ÉCOLE POLYTECHNIQUE
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Higgs inflation, Planck and LHC

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Cosmology and Fundamental Physics with Planck

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Outline

- Higgs inflation, tree approximation
- Radiative corrections
- LHC, Planck and Higgs inflation
- Self-consistency of Higgs inflation
- Higher dimensional operators and relation between low energy and high energy parameters
- Conclusions

Higgs inflation: tree analysis

Bezrukov, M.S.

Main idea of Higgs inflation: non-minimal coupling of the Higgs field to gravity:

$$S = \int d^4x \sqrt{-g} \left\{ -\frac{M_P^2}{2} R + g_{\mu\nu} \frac{\partial^\mu h \partial^\nu h}{2} - \frac{\lambda}{4} \left(h^2 - v^2\right)^2 \right\} + \Delta S$$

Extra term, necessary for renormalizability:

$$\Delta S = \int d^4x \sqrt{-g} \left\{ -\frac{\xi h^2}{2} R \right\}$$

Feynman, Brans, Dicke,...

Standard Model Higgs boson as inflaton

Consider large Higgs fields h.

- Gravity strength: $M_P^{\rm eff} = \sqrt{M_P^2 + \xi h^2} \propto h$
- All particle masses are $\propto h$

For $h > \frac{M_P}{\xi}$ (classical) physics is the same ($M_W / M_P^{\rm eff}$ does not depend on h)!

Existence of effective flat direction, necessary for successful inflation.

Formalism: go from Jordan frame to Einstein frame with the use of conformal transformation:

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}\ , \quad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2}$$

Redefinition of the Higgs field to make canonical kinetic term

$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi_h^2 h^2/M_P^2}{\Omega^4}} \quad \Longrightarrow \begin{cases} h \simeq \chi & \text{for } h < M_P/\xi \\ h \simeq \frac{M_P}{\sqrt{\xi}} \exp\left(\frac{\chi}{\sqrt{6}M_P}\right) & \text{for } h > M_P/\sqrt{\xi} \end{cases}$$

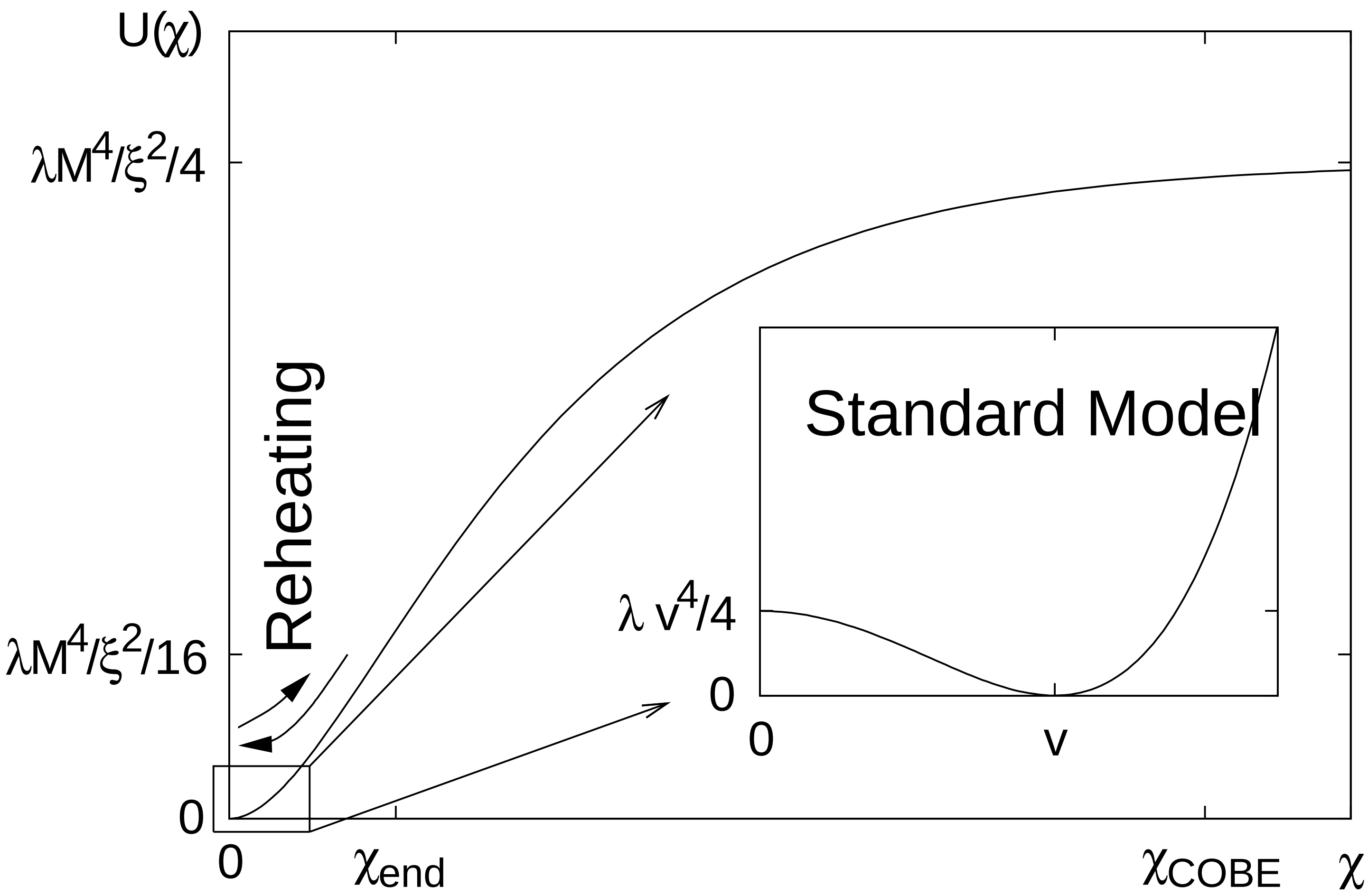
Resulting action (Einstein frame action)

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} - \frac{1}{\Omega(\chi)^4}\frac{\lambda}{4}h(\chi)^4 \right\}$$

Potential:

$$U(\chi) = \begin{cases} \frac{\lambda}{4}\chi^4 & \text{for } h < M_P/\xi \\ \frac{\lambda M_P^4}{4\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2 & \text{for } h > M_P/\xi \end{cases} .$$

Potential in Einstein frame



Inflaton potential and observations

If inflaton potential is known one can make predictions and compare them with observations.

- $\delta T/T$ at the WMAP normalization scale ~ 500 Mpc
- The value of spectral index n_s of scalar density perturbations

$$\left\langle \frac{\delta T(x)}{T}\frac{\delta T(y)}{T}\right\rangle \propto \int \frac{d^3k}{k^3} e^{ik(x-y)} k^{n_s-1}$$

- The amplitude of tensor perturbations $r = \frac{\delta \rho_s}{\delta \rho_t}$

These numbers can be extracted from WMAP observations of cosmic microwave background. Higgs inflation: one new parameter, $\xi \Longrightarrow$ two predictions.

Slow roll stage

COBE normalization $U/\epsilon = (0.027 M_P)^4$ gives

$$\xi \simeq \sqrt{\frac{\lambda}{3}} \frac{N_{\rm COBE}}{0.027^2} \simeq 49000\sqrt{\lambda} = 49000 \frac{m_H}{\sqrt{2}v}$$

