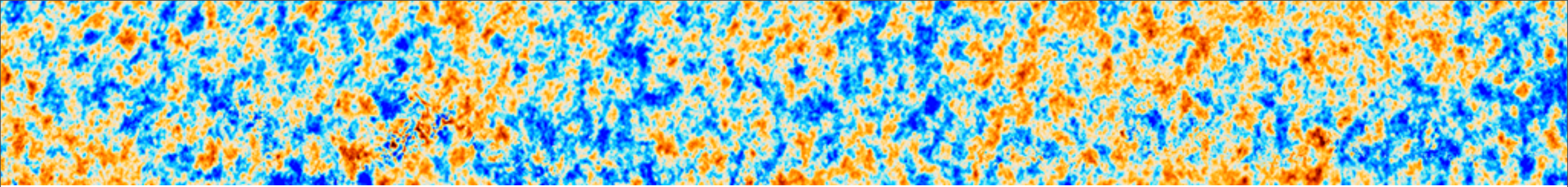


Cosmic Strings post Planck

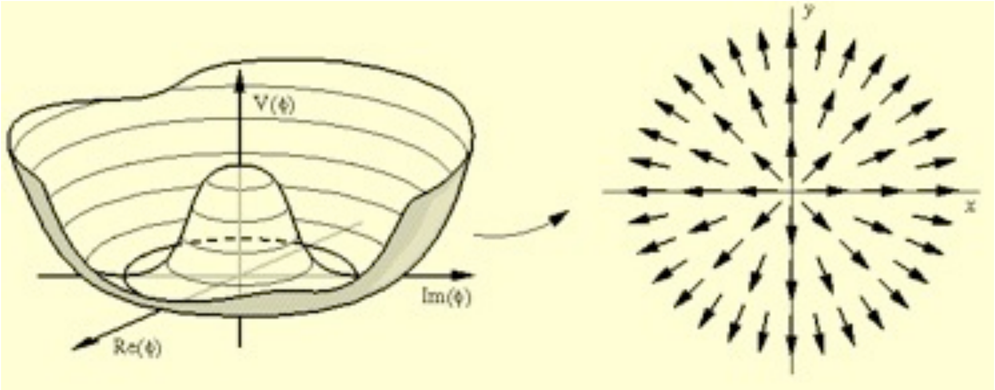
Adam Moss

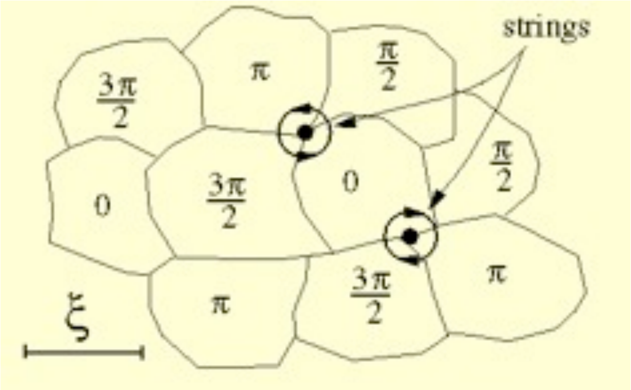
University of Nottingham

Cosmic Strings

planck

- Strings can form during phase transitions in early Universe



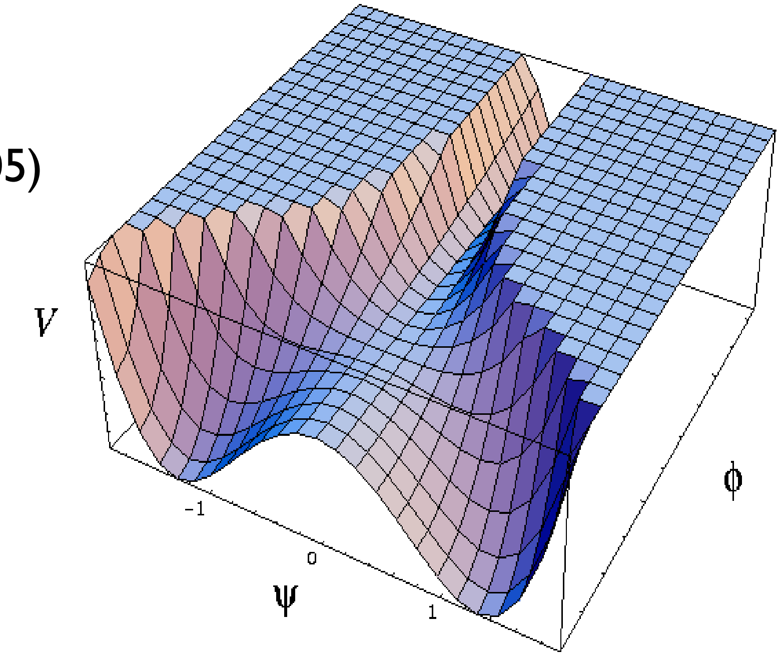


(Kibble 1976)

- Ruled out as primary mechanism to produce anisotropies

(E.g. Polchinski 2005)

- Formed in hybrid, brane inflation etc models
- GUT scale phase transition gives $\Delta T/T \approx 10^{-5}$
- Potential window to high energy physics



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Observables

- CMB power spectrum. Includes recombination and post-recombination physics. Strongest limit on allowed fraction of strings, improved by Planck
- Non Gaussianity in CMB maps. Search for signatures of post-recombination Doppler shift induced by moving strings (Kaiser-Stebbins effect). Strongest limit also from Planck
- CMB B modes. Defects produce comparable scalar, vector and tensor fluctuations.
- Gravitational lensing. Search for microlensing events (not discussed here)
- Pulsar timing/gravitational waves. Stochastic background from loops, waves from cusps (not discussed here)

String Evolution

- Perturbations from strings are **active**, so their evolution is key to understanding CMB anisotropies
- Main idea is strings evolve toward self-similar **scaling** regime
- Average properties of network are (nearly) constant with time
- Dynamics can be studied using numerical simulations
- Two approaches - **Nambu** and **Abelian-Higgs** models
- Both have advantages and disadvantages
- Main issue is **dynamical range** - assumptions have to be made in either case
- Will present Planck constraints for each case

Nambu Model

- Thin string approximation
- Ignore radiation back reaction
- Impose reconnection by hand
- Network characterised by **correlation length** L
- Energy density is $$\rho = \frac{\mu}{L^2},$$
- Observationally, string tension μ is main quantity of interest
- Find scaling solution L ~ t
- Make measurements of correlation length, velocity, small scale structure (wiggliness) ......

(Martin and Shellard)

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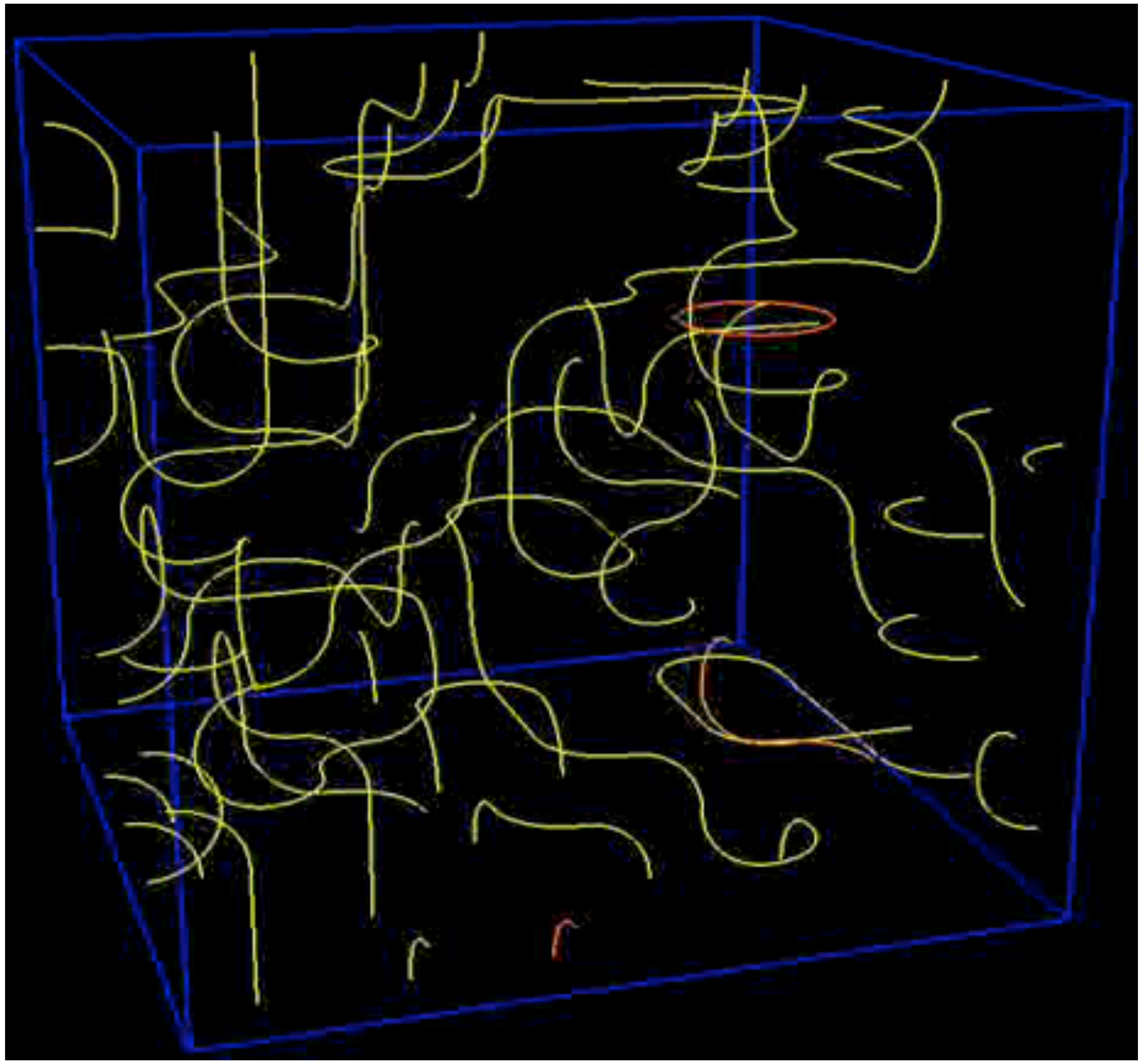

