Outline

- Why bothering about second-order perturbations?
- The Song code
- Reliability
- Conclusions

What is Song?

Song stands for Second-Order Non-Gaussianity and is a code that

- Evolves the Boltzmann-Einstein system of differential equations at second order in the cosmological perturbations
- Computes the value of the transfer functions today
- Builds the observables

It is a second-order version of codes like CMBFAST, Camb, CLASS.

Seljak & Zaldarriaga, 1996
Lewis, Challinor & Lasenby, 2000
Blas, Lesgourgues & Tram, 2011

Why is Song useful?

Song includes/will include many physical effects cannot be predicted by linear theory

- Mode coupling
- Non-linear scattering
- Propagation in an inhomogeneous Universe
- Scalar-Vector-Tensor mixing

Each of these effects affect the observables of the CMB in a way that linear theory cannot predict.

Mode coupling

All scales are coupled throught the convolution integral

$$a_{lm}(\mathbf{k}) = \mathcal{T}_{lm}{}^{(1)}(k)\,\Phi_{\rm in}(\mathbf{k}) + \mathcal{K}\left\{\mathcal{T}_{lm}{}^{(2)}(\mathbf{k_1},\mathbf{k_2},\mathbf{k})\,\Phi_{\rm in}(\mathbf{k_1})\,\Phi_{\rm in}(\mathbf{k_2})\right\}$$

Main consequence: non vanishing intrinsic bispectrum!

$$\left\langle \Phi_{\rm in}(\mathbf{k_1})\,\Phi_{\rm in}(\mathbf{k_2})\,\Phi_{\rm in}(\mathbf{k_3})\right\rangle = 0 \quad \not\Rightarrow \quad \left\langle a_{l_1m_1}\,a_{l_2m_2}\,a_{l_3m_3}\right\rangle = 0$$

The non-linear dynamics inevitably leads to an intrinsic bispectrum even in the absence of primordial one

The Intrinsic Bispectrum

We quantified this inevitable intrinsic bispectrum for the CMB using Song, and we found that:

GWP, Fidler, Crittenden, Koyama, Wands 2013

- It is just below the detection treshold of Planck
- It does not bias Planck's search for primordial non-Gaussianity ($f_{NL} = 0.5$)

There are other contexts where it is relevant

- It is strongly enhanced in some f(R) modified gravity theories

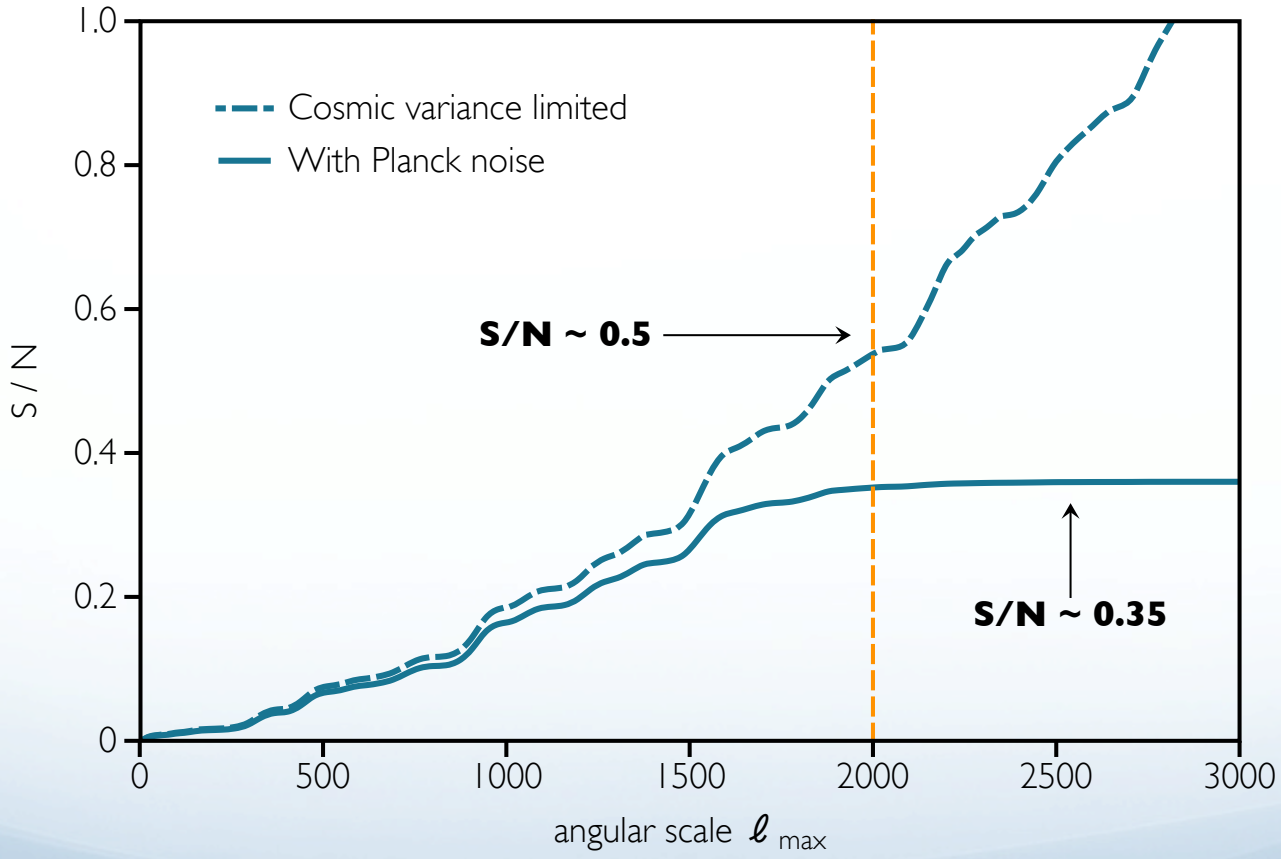
Gao X., 2011, PLB

- It should be accounted for in the initial conditions of non-Gaussian N-body simulations (BAO in the bispectrum)

e.g. Hidalgo, Christopherson & Malik 2013

Intrinsic Bispectrum

Signal of the intrinsic bispectrum for Planck best-fit parameters (scalar contribution)



Intrinsic Bispectrum

Contamination to the various primordial non-Gaussian models

Model	B^R	B^{R+Z}	B^{R+Z+M}	S/N
Local	2.5	0.57	0.50	0.24
Equilateral	6.7	4.7	4.3	0.018
Orthogonal	-5.1	-1.13	-1.05	0.037
S/N	0.77	0.47	0.47	—

Table 1. Correlations between the primordial templates and the intrinsic bispectra, computed as $f_{\rm NL}^{\rm intr} = F^{B,T}/F^{T,T}$. The signal-to-noise S/N is given by the square root of the autocorrelation.