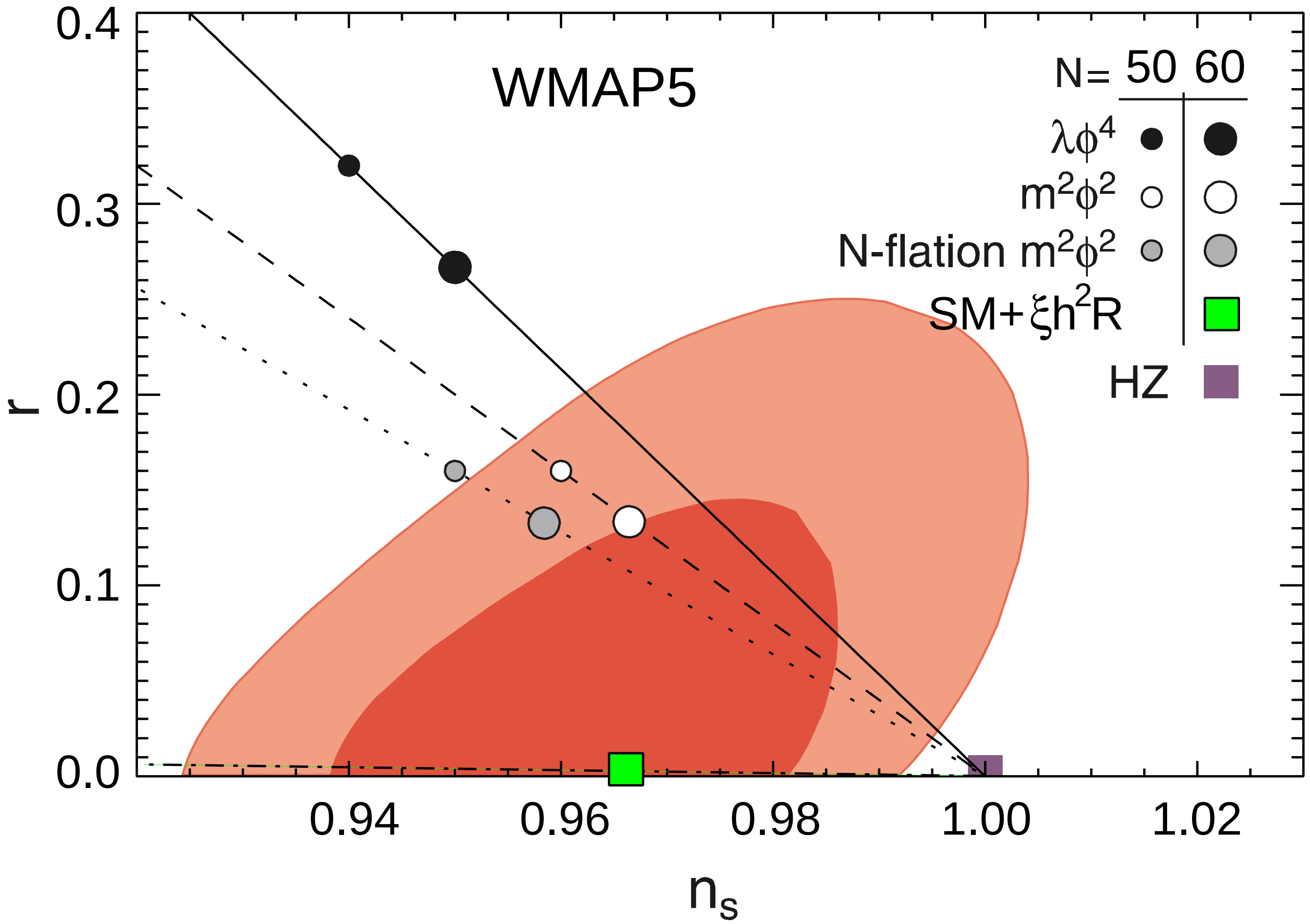
Number of e-folds of inflation at the moment h_N is $N \simeq \frac{6}{8}\frac{h_N^2 - h_{\rm end}^2}{M_P^2/\xi}$

Slow roll ends at $\chi_{\rm end} \simeq M_P$; and “begins” at $\chi_{60} \simeq 5 M_P$

$$\epsilon = \frac{M_P^2}{2}\left(\frac{dU/d\chi}{U}\right)^2, \quad \eta = M_P^2 \frac{d^2U/d\chi^2}{U}$$

$$n_s = 1 - 6\epsilon + 2\eta, \qquad r = 16\epsilon$$

CMB parameters—spectrum and tensor modes



Earlier works: R^2 inflation, non-minimal coupling of scalars in GUTs, etc:

- A. Starobinsky ’80
- B. Spokoiny ’84
- D. Salopek, J. Bond and J. Bardeen ’89
- R. Fakir and W. G. Unruh ’90
- A. O. Barvinsky and A. Y. Kamenshchik ’94, ’98
- E. Komatsu and T. Futamase ’99
- S. Tsujikawa and B. Gumjudpai ’04

Computation of spectral indexes gives the same results in Einstein and Jordan frames.

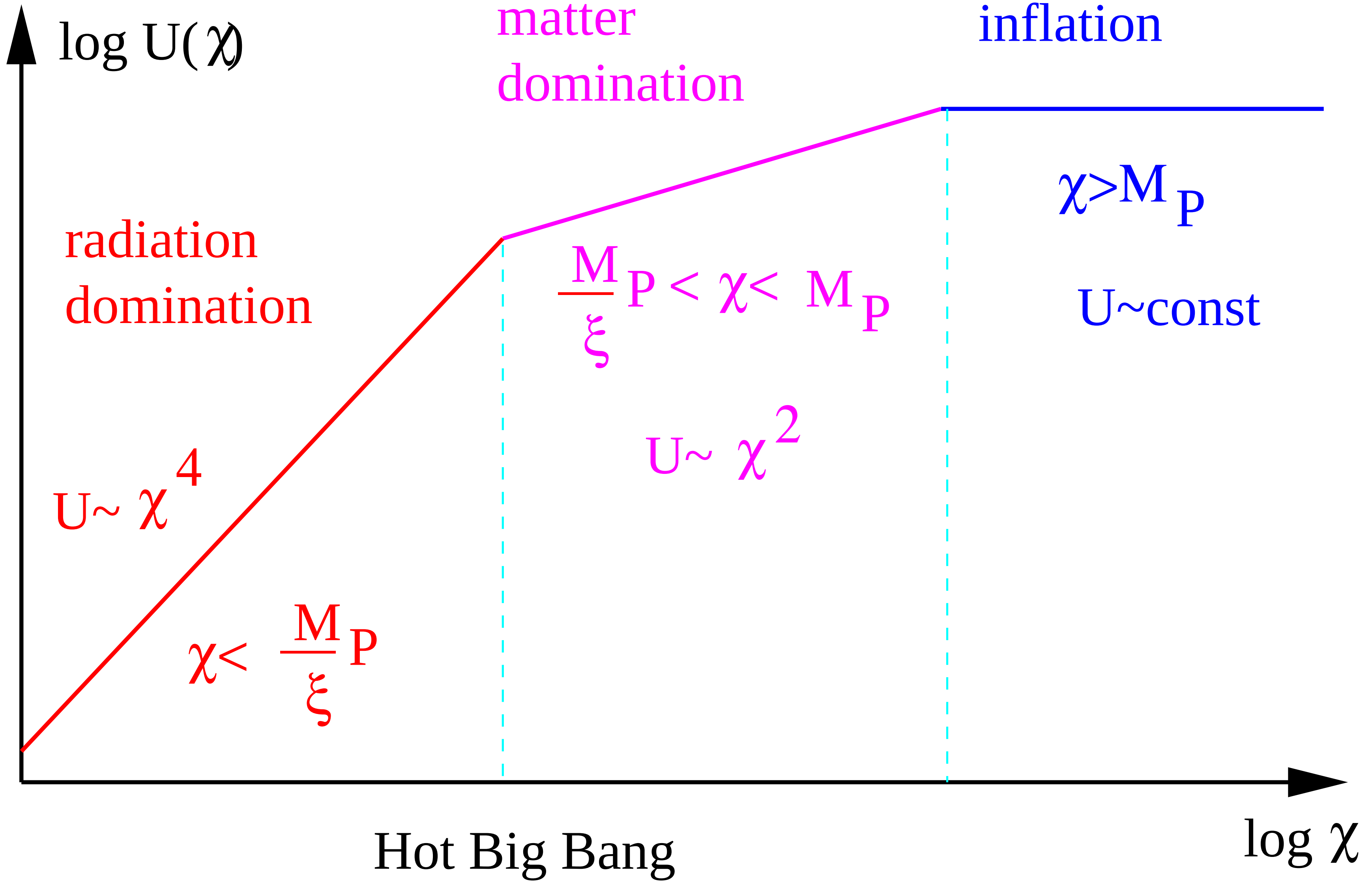
Life after inflation

Two different stages:

- For scalar field $M_P > \chi > \frac{M_P}{\xi}$ the potential for χ is essentially *quadratic*, $m_\chi^2 \sim \lambda M_P^2/\xi^2$. Exponential expansion of the Universe is changed to the power low, corresponding to matter domination. Particle creation takes place when χ passes through zero.
- After $\mathcal{O}(\xi)$ oscillations the scalar field reaches $\chi \simeq \frac{M_P}{\xi}$. The energy is transferred to other fields of the SM, and the radiation-dominated epoch starts,

$$T_r \simeq (3.3 - 8.3) \times 10^{13}\ \mathrm{GeV}.$$

Bezrukov, Gorbunov, M.S.; J. Garcia-Bellido, D. G. Figueroa, J. Rubio


log U(χ)
matter
domination
inflation
χ>M P
radiation
domination
M P / ξ < χ < M P
U~const
U~ χ 2
U~ χ 4
χ< M P / ξ
log χ
Hot Big Bang

Higgs inflation: radiative corrections

Main assumptions

- The SM is a valid effective theory up to the Planck (or, to be more modest, up to the inflationary scale)
- Its Lagrangian is as it is - no any higher dimensional operators are added
- The quadratic divergences are ignored (the valid procedure at small energies). Technically, this corresponds to the use of the minimal subtraction scheme.

Effective field theory

Electroweak theory in the inflationary region, for

$h \sim M_P/\sqrt{\xi}, \;\; h \gg M_P/\xi$:

Take the SM, freeze the radial mode of the Higgs field, and add to Lagrangian almost massless and almost non-interacting scalar: chiral SM.

Why the Higgs decouples?