(Martin and Shellard)

VOS Model

planck

- VOS = Velocity One-Scale model (Kibble 1985, Martin and Shellard 1996/2000)
- Expansion stretches strings, long strings reconnect and loops decay
- From Nambu action

Loop production, calibrated from simulations

Curvature term, proportional to RMS velocity v

$$\frac{1}{L}\frac{dL}{dt} = (1+v^2)H + \frac{\tilde{c}v}{2L}, \qquad \frac{dv}{dt} = (1-v^2)\left(\frac{\tilde{k}}{L} - 2Hv\right),$$

- For power law expansion $a(t) \propto t^\beta$, find attractor solution with scaling

$$L = \epsilon t, \qquad \epsilon = \sqrt{\frac{\tilde{k}(\tilde{k}+\tilde{c})}{4\beta(1-\beta)}}, \qquad v = \sqrt{\frac{\tilde{k}(1-\beta)}{\beta(\tilde{k}+\tilde{c})}},$$

- More complicated VOS models can be constructed for superstrings, networks with junctions etc.
- Will use comoving correlation length $l = \frac{L}{a} = \xi\tau,$

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VOS Parameters

- Simulations find network parameters vary between matter and radiation eras
- Density in radiation era is greater than in matter era
- Simulations of Martins and Shellard find

$$\xi_{\rm rad} = 0.13 \rightarrow \xi_{\rm mat} = 0.21 \qquad v_{\rm rad} = 0.65 \rightarrow v_{\rm mat} = 0.60$$

- Ringeval et al find

$$\xi_{\rm rad} = 0.16 \rightarrow \xi_{\rm mat} = 0.19$$

- Strings also have small scale structure, or 'wiggliness' (Carter 2000)
- Effective coarse grained energy momentum tensor gives rescaled mass per unit length $U = \alpha\mu,$
- Estimated to be

$$\alpha_{\rm rad} = 1.5 \rightarrow \alpha_{\rm mat} = 1.9$$

Abelian Higgs

- Lagrangian $\mathcal{L} = (D_\mu \Phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\Phi)$
- Solve equations of motion on 3D lattice (Bevis et al)
- Extract unequal time correlator (UETC) of energy momentum tensor

$$\langle \Theta(k, \tau_1) \Theta(k, \tau_2) \rangle$$

- Radiation into propagating modes included
- However, simulation box sizes limited to ~ 300x string core width
- Requires slowing down rate of growth of core
- Found only very small loop production - most of energy loss to radiation
- See no observed small scale structure
- Similar network parameters in matter and radiation eras

$$\xi = 0.3 \qquad v = 0.5$$ (Hindmarsh et al 2009)

- More work required to resolve differences with Nambu model

CMB Anisotropies

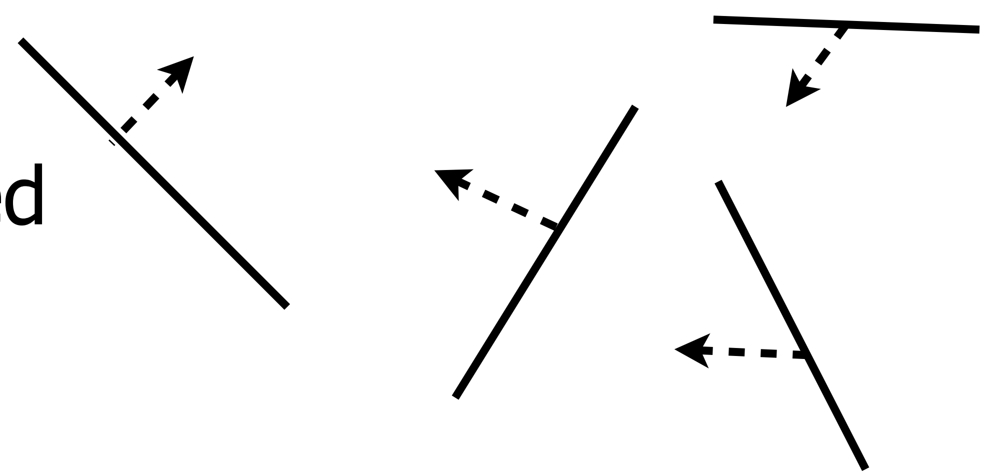
planck

- **Key** ingredient is the UETC
- Use stiff source approximation

String energy momentum tensor

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \qquad \delta G_{\mu\nu} = 8\pi G \left(\delta T_{\mu\nu} + \theta_{\mu\nu}\right)$$

- Can estimate UETC directly from simulations, and use as sources in CMB codes
- For thin strings there is a useful intermediate framework called Unconnected Segment Model (USM) (Vincent et al, Albrecht et al, Pogosian and Vachaspati)
- Model strings are ensemble of uncorrelated straight segments, each moving with random velocity
- Inputs are correlation length, velocity and wiggliness, as measured from simulations