Scalar-Vector-Tensor coupling

Vorticity and anisotropic stresses are naturally induced at second order.

Example: vector modes in Newtonian gauge

$$\omega\left(\mathbf{k},\mathbf{k_1},\mathbf{k_2}\right) \supset \frac{2\,\mathbf{k_1}}{4\mathcal{H}^2-4\dot{\mathcal{H}}+k^2}\left[2\,\mathcal{H}\,\Psi_1\left(\Psi_2-\Phi_2\right)-2\,\Phi_1\,\dot{\Phi}_2-4\,\Phi_2\,\dot{\Phi}_1+\Psi_1\,\dot{\Phi}_2\right]+\text{symm}$$

Magnetic fields are generated by the vortical currents at recombination

Fenu, Pitrou & Maartens 2011

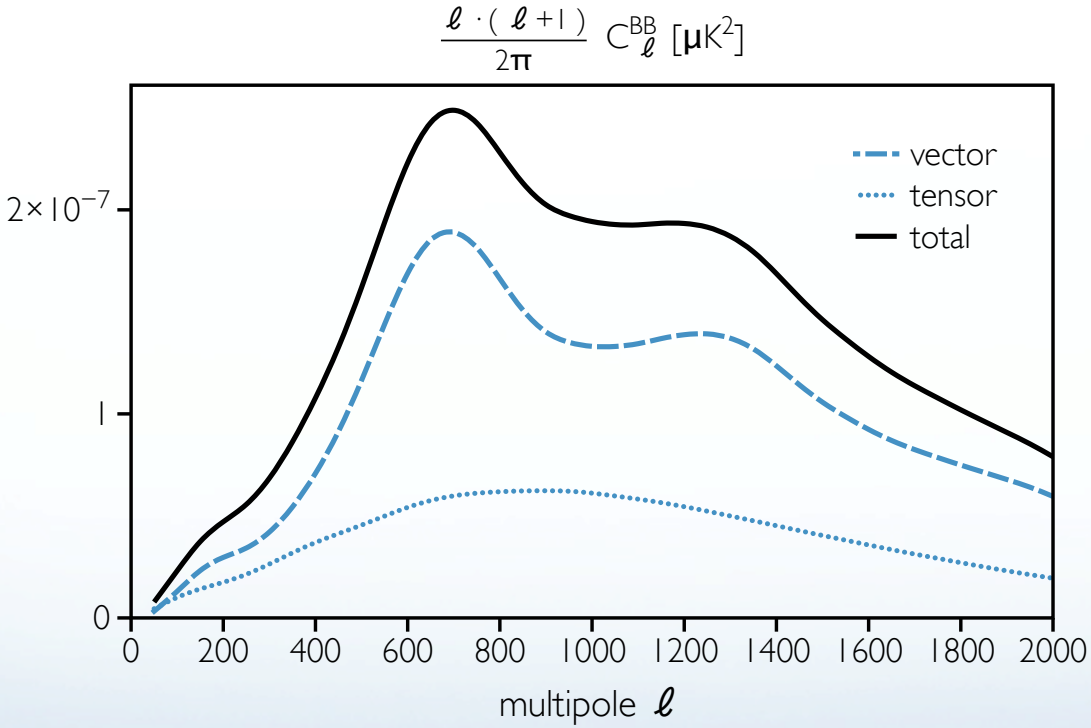
B-mode polarisation is sourced by the vector and tensor modes

Mollereach, Harari & Matarrese 2004
Beneke, Fidler & Klingmuller 2011

The B-mode polarisation biases the search for primordial gravitational waves

B-mode polarisation

We find that the B-mode polarisation is comparable in amplitude to a primordial signal of $r \sim 10^{-5}$.



To be submitted in 2013, Fidler C, **GWP**, Crittenden R, Koyama K, Wands D.

Propagation through inhomogeneities

Line of sight effects are small in principle but they are integrated over large distances (~ 14.000 Mpc).

At first order, there is only ISW. At second order we have extra ones, one of which being gravitational lensing.

- The ISW-lensing correlation is particularly important for the squeezed shape and it is known very well.

  Goldberg & Spergel 1999
  Seljak & Zaldarriaga 1999
  Lewis, Challinor & Hanson 2011
  Lewis 2012

- The lensing correlates with more terms in the bispectrum; it is important to quantify the importance of these correlations and eventually subtract them (or measure them!) from the data.

First item in the to do list!

Spectral distortions

Unlike the first order case, at second order the blackbody shape of the spectrum is not preserved because of the energy transfer between photons and electrons.

During recombination and reionization, spectral y-distortions are induced in the photon temperature distribution that are similar to those induced by the thermal SZ effect.

The y-distortions can be implemented in Song, and studying them can help answering the following questions:

- Can they be measured (e.g. by PIXIE or PRISM)?
- What can they tell us about the details of reionization?
- How do they behave in different cosmological models?

Why is Song useful?

Song is not meant to be a high-precision replacement of the linear Boltzmann codes.

Song is a tool that, given a cosmological model, provides predictions for "new" observables.

Technical details about Song

- Based on CLASS Blas, Lesgourgues & Tram JCAP07(2011)034

  written in C from scratch
  uses a brand new evolver optimised to solve stiff equations (*ndf15*)
  comes with an python frontend + MCMC

- Fast

  parallelised with OpenMP
  uses novel algorithms for Bessel convolution, bispectrum integration, 3D interpolation
  takes 8 CPU hours to generate a full-sky bispectrum (6 minutes on 60 cores)

- Easy to use

  modular structure
  PhD-student-friendly, with more than 10.000 lines of comments!

Power is nothing without control

The computation of perturbations at second order is a messy business, with many numerical and analytical pitfalls. To reduce the risk to a minimum, we performed many tests on Song.

- We match numerous analytical limits
- We performed rigorous convergence and consistency checks
- Song matches the results of the code used in *Beneke & Fidler 2011*, which uses a completely different approach based on Green functions.

