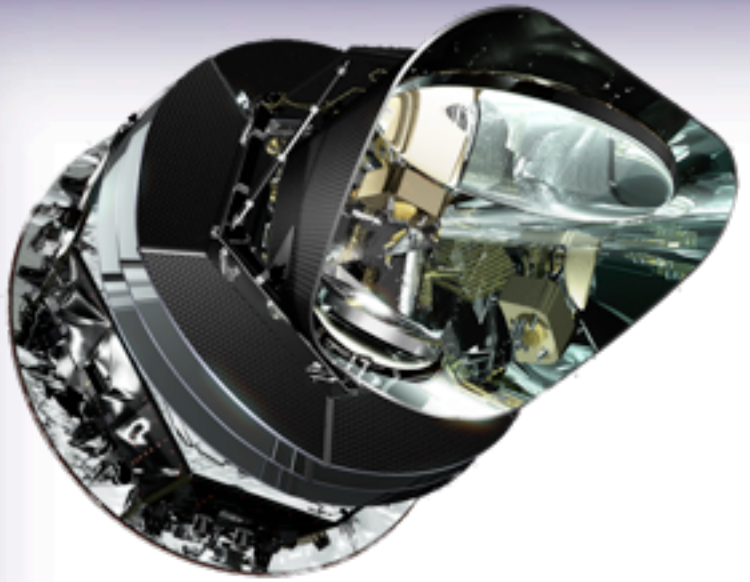


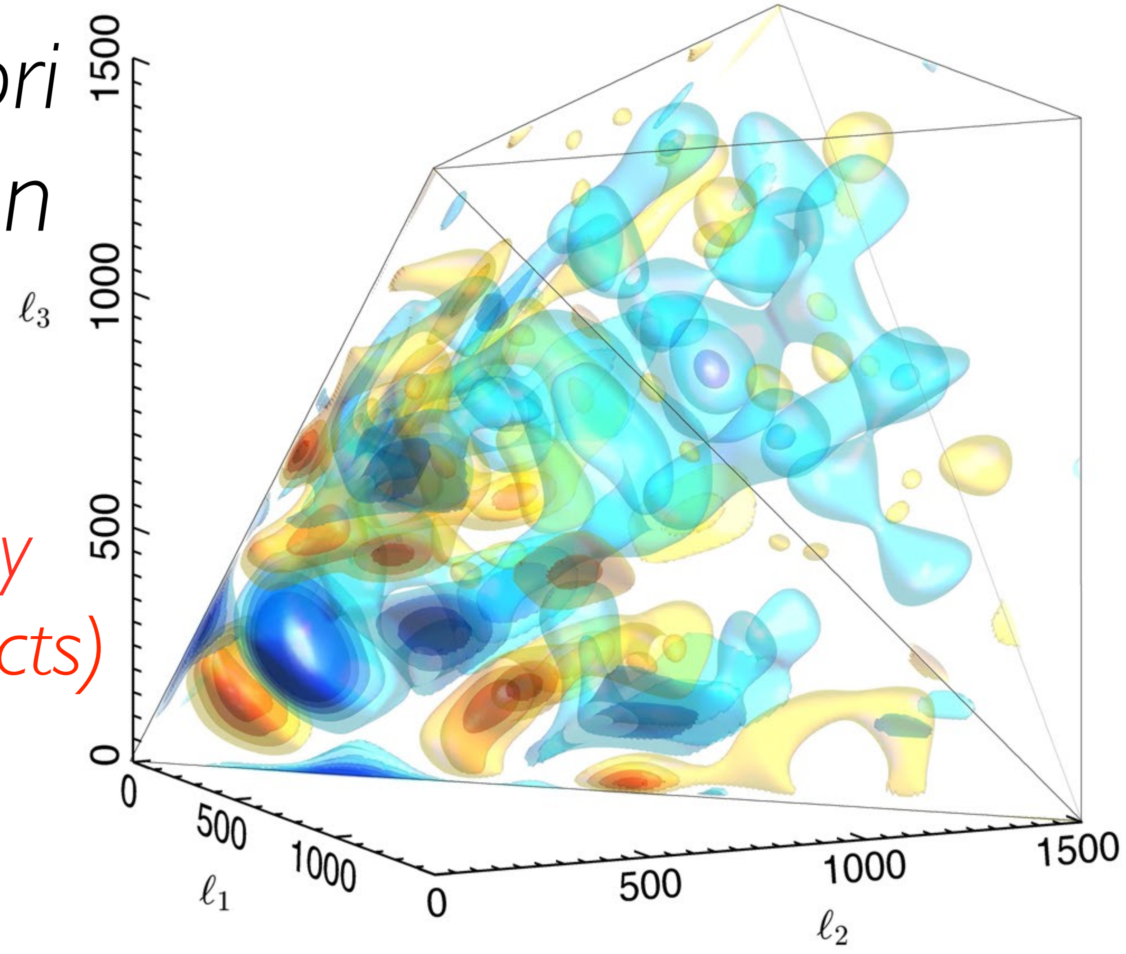
Planck Cosmology
Results 2013

Planck, Primordial Non-Gaussianity, and Prospects

Paul Shellard
with James Fergusson & Michele Liguori
on behalf of the Planck collaboration
(Centre for Theoretical Cosmology,
DAMTP, Cambridge University)

XXIV. Constraints on primordial non-Gaussianity
(XXV. Searches for cosmic strings & other defects)

Implications of Planck Meeting
CERN, 28 June 2013



Acknowledgements

esa

planck

cnes

asi
agenzia spaziale italiana

NASA

CNRS

DTU Space
National Space Institute

INAF ISTITUTO NAZIONALE DI ASTROFISICA

HFI PLANCK

PLANCK LFI

Science & Technology Facilities Council

CSIC

National Research Council of Italy

GOBIERNO DE ESPAÑA
MINISTERIO DE CIENCIA E INNOVACIÓN

IAS orsay

IAP

INAF - IASF BO
ISTITUTO NAZIONALE DI ASTROFISICA
ISTITUTO DI ASTROFISICA SPAZIALE E FISICA COSMICA DI BOLOGNA

CSA ASC

DLR Deutsches Zentrum für Luft- und Raumfahrt e.V.

UK SPACE AGENCY

MAX-PLANCK-GESELLSCHAFT

A"

ioa

KICC Kavli Institute for Cosmology Cambridge

UNIVERSITY OF CAMBRIDGE

CARDIFF UNIVERSITY
PRIFYSGOL CAERDYDD

irfu cea saclay

cea

CESR

CITA–ICAT

CNRS IN2P3

ipac Infrared Processing and Analysis Center

HELSINKI INSTITUTE OF PHYSICS

IFCA

Imperial College London

UNIVERSITÀ DEGLI STUDI DI MILANO

NÉEL Institut

IPAG

ISDC

JPL

LAL

l'Observatoire de Paris LERMA

LPSC Grenoble

MilliLab

NUI MAYNOOTH

INAF OSSERVATORIO ASTRONOMICO DI PADOVA

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Rutherford Appleton Laboratory

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UCSB

SISSA

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ESO

US University of Sussex

esa

UNIVERSITÉ DE GENÈVE

UNIVERSITY OF TORONTO

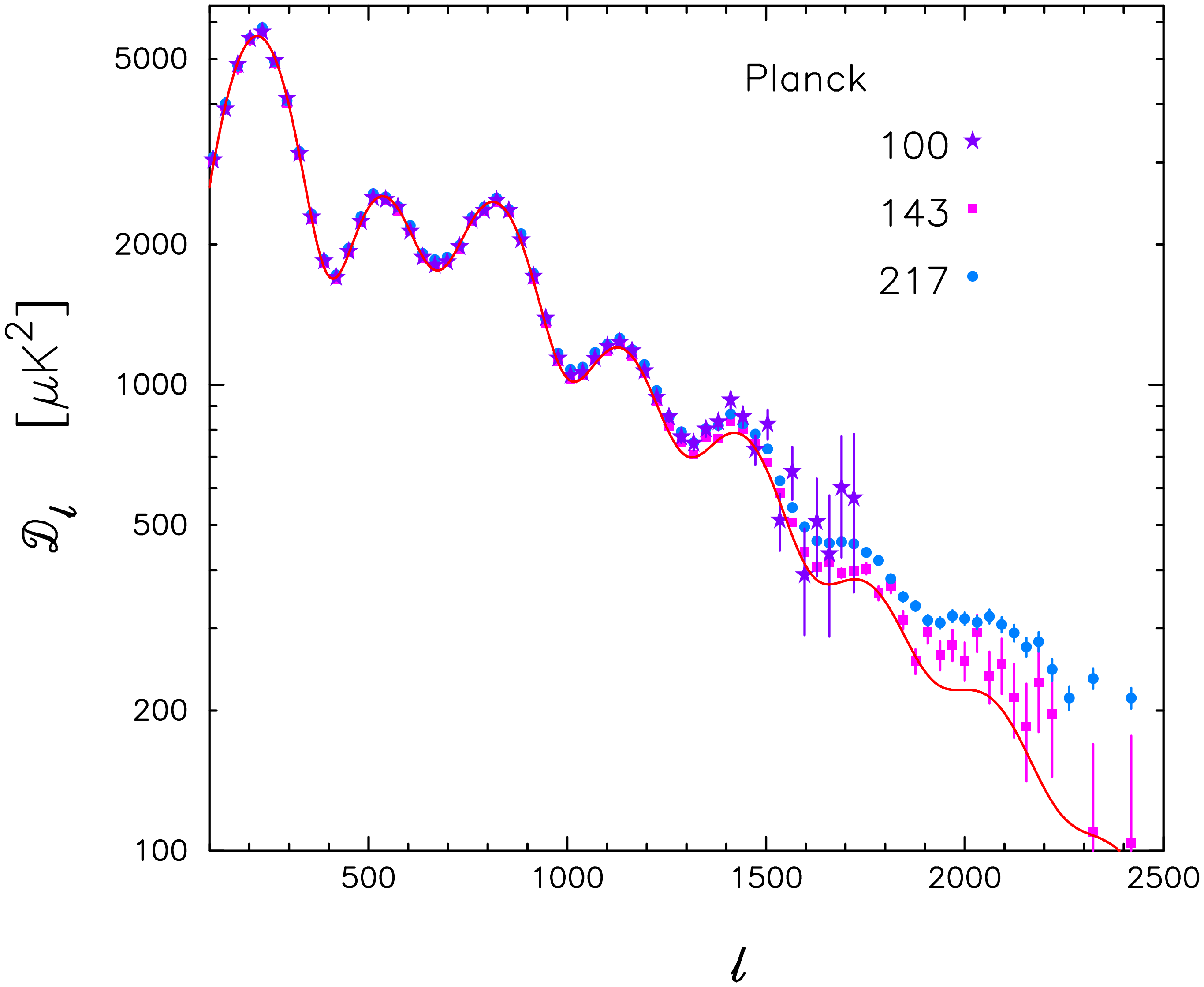
UNIVERSITÉ DE PARIS-SUD XI

asdc

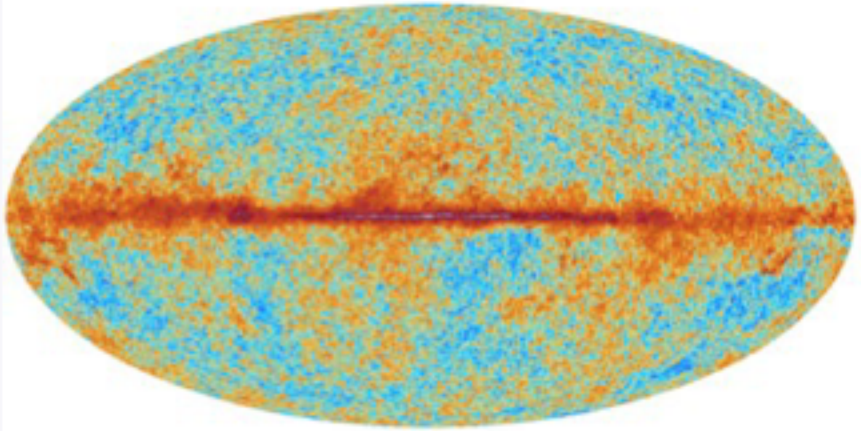
Planck power spectrum

Conservative spectral analysis

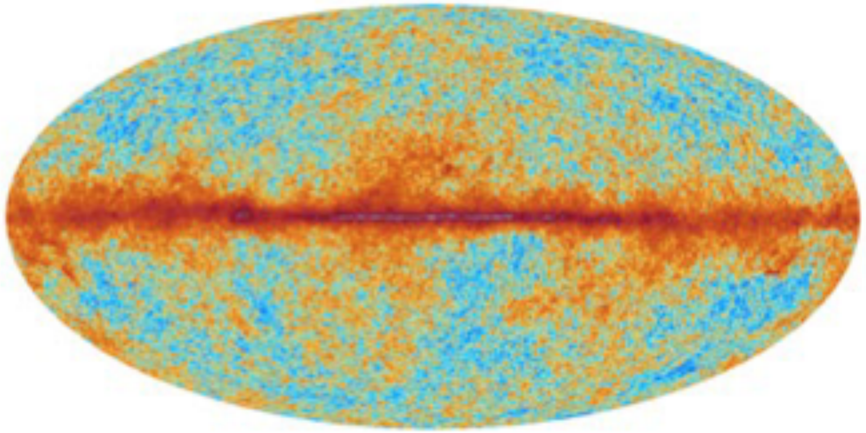
- *Uses small portions of the sky with minimal foreground contamination*
- *Uses detector cross-spectra to remove uncorrelated noise from power*
- *Non-CMB spectra at small-scales are modeled with extra parameters (dust, SZ, CIB etc)*
- *CMB likelihoods published ...*



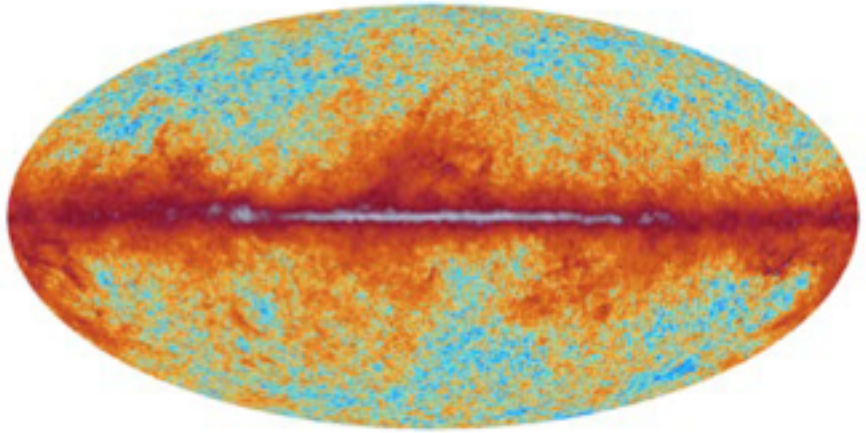
100 GHz (49%)



143 GHz (31%)

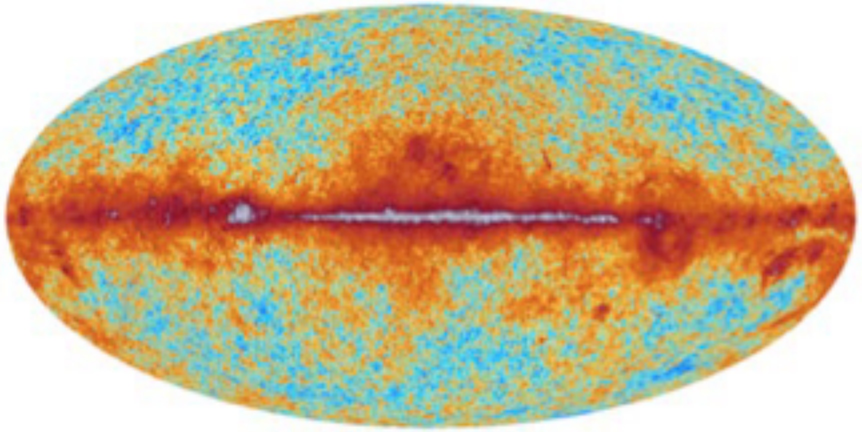


217 GHz (31%)

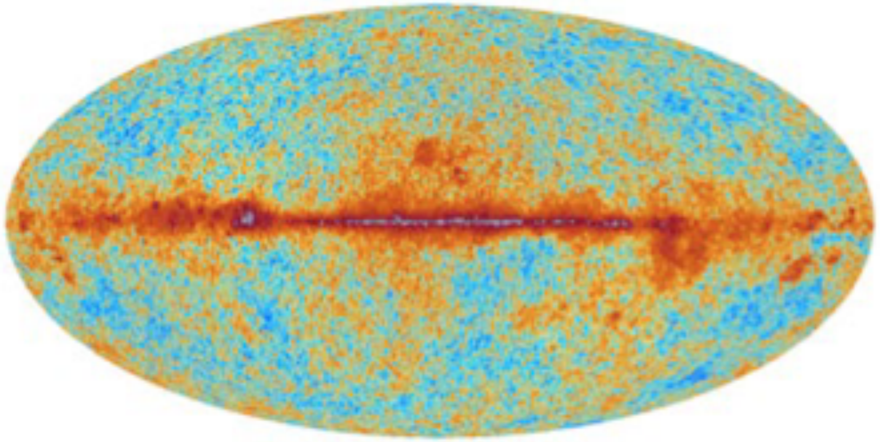


Planck frequency maps

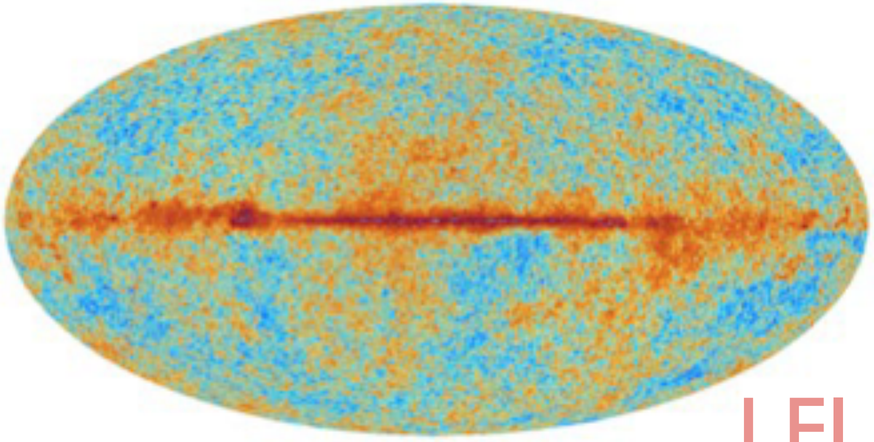
30 GHz



44 GHz

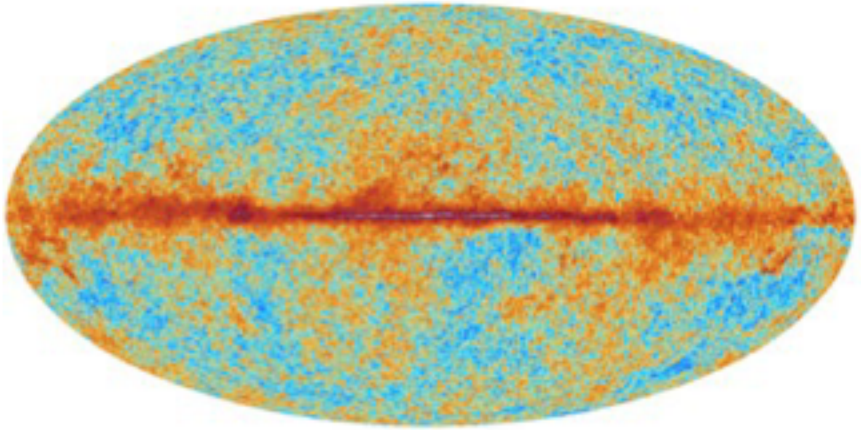


70 GHz

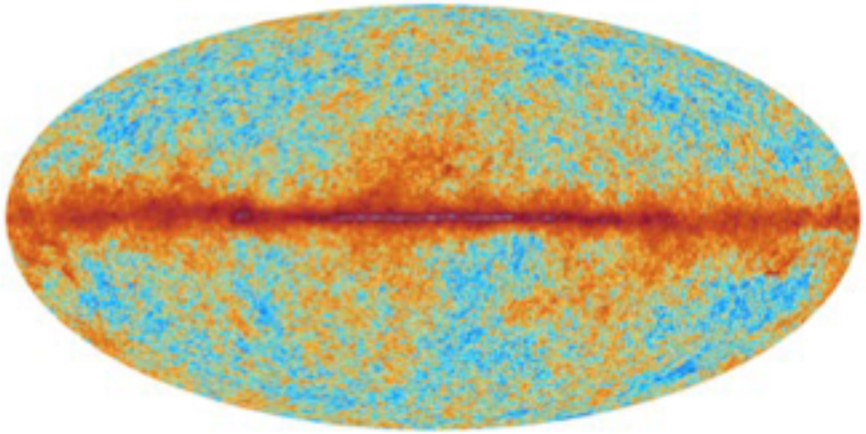


LFI

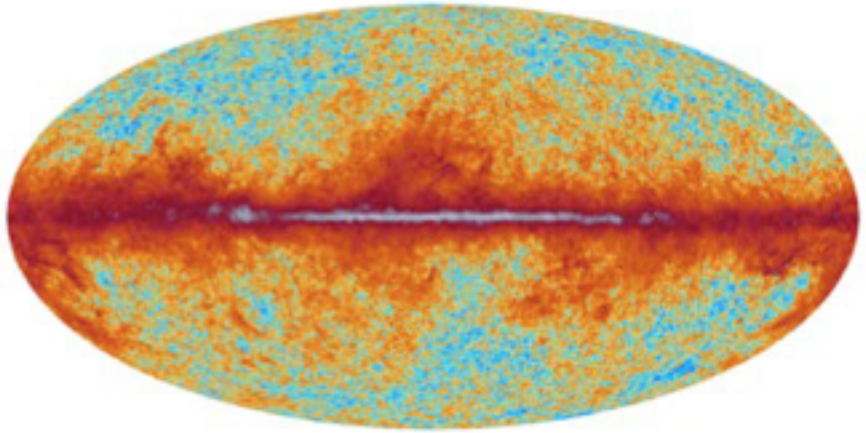
100 GHz



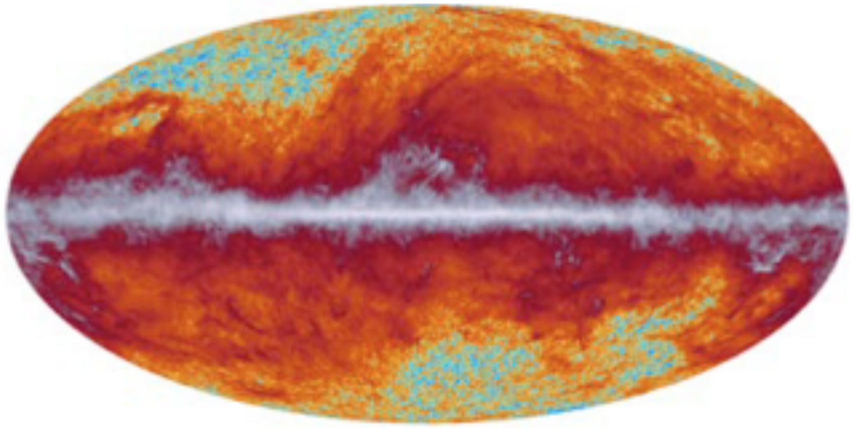
143 GHz



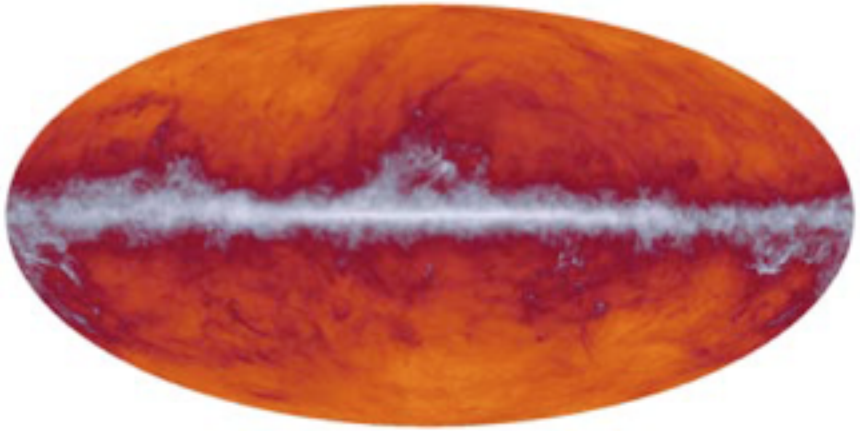
217 GHz



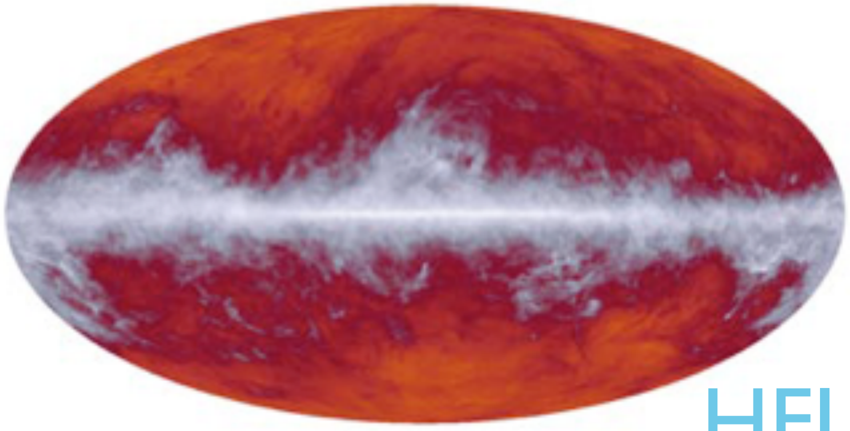
353 GHz



545 GHz



857 GHz

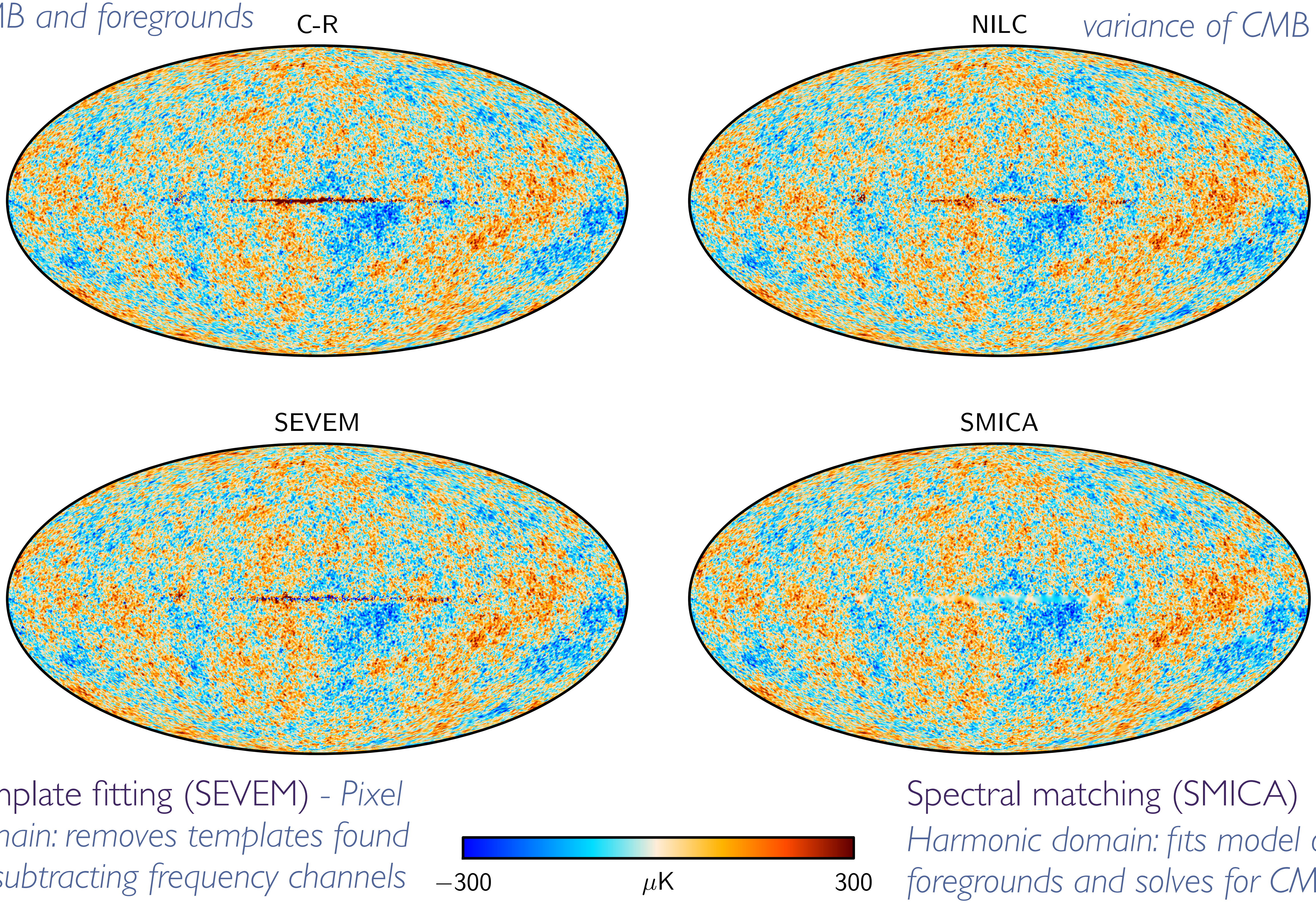


HFI

Foreground-cleaned CMB maps

Commander-Ruler (C-R) - *Pixel domain: fits parametrized model of CMB and foregrounds*

Internal linear combination (NILC) *Needlet (wavelet) domain: minimizes variance of CMB signal*

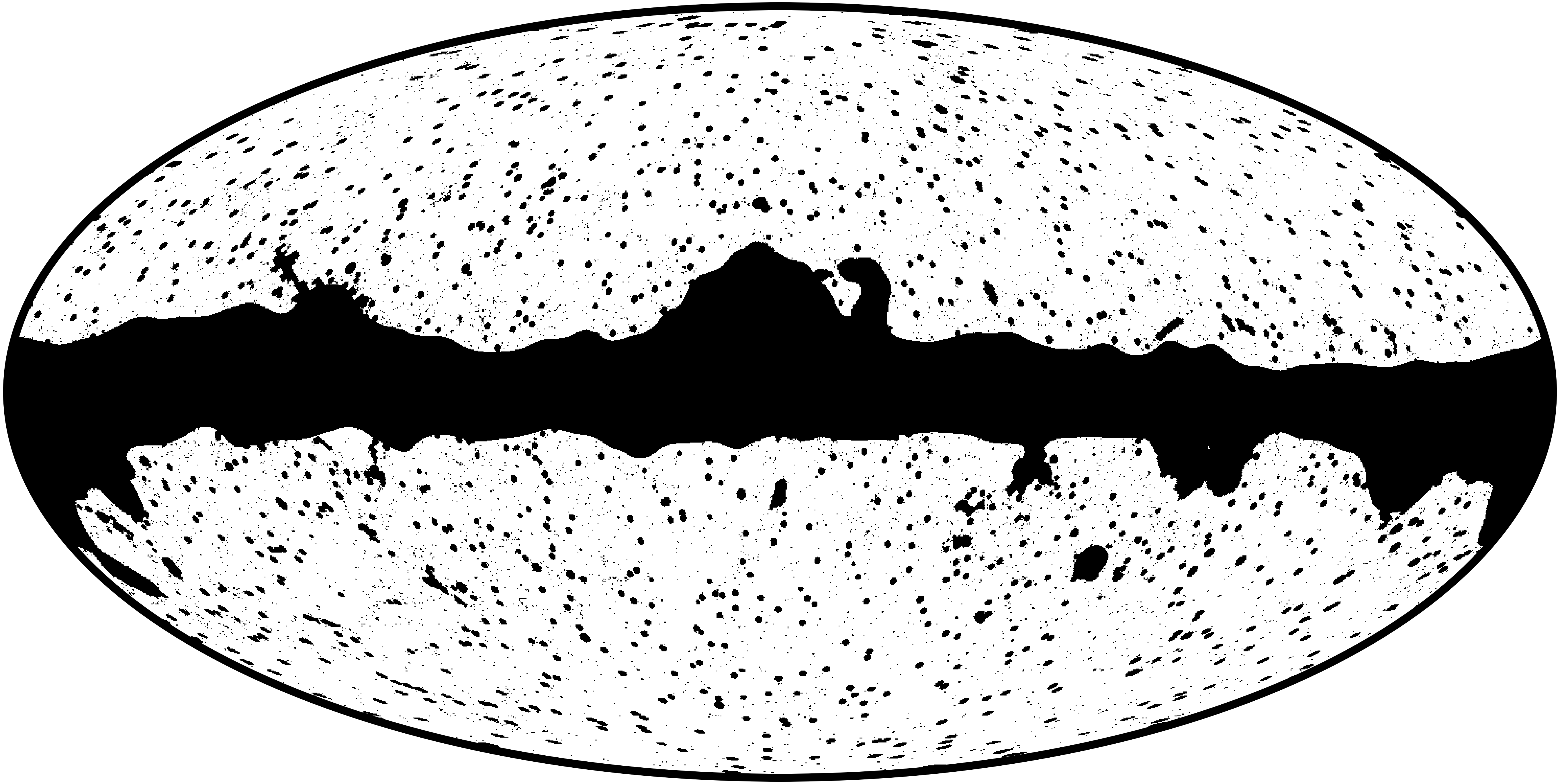


Template fitting (SEVEM) - *Pixel domain: removes templates found by subtracting frequency channels*

Spectral matching (SMICA) *Harmonic domain: fits model of foregrounds and solves for CMB*

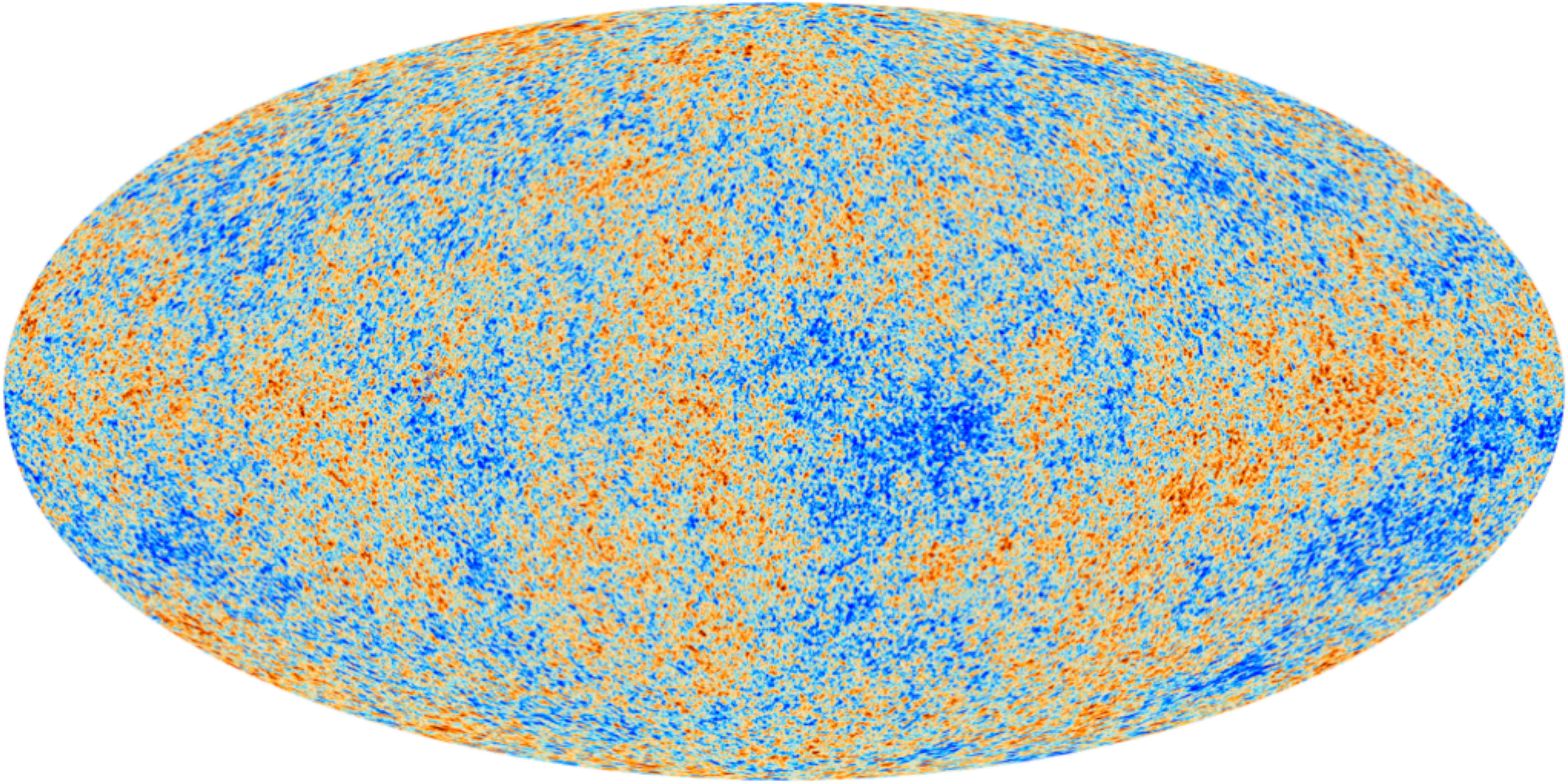
Union Mask

Union of confidence masks for all four methods (U73), leaving 73% of the sky

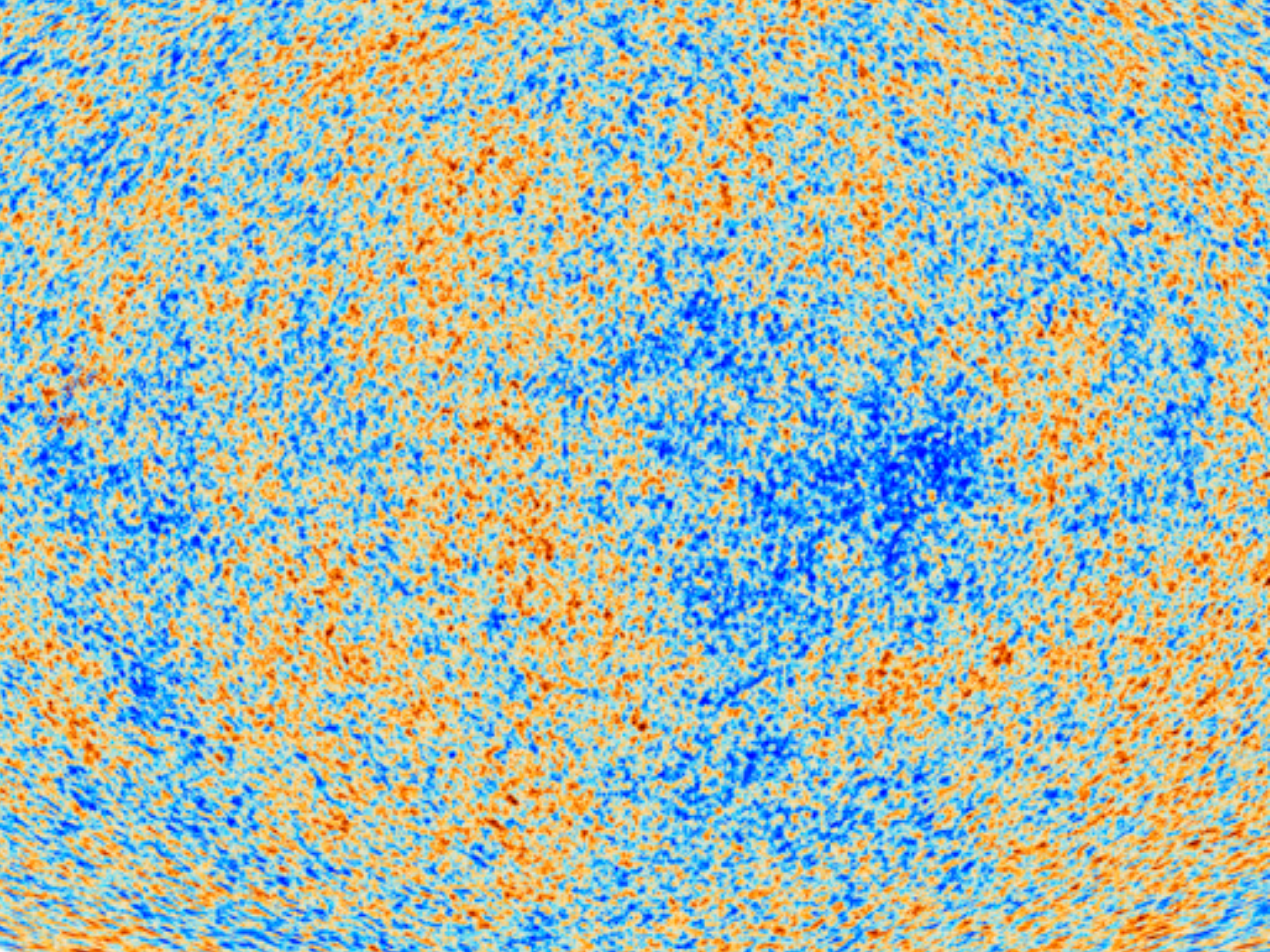


Planck SMICA CMB map

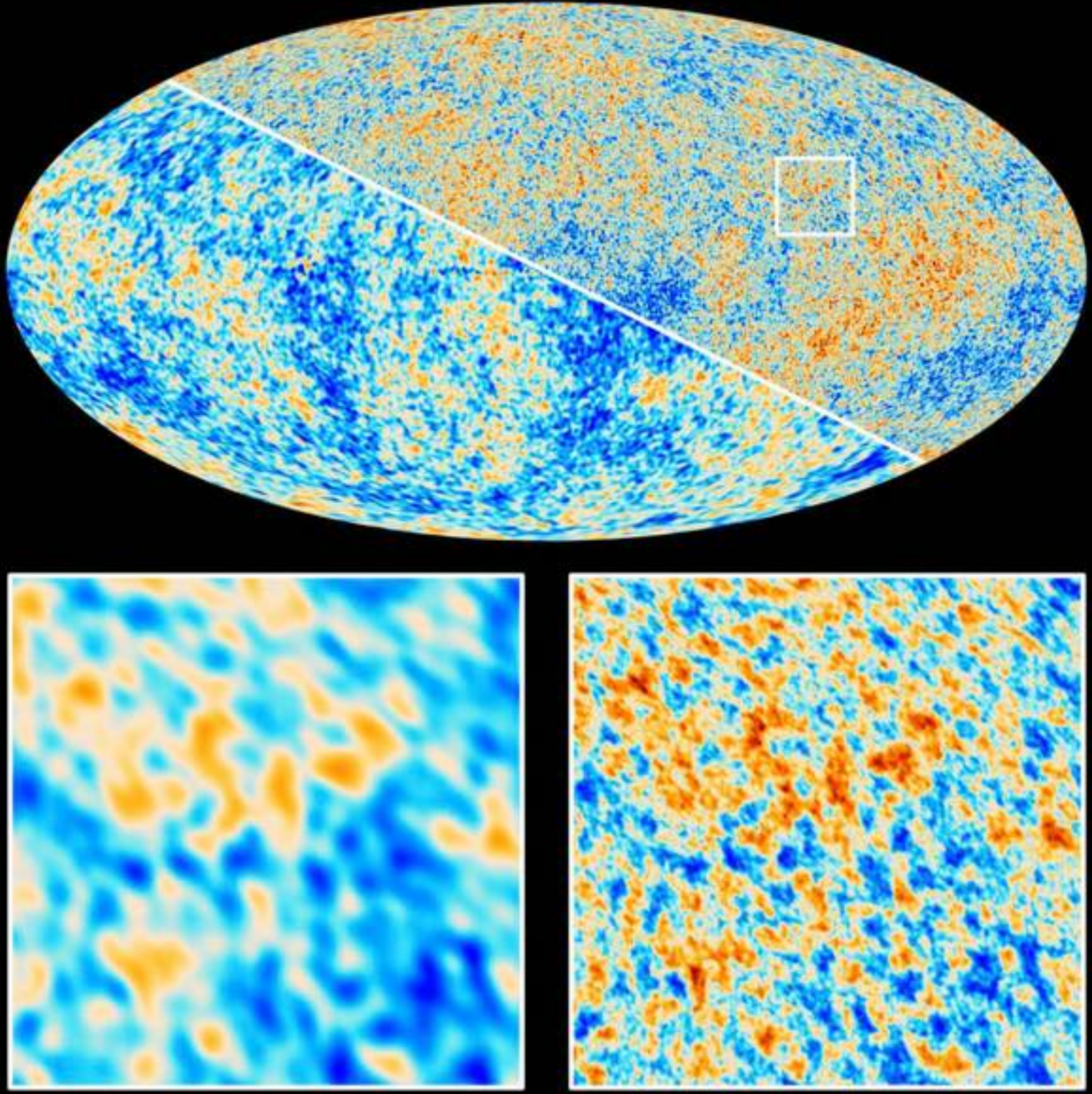
Leading method for high-l analysis - min. foreground residuals and preserves non-Gaussianity
- the 3% processing mask has been filled in with a constrained realization

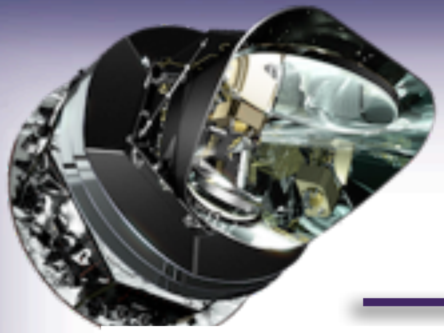


Key public data product from the Planck mission, refer to:
http://www.sciops.esa.int/index.php?project=planck&page=Planck_Legacy_Archive

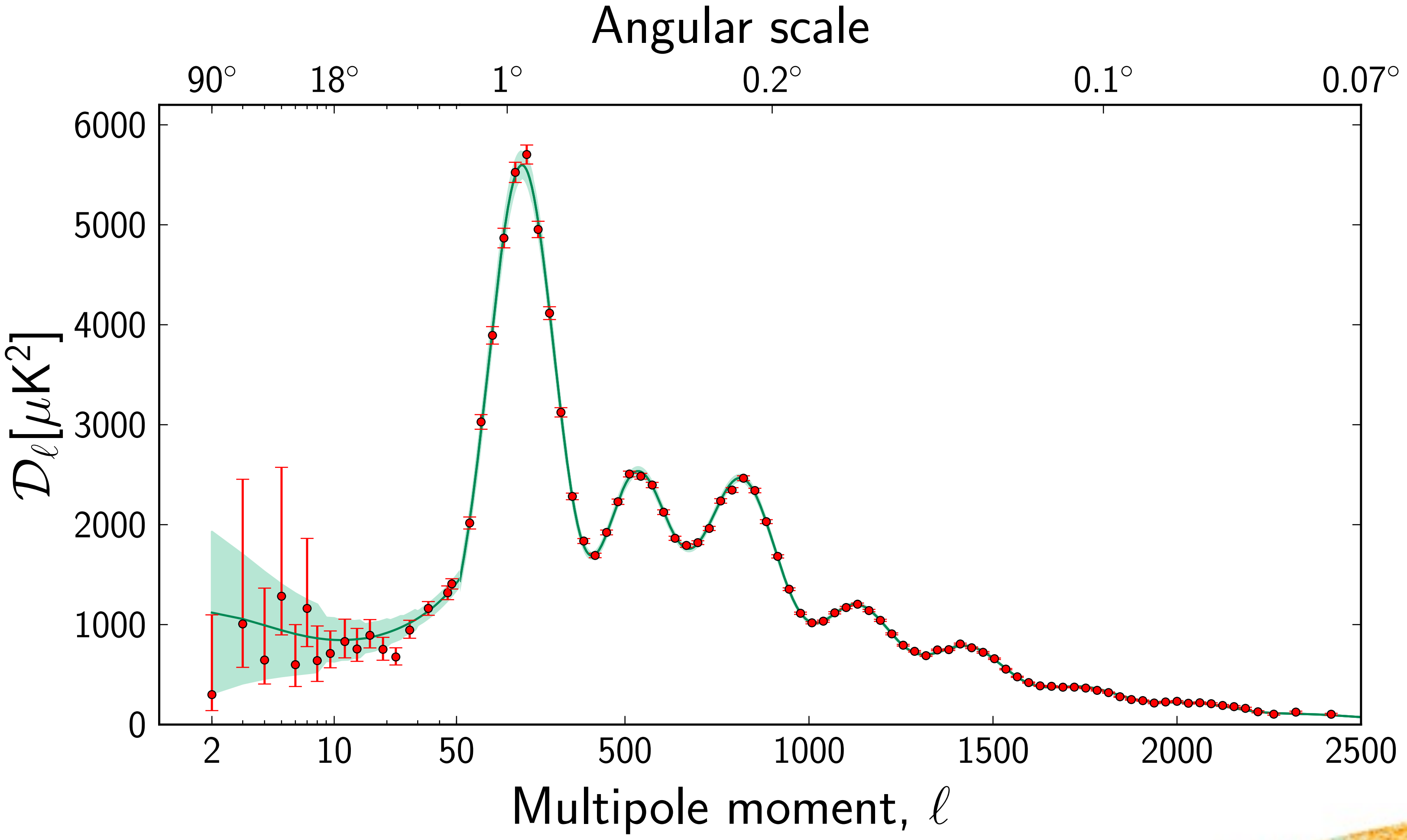


WMAP vs Planck





Non-Gaussianity (NG)

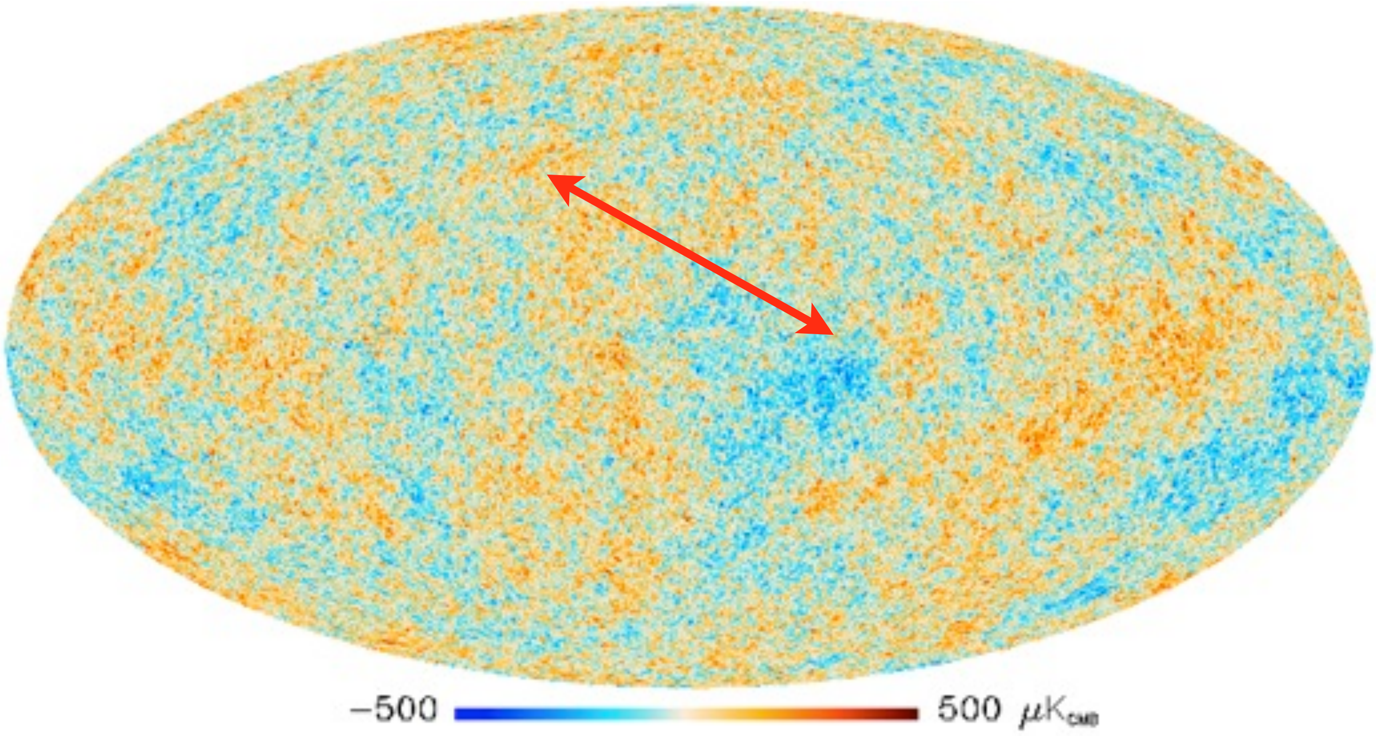


Triumph of inflation

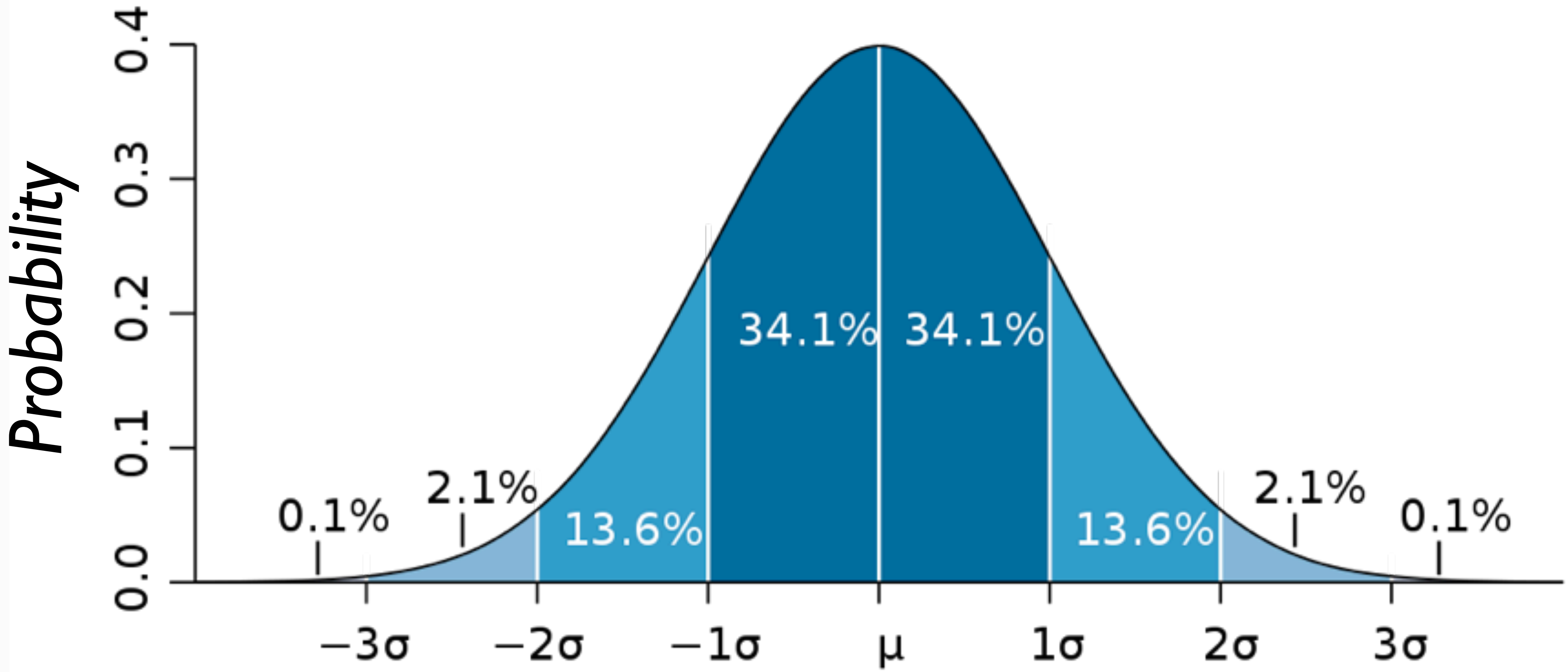
A self-consistent concordance model with 68.3% dark energy, 26.8% dark matter, and 4.9% ordinary matter.

