A Non-Gaussian Landscape

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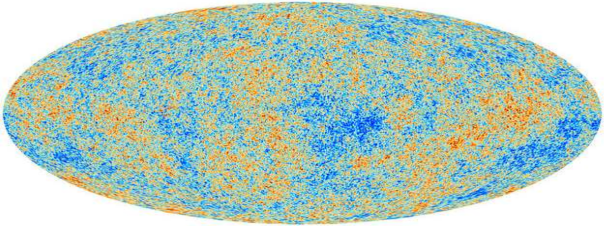
Cosmology and Fundamental Physics with Planck

SN, Byrnes, Tasinato: 1301.3128
Byrnes, SN, Tasinato, Wands: 1306.2370

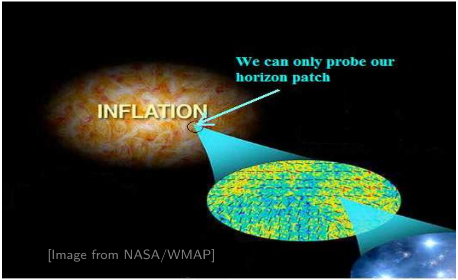
Introduction



- No sign of primordial non-Gaussianity
- Stringent constraints by Planck: $-8.9 < f_{\rm NL} < 14.3$, $\tau_{\rm NL} < 2800$ (95% CL) many talks: Bouchet, Martinez Gonzalez, Finelli, Peiris, Wandelt ...
- $g_{\rm NL}$ constrained by several works using WMAP and LSS data: $|g_{\rm NL}| \lesssim 10^5$ [Desjacques, Seljak 09 ... Sekiguchi, Sugiyama 13]
- Bispectrum has to be small, trispectrum could still be large, can we say more than that?

One approach to the question

- What do we expect to see if inflation lasted more than $N_{\mathrm{CMB}} \sim 60$ e-folds?



- Observable perturbations: statistics of the underlying inflationary model for N_{TOT} e-folds (imagine this is known) + a contribution from long-wavelength modes specific to our location (unknown)

Separate the local and global contributions

- Fluctuations grow proportional to e-folds $\langle\sigma^2\rangle \propto H^2 \ln a$
- Modes $k > aH$ observable, $k < aH$ count as a background in patches $L_{\rm obs} < L_0$

$$\delta\sigma(\mathbf{x}) = \int_{k>aH} d\mathbf{k}\, e^{i\mathbf{k}\cdot\mathbf{x}}\delta\sigma_{\mathbf{k}} + \int_{k<aH} d\mathbf{k}\, e^{i\mathbf{k}\cdot\mathbf{x}}\delta\sigma_{\mathbf{k}} \equiv \delta\sigma_{\rm obs}(\mathbf{x}) + \Delta\sigma(\mathbf{x})$$

- Long-wavelength modes render spatial averages over $L_{\rm obs}$ location dependent

$$\langle\Delta\sigma(\mathbf{x})\rangle_{L(\mathbf{x}_{\rm obs})} \simeq \Delta\sigma(\mathbf{x}_{\rm obs})$$

Impacts on the observable sky

- Semiclassical approach: location dependent shifts of local vevs

$$\mathcal{L}_{\rm obs} = \mathcal{L}_0(\sigma_0 + \Delta\sigma(\mathbf{x}_{\rm obs}))$$

- Shifts proportional to the e-folds before our horizon exit

$$\langle \Delta\sigma^2 \rangle = \left(\frac{H^2}{2\pi}\right)^2 N_{\rm in} \ , \qquad N_{\rm in} = \ln\left(\frac{k_{\rm obs}}{a_0}\right)$$

- Concentrate on the regime $N_{\rm in} \lesssim \mathcal{O}(10^2)$, neglect slow roll corrections and backreaction

Single source models with local non-Gaussianity

- Curvature perturbation over the entire inflating patch L_0

$$\begin{aligned}\zeta_0(\mathbf{x}) &= \zeta_{\rm G}(\mathbf{x}) + \frac{3}{5} f_{\rm NL}^{0} \zeta_{\rm G}^{2}(\mathbf{x}) + \left(\frac{3}{5}\right)^2 g_{\rm NL}^{0} \zeta_{\rm G}^{3}(\mathbf{x}) + \ldots \\ &= \sum \frac{N^{(n)}}{n!}(t,\sigma_0)\delta\sigma(\mathbf{x})\end{aligned}$$

- Do the split $\delta\sigma(\mathbf{x}) = \delta\sigma_{\rm obs}(\mathbf{x}) + \Delta\sigma(\mathbf{x})$
- ζ in our observable patch $L_{\rm obs}(x_0)$ becomes

$$\zeta_{\rm obs}(\mathbf{x}) \simeq \sum \frac{N^{(n)}(\sigma_0 + \Delta\sigma(\mathbf{x}_0))}{n!} \left(\delta\sigma_{\rm obs}(\mathbf{x})\right)^n .$$

