# Second-order Boltzmann Code and CMB Bispectrum from Recombination

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# Outline

Introduction

Second-order Boltzmann Code

CMB bispectrum

Results

# Planck Results on Primordial Non-Gaussianity

Foreground and ISW-lensing subtracted:

$$f_{\rm NL}^{\rm local} = 2.7 \pm 5.8$$

$$f_{\rm NL}^{\rm equilateral} = -42 \pm 75$$

$$f_{\rm NL}^{\rm orthogonal} = -25 \pm 39$$

# In a Nutshell

- observed bispectrum
  - primordial
    - local
    - equilateral
    - orthogonal
    - ...
  - secondary
    - Sachs-Wolfe
    - Doppler
    - Rees-Sciama
    - lensing (ref: Lewis et al)
    - integrated lensing
    - time delay(ref: Hu and Cooray)
    - integrated time delay
  - foreground
    - synchrotron
    - thermal dust
    - thermal/kinetic SZ
    - point sources
    - ...

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# The debate about $f_{\rm NL}^{\rm local}$ contamination

CMBquick era:

Khatri et al 2009 : $-1$

Nitta et al 2009 (only quadratic terms): 1

Pitrou et al 2010: $\sim 5$

Senatore et al 2010: $-3.5$ (*)

Creminelli et al 2011: 0.94

Bartolo et al. 2011: $O(1)$

Post-CMBquick era:

**Huang and Vernizzi:** 0.82 **(for** $\ell_{\rm max} = 2000$**)**

**Su et al. 2012:** 0.88 **(for** $\ell_{\rm max} = 2000$**)**

**Pettinari et al. 2013:** 0.5 **(for** $\ell_{\rm max} = 2000$**)**

# Second-order Boltzmann code

# Linear-order perturbations

$$\frac{d\delta X_i}{d\eta} + A_{ij}(\eta)\delta X_i = 0$$

$\delta X_i$ $(i = 1, 2, \ldots)$ are linear-order perturbations and $A_{ij}(\eta)$ are known background functions.

Perturbations include: baryon (density & velocity), CDM (density & velocity), neutrinos (phase-space distribution), radiation (phase-space distribution), and also DE if not a cosmological constant.

codes: CMBfast, CAMB, CMBEasy, CLASS, CosmoLib, CMBquick, ...

# Second-order perturbations

$$\frac{d\delta X_i^{(2)}}{d\eta} + A_{ij}(\eta)\delta X_j^{(2)} = S_i$$

$\delta X_i^{(2)}$ $(i = 1, 2, \ldots)$ are second-order perturbations (in Fourier space), $A_{ij}(\eta)$ remain the same, and the sources $S_i$ are convolutions of linear order perturbations.

Bruni *et al* 97; Pitrou *et al* 09, 10; Beneke & Fidler 10; Christopherson *et al* 08, 09, 11; Bartolo 07, 11; Senatore 08; Nitta *et al* 09; Khatri *et al* 09; Creminelli, Pitrou, and Vernizzi 11; Lewis 12 . . .

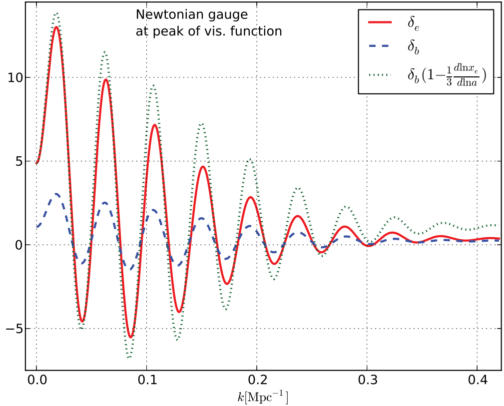
mathematica code CMBquick2 by Cyril Pitrou

