Statistical tools for nuclear experiments

Karsten Riisager

Aarhus University

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Contents

Main topics

- Preliminaries
- Fitting
- Testing
- Other topics

Subtopics

- Why statistics ?
- Aim of lectures
- Frequentist/Bayesian ?
- Probability distributions
- Data description
  - errors
  - covariance

General references

B — Roger Barlow: Statistics (Wiley)

C — Glen Cowan: Statistical Data Analysis (Oxford)

F — Fredrick James: Statistical Methods in Experimental Physics (World Scientific)

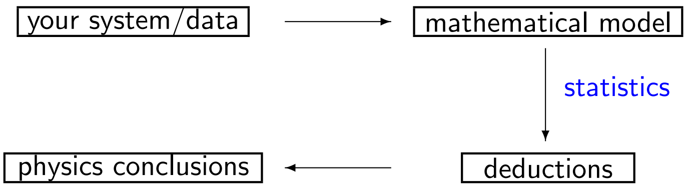
P — Particle Data Group 2012: J Beringer et al, Phys.Rev. D86, 010001 (2012) http://pdg.lbl.gov

Why statistics at all ?

Fluctuations due to:

- non-deterministic processes
- influences *not* under experimental control

Compare: error bar, uncertainty, systematic error, bias



What is special for nuclear physics ?

Often emphasis on count numbers
Frequently multi-parameter problems
Most practitioners meet statistics problems (few specialists)
Still more frequentist than Bayesian (long tradition/prehistory)

Compare to particle physics/astrophysics — statistics as used in biology/medicin/social science

A general overview

Assumptions: some acquaintance with statistics and data analysis

- Review of the important concepts
- Sketch of possible implementations
- Illustration with examples
- Excercises
  - train concepts
  - turn theory into practice

Aim: make you understand what you do when you fit — and why...

If all you have is a hammer, everything looks like a nail
A Maslow / M Twain/ ...

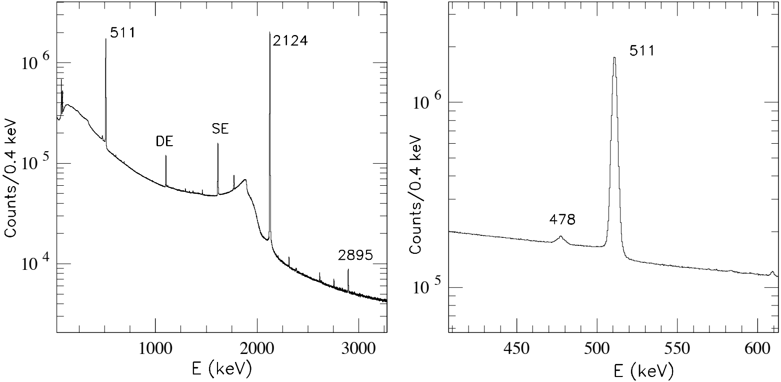
Type of questions you may find answers to...

- How do I turn my problem into a model ?
- Is my model compatible with the data ?
- What is the "measured value of a parameter" ?
- What is its error bar ?

And the more tricky ones:

- My fit is on top of the data, but χ^2 is bad. Why ?
- My χ^2 is good, but the fit looks funny...
- One data point is way off the rest. Can I throw it away ?
- Are error bars the same from multi-parameter fits as from single-parameter ?
- What to do in between low statistics and no statistics ?

The frustration of too many counts



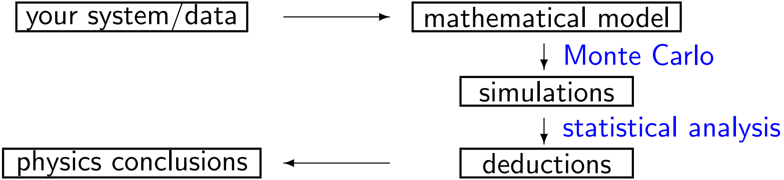
Describing detector physics rather than “wanted” physics ?!

How detailed do you need to be ?

More statistical tools → easier to focus on the essentials

How to avoid learning statistics

- build a good model
- do Monte Carlo simulations
- learn how to interpret them



So some statistics, sorry. . .

Frequentist or Bayesian statistics ?

Two different approaches/philosophies, in practice often similar answers.