- Captures models with one inhomogeneous field (curvaton, modulated reheating ...) generalization to multiple fields straightforward

Distribution of $\zeta_{\rm obs}$

- The curvature perturbation depends on the location $\mathbf{x}_0$ of the patch $L_{\rm obs.}$

$$\zeta_{\rm obs}(\mathbf{x}) \simeq \sum \frac{N^{(n)}(\sigma_0 + \Delta\sigma(\mathbf{x}_0))}{n!} \left(\delta\sigma_{\rm obs}(\mathbf{x})\right)^n \ .$$

- Distribution of $\Delta\sigma(\mathbf{x}_0)$ over the inflating patch determined by theory
- Straightforward to work out the distributions for $\zeta_{\rm obs}$ and its moments in a randomly located patch $L_{\rm obs.}$

The spectrum in our observable patch

- Expand the local result $\mathcal{P}_{\rm obs} = N'^2(\sigma_0 + \Delta\sigma)\mathcal{P}_\sigma$ in $\Delta\sigma$

$$\mathcal{P}_{\rm obs.} = \mathcal{P}_0 \left(1 + \frac{12}{5} f_{\rm NL}^0 N'(\sigma_0)\Delta\sigma(\mathbf{x}_0) + \ldots \right)$$

- Gaussian distribution for $\mathcal{P}_{\rm obs.}$

$$P(\mathcal{P}_{\rm obs.}) = \frac{\exp\left(-\frac{(\mathcal{P}_{\rm obs.} - \mathcal{P}^0)^2}{2\sigma_\mathcal{P}^2}\right)}{\sqrt{2\pi\sigma_\mathcal{P}^2}} \,, \quad \sigma_\mathcal{P}^2 = \left(\frac{12}{5} f_{\rm NL}^0 \mathcal{P}_0\right)^2 \mathcal{P}_0 N_{\rm in.}$$

- For $N_{\rm in} \lesssim 10^2$ the variance is small

$$\frac{\sigma_\mathcal{P}}{\mathcal{P}_0} \lesssim 10^{-3} f_{\rm NL}^0 \sqrt{\frac{\mathcal{P}_0}{10^{-9}}}$$

- Hence we can use $\mathcal{P}_{\rm obs.} \simeq \mathcal{P}_0$

The bispectrum

- Expand: $f_{\rm NL}^{\rm obs.} = f_{\rm NL}^{0} + (f_{\rm NL}^{0})'\Delta\sigma + \mathcal{O}(f_{\rm NL}''\Delta\sigma^2)$

$$P(f_{\rm NL}^{\rm obs.}) = \frac{\exp\left(-\frac{(f_{\rm NL}^{\rm obs.}-f_{\rm NL}^{0})^2}{2\sigma_{f_{\rm NL}}^2}\right)}{\sqrt{2\pi\sigma_{f_{\rm NL}}^2}}\,, \quad \sigma_{f_{\rm NL}}^2 = \left(\frac{9}{5}g_{\rm NL}^{0} - \frac{12}{5}(f_{\rm NL}^{0})^2\right)^2 \mathcal{P}_0 N_{\rm in.}$$

- If $g_{\rm NL}^{0} \gg (f_{\rm NL}^{0})^2$ the distribution becomes broad $\sigma_{f_{\rm NL}} \gtrsim f_{\rm NL}^{0}$
- $f_{\rm NL}^{\rm obs}$ in the observable patch can be quite different from the global value $f_{\rm NL}^{0}$

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- If $g_{\rm NL}^0 \gg (f_{\rm NL}^0)^2$ the distribution becomes broad $\sigma_{f_{\rm NL}} \gtrsim f_{\rm NL}^0$
- The bispectrum in our observable patch $f_{\rm NL}^{\rm obs}$ can be quite different from $f_{\rm NL}^0$
- No breakdown of perturbation theory implied by $g_{\rm NL}^0 \gg (f_{\rm NL}^0)^2$

The bispectrum

- Expand: $f_{\rm NL}^{\rm obs.} = f_{\rm NL}^0 + (f_{\rm NL}^0)'\Delta\sigma + \mathcal{O}(10^{-5} N_{\rm in.}^{1/2} h_{\rm NL}^0/g_{\rm NL}^0)$

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- If $g_{\rm NL}^0 \gg (f_{\rm NL}^0)^2$ the distribution becomes broad $\sigma_{f_{\rm NL}} \gtrsim f_{\rm NL}^0$
- The bispectrum in our observable patch $f_{\rm NL}^{\rm obs}$ can be quite different from $f_{\rm NL}^0$
- No breakdown of perturbation theory implied by $g_{\rm NL}^0 \gg (f_{\rm NL}^0)^2$
- Variance of $g_{\rm NL}^{\rm obs}$ small: $\sigma_{g_{\rm NL}}/g_{\rm NL}^0 \sim 10^{-5} h_{\rm NL}^0/g_{\rm NL}^0 N_{\rm in.}^{1/2}$, can identify $g_{\rm NL}^0 \sim g_{\rm NL}^{\rm obs.}$