Based on the two-point correlator or angular power spectrum C_l

But there is more information …



Gaussian distribution

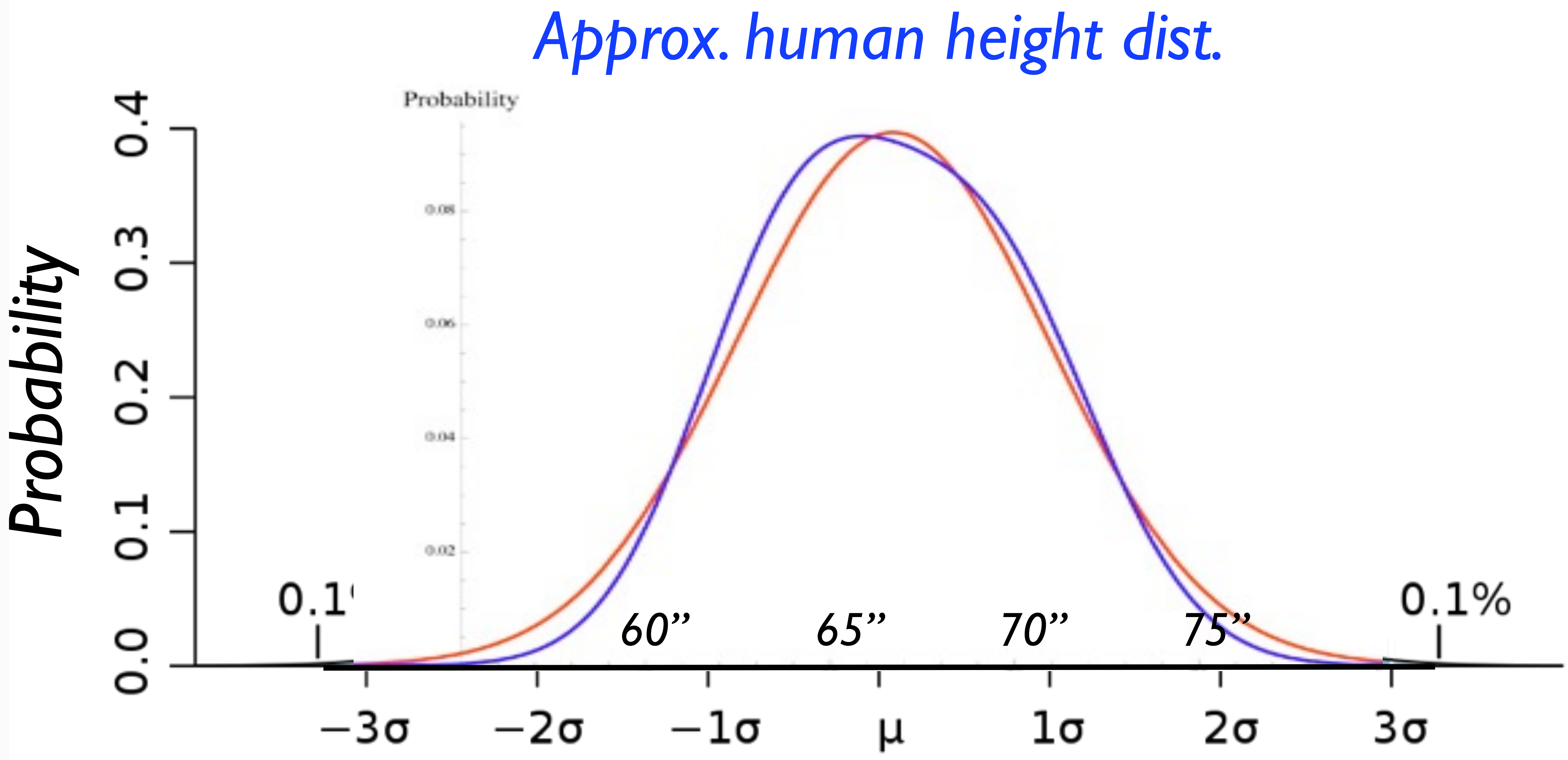


Determined only by mean μ and standard deviation σ

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

Central limit theorem: any independent random process

Gaussian distribution



Determined only by mean μ and standard deviation σ

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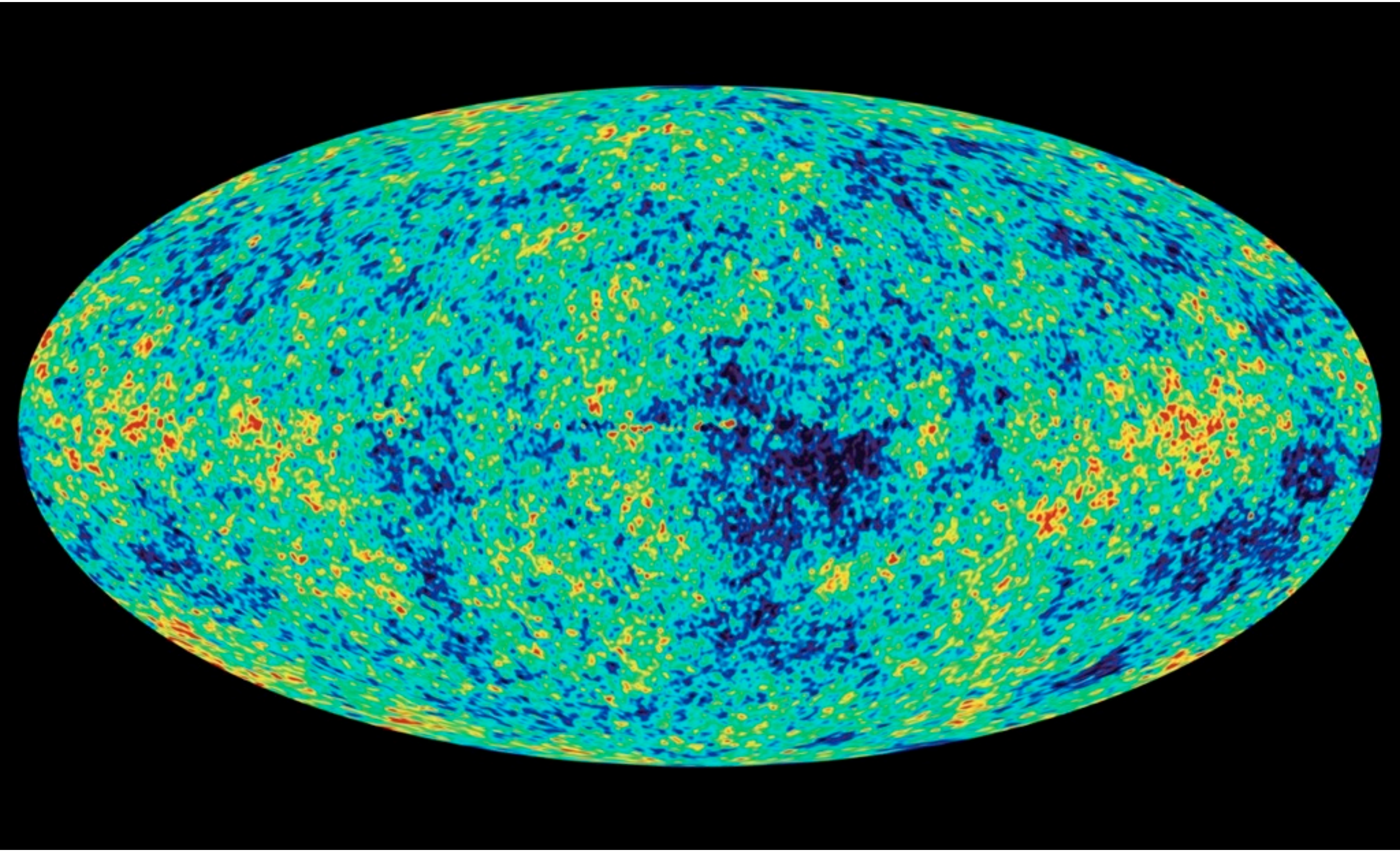
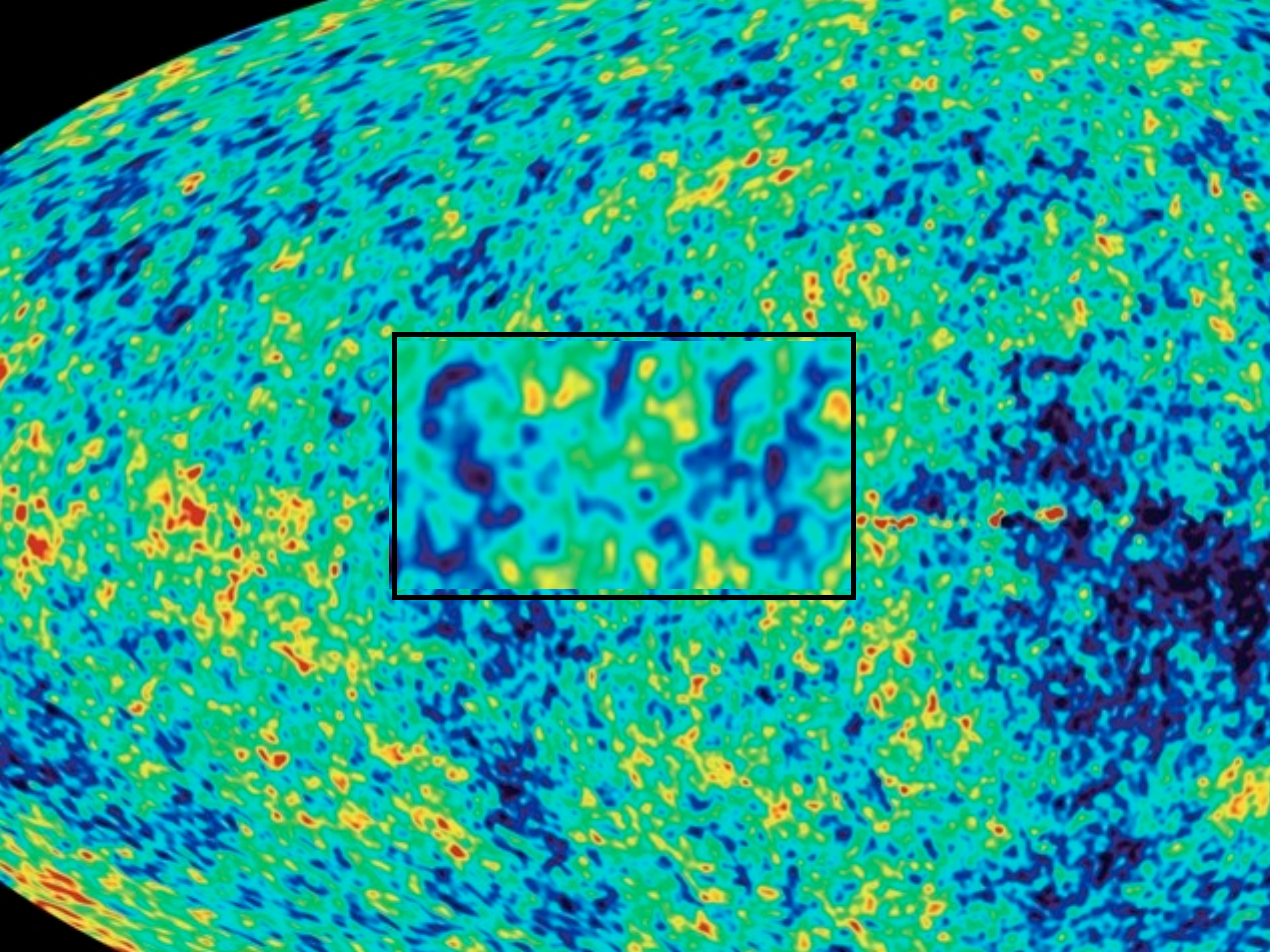
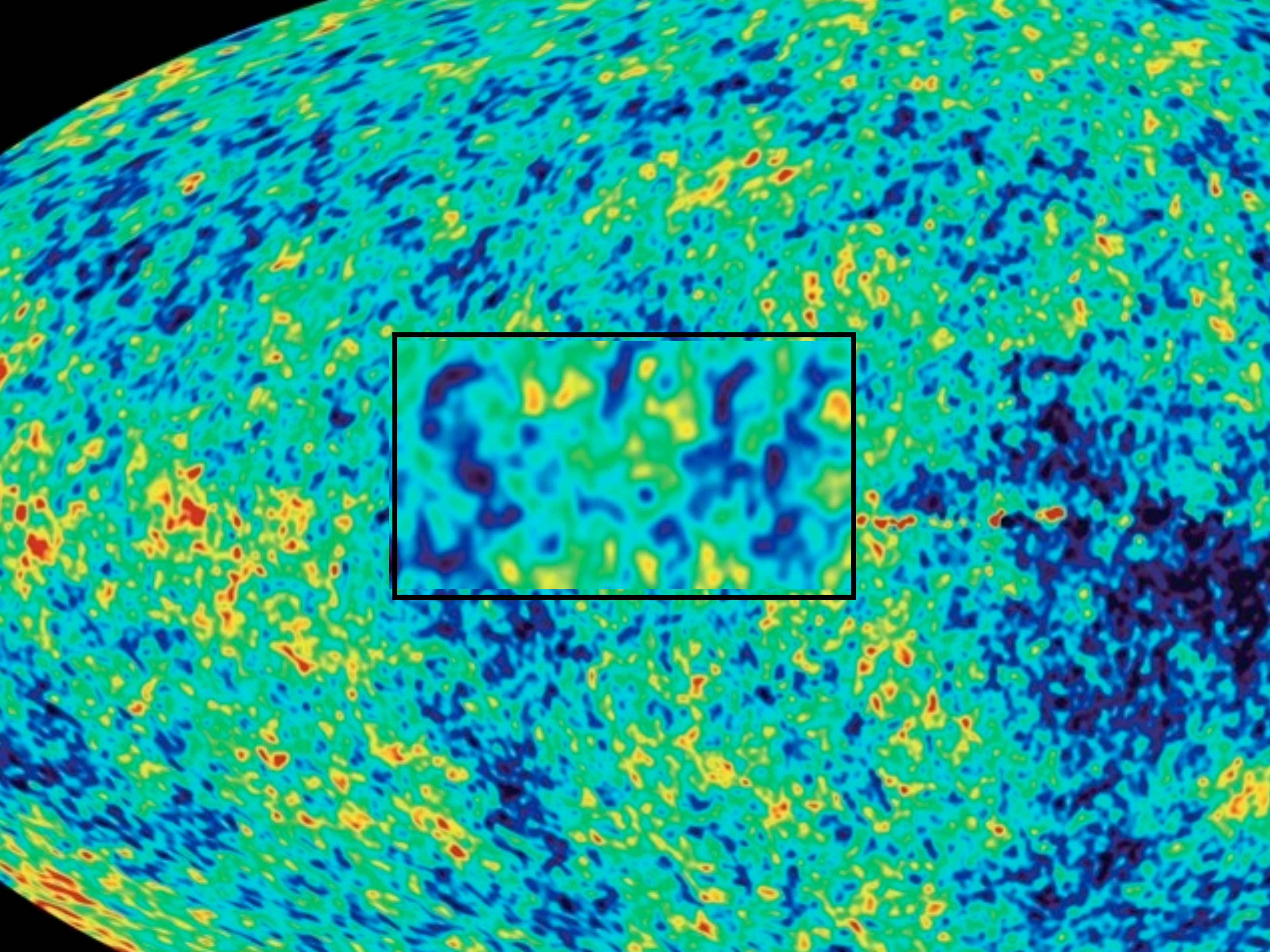
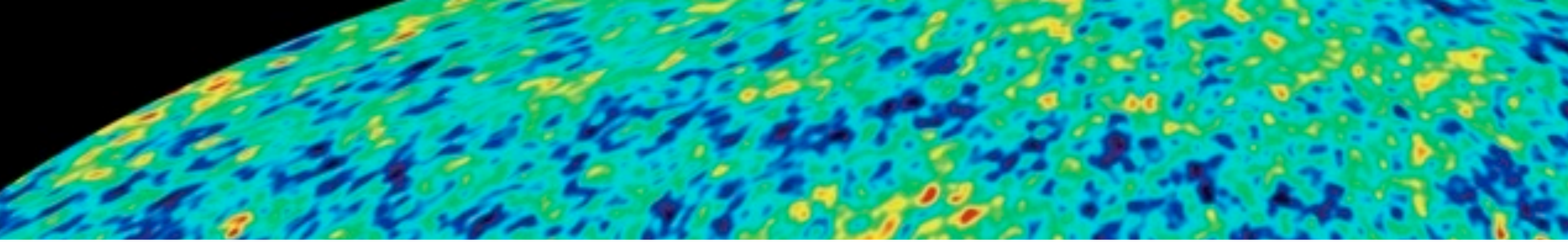
Central limit theorem: any independent random process

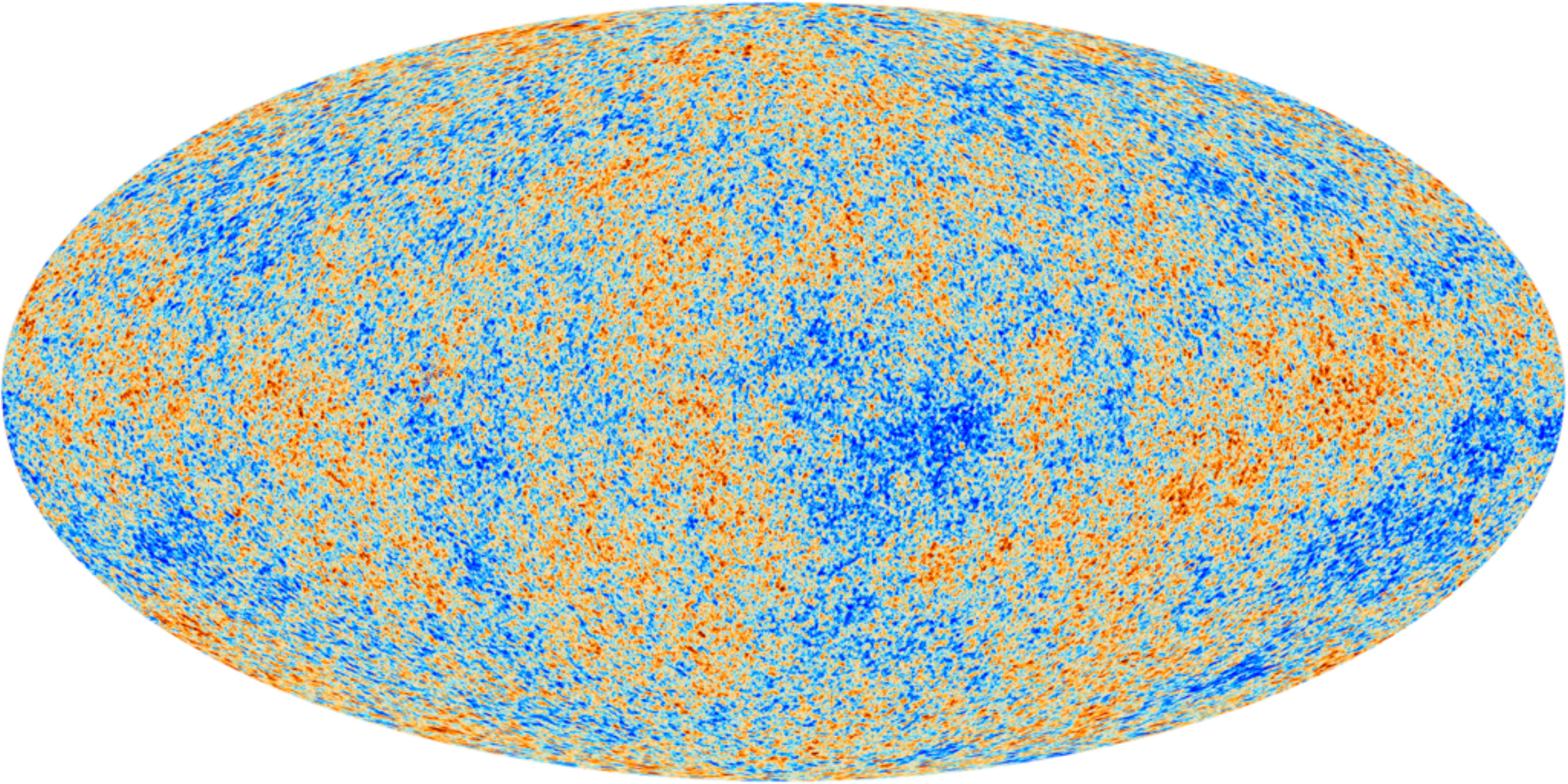
WMAP anomalies II

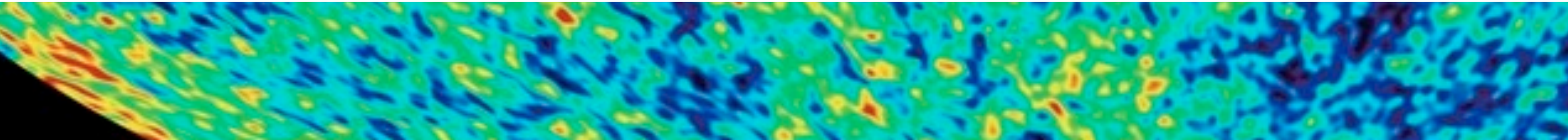


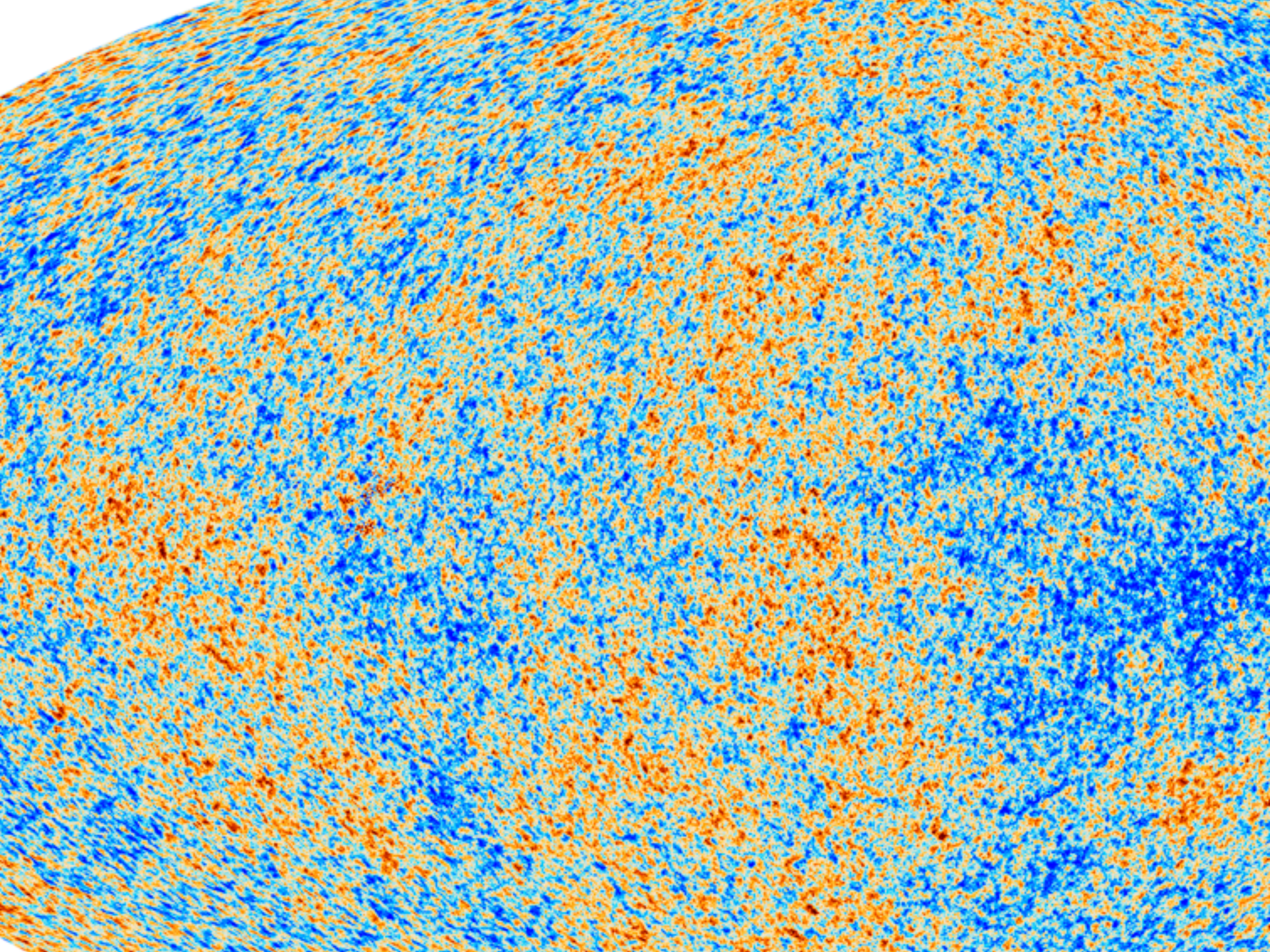


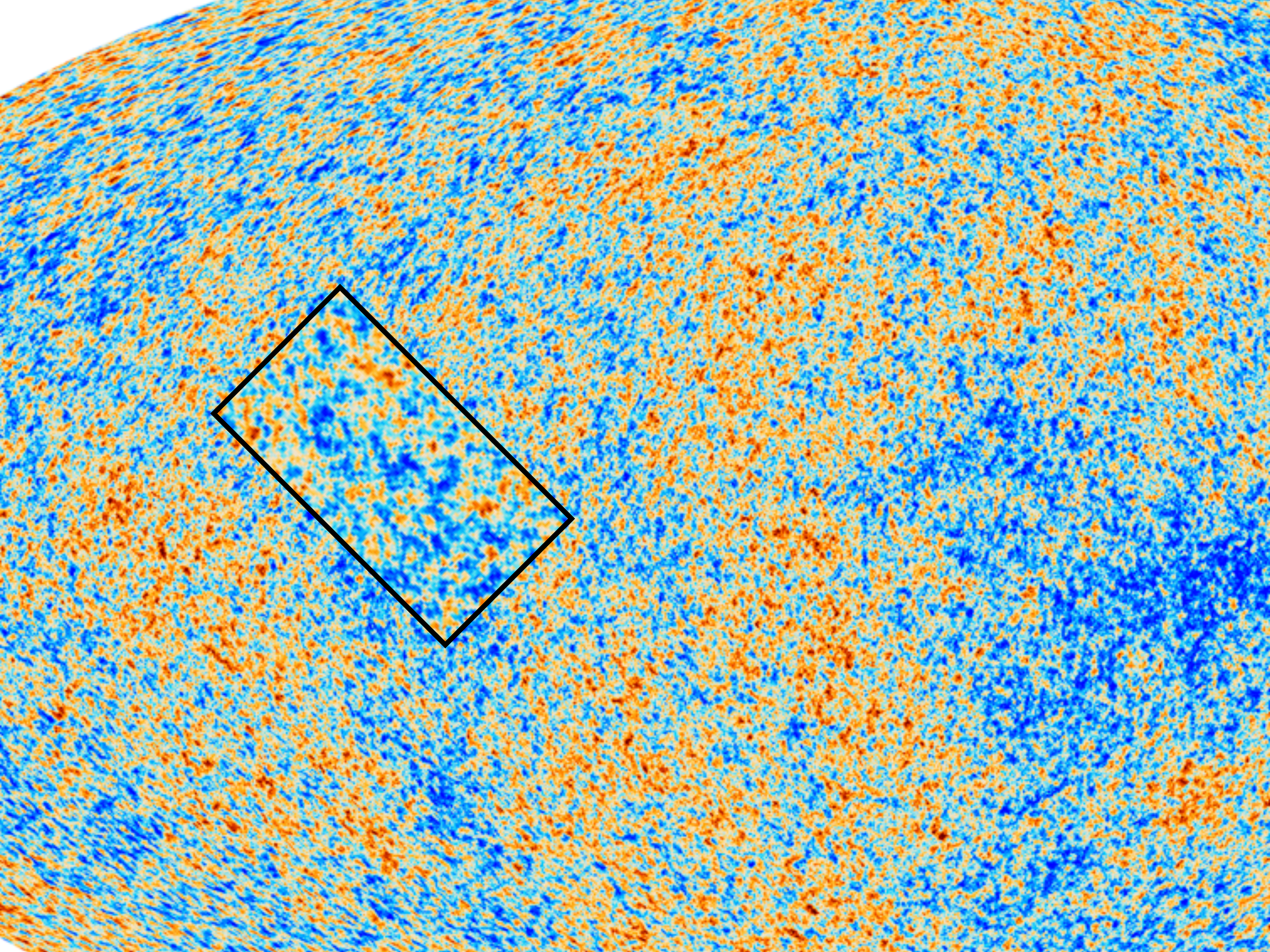




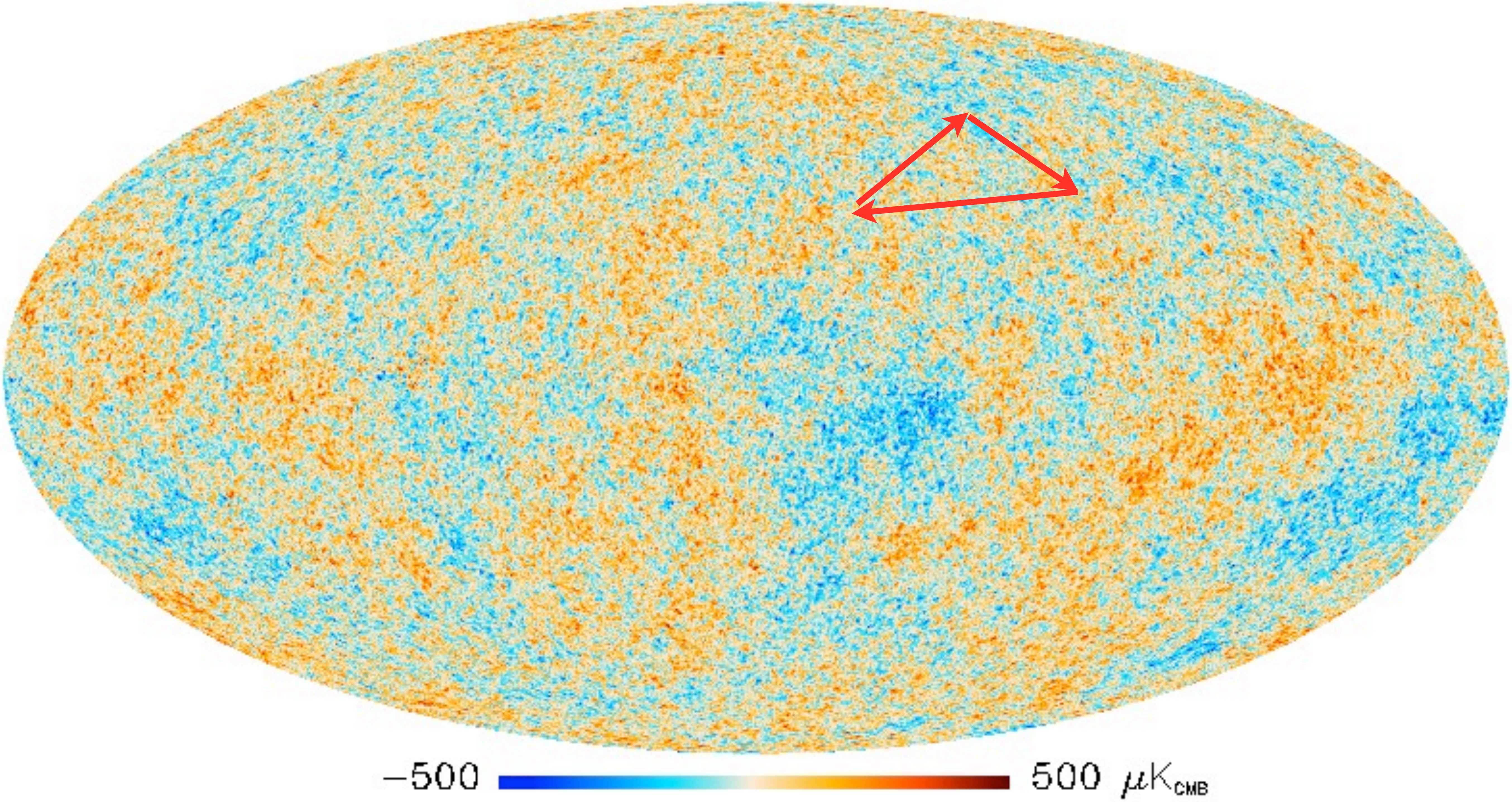






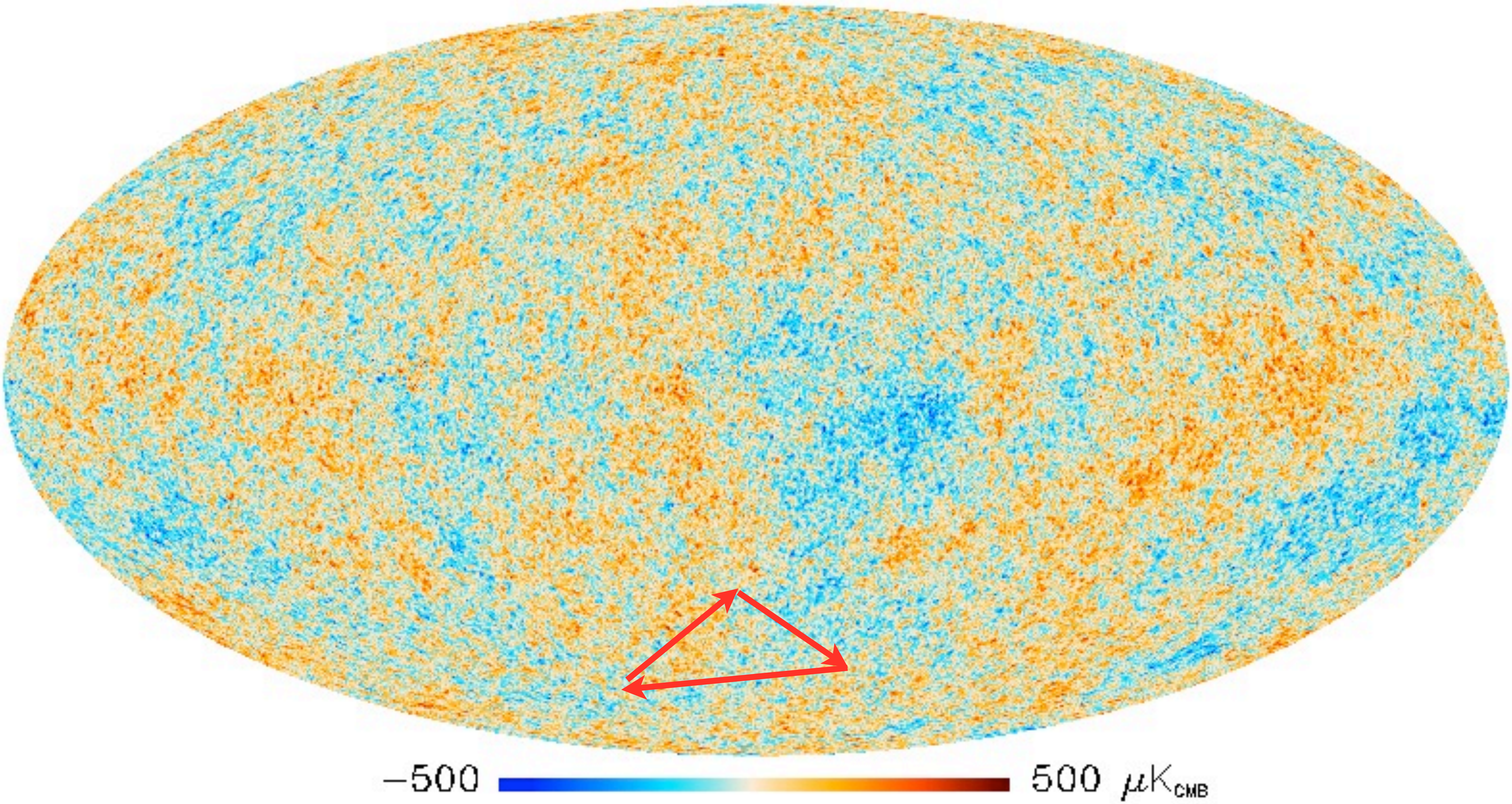


Triangles in the Sky



The CMB Bispectrum

Triangles in the Sky

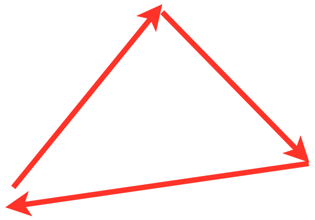


The CMB Bispectrum

Tetrapyd - Bispectrum domain

Allowed multipoles l_1, l_2, l_3 for the CMB bispectrum live in the domain

$$\begin{array}{ll}
\text{Resolution:} & l_1, l_2, l_3 \le l_{\max}\,, \quad l_1, l_2, l_3 \in \mathbb{N}\,, \\
\text{Triangle condition:} & l_1 \le l_2 + l_3 \;\; \text{for} \;\; l_1 \ge l_2,\, l_3, \quad + \text{cyclic perms.} \\
\text{Parity condition:} & l_1 + l_2 + l_3 = 2n\,, \quad n \in \mathbb{N}\,.
\end{array}$$



Reduced bispectrum $b_{l_1 l_2 l_3}$ from primordial bispectrum $B(k_1,k_2,k_3)$

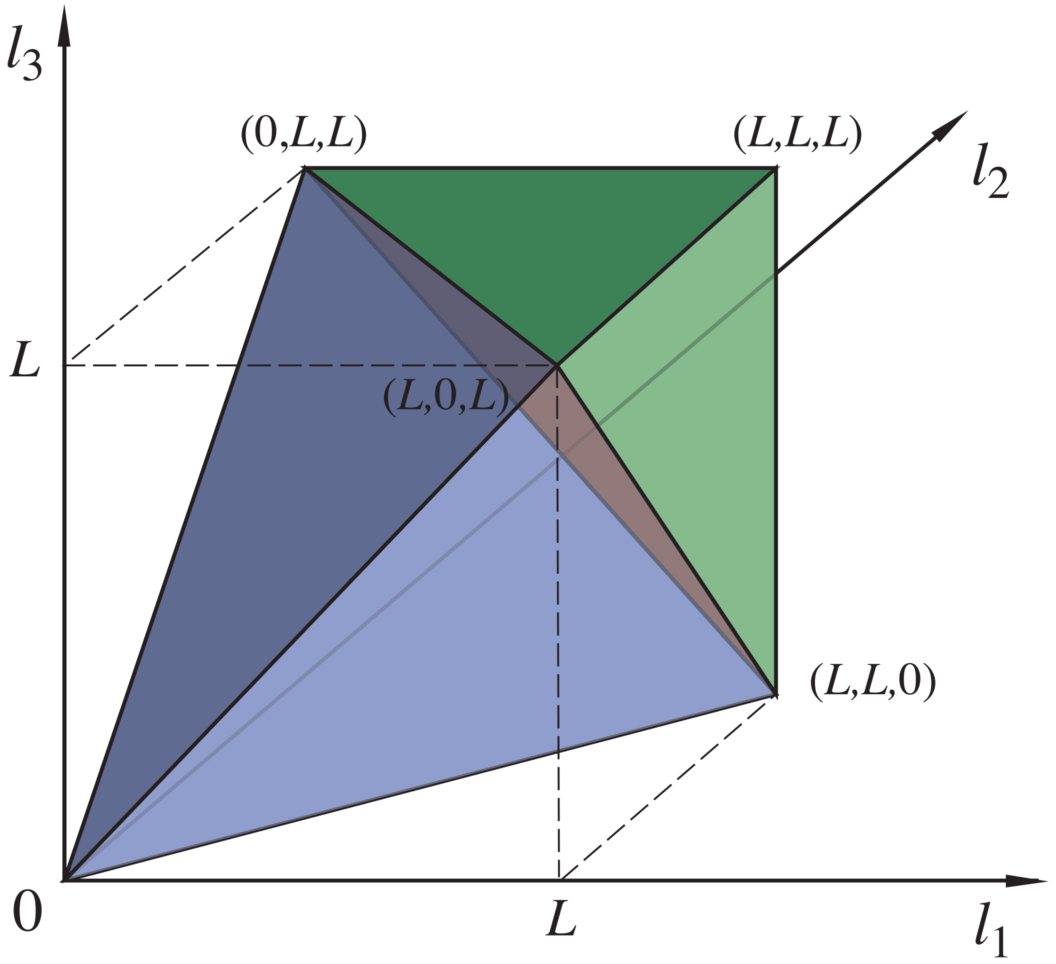
$$b_{l_1 l_2 l_3} = \left(\frac{2}{\pi}\right)^3 \int x^2 dx \int dk_1 dk_2 dk_3 (k_1 k_2 k_3)^2 B(k_1,k_2,k_3) \Delta_{l_1}(k_1) \Delta_{l_2}(k_2) \Delta_{l_3}(k_3) j_{l_1}(xk_1) j_{l_2}(xk_2) j_{l_3}(xk_3)$$

Inner product:

Defined by estimator sum

$$\langle b,\, b' \rangle \;\equiv \sum_{l_1, l_2, l_3 \in \mathcal{V}_{\mathcal{T}}} w_{l_1 l_2 l_3}\, b_{l_1 l_2 l_3}\, b'_{l_1 l_2 l_3}$$

with weight $\quad w_{l_1 l_2 l_3} = h^2_{l_1 l_2 l_3}$



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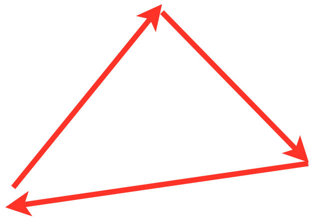

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Primordial bispectrum

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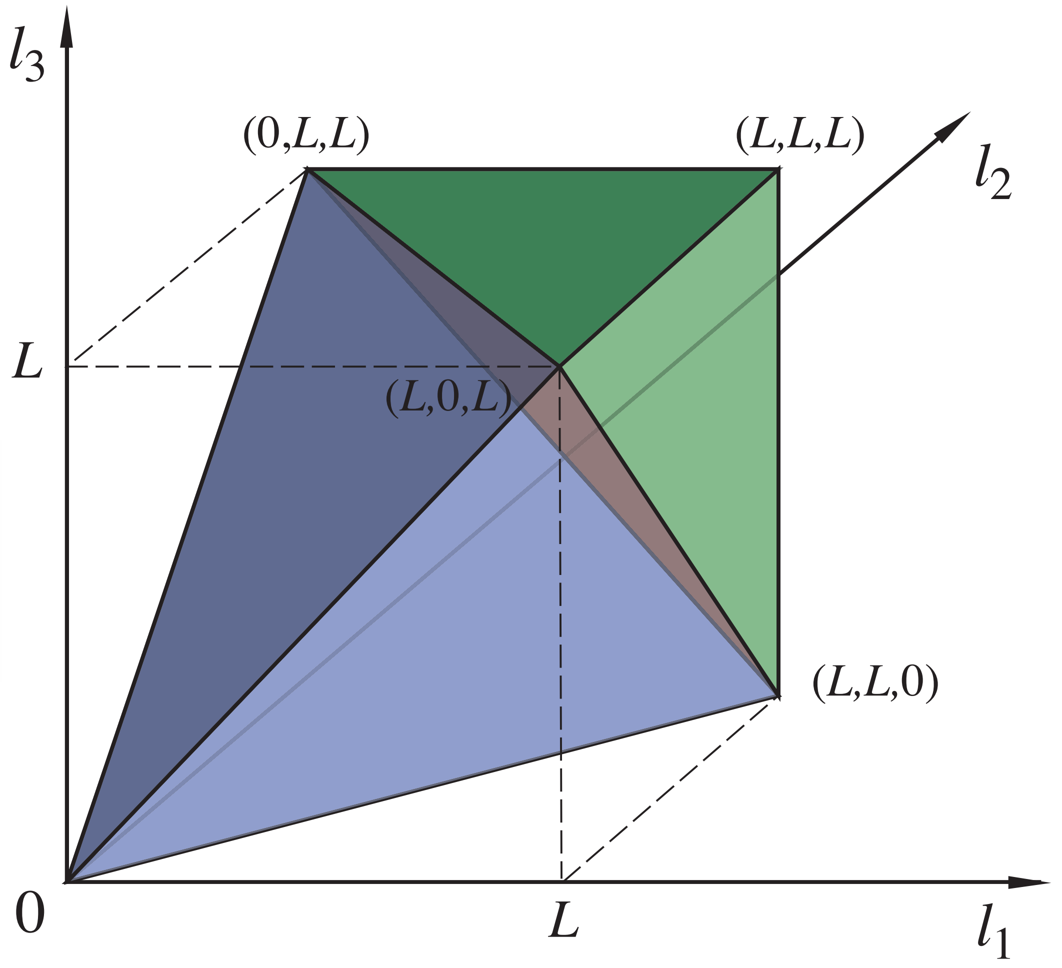

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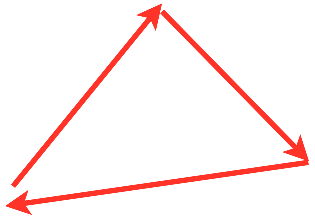

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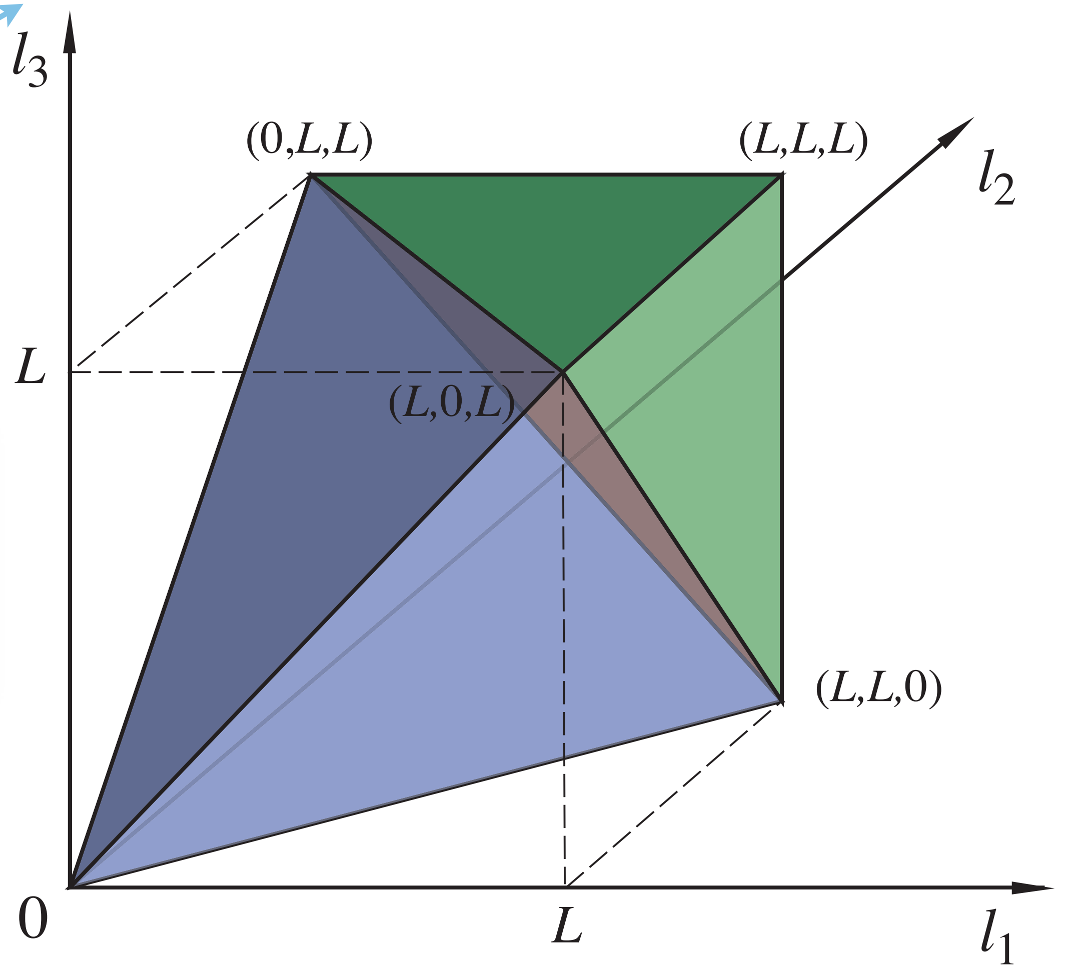
Primordial bispectrum

Transfer functions



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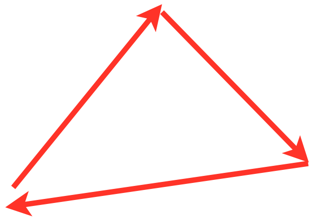

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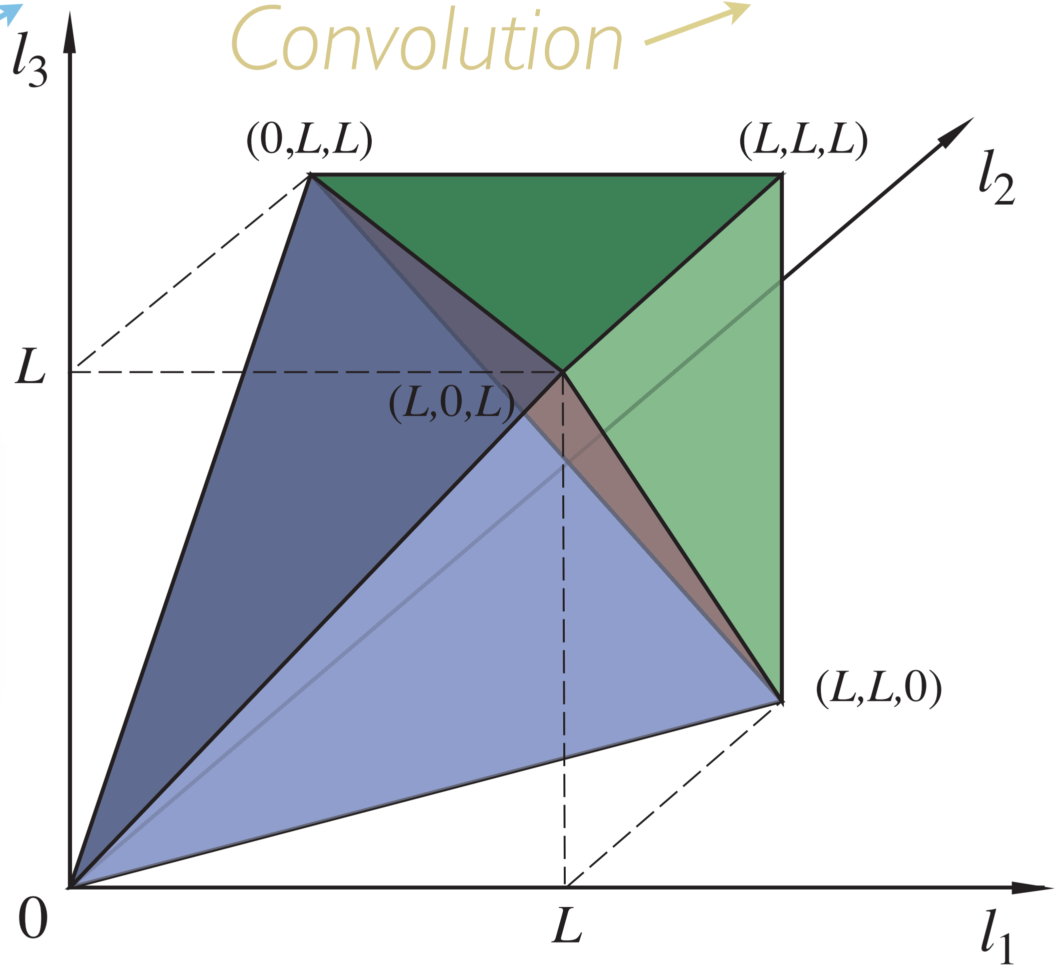
Primordial bispectrum

Transfer functions

Convolution



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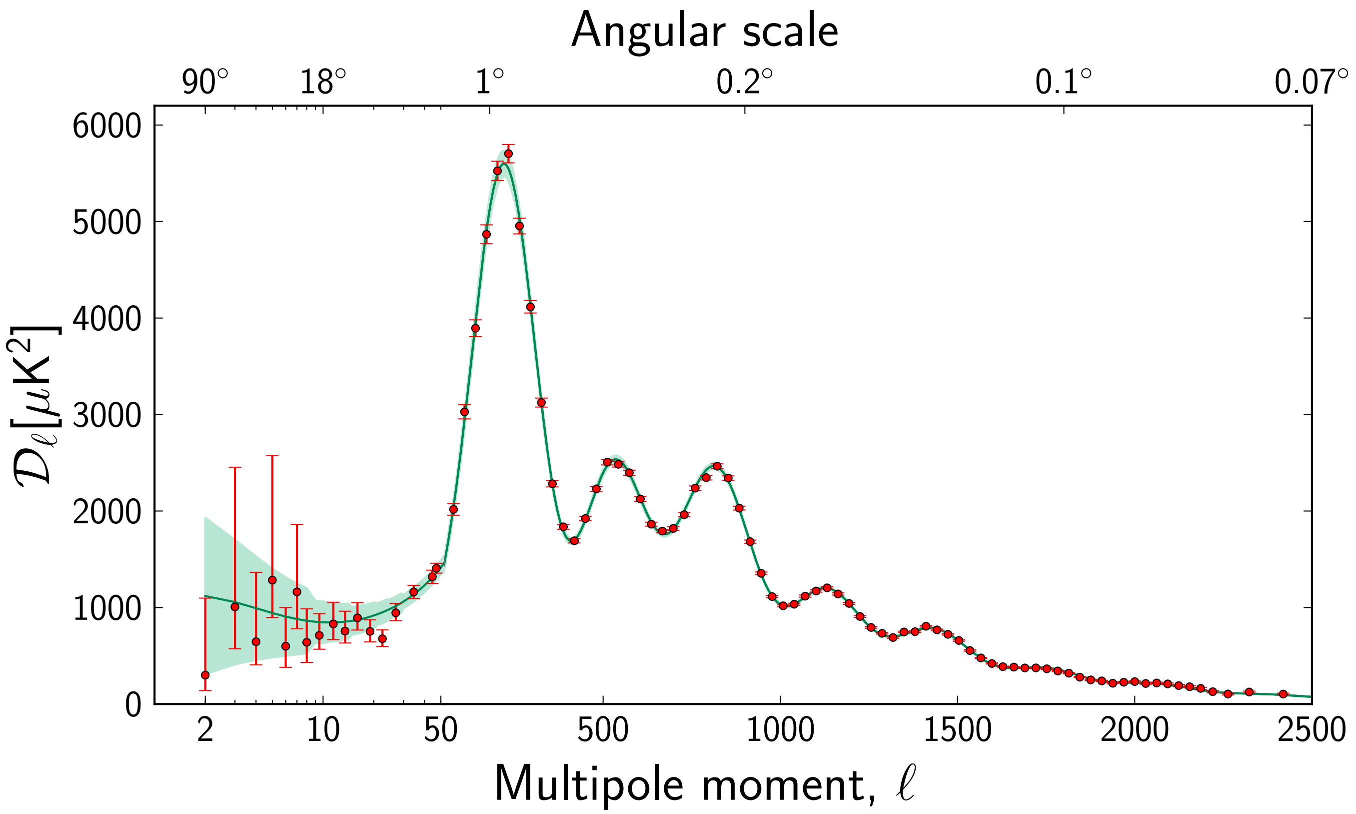
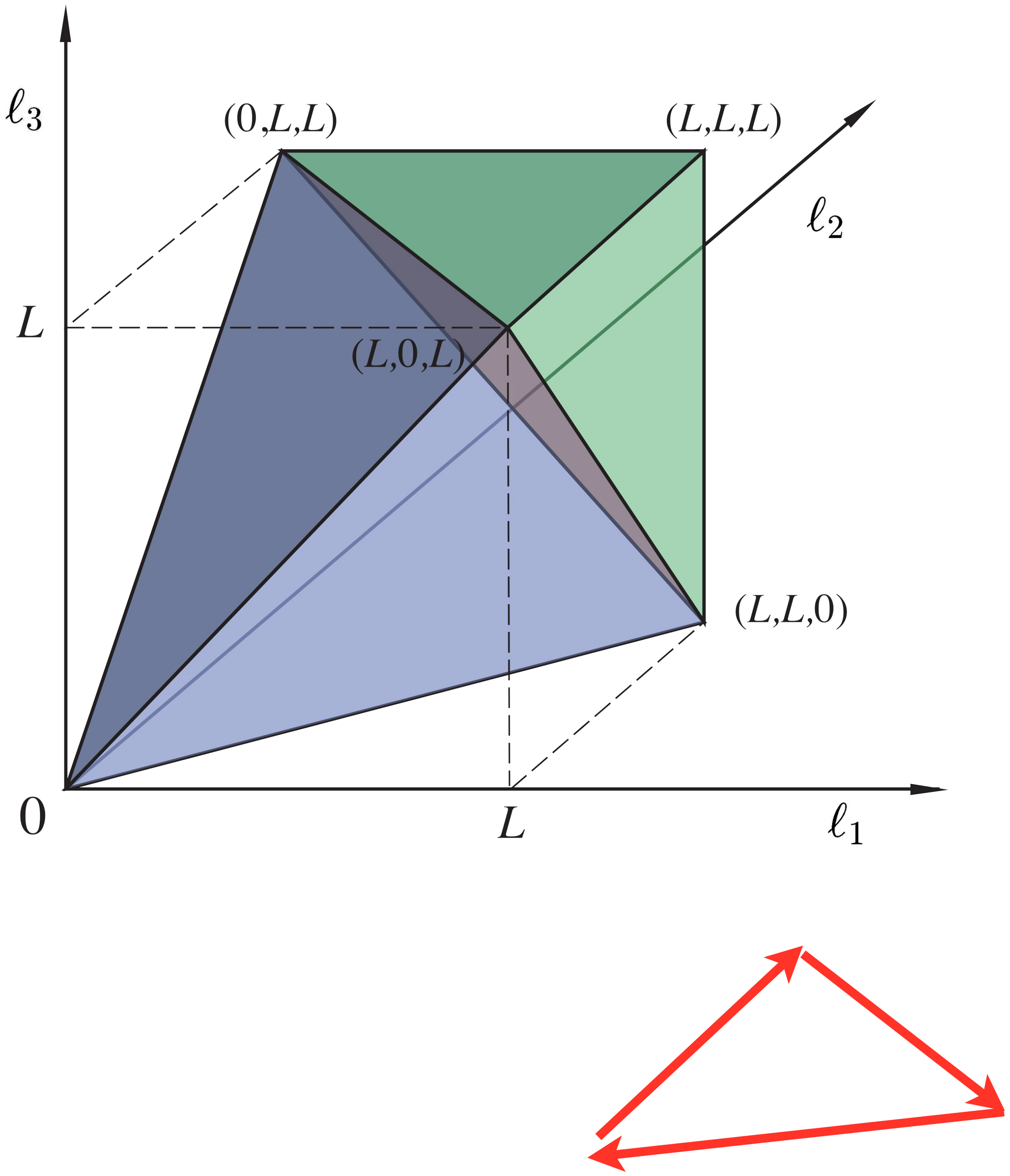
with weight $\quad w_{l_1l_2l_3} = h^2_{l_1l_2l_3}$

Inflation and the bispectrum

Hot plasma oscillations create patterns of acoustic peaks:

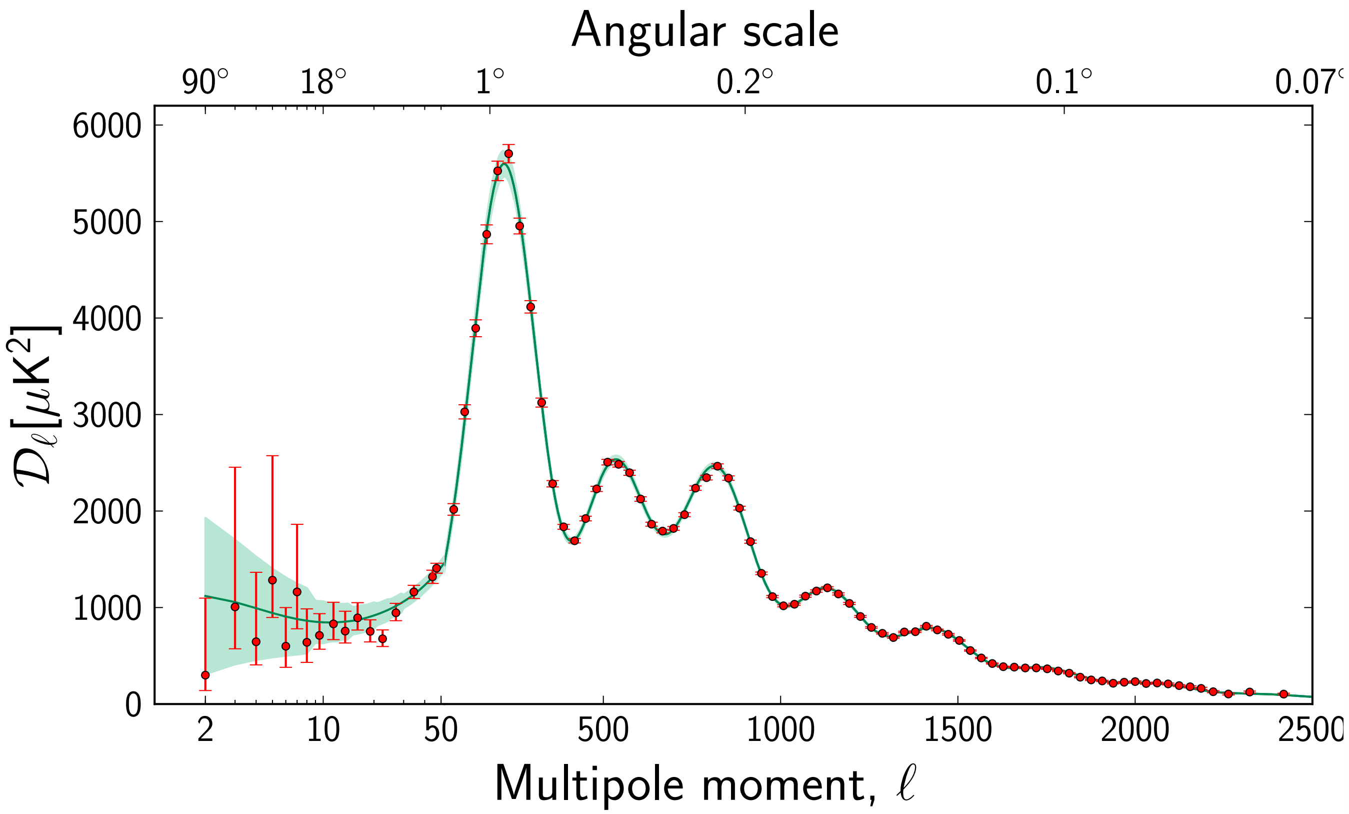


Power spectrum (2pt correlator)

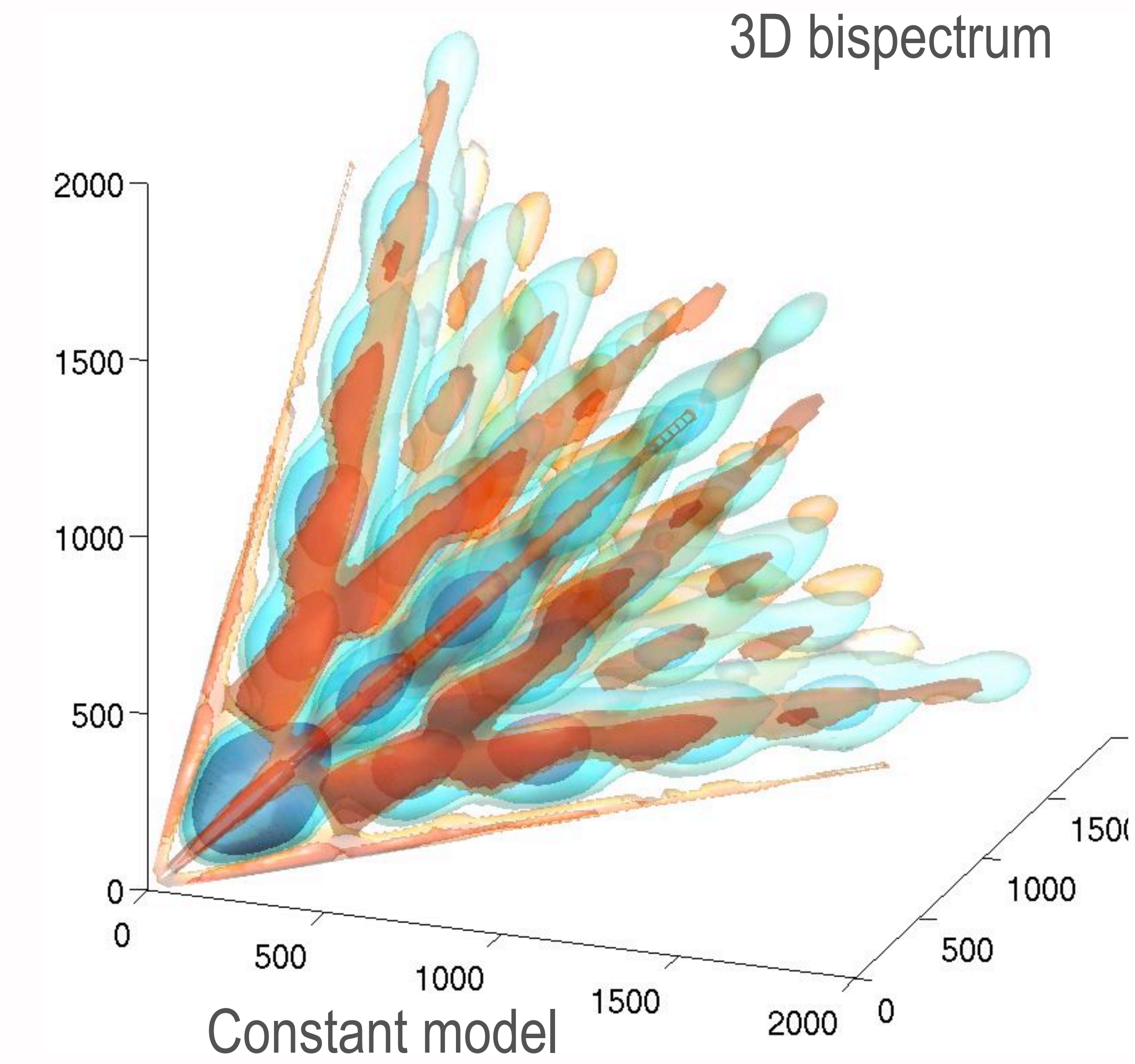


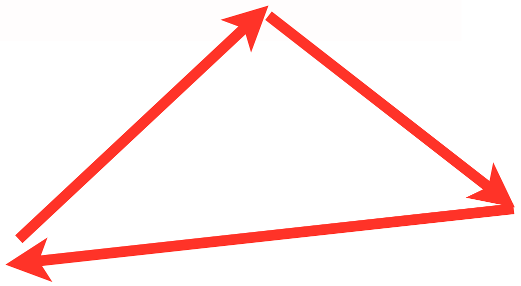
Inflation and the bispectrum

Hot plasma oscillations create patterns of acoustic peaks:



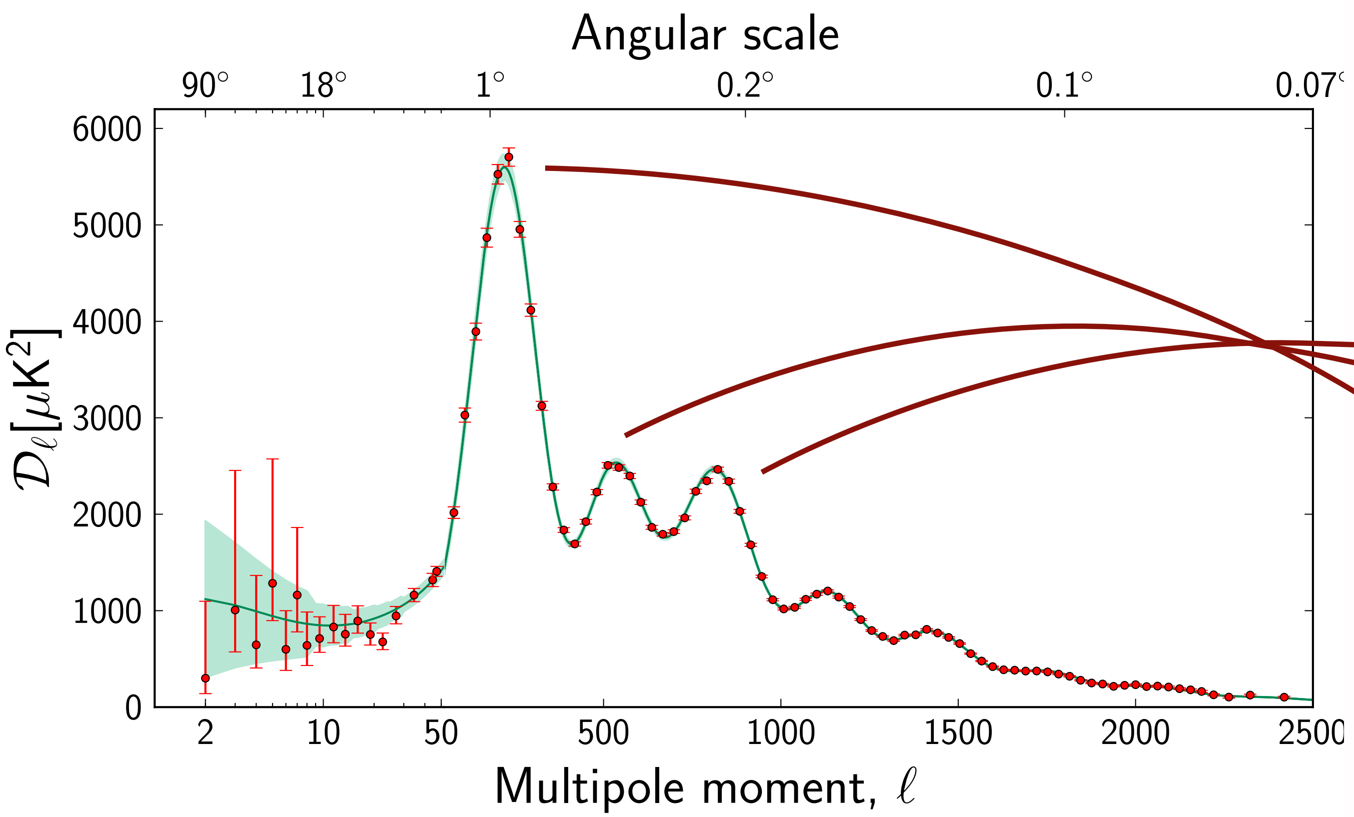
Power spectrum (2pt correlator)





Inflation and the bispectrum

Hot plasma oscillations create patterns of acoustic peaks:



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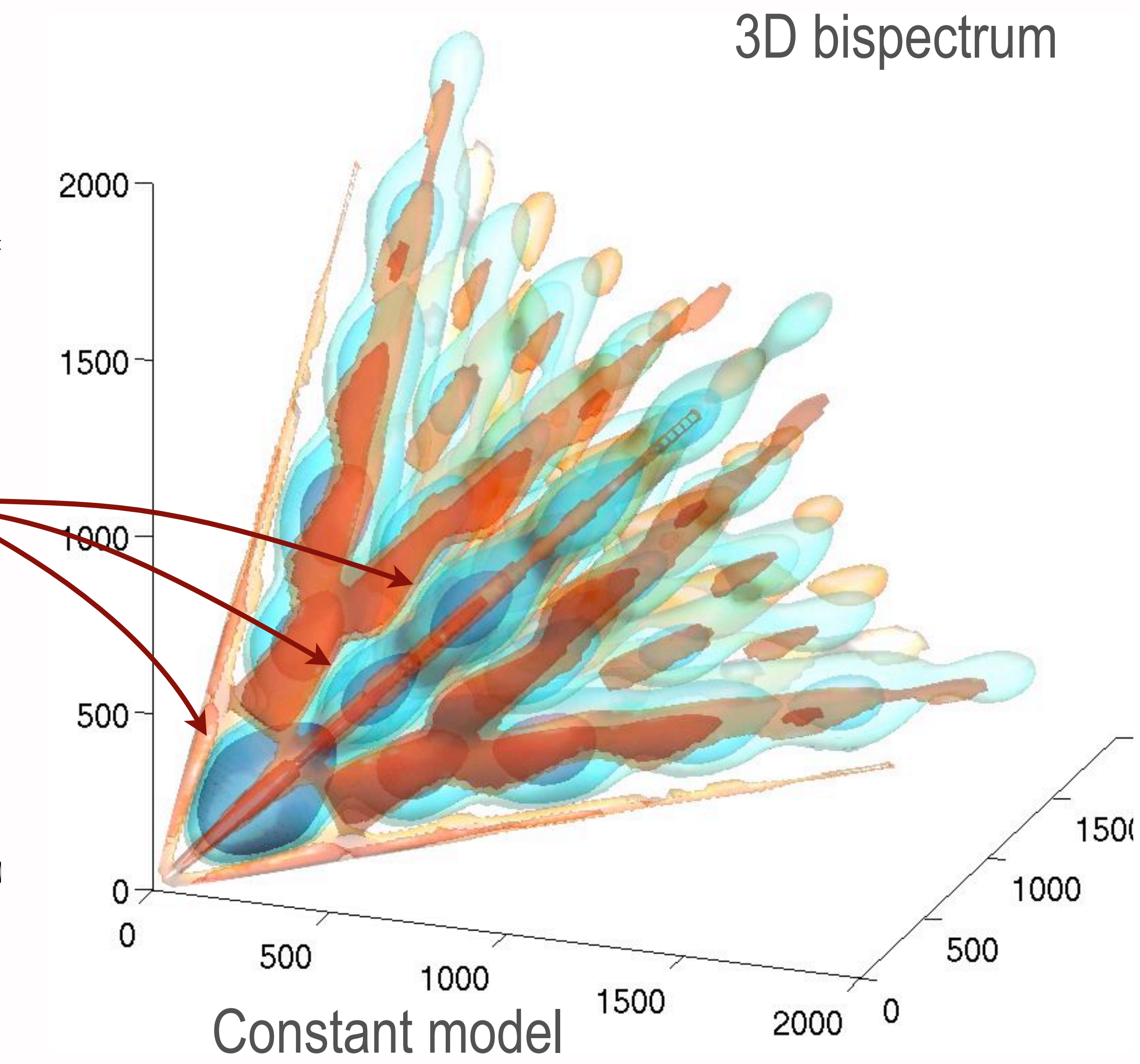
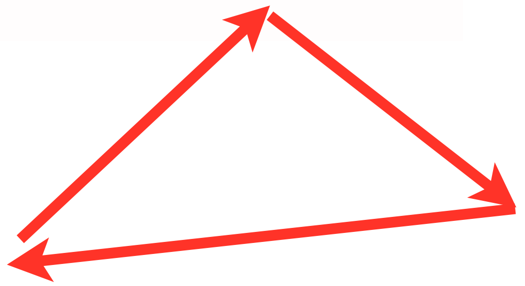
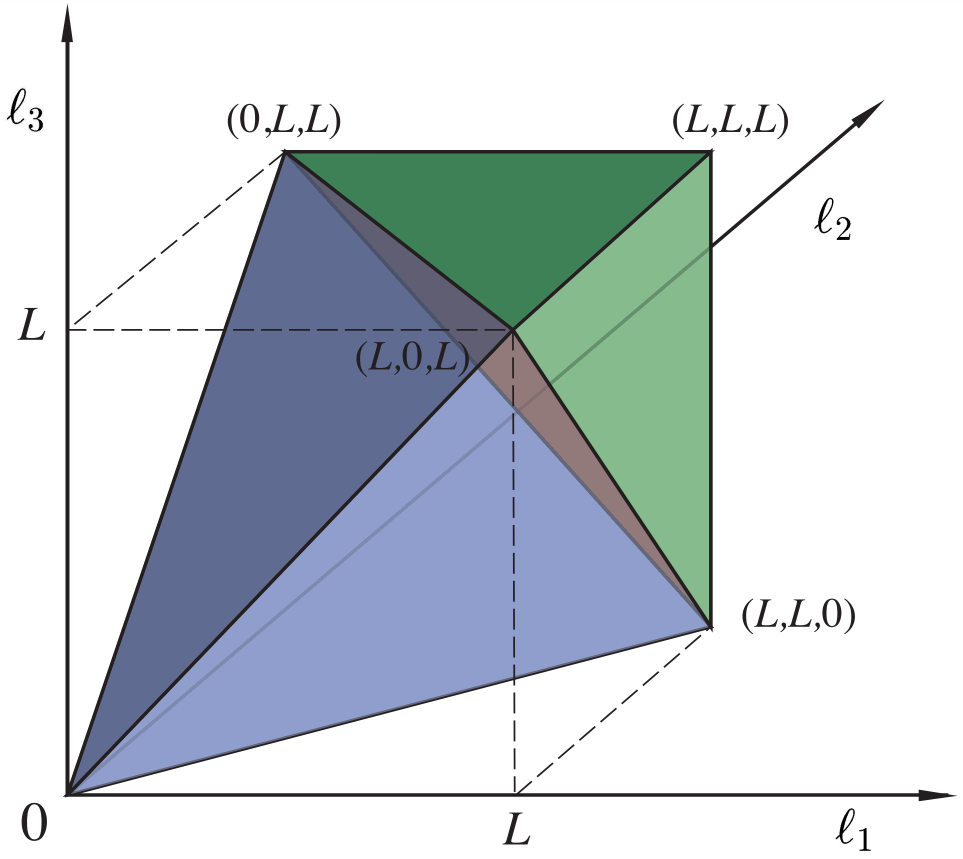




Aside: tetrapyd triangles

Tetrapyd



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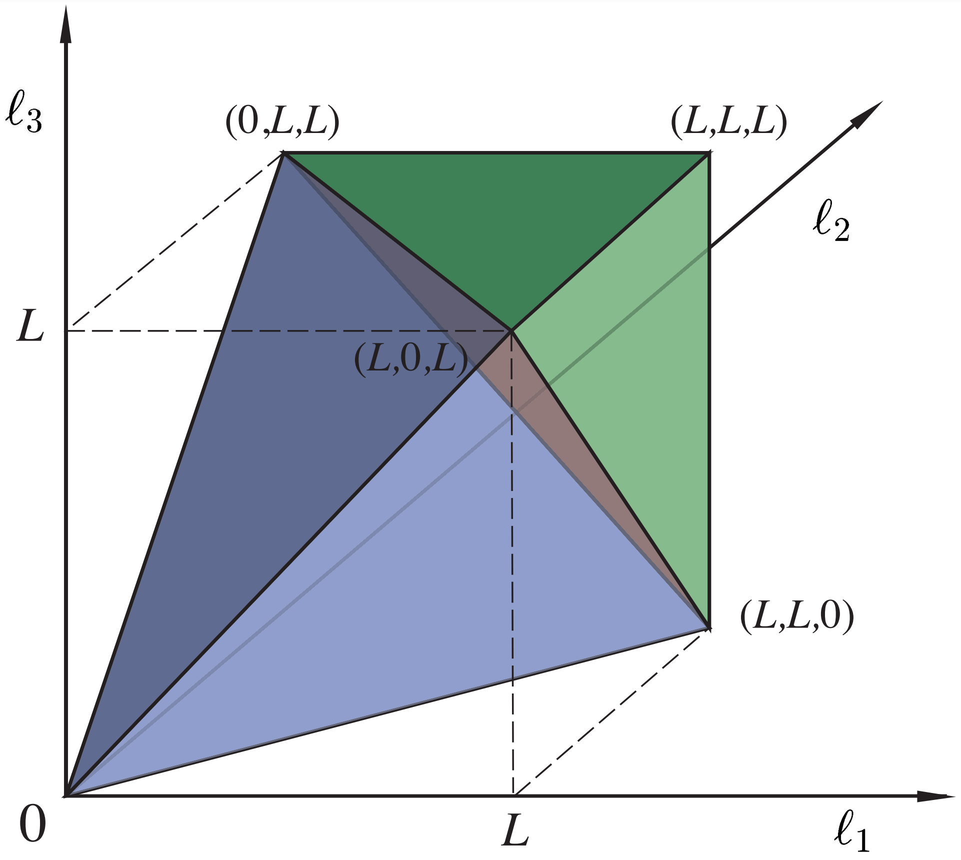
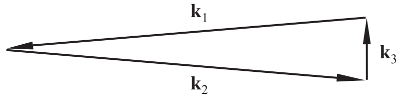
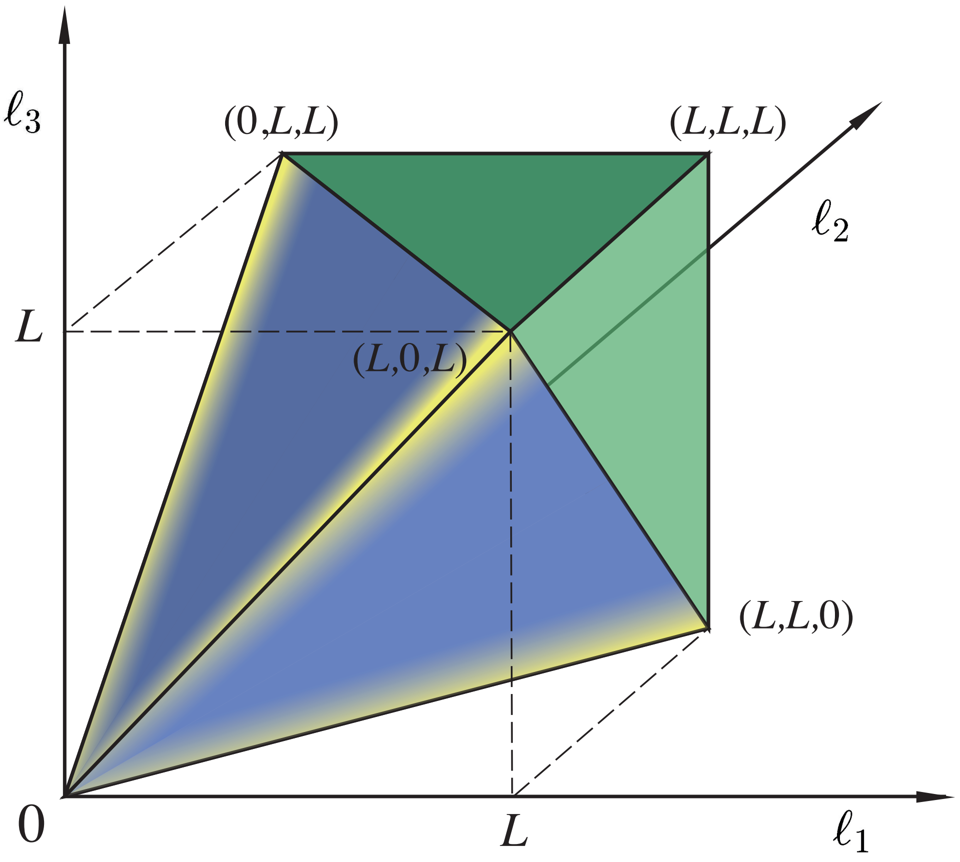


Squeezed





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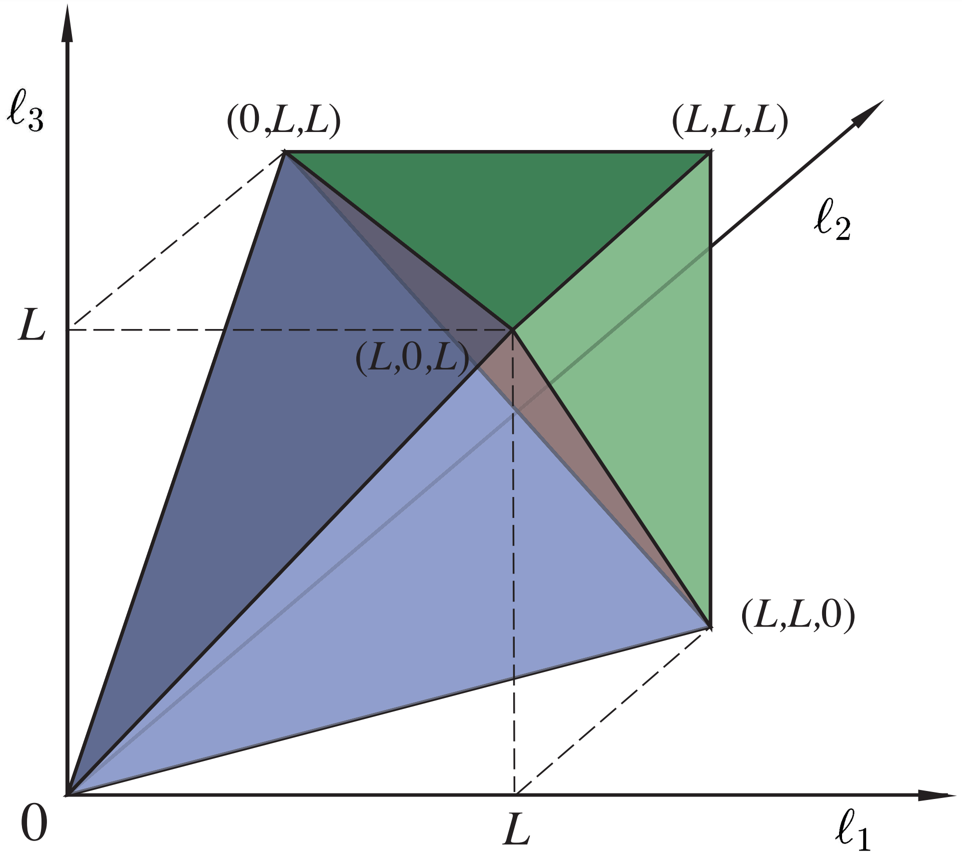

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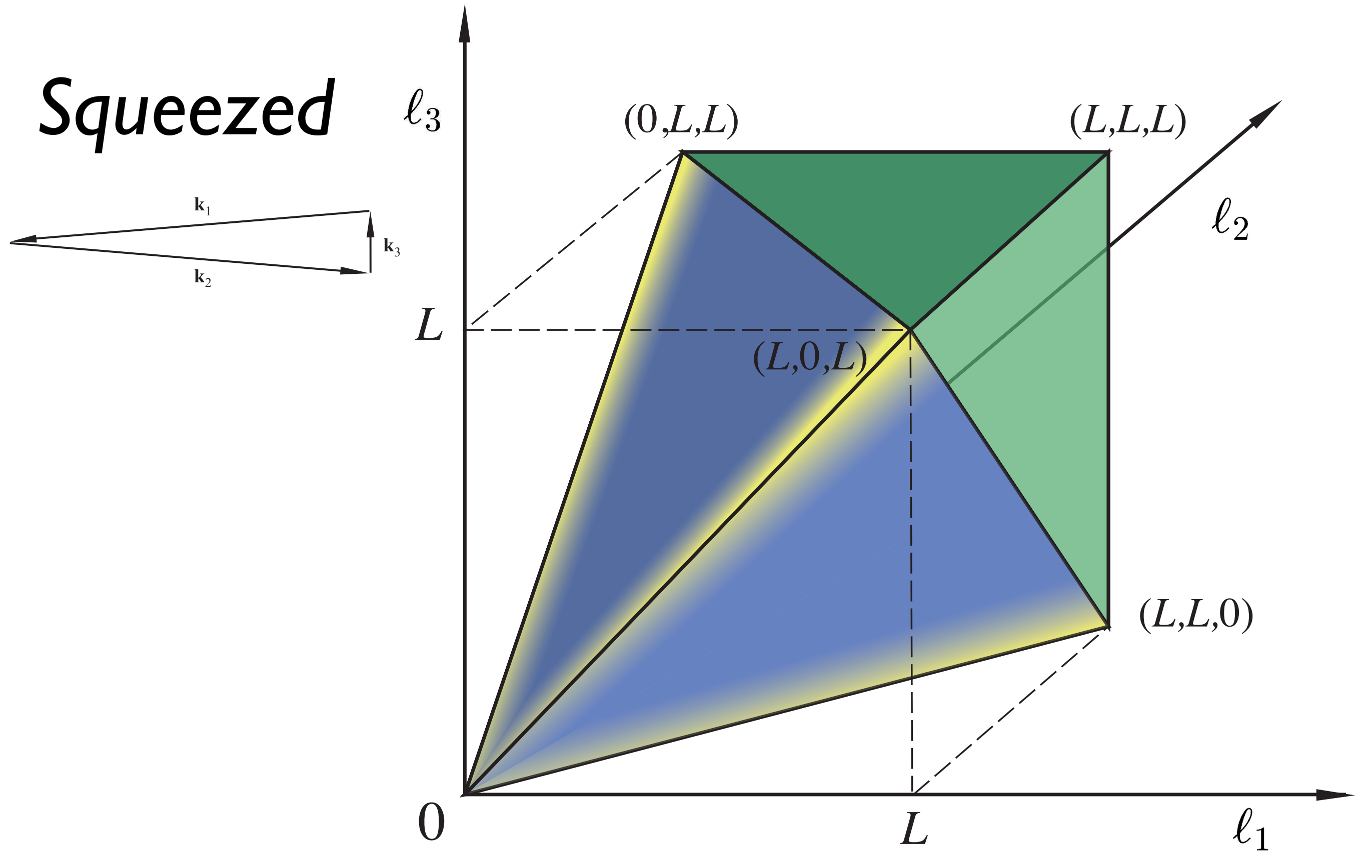
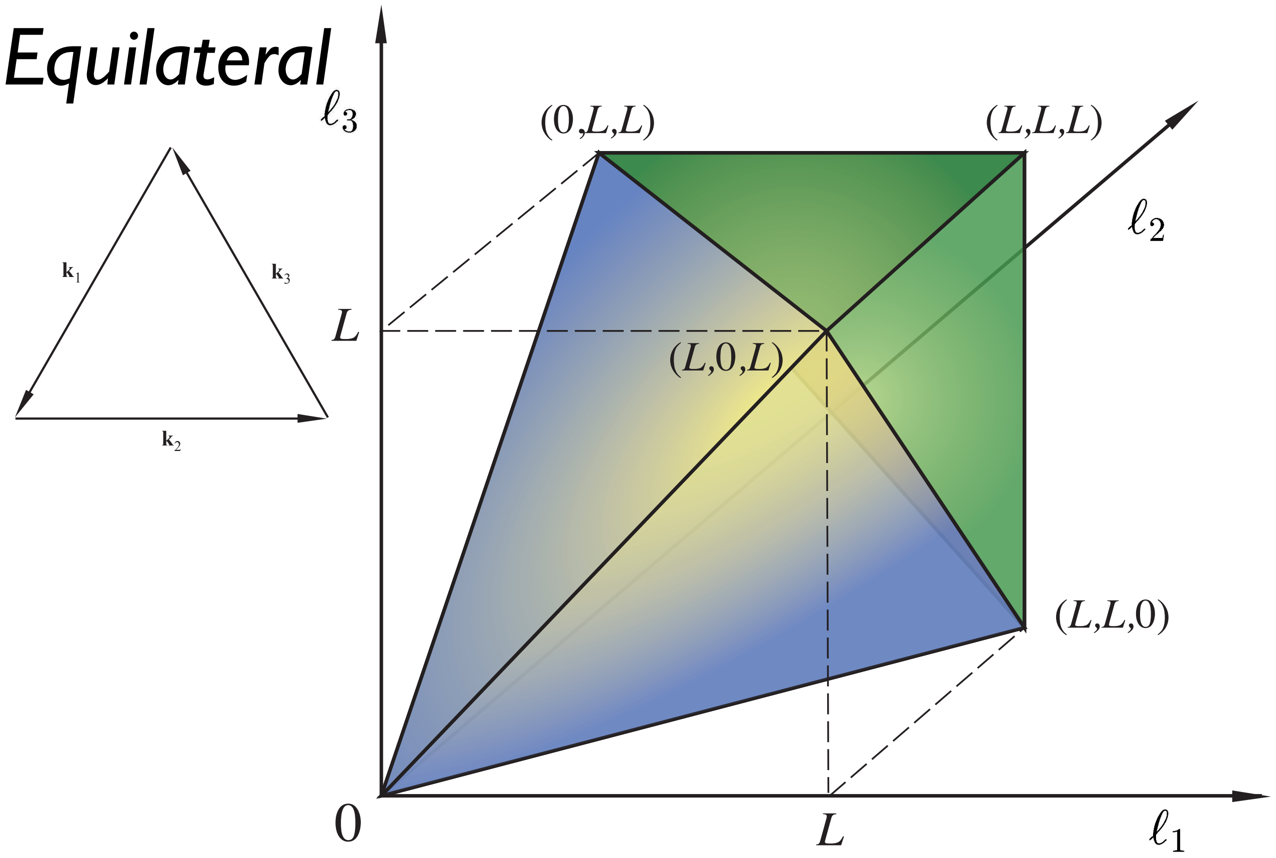


Equilateral



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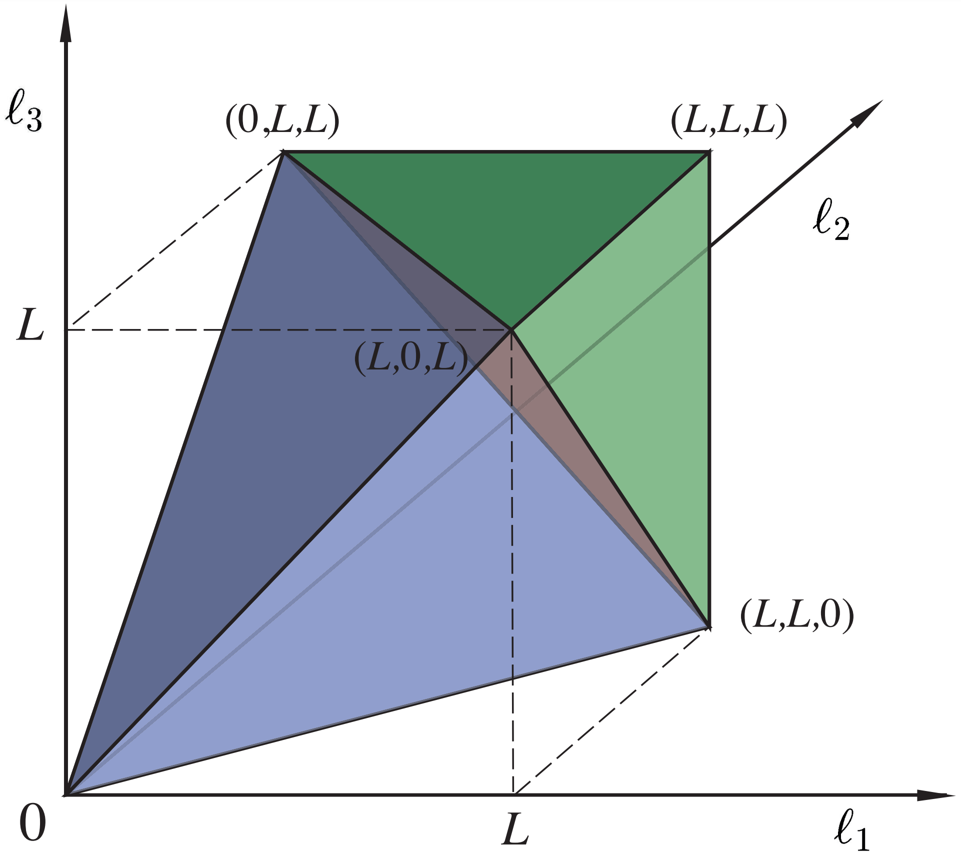

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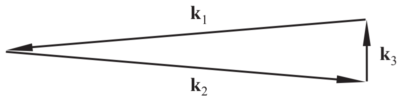

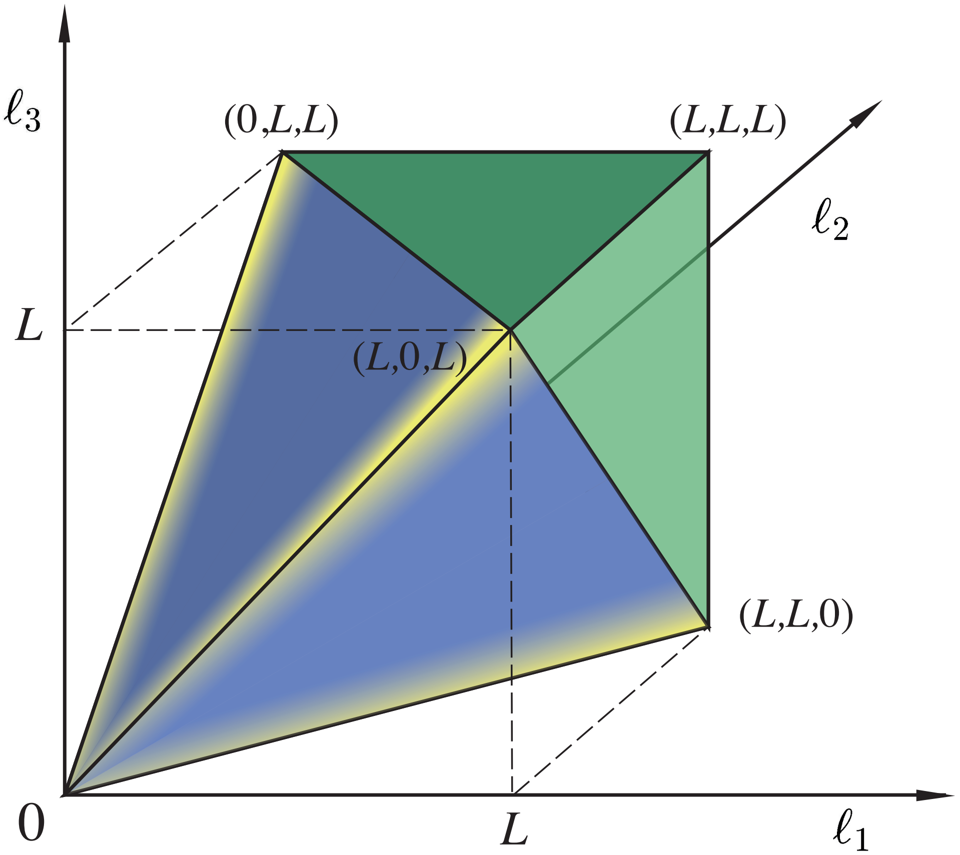

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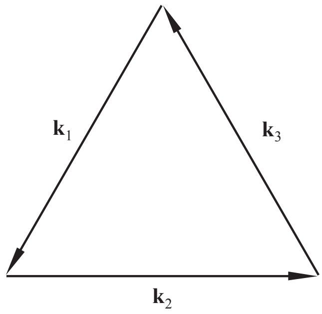
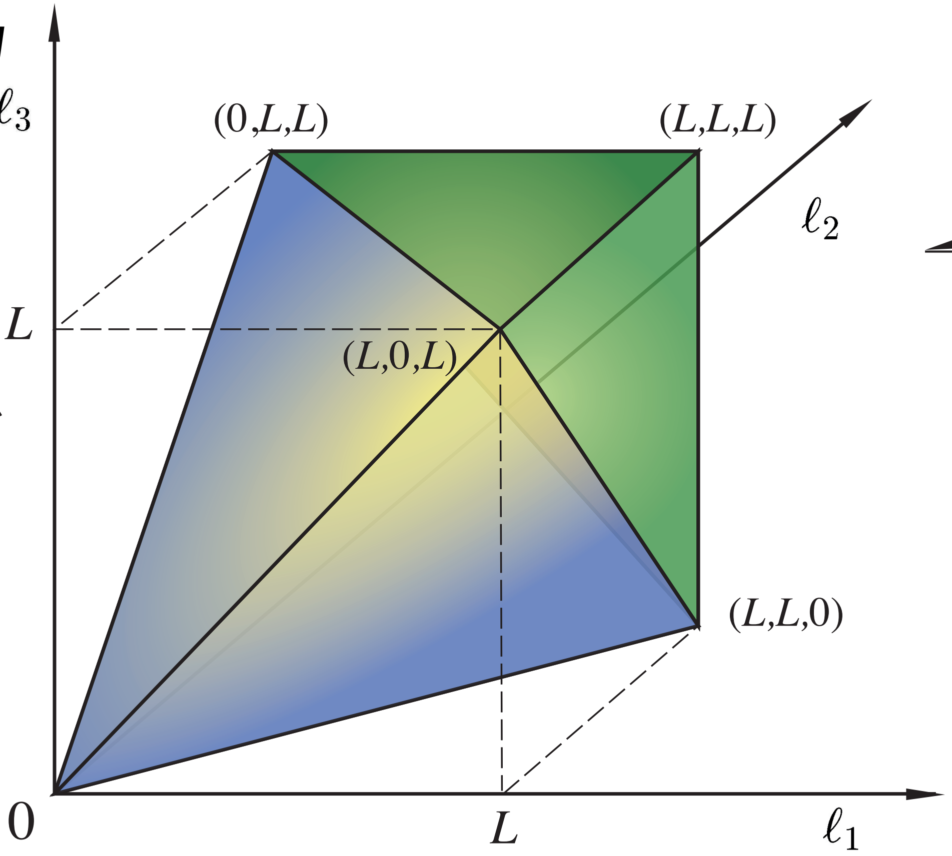
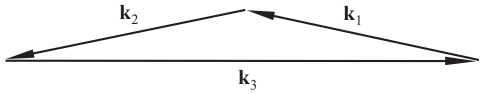
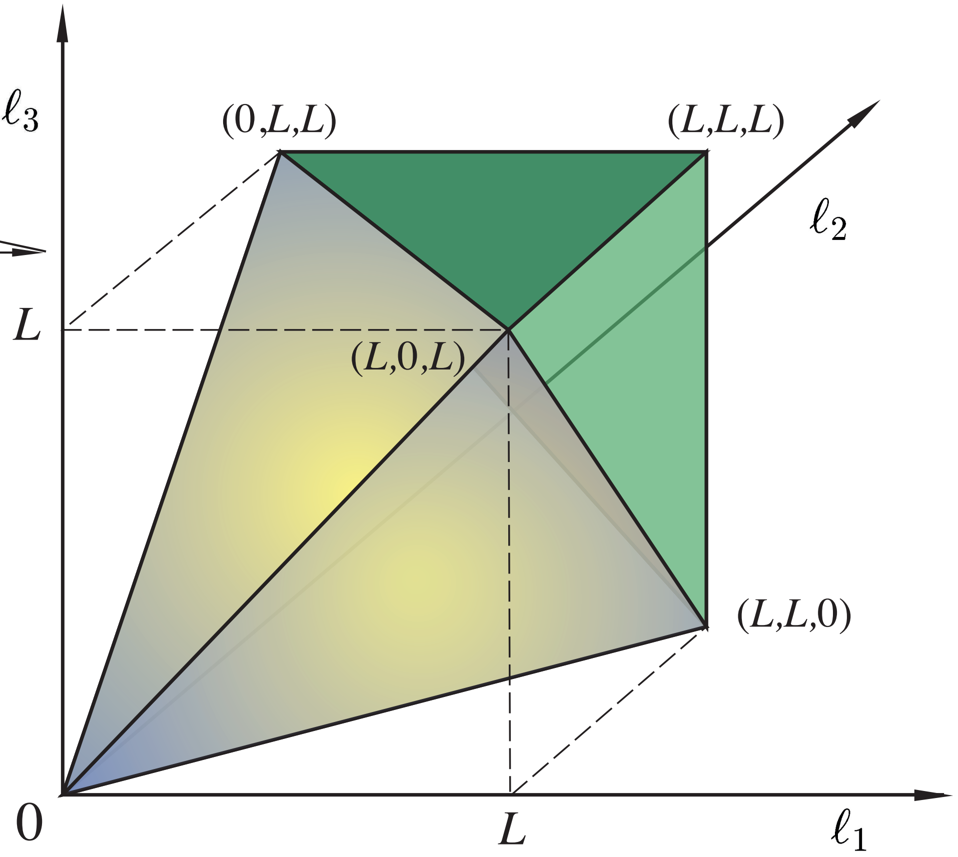




Flattened





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Simple inflation models *cannot* generate observable non-Gaussianity:

- single scalar field
- canonical kinetic terms
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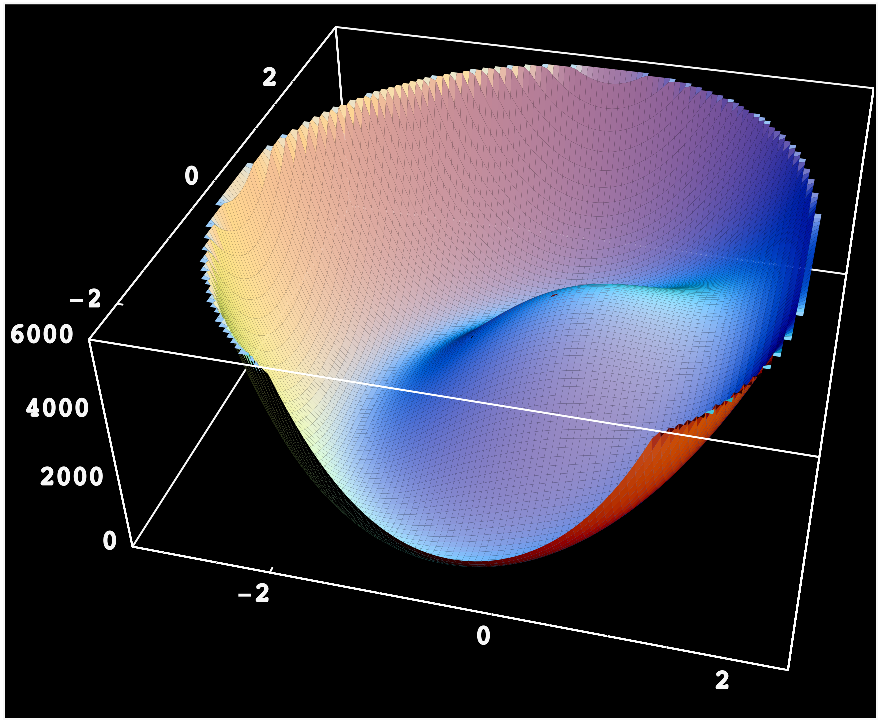
But simple inflation model-building faces rigorous challenges in fundamental theory (e.g. *eta problem and super-Planckian field values*). Many fundamental cosmology ideas/solutions violate these conditions!

Multifield inflation

NG from interacting potentials

$$V(\phi_1, \phi_2) = \frac{1}{4}\lambda(\phi_1^2 + \phi_2^2 - m^2)^2 + \nu(\phi_1 + m)^3$$



Significant final f_{NL} ingredients:

- *corner turning*
- *nontrivial potential*
- *or breakout (hybrid models)*

Rigopoulos, EPS, van Tent 05, 06;
Vernizzi & Wands 06,
and Bernadeau & Uzan 02 etc etc

- *Curvatons* - *post-inflation eqn of state domination*

e.g. Linde & Mukhanov 96; Enqvist & Sloth 01; Lyth & Wands 01; Moroi &Tak

- *End of inflation, reheating and preheating*

Modulated reheating *e.g. Kofman et al 05; Dvali et al 06; etc*

Nonlinear perturbations from preheating
e.g. Chambers & Rajantie 07,08; Bond, Frolov, Huang & Kofman, 09.

- *Particle production during inflation*

(incl. warm inflation) *Moss & Xiong, 07; Moss & Graham, 07.*

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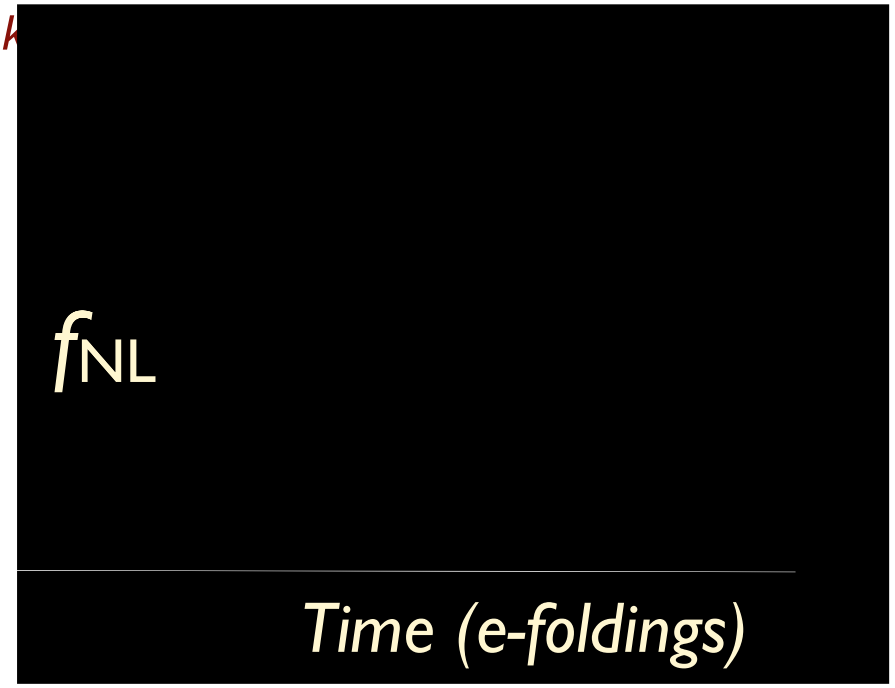

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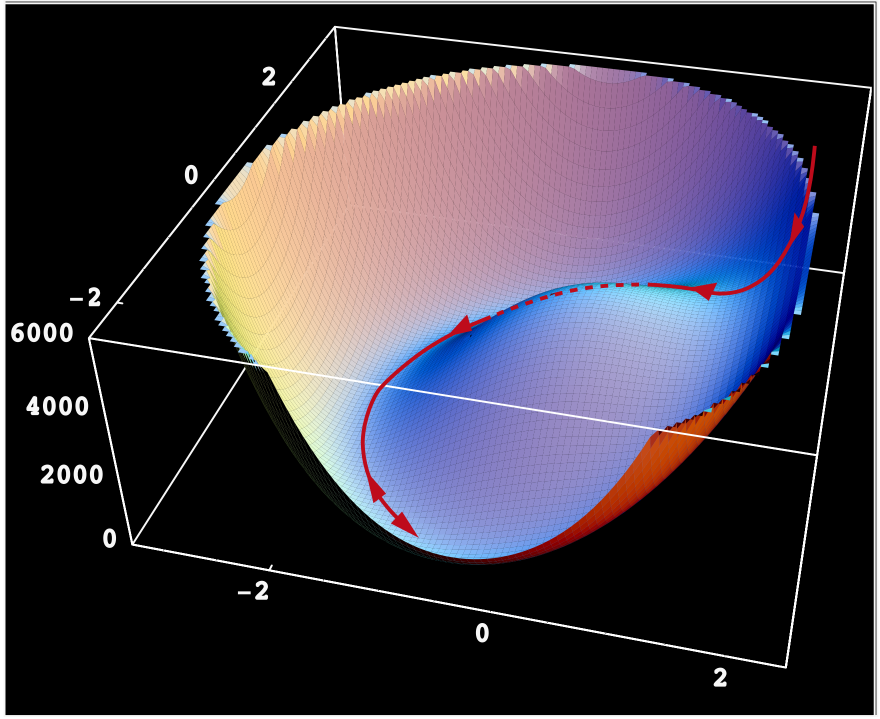

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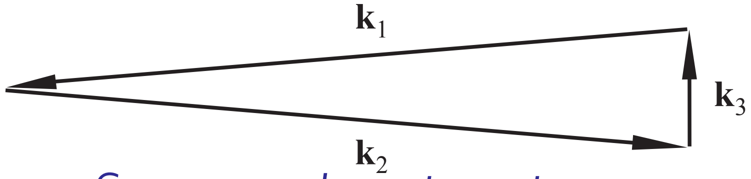

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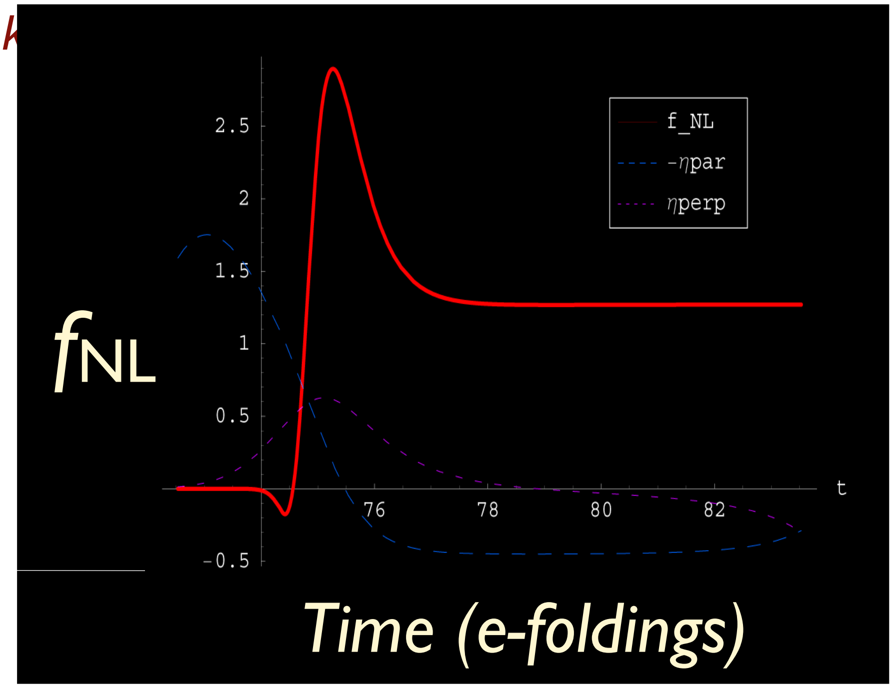

Non-Gaussian Sources (cont.)

Non-Canonical Inflation

- Single field: K-inflation, DBI inflation - *modified sound speed*
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e.g. Chen, et al, 2006; Holman & Tolley, 2008; Meerburg et al 2008

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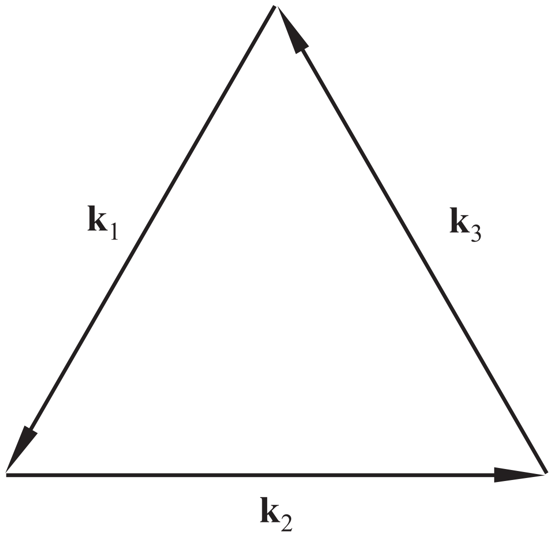

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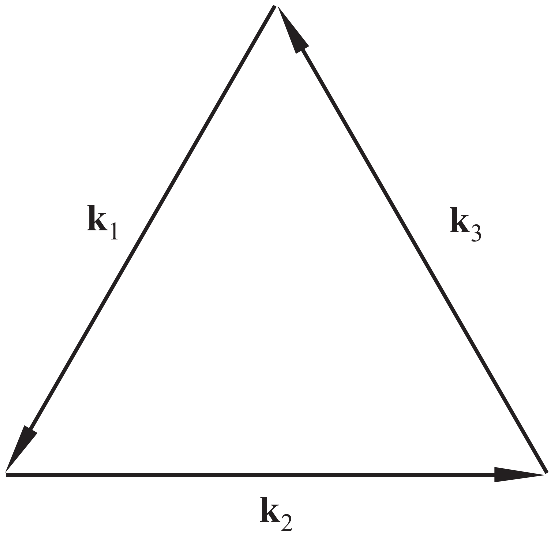

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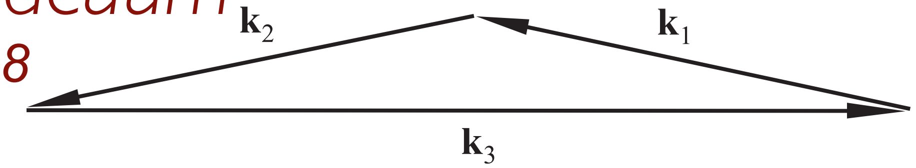

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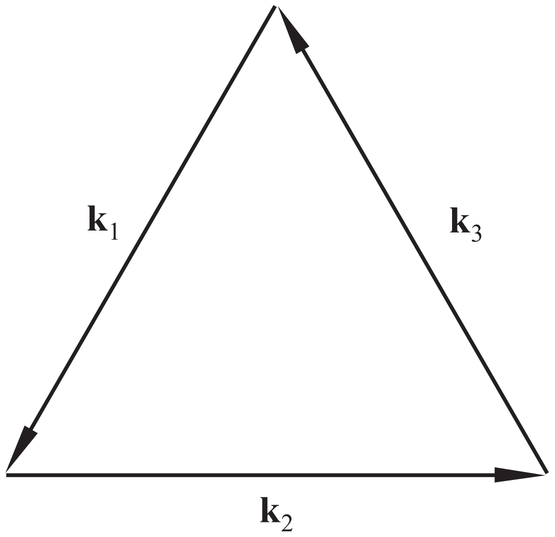

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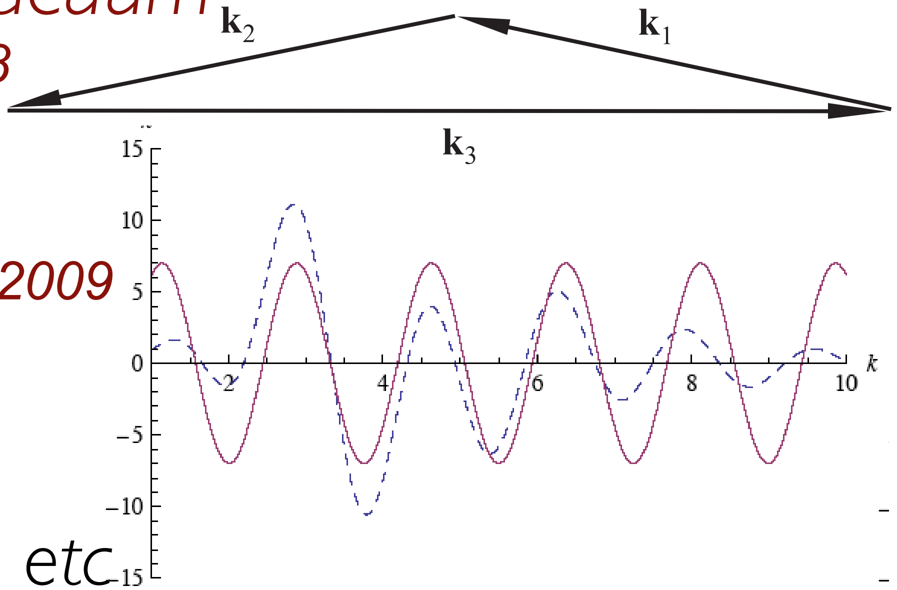

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Axion Monodromy

Large-field inflation predicts gravitational waves - r ~ 0.05 - but ...

- large excursions with a flat potential not natural (corrections)

- slow-roll inflation requires an effective shift symmetry $\Phi \rightarrow \Phi + c$

Ingredients: UV completion - string theory

Shift symmetry - axions a –> a+2π

Axion potential recycled - monodromy

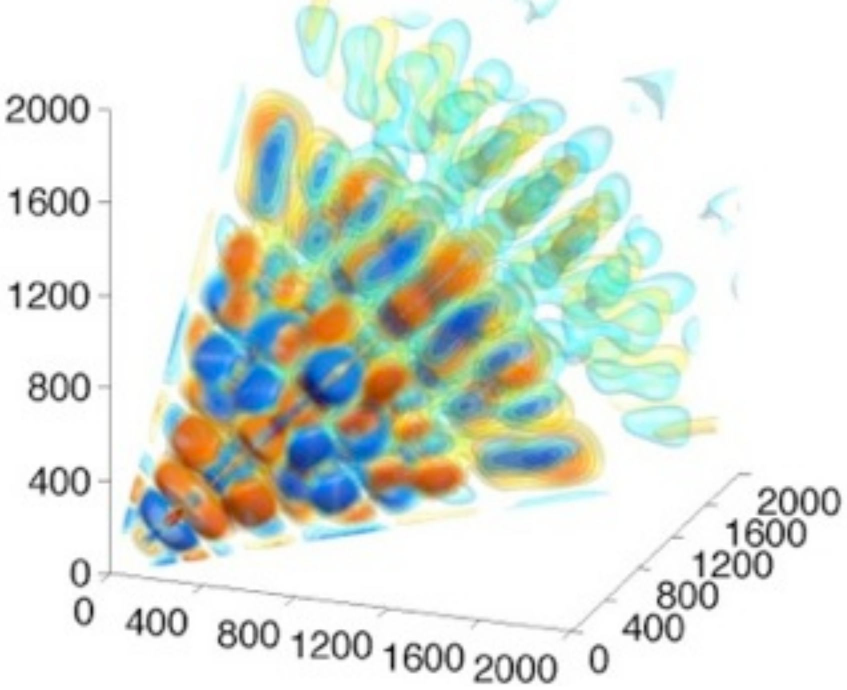
e.g. Silverstein & Westphal 2008

Flauger et al 2009

Predictions: Tensor modes r>0.07

Power spectrum periodicity

Bispectrum oscillations



Cosmic Defects

Cosmic strings and topological defects form at phase transitions

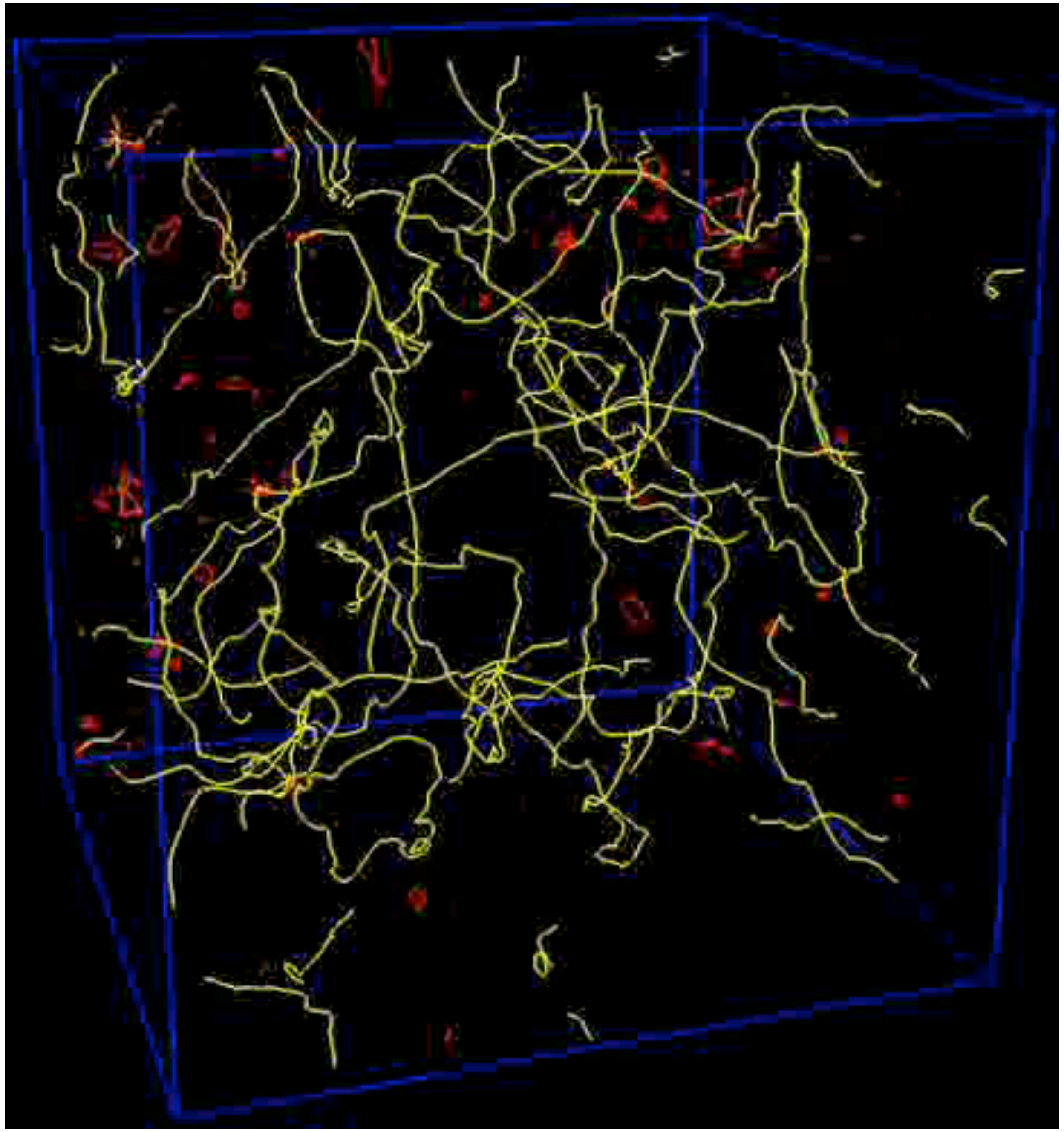
Key parameter **Gμ** = $(\eta/M_{\mathrm{Pl}})^2$

Evolve in a scale-invariant manner

Different varieties:

Local Nambu-Goto (super-)strings - modelled with line-like simulations

Strings with radiative effects - modelled with field theory simulations (Abelian-Higgs or global strings)

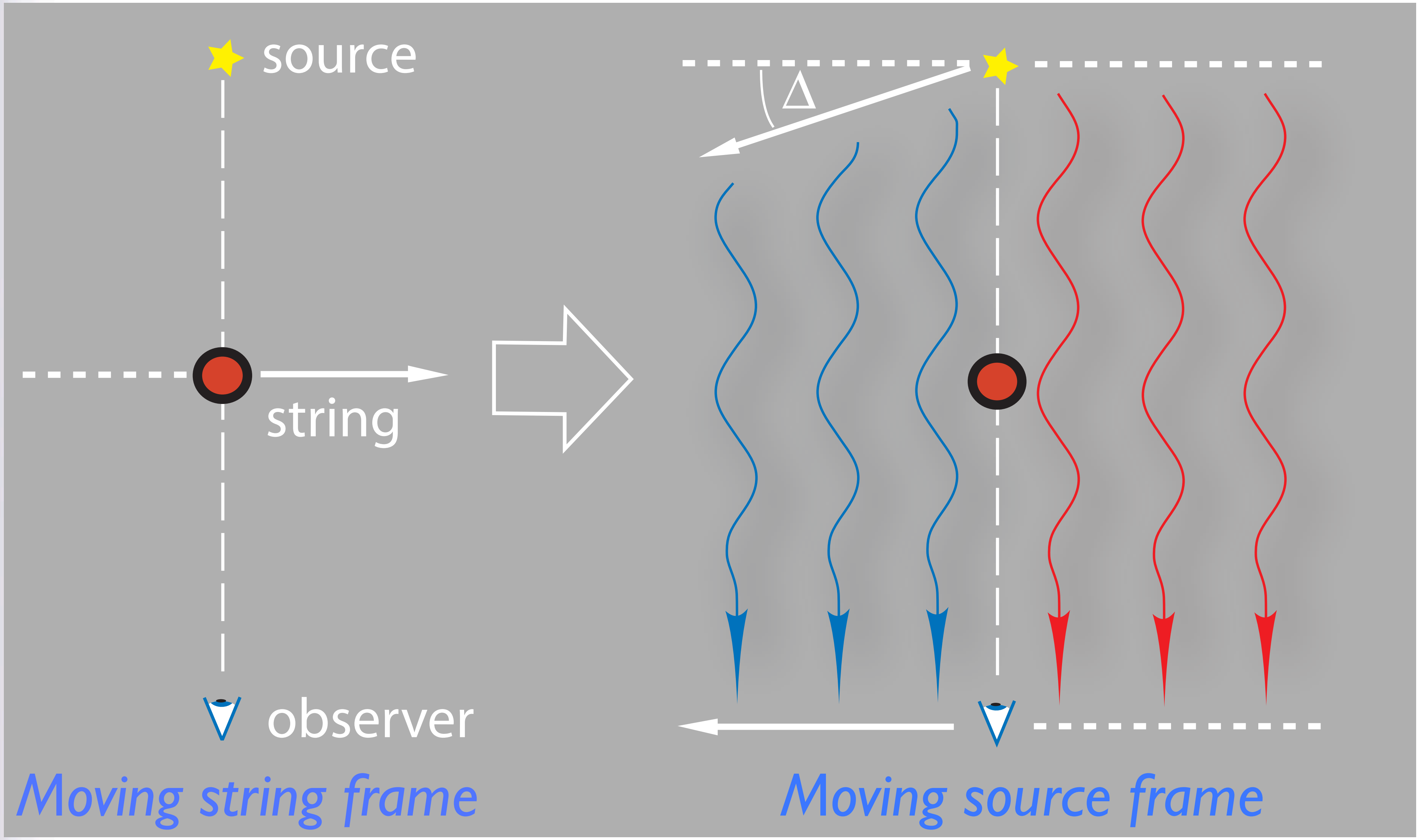


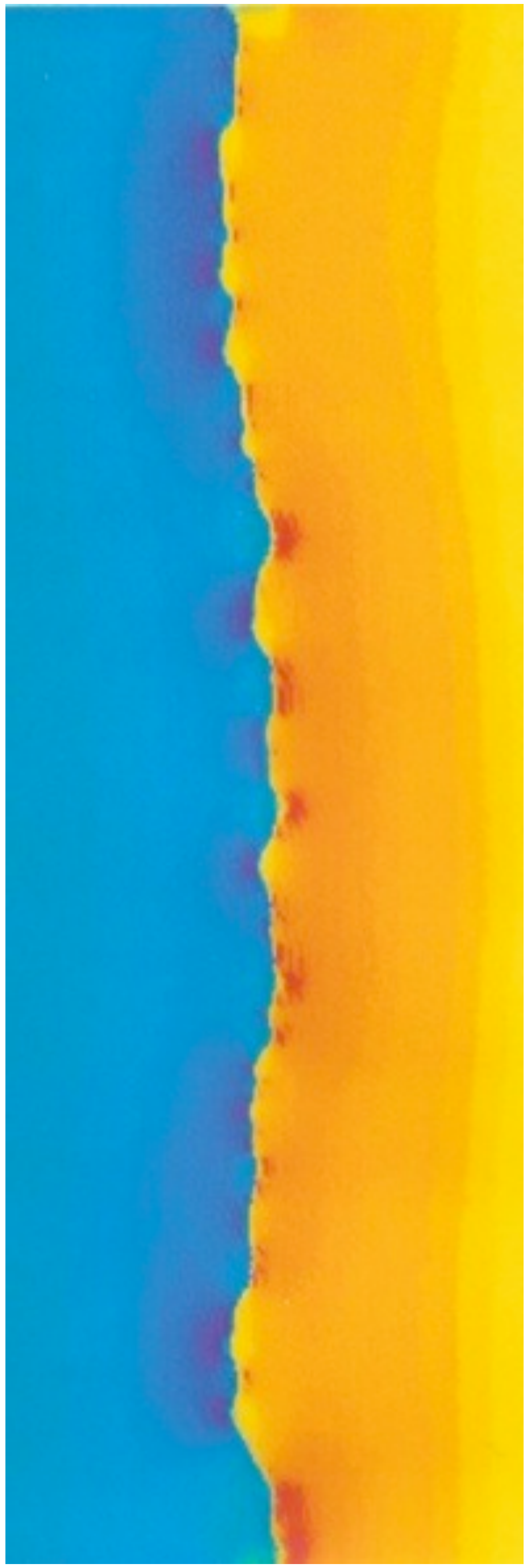
Epoch	MS	RSB	BOS	MSM	BHKU
Radiation	11.5	9.5	11.0	5.0	3.8
Matter	3.0	3.2	3.7	1.5	1.3

Bracket uncertainties by constraining both string varieties (& textures)

Cosmic Microwave Sky

Cosmic strings create line-like CMB discontinuities





Typical CMB temperature jump

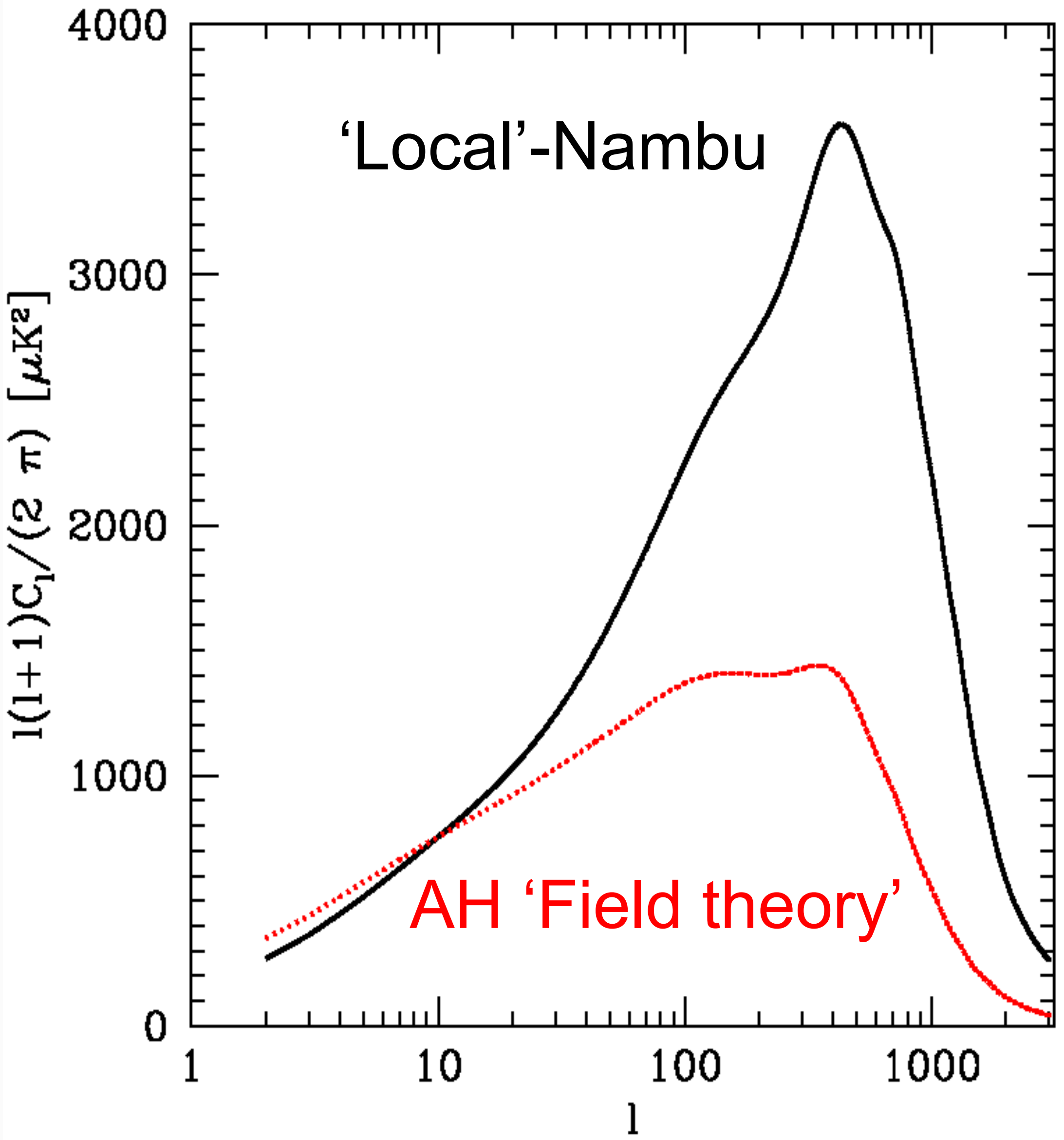
$$\frac{\Delta T}{T} = 8\pi G\mu\,\gamma\,\hat{\mathbf{n}}_\gamma\cdot(\mathbf{v}\times\hat{\mathbf{s}}) \approx 13G\mu$$

with $|\mathbf{v}| \approx 0.6$

Cosmic string power spectra

CMB power spectra

Battye, Kunz & Moss



Reliable power spectra available (within string uncertainties)