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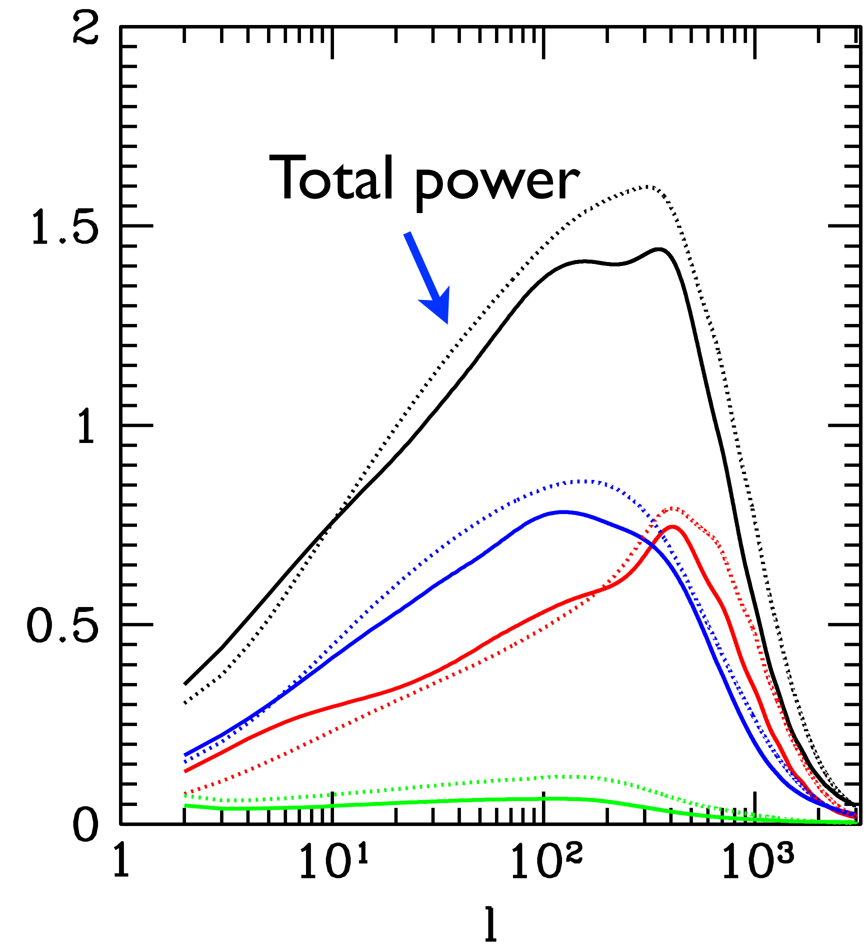
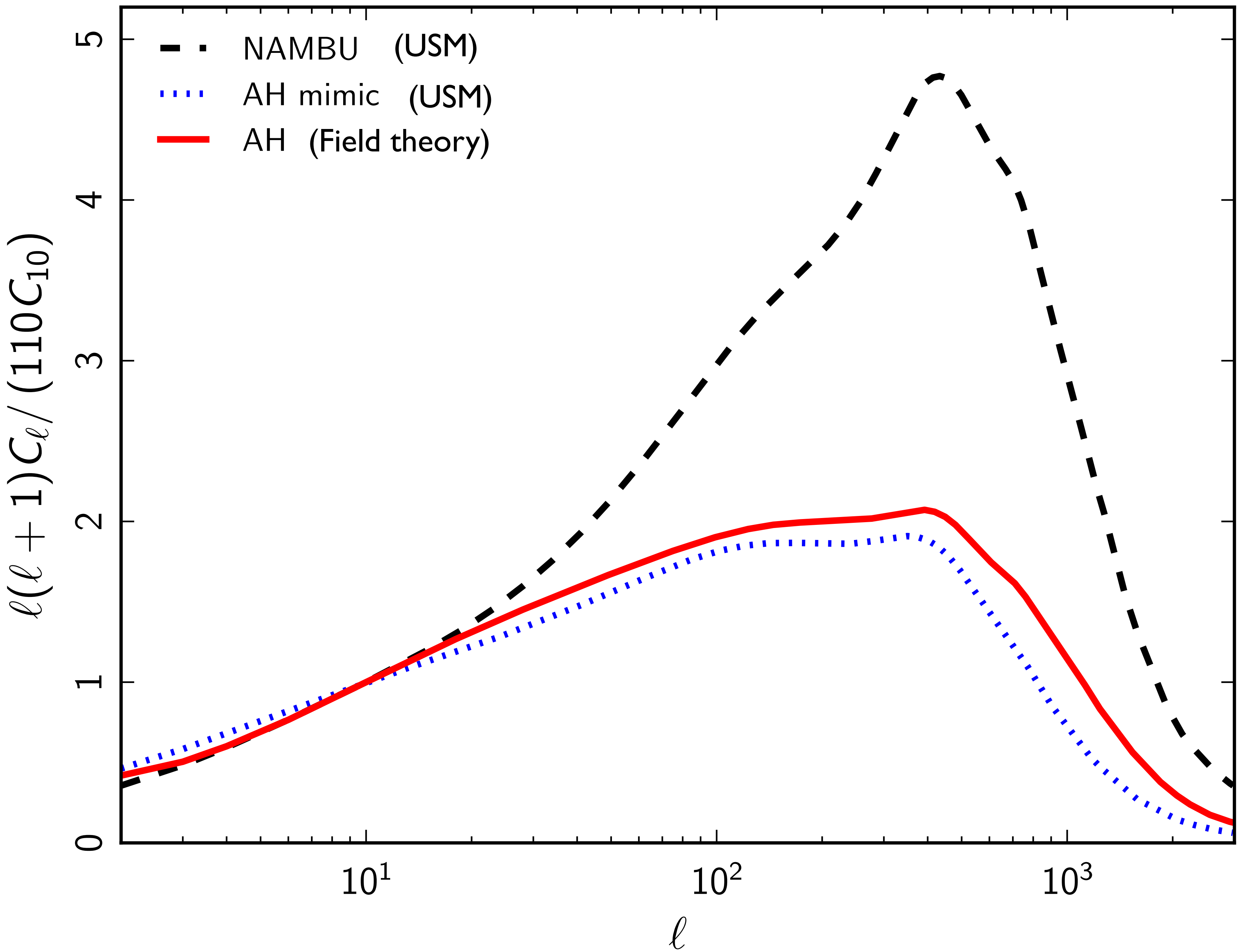
Mimic Model

- Mimic model computed with USM, choosing parameters to more closely resemble Abelian-Higgs
- Turn off evolution of network parameters between matter and radiation eras
- Choose $\xi = 0.35 \quad v = 0.4 \quad \alpha = 1.05$
- Encouragingly find closer agreement between spectra
- Remember Nambu suffers less of an issue with dynamical range



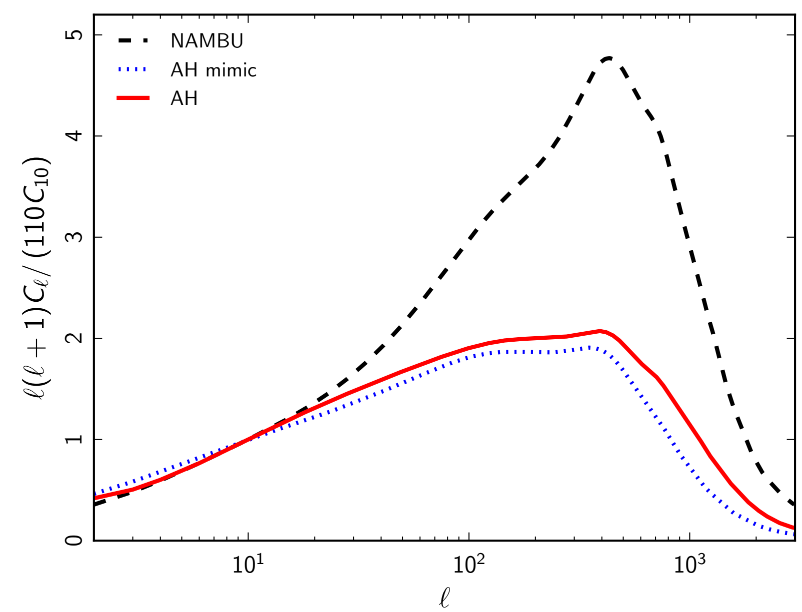
- Abelian-Higgs has advantage of including radiation
- Even with simple physics USM does a decent job of fitting both!

Power Spectrum



Power Spectrum

- Note: Amplitude set by mass per unit length $C_\ell^{\rm string} \propto (G\mu)^2$
- Two primary contributions to spectrum - density perturbations at last scattering, Kaiser-Stebbins effect along line of sight
- Peak position set by correlation length at last scattering
- Why the difference between Nambu and Abelian-Higgs?
- Normalisation determined by correlation length
- Nambu model has more small scale power - smaller string correlation length in radiation era, by factor of **~2**
- Different split into scalar, vector and tensors - small scale structure of strings



USM

- Let's now revisit the USM. In existing codes (CMBACT) the UETC is **not** computed
- Rather an ensemble of source histories are created, then averaged to find power spectra
- The EM tensor of a straight string segment is

$$\Theta_{00} = \frac{\mu\alpha}{\sqrt{1-v^2}} \frac{\sin(k\hat{X}_3\xi\tau/2)}{k\hat{X}_3/2} \cos(\chi + k\hat{\dot{X}}_3 v\tau), \qquad \Theta_{ij} = \left[v^2\hat{\dot{X}}_i\hat{\dot{X}}_j - \frac{1-v^2}{\alpha}\hat{X}_i\hat{X}_j\right]\Theta_{00},$$

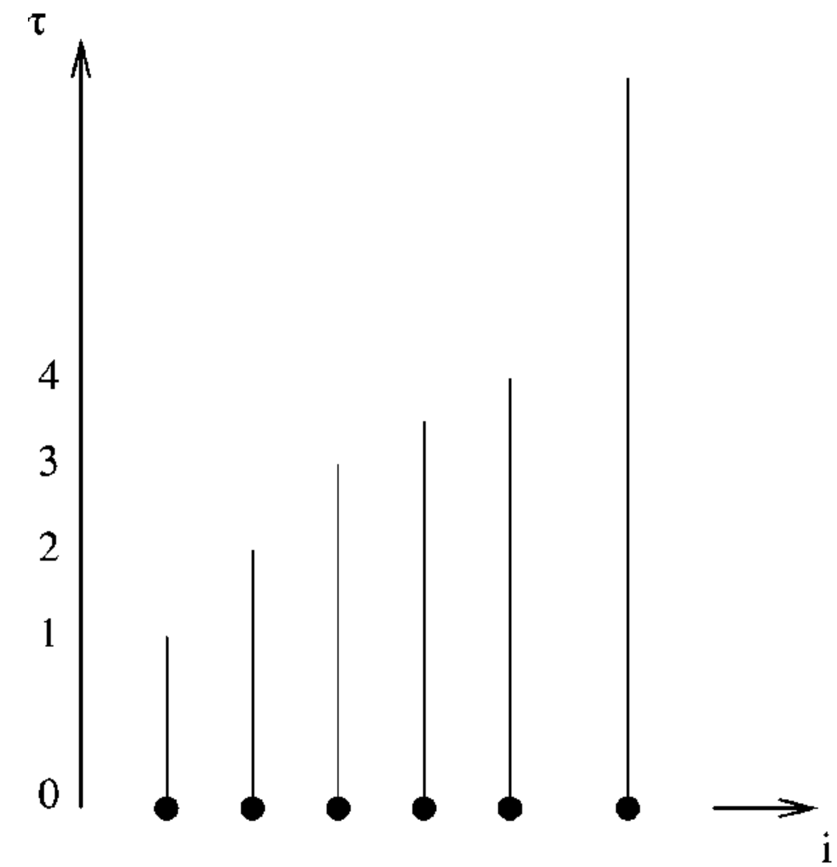
- $\hat{X}$ and $\hat{\dot{X}}$ are randomly orientated unit vectors satisfying $\hat{X}\cdot\hat{\dot{X}} = 0$
- Phase χ set by location of string
- Orientated wave vector as $\mathbf{k} = k\hat{k}_3$, perform scalar, vector, tensor split

$$\Theta^S = (2\Theta_{33} - \Theta_{11} - \Theta_{22})/2, \qquad \Theta^V = \Theta_{13}, \qquad \Theta^T = \Theta_{12}.$$