Analytical limits

The behaviour of the CDM density contrast during matter domination on sub-Horizon scales is known analytically

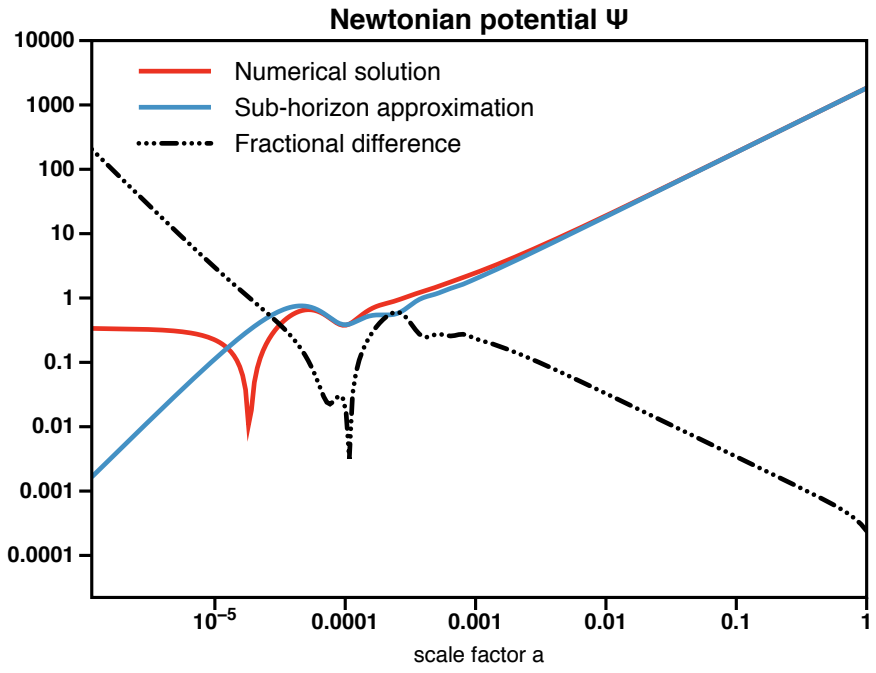
$$\delta \simeq 2\,\delta(k_1)\,\delta(k_2) \times \mathcal{K}\left[\frac{5}{7} + \frac{1}{2}\,\mathbf{k_1}\cdot\mathbf{k_2}\left(\frac{1}{k_1^2}+\frac{1}{k_2^2}\right) + \frac{2}{7}\,\frac{(\mathbf{k_1}\cdot\mathbf{k_2})^2}{k_1^2\,k_2^2}\right]$$

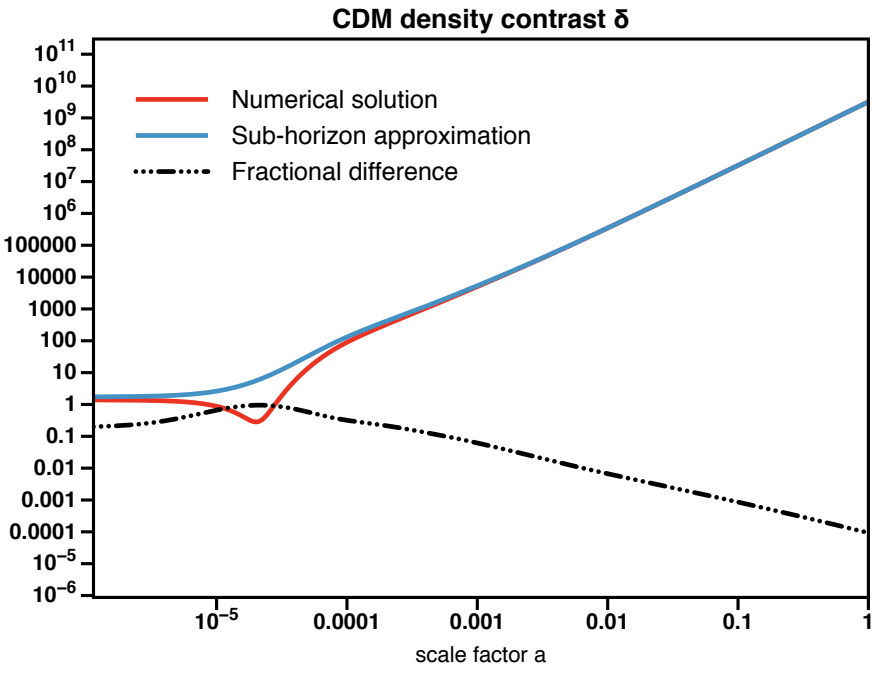
Goroff, Grinstein, et al., 1986
Makino, Sasaki & Suto, 1992
Jain & Bertschinger, 1994
Bernardeau et al., 2002

Similar formula exist for the scalar, vector and tensor potentials.

$$\Psi^{(2)}, \quad \omega^{(2)}, \quad \gamma^{(2)}$$

Bartolo, Matarrese & Riotto 2006
Matarrese, Mollerach & Bruni 1998
Boubeker et al. 2009


Newtonian potential Ψ
Numerical solution
Sub-horizon approximation
Fractional difference
10000
1000
100
10
1
0.1
0.01
0.001
0.0001
10^−5
0.0001
0.001
0.01
0.1
1
scale factor a


CDM density contrast δ
Numerical solution
Sub-horizon approximation
Fractional difference
10^11
10^10
10^9
10^8
10^7
10^6
100000
10000
1000
100
10
1
0.1
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10^−5
10^−6
10^−5
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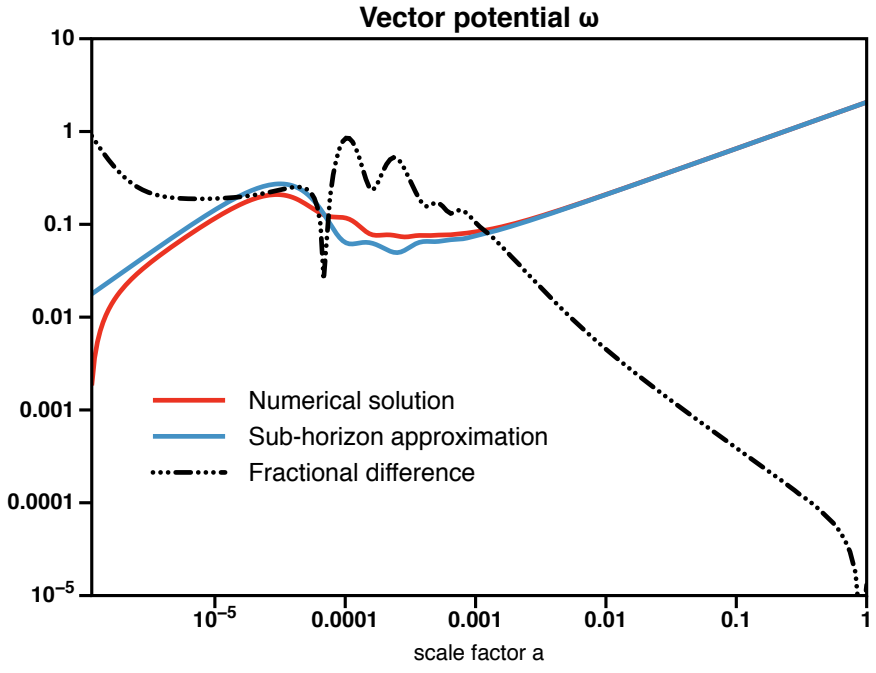
scale factor a