Einstein frame: masses of all the particles

$$M_W, ..., \; m_t \propto v = \frac{h}{\Omega(h)} \rightarrow \textbf{const for } h \rightarrow \infty$$

The procedure for computations of inflationary parameters

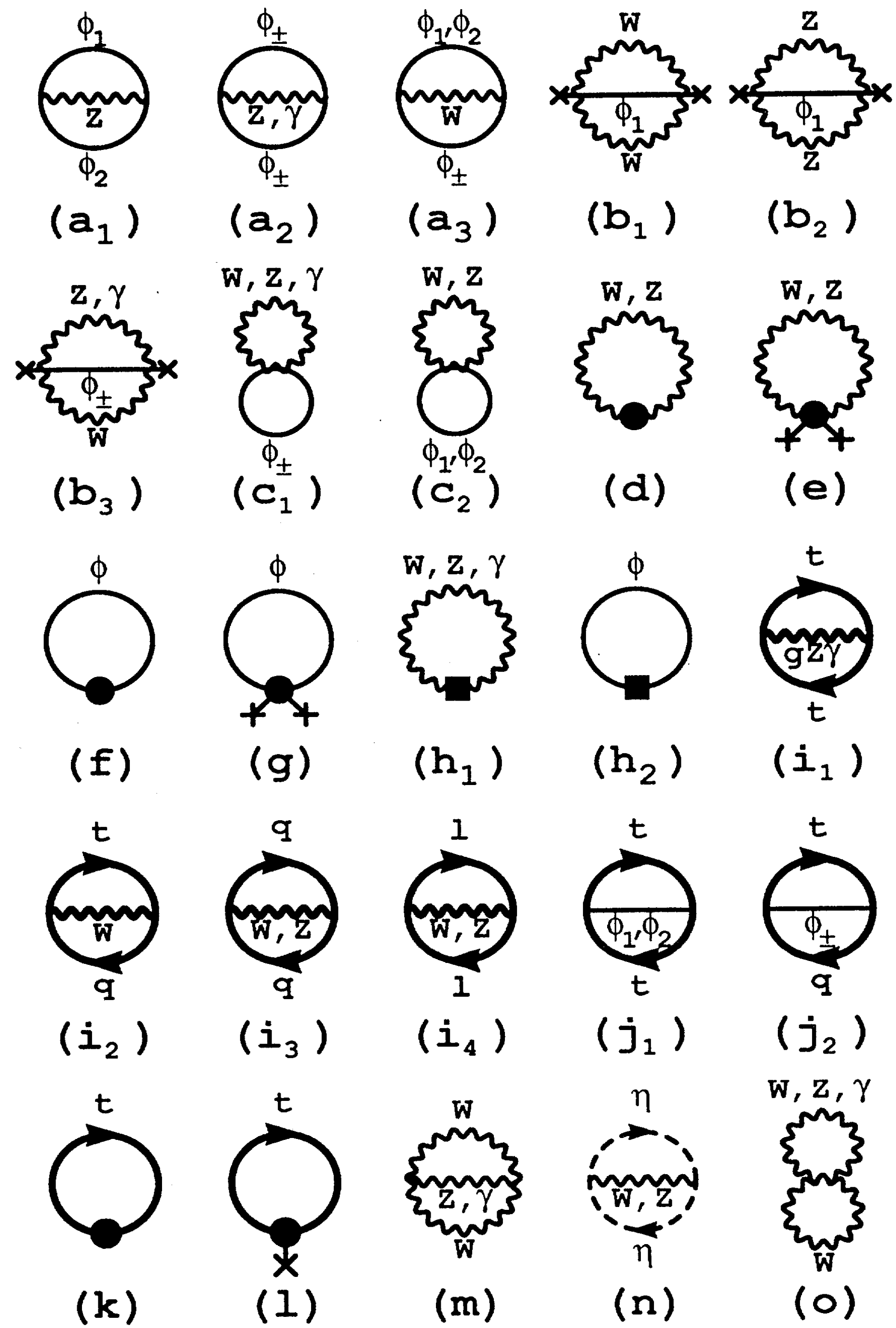
- Compute the effective potential in the inflationary region (tree, one-loop, two-loop,... with the use of the chiral SM
- Choose the normalization point μ to minimize higher order terms (normally, $\mu \sim M_W$ or M_Z or m_t)
- To find values of different coupling constants in inflationary region, solve one-loop, two-loop ... RG equations, getting initial conditions from tree, one-loop, two-loop,.. relations mapping physical SM parameters to couplings.
- Find ξ from COBE normalization and compute n_s and r as a function of the Higgs mass

Subtle point: renormalization

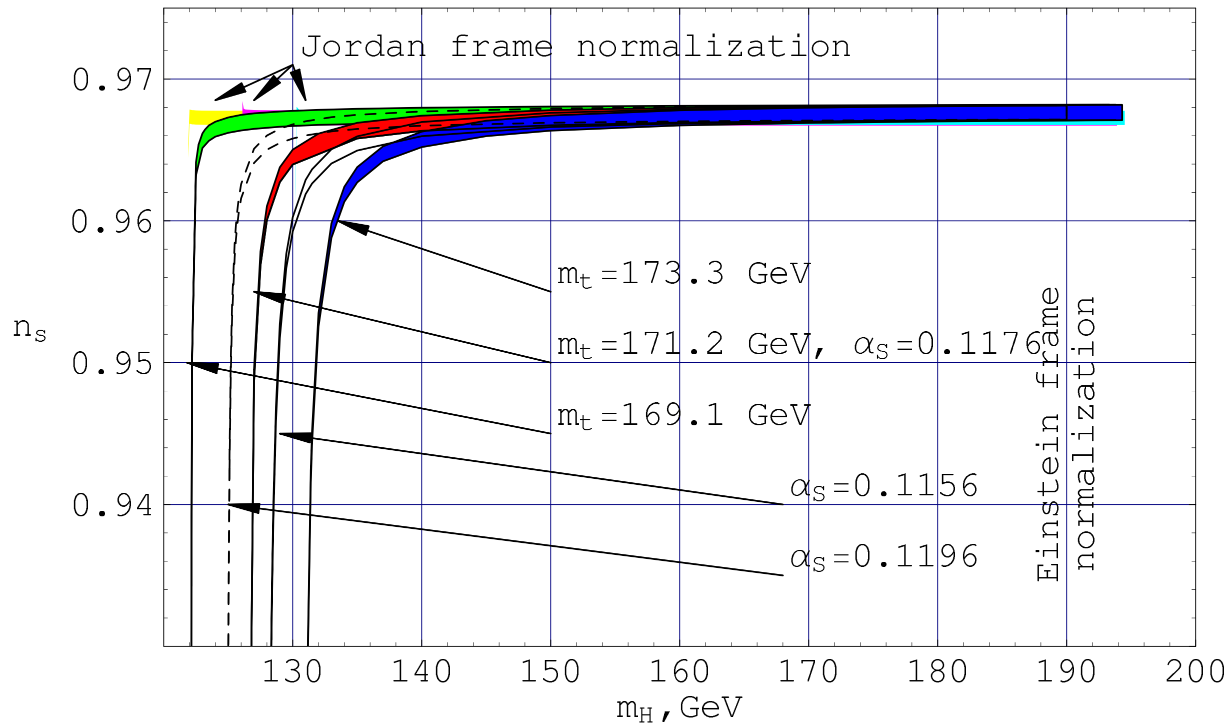
The straightforward computation in Einstein or Jordan frames leads to different results. μ in the different frames

	I BS	II BKS
Jordan frame	$M_P^2 + \xi h^2$	M_P^2
Einstein frame	M_P^2	$\frac{M_P^4}{M_P^2 + \xi h^2}$

The prescription I is “standard” (field-independent) in the Einstein frame, whereas the prescription II is “standard”(field-independent) in the Jordan frame. To be fixed by (unknown) physics at the Planck scale.

