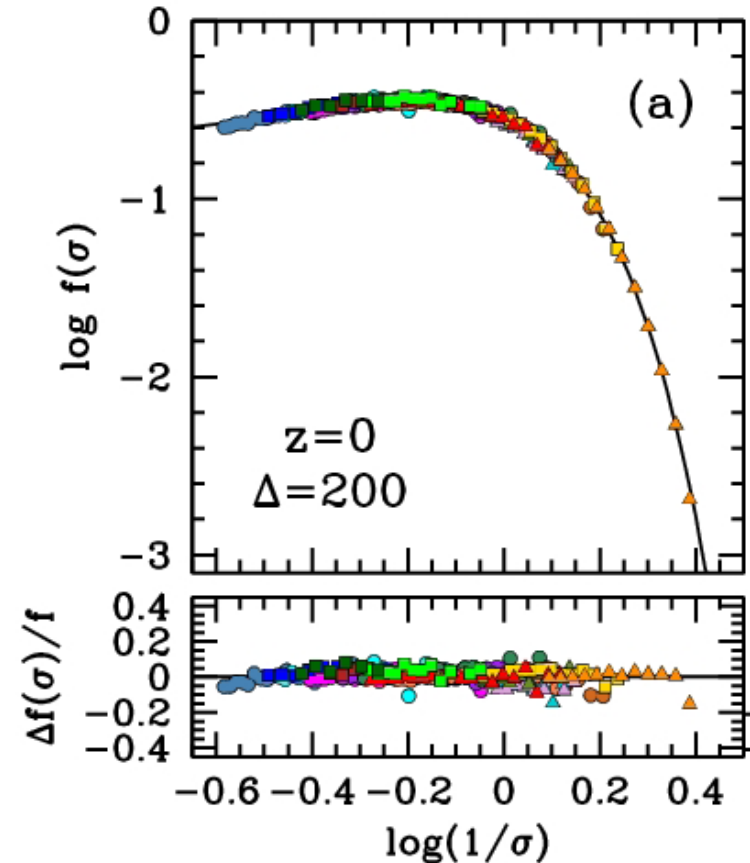
mean matter density

$$f_T(\nu) = 0.368 \left[ 1 + (\hat{\beta}\nu)^{-2\hat{\phi}} \right] \nu^{2\hat{\eta}+1} e^{-\hat{\gamma}\nu^2/2}$$

$$\nu = \frac{\delta_c}{\sigma(M)}$$

$$\hat{\beta} = 0.589 (1+z)^{0.20}, \quad \hat{\phi} = -0.729 (1+z)^{-0.08},$$

$$\hat{\eta} = -0.243 (1+z)^{0.27}, \quad \hat{\gamma} = 0.864 (1+z)^{-0.01}.$$



Mass function calibrated with simulations.  
Credit: Tinker et al. (2008).

# Theoretical modelling: Number Counts

- Clusters abundances and total masses in richness bins

Average  
cluster nr.  
density  
in a bin

$$n_i = \int_{N_{\text{gal},i}^{\text{obs}}}^{N_{\text{gal},i+1}^{\text{obs}}} d \ln N_{\text{gal}}^{\text{obs}} \int d \ln N_{\text{gal}} \frac{dn}{d \ln N_{\text{gal}}} p(N_{\text{gal}}^{\text{obs}} | N_{\text{gal}}) =$$

$$= \int d \ln N_{\text{gal}} \frac{dn}{d \ln N_{\text{gal}}} \frac{1}{2} [\text{erfc}(x_i) - \text{erfc}(x_{i+1})],$$

$$x_i \equiv x(N_{\text{gal},i}^{\text{obs}})$$

$$\frac{dn}{d \ln N_{\text{gal}}} = \frac{dn}{d \ln M} \cdot \frac{d \ln M}{d \ln N_{\text{gal}}} = \alpha_N \frac{dn}{d \ln M}$$