Vector potential ω
Numerical solution
Sub-horizon approximation
Fractional difference
10
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0.1
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0.001
0.0001
10^−5
10^−5
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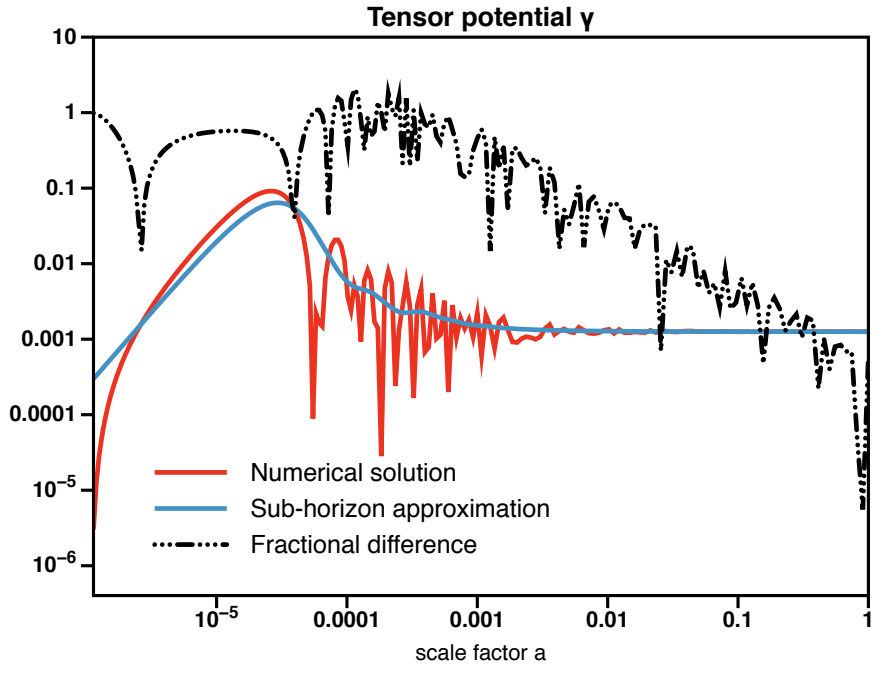
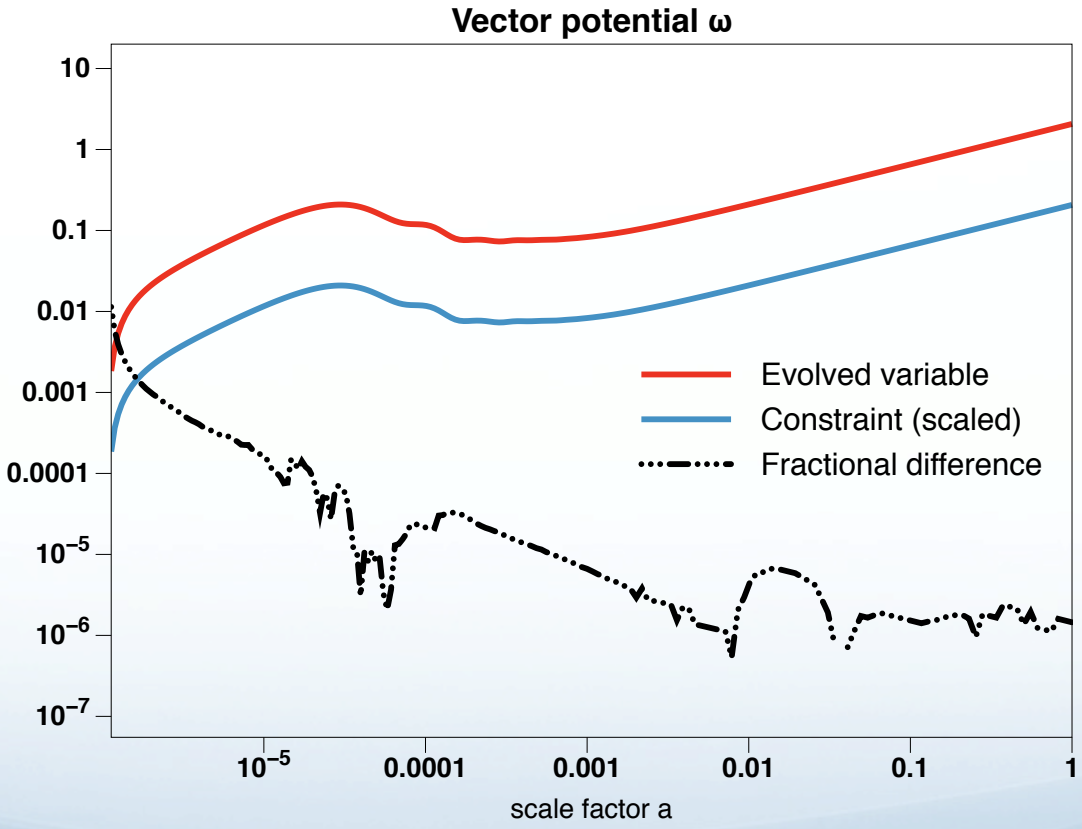
1
scale factor a


Tensor potential γ
Numerical solution
Sub-horizon approximation
Fractional difference
10
1
0.1
0.01
0.001
0.0001
10^−5
10^−6
10^−5
0.0001
0.001
0.01
0.1
1
scale factor a