Bayesian: "not a fit, but update of information"
Prior/posterior distributions, natural to incorporate restrictions and previous knowledge, testing complicated, decision-making straightforward.

Frequentist: "estimate general properties from one data set"
Focus on behaviour at maximum, significance test and goodness-of-fit test natural, hard to include extra information.

Arbitrary prior $\leftrightarrow$ arbitrary analysis method

P chap 36, **J**, F James PHYSTAT2003
http://www.slac.stanford.edu/econf/C030908/papers/THAT002.pdf

Basic Bayesian. . .

Bayes theorem: $P(a|b) = P(b|a)P(a)/P(b)$

Naively: $P(\mathrm{theory}|\mathrm{result}) = P(\mathrm{result}|\mathrm{theory})P(\mathrm{theory})/P(\mathrm{result})$

With prior ($\pi(\theta)$) and posterior ($p(\theta|x)$) distributions and the likelihood $L(x|\theta)$ this becomes:

$$p(\theta|x) = \frac{L(x|\theta)\pi(\theta)}{\int L(x|\theta')\pi(\theta')\mathrm{d}\theta'}$$

Bayesian inference in physics, U von Toussaint, Rev.Mod.Phys. 83 (11) 943

Bayesian inference in processing experimental data: principles and basic applications, G D'Aostini, Rep.Prog.Phys. 66 (03) 1383

Bayesian inference in physics: case studies, V Dose, Rep.Prog.Phys. 66 (03) 1421

Ingredients to build a model (1)

Need probability distributions
Gaussian (continuous) and Poisson (discrete) not sufficient
Overview in: **B** 3, **C** 2, **J** 4, **P** chap 35

Should know about
discrete: binomial, multinomial, Poisson
continuous: Gaussian, chi-square, uniform, exponential, (Breit-Wigner, Landau)

Central Limit Theorem = "Everything turns Gaussian"
Convergence in distribution (weak !), be careful with tails

Example: Poisson process

Def: process with independent increments at constant intensity λ

Time between counts has an exponential distribution.
The number of counts after time t is Poisson distributed with mean λt.

NB! Counting until a total given number of counts gives a multinomial distribution !

DAQ/detector deadtime $\rightarrow$ recorded count number never strictly Poisson. . .

Dead time: JW Müller, NIM A301 (91) 543 and refs therein

Ingredients to build a model (2)

What type of errors / uncertainties ?

error	statistical/intrinsic	systematic
caused by	inate imprecision	bias of setup
example	count number	calibration uncertainty
scales as	$\sqrt{N}$	const
points	uncorrelated	correlated

Need to treat correlations → covariance

See also **B** 4.4 (J Heinrich and L Lyons, Ann.Rev.Nucl.Part.Sci. 57 (07) 145)

Example: figure 21.1 in **P** (distances to Type 1a supernovae as function of redshift) – what errors/distribution is relevant here ?

How to summarize data/ results/ distributions

Location: mean median mode

Spread: variance/standard deviation MAD IR
why actually $(\Delta x)^2$ and not $|\Delta x|$?

Higher order moments ? skew curtosis

Much more important: covariance normalized = correlation coef

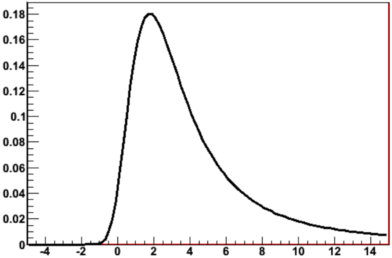
$$\mathrm{cov}(x, y) = \overline{(x - \overline{x})(y - \overline{y})} = \overline{xy} - \overline{x}\,\overline{y}$$

NB! “Exploratory data analysis” used in other fields

Example: Landau distribution

One example to show the limitations of the standard concepts
a less extreme example is given in the problems

The Landau distribution
used to describe energy loss



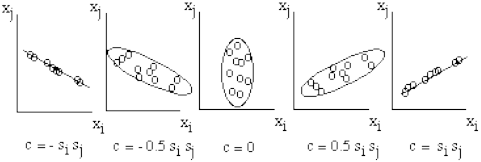
Mean and variance are undefined (median exists).