Mass function      Jacobian

# Theoretical modelling: Number Counts

- Clusters abundances and total masses in richness bins

Average cluster nr. density in a bin

$$n_i = \int_{N_{\text{gal},i}^{\text{obs}}}^{N_{\text{gal},i+1}^{\text{obs}}} d \ln N_{\text{gal}} \int d \ln N_{\text{gal}} \frac{dn}{d \ln N_{\text{gal}}} p(N_{\text{gal}}^{\text{obs}} | N_{\text{gal}}) =$$

$$= \int d \ln N_{\text{gal}} \frac{dn}{d \ln N_{\text{gal}}} \frac{1}{2} [\text{erfc}(x_i) - \text{erfc}(x_{i+1})],$$

$$x_i \equiv x(N_{\text{gal},i}^{\text{obs}})$$

$$\frac{dn}{d \ln N_{\text{gal}}} = \frac{dn}{d \ln M} \cdot \frac{d \ln M}{d \ln N_{\text{gal}}} = \alpha_N \frac{dn}{d \ln M}$$

Mass function      Jacobian

Total cluster number per bin

$$\Delta N_i = \Delta \Omega \int_{z_{\text{min}}}^{z_{\text{max}}} dz \frac{d^2 V}{dz d \Omega} n_i$$

Survey sky coverage

Volume element

Total mass of clusters per bin

$$(\Delta N \bar{M})_i = \beta \Delta \Omega \int_{z_{\text{min}}}^{z_{\text{max}}} dz \frac{d^2 V}{dz d \Omega} (nm)_i$$

Nuisance parameter

Average total mass

# Theoretical modelling: Power Spectrum

---

- **Tinker's analytical bias**  $b_T(M, z) \simeq 1 + \frac{\hat{\gamma}v^2 - (1 + 2\hat{\eta})}{\delta_c} + \frac{2\hat{\phi}/\delta_c}{1 + [\hat{\beta}v]^{2\hat{\phi}}}$   
*[Tinker et al. 2010]*
- **Average bias** *[Lima, Hu 2005]*  $\bar{b}(z) = \frac{1}{\Delta N} \int_{M_{\min}}^{M_{\max}} d \ln M \frac{dn(M, z)}{d \ln M} b_T(M, z)$

# Theoretical modelling: Power Spectrum

- **Tinker's analytical bias**  $b_T(M, z) \simeq 1 + \frac{\hat{\gamma}v^2 - (1 + 2\hat{\eta})}{\delta_c} + \frac{2\hat{\phi}/\delta_c}{1 + [\hat{\beta}v]^2}$   
[Tinker et al. 2010]

- **Average bias** [Lima, Hu 2005]  $\bar{b}(z) = \frac{1}{\Delta N} \int_{M_{\min}}^{M_{\max}} d \ln M \frac{dn(M, z)}{d \ln M} b_T(M, z)$

- **Power spectrum** [Hütsi 2010]

$$P_{\text{NL}}(k) = \left( b^{\text{obs}} \right)^2 \left( 1 + q_{\text{NL}} k^{3/2} \right) s(k) P_{\text{lin}}(k) \left[ 1 + \frac{2}{3} \beta_z + \frac{1}{5} \beta_z^2 \right] + \text{Convolution} + \text{AP effect}$$

[AP 1979]

Effective bias

$$b^{\text{obs}} = \bar{b} \cdot B$$

Nuisance parameter

Non-linearity  
correction  
( $q_{\text{NL}}$  nuisance  
parameter)

Photo-z smoothing  
(Gaussian photo-z)

$$s(k) = \left( \frac{\sqrt{\pi}}{2\sigma_z k} \right) \text{erf}(\sigma_z k)$$

Nuisance parameter

Redshift-space  
distortions  
[Kaiser 1987]

# Theoretical modelling: PNG

- Local type ( $z^*$  primordial)

$$\Phi(\mathbf{x}, z_*) = \varphi(\mathbf{x}, z_*) + f_{\text{NL}} \left[ \varphi^2(\mathbf{x}, z_*) - \langle \varphi^2 \rangle(z_*) \right]$$