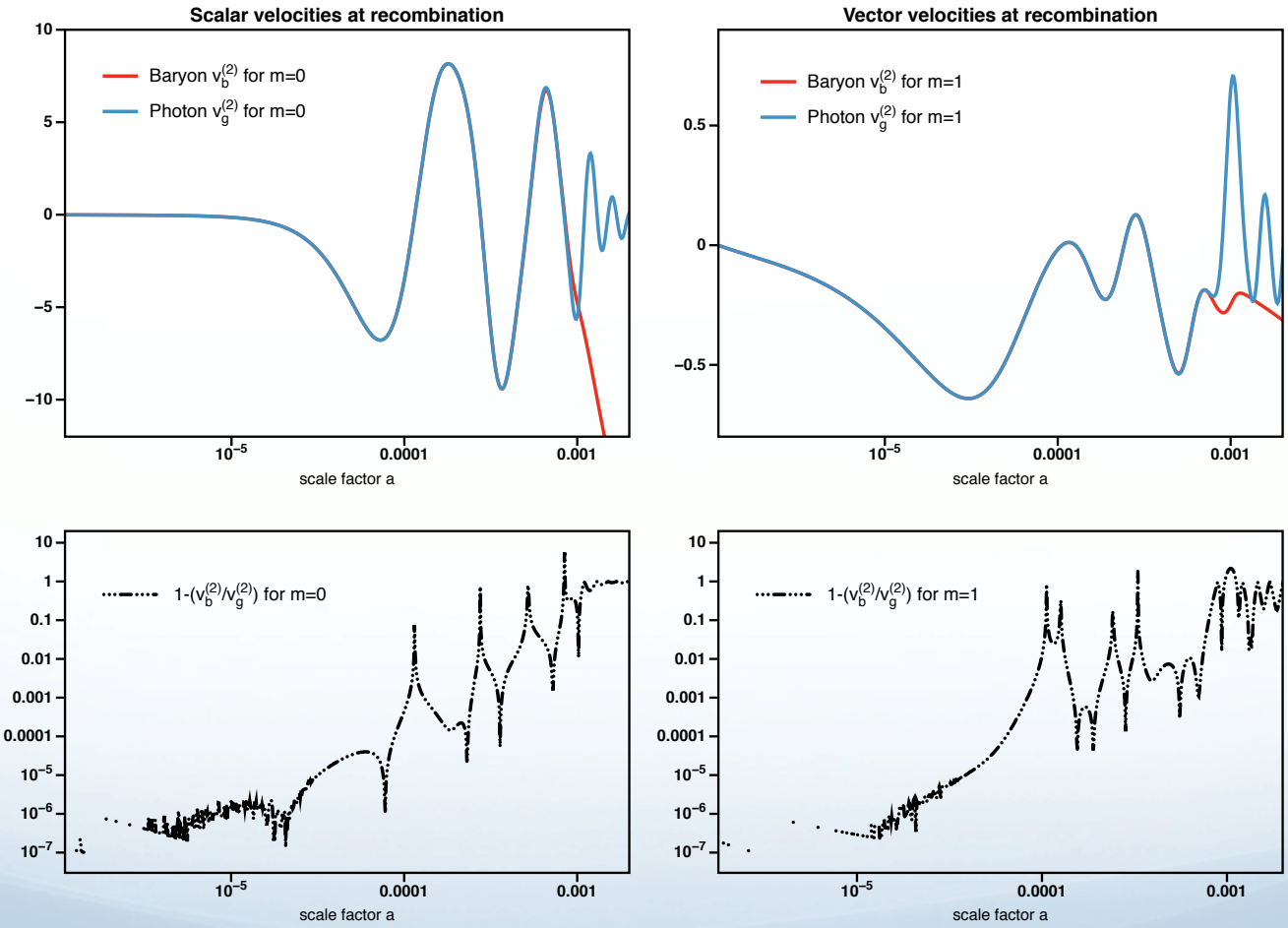
Analytical limits

We checked the consistency of the differential system by plugging the evolved variables into the Einstein constraint equations.



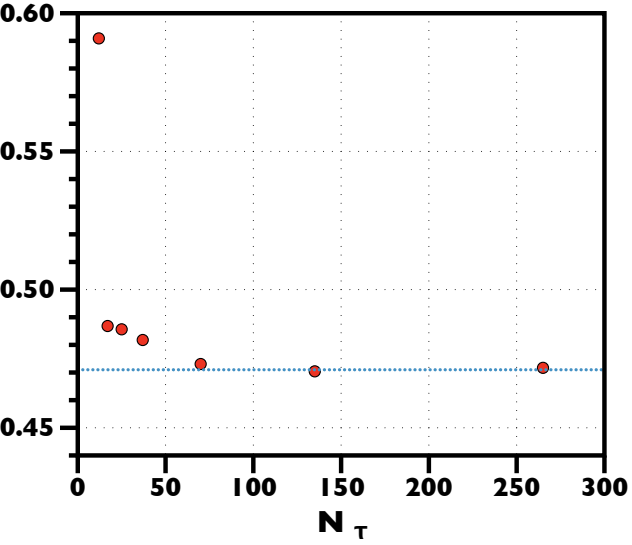
Tight coupling

We recover perfect tight coupling before recombination, as it should be.