Battye & Moss, 2010

Urrestilla, et al, 2011

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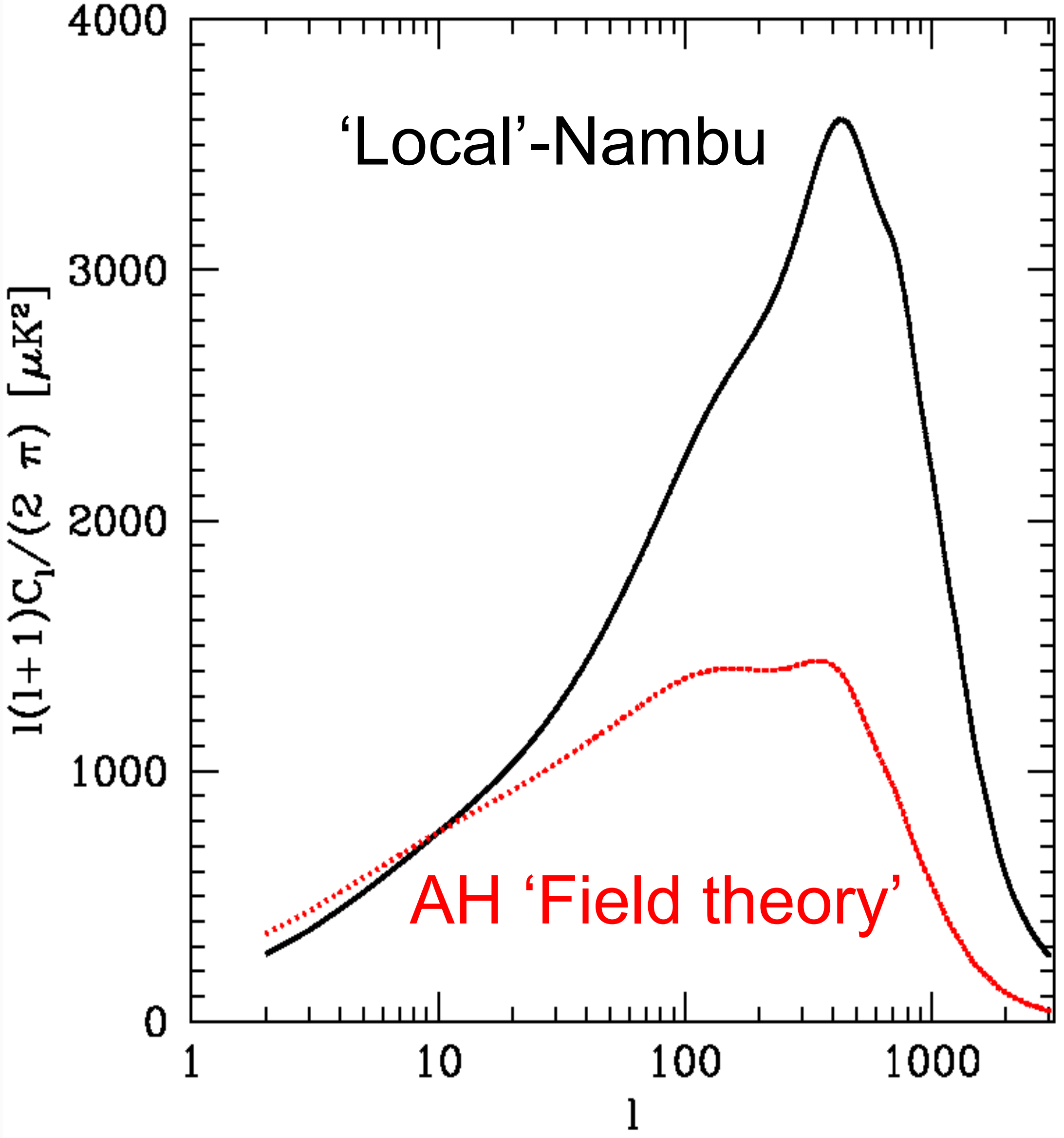

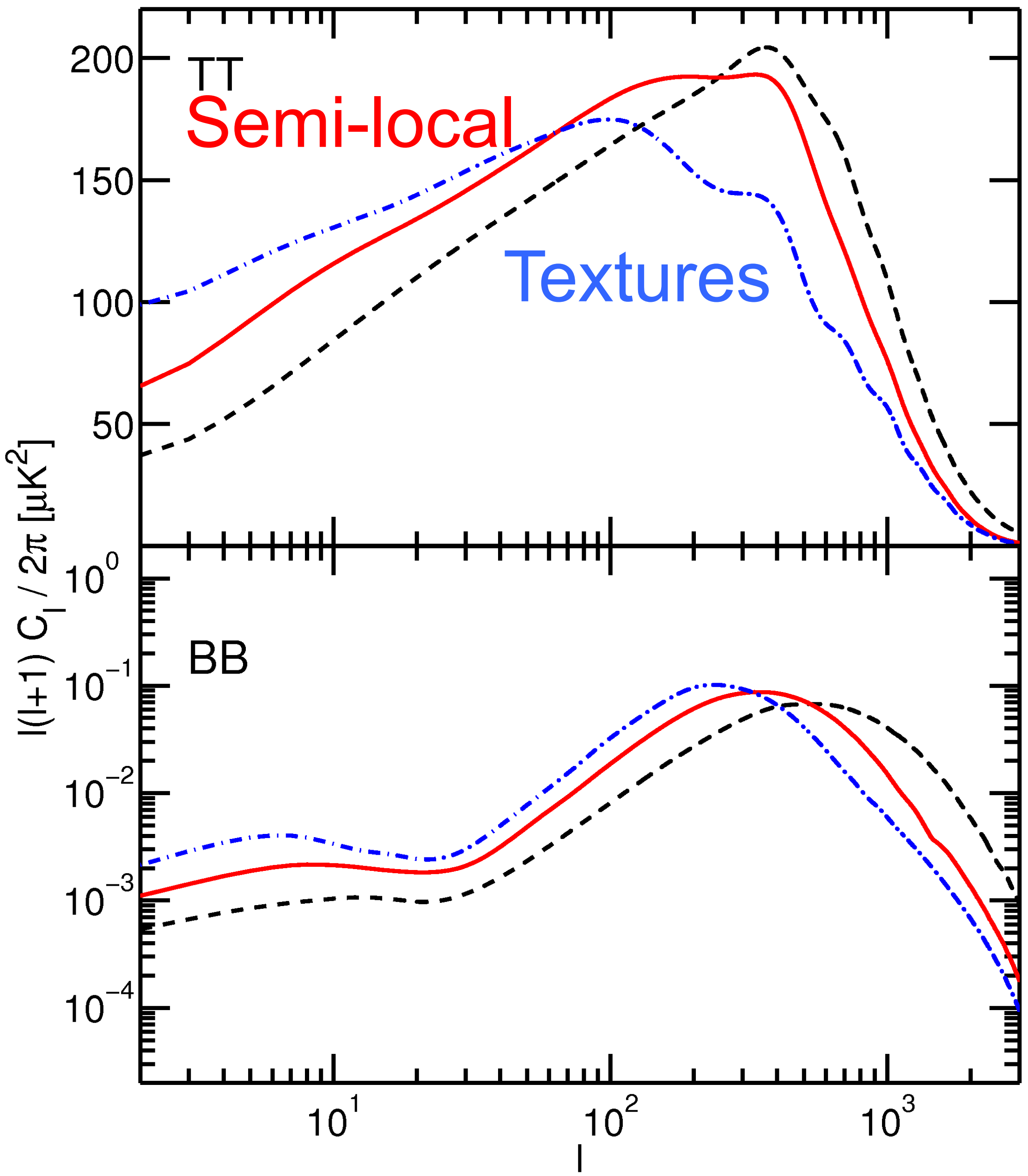

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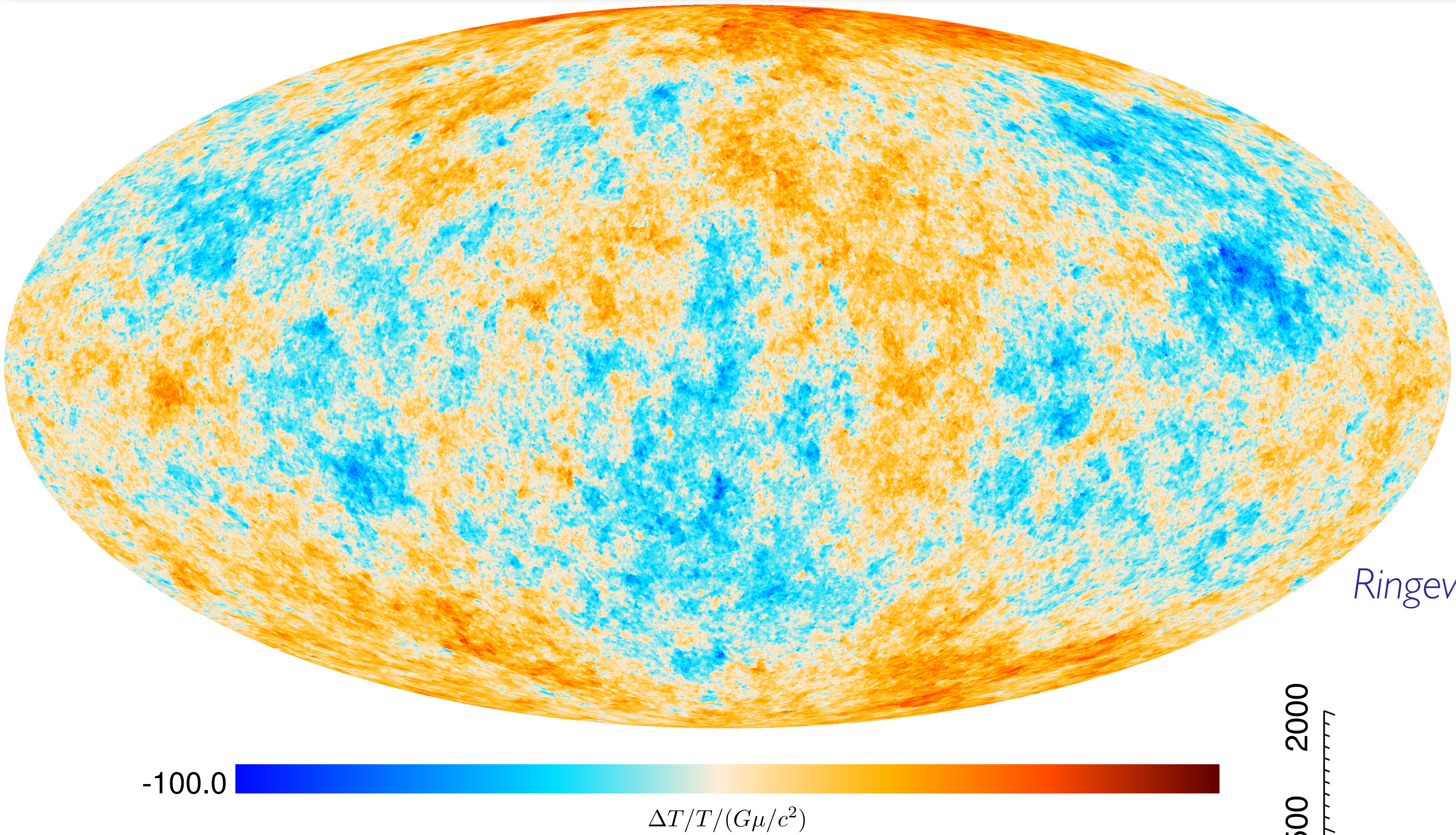
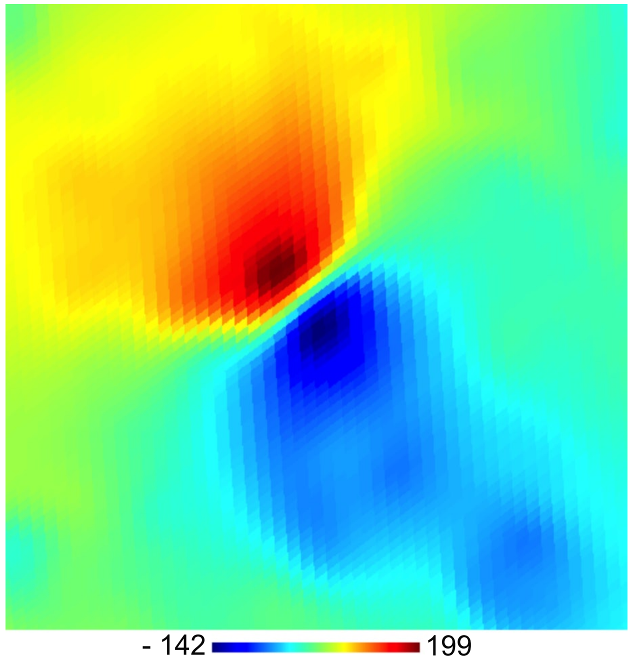
Cosmic string non-Gaussianity

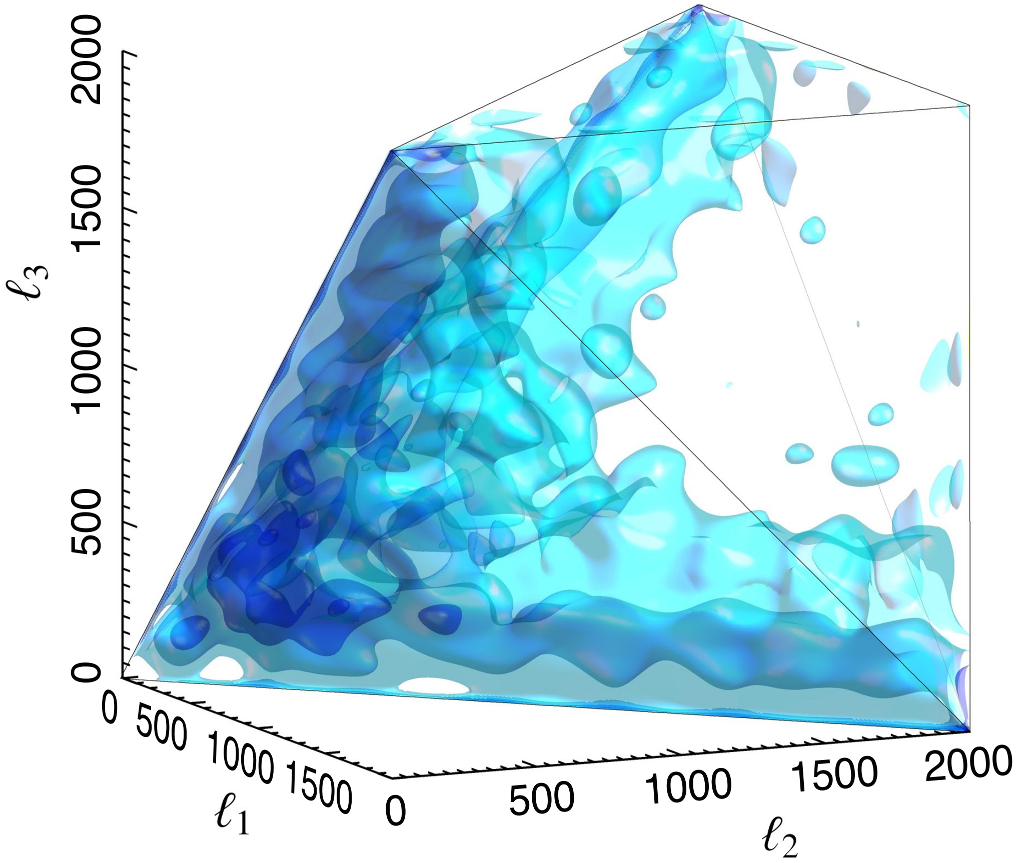




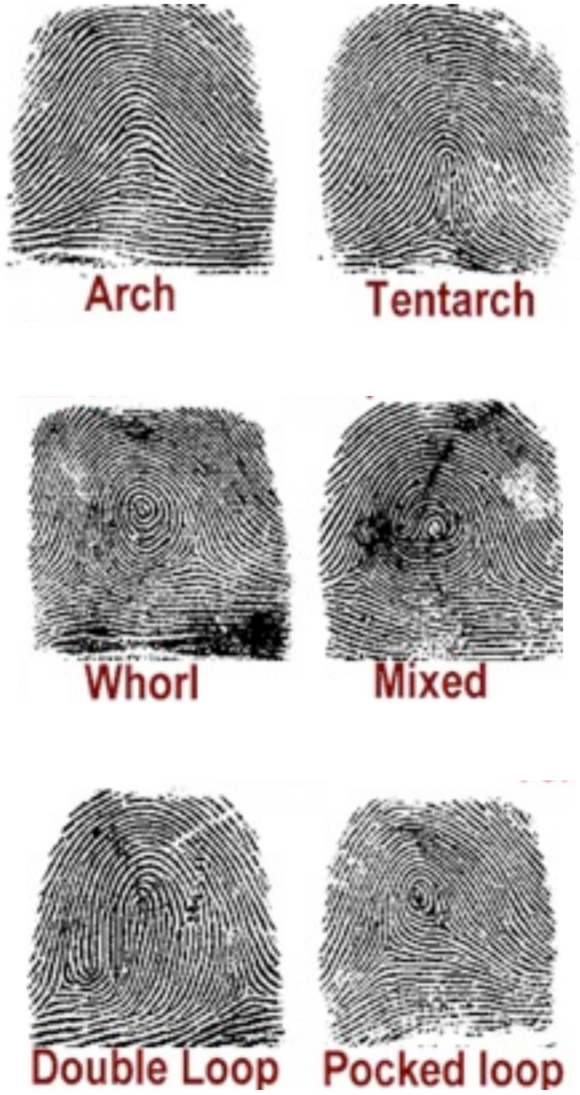
Ringeval & Bouchet, 2012

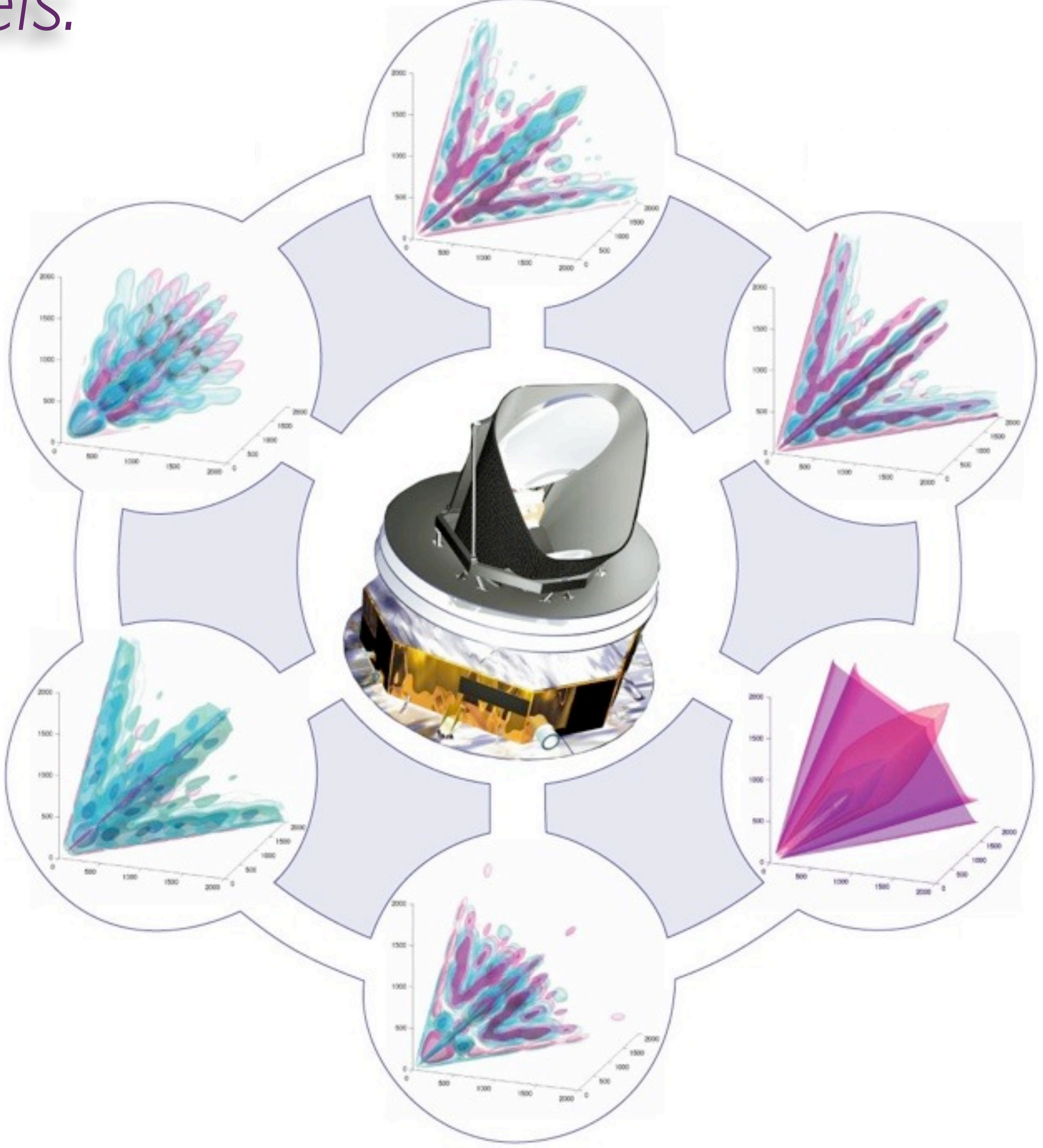
Post-recombination string simulation used as gravitational sources for CMB maps (Green's functions in flat sky approximation)

Extract the CMB string bispectrum from simulations - nearly constant & negative



Alternative models: Fingerprints of the very early Universe?





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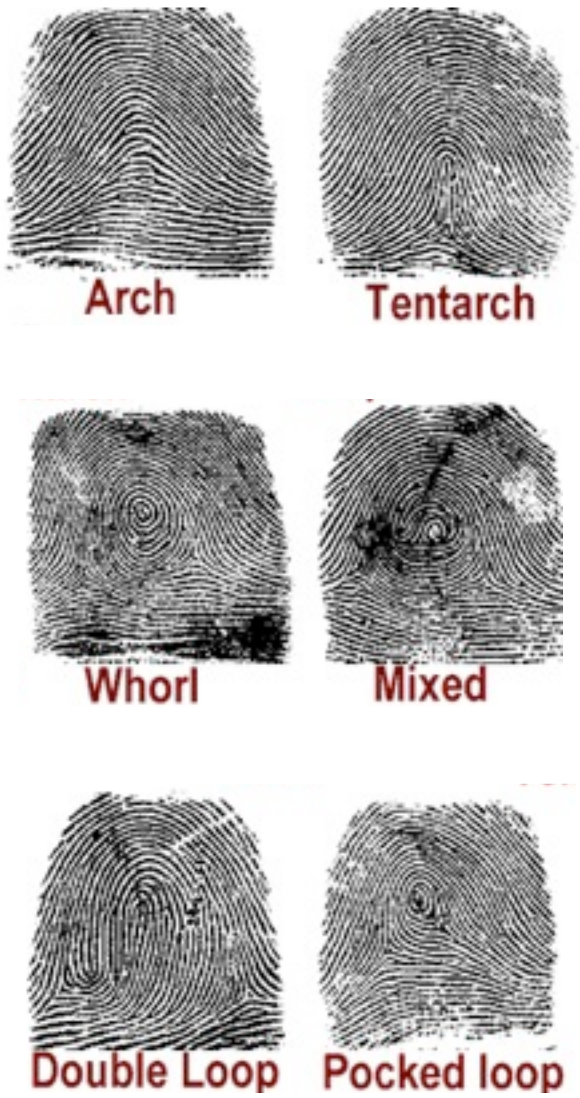

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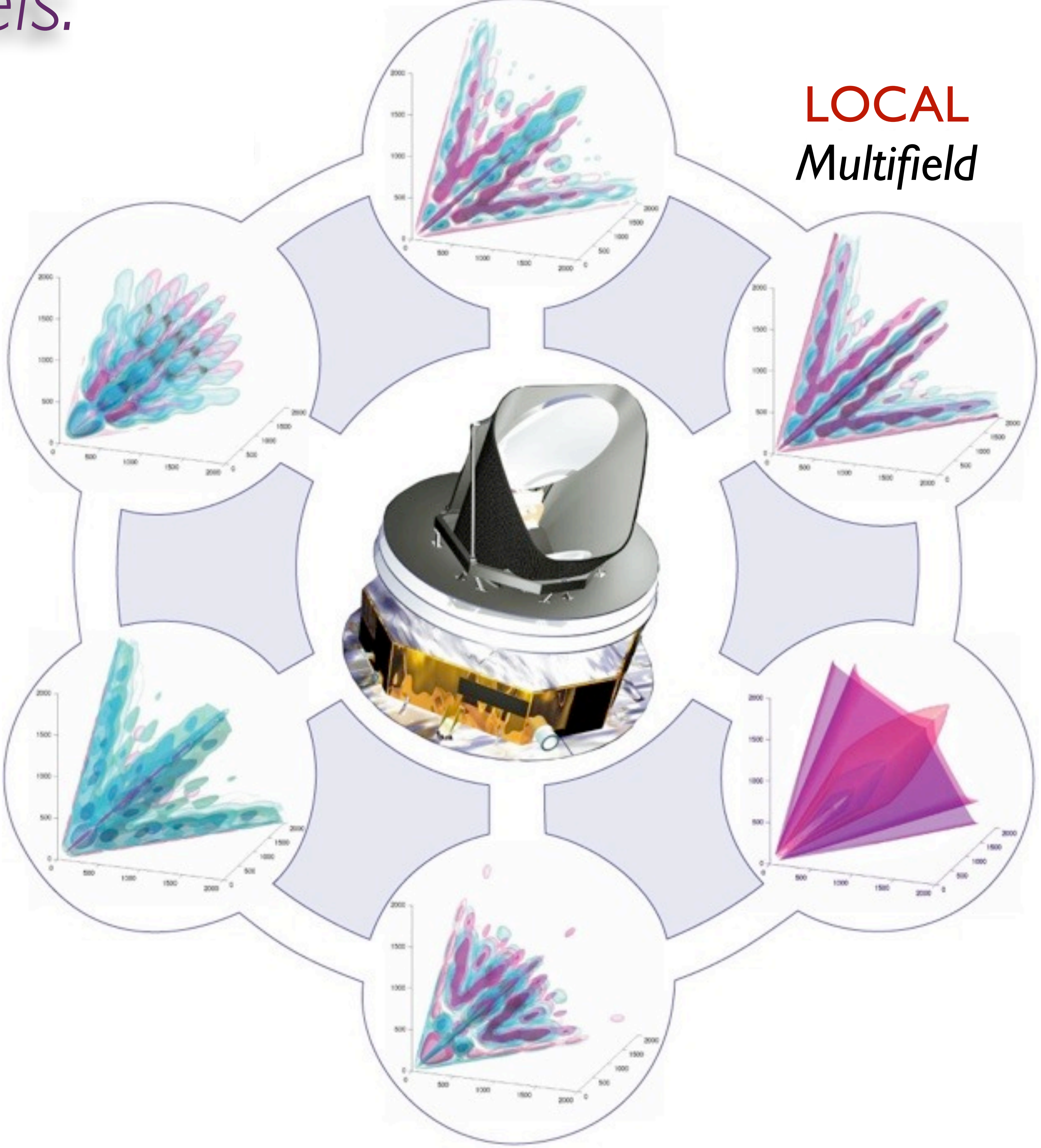

LOCAL

Multifield

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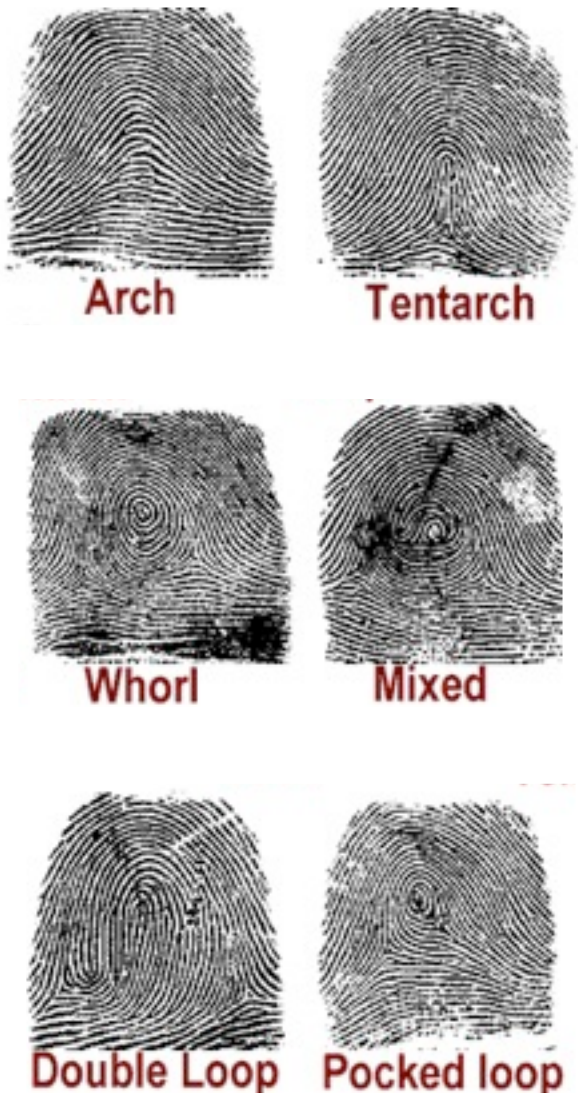

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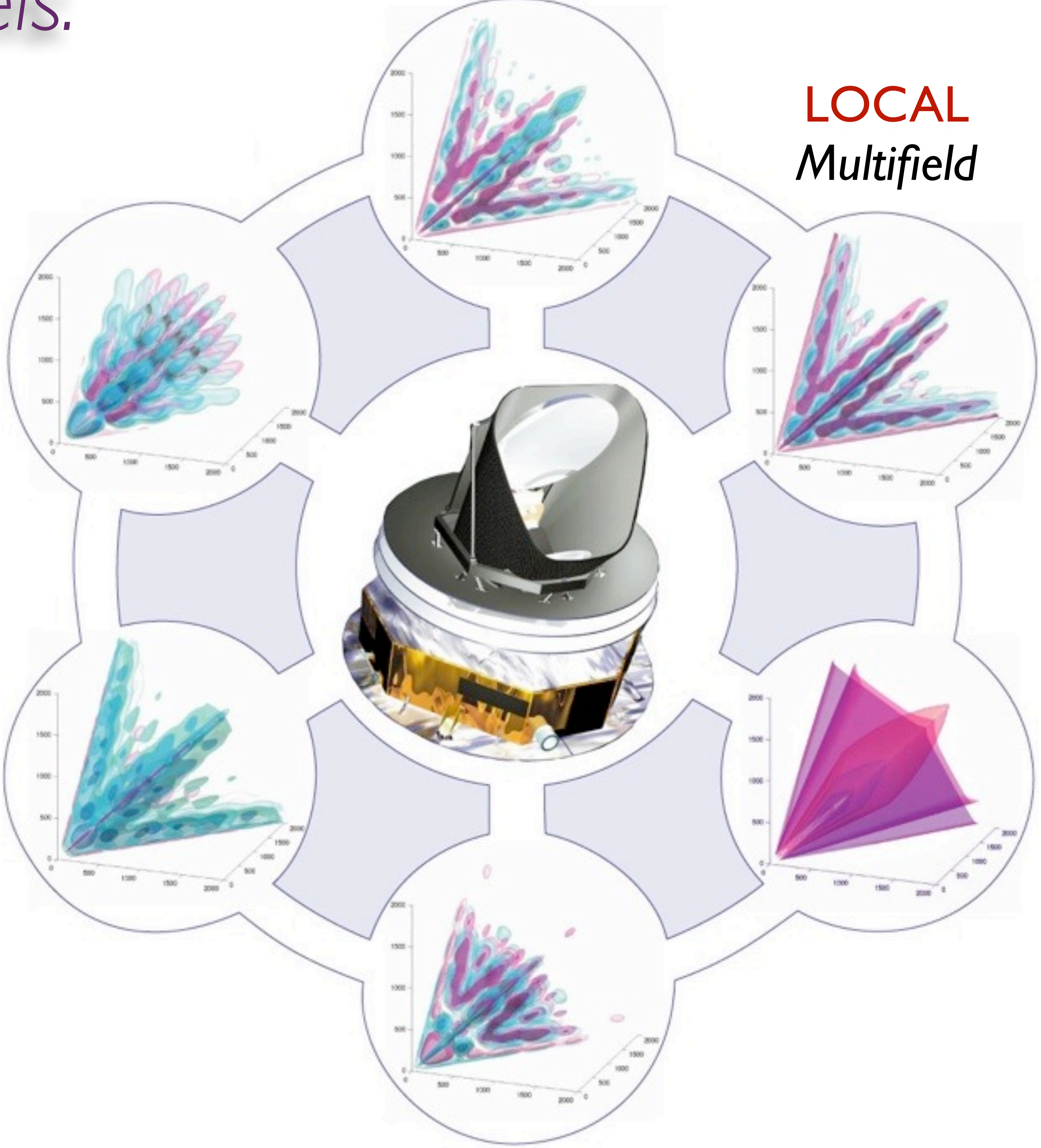


FLAT *Excited states*

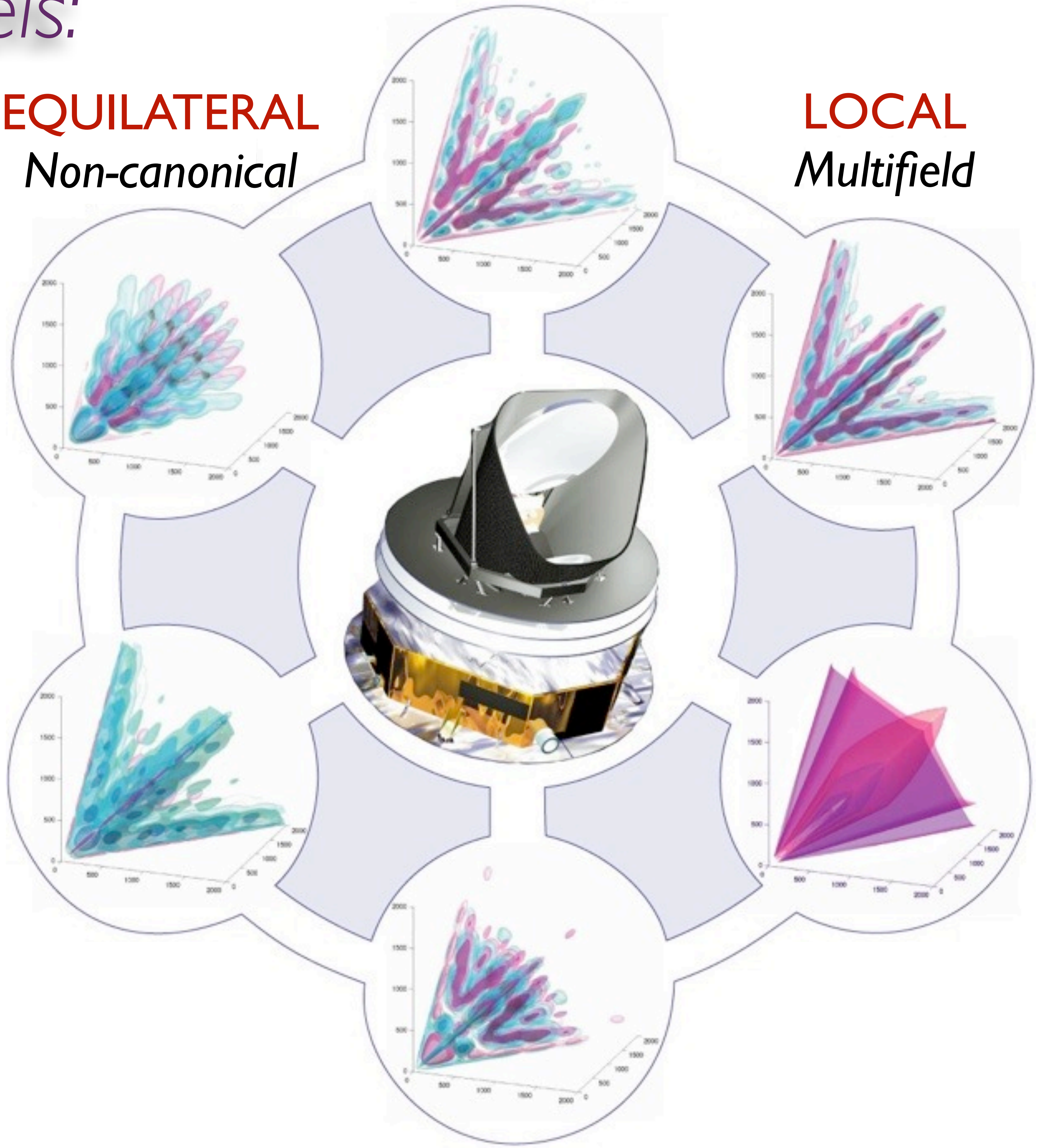
LOCAL *Multifield*

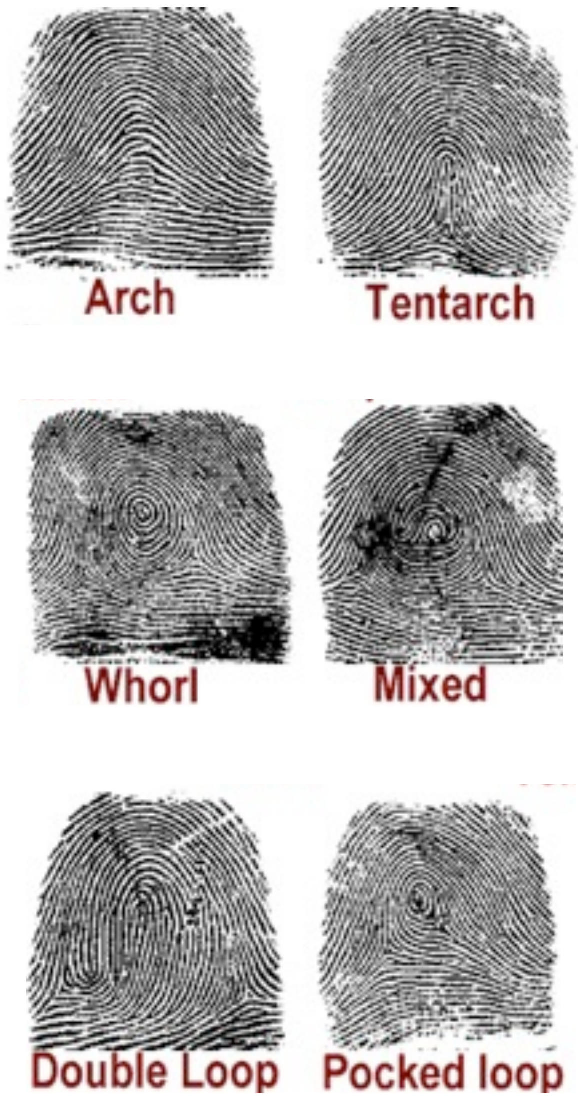
Alternative models: Fingerprints of the very early Universe?

FLAT *Excited states*

EQUILATERAL
Non-canonical

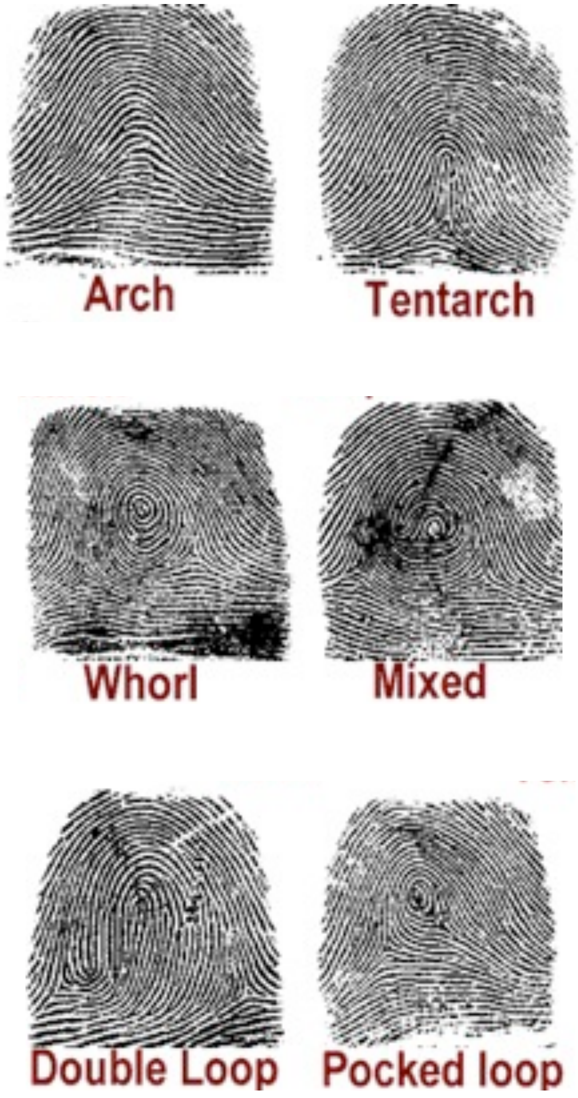
LOCAL
Multifield



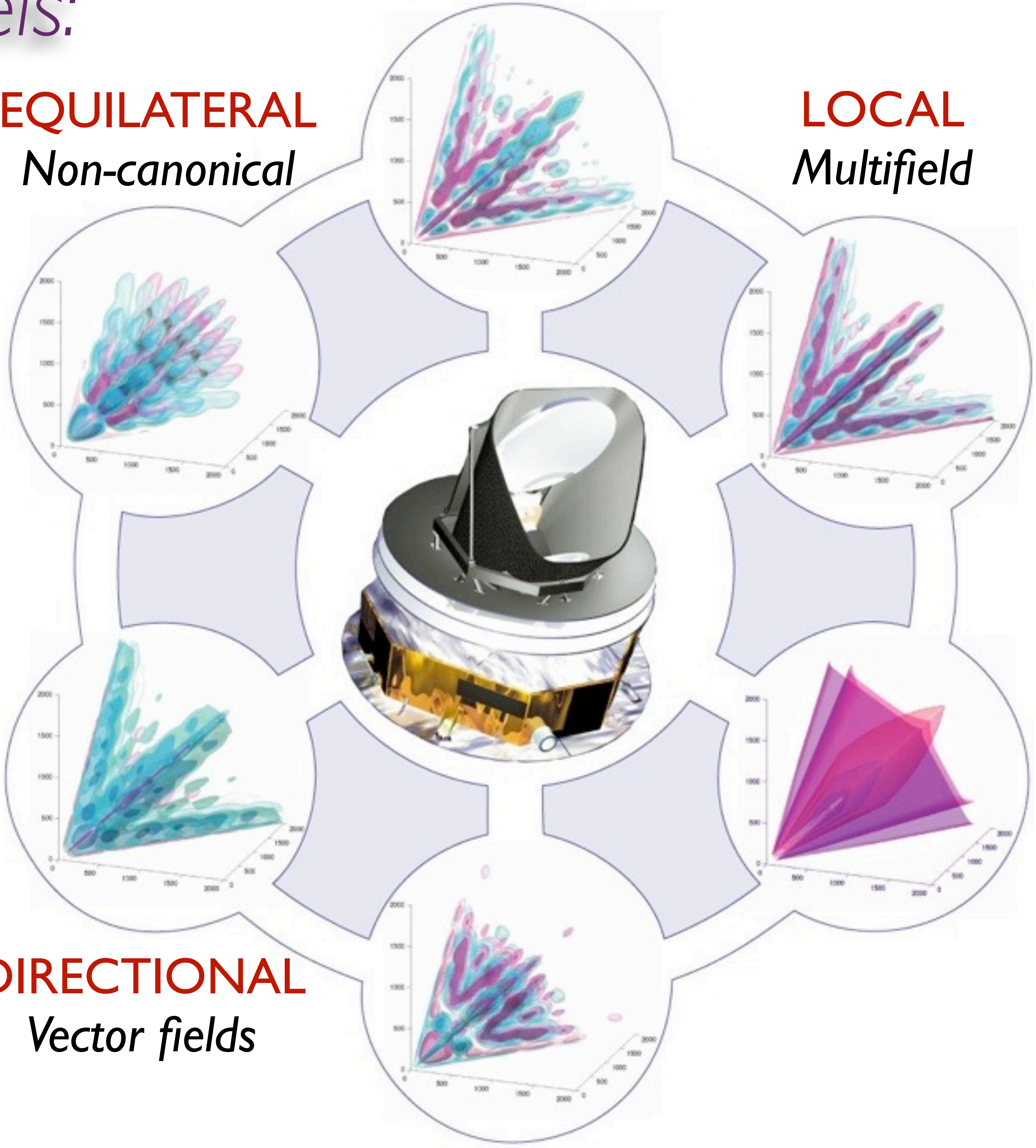


Fergusson and EPS, 2008

Alternative models: Fingerprints of the very early Universe?



Fergusson and EPS, 2008



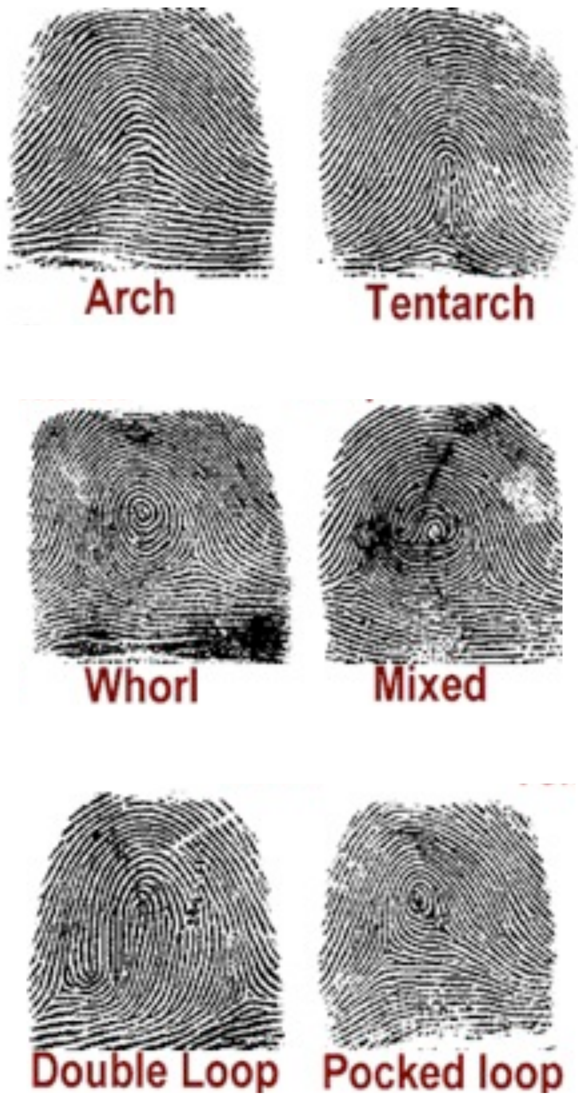
FLAT *Excited states*

EQUILATERAL
Non-canonical

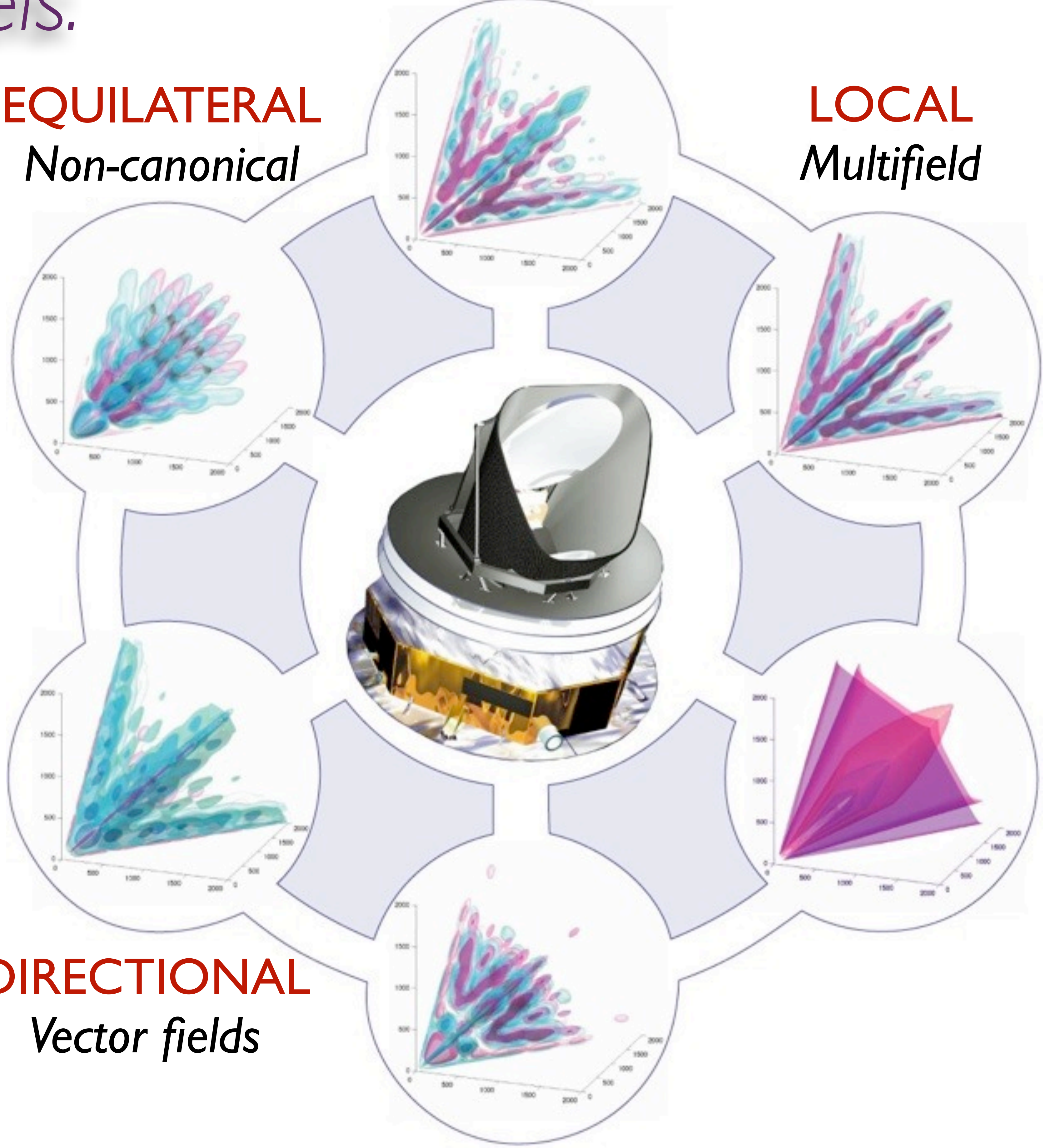
LOCAL
Multifield

DIRECTIONAL
Vector fields

Alternative models: Fingerprints of the very early Universe?



Fergusson and EPS, 2008



FLAT *Excited states*

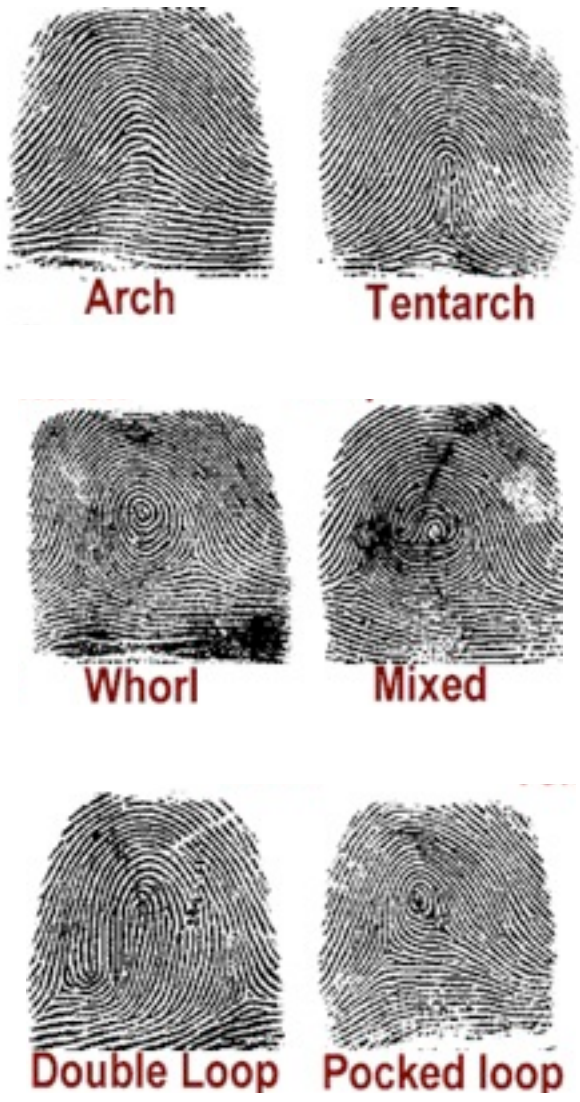
EQUILATERAL
Non-canonical

LOCAL
Multifield

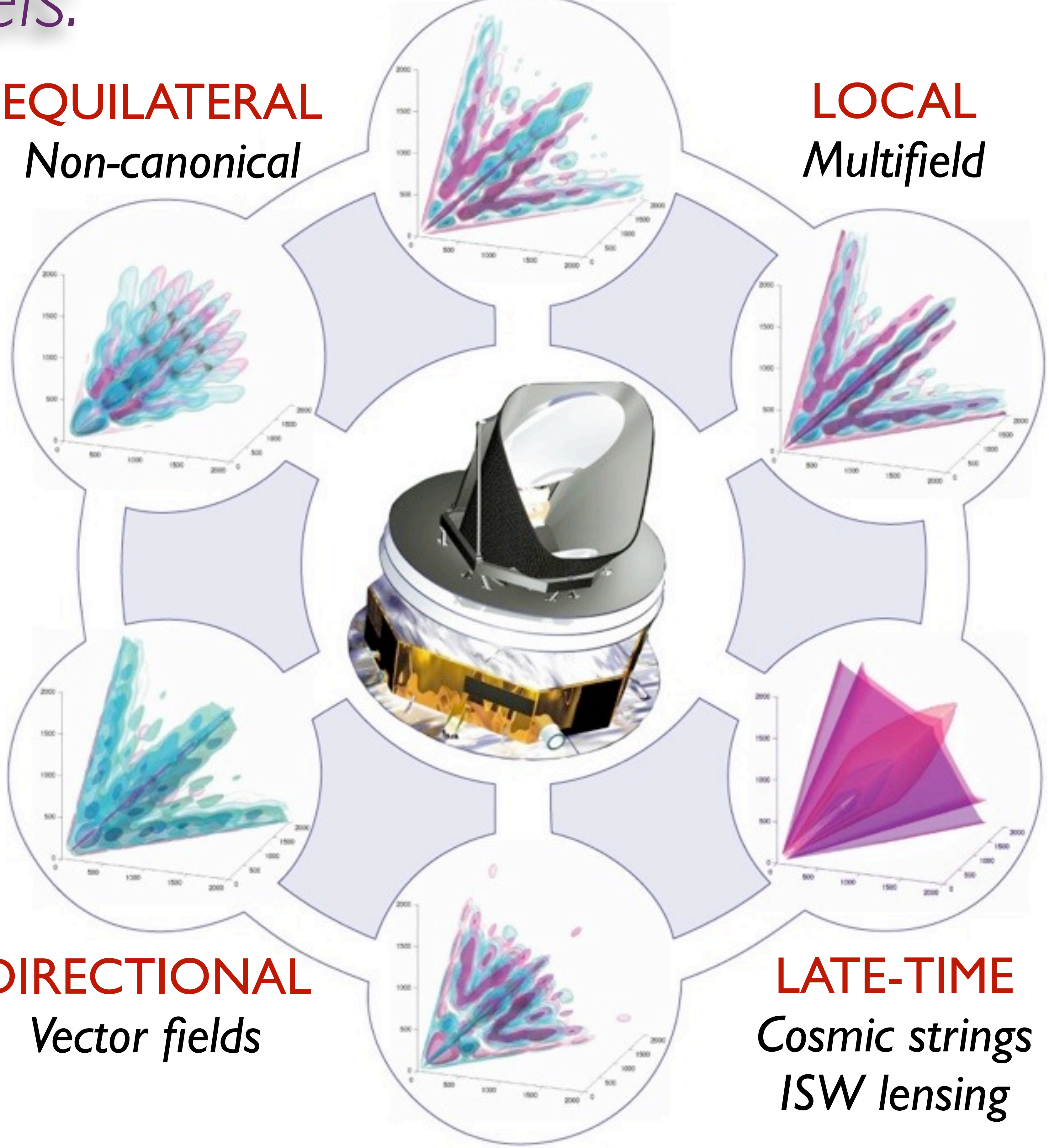
DIRECTIONAL
Vector fields

NON-SCALING *Oscillatory features*

Alternative models: Fingerprints of the very early Universe?



Fergusson and EPS, 2008



FLAT *Excited states*

EQUILATERAL
Non-canonical

LOCAL
Multifield

DIRECTIONAL
Vector fields

LATE-TIME
Cosmic strings
ISW lensing

NON-SCALING *Oscillatory features*

Bispectrum estimator

Mainly work done in Planck with James Fergusson and Michele Liguori

Purpose: Test a model with predicted theoretical bispectrum

$$b^{\rm th}_{l_1 l_2 l_3} = \sum_{m_i} \mathcal{G}^{\,l_1\,l_2\,l_3}_{m_1 m_2 m_3} \langle a^{\rm th}_{l_1 m_1} a^{\rm th}_{l_2 m_2} a^{\rm th}_{l_3 m_3} \rangle$$

Estimator gives a least squares fit to the data

$$\begin{aligned}\mathcal{E} &= \frac{1}{N^2} \sum_{l_i, m_i} \langle a^{\rm th}_{l_1 m_1} a^{\rm th}_{l_2 m_2} a^{\rm th}_{l_3 m_3} \rangle (C^{-1}a)_{l_1 m_1} (C^{-1}a)_{l_2 m_2} (C^{-1}a)_{l_3 m_3} \\ &= \frac{1}{N^2} \sum_{l_i m_i} \frac{\mathcal{G}^{l_1 l_2 l_3}_{m_1 m_2 m_3}\; b^{\rm th}_{l_1 l_2 l_3}\; a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}}{C_{l_1} C_{l_2} C_{l_3}}\end{aligned}$$

Babich, 2005; see also KSW etc

with covariance matrix $C_{lm,l'm'} = \langle a_{lm} a_{l'm'} \rangle$

with inverse weighting $(C^{-1}a)_{lm} = C^{-1}_{lm,l'm'} a_{l'm'} \approx \frac{a_{lm}}{C_l}$ (ideal case)

(Neglected discussion of 'linear term', incorporating systematic effects.)

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Model

$$= \frac{1}{N^2} \sum_{l_i m_i} \frac{\mathcal{G}^{l_1 l_2 l_3}_{m_1 m_2 m_3}\, b^{\rm th}_{l_1 l_2 l_3}\ a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}}{C_{l_1} C_{l_2} C_{l_3}}$$

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Estimator gives a least squares fit to the data

$$\begin{aligned}\mathcal{E} &= \frac{1}{N^2}\sum_{l_i,m_i}\langle a^{\rm th}_{l_1m_1}a^{\rm th}_{l_2m_2}a^{\rm th}_{l_3m_3}\rangle (C^{-1}a)_{l_1m_1}(C^{-1}a)_{l_2m_2}(C^{-1}a)_{l_3m_3}\\ &= \frac{1}{N^2}\sum_{l_im_i}\frac{\mathcal{G}^{l_1l_2l_3}_{m_1m_2m_3}\,b^{\rm th}_{l_1l_2l_3}\;\;a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}}{C_{l_1}C_{l_2}C_{l_3}}\end{aligned}$$

Model

Signal

Babich, 2005; see also KSW etc

with covariance matrix $C_{lm,l'm'} = \langle a_{lm}a_{l'm'}\rangle$

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Estimator gives a least squares fit to the data

$$\mathcal{E} = \frac{1}{N^2}\sum_{l_i,m_i} \langle a^{\text{th}}_{l_1 m_1} a^{\text{th}}_{l_2 m_2} a^{\text{th}}_{l_3 m_3}\rangle (C^{-1}a)_{l_1 m_1}(C^{-1}a)_{l_2 m_2}(C^{-1}a)_{l_3 m_3}$$

Model

Signal

$$= \frac{1}{N^2}\sum_{l_i m_i} \frac{\mathcal{G}^{l_1 l_2 l_3}_{m_1 m_2 m_3}\, b^{\text{th}}_{l_1 l_2 l_3}\; a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}}{C_{l_1} C_{l_2} C_{l_3}}$$

Noise

Babich, 2005; see also KSW etc

with covariance matrix $C_{lm,l'm'} = \langle a_{lm} a_{l'm'} \rangle$

with inverse weighting $(C^{-1}a)_{lm} = C^{-1}_{lm,l'm'} a_{l'm'} \approx \frac{a_{lm}}{C_l}$ (ideal case)

(Neglected discussion of 'linear term', incorporating systematic effects.)