- LoVerde mass function [LoVerde et al. 2008] and derived bias

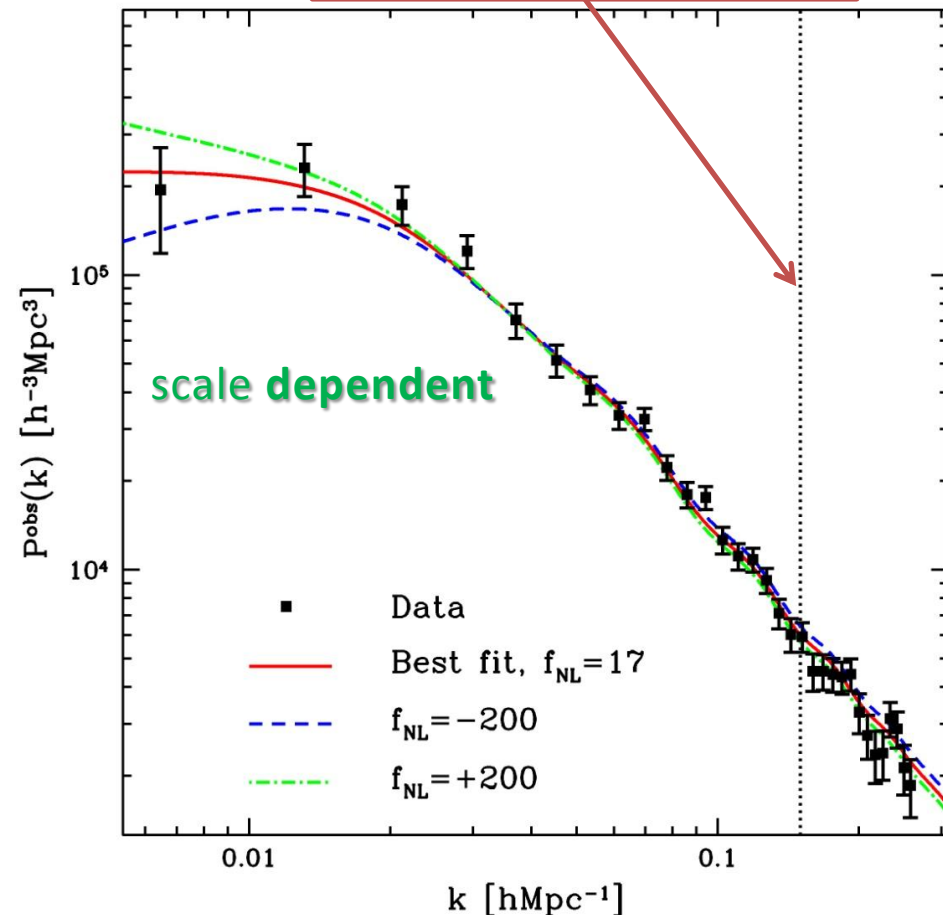
$$f_{\text{LV}}(\nu) = \sqrt{\frac{2}{\pi}} e^{-\frac{\nu^2}{2}} \left[ \nu + S_3 \frac{\sigma}{6} (\nu^4 - 2\nu^2 - 1) + \frac{dS_3}{d \ln \sigma} \frac{\sigma}{6} (\nu^2 - 1) \right]$$

$$b_{\text{LV}}^L(\nu) = \frac{\delta_c}{\sigma^2} - \frac{1}{\sigma} \frac{6 + S_3 \sigma (4\nu^3 - 4\nu) + 2 \frac{dS_3}{d \ln \sigma} \sigma \nu}{6\nu + S_3 \sigma (\nu^4 - 2\nu^2 - 1) + \frac{dS_3}{d \ln \sigma} \sigma (\nu^2 - 1)}$$

- Scale independent (small scales)  
+ scale dependent (large scales)  
corrections to the cluster bias

Quasi-linear regime only:

$$k_{\text{max}} = 0.15 \text{ hMpc}^{-1}$$



# Monte Carlo Markov Chain analysis

- 3 data sets + cov. matrices  $\pm$  CMB (WMAP7)
- assume flat  $\Lambda$ CDM cosmology