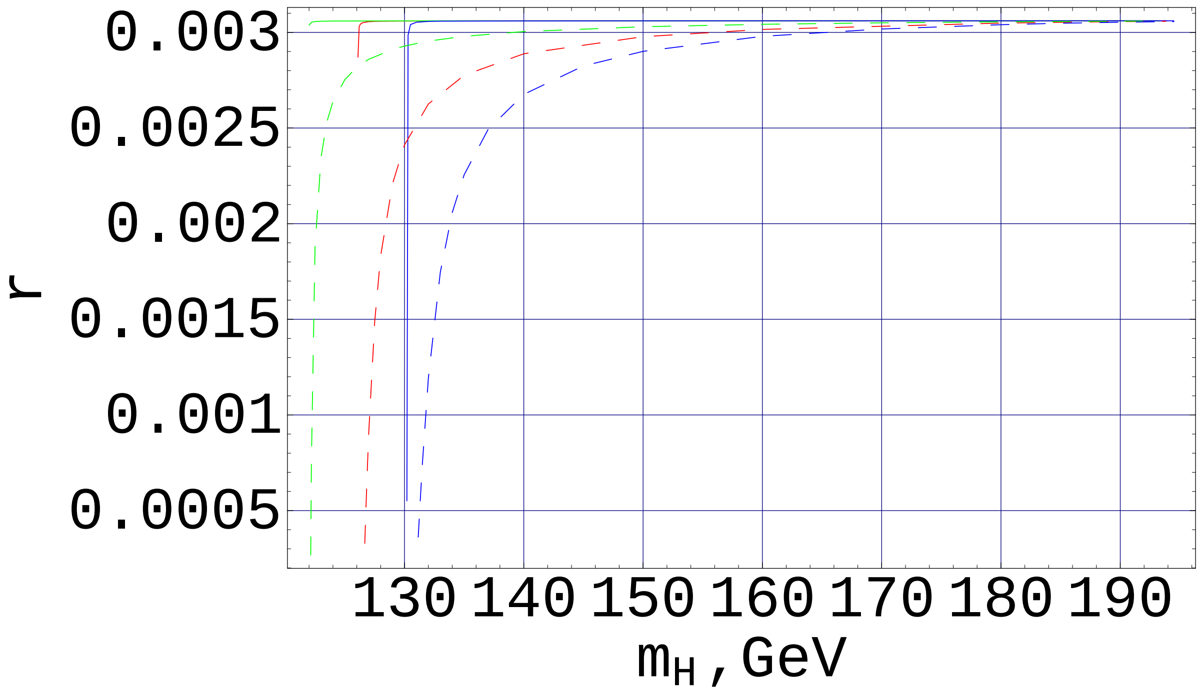

φ1
Z
φ2
(a1)
φ±
Z, γ
φ±
(a2)
φ1, φ2
W
φ±
(a3)
W
φ1
W
(b1)
Z
φ1
Z
(b2)
Z, γ
φ±
W
(b3)
W, Z, γ
φ±
(c1)
W, Z
φ1, φ2
(c2)
W, Z
(d)
W, Z
(e)
φ
(f)
φ
(g)
W, Z, γ
(h1)
φ
(h2)
t
gZγ
t
(i1)
t
W
q
(i2)
q
W, Z
q
(i3)
l
W, Z
l
(i4)
t
φ1, φ2
t
(j1)
t
φ±
q
(j2)
t
(k)
t
(l)
W
Z, γ
W
(m)
η
W, Z
η
(n)
W, Z, γ
W
(o)

Two-loop results



Nearly horizontal coloured stripes correspond to the normalization prescription I. Green, red, and blue stripes give the result with normalization prescription II for different m_t and $\alpha_s = 0.1176$, two white regions correspond to different α_s and $m_t = 171.2\mathrm{GeV}$. The width of the stripes corresponds to changing the number of e-foldings between 58 and 60, or approximately one order of magnitude in reheating temperature.

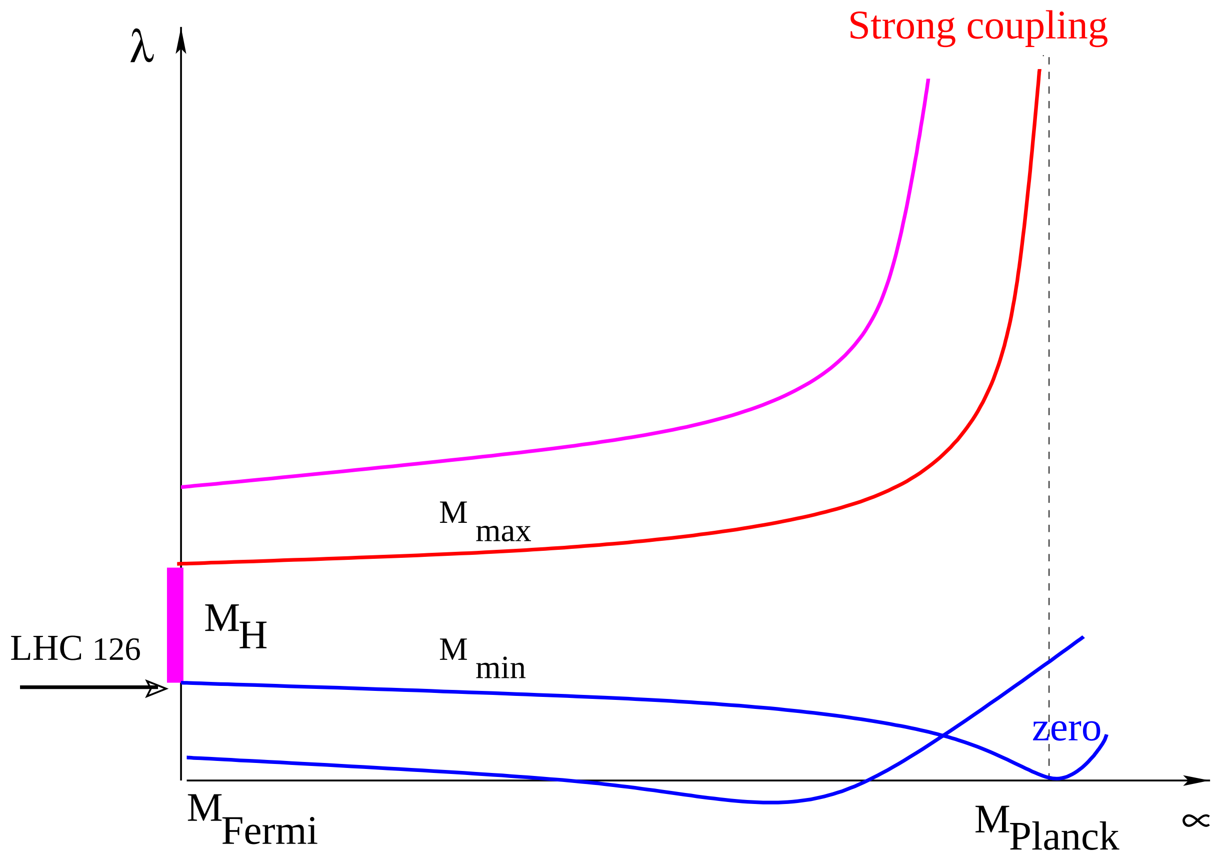
Two-loop results

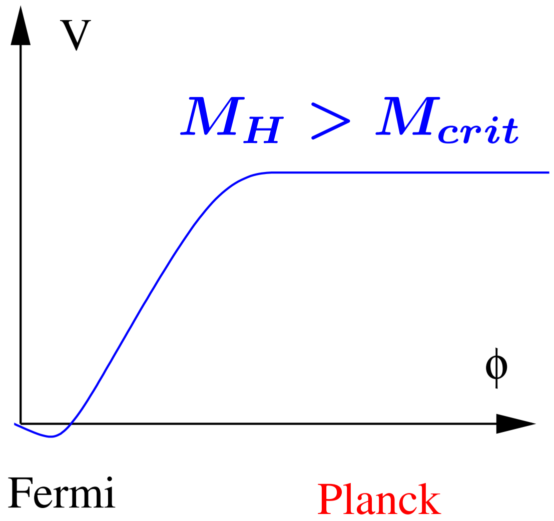


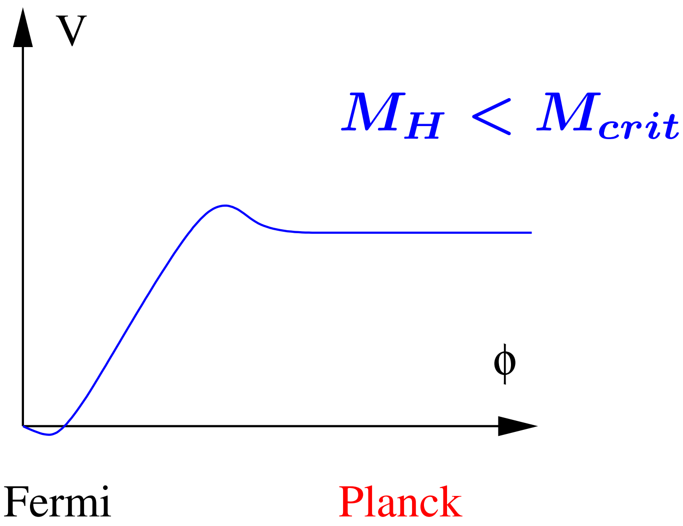
Tensor-to-scalar ratio r depending on the Higgs mass m_H, calculated with the RG enhanced effective potential. Nearly horizontal solid lines correspond to the normalization prescription I. Green, red, and blue dashed lines give the result with normalization prescription II for $m_t = 169.1, 171.2, 173.3$ GeV. Dependence on the number of e-foldings is very small.

Inflation and the Higgs mass

Higgs inflation works only for $\lambda(M_P/\sqrt{(\xi)}) > 0$, provided the Landau pole in λ is above the inflation scale.