CMB modal decomposition

$$\mathcal{E} = \sum_{l_i,m_i}\sum_{n\leftrightarrow prs}\bar{\alpha}^{\mathcal{Q}}_n\bar{q}_{\{p}\bar{q}_r\bar{q}_{s\}}\int d^2\hat{\mathbf{n}}\,Y_{l_2m_2}(\hat{\mathbf{n}})Y_{l_1m_1}(\hat{\mathbf{n}})\,Y_{l_3m_3}(\hat{\mathbf{n}})\,\frac{a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}}{v_{l_1}v_{l_2}v_{l_3}\sqrt{C_{l_1}C_{l_2}C_{l_3}}}$$

$$= \sum_{n\leftrightarrow prs}\bar{\alpha}^{\mathcal{Q}}_n\int d^2\hat{\mathbf{n}}\left(\sum_{l_1,m_1}\bar{q}_{\{p}\frac{a_{l_1m_1}Y_{l_1m_1}}{v_{l_1}\sqrt{C_{l_1}}}\right)\left(\sum_{l_2,m_2}\bar{q}_r\frac{a_{l_2m_2}Y_{l_2m_2}}{v_{l_2}\sqrt{C_{l_2}}}\right)\left(\sum_{l_3,m_3}\bar{q}_{s\}}\frac{a_{l_3m_3}Y_{l_3m_3}}{v_{l_3}\sqrt{C_{l_3}}}\right)$$

$$\bar{M}_p(\hat{\mathbf{n}}) = \sum_{lm} q_p(l)\frac{a_{lm}}{v_l\sqrt{C_l}}Y_{lm}(\hat{\mathbf{n}})$$

$$\bar{\mathcal{M}}_n(\hat{\mathbf{n}}) = \bar{M}_p(\hat{\mathbf{n}})\bar{M}_r(\hat{\mathbf{n}})\bar{M}_s(\hat{\mathbf{n}})$$

$$\beta_n = \int d^2\hat{\mathbf{n}}\mathcal{M}_n(\hat{\mathbf{n}})$$

$$\mathcal{E} = \frac{1}{N}\sum_{n=0}^{n_{\max}}\bar{\alpha}^{\mathcal{Q}}_n\bar{\beta}^{\mathcal{Q}}_n$$

Now the projection is in alpha rather than beta

Fergusson, Liguori and EPS, 2009, 2010; see also KSW

CMB modal decomposition

$$\mathcal{E} = \sum_{l_i,m_i}\sum_{n\leftrightarrow prs} \bar{\alpha}^{\mathcal{Q}}_n \bar{q}_{\{p}\bar{q}_r\bar{q}_{s\}} \int d^2\hat{\mathbf{n}}\, Y_{l_2m_2}(\hat{\mathbf{n}})Y_{l_1m_1}(\hat{\mathbf{n}})\, Y_{l_3m_3}(\hat{\mathbf{n}}) \frac{a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}}{v_{l_1}v_{l_2}v_{l_3}\sqrt{C_{l_1}C_{l_2}C_{l_3}}}$$

$$= \sum_{n\leftrightarrow prs} \bar{\alpha}^{\mathcal{Q}}_n \int d^2\hat{\mathbf{n}} \left(\sum_{l_1,m_1} \bar{q}_{\{p} \frac{a_{l_1m_1}Y_{l_1m_1}}{v_{l_1}\sqrt{C_{l_1}}}\right)\left(\sum_{l_2,m_2} \bar{q}_r \frac{a_{l_2m_2}Y_{l_2m_2}}{v_{l_2}\sqrt{C_{l_2}}}\right)\left(\sum_{l_3,m_3} \bar{q}_{s\}} \frac{a_{l_3m_3}Y_{l_3m_3}}{v_{l_3}\sqrt{C_{l_3}}}\right)$$

$$\bar{M}_p(\hat{\mathbf{n}}) = \sum_{lm} q_p(l)\frac{a_{lm}}{v_l\sqrt{C_l}}Y_{lm}(\hat{\mathbf{n}})$$

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$$\boxed{\mathcal{E} = \frac{1}{N}\sum_{n=0}^{n_{\max}} \bar{\alpha}^{\mathcal{Q}}_n \bar{\beta}^{\mathcal{Q}}_n}$$

Now the projection is in alpha rather than beta

Fergusson, Liguori and EPS, 2009, 2010; see also KSW

$B_{l_1l_2l_3}$ reconstruction

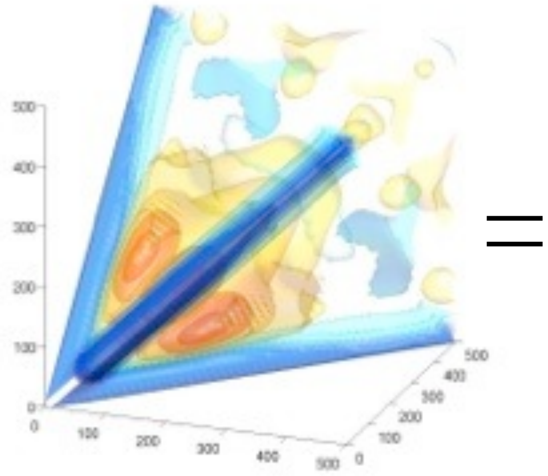
SEPARABILITY = TRACTABILITY, so create a basis of separable modes

$$\begin{aligned}\bar{\mathcal{Q}}_n(l_1,l_2,l_3) &= \tfrac{1}{6}[\bar q_p(l_1)\,\bar q_r(l_2)\,\bar q_s(l_3) + \bar q_r(l_1)\,\bar q_p(l_2)\,\bar q_s(l_3) + \text{cyclic perms in } prs] \\ &\equiv \bar q_{\{p}q_r q_{s\}} \quad \text{with} \quad n \leftrightarrow \{prs\}\,,\end{aligned}$$

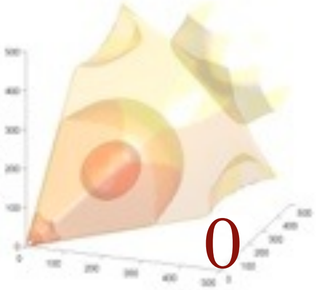
Expand any (nonseparable) bispectrum signal strength in modes as

$$\frac{v_{l_1}v_{l_2}v_{l_3}}{\sqrt{C_{l_1}C_{l_2}C_{l_3}}}\, b_{l_1l_2l_3} = \sum_n \bar\alpha_n^{\mathcal{R}}\,\bar{\mathcal{R}}_n$$

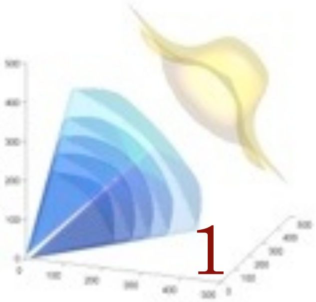
E.g. Local f_{NL} Model expansion for the a_n coefficients:



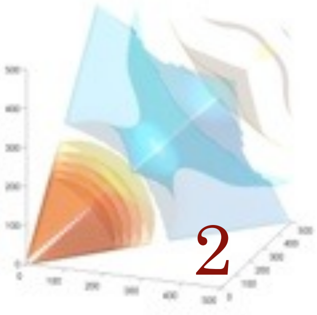
= -0.07



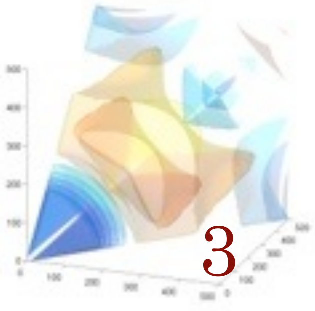
+ 0.14



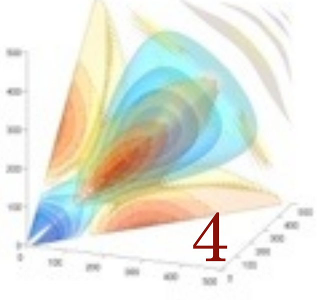
+ 0.30



- 0.43



+ 0.14



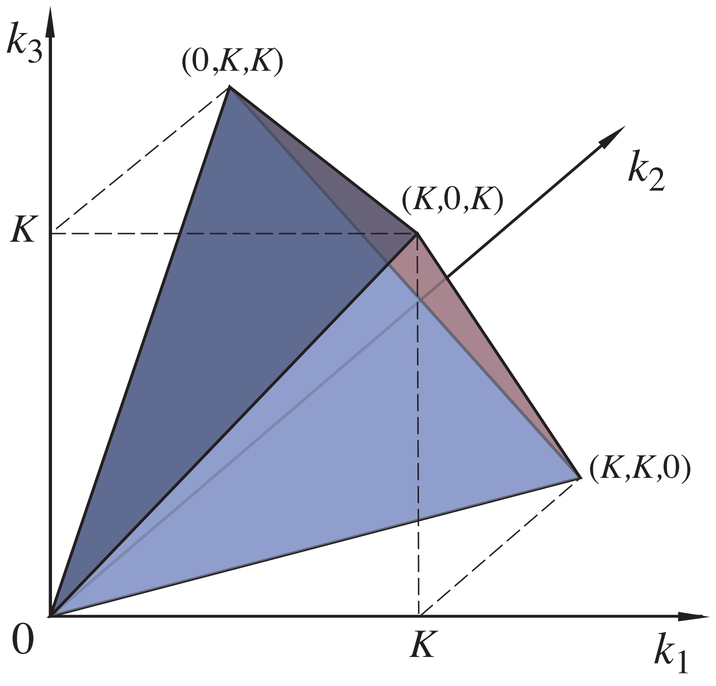
+ ....

Fergusson, Liguori and EPS, 2009, 2010

Modal Polyspectra Estimation

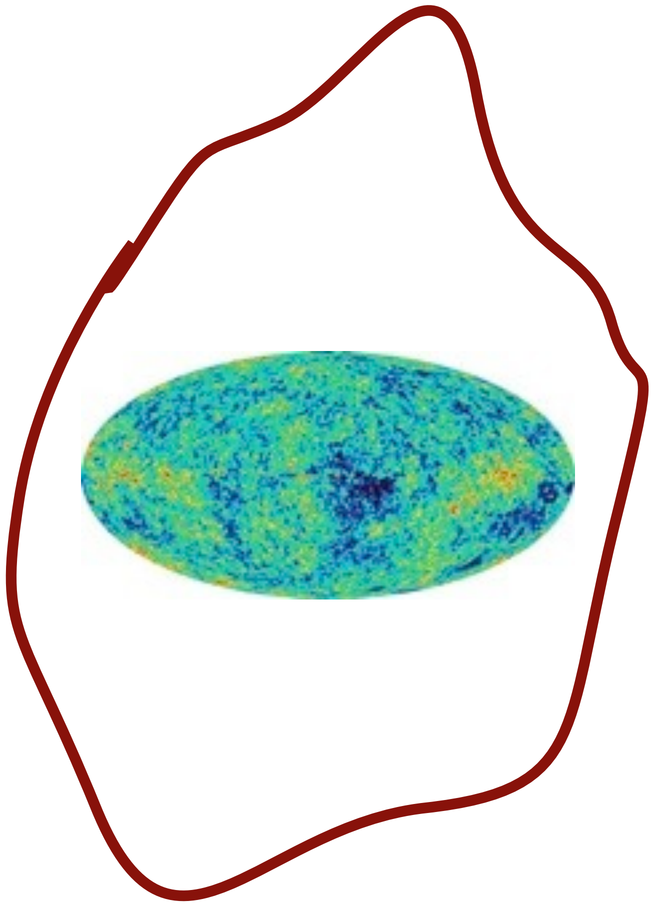
THEORY

Primordial bispectra
(*k*-space)



OBSERVATION

CMB map



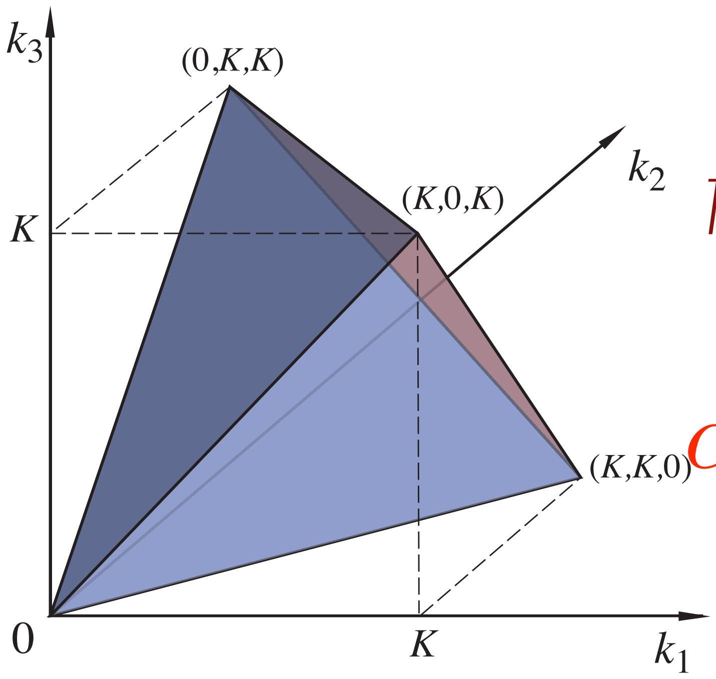
Fergusson, Liguori and EPS, 2009

Modal Polyspectra Estimation

THEORY

Primordial bispectra

(*k*-space)

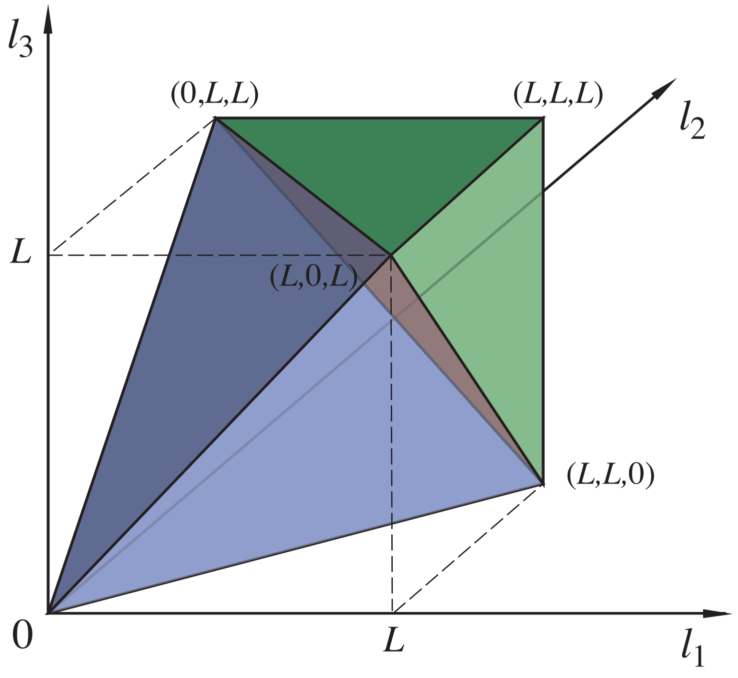


Mode transfer functions

$\alpha_n \rightarrow \bar{\alpha}_n$

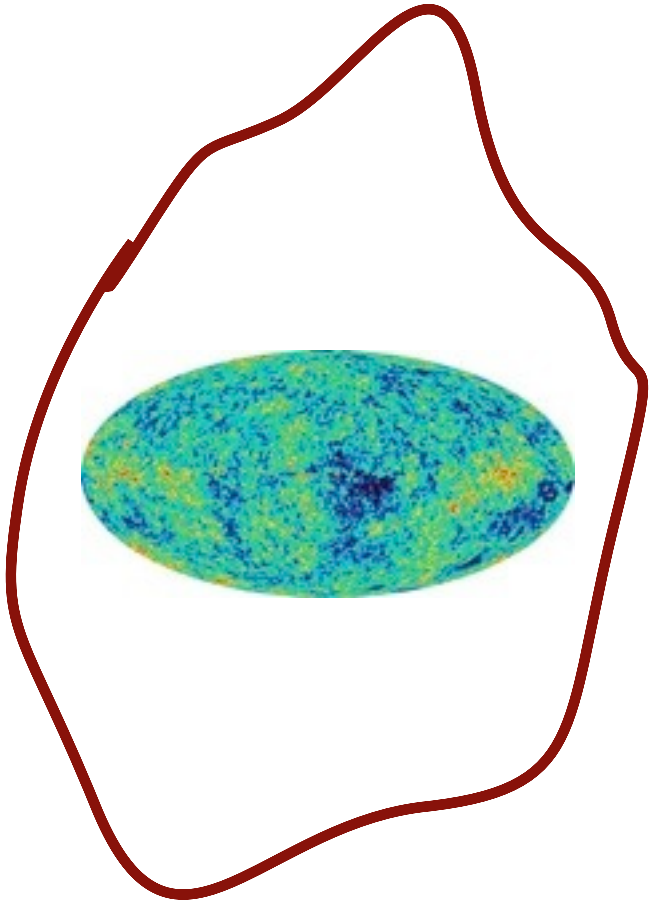
CMB bispectra

(*l*-space)



OBSERVATION

CMB map

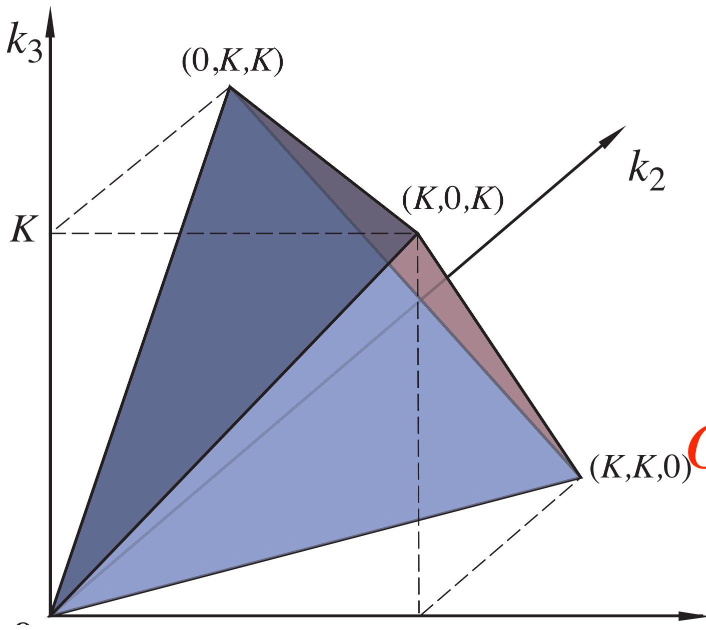


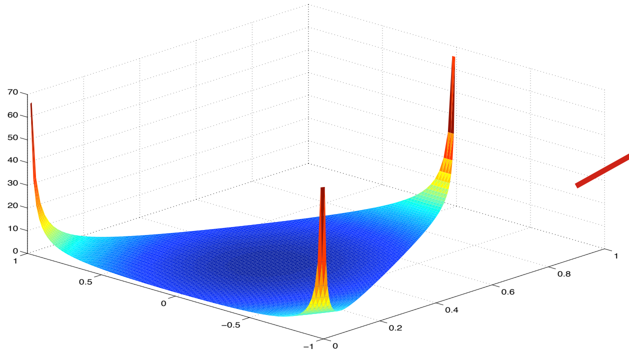
Fergusson, Liguori and EPS, 2009

Modal Polyspectra Estimation

THEORY

Primordial bispectra
(k-space)





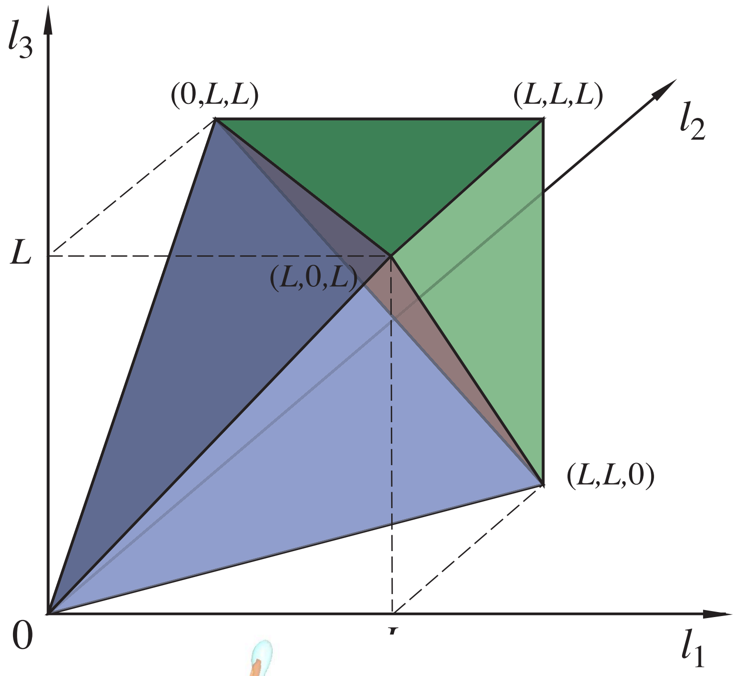
Expand any model with primordial modes α_n

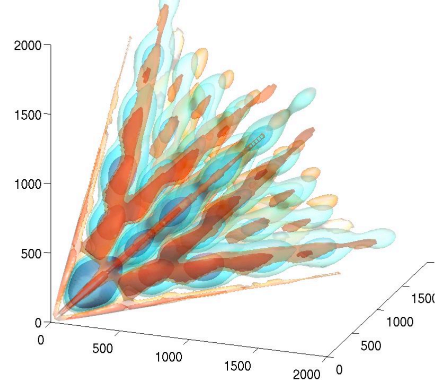
Mode transfer functions

$\alpha_n \rightarrow \bar{\alpha}_n$

CMB bispectra

(l-space)

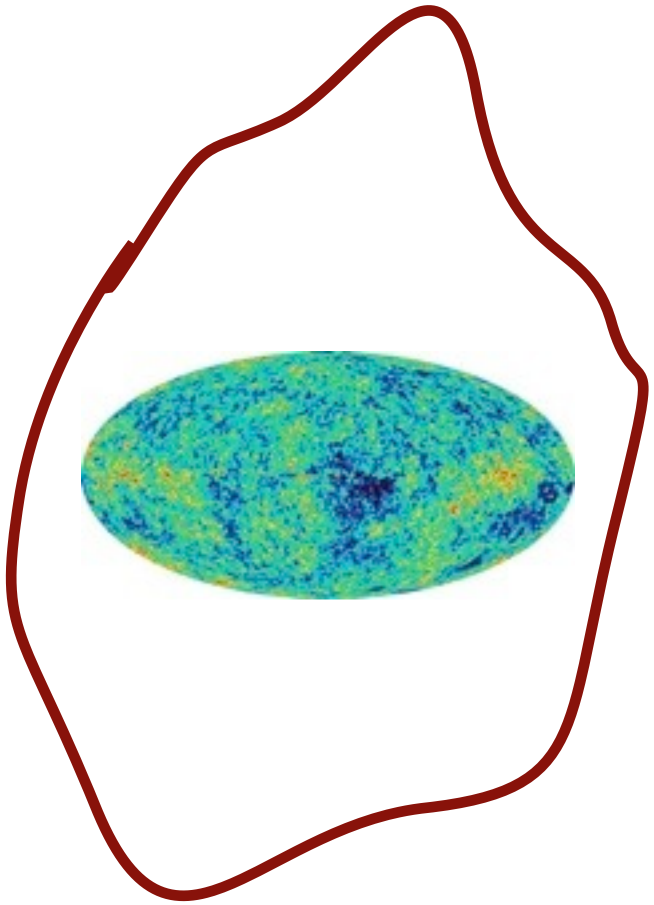




Fergusson, Liguori and EPS, 2009

OBSERVATION

CMB map

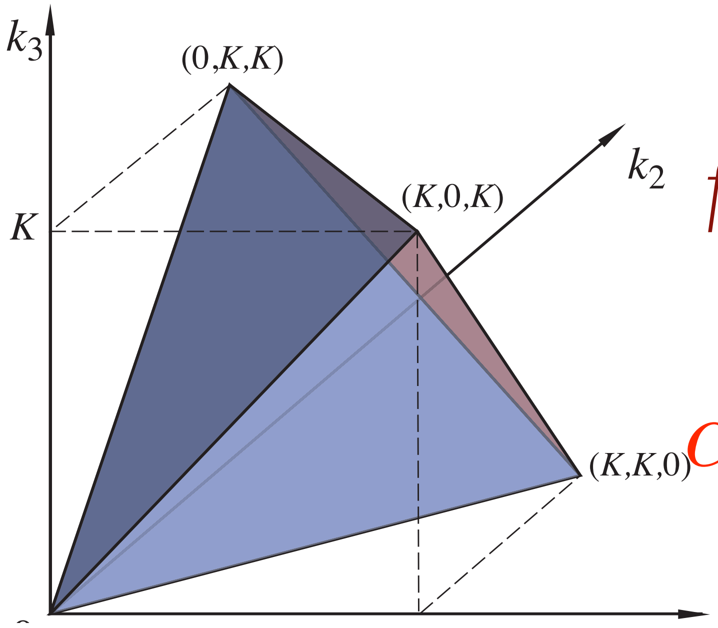


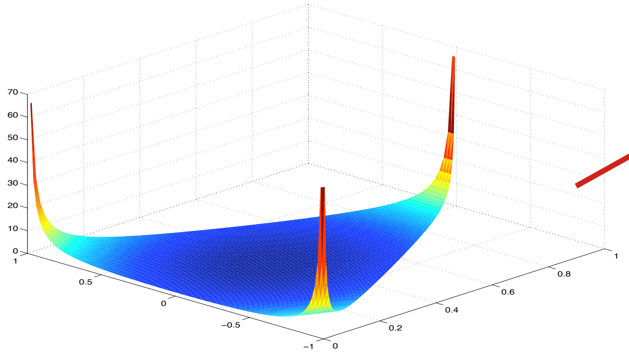
Modal Polyspectra Estimation

THEORY

Primordial bispectra

(k-space)





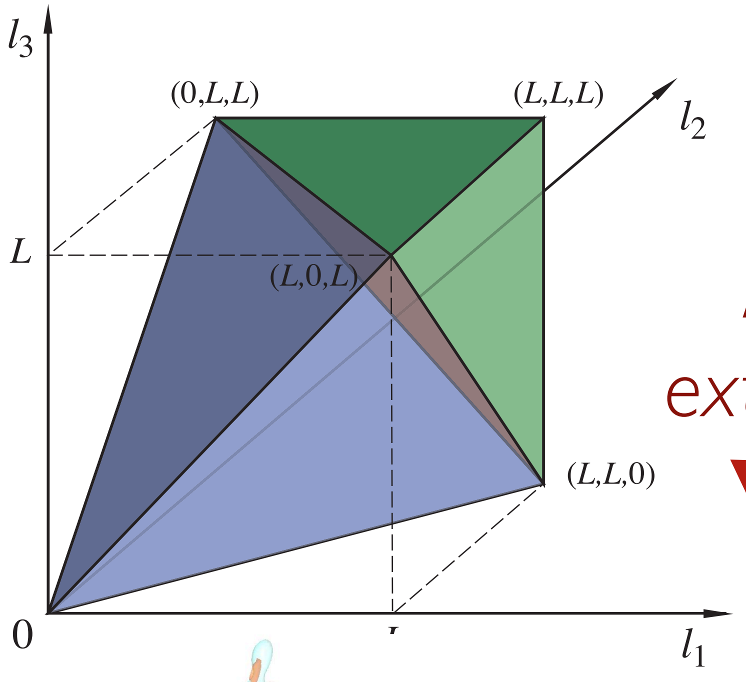
Expand any model with primordial modes α_n

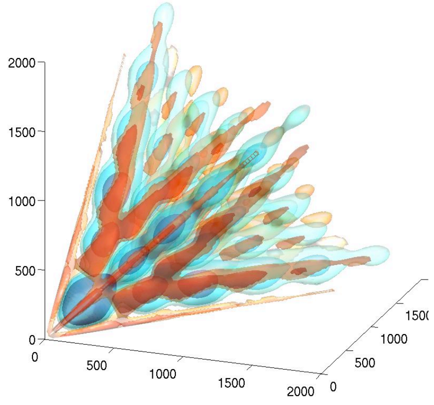
Mode transfer functions

$\alpha_n \rightarrow \bar{\alpha}_n$

CMB bispectra

(l-space)





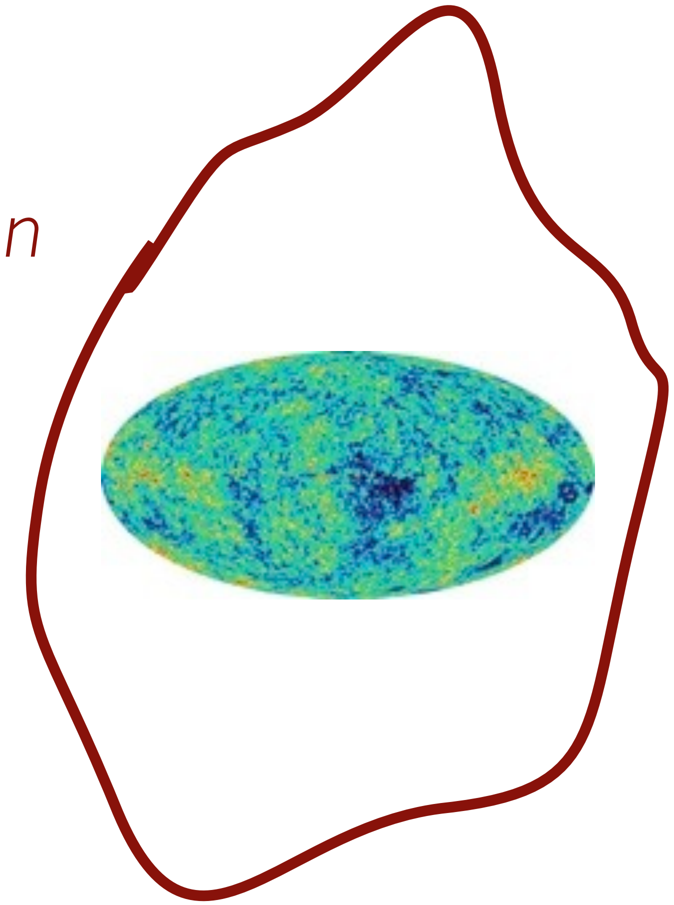
Fergusson, Liguori and EPS, 2009

OBSERVATION

CMB map

Map extraction

$\bar{\beta}$

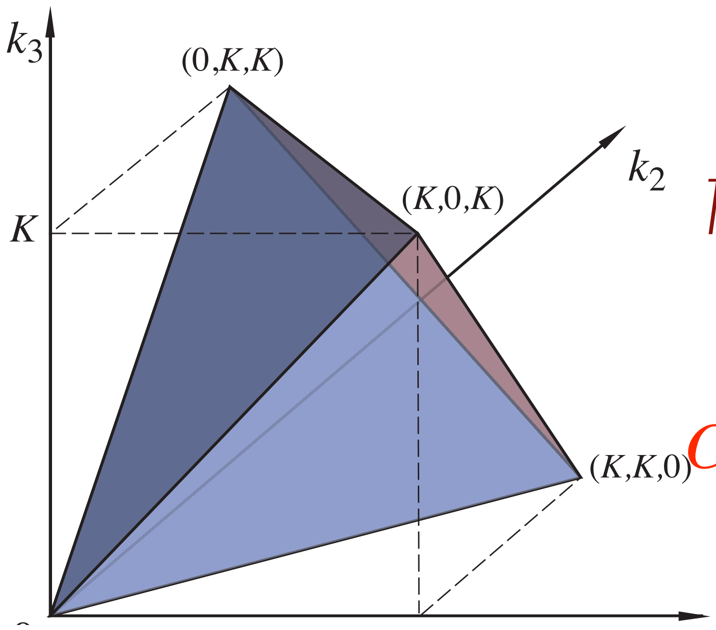


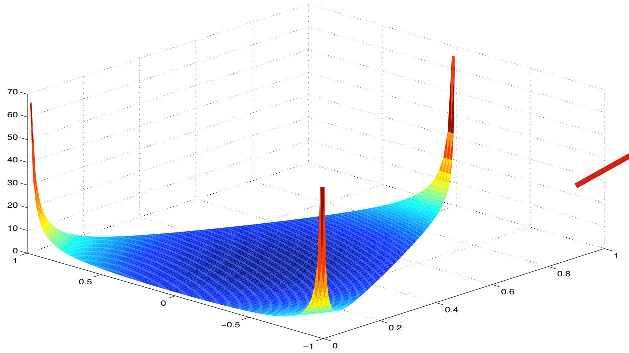
Filter with sufficient separable eigenmodes

Modal Polyspectra Estimation

THEORY

Primordial bispectra
(*k*-space)



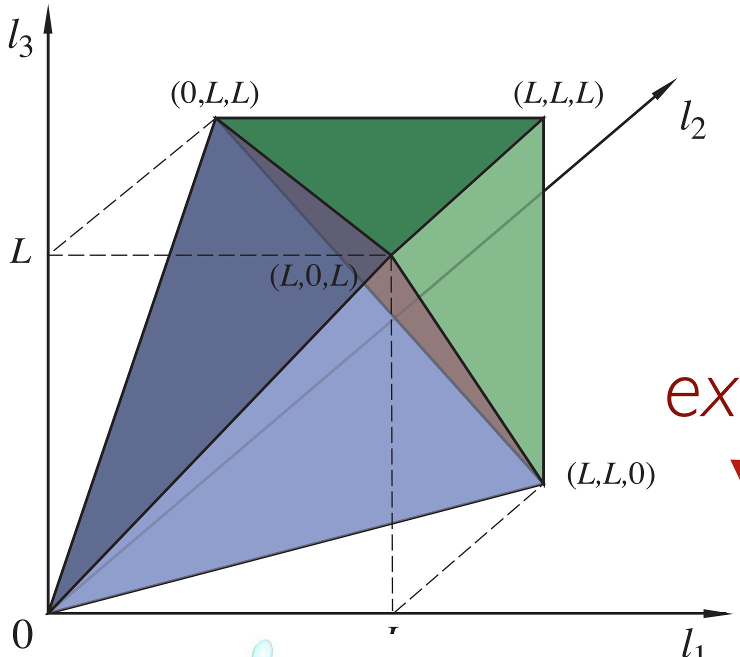


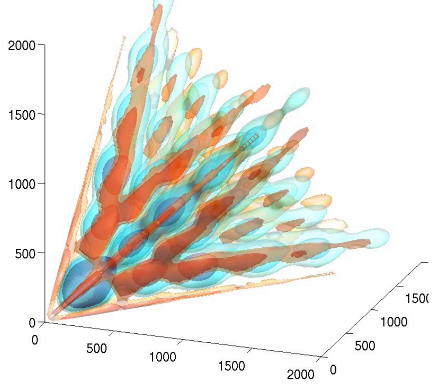
Mode transfer functions

$\alpha_n \rightarrow \bar{\alpha}_n$

CMB bispectra

(*l*-space)



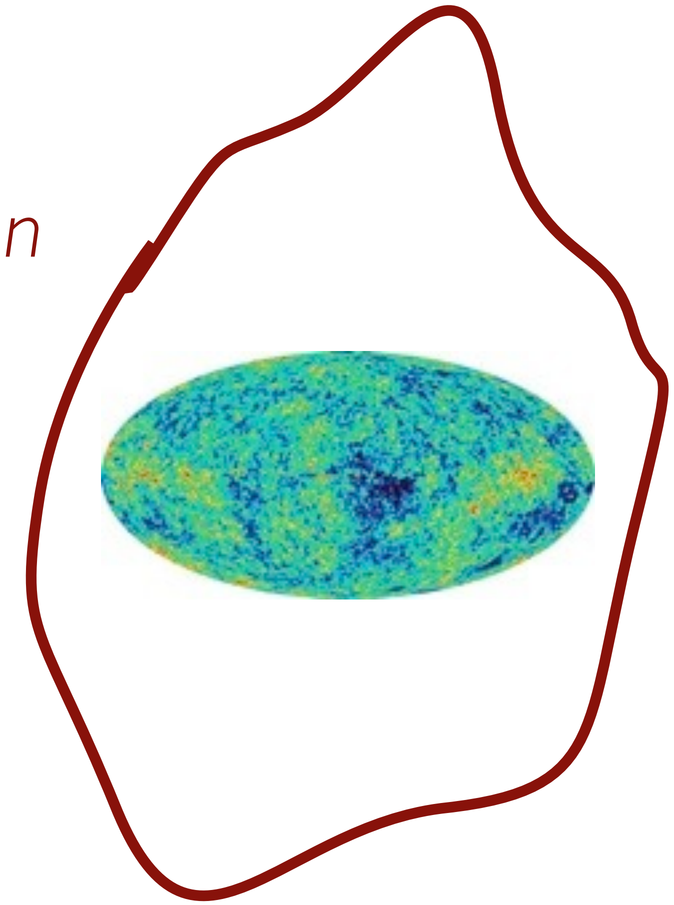


Map extraction

$\bar{\beta}$

OBSERVATION

CMB map



Expand any model with primordial modes α_n

Modal estimator

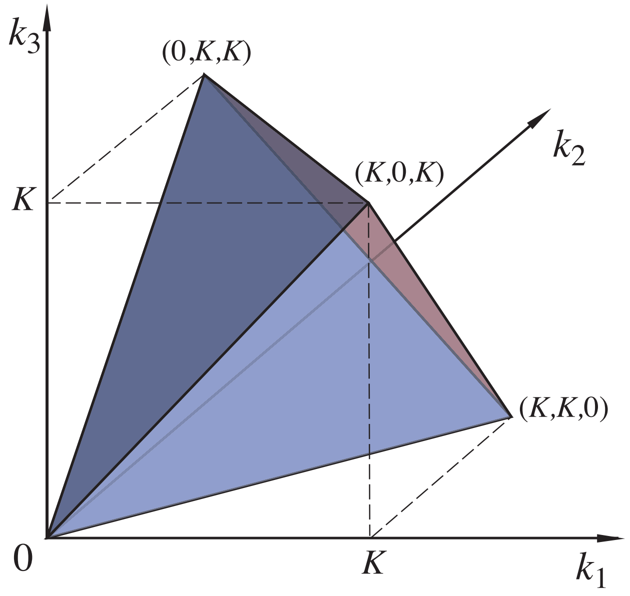
$$\mathcal{E} = \frac{\sum_n \bar{\alpha}_n^R \bar{\beta}_n^R}{\sum_n (\bar{\alpha}_n^R)^2}$$

Fergusson, Liguori and EPS, 2009

Filter with sufficient separable eigenmodes

Primordial to CMB basis

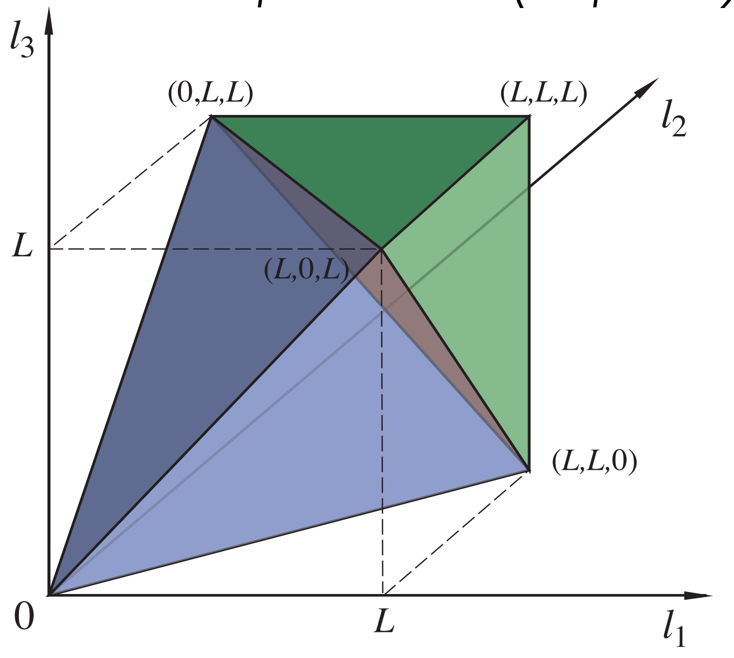
Primoridial bispectrum (k-space)



Transfer functions

$\Delta_l(k)$

CMB bispectrum (l-space)



Use transfer functions *once* to project forward primordial modes so we calculate

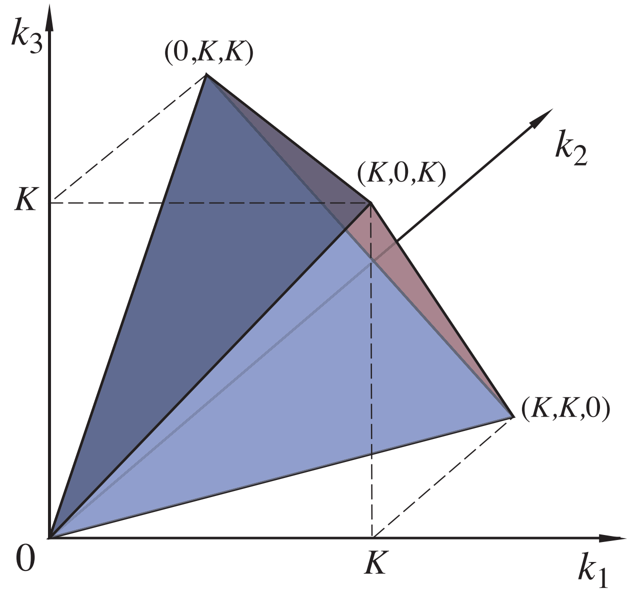
$$\Gamma_{nm} = \left\langle \bar{Q}^n \frac{vvv\tilde{Q}^m}{\sqrt{CCC}} \right\rangle$$

Then we can transform between the primordial and CMB expansions

$$\bar{\alpha}^Q = \bar{\gamma}^{-1}\Gamma\alpha^Q$$

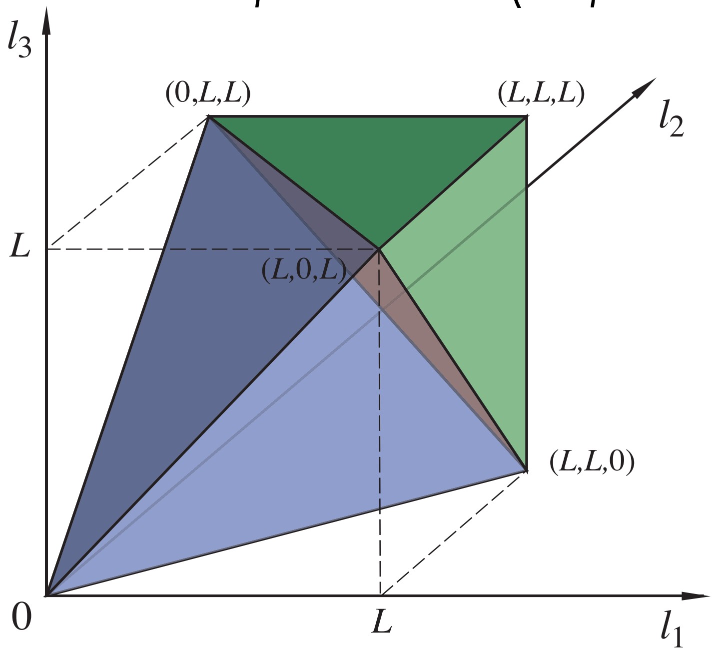
Primordial to CMB basis

Primoridial bispectrum (k-space)



Transfer functions
$\Delta_l(k)$

CMB bispectrum (l-space)



Use transfer functions *once* to project forward primordial modes so we calculate

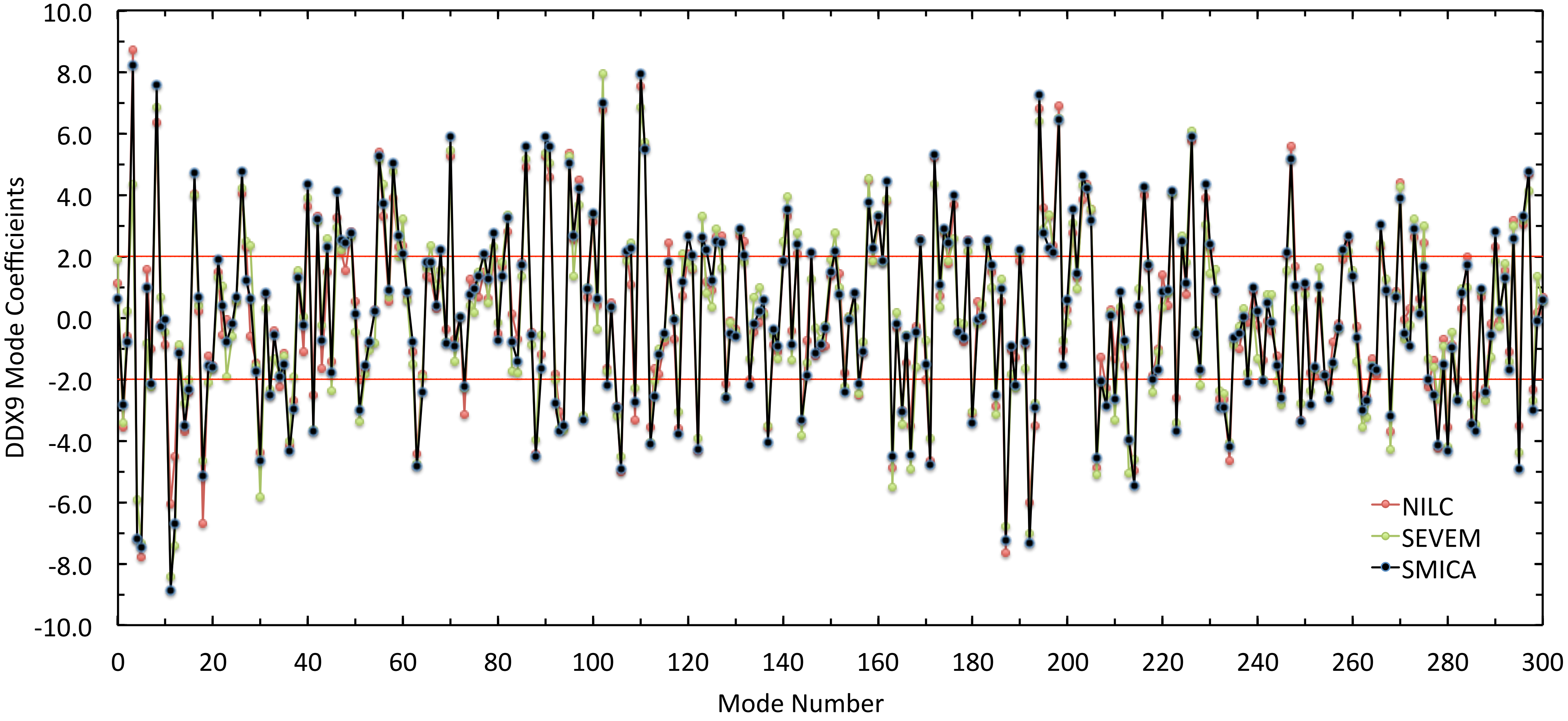
$$\Gamma_{nm} = \left\langle \bar{Q}^n \frac{vvv\tilde{Q}^m}{\sqrt{CCC}} \right\rangle$$

Then we can transform between the primordial and CMB expansions

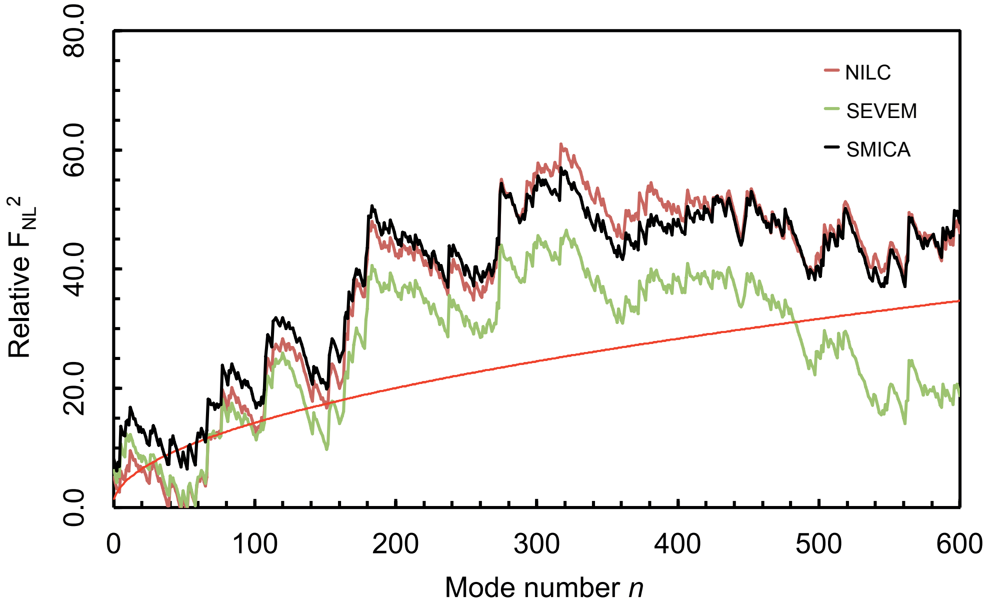
$$\bar{\alpha}^Q = \bar{\gamma}^{-1}\Gamma\alpha^Q$$

Bispectrum reconstruction modes

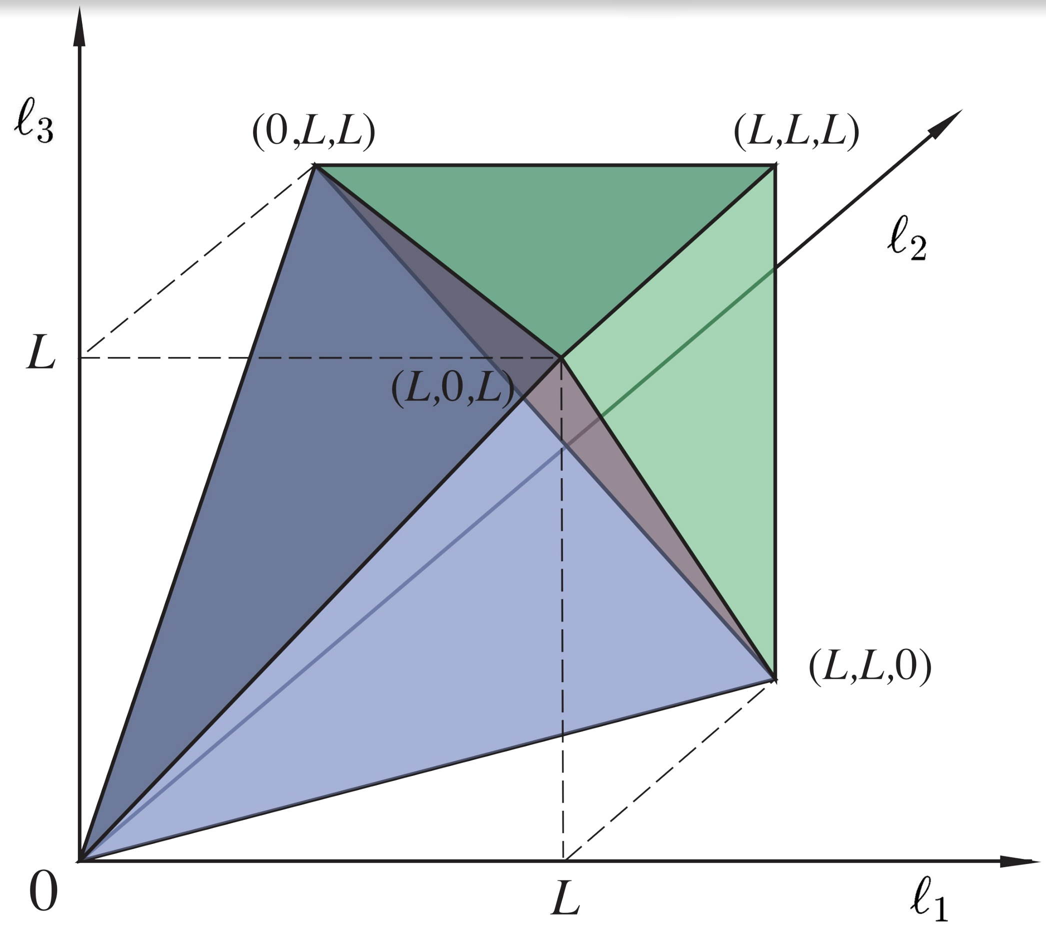
Reconstructed β_n modes from filtering Planck data

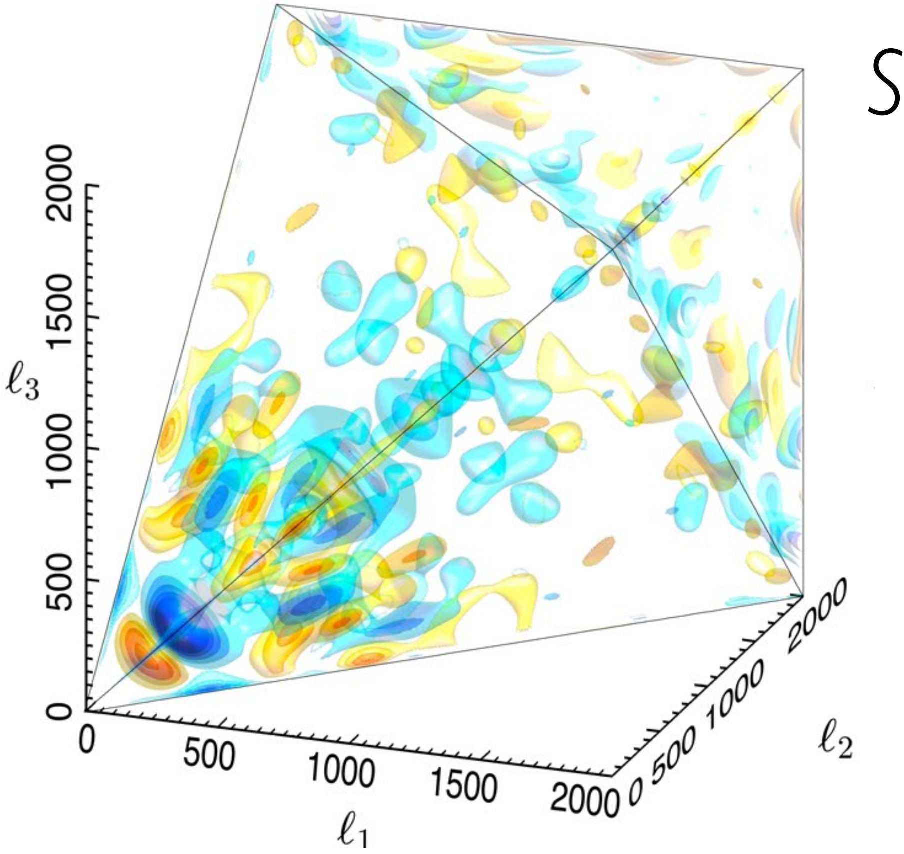


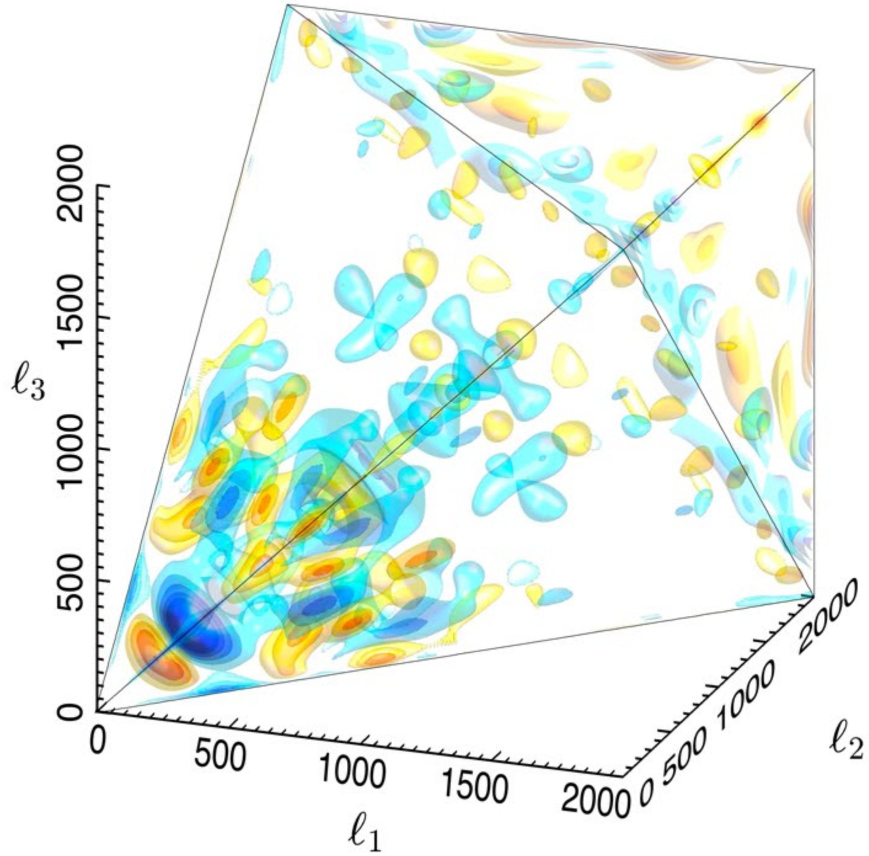
High signal from comparison with 200 Gaussian maps (χ^2-tests see later)