**Cosmological Monte Carlo\***

Type	Symbol	Definition	Prior without CMB	Prior with CMB
Cosmology	$h$	Dimensionless Hubble parameter	0.7	[0.4, 0.9]
	$n_s$	Scalar spectral index	0.96	[0.5, 1.5]
	$\Omega_b$	Baryon energy density	0.04397	[0.01, 0.2]
	$\Omega_c$	Cold dark matter energy density	[0.1, 0.9]	[0.1, 0.9]
	$\log(10^{10} A_s)$	Amplitude of primordial perturbations	[0.1, 6.0]	[0.1, 6.0]
	$\tau$	Optical depth	0.09	[0.01, 0.125]
	$f_{NL}$	Primordial non-Gaussianity amplitude	[-900, 900]	[-900, 900]
Scaling relation	$\ln N_1 \equiv \ln N_{gal} M_1$	Richness at $M_1 = 1.3 \times 10^{14} M_\odot$	[1.0, 4.0]	[1.0, 4.0]
	$\ln N_2 \equiv \ln N_{gal} M_2$	Richness at $M_2 = 1.3 \times 10^{15} M_\odot$	[3.0, 6.0]	[3.0, 6.0]
	$\sigma_{\ln M N_{gal}^{obs}}$	Scatter	$0.45 \pm 0.1$	$0.45 \pm 0.1$
Nuisance	$\beta$	Weak lensing mass measurements bias	$1.0 \pm 0.06$	$1.0 \pm 0.06$
	$B$	Scatter on bias derived from mass function	$1.0 \pm 0.15$	$1.0 \pm 0.15$
	$q_{NL}$	Non-linear correction to power spectrum	[0.0, 50.0]	[0.0, 50.0]
	$\sigma_z$	Photo-Z errors	[0, 120]	[0, 120]
	$A_{SZ}$	Amplitude of CMB SZ template	1	[0, 2]
Derived	$\Omega_m$	Total matter energy density	—	—
	$\sigma_8$	Amplitude of density perturbations	—	—

[Mana et al. 2013]