# Fortran code CosmoLib$^{\mathrm{2nd}}$

Comparison with CMBQuick2 by Cyril Pitrou

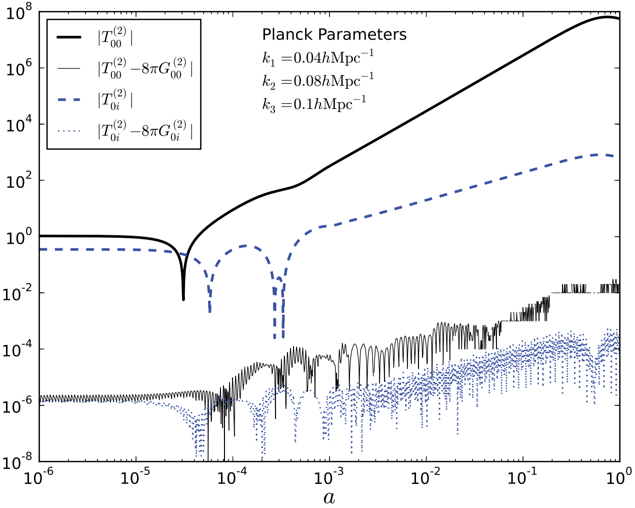
- Written in Fortran, no license constraint.
- Faster and parallelized.
- Much more accurate (energy and momentum constraint $\sim 10^{-6}$).
- Consistent treatment of perturbed RECFast (including Helium)
- Better scheme to integrate the CMB bispectrum (split out lensing and time delay boundary terms).
- Full-sky bispectrum.

# Perturbed electron number density $\delta_e \equiv \delta n_e / n_e$

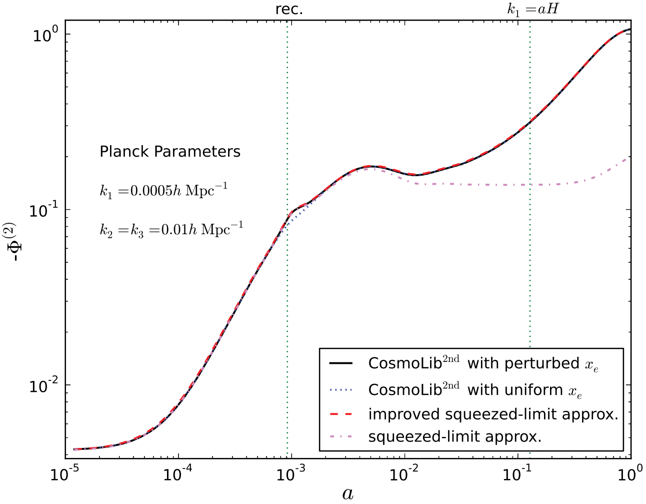



- Agree with the calculation in Senatore et al.
- $\delta_e \sim 5$ is large (expect large effective $f_{\rm NL}$?).

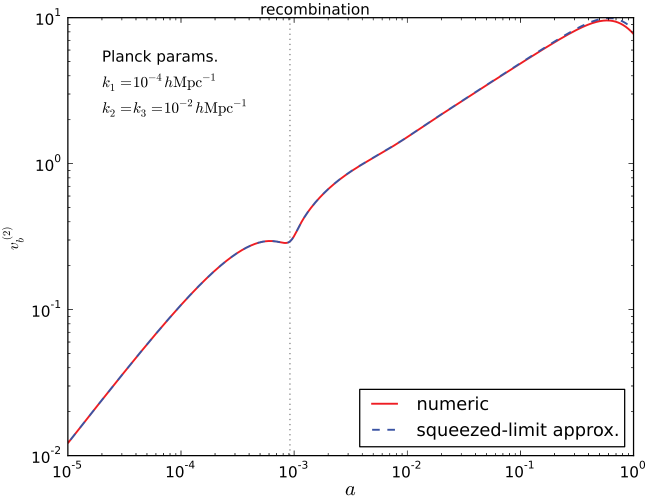
# Einstein equations: energy/momentum constraints



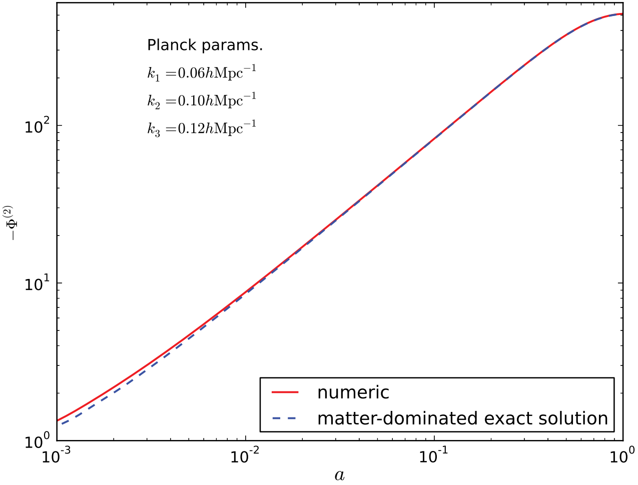
# Squeezed limit: gravitational potential