USM

- During scaling number density of strings scales as $n(\tau) \propto \tau^{-3}$, requires tracking very large number of segments
- USM consolidates all segments that decay at some time into a **single** segment
- Number of segments that decay between τ_i and τ_{i-1} is

$$N_d(\tau_i) = V[n(\tau_{i-1}) - n(\tau_i)],$$



- EM tensor of network (K consolidated segments) is

$$\Theta_{\mu\nu} = \sum_{i=1}^{K} [N_d(\tau_i)]^{1/2} \Theta^i_{\mu\nu} T^{\rm off}(\tau, \tau_i, L_f),$$

- Note consolidated segment has weight $\sqrt{N_d}$
- $T^{\rm off}$ is segment decay function

Analytic UETC

planck

- From these it is possible to work out the USM UETC **analytically**

(Avogoustidis et al 2012)

$$\langle\Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \frac{2f(\tau_1,\tau_2,\xi,L_f)}{16\pi^3}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\psi \int_0^{2\pi} d\chi \Theta(k,\tau_1)\Theta(k,\tau_2),$$

- The scaling function can be computed analytically from the segment decay

$$f(\tau_1,\tau_2,\xi,L_f\to 1) = \frac{1}{[\xi \mathrm{Max}(\tau_1,\tau_2)]^3}.$$

- UETC involves several integrals which are doable, e.g.

$$\frac{1}{2}\int_0^{\pi} d\theta \sin^3\theta \cos(x\cos\theta) J_0(\rho\sin\theta) = \left[1+\frac{\partial^2}{\partial x^2}\right]\left(\frac{\sin\sqrt{\rho^2+x^2}}{\sqrt{\rho^2+x^2}}\right),$$

- And only **two** which weren't ☹ , e.g.

$$\frac{1}{2}\int_0^{\pi} d\theta \sin\theta \cos(x\cos\theta) J_0(\rho\sin\theta)\sec^2\theta = \sum_{c=0}^{\infty}\frac{1}{c!}\frac{\rho}{(2c-1)}\left(-\frac{x^2}{2\rho}\right)^c j_{c-1}(\rho),$$

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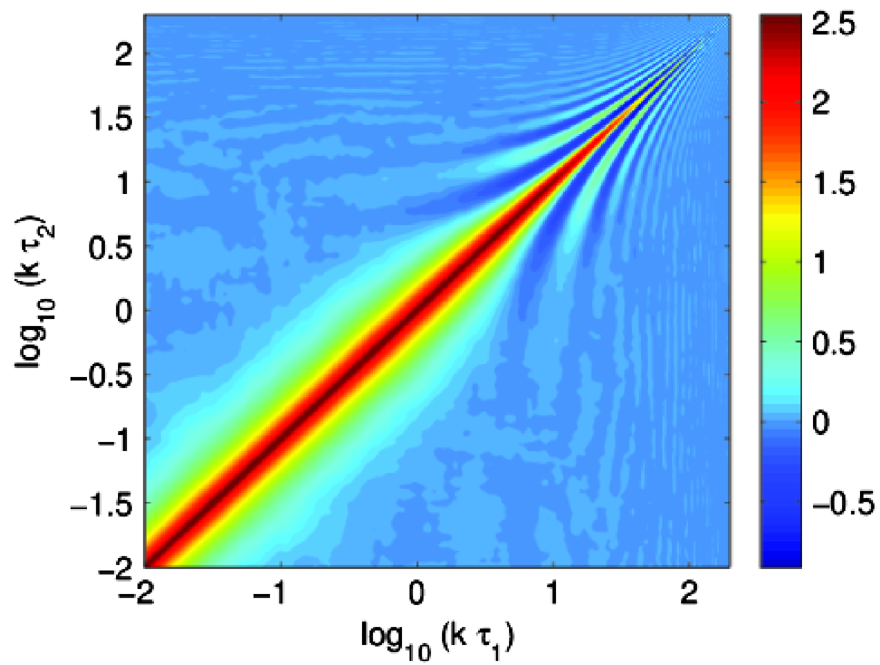
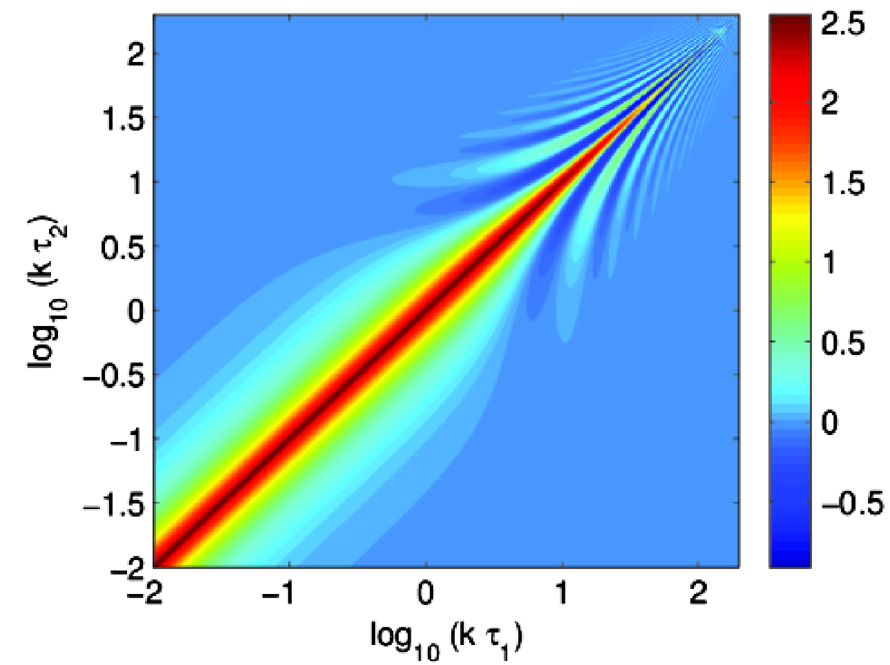
Comparison

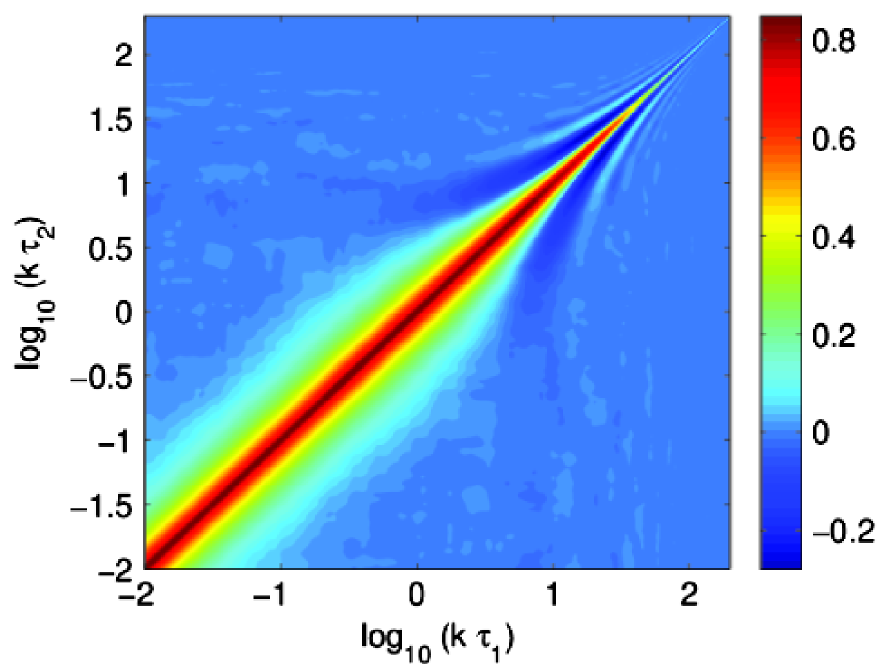
Scalar anisotropic stress, numerical simulations