V
$M_H > M_{crit}$
ϕ
Fermi
Planck


V
$M_H < M_{crit}$
ϕ
Fermi
Planck

Cosmological constraints on the Higgs mass, 2009

tree matching 1 loop running

$$m_{\min} = [136.7 + (m_t - 171.2) \times 1.95]\ \mathrm{GeV}$$
$$m_{\max} = [184.5 + (m_t - 171.2) \times 0.5\ ]\ \mathrm{GeV}$$

1 loop matching 2 loop running

$$m_{\min} = [126.1 + \frac{m_t - 171.2}{2.1} \times 4.1 - \frac{\alpha_s - 0.1176}{0.002} \times 1.5]\ \mathrm{GeV}\ ,$$
$$m_{\max} = [193.9 + \frac{m_t - 171.2}{2.1} \times 0.6 - \frac{\alpha_s - 0.1176}{0.002} \times 0.1]\ \mathrm{GeV}\ .$$

Spectral index behaviour is the same in one and two loops.

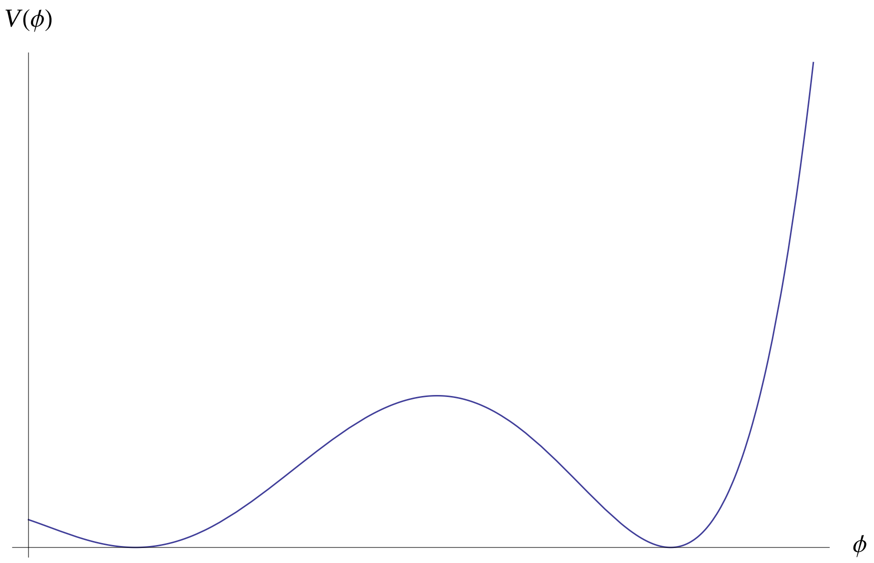
Summary of predictions

- Certain interval for the Higgs mass, $m_{\mathrm{min}} < M_H < m_{\mathrm{max}}$
- $n_s = 0.968, \ \ r = 0.003$, for scale-invariant regularisation
- $n_s < 0.968, \ \ r < 0.003$, for scale-noninvariant regularisation
- Gaussian perturbations

LHC, Planck and Higgs inflation

Theoretical progress

Advanced computations of the absolute stability bound on the Higgs mass M_{crit}



Relation between $m_{\min}$ and M_{crit}:

$$m_{\min} = M_{crit} - 200\text{MeV}$$

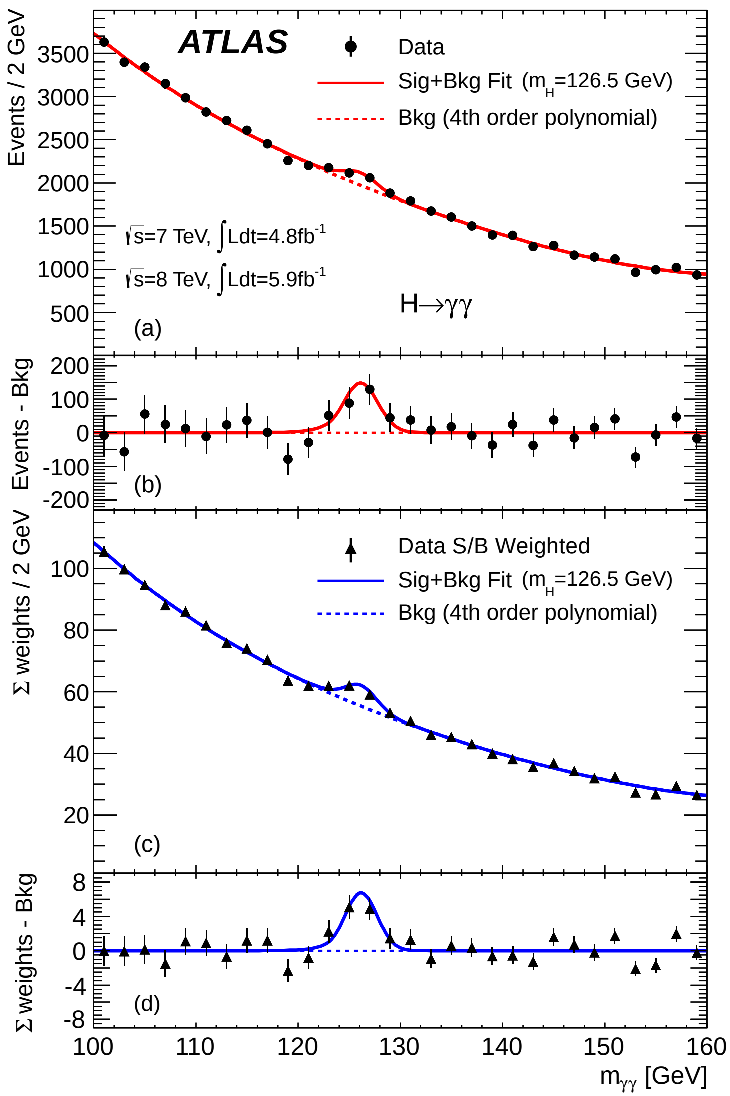
- (Bezrukov et al, May 13, 2012), incorporating $\mathcal{O}(\alpha\alpha_s)$ two-loop matching and 3-loop running of coupling constants (Chetyrkin, Zoller, May 13, 2012)

  $$m_{crit} = [129.0 + \frac{m_t - 172.9}{1.1} \times 2.2 - \frac{\alpha_s - 0.1184}{0.0007} \times 0.56]\ \mathrm{GeV}\,,$$

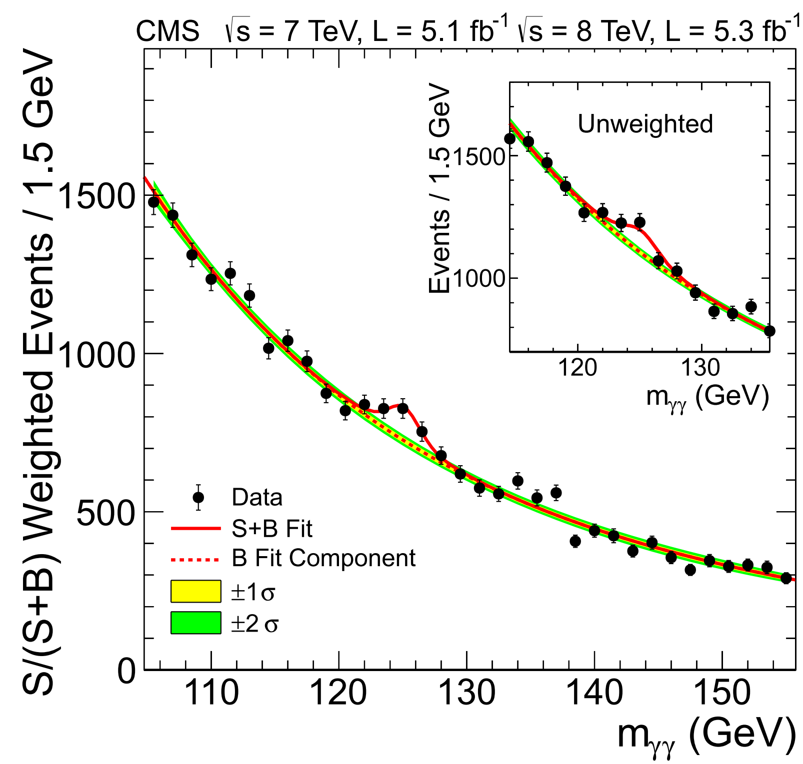
  Theoretical uncertainties: ± 1.2 GeV (different sources are summed quadratically) or ± 2.3 GeV (different sources are summed linearly).

- Degrassi et al., May 29, 2012 Account for extra contributions $\propto y_t^4, y_t^2\lambda^2, \lambda^4$ in the matching procedure. Effect:: shift of the Higgs mass by $100 - 200$ MeV. Quadratic theoretical uncertainty is reduced to ~ 0.8 GeV.