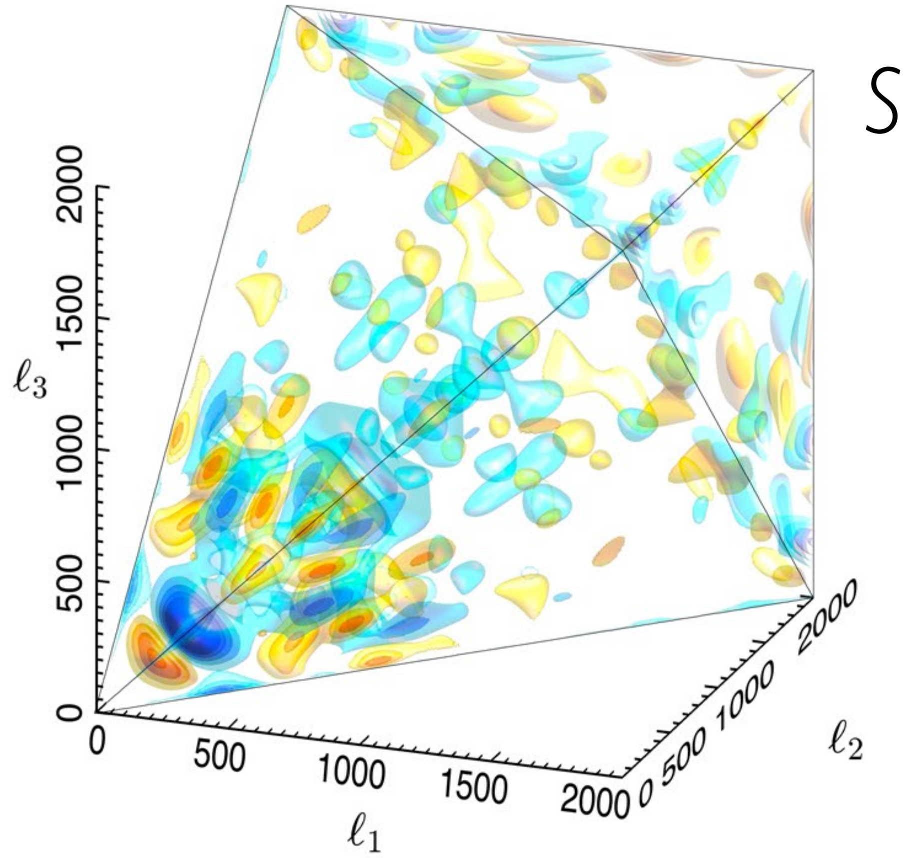


Planck Bispectrum Reconstruction



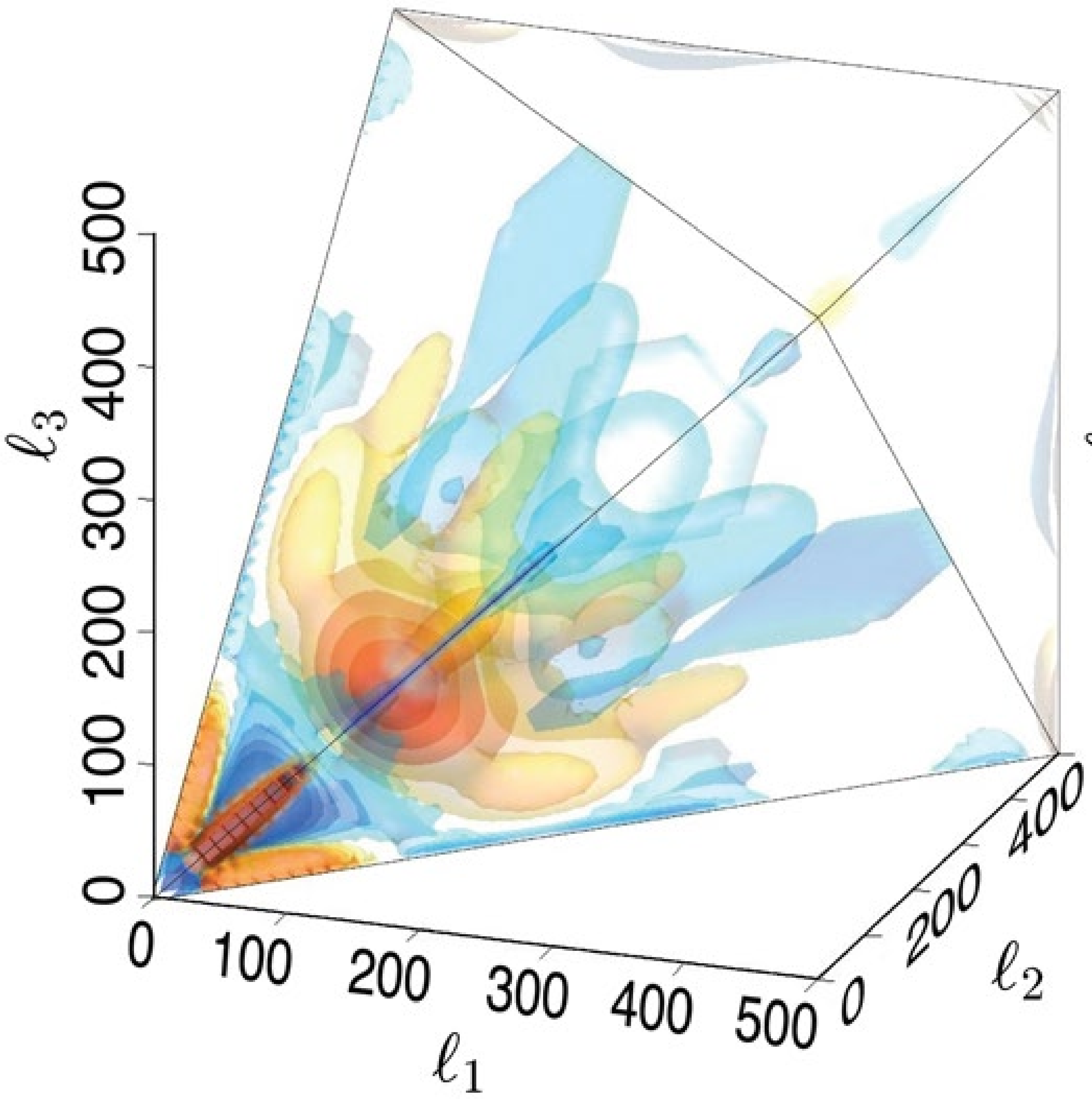

SMICA


NILC


SEVEM

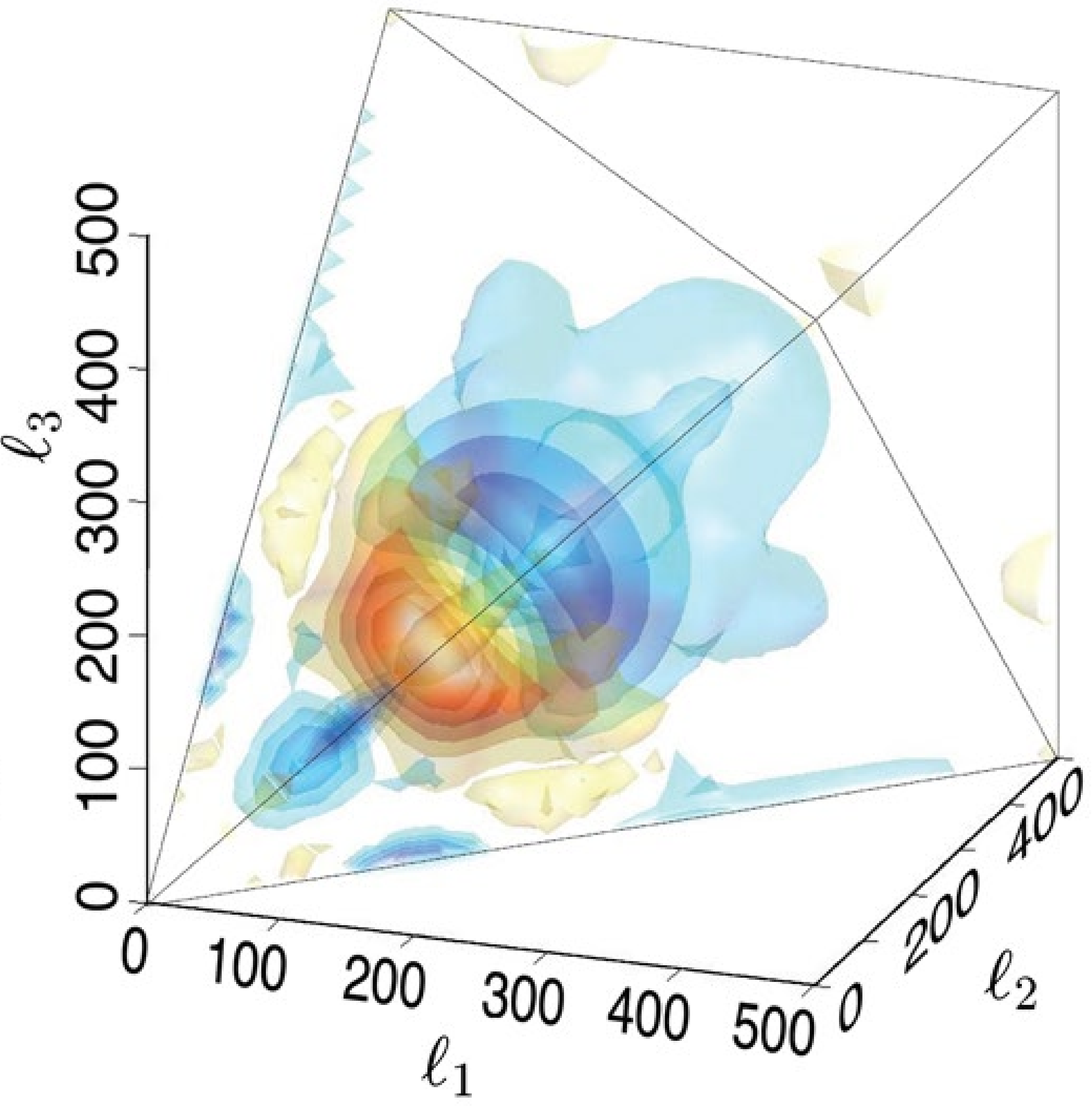
WMAP vs Planck

WMAP 7 year



Fergusson, Liguori and EPS, 2010

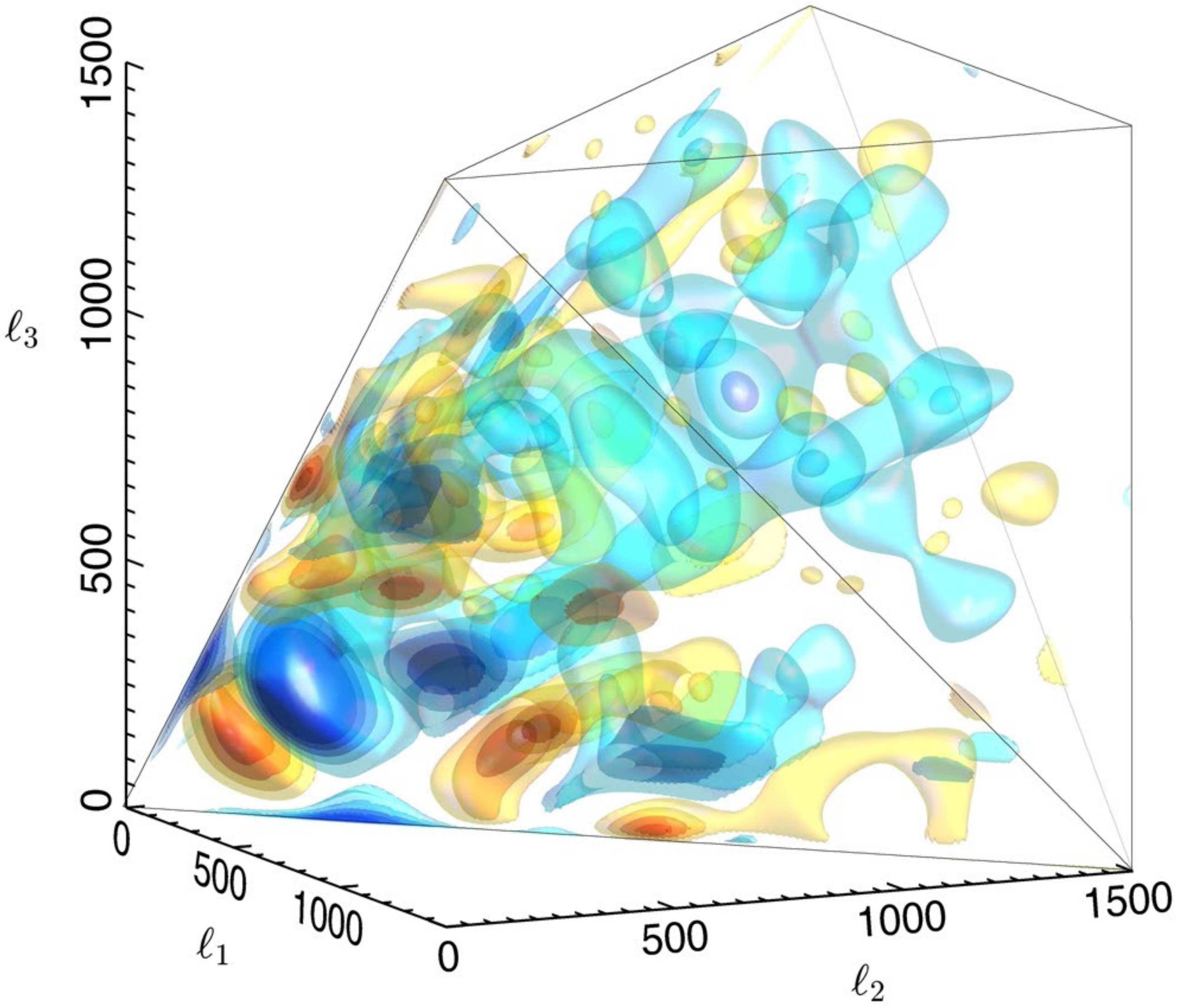
Planck SMICA

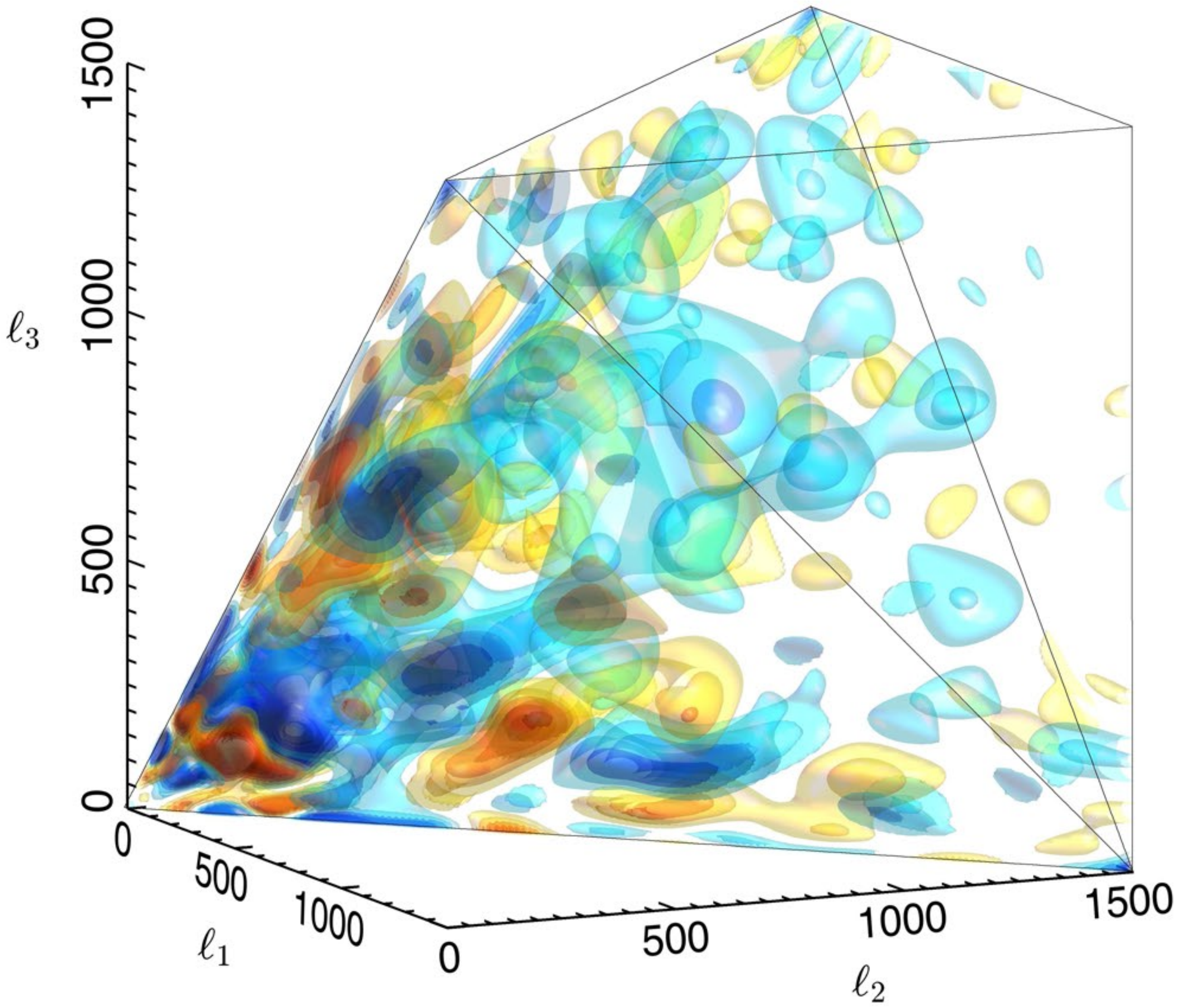


Paper XXIII. Isotropy and statistics of the CMB

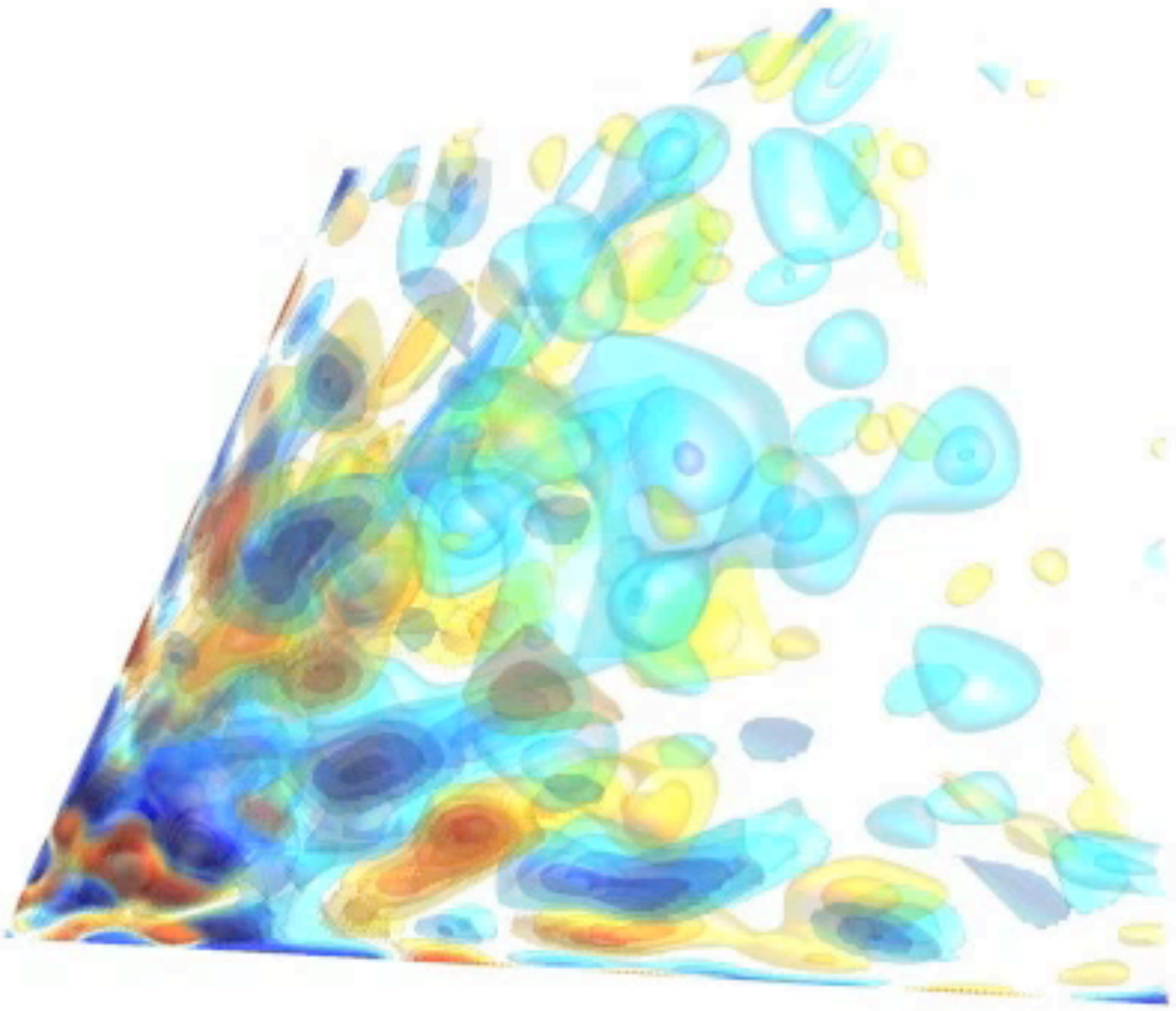
The Planck Bispectrum

Modal reconstruction of the full 3D Planck bispectrum





Fourier modes vs Polynomials

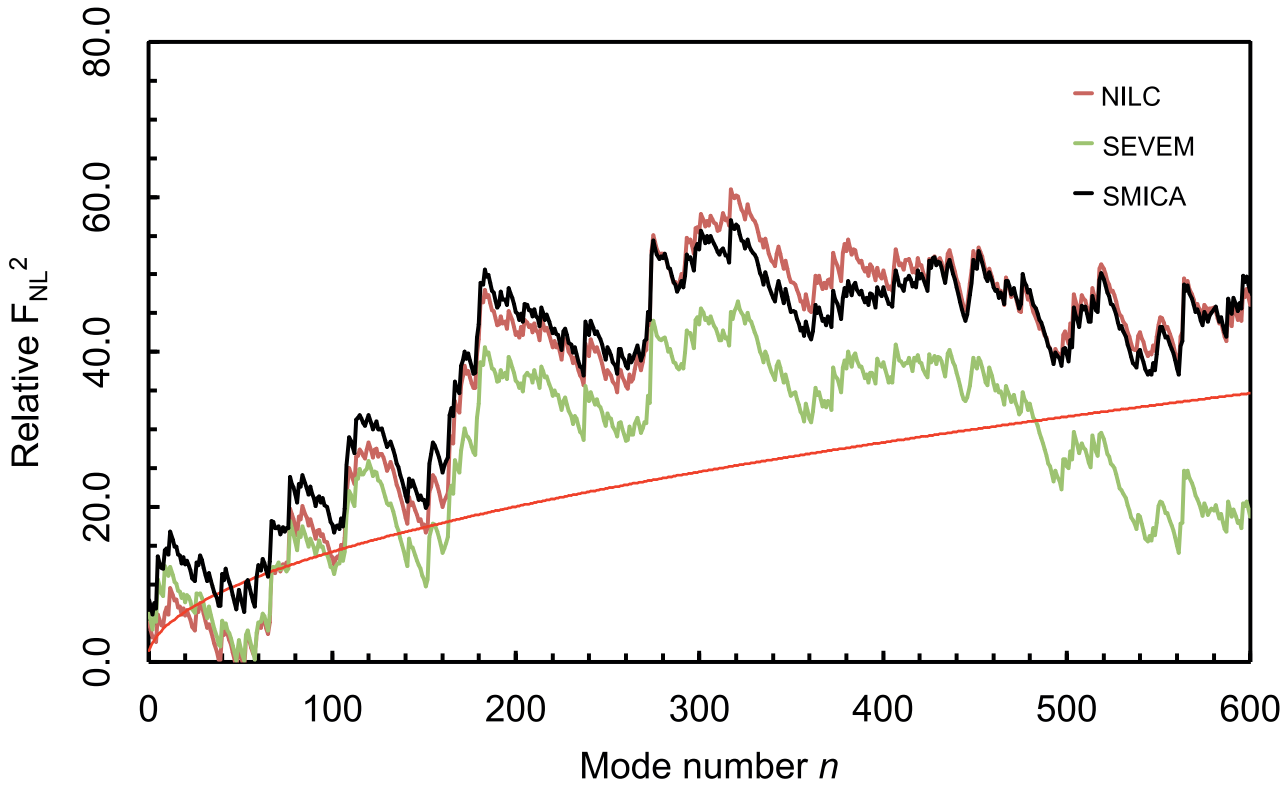


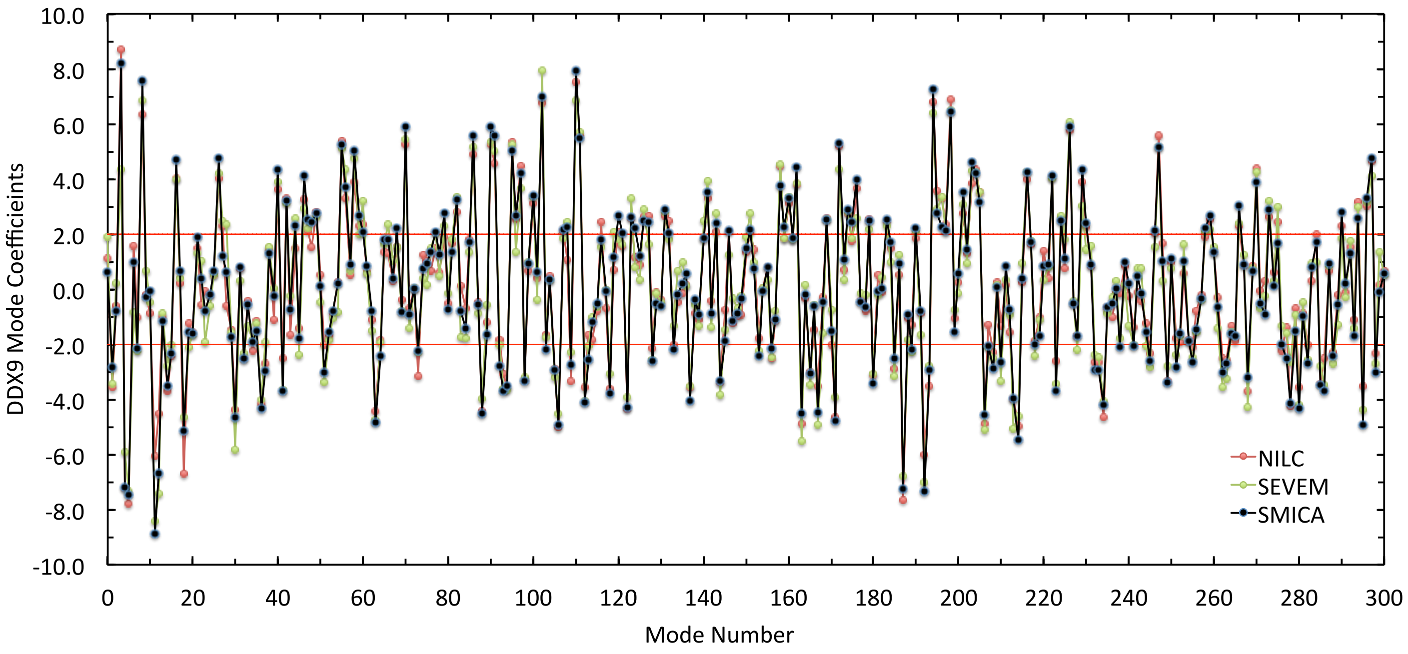
Modal FLS Bispectrum Reconstruction (Planck Collaboration 2013)

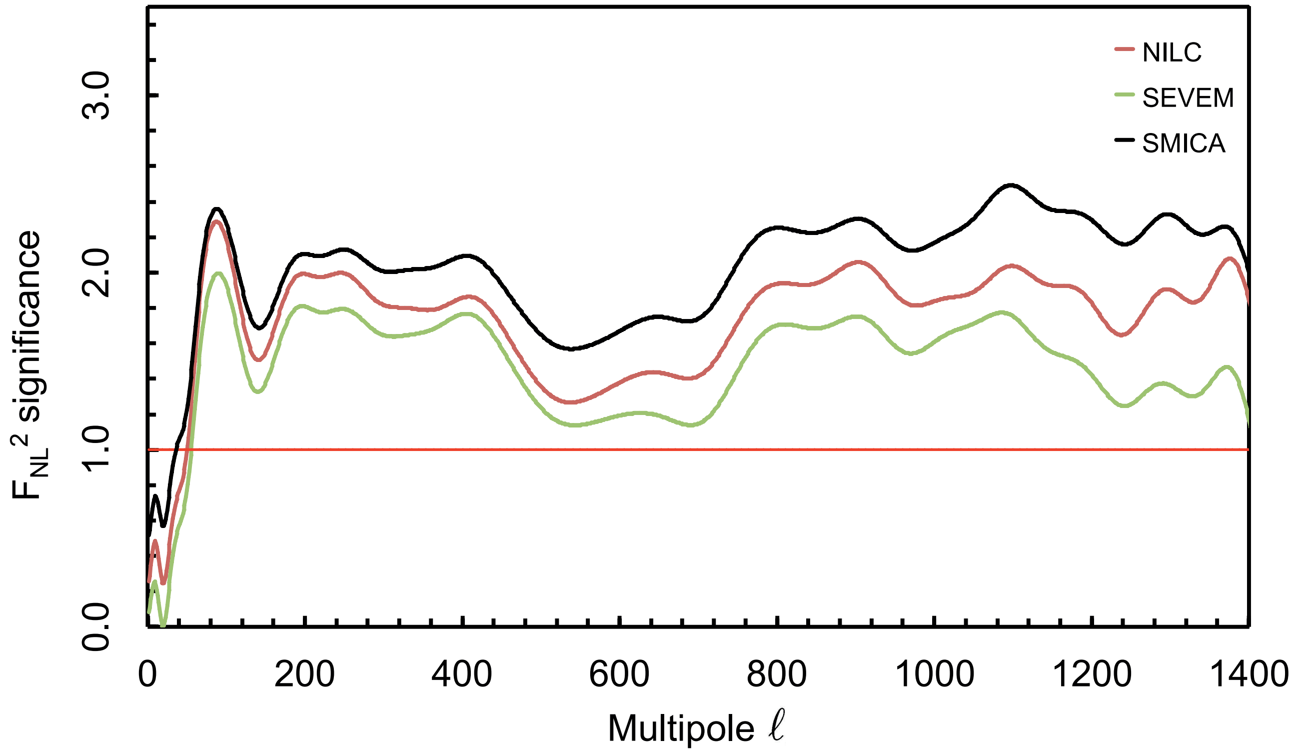
High bispectrum signal

χ^2-tests for integrated bispectrum consistent with Gaussianity, but signal always high.

Comparison with 200 lensed CMB Gaussian maps with Planck noise.

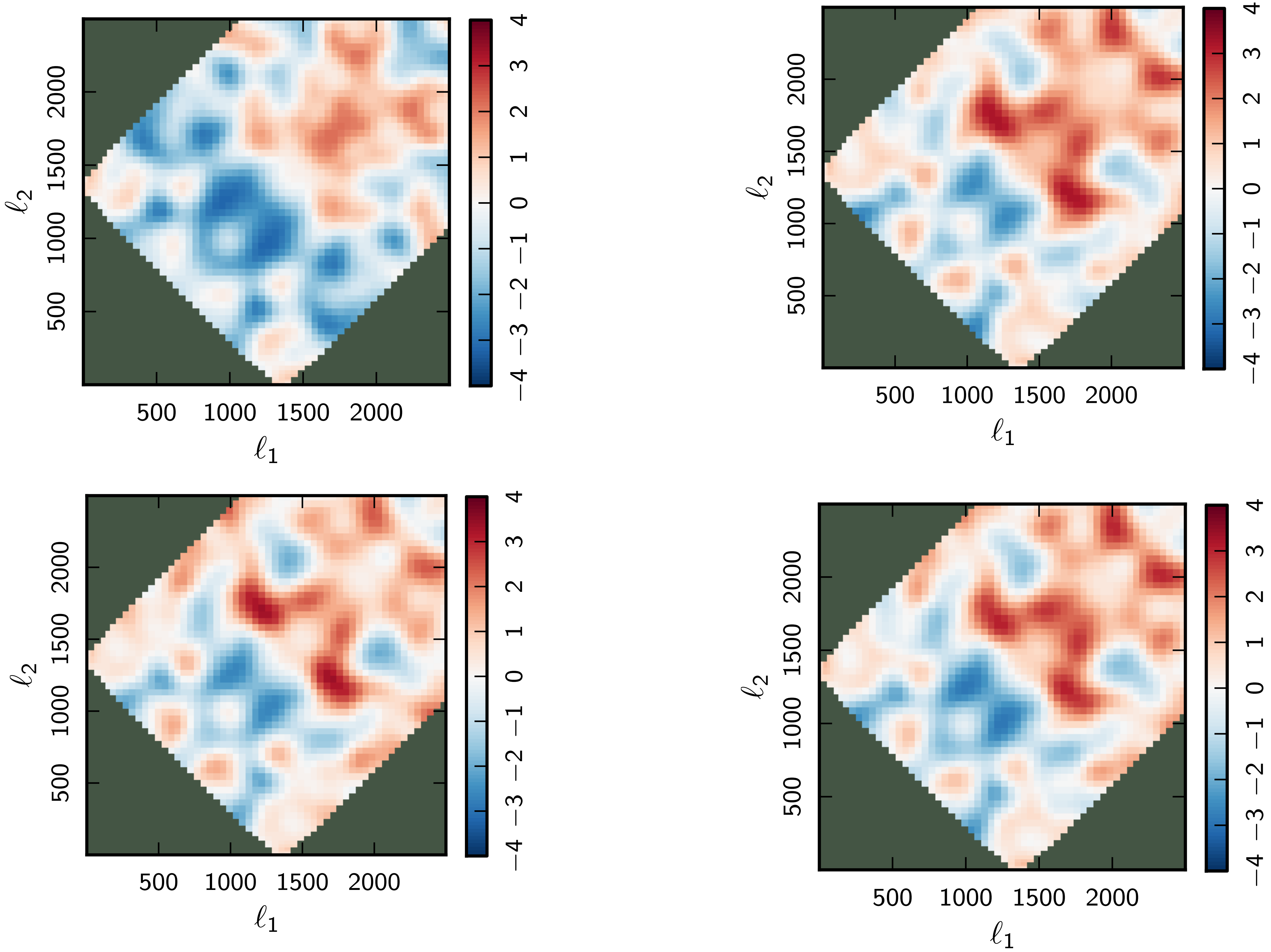






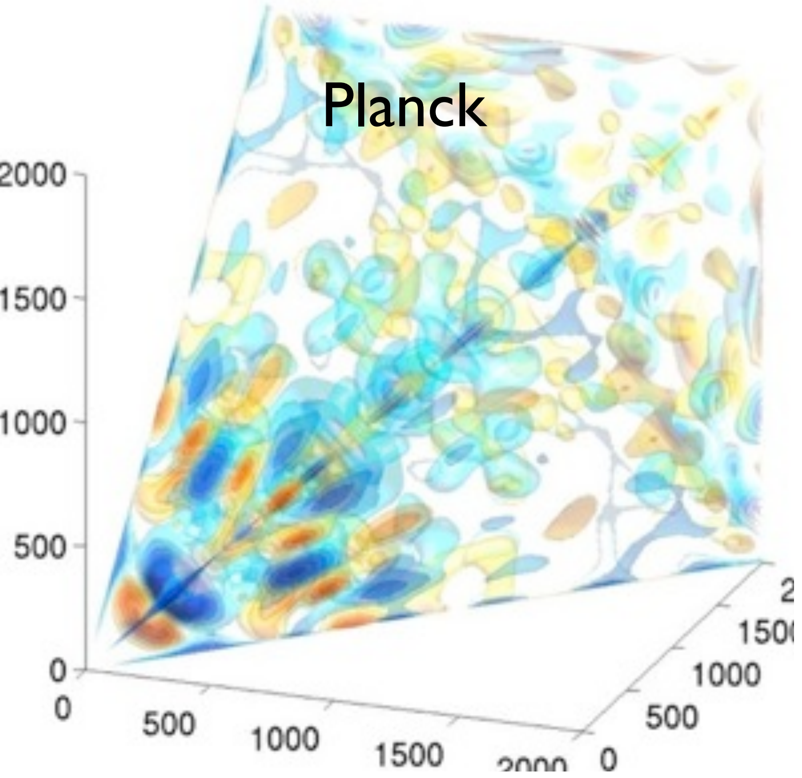
Binned slice reconstruction

Binned estimator S/N weighting - comparison of comp-sep maps

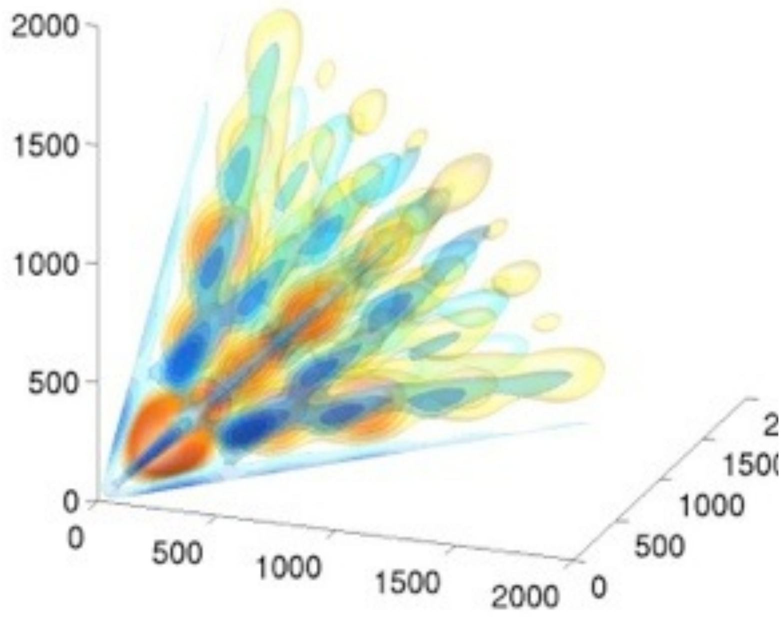


Bispectrum in detail

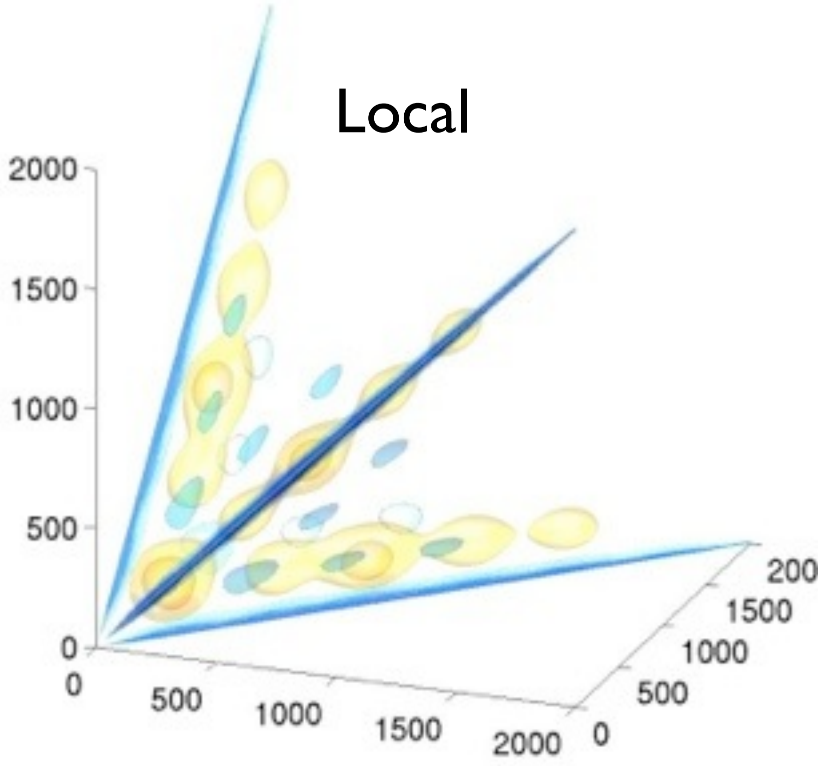
Planck



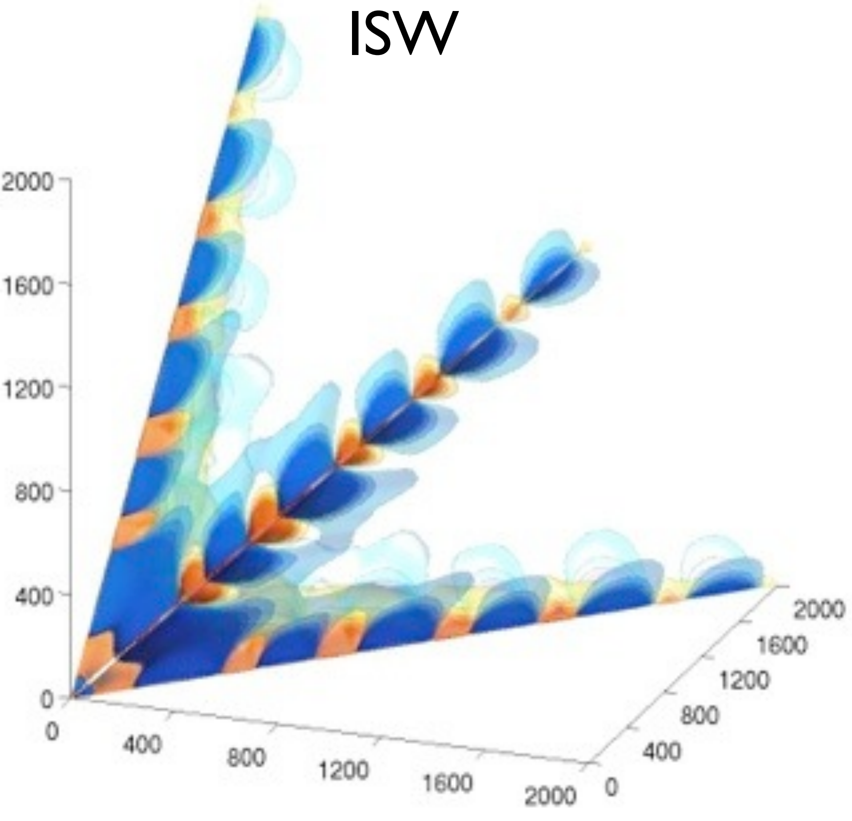
Constant



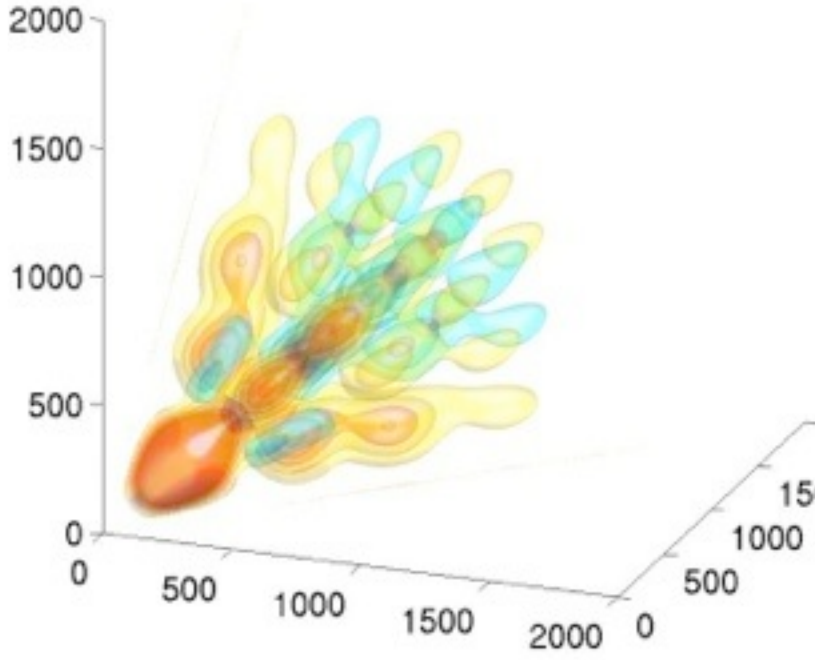
Local



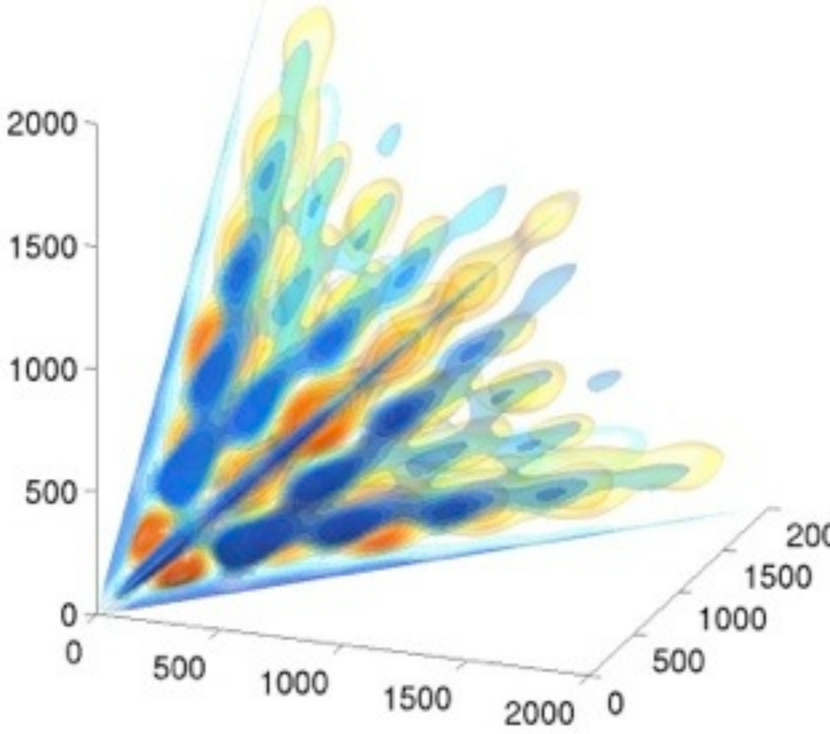
ISW



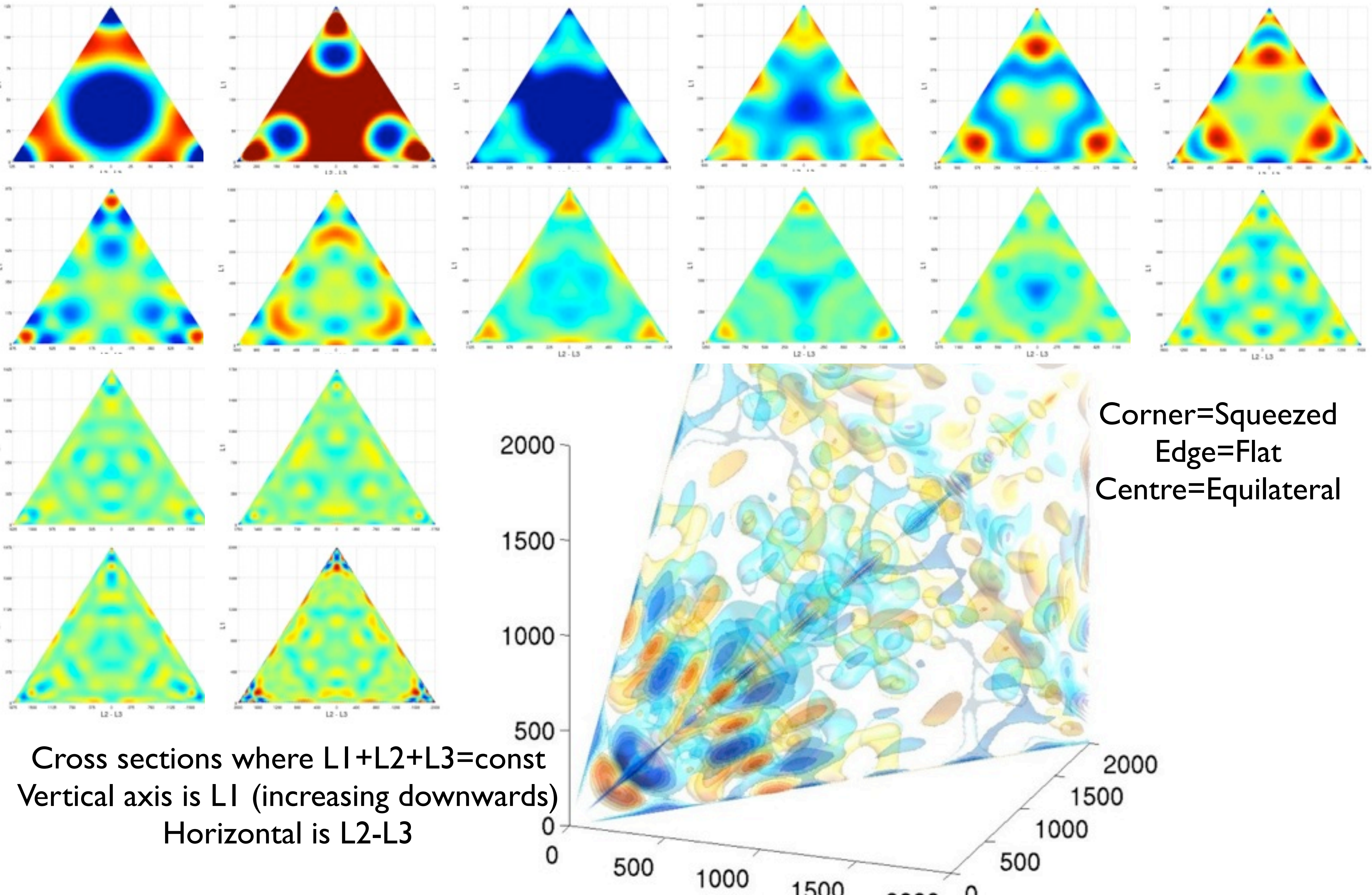
Equilateral



Flat



Bispectrum in detail



Corner=Squeezed
Edge=Flat
Centre=Equilateral

Cross sections where L1+L2+L3=const
Vertical axis is L1 (increasing downwards)
Horizontal is L2-L3

ISW-Lensing

Weak detection of Integrated Sachs-Wolfe (ISW) lensing bispectrum, i.e. correlation between CMB and large-scale evolving grav. potential.

Estimator		SMICA		SEVEM		C-R		NILC	
$T\phi$	$\ell \geq 10$	0.68 ± 0.30	2.3	0.58 ± 0.31	1.9	0.52 ± 0.33	1.5	0.72 ± 0.30	2.4
	$\ell \geq 2$	0.70 ± 0.28	2.5	0.62 ± 0.29	2.1	0.52 ± 0.32	1.6	0.75 ± 0.28	2.7
KSW		0.81 ± 0.31	2.6	0.68 ± 0.32	2.1	0.75 ± 0.32	2.3	0.85 ± 0.32	2.7
binned		0.91 ± 0.37	2.5	0.83 ± 0.39	2.1	0.80 ± 0.40	2.0	1.03 ± 0.37	2.8
modal		0.77 ± 0.37	2.1	0.60 ± 0.37	1.6	0.68 ± 0.39	1.7	0.93 ± 0.37	2.5

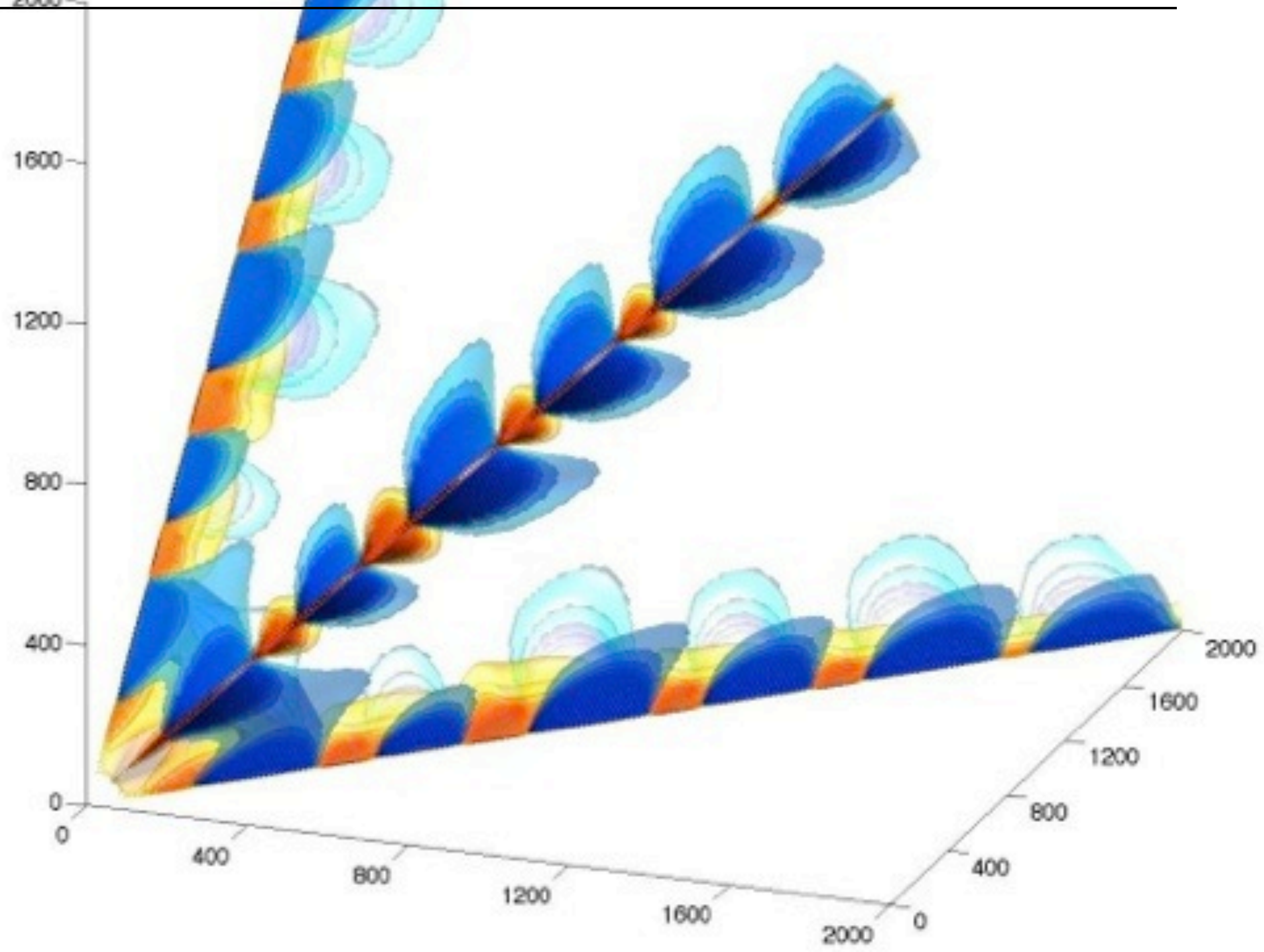
Significance ~ 2.5σ

weak detection ...

Important as correlated with local model f_{NL} ~ 7

Second-order recombination contributions: Total f_{NL} ~ 3

Local f_{NL} ~ 0.88



ISW-Lensing

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KSW		0.81 ± 0.31	2.6
binned		0.91 ± 0.37	2.5
modal		0.77 ± 0.37	2.1

Significance ~ 2.5σ

weak detection ...

Important as correlated with local model f_{NL} ~ 7

Second-order recombinatio contributions: Total f_{NL} ~ 3

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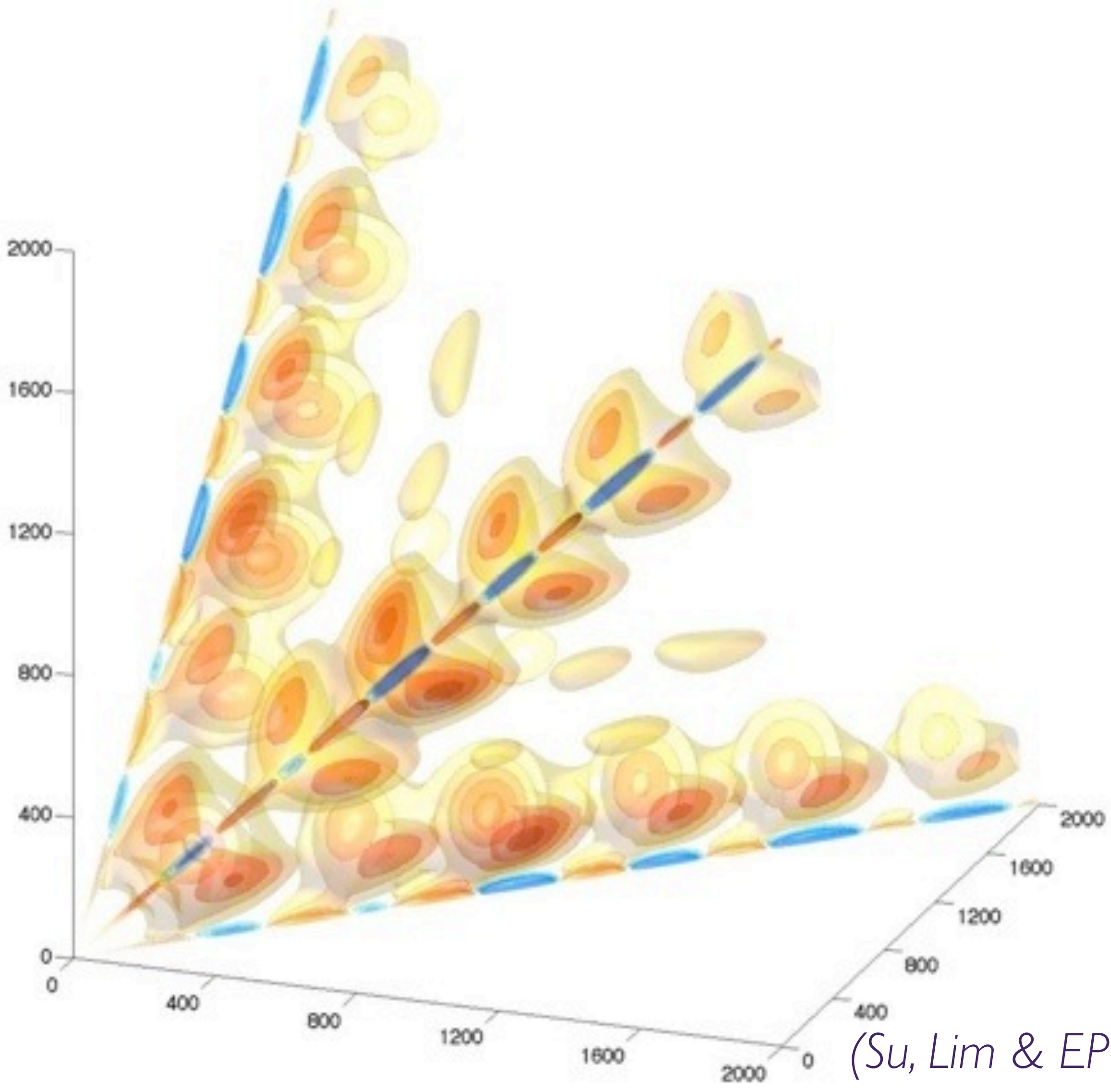
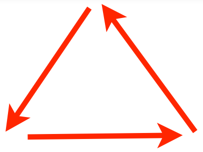


(Su, Lim & EPS, 2012, Huang & Vernizzi, 2012; Pettinari al, 2013)

Standard Bispectra



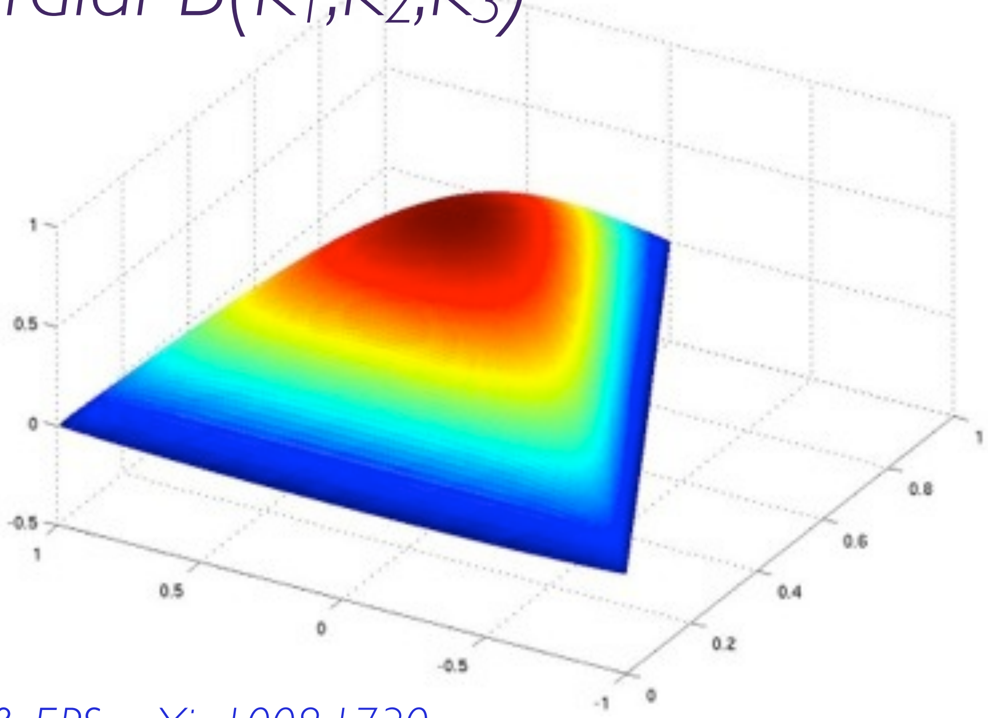
Equilateral bispectra

$$f_{\rm NL}^{\rm equil} = -42 \pm 75$$

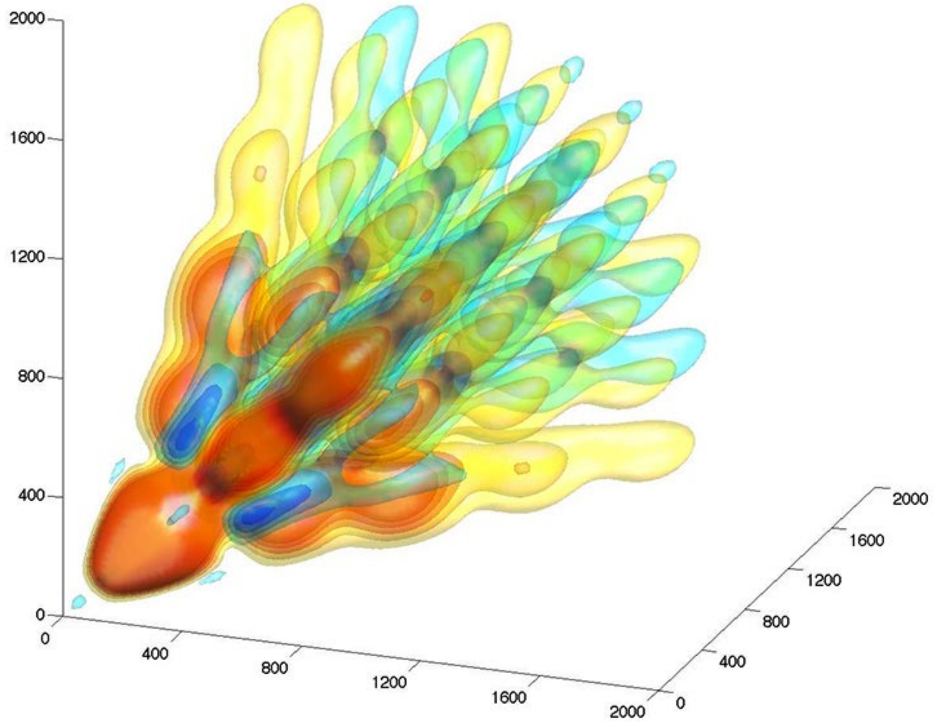
Inflation from higher dimensions

Single-field - sound speed $c_s << c$

Primordial B(k₁,k₂,k₃)



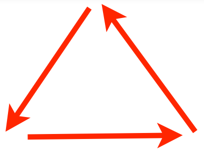
CMB $B_{l_1 l_2 l_3}$



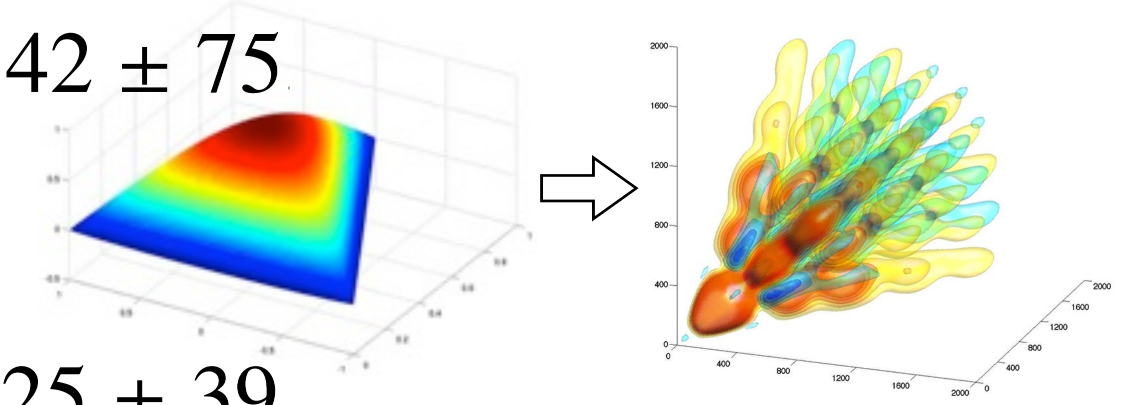
Fergusson & EPS, arXiv:1008.1730

Standard Bispectra

Equilateral bispectra



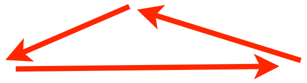
$$f_{\rm NL}^{\rm equil} = -42 \pm 75$$



Inflation from higher dimensions

Single-field - sound speed $c_s << c$

Orthogonal bispectra

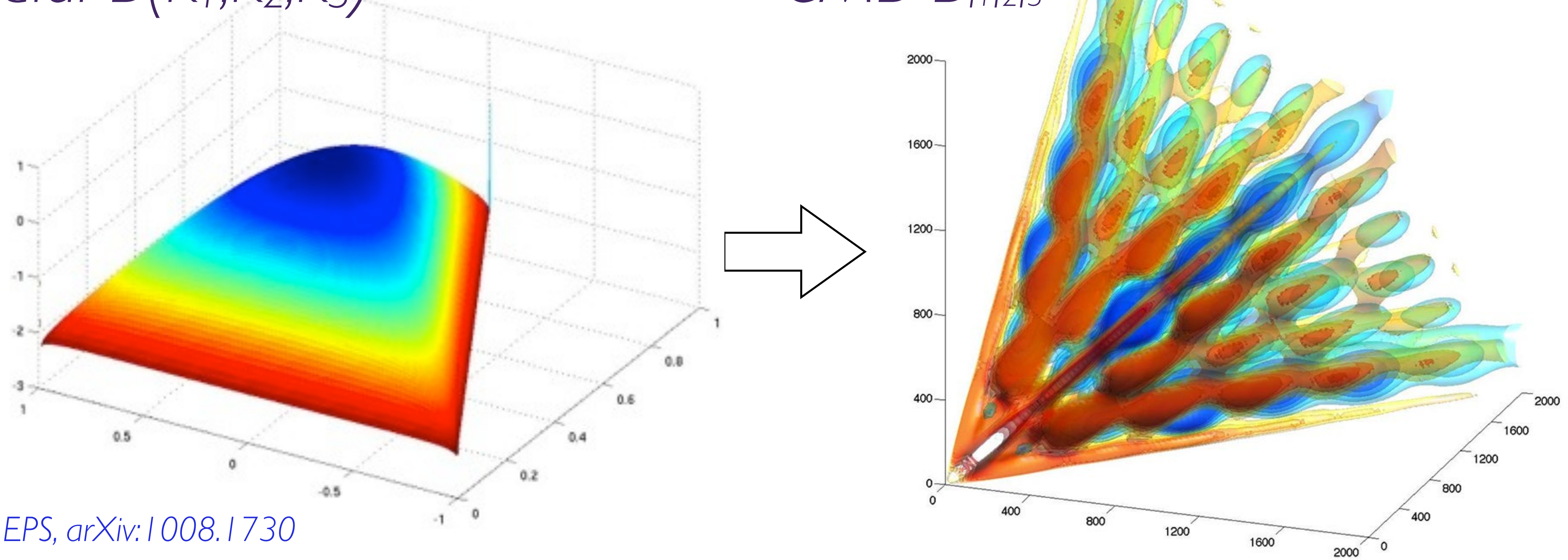


$$f_{\rm NL}^{\rm ortho} = -25 \pm 39$$

Single-field, complement of equilateral

Primordial $B(k_1,k_2,k_3)$

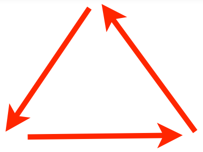
CMB $B_{l_1 l_2 l_3}$



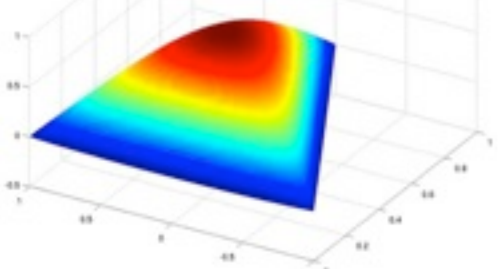
Fergusson & EPS, arXiv:1008.1730

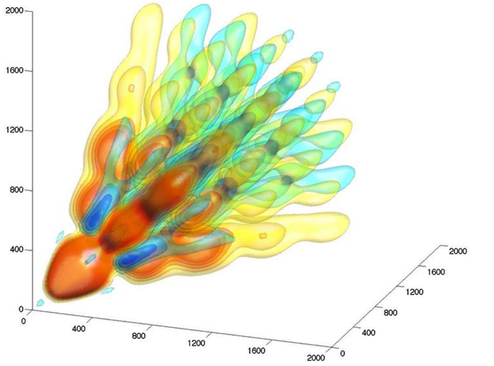
Standard Bispectra

Equilateral bispectra



$$f_{NL}^{\rm equil} = -42 \pm 75$$

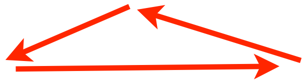




Inflation from higher dimensions

Single-field - sound speed $c_s << c$

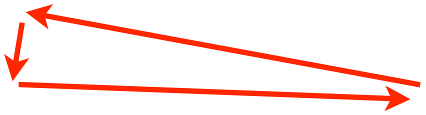
Orthogonal bispectra



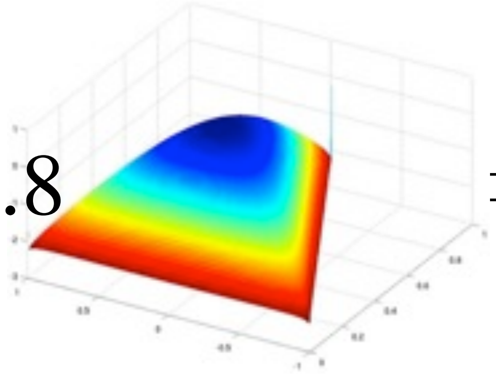
$$f_{NL}^{\rm ortho} = -25 \pm 39$$

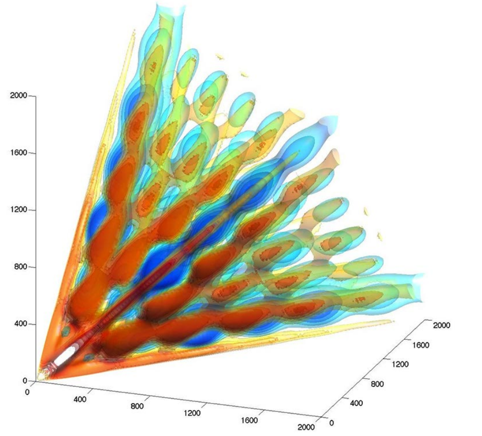
Single-field, complement of equilateral

Local bispectra



$$f_{NL}^{\rm local} = 2.7 \pm 5.8$$

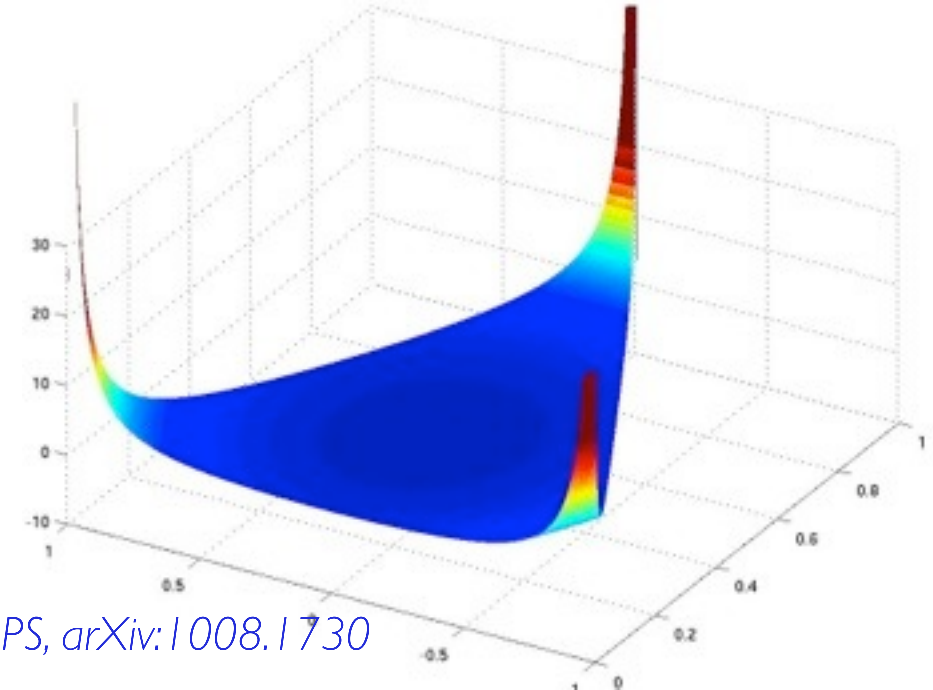


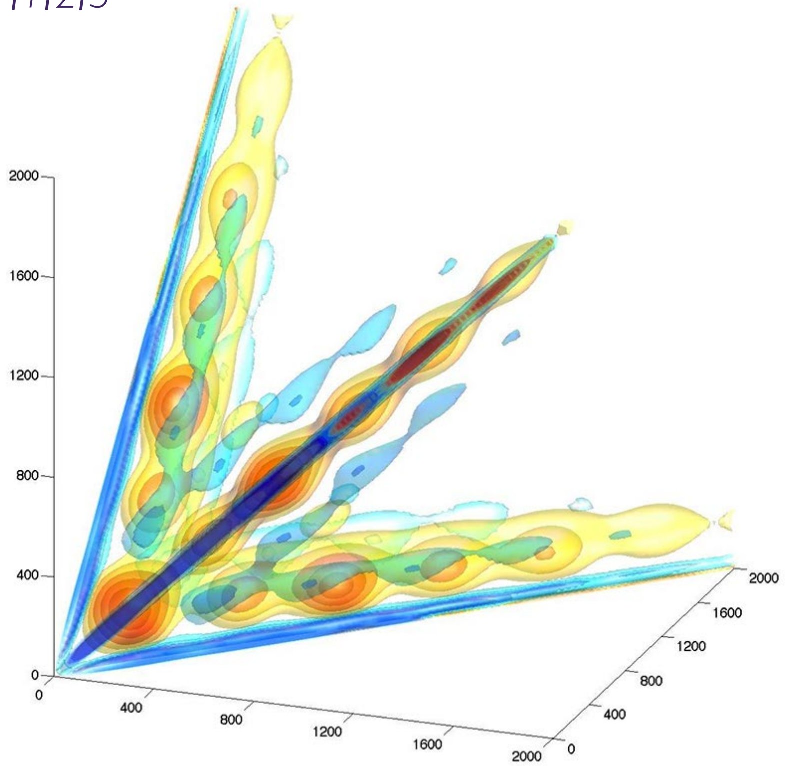


Multifield inflation, curvaton etc.