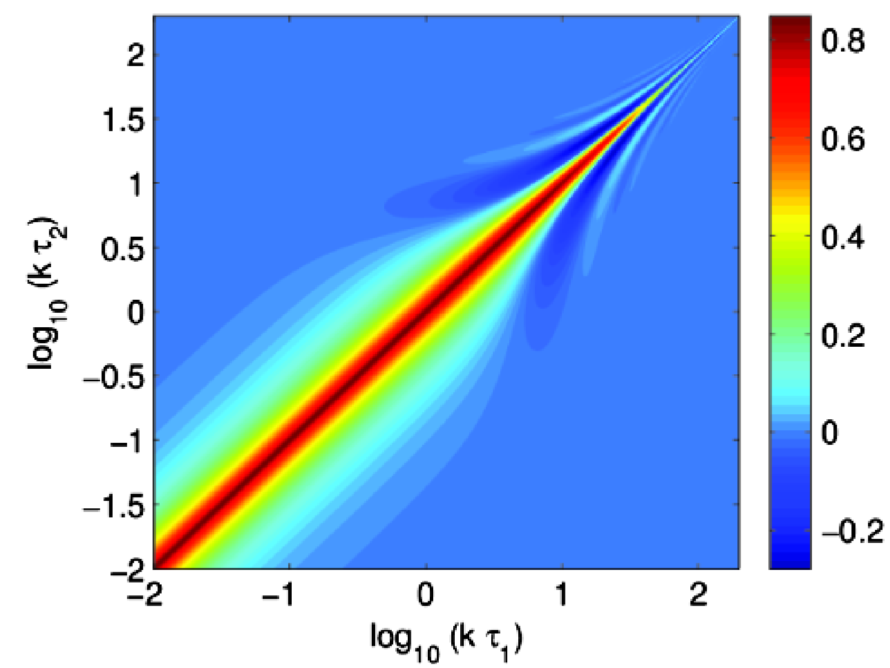




Scalar anisotropic stress, analytic

Vector anisotropic stress, numerical simulations





Vector anisotropic stress, analytic

(Hours)

(Seconds)

Eigenmodes

- UETC can be decomposed into eigenmodes - each of these are **coherent**, can be used as source functions in CMB code

(Pen et al 1997)

$$(k^2\tau_1\tau_2)^\gamma(\tau_1\tau_2)^{1/2}\langle\Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \sum_{i=1}^{n}\lambda_i u_i(k\tau_1)\otimes u_i(k\tau_2),$$

- Diagonalization introduces a change of basis, but since modes are orthogonal and perturbations are linear, CMB sources are

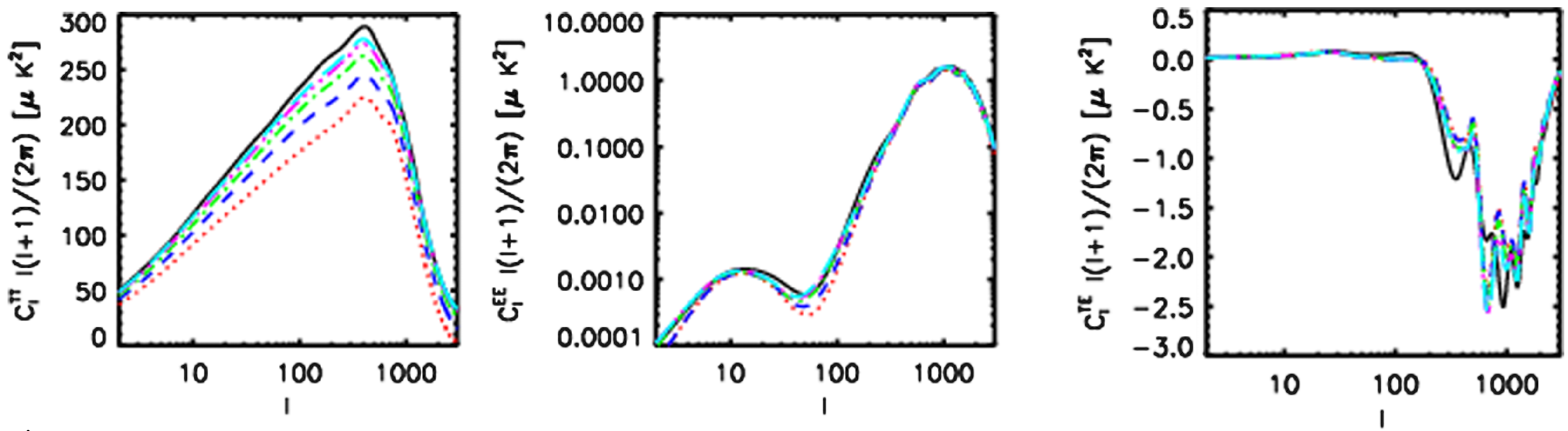
$$\Theta(k\tau) \longrightarrow \frac{u(k\tau)}{(k\tau)^\gamma\tau^{1/2}}.$$

- Power spectra found by summing over eigenmodes (ordered from highest to lowest) and truncated at some number

$$C_\ell = \sum_{i=1}^{n}\lambda_i C_\ell^i,$$

CMB Spectra

- CMB spectra are $$C_\ell^{i(I)} = \frac{2}{\pi}\int k^2 dk \Delta_\ell^{i(I)}(k,\tau_0)\Delta_\ell^{i(I)}(k,\tau_0),$$
- Incorporated into CAMB - CAMBACT (Avogoustidis et al 2012)
- E.g. scalar spectra



$$C_\ell^{\rm string} \propto (G\mu)^2$$

Black - CMBACT, 2000 realizations
Red - CAMBACT, 16 eigenmodes
Green - CAMBACT, 64 eigenmodes
Magenta - CAMBACT, 128 eigenmodes

Some Numbers

- Running CMBACT for 2000 source realisations takes ~ 20 hours
- Obviously this makes it impossible to run any sort of MCMC with Planck (other than fitting for the overall amplitude of a given spectrum)
- With CAMBACT we need somewhere between 50-100 eigenmodes for reasonable accuracy
- Each mode requires running CAMB for scalars, vectors and tensors (i.e. 150-300 CAMB evaluations)
- CAMB takes ~ 1 second to run, hence total CAMBACT computational time is several minutes
- A big improvement, but currently still rather prohibitive for Planck MCMC

Abelian Higgs

planck

- So far focused on USM case. Idea is to use simulations (both Nambu and Abelian-Higgs) to inform network parameters
- State of the art Abelian-Higgs simulations have also been used for the Planck analysis (Bevis et al)
- Fields evolved on 1024^3 grid, starting from random initial conditions designed to mimic a phase transition
- Brief diffusive period ensures system rapidly reaches scaling
- String cores are partially fattened to enlarge dynamical range
- Various runs performed to check results insensitive to string fattening parameter
- UETC's calculated at regular intervals and used in CMBEASY code

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Planck Constraints

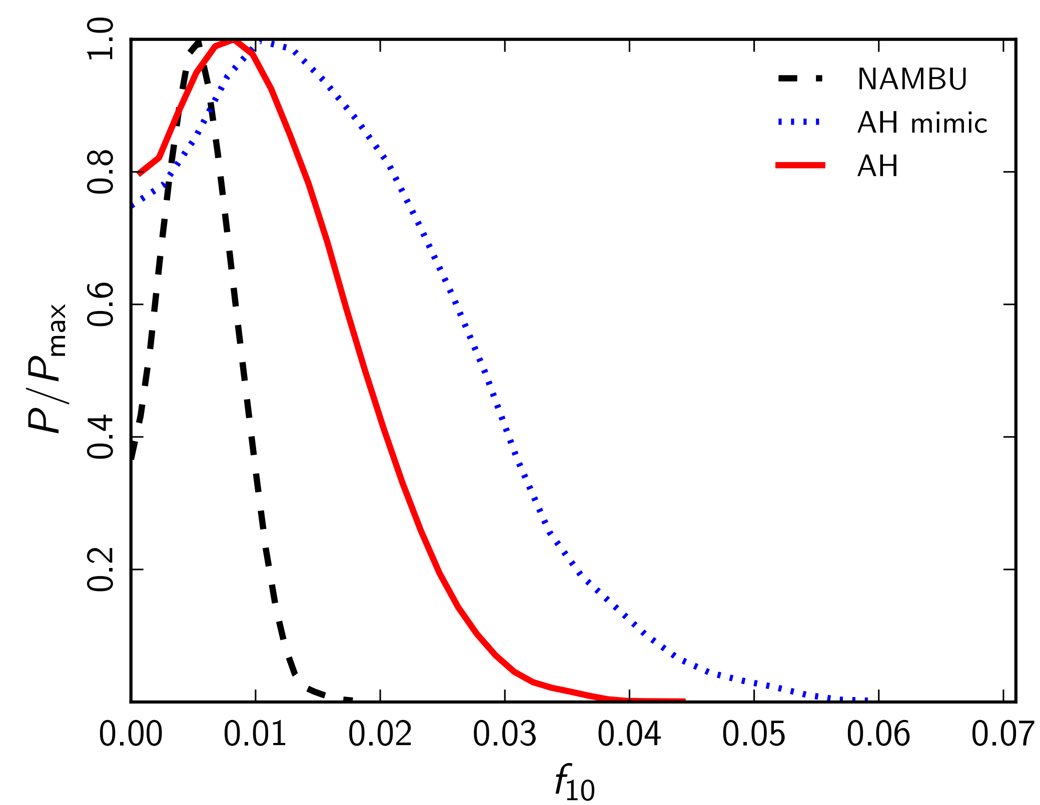
▸ Additional parameter added to LCDM, fraction of strings at $\ell = 10$

$$f_{10} = \frac{C_\ell^{\mathrm{string}}}{C_\ell^{\mathrm{string}} + C_\ell^{\mathrm{inflation}}}$$