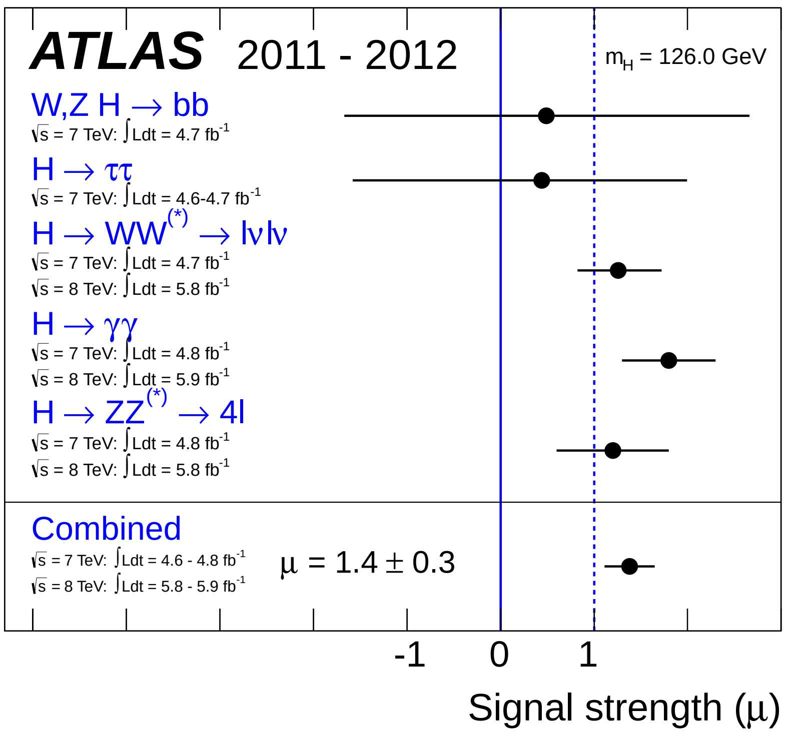
Experimental progress, LHC

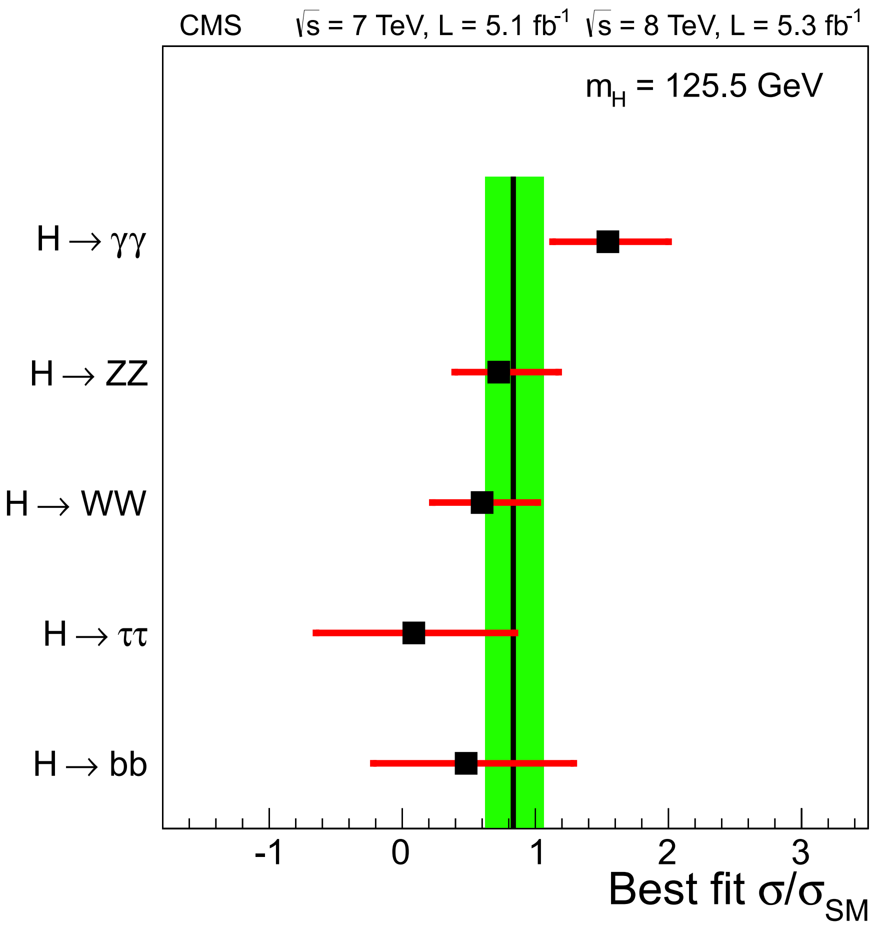


CMS



Higgs at ATLAS and CMS





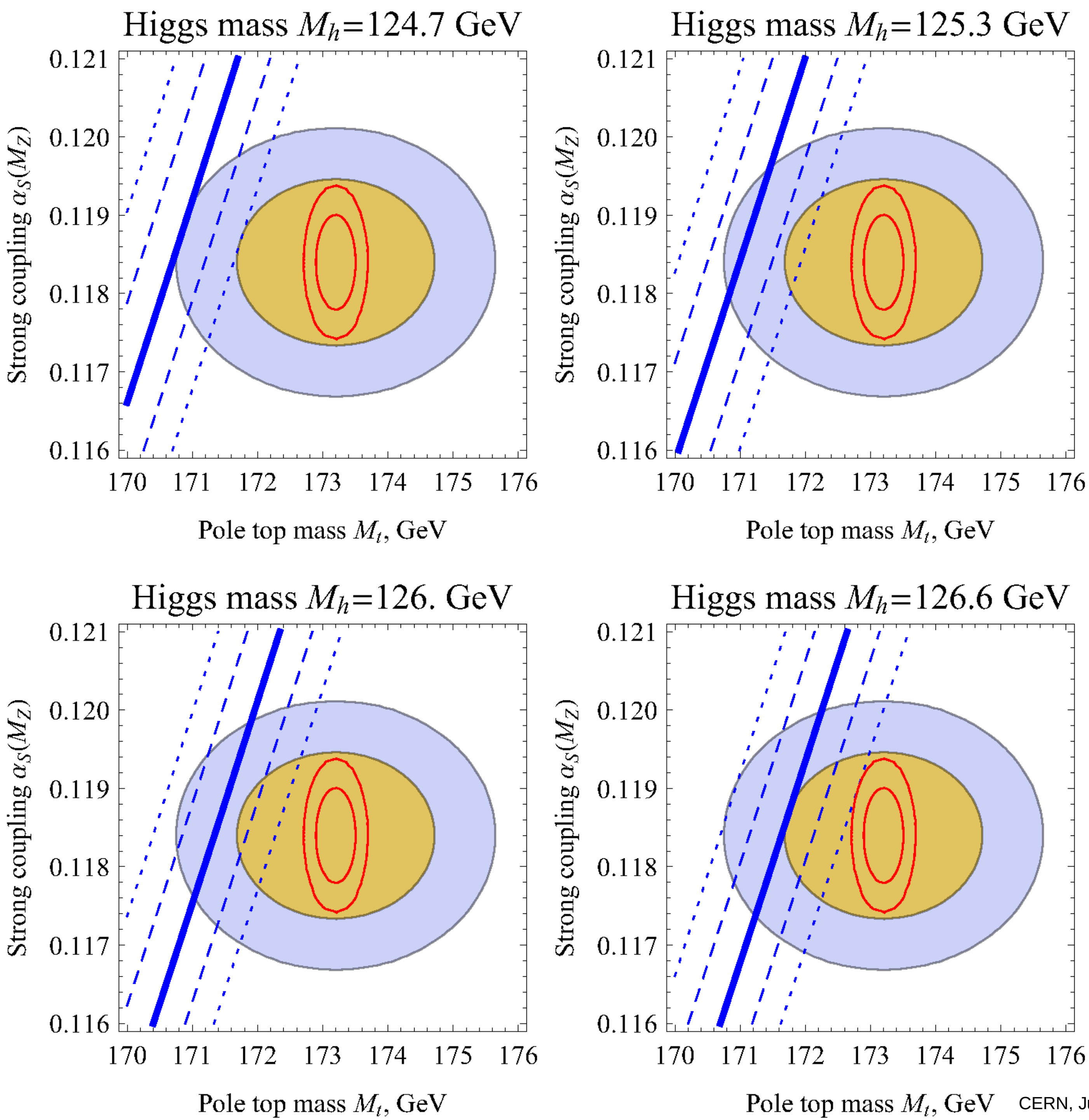
Recent data

According to CMS,

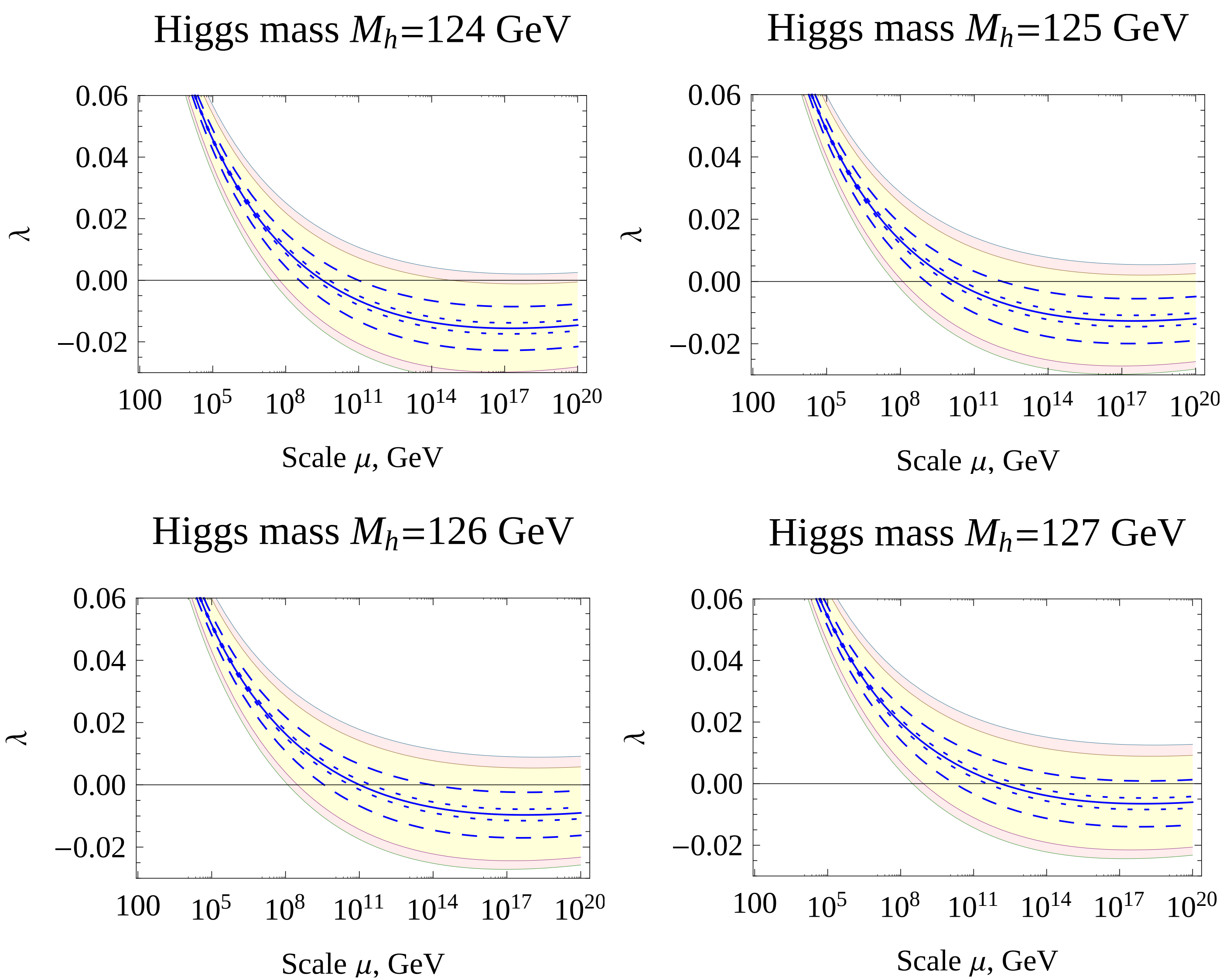
$$M_H = 125.7 \pm 0.4\,\mathrm{GeV},$$

According to ATLAS,

$$M_H = 125.5 \pm 0.6\,\mathrm{GeV}.$$


Higgs mass M_h=124.7 GeV