An example: $f_{\rm NL}^0 = 0$ but $g_{\rm NL}^0$ is large

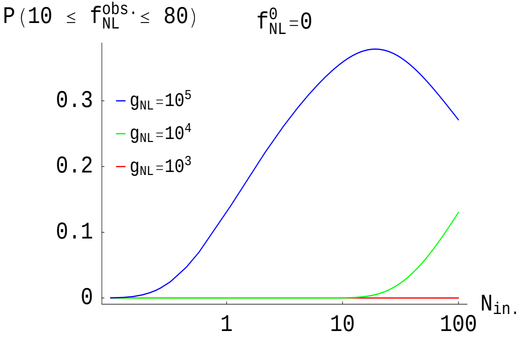

Figure: Probability for $f_{\rm NL}^{\rm obs}$ being large in our patch.

Use the Planck bound $-8.9 < f_{\rm NL}^{\rm obs.} < 14.3$

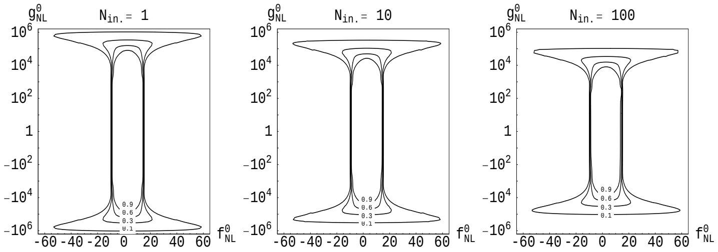

Figure: Probability to get $-8.9 < f_{\rm NL}^{\rm obs} < 14.3$ on observable scales as a function of $f_{\rm NL}^0, g_{\rm NL}^0, N_{\rm in}$

- $\sigma^2_{f_{\rm NL}} \sim (g_{\rm NL}^{\rm obs})^2 \mathcal{P} N_{\rm in}$, large $g_{\rm NL}^{\rm obs}$ tends to make $f_{\rm NL}^{\rm obs}$ too large
- "Tension" between a large $g_{\rm NL}$ and a small $f_{\rm NL}$, the more e-folds the bigger the tension

Is it natural to have $g_{\rm NL} \gg (f_{\rm NL})^2$?

- Specific models with $g^0_{\rm NL} \gg (f^0_{\rm NL})^2$ typically fine-tuned
- Model-independent tuning: $|f^{\rm obs}_{\rm NL}| \lesssim 10$, $|g^{\rm obs}_{\rm NL}| \gtrsim 10^4$ favour $N_{\rm in} \lesssim 10$

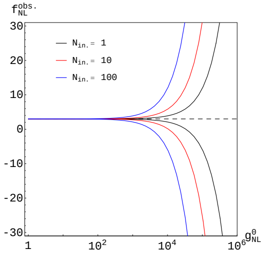

Figure: $f^{\rm obs.}_{\rm NL} = f^0_{\rm NL} \pm \sigma_{f_{\rm NL}}$ shown for $f^0_{\rm NL} = 3$.

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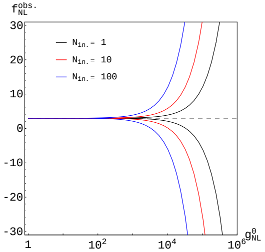

Figure: $f^{\rm obs.}_{\rm NL} = f^0_{\rm NL} \pm \sigma_{f_{\rm NL}}$ shown for $f^0_{\rm NL} = 3$.

- What if we nevertheless observe $|g_{\rm NL}| \gtrsim 10^4$?
  - We live in an atypical realization?
  - Just the minimum number of e-folds?

Generalization to multiple fields

- The variances always bigger than for the single source case, e.g.

$$\sigma^2_{f_{\rm NL}} = f_{{\rm NL},a} f_{{\rm NL},b} \langle \delta_{\rm L}\sigma_a \delta_{\rm L}\sigma_b \rangle \geq \left(\frac{9}{5} g^0_{\rm NL} - \frac{12}{5} (f^0_{\rm NL})^2 \right)^2 \mathcal{P}_0 N_{\rm in.}$$

- The single source constraints are the most conservative ones
- The idea also generalizes to non-local non-Gaussianity

Conclusions

- The observable primordial perturbations affected by random long-wavelength fluctuations, need to relate $\mathcal{L}_0$ (theory) and $\mathcal{L}_{\rm obs.}$ (observations)
- Given a model ($\mathcal{L}_0$) can work out the probabilities for different observational signatures
- Apply to non-Gaussianity: Planck constrains $f_{\rm NL}^{\rm local} = 2.7 \pm 5.8$
- A large $g_{\rm NL}$ allowed by observations but long-wavelength effects tend to generate too large $f_{\rm NL}$ – generic tuning of the number of e-foldings implied
- Signatures $|f_{\rm NL}| \lesssim 10, |g_{\rm NL}| \gtrsim 10^4$ atypical unless $N_{\rm tot} \simeq N_{\rm CMB}$