C2.9

Covariance matrix

Systematic effects can give covariance: $\begin{pmatrix} \sigma_1^2 + s^2 & s^2 \\ s^2 & \sigma_2^2 + s^2 \end{pmatrix}$

Data sets can have covariance



Also covariance between derived parameters, including fit parameters !
MINUIT gives correlations between fit parameters

B2.6+4.4.2, **C**1.5

Example: ^{12}Be halflife strongly correlated with daughter halflife, see figure 1 and 2 of U.C. Bergmann et al., Eur. Phys. J. A 11 (01) 279

Propagation of errors

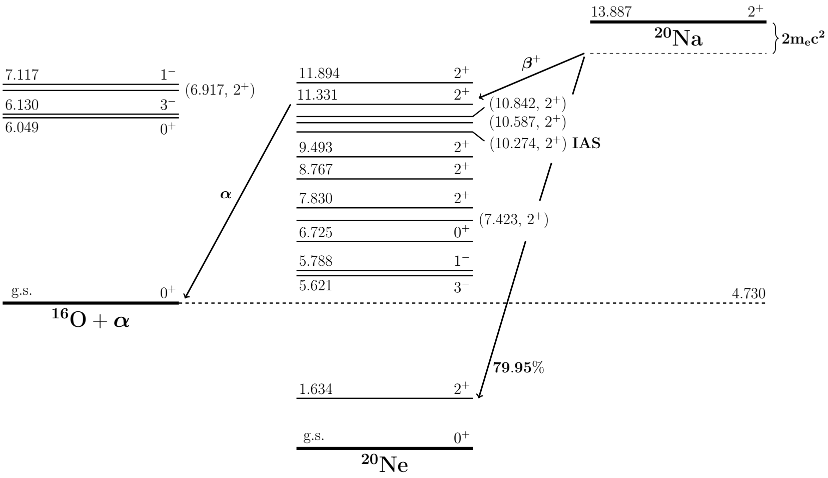
First method: do Taylor expansion to first order and take expectation values

$$\mathrm{cov}(f_k, f_l) = \sum_i \sum_j \frac{\partial f_k}{\partial x_i} \frac{\partial f_l}{\partial x_j} \mathrm{cov}(x_i, x_j)$$

- standard expression for $\sigma(f)$ when $\mathrm{cov}(x_i, x_j)$ is "diagonal"
- possible covariance between f's even with diagonal $\mathrm{cov}(x_i, x_j)$
  trivial example: area and circumference of rectangle

Second method: simulate input parameters, evaluate output parameters

^{20}Na $\beta\alpha$ decay, IGISOL at JYFL



K.L Laursen, O.S. Kirsebom et al., to be submitted

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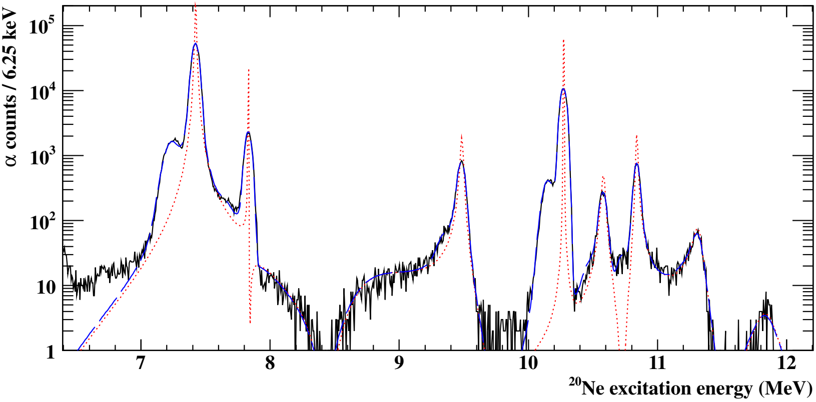

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^{20}Na $\beta\alpha$ decay: IAS/total α-branch

Four measurements of $I^{\alpha}_{IAS}/I^{\alpha}_{tot}$ (four detectors):

Detector 1: $13.848 \pm 0.038 \pm 0.018$ %
Detector 2: $13.869 \pm 0.033 \pm 0.020$ %
Detector 3: $13.824 \pm 0.036 \pm 0.024$ %
Detector 4: $13.923 \pm 0.039 \pm 0.020$ %

Check consistency, σ statistical, $\chi^2 = 3.72$.
Weighted average, simple average, add count numbers ??
Add statistical and systematic errors:
$= 13.864 \pm 0.018 \pm 0.024 = 13.864 \pm 0.030$.