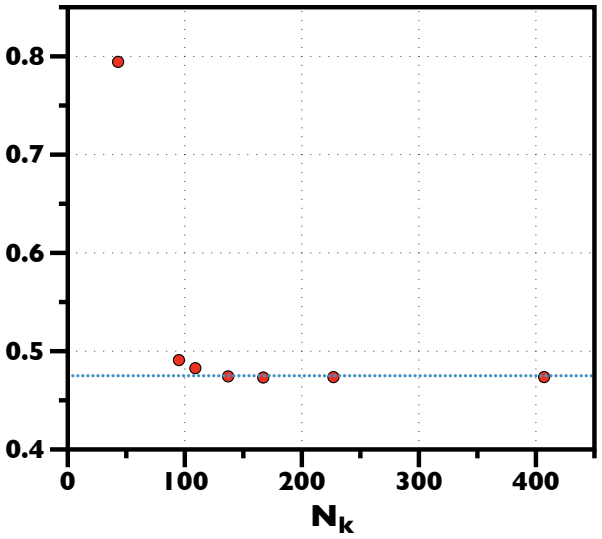


Convergence Tests on the S/N

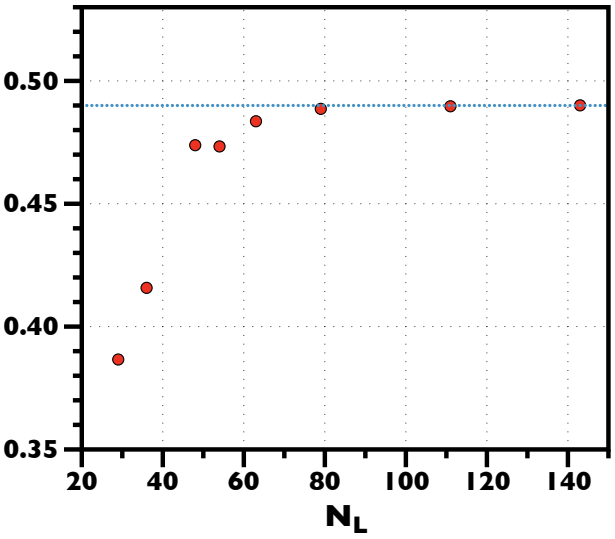
of time sampling points



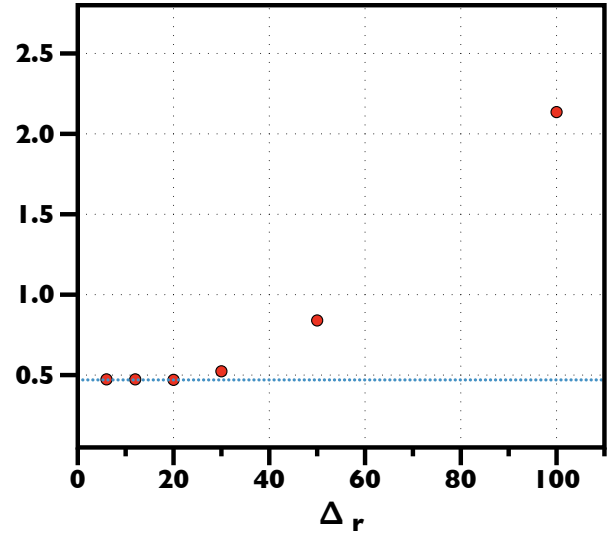
of points per k-direction



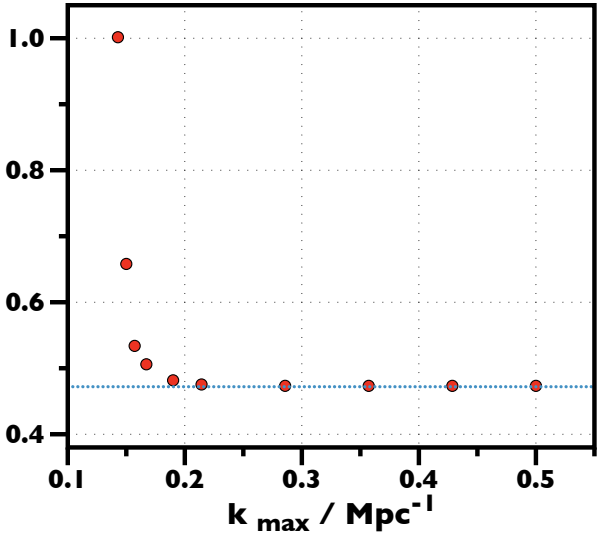
of points per l-direction



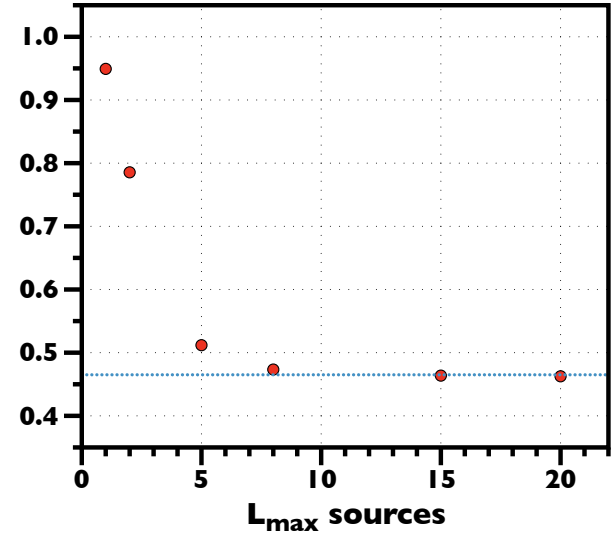
density of bispectrum integ. grid



maximum k considered



of included line of sight sources



Squeezed limit bispectrum

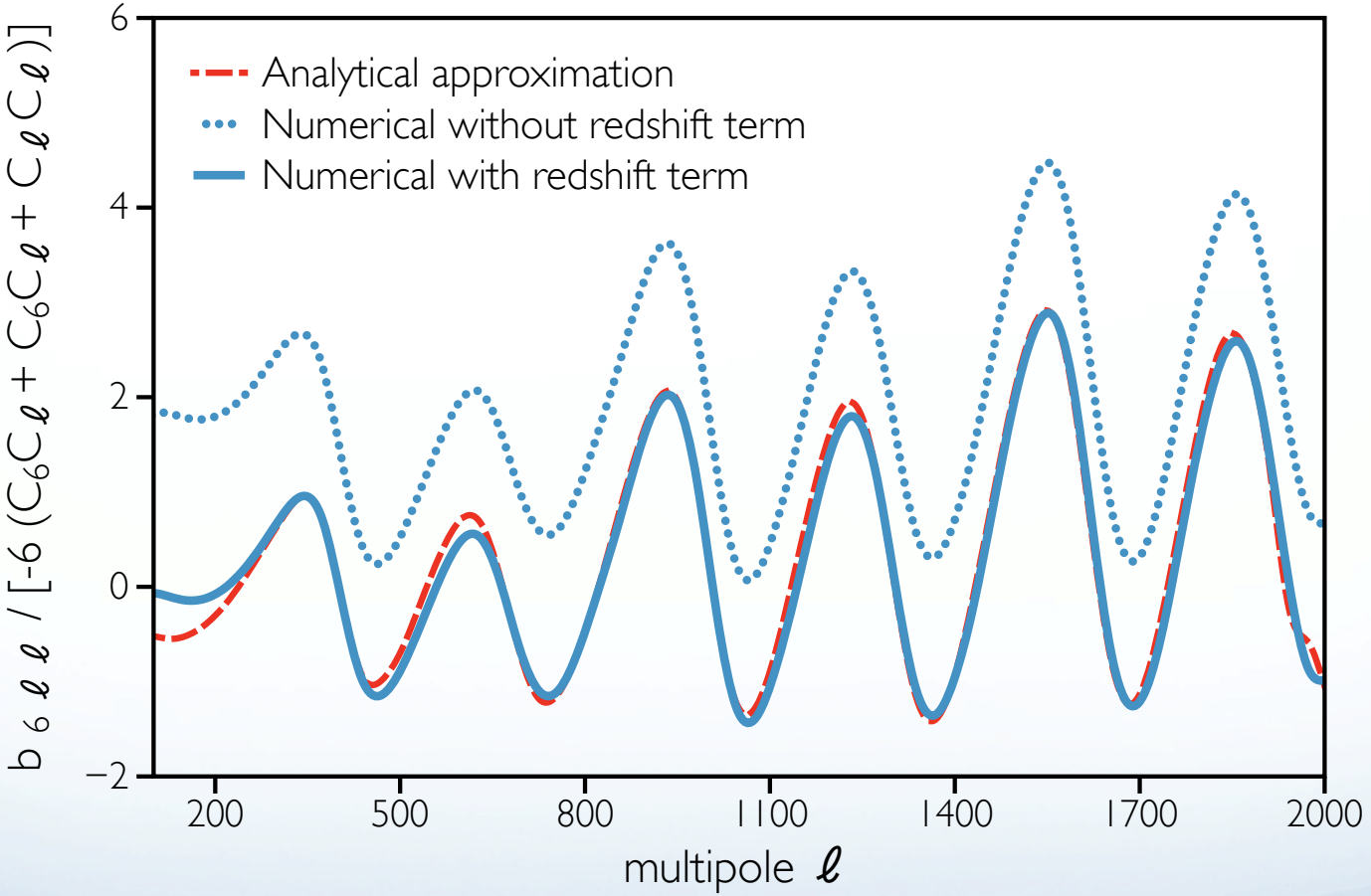
Analytical approximation in the squeezed limit :

$$b_{\ell_1 \ell_2 \ell_3} = -C_{\ell_1}^{T\zeta} \frac{1}{2} \left(C_{\ell_2} \frac{d\ln\left(l_2^2\, C_{\ell_2}\right)}{d\ln l_2} + C_{\ell_3} \frac{d\ln\left(l_3^2\, C_{\ell_3}\right)}{d\ln l_3} \right)$$