# Results - *Mana et. al 2013*

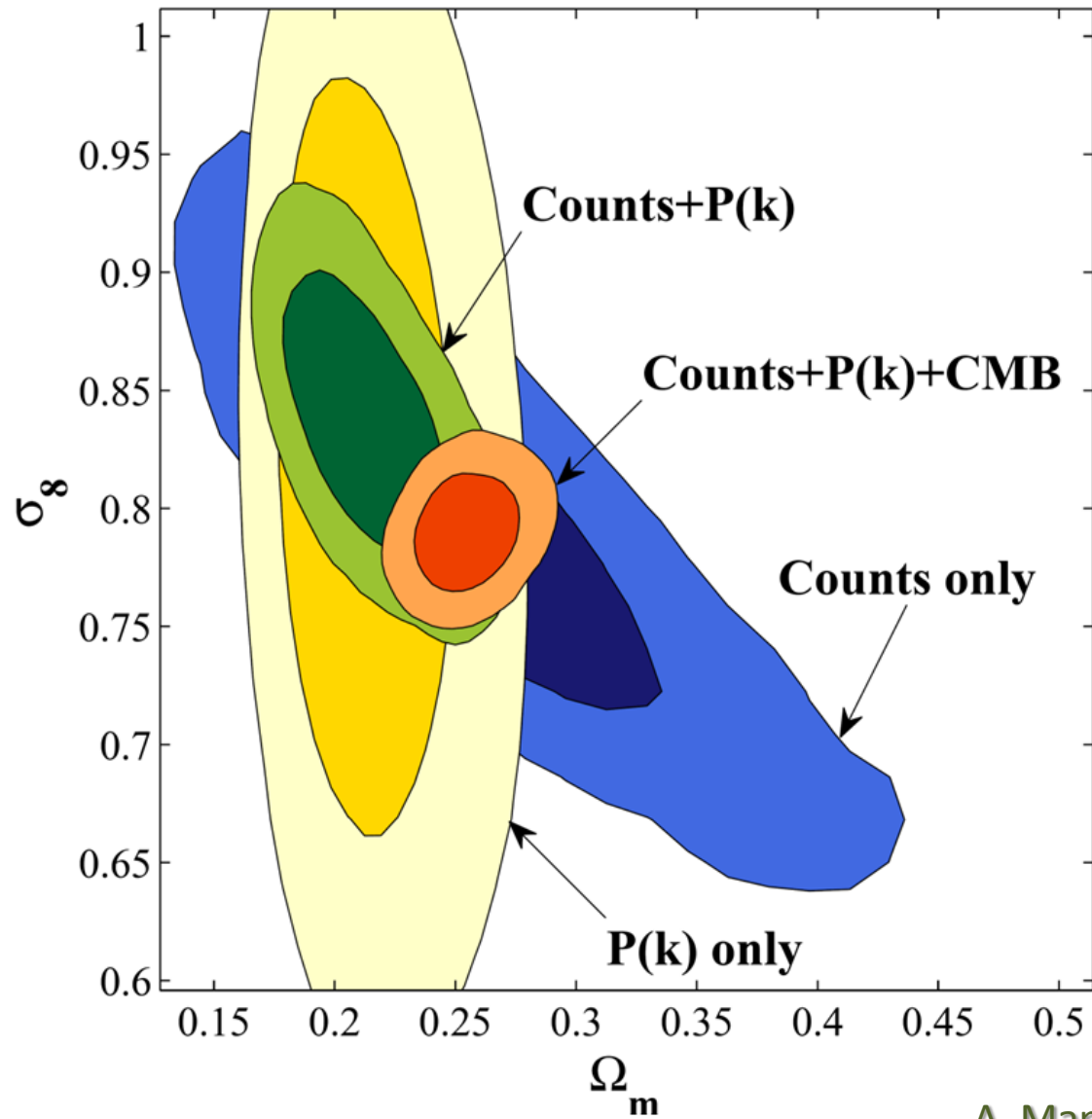
Params	Counts only		Counts+ $P(k)$		Clusters+CMB	
	no $f_{NL}$	+ $f_{NL}$	no $f_{NL}$	+ $f_{NL}$	no $f_{NL}$	+ $f_{NL}$
$\Omega_m$	0.25 $\pm$ 0.06	0.25 $\pm$ 0.06	0.215 $\pm$ 0.022	0.209 $\pm$ 0.022	0.255 $\pm$ 0.014	0.248 $\pm$ 0.013
$\sigma_8$	0.80 $\pm$ 0.06	0.77 $\pm$ 0.07	0.84 $\pm$ 0.04	0.85 $\pm$ 0.05	0.790 $\pm$ 0.016	0.780 $\pm$ 0.016
$\ln N_1$	2.44 $\pm$ 0.11	2.44 $\pm$ 0.11	2.49 $\pm$ 0.09	2.49 $\pm$ 0.08	2.44 $\pm$ 0.08	2.43 $\pm$ 0.08
$\ln N_2$	4.16 $\pm$ 0.15	4.15 $\pm$ 0.15	4.13 $\pm$ 0.13	4.11 $\pm$ 0.12	4.19 $\pm$ 0.11	4.15 $\pm$ 0.11
$\sigma_{\ln M}$	0.38 $\pm$ 0.06	0.38 $\pm$ 0.06	0.36 $\pm$ 0.06	0.37 $\pm$ 0.06	0.378 $\pm$ 0.059	0.38 $\pm$ 0.06
$\beta$	1.00 $\pm$ 0.06	1.01 $\pm$ 0.06	1.01 $\pm$ 0.06	1.01 $\pm$ 0.06	1.01 $\pm$ 0.06	1.00 $\pm$ 0.06
$q_{NL}$	-	-	26 $\pm$ 10	27 $\pm$ 10	14 $\pm$ 6	16 $\pm$ 7
$\sigma_z$	-	-	46 $\pm$ 12	42 $\pm$ 8	43 $\pm$ 10	31 $\pm$ 5
$B$	-	-	1.07 $\pm$ 0.13	1.01 $\pm$ 0.15	1.19 $\pm$ 0.11	1.00 $\pm$ 0.14
$f_{NL}$	-	282 $\pm$ 317	-	12 $\pm$ 157	-	194 $\pm$ 128

Marginalised mean values and  $1\sigma$  errors on the cosmological parameters.