▸ Use same dataset/priors as in main cosmology paper

Defect type	*Planck*+WP f_{10}	*Planck*+WP $G\mu/c^2$	*Planck*+WP+highL f_{10}	*Planck*+WP+highL $G\mu/c^2$
NAMBU	0.015	1.5×10^{-7}	0.010	1.3×10^{-7}
AH-mimic	0.033	3.6×10^{-7}	0.034	3.7×10^{-7}
AH	0.028	3.2×10^{-7}	0.024	3.0×10^{-7}
SL	0.043	11.0×10^{-7}	0.041	10.7×10^{-7}
TX	0.055	10.6×10^{-7}	0.054	10.5×10^{-7}

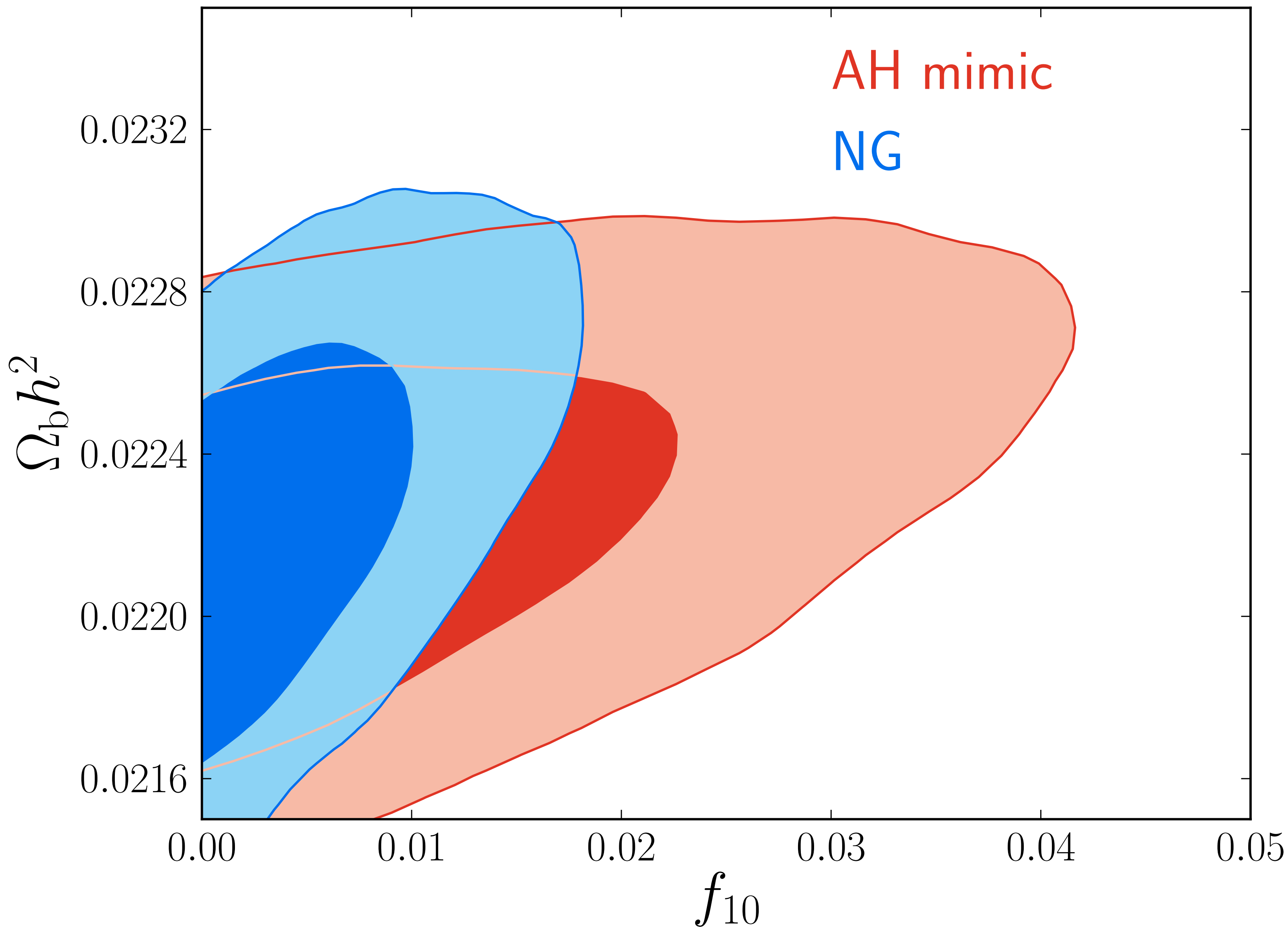
▸ Limits on string tension significantly improved, e.g. in Nambu model string fraction <1%



Planck + WP + highL

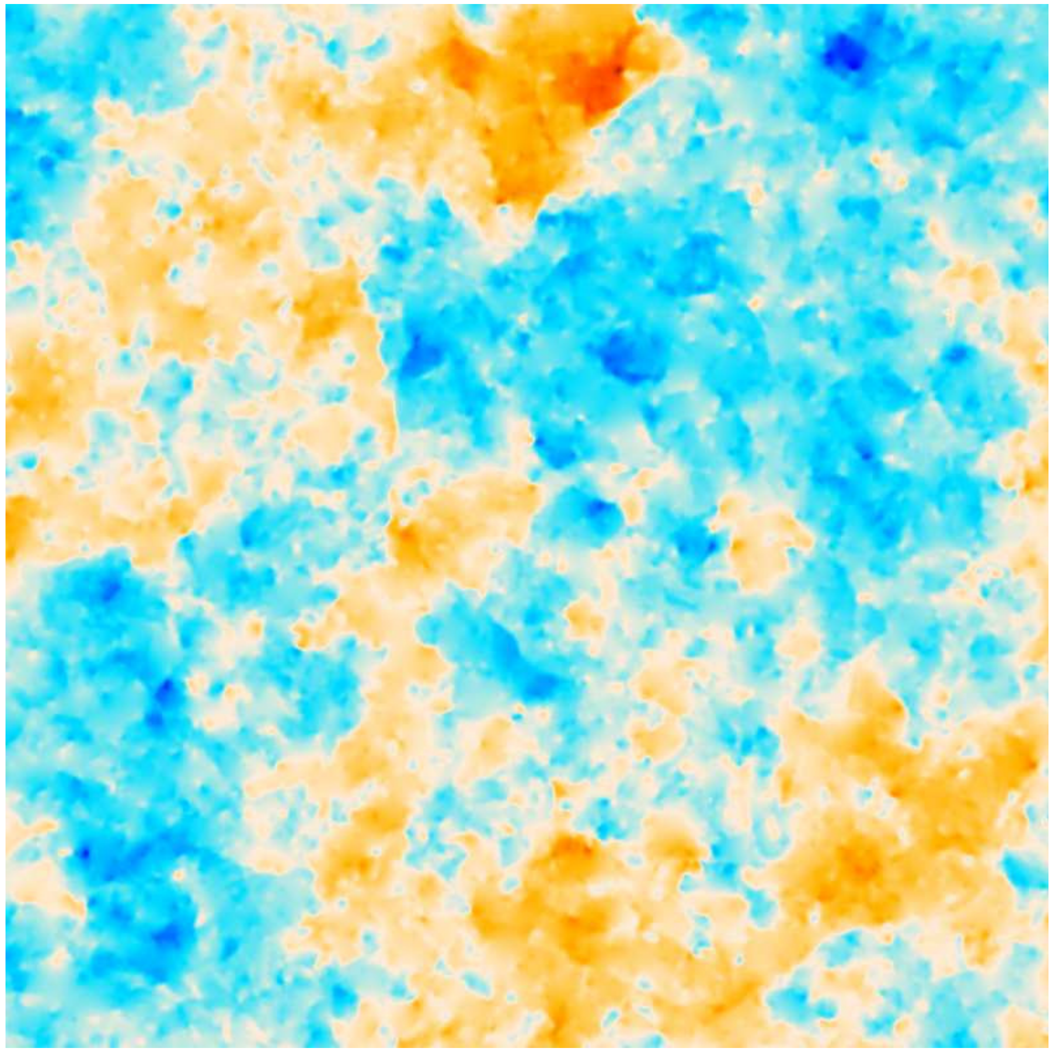
Planck Constraints

- Strings exhibit no significance correlations with any other cosmological parameter (as for WMAP)