Creminelli et al. 2011
Bartolo et al. 2011
Lewis 2012

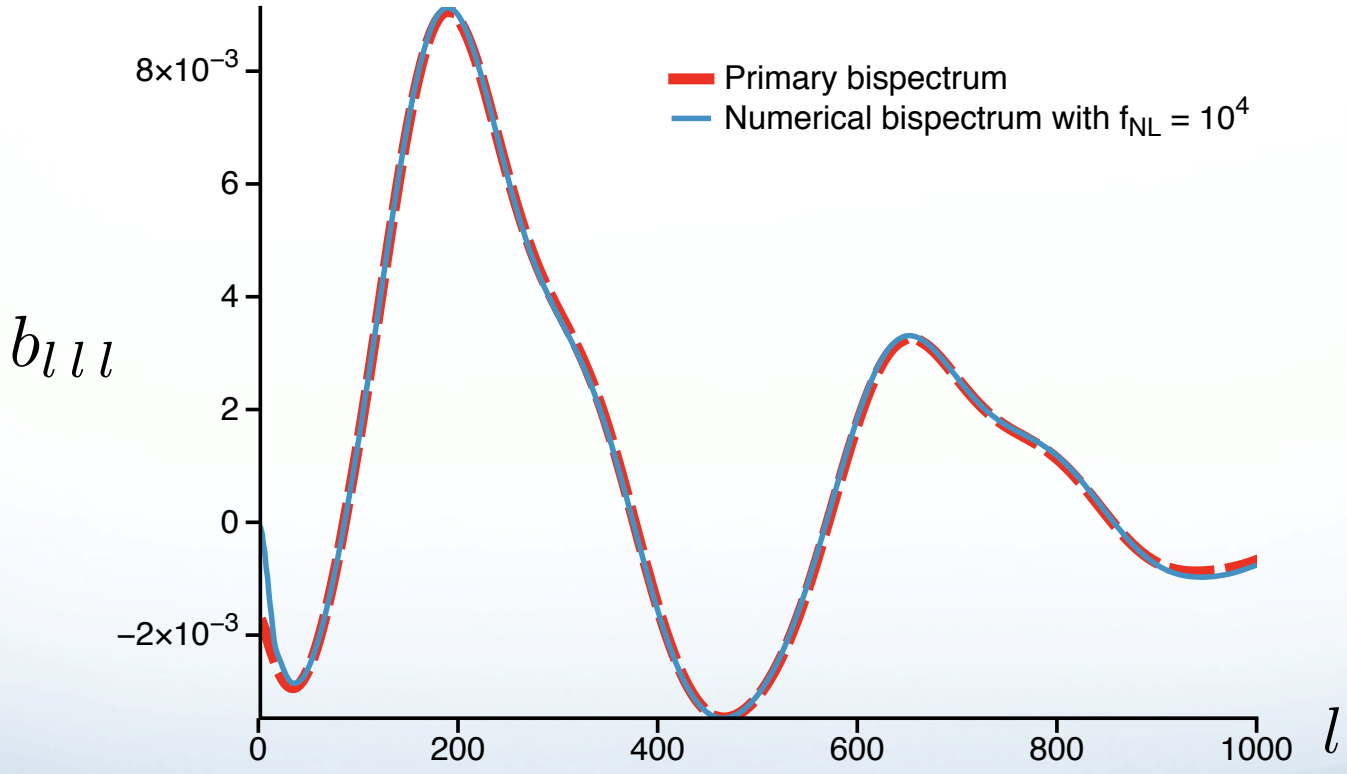
(assuming cold dark matter dominance and instantaneouos recombination)

Squeezed limit bispectrum



Full-sky bispectrum

Non-trivial check: primordial bispectrum versus numerical highly non-Gaussian one.



Conclusions

- We have developed Song, a second-order Boltzmann code that provides predictions for many new non-linear effects (not only in the CMB!)
- We find good news for Planck: the intrinsic bispectrum (excluding lensing) does not bias the estimates of primordial non-Gaussianity
- Similarly the B-mode polarisation from second order does not seem to contaminate the search for primordial gravitational waves
- Song can tell us something also about spectral distortions, magnetic fields, lensing.
- Song has passed many different analytical and numerical tests.

Current work

- Pre-release plan
  - Estimate full power spectrum of the B-mode polarisation
  - Estimate the contribution to the intrinsic bispectrum by vector and tensor modes
  - Include lensing and time-delay in the line of sight integration