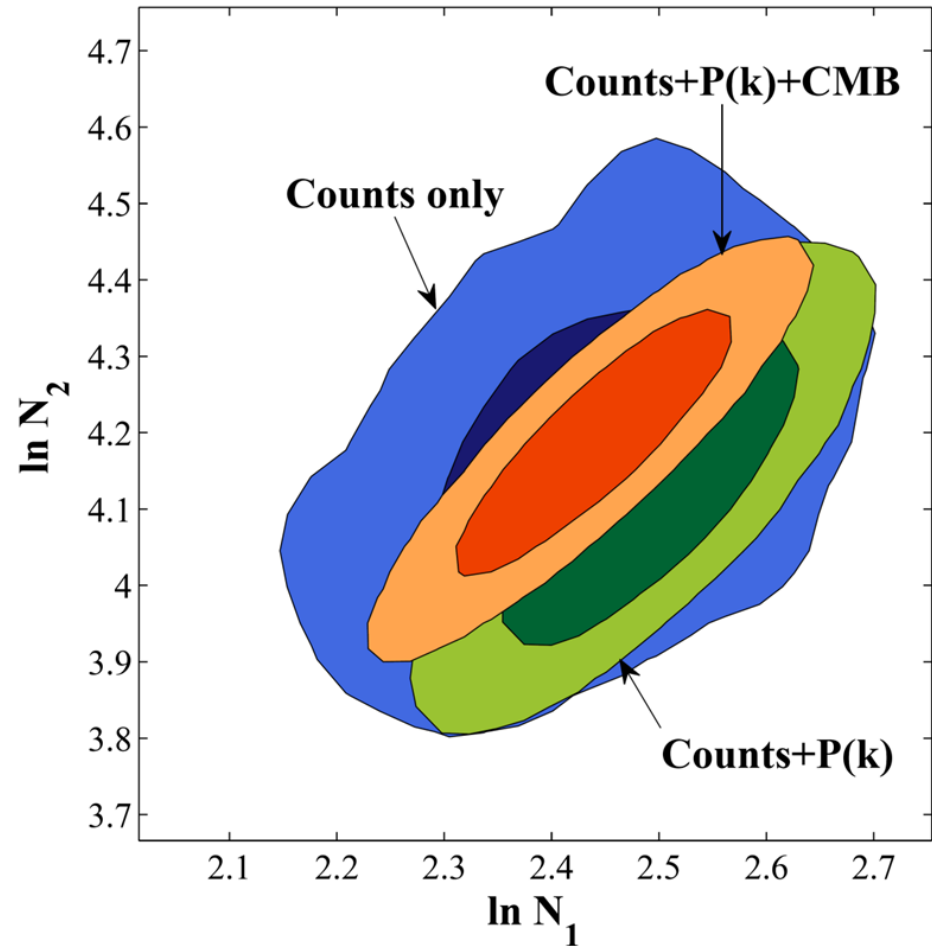
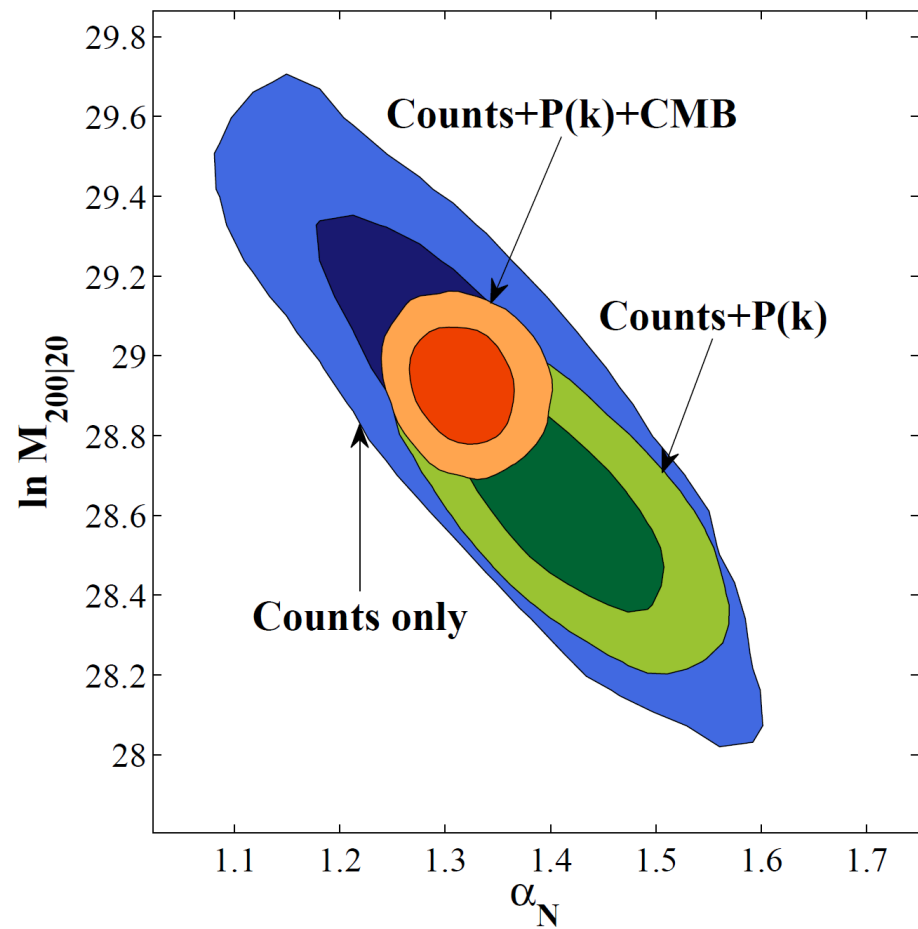
**The errors are significantly reduced by the addition of  $P(k)$ .**