# Squeezed limit: baryon velocity



# Late-time exact solution



# CMB bispectrum

# The bispectrum: definition

$$\frac{\Delta T}{T}(\mathbf{n}) = \sum_{l,m} a_{lm} Y_{lm}(\mathbf{n}) \,.$$

$$\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = B_{l_1 l_2 l_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \,.$$

$$B_{l_1 l_2 l_3} = b_{l_1 l_2 l_3} \sqrt{\frac{(2l_1+1)(2l_2+1)(2l_3+1)}{4\pi}} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix}$$

# Primordial non-Gaussianity: a difficult integral

Primordial bispectrum $\Rightarrow$ CMB angular bispectrum

$$b_{l_1 l_2 l_3} = \int dx dk_1 dk_2 dk_3 (x k_1 k_2 k_3)^2 B_{\rm prim}(k_1, k_2, k_3) \Delta_{l_1}(k_1) \Delta_{l_2}(k_2) \Delta_{l_3}(k_3) j_{l_1}(k_1 x) j_{l_2}(k_2 x) j_{l_3}(k_3 x)$$

Solution: Reduce the 4D integral into products of 1D integrals by factorizing the primordial bispectrum $B_{\rm prim}(k_1, k_2, k_3) = \sum_i X_i(k_1) Y_i(k_2) Z_i(k_3)$.

See e.g. Fergusson *et al.* 09.

# Second-order perturbs: a more difficult integral

$$b_{l_1 l_2 l_3} \propto \sum_{m_3} \int dk_1 dk_2 d\mu d\eta S_{l_3' m_3}(k_1, k_2, \mu, \eta) j_{l_3}^{(l_3' m_3)}[k(\eta_0 - \eta)] \sum_{m_1 m_2} Y^*_{l_1 m_1} Y^*_{l_2 m_2} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \ldots + \mathrm{perms.}$$

Brute force almost impossible. However, the following tricks can be used to make it work:

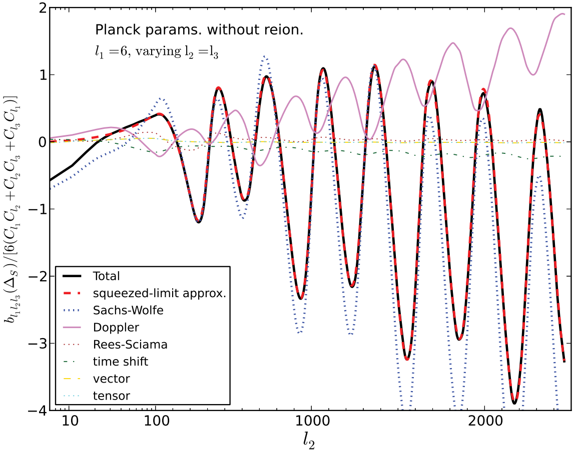
- The cosmology-independent geometrical factors (product of $Y_{\rm lm}$ and 3-$j$ symbols) can be precomputed and saved for interpolation.
- $j_l^{(l' m)}$ can be replaced with $j_l$ if we integrate by part (in a discrete way, using SVD decomposition).
- Use adaptive mesh.

# The squeezed-limit consistency relation

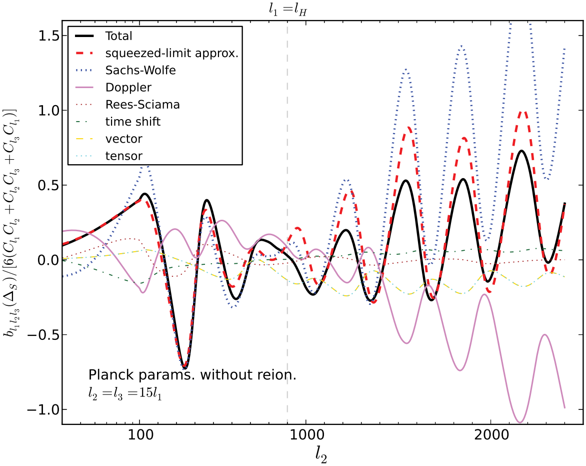
$$b_{l_S l_L l_L} = 2 C_{l_L}^{TT} C_{l_S}^{TT} - C_{l_L}^{T\zeta} \tilde{C}_{l_S}^{TT} \frac{d \ln \left[ \left( l_S + \frac{1}{2} \right)^2 \tilde{C}_{l_S}^{TT} \right]}{d \ln \left( l_S + \frac{1}{2} \right)}$$

,
if $l_L \ll l_S$ and $l_L \ll l_H \sim 60$.