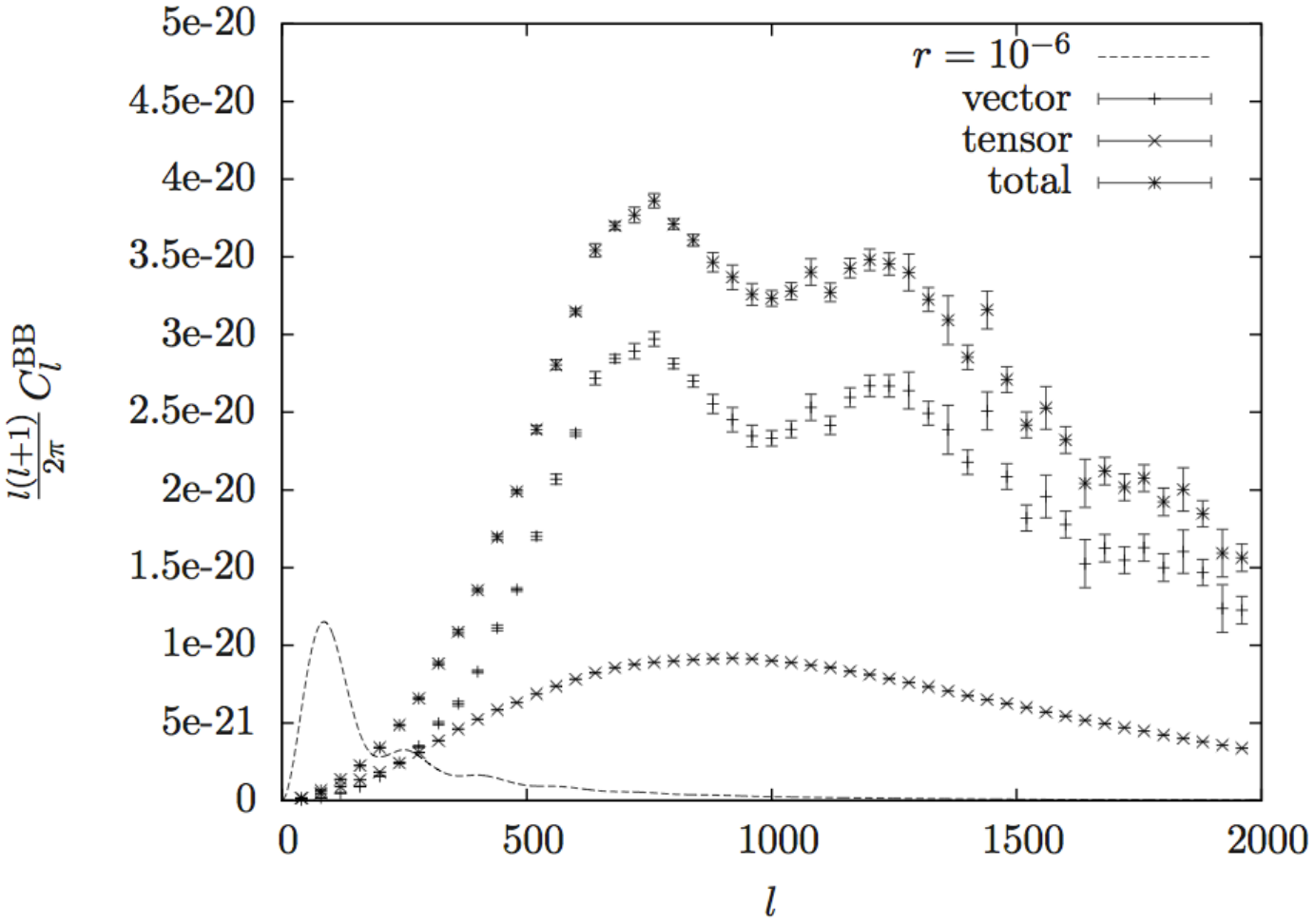
- Future work
  - Estimate magnetic field generation at recombination
  - Quantify spectral distortions at recombination and reionization
  - Estimate effect of initial bispectrum for N-Body simulations
  - Implement Synchronous gauge as a double check of the computation

Thank you for your attention!

What it currently does

- ✓ Transfer functions for photons, baryons, neutrinos and CDM
- ✓ Second-order spectra (T, E, B)
- ✓ Intrinsic & primordial full-sky bispectra (T, E, B)
- ✓ Fisher matrix module including noise model

B-mode polarization



Full-sky bispectrum

Involved integral on 6 highly oscillatory functions

Fergusson & Shellard, 2007, 2009
Fregusson, Regan & Shellard 2011

$$b_{l_1 l_2 l_3} = \int dr\, r^2 \int dk_1\, k_1^2\, T_{l_1}(k_1)\, P(k_1)\, j_{l_1}(r\, k_1) \int dk_2\, k_2^2\, T_{l_2}(k_2)\, P(k_2)\, j_{l_2}(r\, k_2) \int dk_3\, k_3^2\, T_{l_3}(k_1, k_2, k_3)\, j_{l_3}(r\, k_3)$$

Second-order transfer function

Lensing

Planck Paper XXIV, Planck Collaboration (2013)

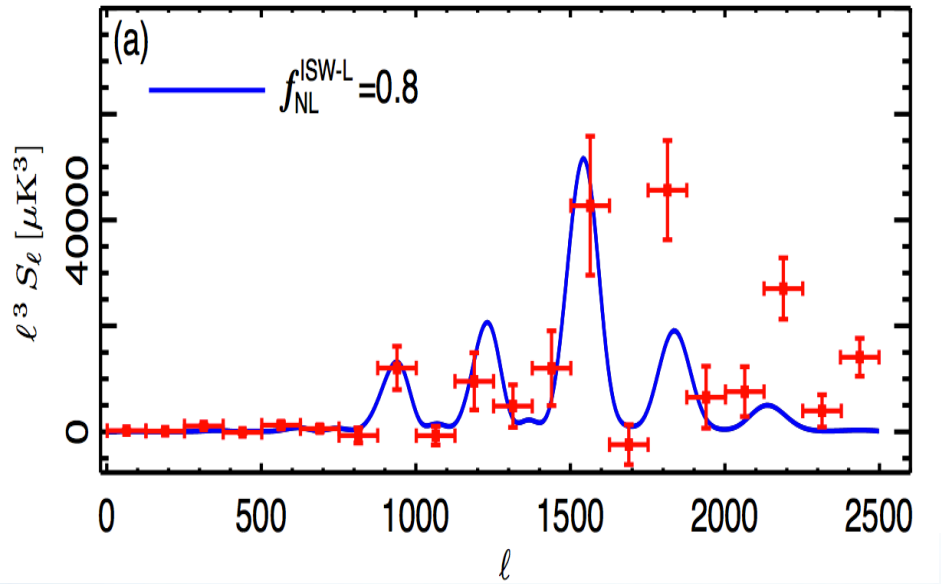


Fig. 2. The binned skew-C_ℓ statistics from the SMICA map for (a) ISW-lensing and (b) Poisson point sources. Theoretical curves are not fitted to the data shown, but are plotted with the amplitude (the only free parameter) determined from the KSW technique. The Poisson point-source foreground is clearly detected, and the ISW-lensing skew-spectrum is evident for $\ell < 1750$, with a suggestion of another source of NG at high ℓ. b_{ps} is the Poisson point-source amplitude in dimensionless units of 10^{-29}, and f_{NL}^{ISW-L} is the ISW-lensing amplitude in units of that expected from the *Planck* best-fit cosmology. Note that error bars are from data-averaging, and as a consequence are underestimates.

Why is Song useful?

Many physical effects cannot be predicted by linear theory

- Mode coupling
- Scalar-Vector-Tensor mixing
- Non-linear scattering
- Propagation in an inhomogeneous Universe

Each of these effects affect the observables of the CMB in a way that linear theory cannot predict. They are encoded in the second-order transfer function

$$a_{lm}(\mathbf{k}) = \mathcal{T}_{lm}{}^{(1)}(k)\,\Phi_{\rm in}(\mathbf{k}) + \mathcal{K}\left\{\mathcal{T}_{lm}{}^{(2)}(\mathbf{k_1},\mathbf{k_2},\mathbf{k})\,\Phi_{\rm in}(\mathbf{k_1})\,\Phi_{\rm in}(\mathbf{k_2})\right\}$$

which is computed by Song.