Primordial $B(k_1,k_2,k_3)$

CMB $B_{l_1l_2l_3}$





Fergusson & EPS, arXiv:1008.1730

"Main Planck NG results"

Achieved forecast local variance $\Delta f_{\rm NL} = 5.8$ (*Planck* nominal mission)

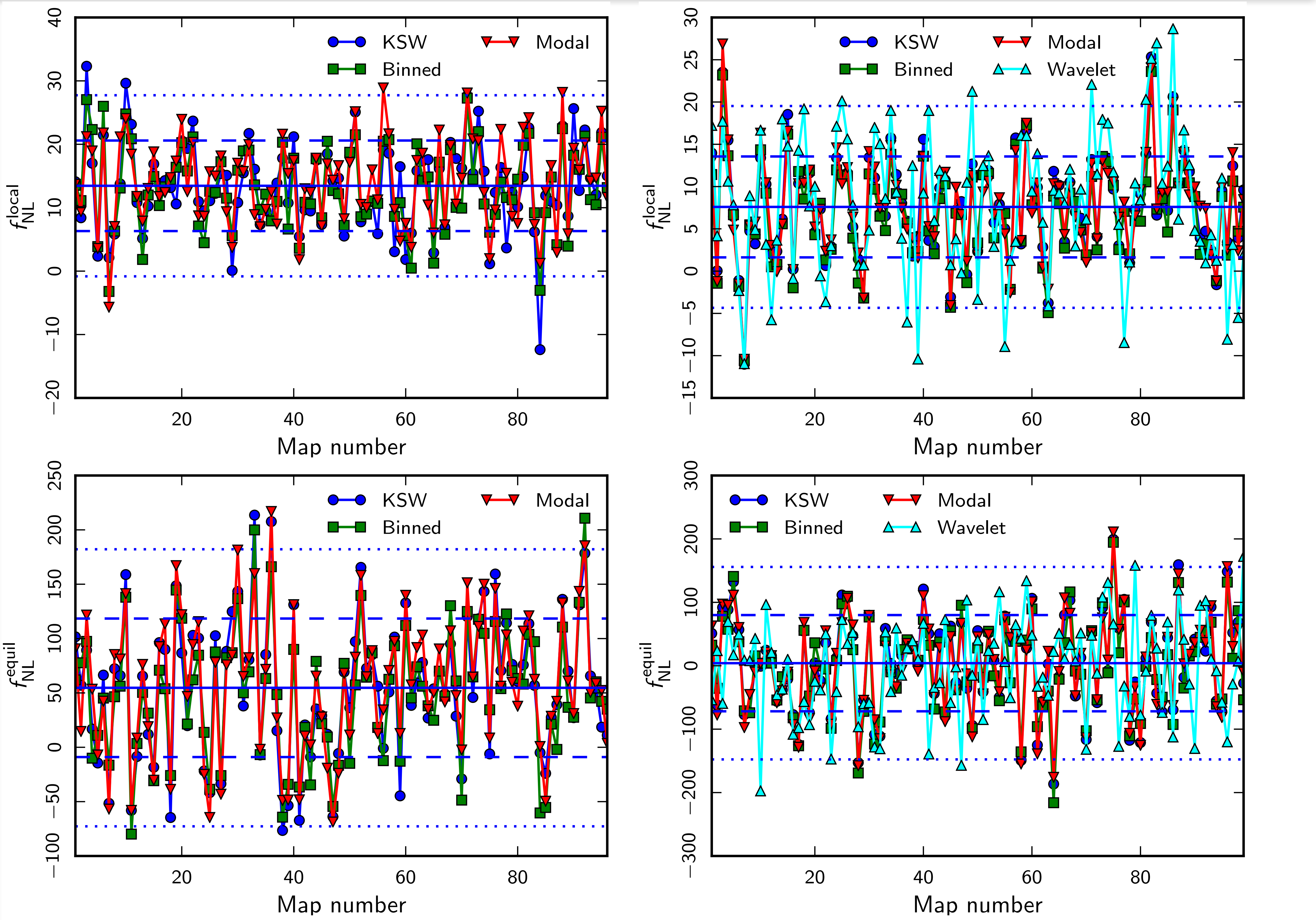
Most stringent constraints for the standard separable shapes:

$$f_{\rm NL}^{\rm local} = 2.7 \pm 5.8,\ f_{\rm NL}^{\rm equil} = -42 \pm 75, \text{ and } f_{\rm NL}^{\rm ortho} = -25 \pm 39$$

i.e., no evidence for local, equilateral or orthogonal bispectra

		Independent				**ISW-lensing subtracted**	
	KSW	Binned	Modal		KSW	Binned	Modal
`SMICA`							
Local	9.8 ± 5.8	9.2 ± 5.9	8.3 ± 5.9	. . . . .	**2.7 ± 5.8**	2.2 ± 5.9	1.6 ± 6.0
Equilateral	−37 ± 75	−20 ± 73	−20 ± 77	. . . . .	**−42 ± 75**	−25 ± 73	−20 ± 77
Orthogonal	−46 ± 39	−39 ± 41	−36 ± 41	. . . . .	**−25 ± 39**	−17 ± 41	−14 ± 42
`NILC`							
Local	11.6 ± 5.8	10.5 ± 5.8	9.4 ± 5.9	. . . . .	4.5 ± 5.8	3.6 ± 5.8	2.7 ± 6.0
Equilateral	−41 ± 76	−31 ± 73	−20 ± 76	. . . . .	−48 ± 76	−38 ± 73	−20 ± 78
Orthogonal	−74 ± 40	−62 ± 41	−60 ± 40	. . . . .	−53 ± 40	−41 ± 41	−37 ± 43
`SEVEM`							
Local	10.5 ± 5.9	10.1 ± 6.2	9.4 ± 6.0	. . . . .	3.4 ± 5.9	3.2 ± 6.2	2.6 ± 6.0
Equilateral	−32 ± 76	−21 ± 73	−13 ± 77	. . . . .	−36 ± 76	−25 ± 73	−13 ± 78
Orthogonal	−34 ± 40	−30 ± 42	−24 ± 42	. . . . .	−14 ± 40	−9 ± 42	−2 ± 42
`C-R`							
Local	12.4 ± 6.0	11.3 ± 5.9	10.9 ± 5.9	. . . . .	6.4 ± 6.0	5.5 ± 5.9	5.1 ± 5.9
Equilateral	−60 ± 79	−52 ± 74	−33 ± 78	. . . . .	−62 ± 79	−55 ± 74	−32 ± 78
Orthogonal	−76 ± 42	−60 ± 42	−63 ± 42	. . . . .	−57 ± 42	−41 ± 42	−42 ± 42

Quantitative non-Gaussianity - validation



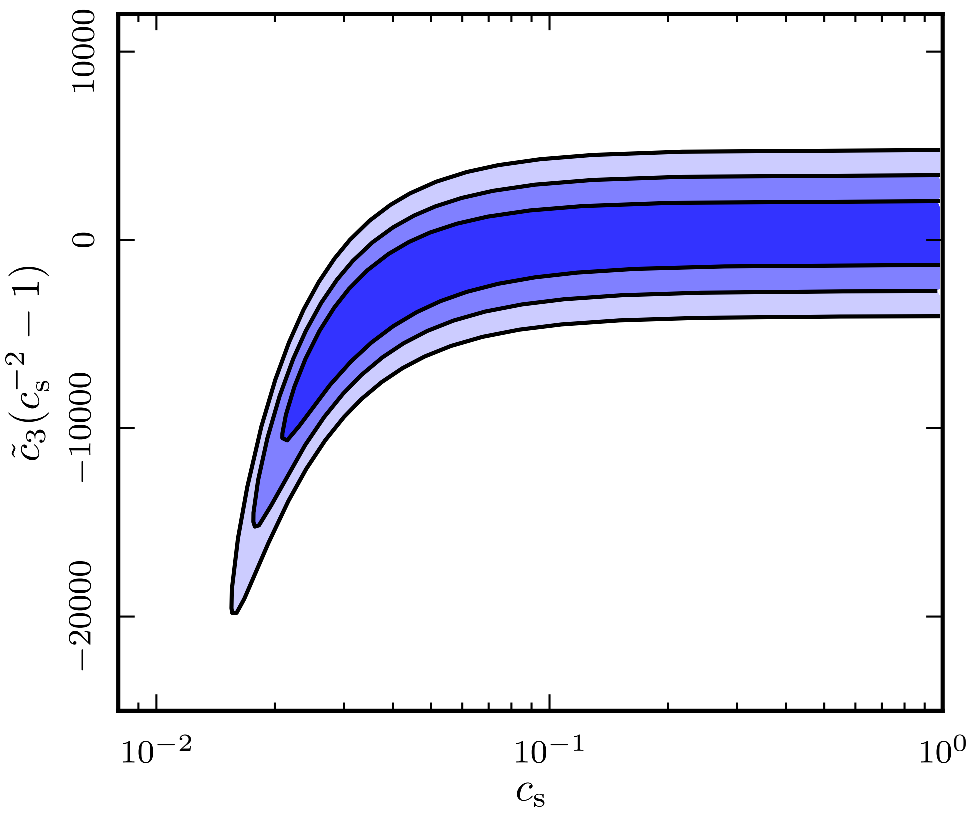
Implications for scale-invariant NG models

Planck Paper XXIV. Constraints on primordial non-Gaussianity

Equilateral and orthogonal shapes implications:

$$f_{\rm NL}^{\rm equil} = -42 \pm 75, \qquad f_{\rm NL}^{\rm ortho} = -25 \pm 39$$

- Effective field theory sound speed $c_s > 0.02$
- For DBI inflation sound speed $c_s > 0.07$
- Ultraviolet DBI models parameter $\beta < 0.7$
- Higher derivative models constrained
- Power law K-inflation ruled out (cf power spectrum)



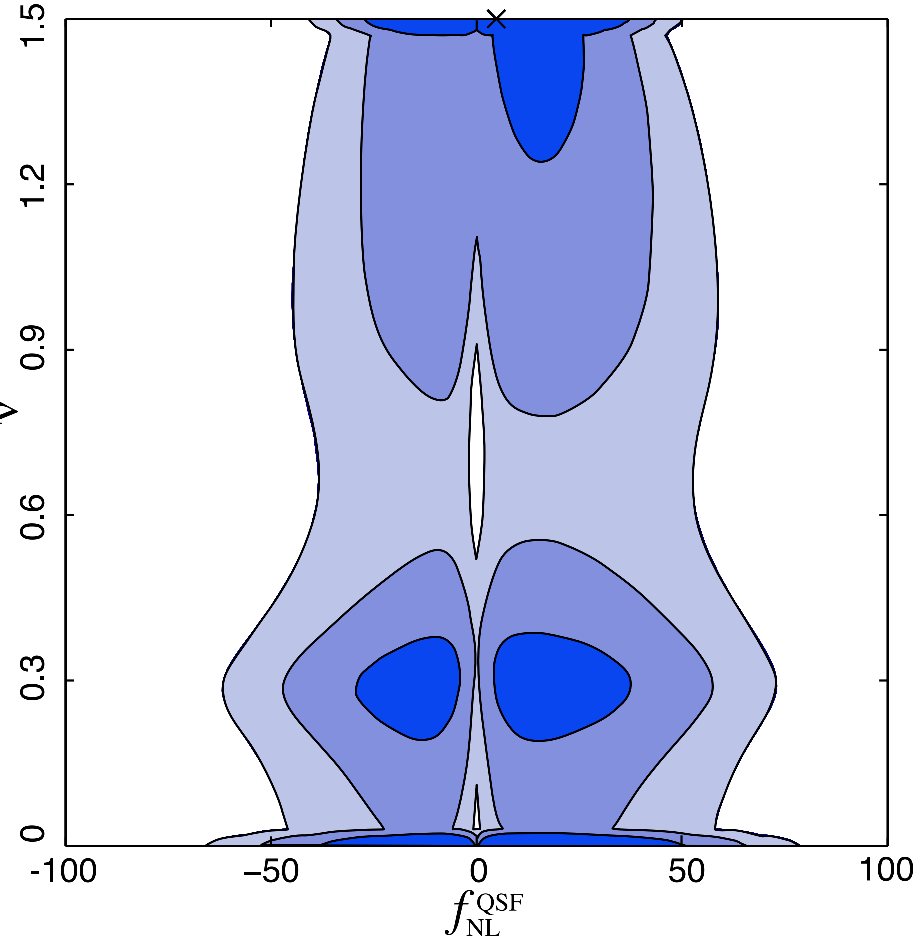
Local (squeezed) constraints:

$$f_{\rm NL}^{\rm local} = 2.7 \pm 5.8$$

- Curvaton model constraint on "decay fraction" $r_D > 0.15$
- Ekpyrotic/cyclic "conversion mechanism" ruled out

Local and equilateral in combination

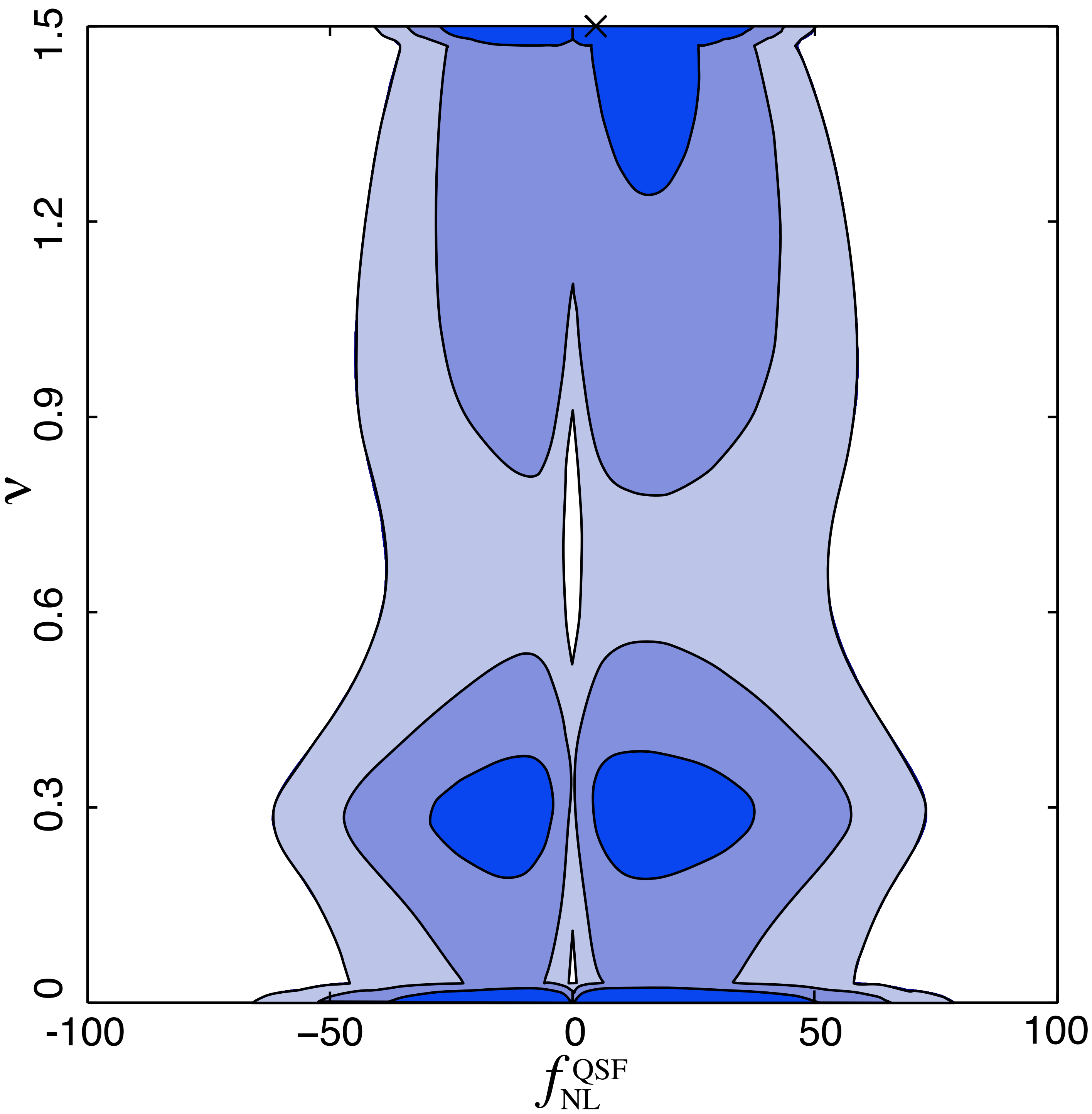
- Quasi-single-field inflation constrained ...



Quasi-Single field

$$B_{\Phi}^{\rm QSI}(k_1,k_2,k_3) = \frac{6A^2 f_{\rm NL}^{\rm QSI}}{(k_1k_2k_3)^{3/2}} \frac{3^{3/2}N_\nu[8k_1k_2k_3/(k_1+k_2+k_3)^3]}{N_\nu[8/27](k_1+k_2+k_3)^{3/2}}($$

Alpha were calculated for 150 values of ν and the Beta covariance matrix was used to produce 2 billion simulations around the measured value of ν and f_{NL} which were used to produce the likelihood plot



Non-separable bispectra

Specific *key* single-field models constrained

DBI inflation, effective field theory and higher derivative models ...

$$f_{\rm NL}^{\rm DBI} = 11 \pm 69$$

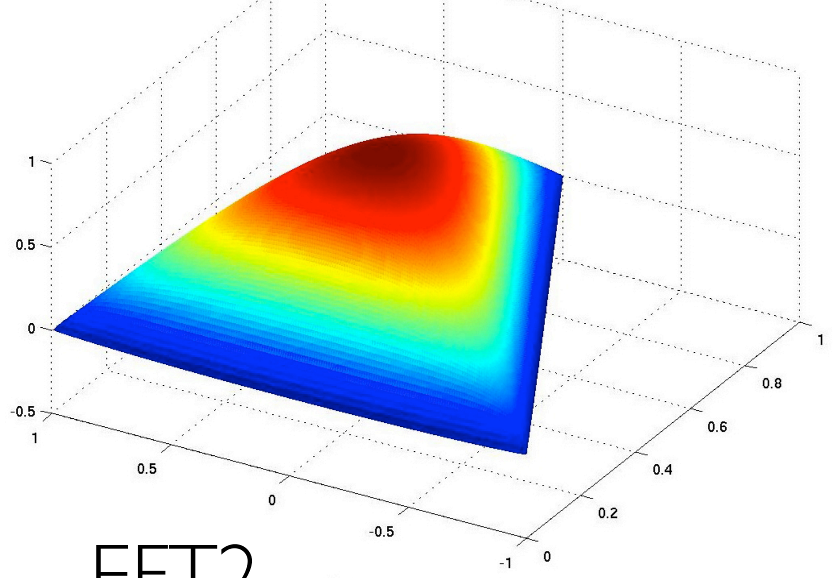
$$f_{\rm NL}^{\rm EFT1} = 8 \pm 73$$

$$f_{\rm NL}^{\rm EFT2} = 19 \pm 57$$

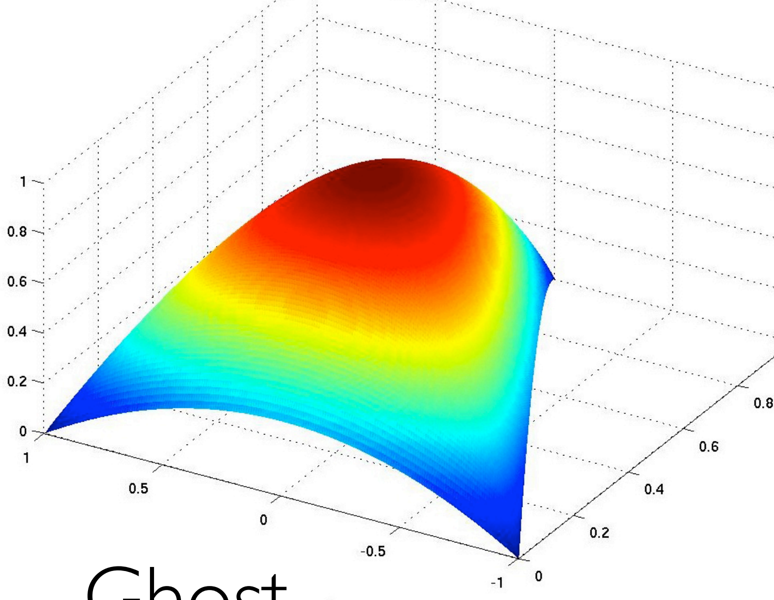
$$f_{\rm NL}^{\rm Ghost} = -23 \pm 88$$

Equilateral/orthogonal constraint on sound speed c$_s$ > 0.02.

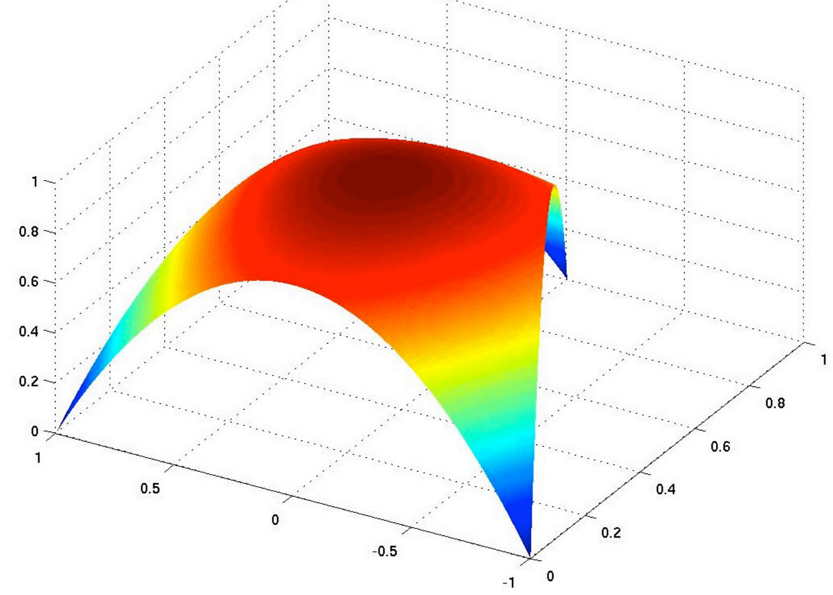
Equil



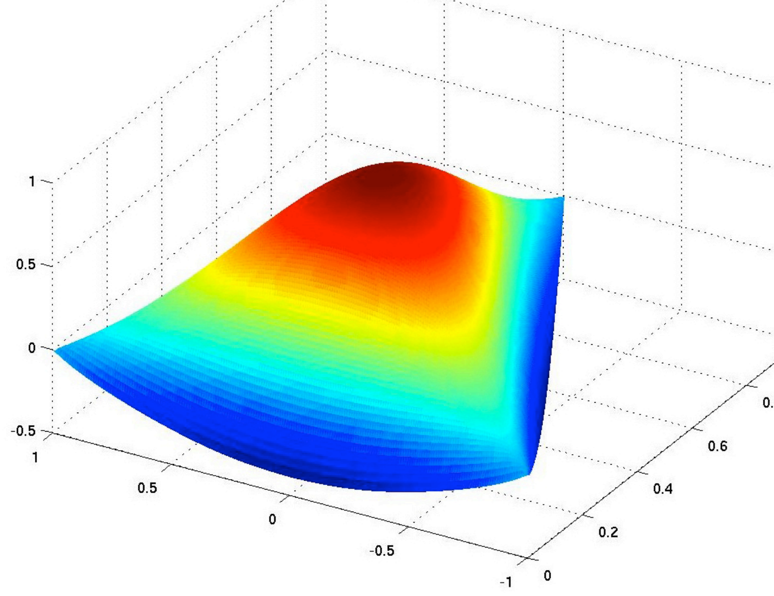
DBI



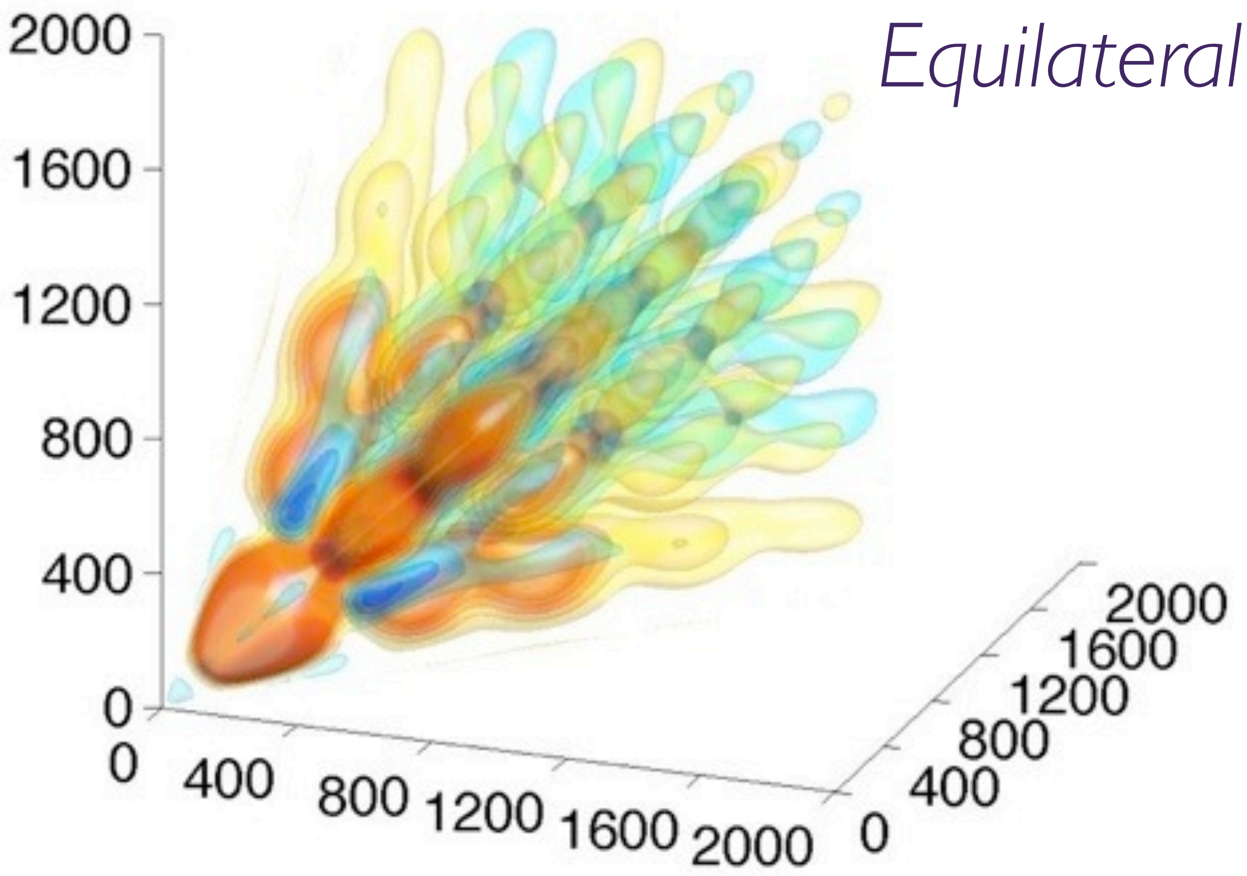
EFT2

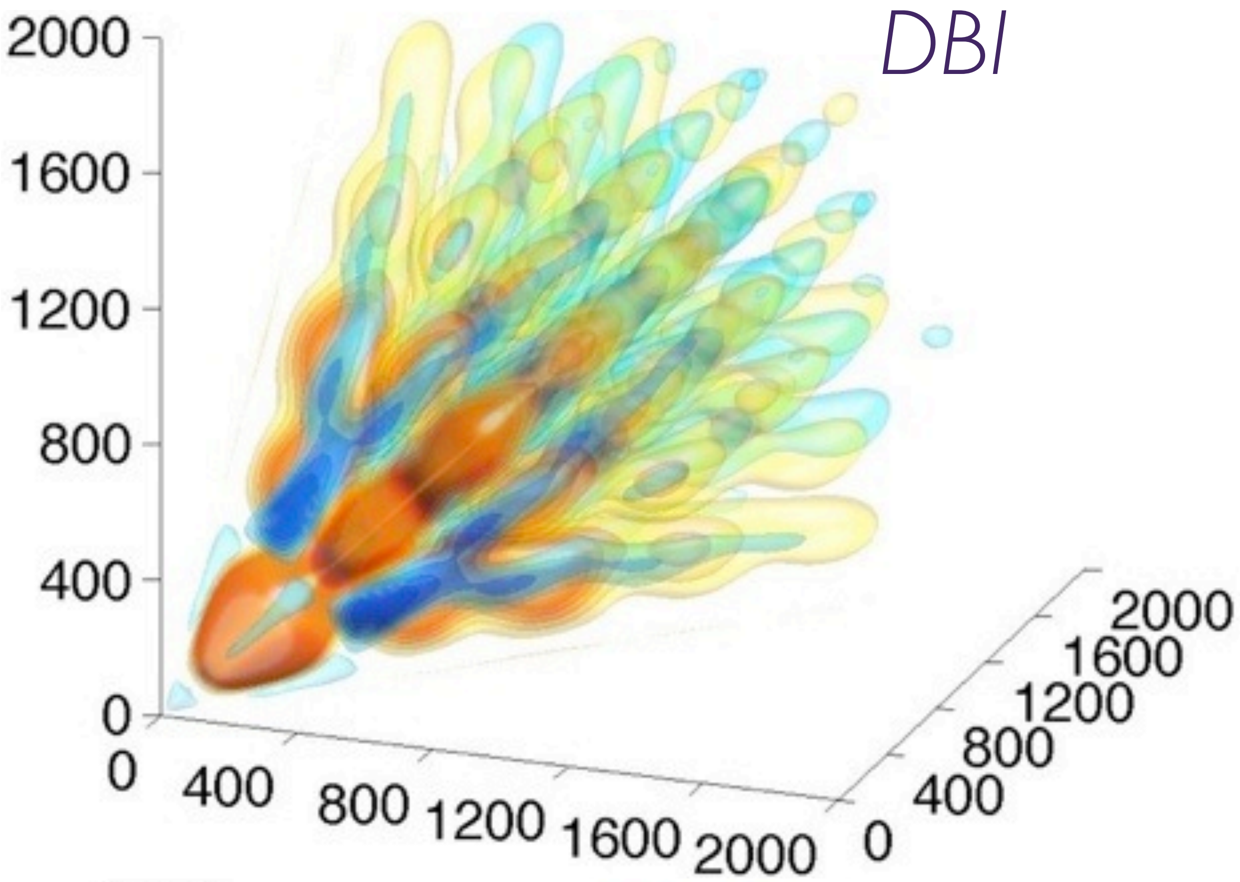


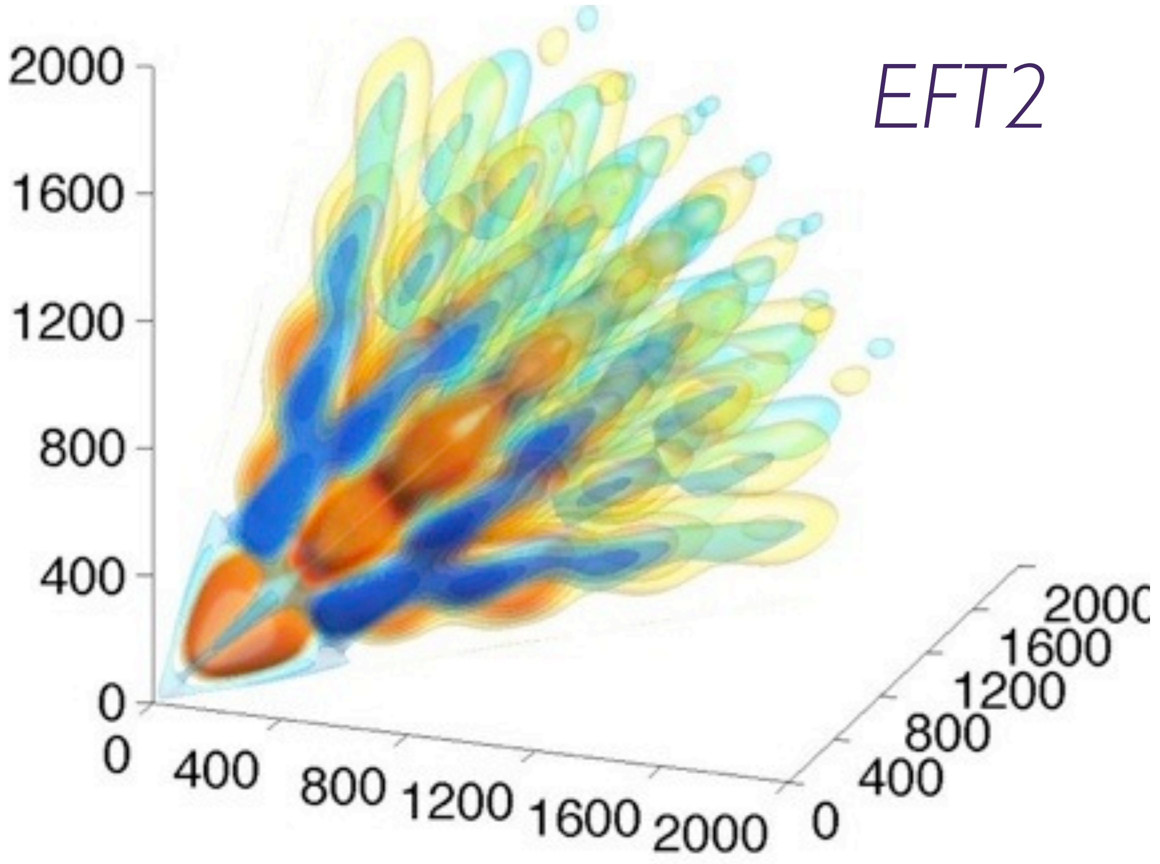
Ghost

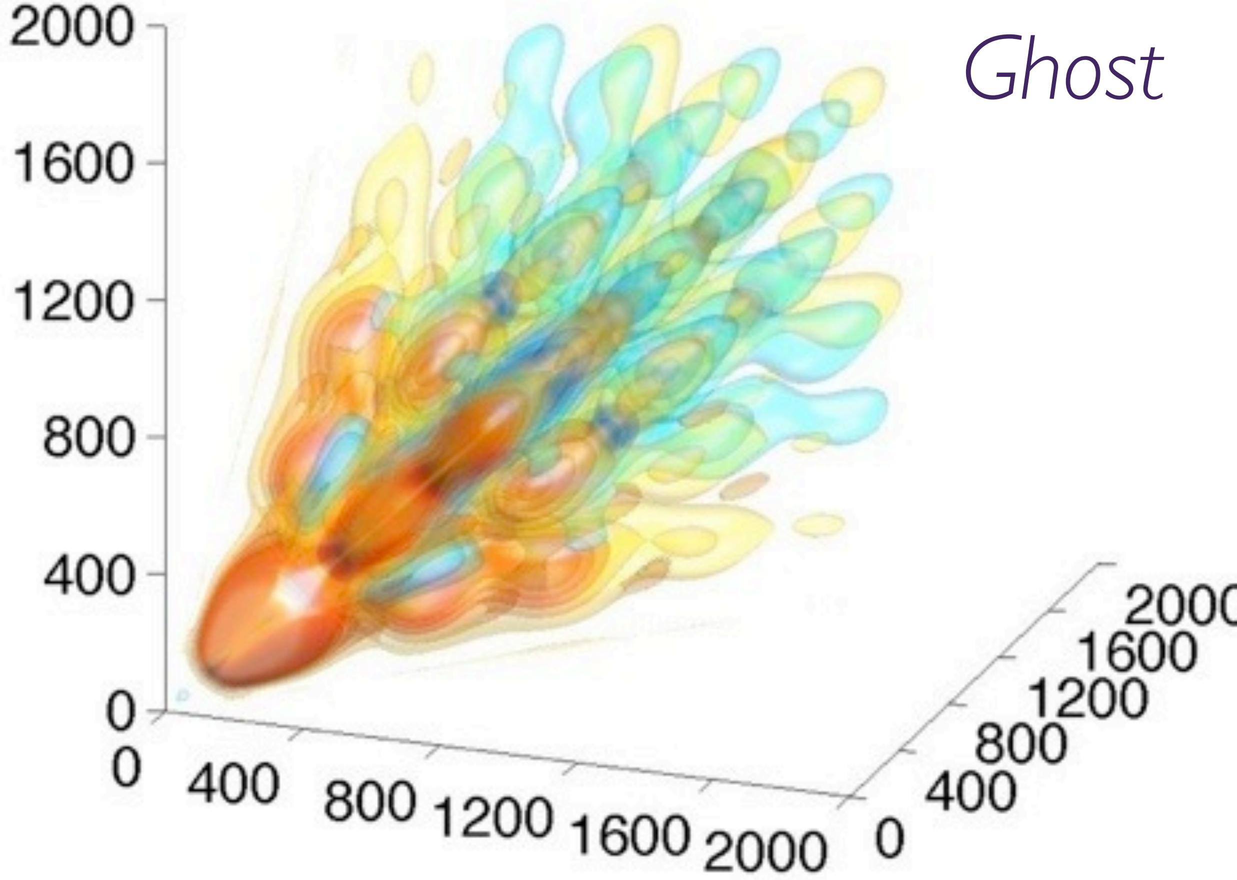


Equilateral shapes



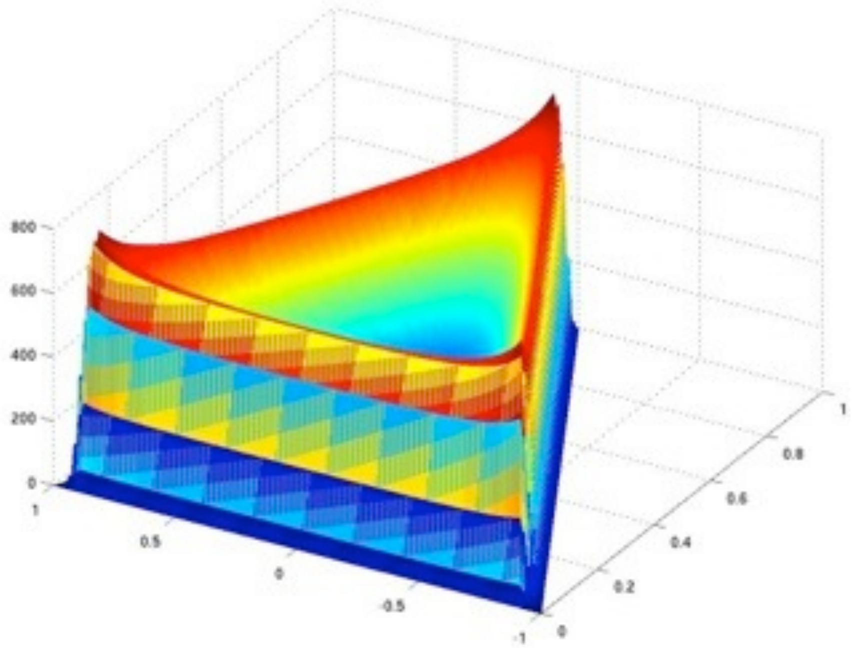


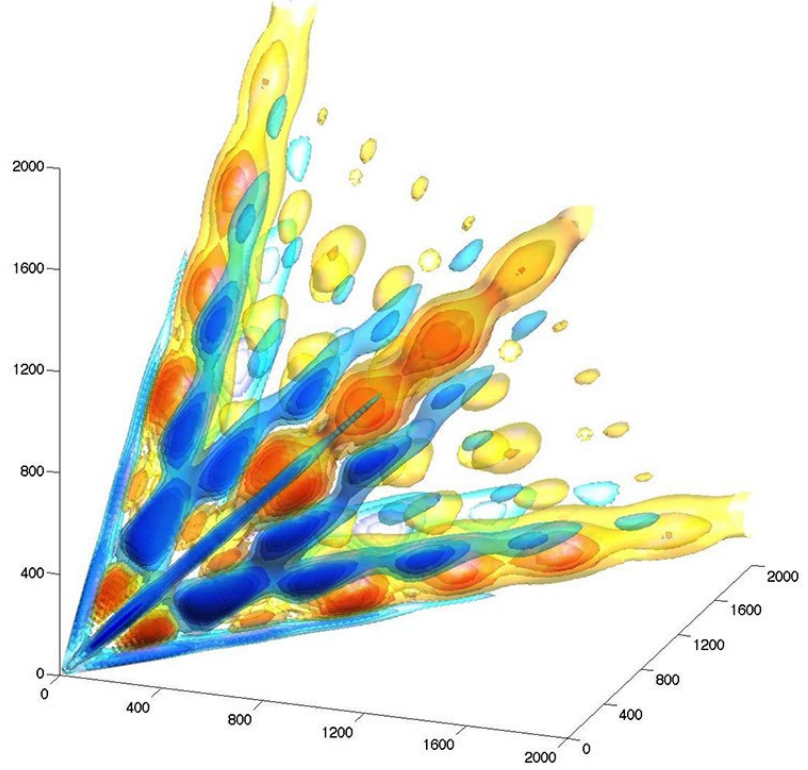




Excited Initial States

Non-Bunch-Davies vacua from trans-Planckian effects or features





Five exemplar flattened models constrained (plus vector models)

Flattened model (Eq. number)	Raw $f_{\rm NL}$	Clean $f_{\rm NL}$	$\Delta f_{\rm NL}$	σ	Clean σ
Flat model (13)	70	37	77	0.9	0.5
Non-Bunch-Davies (NBD)	178	155	78	2.2	2.0
Single-field NBD1 flattened (14)	31	19	13	2.4	1.4
Single-field NBD2 squeezed (14)	0.8	0.2	0.4	1.8	0.5
Non-canonical NBD3 (15)	13	9.6	9.7	1.3	1.0
Vector model $L = 1$ (19)	−18	−4.6	47	−0.4	−0.1
Vector model $L = 2$ (19)	2.8	−0.4	2.9	1.0	−0.1

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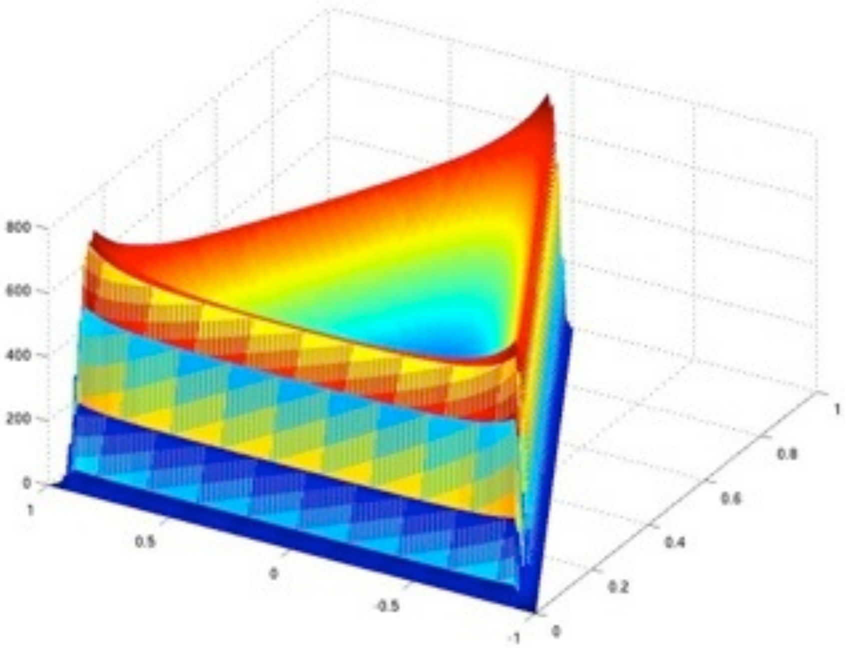

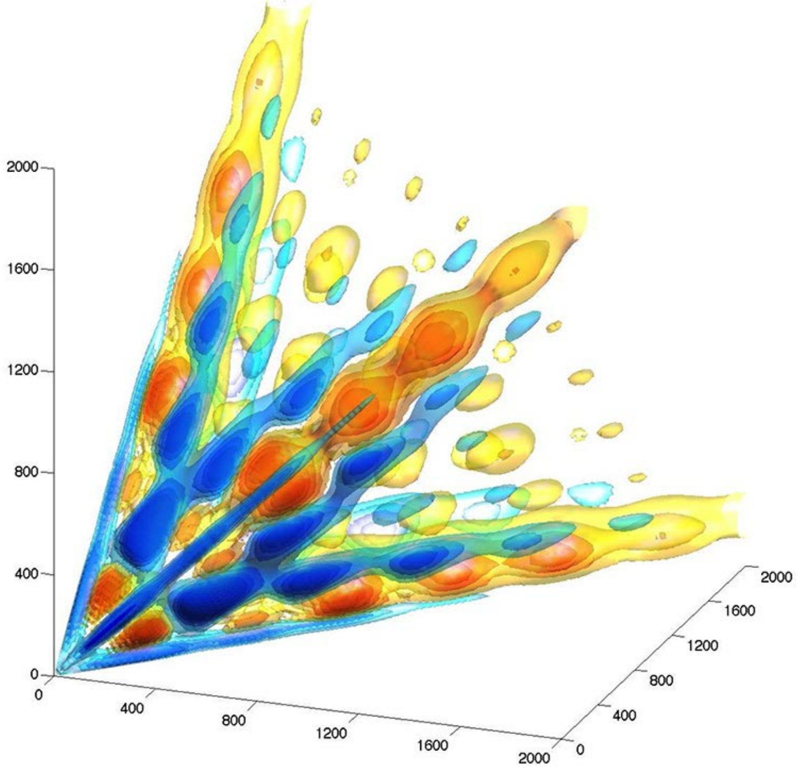

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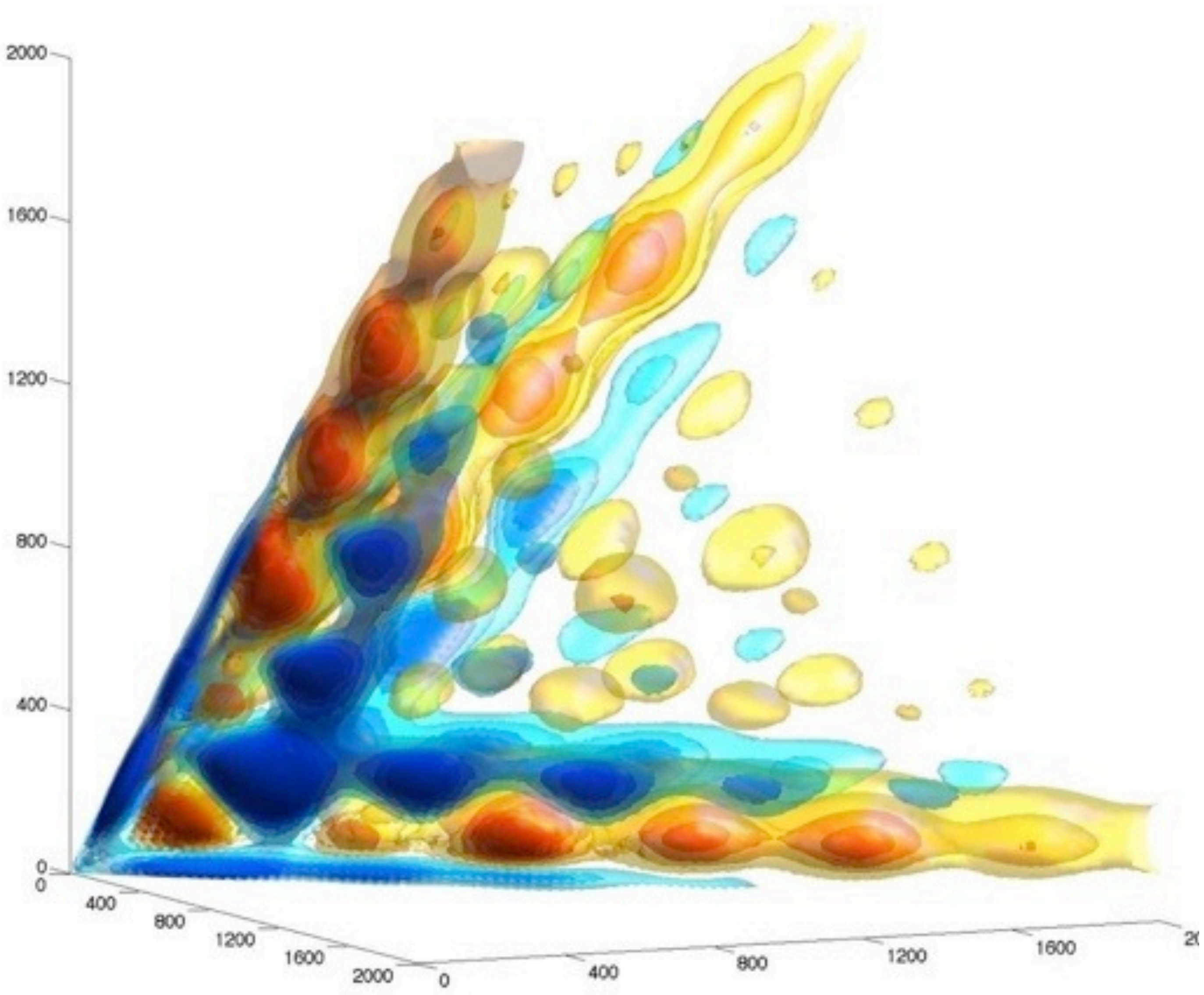
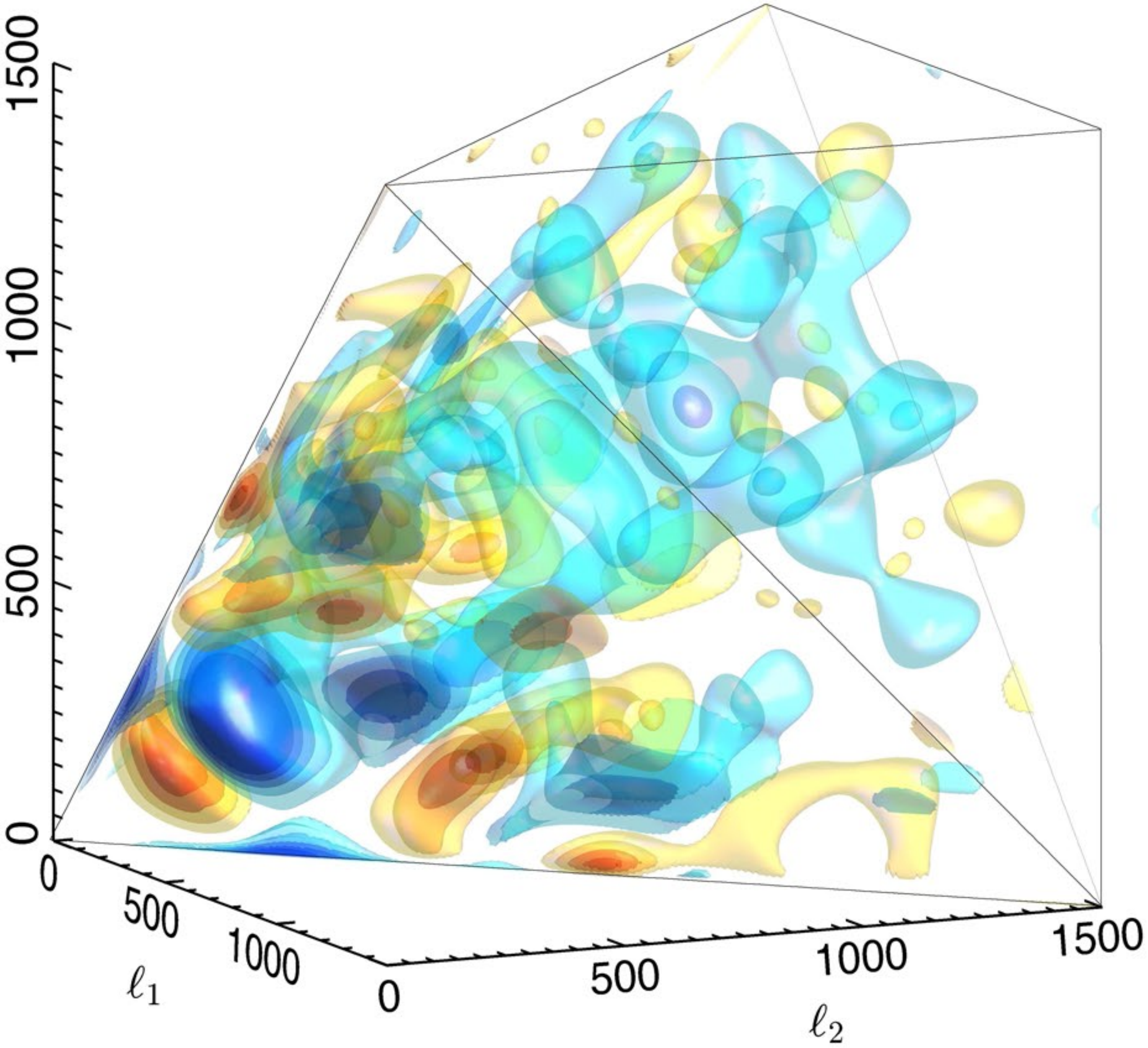
Non-Bunch-Davies vs Planck

Comparison/similarities of non-Bunch-Davies and Planck bispectra





Vector Inflation/Warm inflation

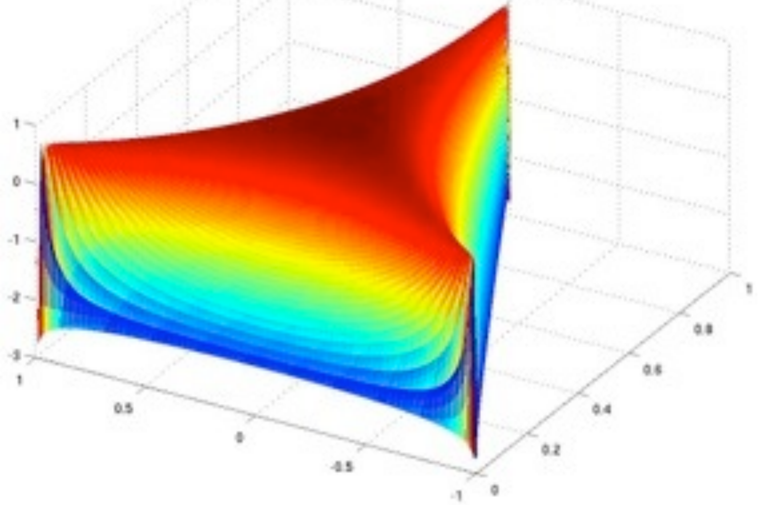
Inflation with gauge/vector fields can have non-trivial directional dependencies

$$B_\Phi(k_1, k_2, k_3) = \sum_L c_L[P_L(\mu_{12})P_\Phi(k_1)P_\Phi(k_2) + 2\,\mathrm{perm}].$$

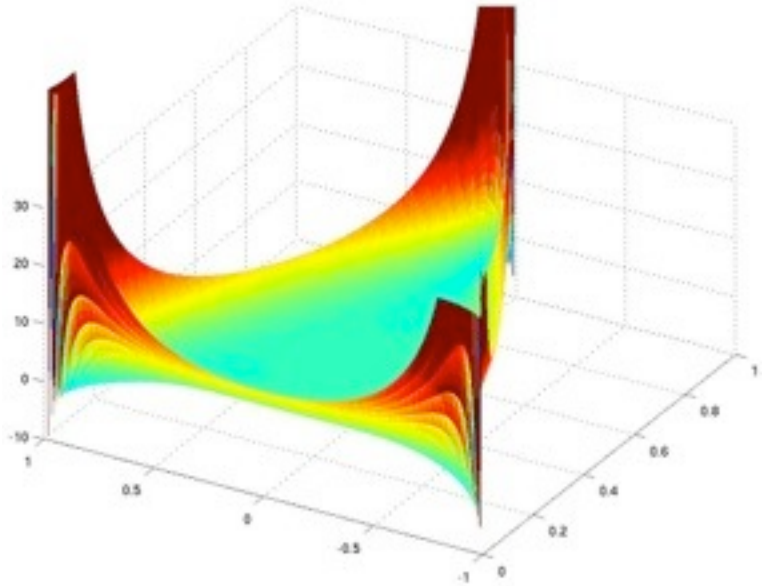
(see e.g. Shiraishi et al, 2012)

Similarly 'twisted' bispectrum for warm inflation

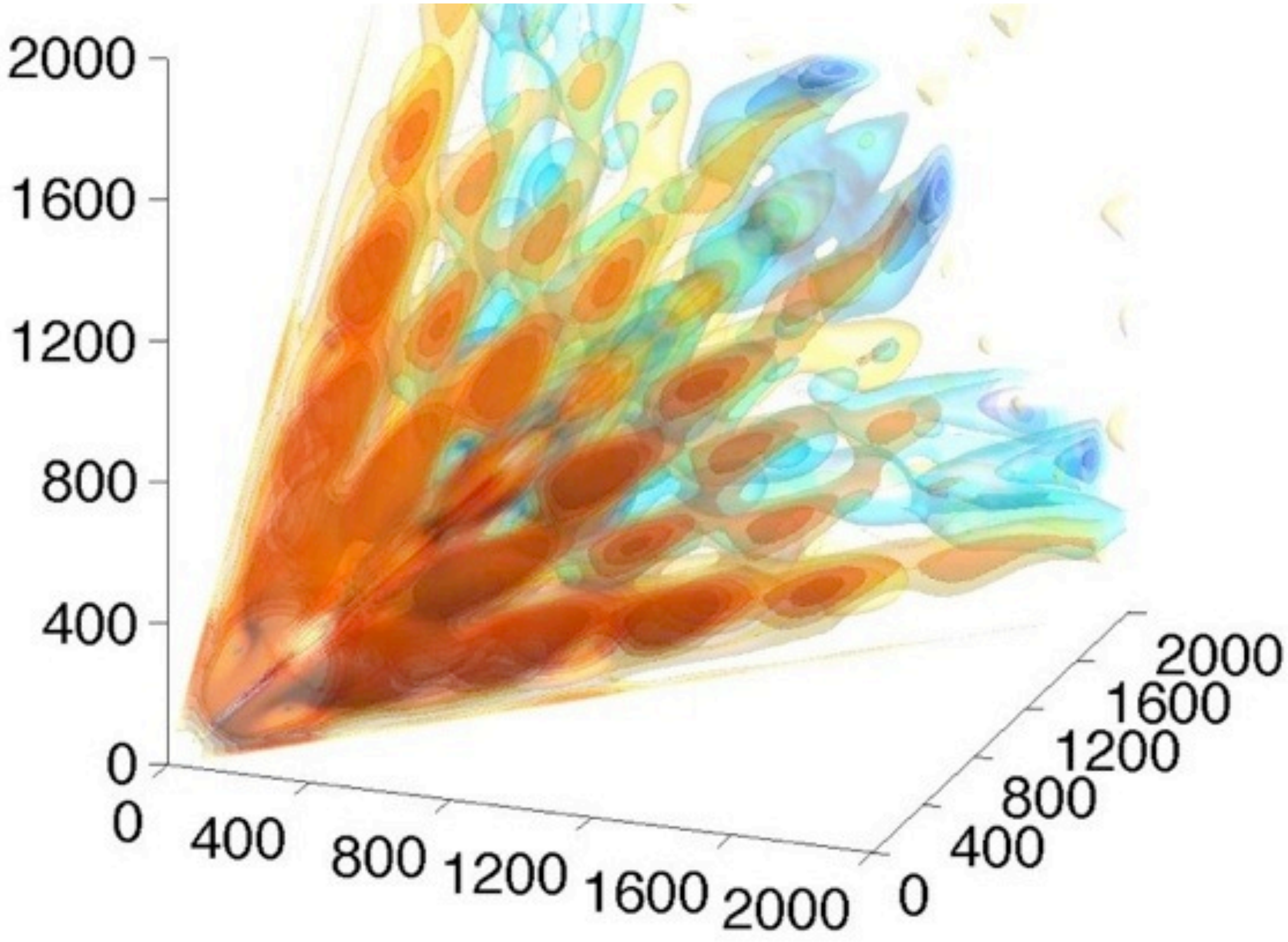
No directional evidence but modal correlation could be improved ...