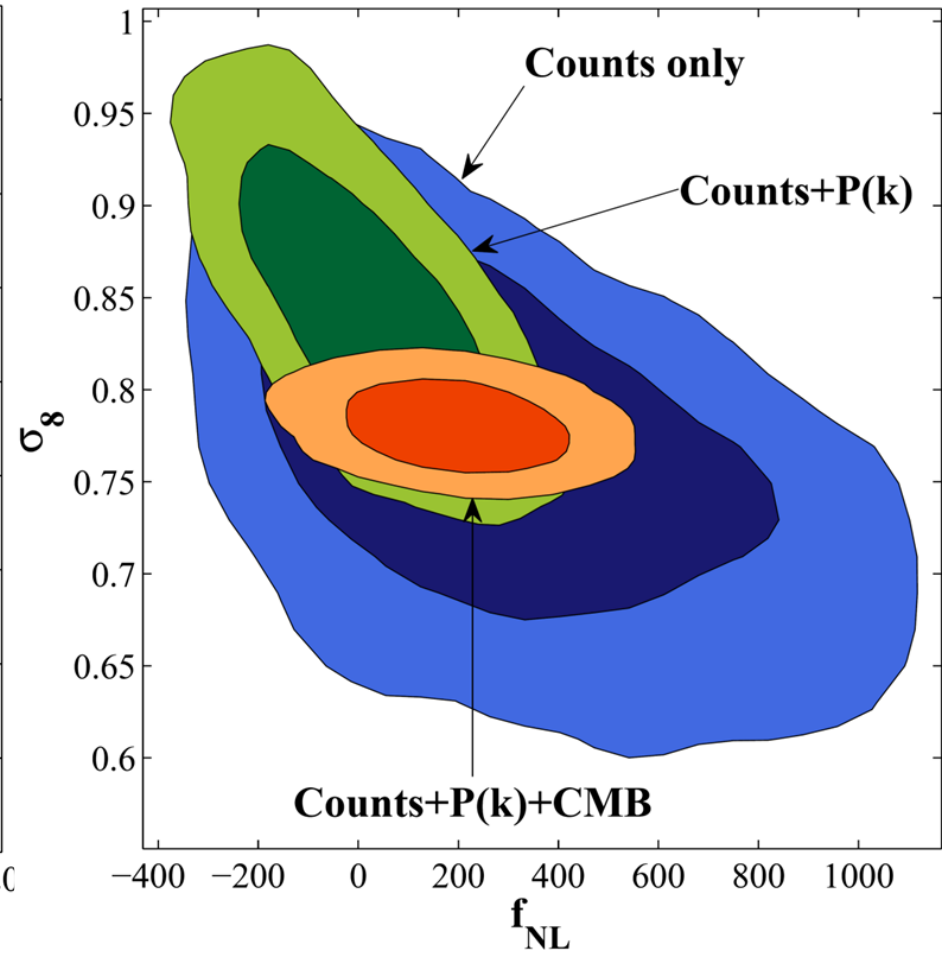
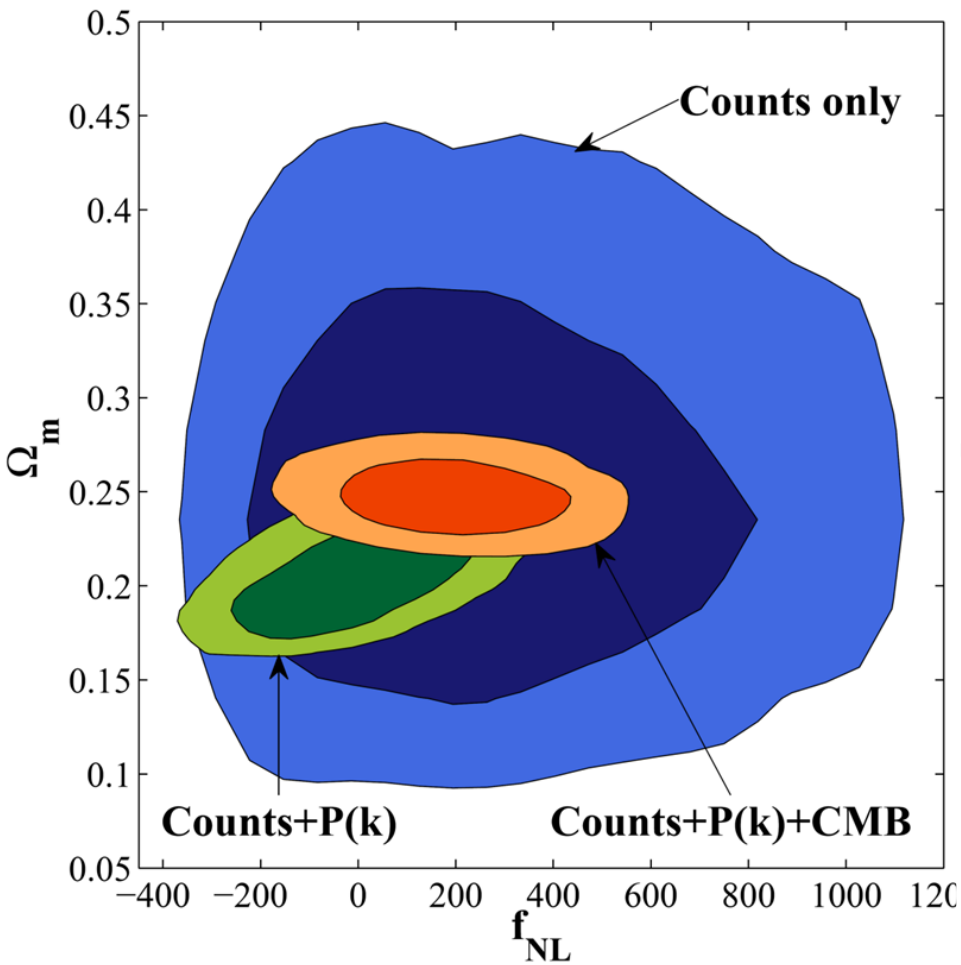
# Results - *Mana et. al 2013*



# Results - *Mana et. al 2013*



# Results - *Mana et. al 2013*



# Conclusions and future work

---

- Combining one and two points statistics, we achieved a factor 1.5 to 3 **improvement on the errors on cosmological constraints**, if compared with previous analyses using counts and masses only (Rozo et al. 2010).
- **PNG** has been tested through non-Gaussian halo mass function and scale-dependent cluster bias, providing **independent and consistent constraints** with the clustering of clusters.
- We are currently updating CosmoMC to include Planck likelihoods.
- We will possibly investigate other catalogues and add new probes (i.e. cross-correlation galaxies-clusters).

# Thanks for the attention

## References

**Mana, A.\*, Giannantonio, T., Weller, J., et al., 2013, arXiv:1303.0287, MNRAS accepted**

Rozo E., Wechsler R. H., Rykoff E. S., et al., 2010, ApJ, 708, 645

Sheldon E. S., Johnston D. E., Scranton R., et al., 2009, ApJ, 703, 2217

Hütsi G., 2010, MNRAS, 401, 2477

Koester B. P., McKay T. A., Annis J., et al., 2007, ApJ, 660, 221 and 239

Tinker J. L., Robertson B. E., Kravtsov A. V., et al., 2010, ApJ, 724, 878