Non Gaussianity

- To go beyond 2-point function Planck has used Nambu simulations to compute post-recombination string maps (Ringeval and Bouchet 2012)
- Maps expected to be accurate and large and small scales, will underestimate power at intermediate scales
- Maps produced at 0.85′ and 1.7′ resolution
- For lower resolution maps aliasing effects carefully taken into account
- Map has Gaussian component, but non Gaussian step like features clearly apparent



NG Searches

planck

- Use 4 different Planck foreground cleaned maps
- Check results are consistent with each map, insensitive to choice of mask

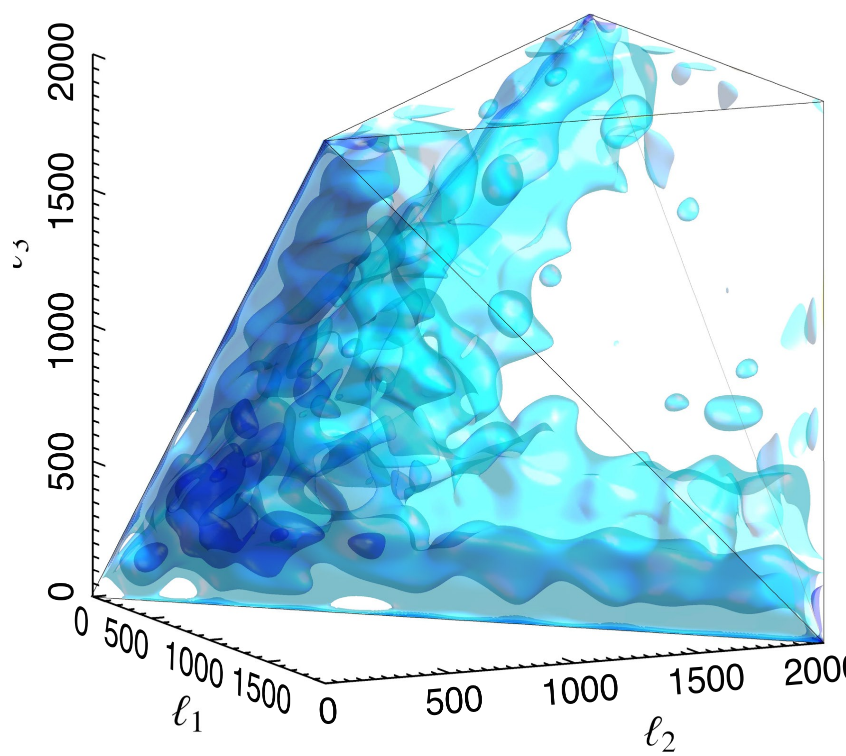
(Fergusson and Shellard)

- Bispectrum computed using **modal** method. Expands reduced bispectrum $b^{\text{string}}_{\ell_1\ell_2\ell_3}$ in separable modes

$$\frac{b^{\text{string}}_{\ell_1\ell_2\ell_3}}{\sqrt{C_{\ell_1}C_{\ell_2}C_{\ell_3}}} = \sum_n \beta^Q_n \, Q_n(\ell_1,\ell_2,\ell_3)\,,$$

- Express bispectrum amplitude estimate in terms of modes $Q_n(\ell_1,\ell_2,\ell_3)$

- Modal reconstruction performed on post-recombination string maps
- Correlation with primordial shapes is since it does not contain an oscillatory feature



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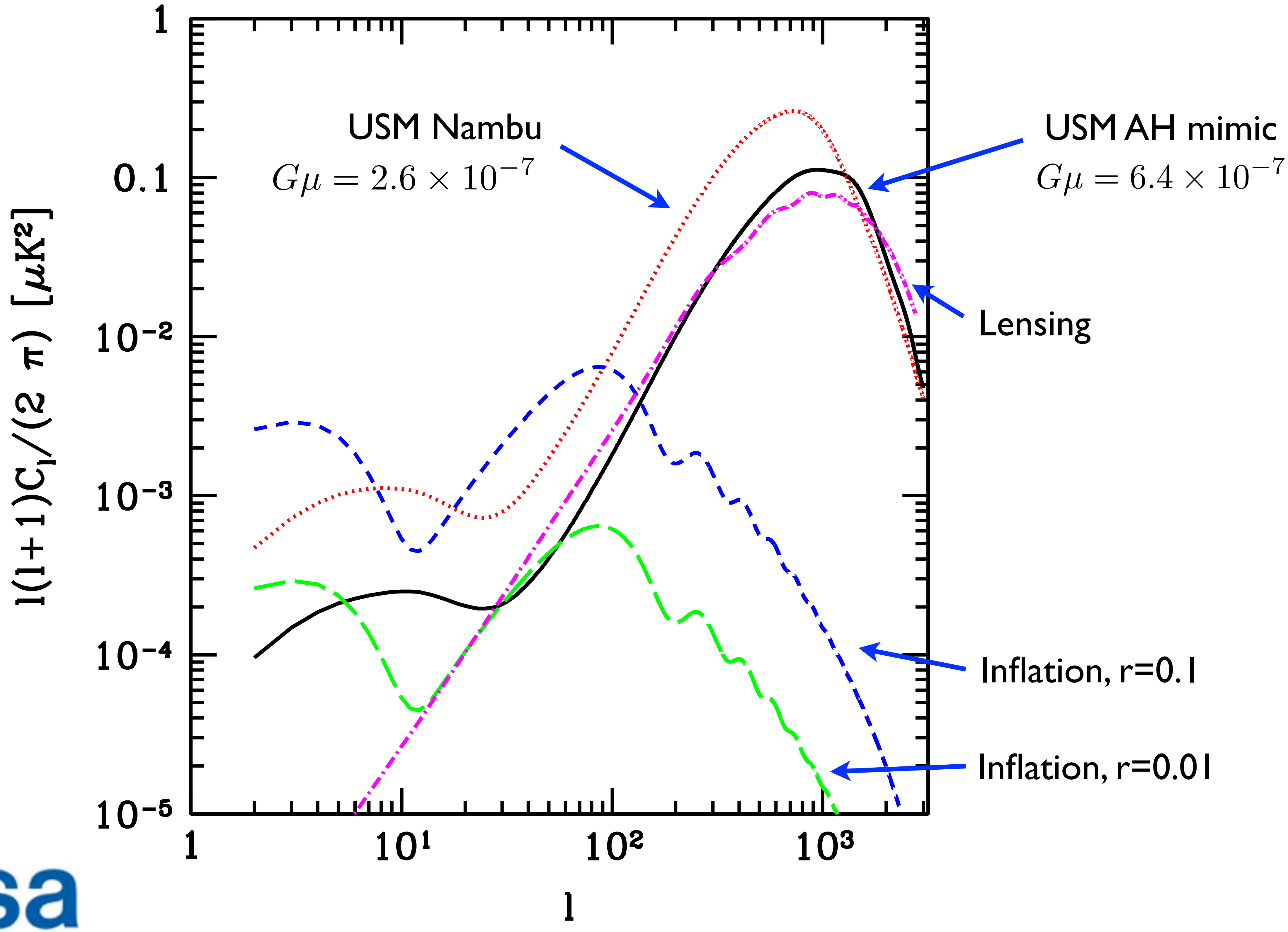
NG Result

Method	Bispectrum signal type	Independent analysis $f_{\rm NL}$	ISW subtract Joint $f_{\rm NL}$	ISW/FG res. Joint $f_{\rm NL}$
SMICA	Lensing ISW	0.75 ± 0.37	–	–
	Diff. PS $\times 10^{28}$	1.05 ± 0.32	1.35 ± 0.34	1.40 ± 0.34
	Cosmic strings	0.19 ± 0.20	0.50 ± 0.21	0.37 ± 0.21
	$G\mu/c^2$ (95%)	8.4×10^{-7}	9.7×10^{-7}	9.3×10^{-7}
NILC	Lensing ISW	0.91 ± 0.36	–	–
	Diff. PS $\times 10^{28}$	1.16 ± 0.32	1.44 ± 0.34	1.44 ± 0.34
	Cosmic strings	0.13 ± 0.20	0.46 ± 0.21	0.23 ± 0.21
	$G\mu/c^2$ (95%)	8.1×10^{-7}	9.6×10^{-7}	8.7×10^{-7}
SEVEM	Lensing ISW	0.6 ± 0.36	–	–
	Diff. PS $\times 10^{28}$	1.07 ± 0.35	1.33 ± 0.38	–
	Cosmic strings	0.10 ± 0.20	0.38 ± 0.21	–
	$G\mu/c^2$ (95%)	7.9×10^{-7}	9.3×10^{-7}	–