Higgs mass M_h=125.3 GeV
Higgs mass M_h=126. GeV
Higgs mass M_h=126.6 GeV
Strong coupling $\alpha_S(M_Z)$
Pole top mass M_t, GeV
0.116
0.117
0.118
0.119
0.120
0.121
170
171
172
173
174
175
176

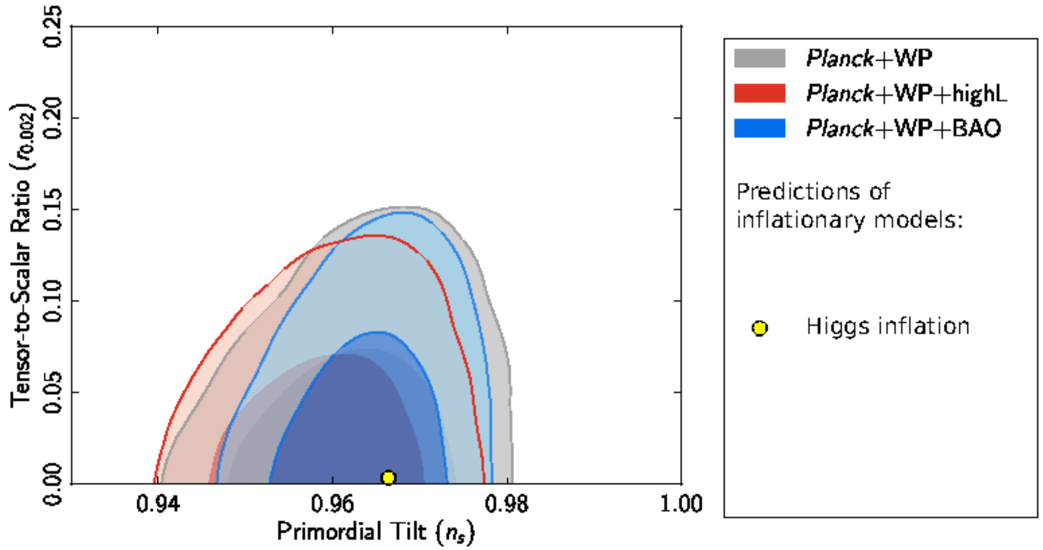
Behaviour of the Higgs self-coupling

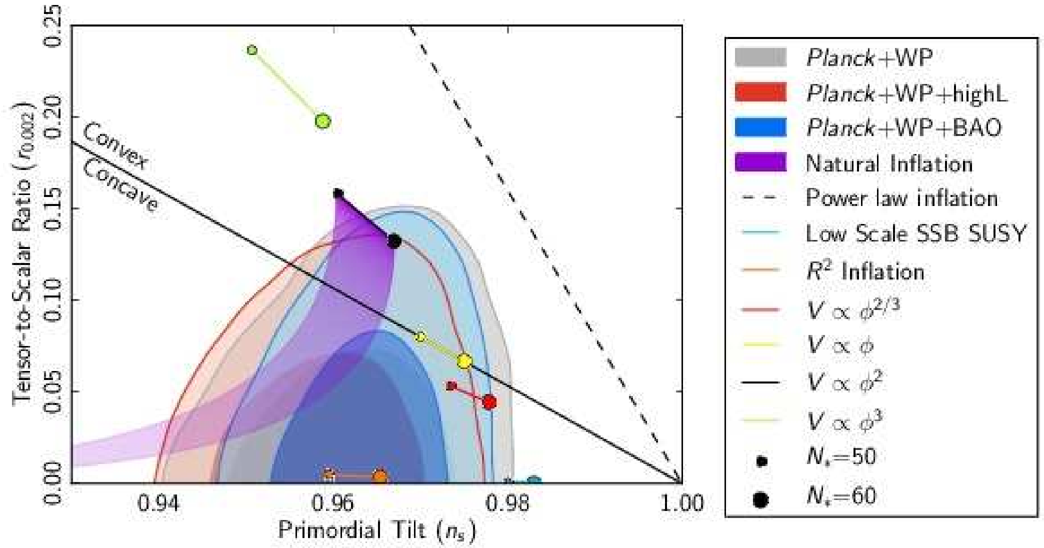


To decrease uncertainty: (the LHC accuracy can be as small as 200 MeV!)