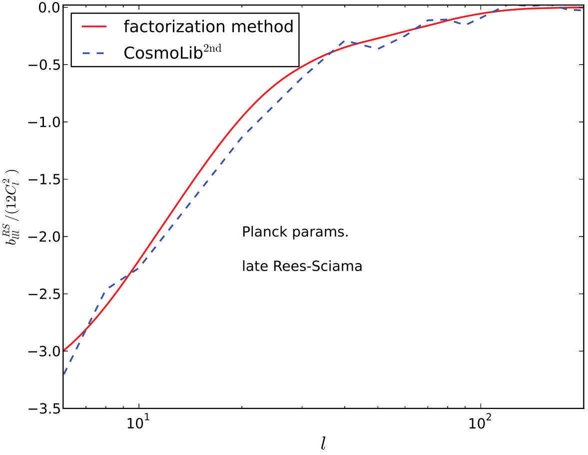
# Testing the consistency relation



# Testing the consistency relation



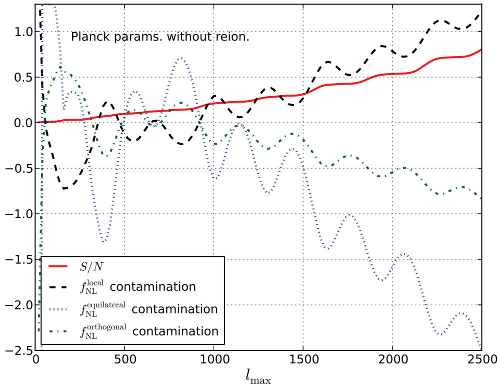
# Testing the integrator in non-squeezed limit



# Results and Conclusions

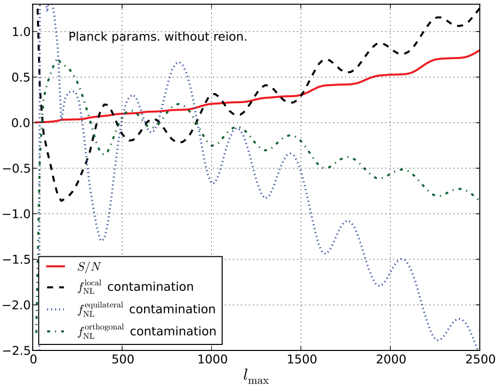
# Results

All contributions $m = 0$ only:



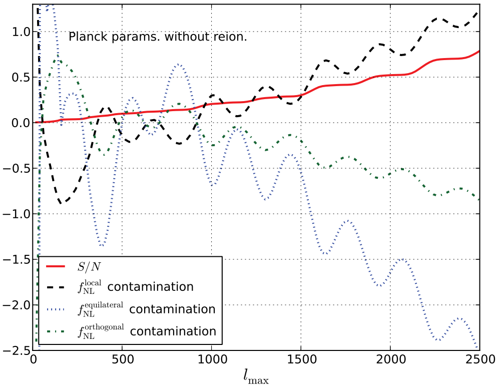
# Results

All contributions $m = 0, 1$:



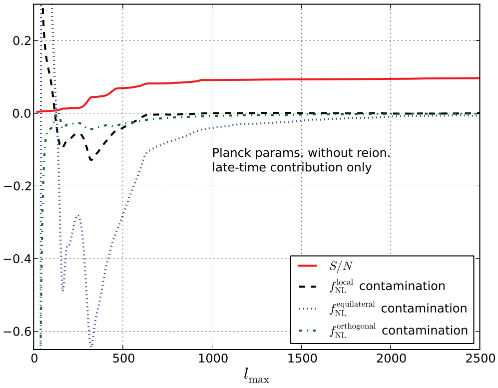
# Results

All contributions $m = 0, 1, 2$:



# Late-time integrated effects

Rees-Sciama + integrated lensing + integrated time delay



# Conclusions

- We did a very complicated calculation and we found nothing important.

# Conclusions

- Secondary effect (not including lensing) biases $f_{\rm NL}$ by $\approx 1$, if we use temperature data only. This bias can be removed using our code.
- The measurement of secondary effect itself is limited by cosmic variance if we use temperature data only.
- What if we use polarization? (to be done)

# The treatment of “lensing” and “time-delay” terms

The line of sight source:

$$\frac{d\Delta}{d\eta} = C + \partial_\eta \Delta + \partial_i \Delta \frac{dx^i}{d\eta} + \partial_{n^i} \Delta \frac{dn^i}{d\eta}$$