- No evidence (currently) for NG string signal assuming post-recombination string maps

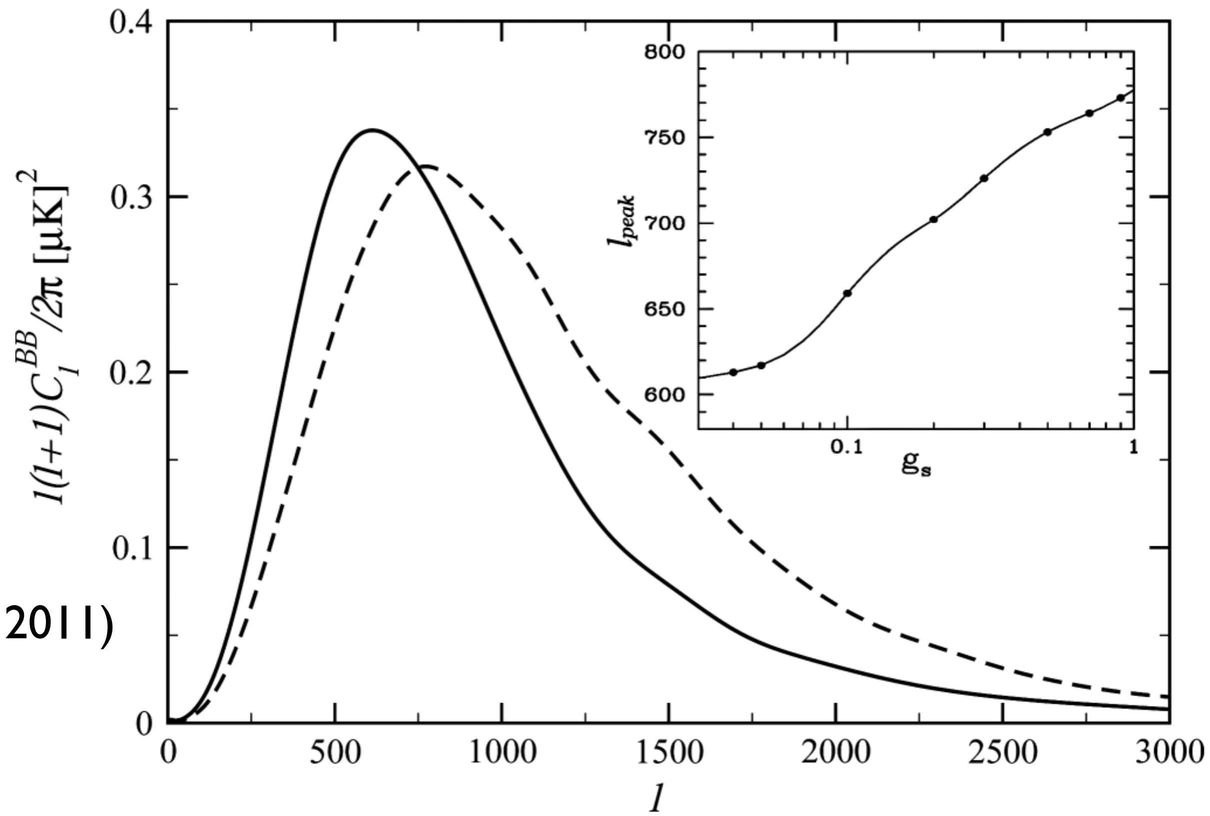
B Modes



1
0.1
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1
10^{1}
10^{2}
10^{3}
l
$l(l+1)C_l/(2\pi)$ [μK^2]
USM Nambu
$G\mu = 2.6 \times 10^{-7}$
USM AH mimic
$G\mu = 6.4 \times 10^{-7}$
Lensing
Inflation, r=0.1
Inflation, r=0.01

Superstrings

- Strings have a fascinating connection with high energy physics
- It is possible more complicated networks may arise from superstrings
- Observational properties depend on fundamental string coupling g_s
- Generally now a spectrum of string tensions determined by g_s and charges which they carry
- Intercommutation properties modified
- Junctions can form between strings of different charge
- Modified VOS equations have been developed
- Find potentially observable shift in B mode spectrum

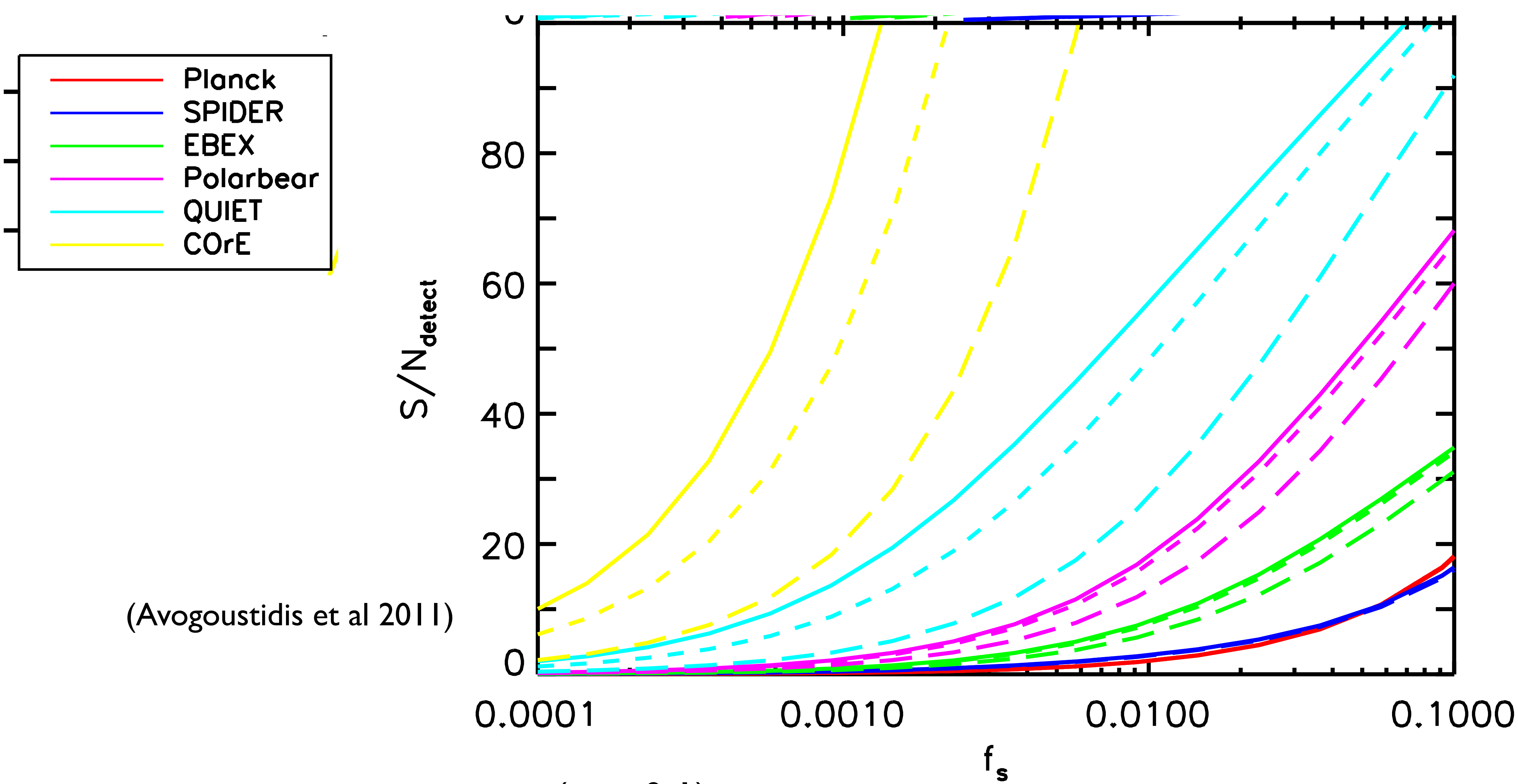


(Avogoustidis et al 2011)

Superstrings

planck

- B mode experiments have the potential to measure a fundamental property of string theory



(Avogoustidis et al 2011)

(g_s = 0.1)

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Conclusions

- Planck has provided stringent limits on strings
- Power spectrum gives

$$G\mu/c^2 < 1.5 \times 10^{-7}\,, \qquad f_{10} < 0.015\,, \qquad \text{(Nambu)}$$
$$G\mu_{\rm AH}/c^2 < 3.2 \times 10^{-7}\,, \qquad f_{10} < 0.028\,. \qquad \text{(Abelian-Higgs)}$$

- Non Gaussian constraints give $G\mu/c^2 < 7.8 \times 10^{-7}$ (95 % CL).
- String evolution is crucial for understanding CMB anisotropies
- Still some uncertainty in modelling approaches - main problem is dynamical range
- Possible that improved USM model could enable MCMC search of string parameter space
- (Optimistic) hope is strings could link to high energy theory