- Compute remaining two-loop $\mathcal{O}(\alpha^2)$ corrections to pole - $\overline{\mathrm{M}S}$ matching for the Higgs mass and top masses. Theoretical uncertainty can reduced to ~ 0.5 GeV, due to irremovable non-perturbative contribution $\sim \Lambda_{QCD}$ to top quark mass.
- Measure better t-quark mass (present error in m_H due to this uncertainty is $\simeq 4$ GeV at 2σ level): construct t-quark factory – e^+e^- or $\mu^+\mu^-$ collider with energy $\simeq 200 \times 200$ GeV. The same conclusion - Alekhin et al, '12. LHC?
- Measure better α_s (present error in m_H due to this uncertainty is $\simeq 1$ GeV at 2σ level)

Experimental progress, Planck



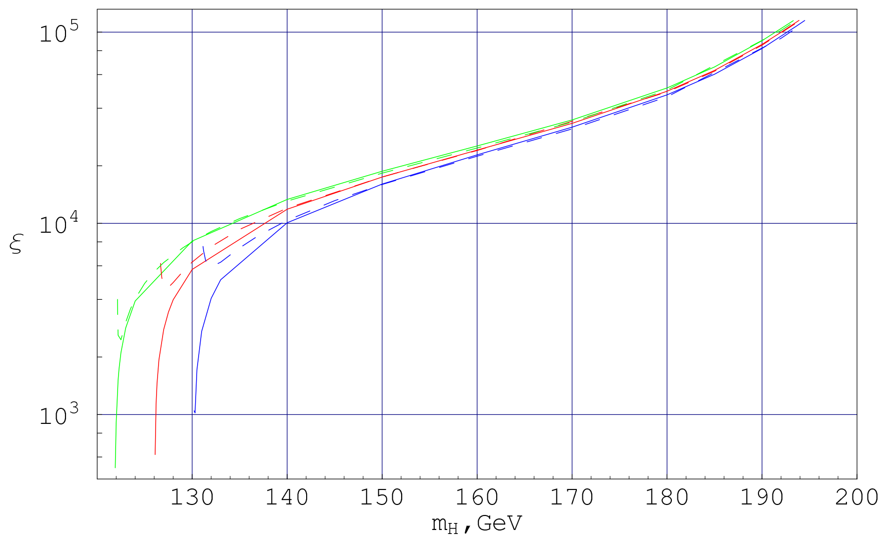


No deviations from Gaussianity

Self-consistency of Higgs inflation

Non-minimal coupling

The LHC has determined the Higgs mass - we can find the non-minimal coupling of the Higgs to gravity, $\xi = 700 - 1000$



ξ at the scale M_P/ξ depending on the Higgs mass m_H for $m_t = 171.2,\ 169.1,\ 173.3$ GeV (from upper to lower graph). Solid lines correspond to prescription I, dashed—to prescription II.

Large ξ - bad or good?

Sibiryakov, '08; Burgess, Lee, Trott, '09; Barbon and Espinosa, '09

Tree amplitudes of scattering of scalars above electroweak vacuum hit the tree unitarity bound at energies

$$E > \Lambda \sim \frac{M_P}{\xi}$$

The typical energy scale at inflation is

$$\frac{M_P}{\sqrt{\xi}} \gg \Lambda$$

Does it mean that the Higgs inflation is inconsistent?

Self-consistency of Higgs inflation

A way to proceed:

Bezrukov, Magnin, M.S., Sibiryakov; Ferrara, Kallosh, Linde, A. Marrani, Van Proeyen

- Take the non-zero Higgs field background, and define the region of applicability of perturbation theory. This region is background dependent.
- Compare the background dependent “cutoff” with the different energy scales important for inflation and for subsequent evolution of the Universe.

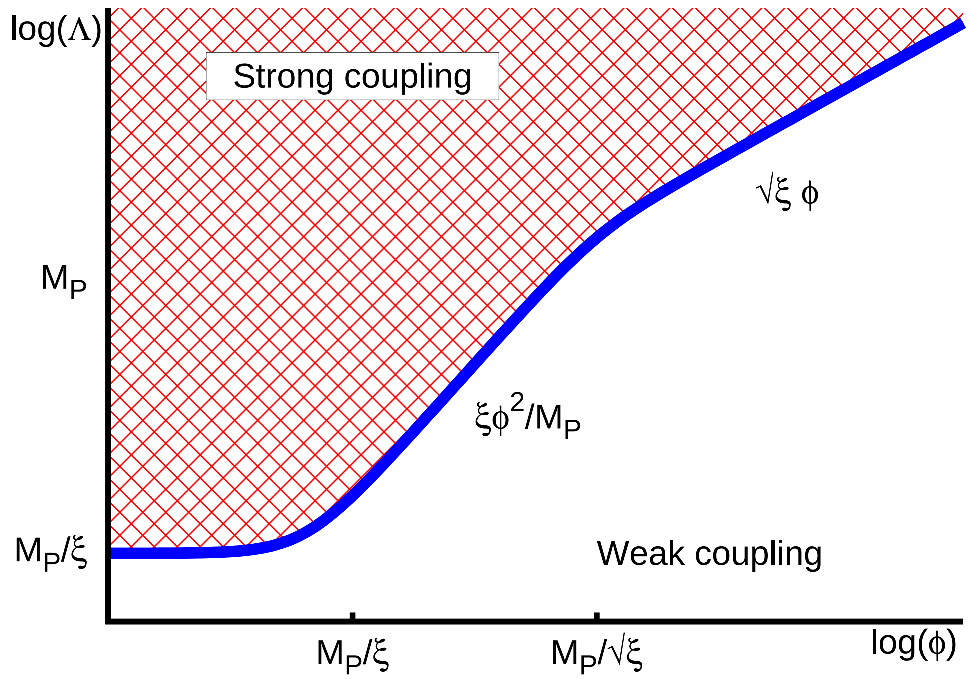
The background dependent “cutoff” - nothing unusual: Fermi interaction - “cutoff” does depend on the Higgs field.

Dynamical cutoff

Computation for the Higgs-gravity part of the SM:

$$\Lambda(h) \simeq \begin{cases} \frac{M_P}{\xi} \;, & \text{for } h \lesssim \frac{M_P}{\xi} \;, \\ \frac{h^2\xi}{M_P} \;, & \text{for } \frac{M_P}{\xi} \lesssim h \lesssim \frac{M_P}{\sqrt{\xi}} \;, \\ \sqrt{\xi} h \;, & \text{for } h \gtrsim \frac{M_P}{\sqrt{\xi}} \;. \end{cases}$$

Higgs-dependent cutoff



Cutoff is higher than the relevant dynamical scales throughout the whole history of the Universe, including the inflationary epoch and reheating!!

The Higgs-inflation is self-consistent.

Higher dimensional operators and relation between low energy and high energy parameters

Effective theory of Higgs inflation

Theory is non-renormalizable (as any theory with gravity).

Let's add to all counter-terms necessary to make it finite with all possible constant parts having the same structure as counter-terms.

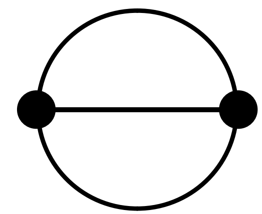
The procedure must respect the classical symmetries of the theory (scale invariance in Jordan frame = shift symmetry in Einstein frame). Technically - use dimensional regularisation and $\overline{\mathrm{MS}}$ subtraction procedure.

Starting Lagrangian:

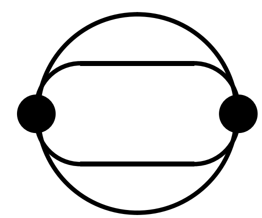
$$L = \frac{(\partial_\mu \chi)^2}{2} - U(\chi)$$

where $U(\chi)$ has at large fields the generic form

$$U(\chi) = U_0 \left(1 + \sum_{n=1}^{\infty} u_n e^{-n\chi/M} \right)$$

$\frac{U_0 u_n}{M^3} e^{-n\bar{\chi}/M}$  $\frac{U_0 u_m}{M^3} e^{-m\bar{\chi}/M}$

$$\propto \frac{1}{\epsilon} \cdot \frac{U_0^2}{M^8} u_n u_m (\partial_\mu \bar{\chi})^2 e^{-(n+m)\bar{\chi}/M} \quad ,$$

$\frac{U_0 u_n}{M^4} e^{-n\bar{\chi}/M}$  $\frac{U_0 u_m}{M^4} e^{-m\bar{\chi}/M}$

$$\propto \frac{1}{\epsilon} \cdot \frac{U_0^2}{M^8} u_n u_m \left(\frac{(\partial^2 \bar{\chi})^2}{M^2} + \frac{(\partial \bar{\chi})^4}{M^4} \right) e^{-(n+m)\bar{\chi}/M}$$

Effective action, incorporating radiative corrections:

$$L = f^{(1)}(\chi)\frac{(\partial_\mu \chi)^2}{2} - U(\chi) + f^{(2)}(\chi)\frac{(\partial^2 \chi)^2}{M^2} + f^{(3)}(\chi)\frac{(\partial \chi)^4}{M^4} + \cdots$$

where dots stand for terms with more derivatives. The coefficient functions are (formal) series in the exponent,

$$f^{(i)}(\chi) = \sum_{n=0}^{\infty} f_n^{(i)} e^{-n\chi/M}$$

Important: asymptotic shift symmetry $\chi \to \chi + const$ in Einstein frame (or scale invariance in the Jordan frame). Prescription I.

Consequences

- Predictions for $n_s, \ r$ remain in force.
- If coefficients in front of higher dimensional operators are of the order of one, the shift in the critical Higgs mass is < 1 GeV

Important assumption: no new physics between the Fermi scale and inflationary scale.

Conclusions

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Conclusions

- Higgs boson of the Standard Model can make the universe flat, homogeneous and isotropic, and can lead to primordial perturbations needed for structure formation
- Higgs inflation is consistent with the LHC results: LHC evidence for the Higgs boson with the mass close to $m_{\min}$ and no indications for new physics
- Higgs inflation is consistent with the Planck results: correct n_s, small r, and no non-Gaussian