L=1



L=2



Feature models

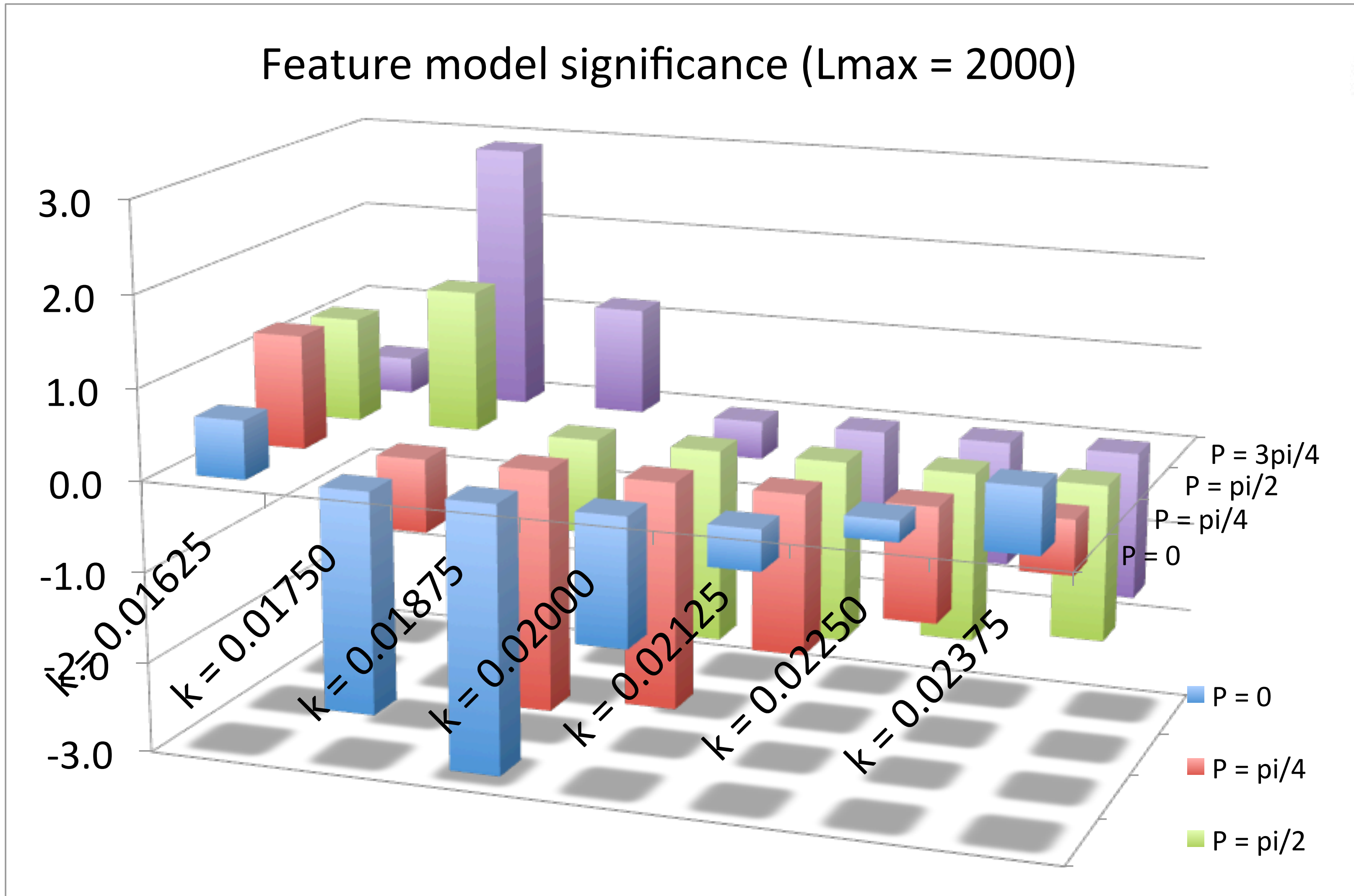
Inflaton potential can have a feature which disturbs slow-roll:

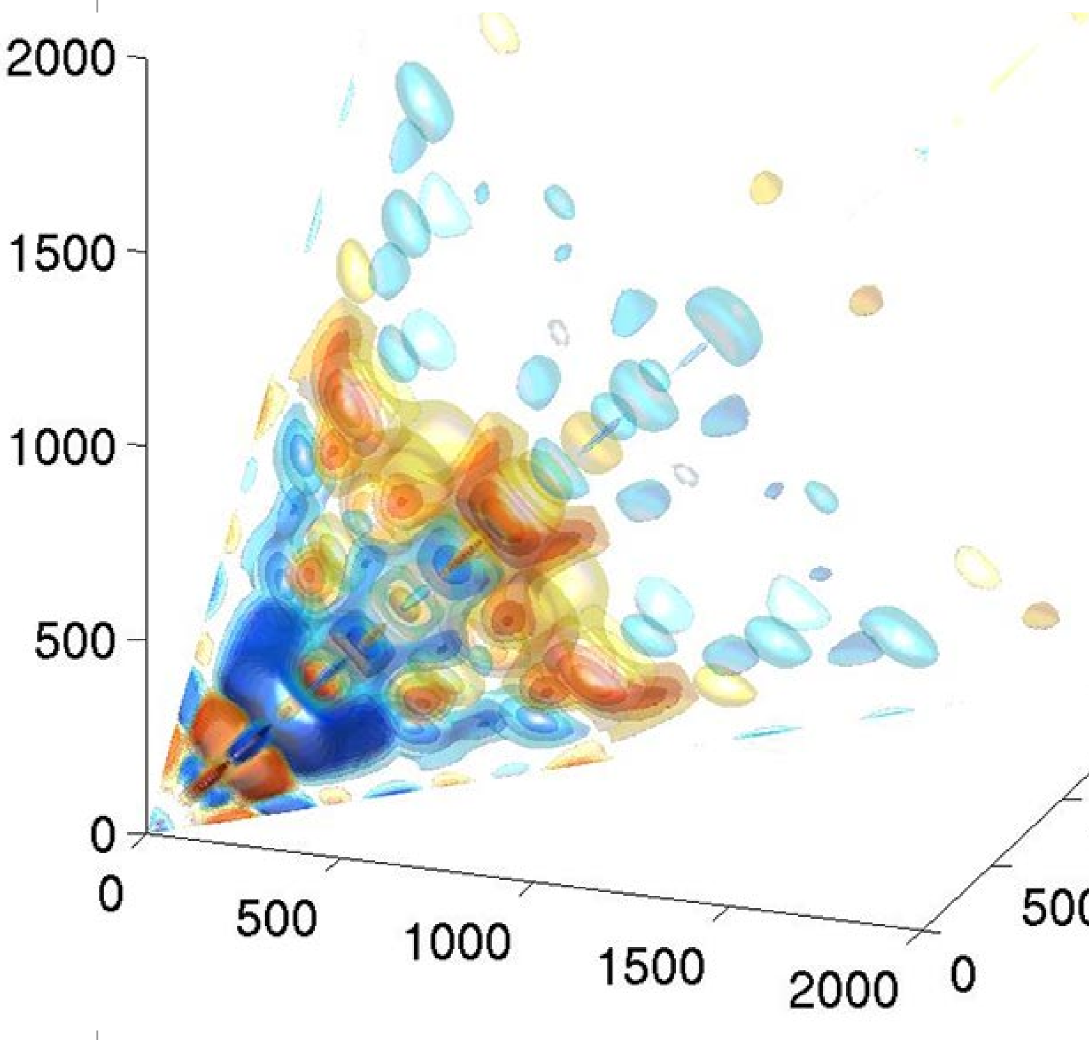
$$B_{\Phi}^{\mathrm{feat}}(k_1,k_2,k_3) = \frac{6A^2 f_{\mathrm{NL}}^{\mathrm{feat}}}{(k_1k_2k_3)^2}\sin\left[\frac{2\pi(k_1+k_2+k_3)}{3k_{\mathrm{c}}}+\phi\right]$$

(Chen et al, 2007)

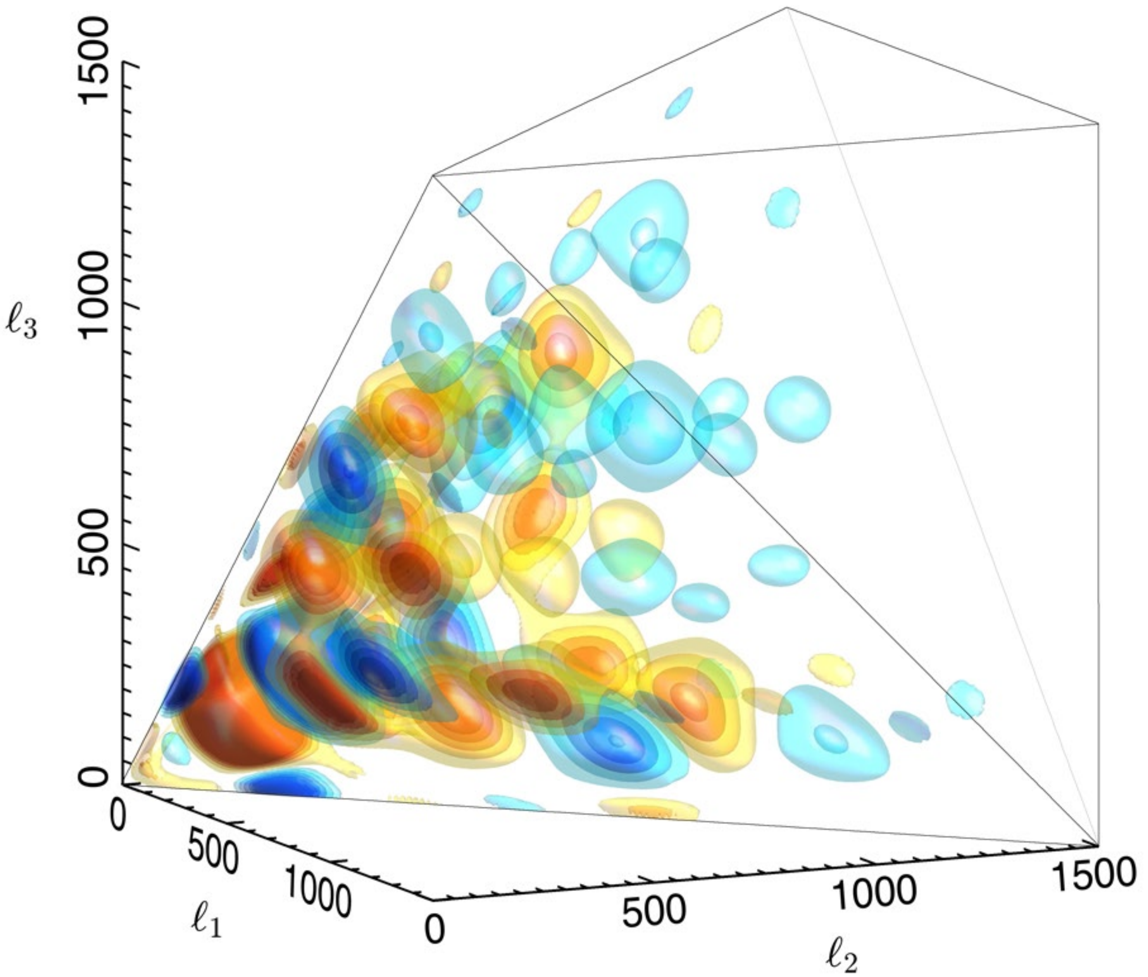
Can match the observed "oscillatory" signal in the Planck bispectrum (consistent with WMAP results)

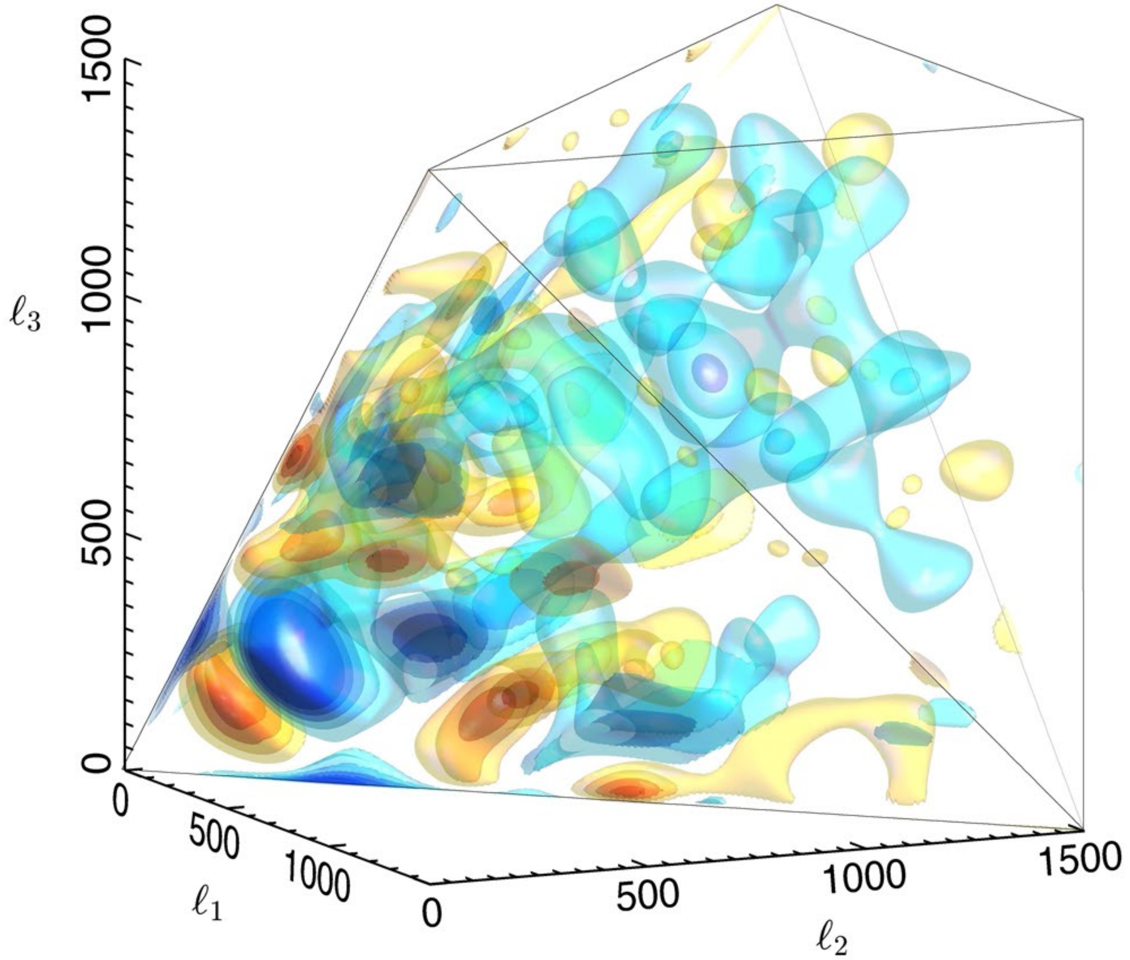
Initial two-parameter survey only (k_c, **Φ**) ..





Feature envelope best-fit





Width Model	$\Delta k = 0.015$ $f_{\rm NL} \pm \Delta f_{\rm NL}$ (σ)	$\Delta k = 0.03$ $f_{\rm NL} \pm \Delta f_{\rm NL}$ (σ)	$\Delta k = 0.045$ $f_{\rm NL} \pm \Delta f_{\rm NL}$ (σ)	Full $f_{\rm NL} \pm \Delta f_{\rm NL}$ (σ)
$k_c = 0.01125$; $\phi = 0$.	765 ± 275 (2.8)	703 ± 241 (2.9)	648 ± 218 (3.0)	434 ± 170 (2.6)
$k_c = 0.01750$; $\phi = 0$.	-661 ± 234 (−2.8)	-494 ± 192 (−2.6)	-425 ± 171 (−2.5)	-335 ± 137 (−2.4)
$k_c = 0.01750$; $\phi = 3\pi/4$	399 ± 207 (1.9)	438 ± 183 (2.4)	442 ± 165 (2.7)	366 ± 126 (2.9)
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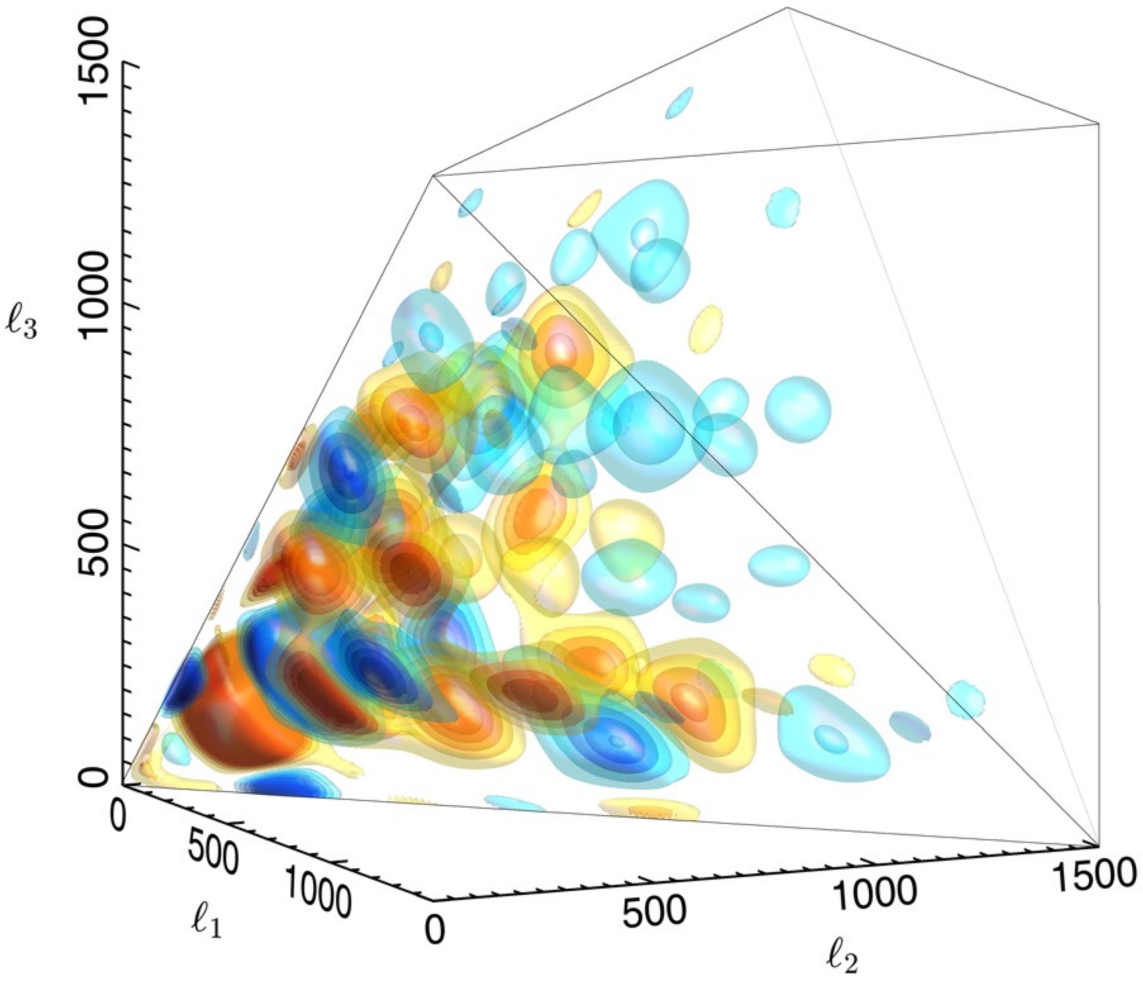
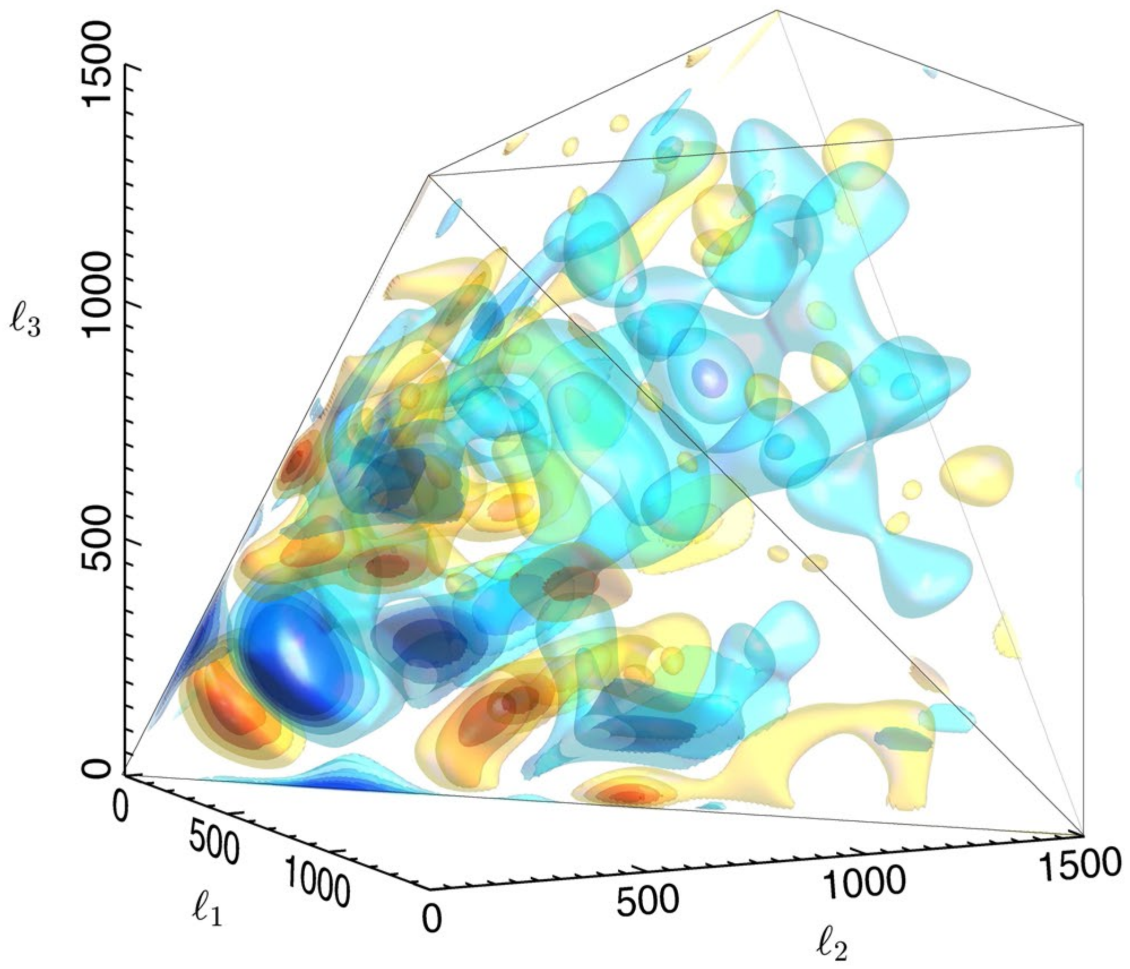
$k_c = 0.02000$; $\phi = \pi/4$	-665 ± 229 (−2.9)	-593 ± 185 (−3.2)	-500 ± 160 (−3.1)	-298 ± 119 (−2.5)

Intriguing 'hints' - but single oscillation 'look elsewhere' effect' analysis < 2σ
Counterparts in power spectrum (initial Planck analysis absent) - ongoing ...

Feature envelope best-fit





Width	$\Delta k = 0.015$	$\Delta k = 0.03$	$\Delta k = 0.045$	Full
Model	$f_{\rm NL} \pm \Delta f_{\rm NL}$ (σ)	$f_{\rm NL} \pm \Delta f_{\rm NL}$ (σ)	$f_{\rm NL} \pm \Delta f_{\rm NL}$ (σ)	$f_{\rm NL} \pm \Delta f_{\rm NL}$ (σ)
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Resonance and NBD Features

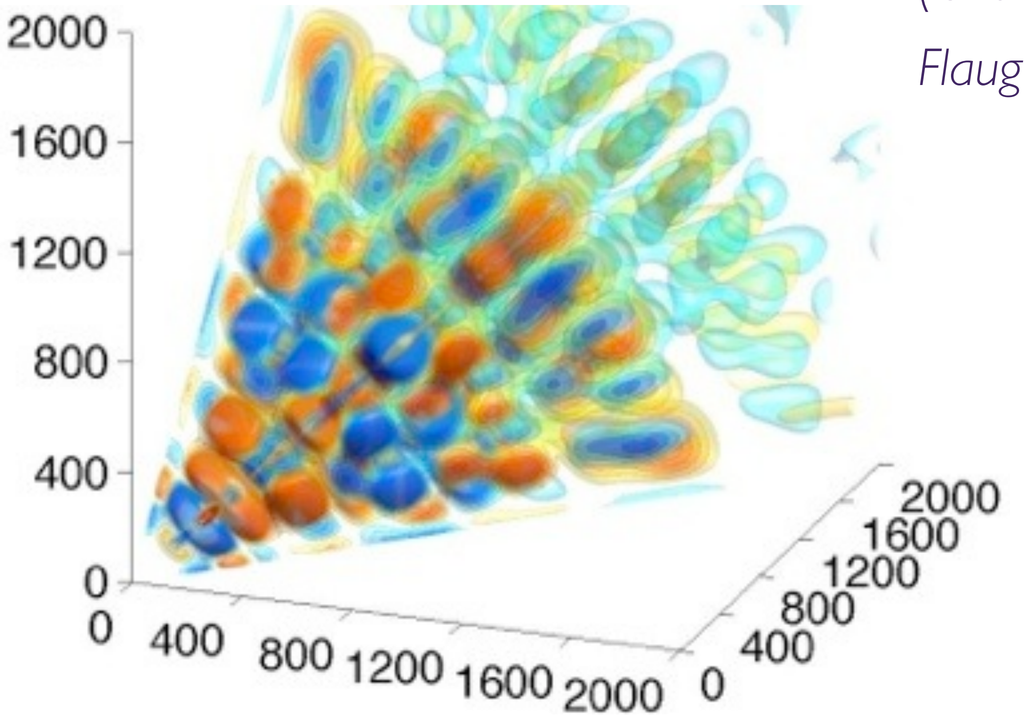
Feature models

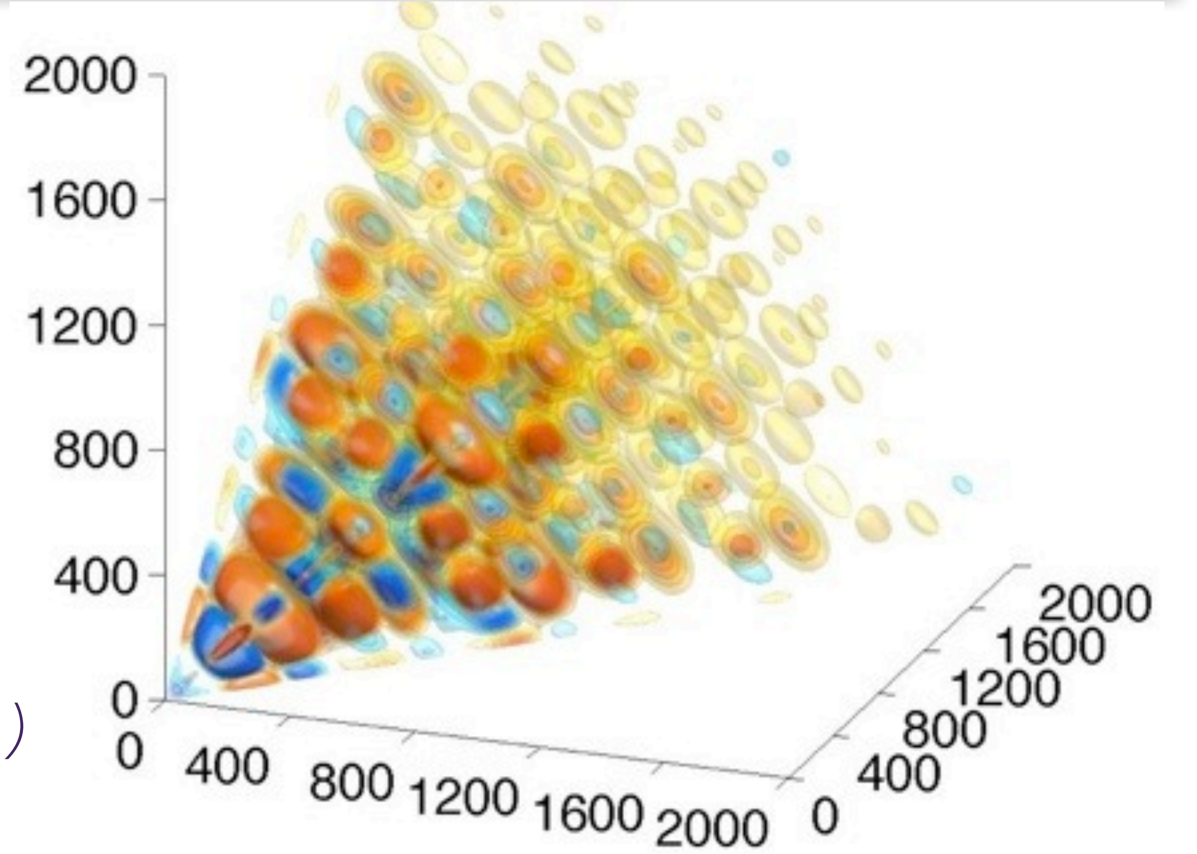
$$B_{\Phi}^{\text{feat}}(k_1,k_2,k_3) = \frac{6A^2 f_{\text{NL}}^{\text{feat}}}{(k_1k_2k_3)^2}\sin\left[\frac{2\pi(k_1+k_2+k_3)}{3k_{\text{c}}}+\phi\right],$$

Resonance models (e.g. axion monodromy)

$$B_{\Phi}^{\text{res}}(k_1,k_2,k_3) = \frac{6A^2 f_{\text{NL}}^{\text{res}}}{(k_1k_2k_3)^2}\sin\left[C\ln(k_1+k_2+k_3)+\phi\right],$$

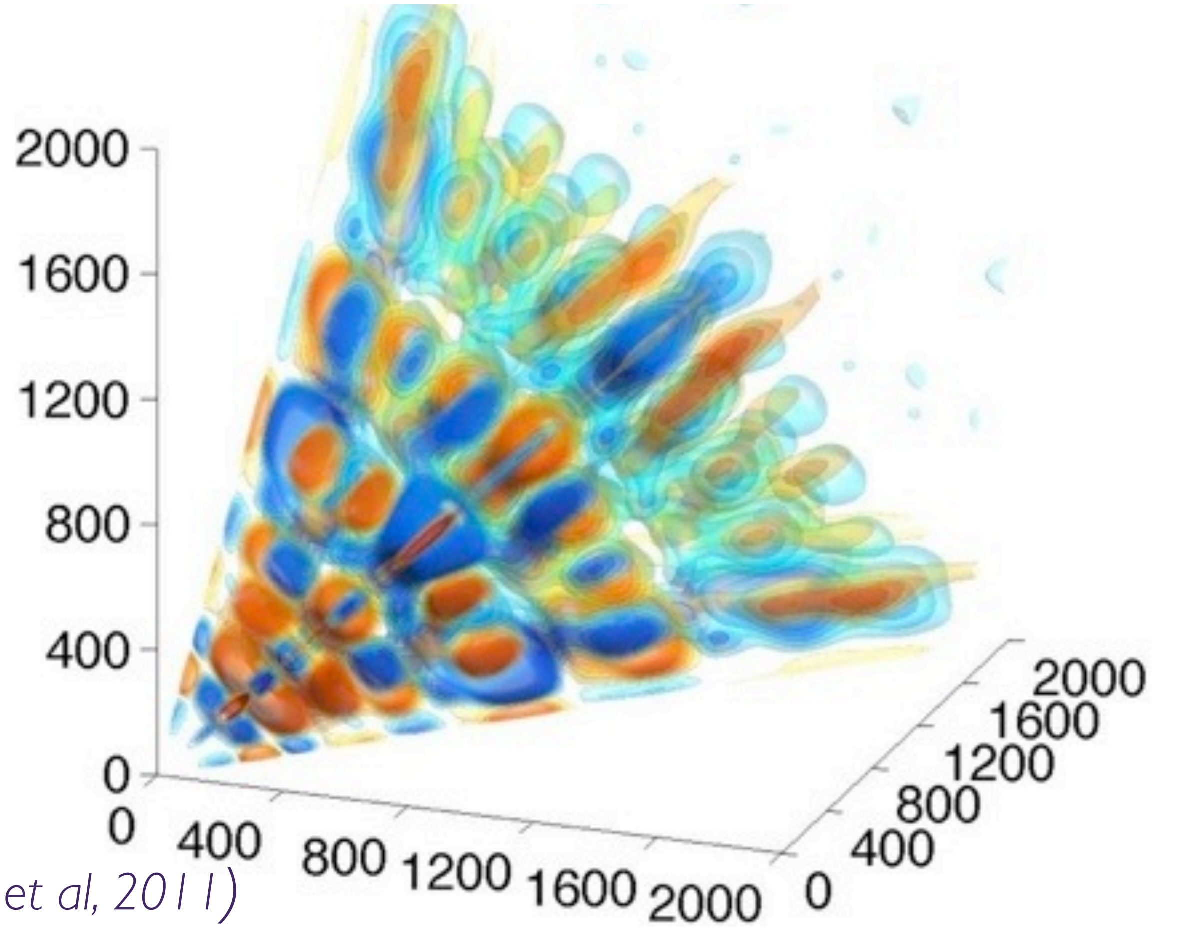
(Chen et al, 2008, Flauger & Pajer 2011)





Enfolded resonance models

$$B_{\Phi}^{\text{resNBD}}(k_1,k_2,k_3) = \frac{2A^2 f_{\text{NL}}^{\text{resNBD}}}{(k_1k_2k_3)^2}\Big\{\exp(-k_{\text{c}}^{3/5}(k_2+k_3-k_1)/2k_1)$$
$$\times\sin[k_{\text{c}}((k_2+k_3-k_1)/2k_1+\ln k_1)+\phi]+2\text{ perm.}\Big\}.\quad(18)$$



(Chen et al, 2011)

Resonance and NBD Features

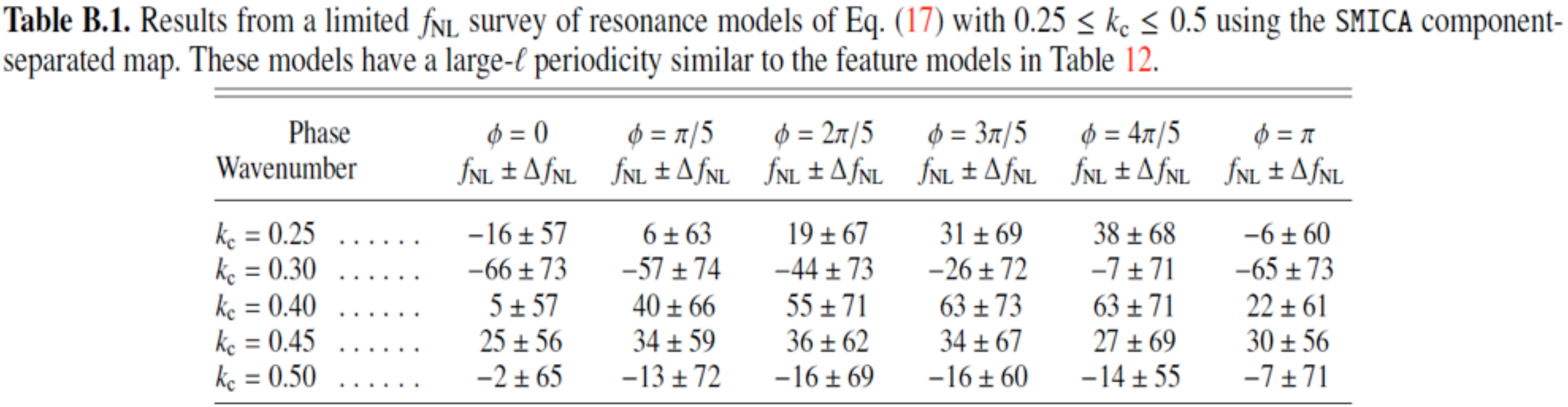


Table B.1. Results from a limited $f_{\rm NL}$ survey of resonance models of Eq. (17) with $0.25 \le k_{\rm c} \le 0.5$ using the `SMICA` component-separated map. These models have a large-ℓ periodicity similar to the feature models in Table 12.

Phase Wavenumber	$\phi = 0$ $f_{\rm NL} \pm \Delta f_{\rm NL}$	$\phi = \pi/5$ $f_{\rm NL} \pm \Delta f_{\rm NL}$	$\phi = 2\pi/5$ $f_{\rm NL} \pm \Delta f_{\rm NL}$	$\phi = 3\pi/5$ $f_{\rm NL} \pm \Delta f_{\rm NL}$	$\phi = 4\pi/5$ $f_{\rm NL} \pm \Delta f_{\rm NL}$	$\phi = \pi$ $f_{\rm NL} \pm \Delta f_{\rm NL}$
$k_{\rm c} = 0.25$	-16 ± 57	6 ± 63	19 ± 67	31 ± 69	38 ± 68	-6 ± 60
$k_{\rm c} = 0.30$	-66 ± 73	-57 ± 74	-44 ± 73	-26 ± 72	-7 ± 71	-65 ± 73
$k_{\rm c} = 0.40$	5 ± 57	40 ± 66	55 ± 71	63 ± 73	63 ± 71	22 ± 61
$k_{\rm c} = 0.45$	25 ± 56	34 ± 59	36 ± 62	34 ± 67	27 ± 69	30 ± 56
$k_{\rm c} = 0.50$	-2 ± 65	-13 ± 72	-16 ± 69	-16 ± 60	-14 ± 55	-7 ± 71

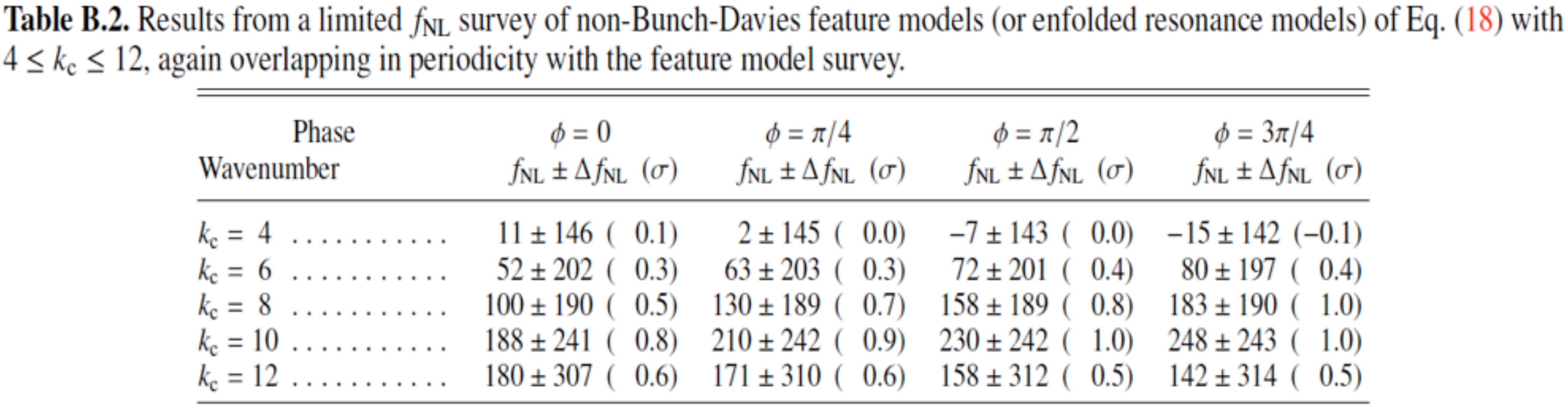
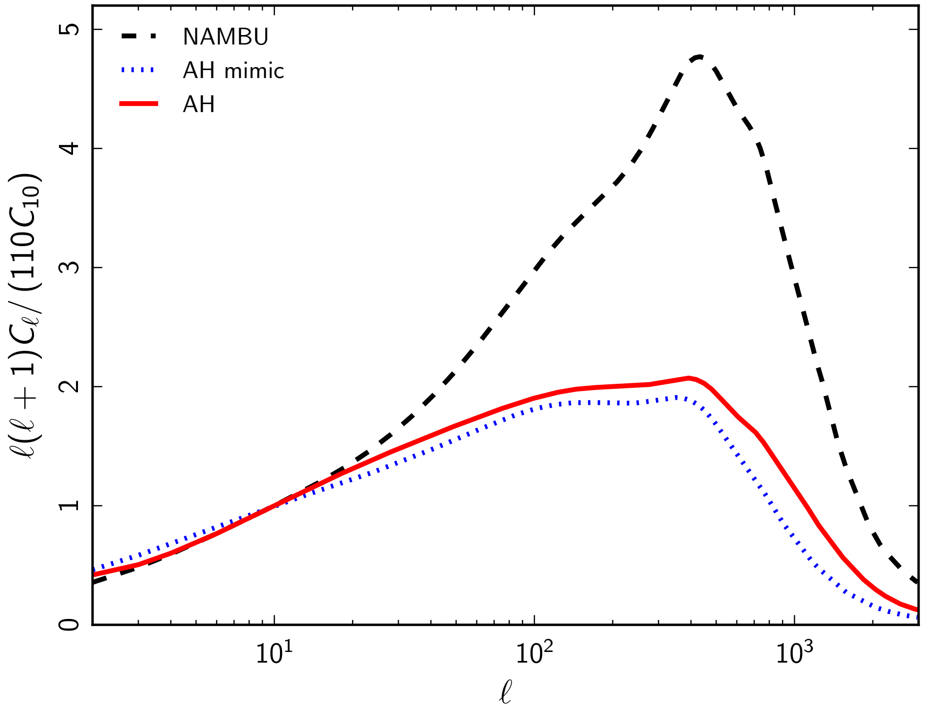


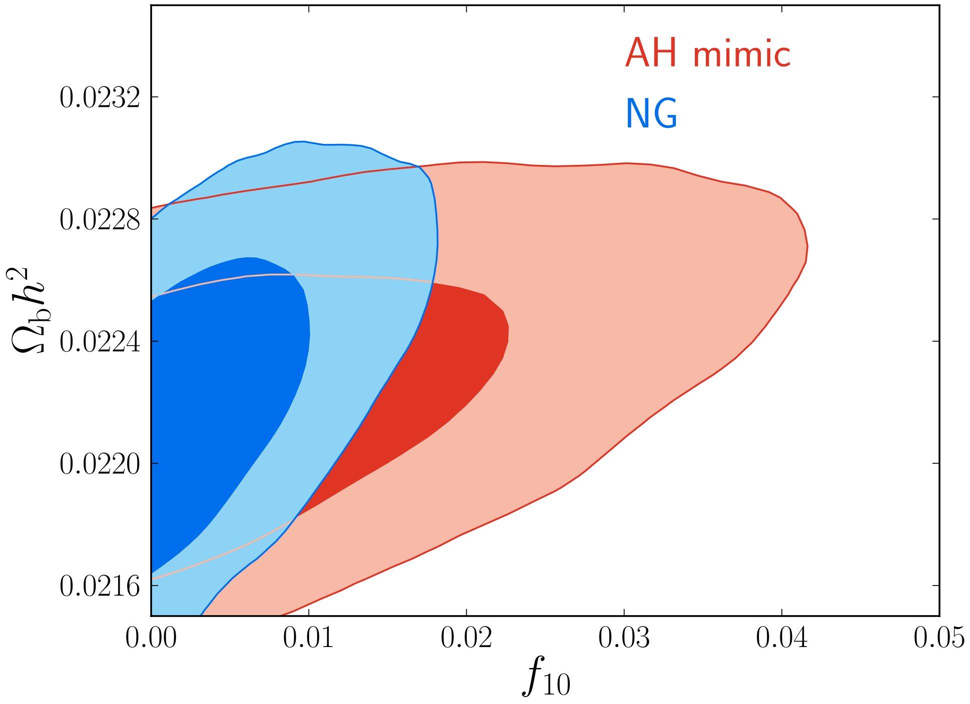
Table B.2. Results from a limited $f_{\rm NL}$ survey of non-Bunch-Davies feature models (or enfolded resonance models) of Eq. (18) with $4 \le k_{\rm c} \le 12$, again overlapping in periodicity with the feature model survey.

Phase Wavenumber	$\phi = 0$ $f_{\rm NL} \pm \Delta f_{\rm NL}$ (σ)	$\phi = \pi/4$ $f_{\rm NL} \pm \Delta f_{\rm NL}$ (σ)	$\phi = \pi/2$ $f_{\rm NL} \pm \Delta f_{\rm NL}$ (σ)	$\phi = 3\pi/4$ $f_{\rm NL} \pm \Delta f_{\rm NL}$ (σ)
$k_{\rm c} = 4$	11 ± 146 (0.1)	2 ± 145 (0.0)	-7 ± 143 (0.0)	-15 ± 142 (−0.1)
$k_{\rm c} = 6$	52 ± 202 (0.3)	63 ± 203 (0.3)	72 ± 201 (0.4)	80 ± 197 (0.4)
$k_{\rm c} = 8$	100 ± 190 (0.5)	130 ± 189 (0.7)	158 ± 189 (0.8)	183 ± 190 (1.0)
$k_{\rm c} = 10$	188 ± 241 (0.8)	210 ± 242 (0.9)	230 ± 242 (1.0)	248 ± 243 (1.0)
$k_{\rm c} = 12$	180 ± 307 (0.6)	171 ± 310 (0.6)	158 ± 312 (0.5)	142 ± 314 (0.5)

String Constraints

Stringent new constraints on cosmic strings and global textures





Defect type . . .	*Planck*+WP		*Planck*+WP+highL	
. . . .	f_{10}	$G\mu/c^2$	f_{10}	$G\mu/c^2$
NAMBU	0.015	1.5×10^{-7}	0.010	1.3×10^{-7}
AH-mimic	0.033	3.6×10^{-7}	0.034	3.7×10^{-7}
AH	0.028	3.2×10^{-7}	0.024	3.0×10^{-7}
SL	0.043	11.0×10^{-7}	0.041	10.7×10^{-7}
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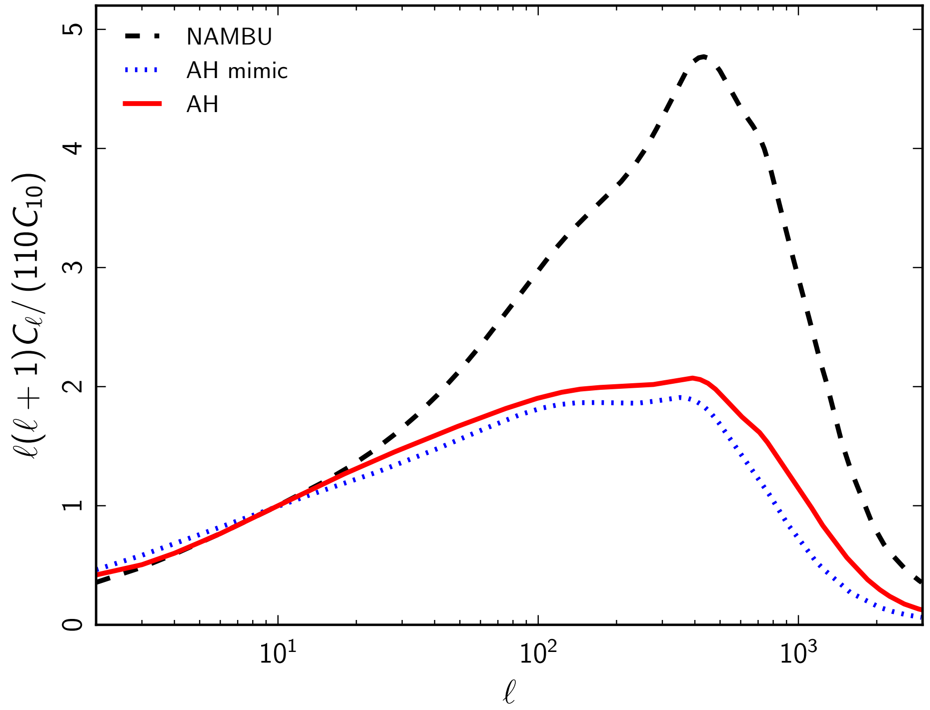

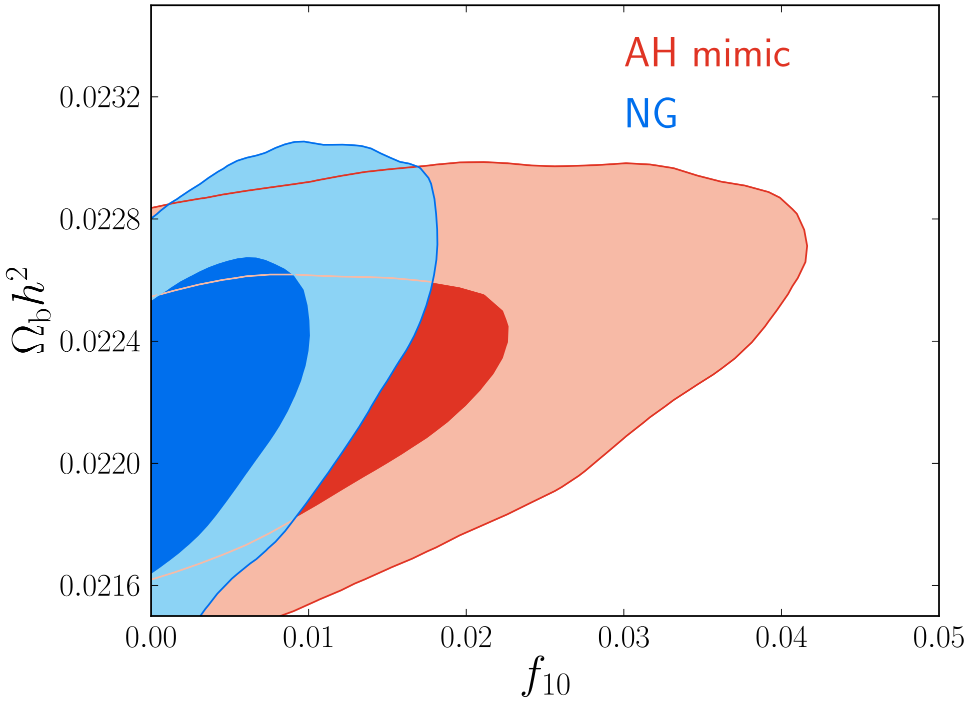

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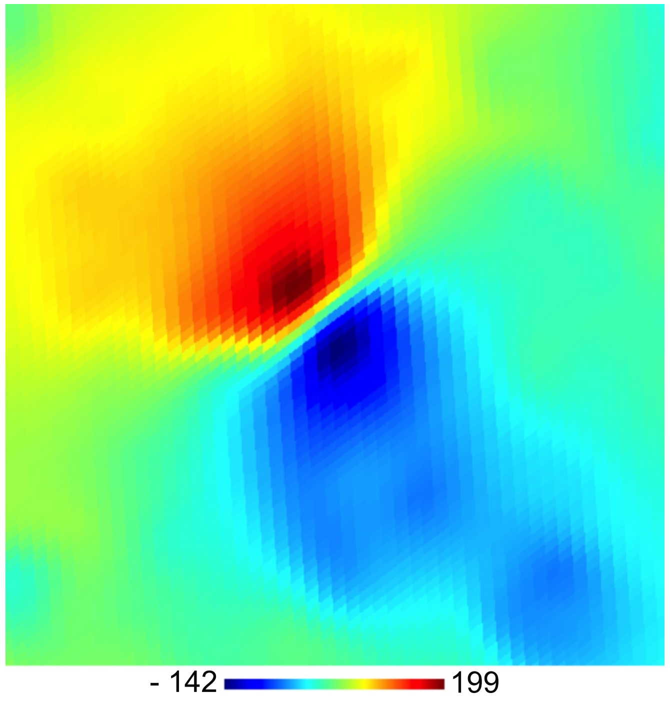
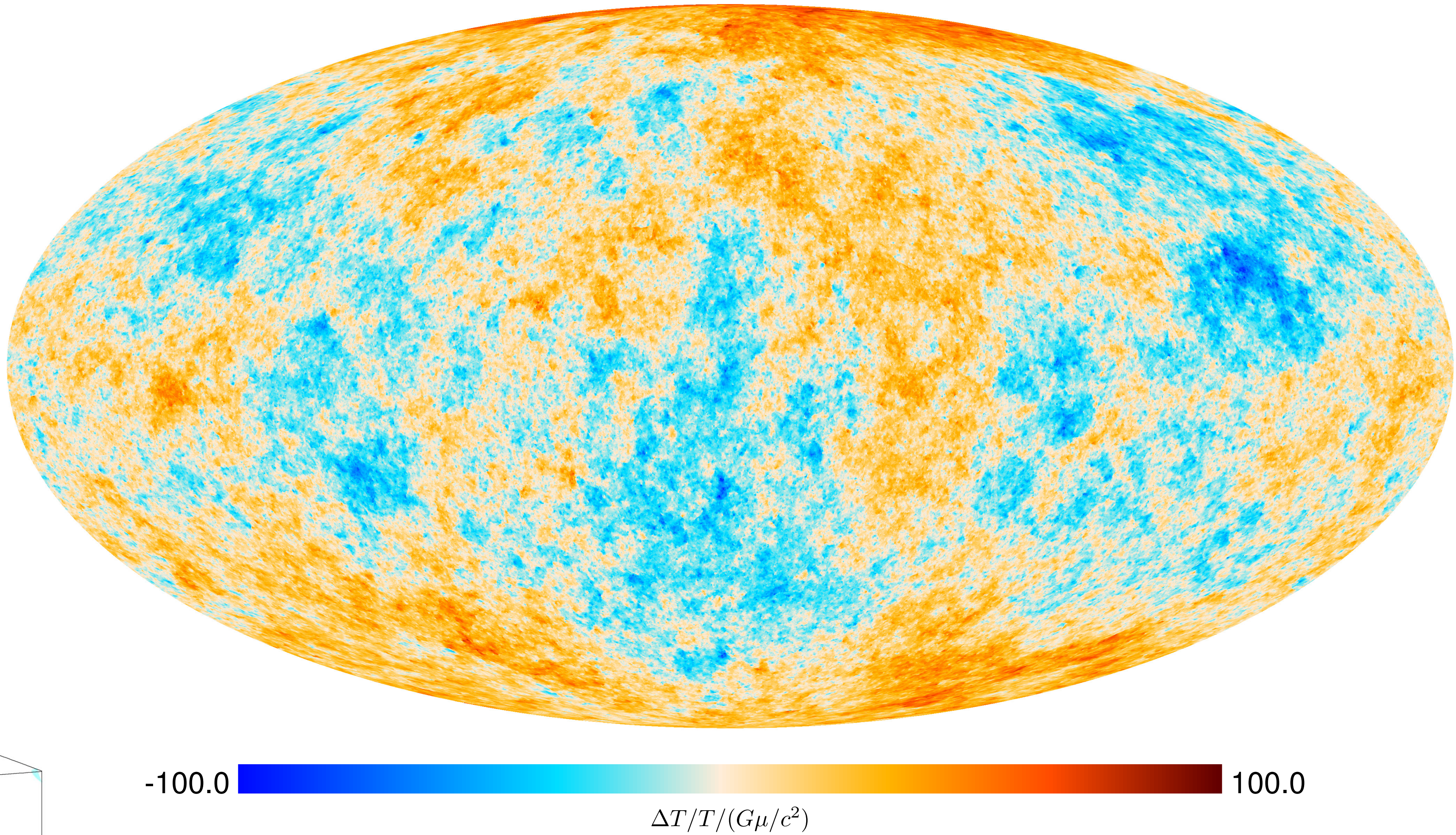
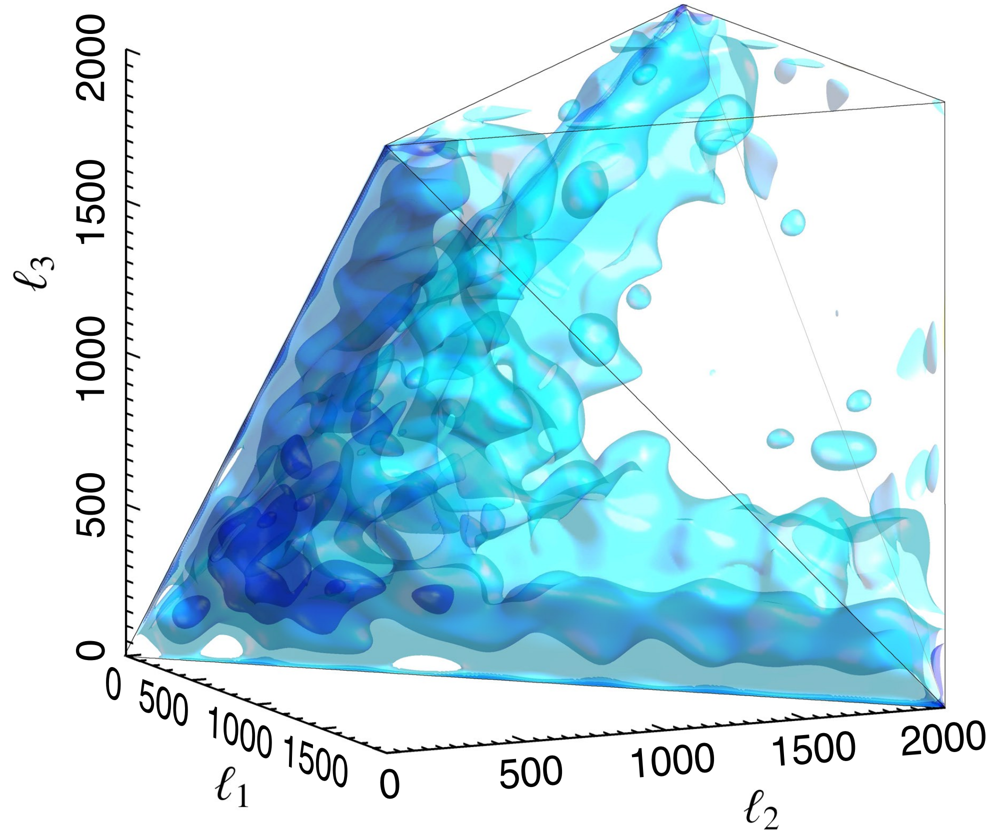
Cosmic string non-Gaussianity







Calibration using post-recombination string simulation maps:

String bispectrum

$$G\mu/c^2 < 8.8 \times 10^{-7}$$

Minkowski functionals

$$G\mu/c^2 < 7.8 \times 10^{-7}$$

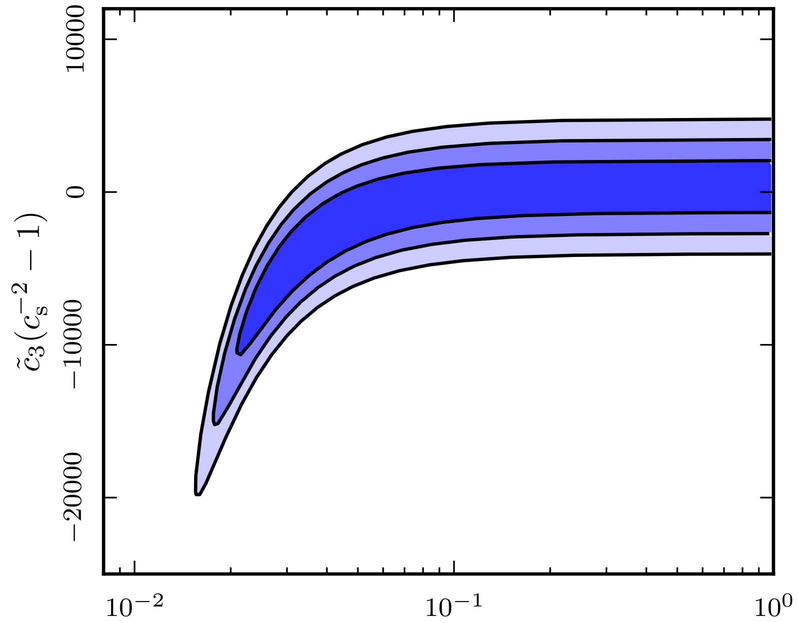
NG Conclusions

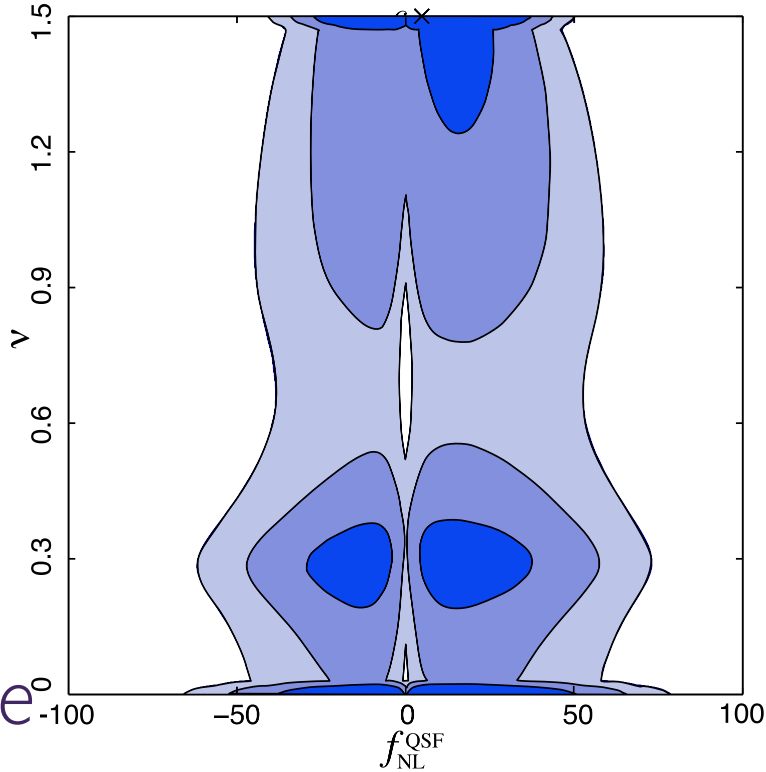
Local, equilateral and orthogonal shapes constrained $f_{\mathrm{NL}}^{\mathrm{local}} = 2.7 \pm 5.8$

All bispectrum paradigms investigated - squeezed, equil, flat, oscillatory

Some implications for fundamental cosmology:

- Effective field theory sound speed $c_s > 0.02$
- For DBI inflation sound speed $c_s > 0.07$
- Power law K-inflation ruled out (cf power spectrum)
- Curvaton model constraint on "decay fraction" r_D
- Ekpyrotic/cyclic "conversion mechanism" ruled out
- Excited initial states and vector inflation constrained
- Feature model results not significant - interesting 'hints'
- Strongest CMB constraints on cosmic defects
- Also first results for trispectrum $\tau_{\mathrm{NL}} < 2800$





Planck bispectrum reconstruction "patterns" appear to have high NG signal

Investigations ongoing ... plus Full Mission data yet to come

NG Prospects

1. New quantitative probe for cosmology - precision achieved by Planck
   - improving with polarization data (i.e. more data 2014) $f_{NL} \approx 4$
   - still early days for the trispectrum, t_{NL} >> g_{NL}, full 5D

2. Most stringent test of inflation - passed scale-invariant NG test
   - but what about other shapes? Is any Planck NG signal primordial?

3. Secondary NG signals - weak ISW lensing will become significant
   - partial recombination-only corrections $f_{NL} \sim 3$
   - 2nd-order theoretical analysis beginning - e.g. corrections to lensing?

4. Primordial NG - stringent constraints on scale-invariant models
   - explore alternative scale-dependent models
   - e.g. oscillatory models, flattened NBD shapes

5. Large-scale structure - galaxy surveys, 21cm
   - explore possibilities (look beyond local NG)!

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