



Combination inputs
Combination model
Model Construction



Combined Higgs model at ATLAS

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Individual channels

Higgs Boson Decay	Subsequent Decay	Sub-Channels	$\int L dt$ [fb $^{-1}$]	Ref.
2011 $\sqrt{s} = 7$ TeV				
$H \rightarrow ZZ^{(*)}$	4ℓ	{ $4e, 2e2\mu, 2\mu2e, 4\mu$ }	4.6	[1]
$H \rightarrow \gamma\gamma$	-	10 categories $\{p_{T\ell} \otimes \eta_\gamma \otimes \text{conversion}\} \oplus \{\text{2-jet VBF}\} \oplus \{\ell\text{-tag}, \text{2-jet VH}\}$	4.8	[5]
$H \rightarrow \tau\tau$	$\tau_{\text{lep}}\tau_{\text{lep}}$ $\tau_{\text{lep}}\tau_{\text{had}}$ $\tau_{\text{had}}\tau_{\text{had}}$	{ $e\mu$ } $\otimes \{\text{0-jet}\} \oplus \{\ell\ell\} \otimes \{\text{1-jet, 2-jet, } p_{T,\tau\tau} > 100 \text{ GeV, VH}\}$ $\{e, \mu\} \otimes \{\text{0-jet, 1-jet, } p_{T,\tau\tau} > 100 \text{ GeV, 2-jet}\}$ $\{\text{1-jet, 2-jet}\}$	4.6 4.6 4.6	[7]
$VH \rightarrow Vbb$	$Z \rightarrow \nu\nu$ $W \rightarrow \ell\nu$ $Z \rightarrow \ell\ell$	$E_T^{\text{miss}} \in \{120 - 160, 160 - 200, \geq 200 \text{ GeV}\} \otimes \{\text{2-jet, 3-jet}\}$ $p_T^W \in \{< 50, 50 - 100, 100 - 150, 150 - 200, \geq 200 \text{ GeV}\}$ $p_T^Z \in \{< 50, 50 - 100, 100 - 150, 150 - 200, \geq 200 \text{ GeV}\}$	4.6 4.7 4.7	[8]
2012 $\sqrt{s} = 8$ TeV				
$H \rightarrow ZZ^{(*)}$	4ℓ	{ $4e, 2e2\mu, 2\mu2e, 4\mu$ }	13	[6]
$H \rightarrow \gamma\gamma$	-	12 categories $\{p_{T\ell} \otimes \eta_\gamma \otimes \text{conversion}\} \oplus \{\text{2-jet VBF}\} \oplus \{\ell\text{-tag}, \text{2-jet VH}\}$	13	[5]
$H \rightarrow WW^{(*)}$	$e\nu\mu\nu$	{ $e\mu, \mu e$ } $\otimes \{\text{0-jet, 1-jet}\}$	13	[9]
$H \rightarrow \tau\tau$	$\tau_{\text{lep}}\tau_{\text{lep}}$ $\tau_{\text{lep}}\tau_{\text{had}}$ $\tau_{\text{had}}\tau_{\text{had}}$	{ $\ell\ell$ } $\otimes \{\text{1-jet, 2-jet, } p_{T,\tau\tau} > 100 \text{ GeV, VH}\}$ $\{e, \mu\} \otimes \{\text{0-jet, 1-jet, } p_{T,\tau\tau} > 100 \text{ GeV, 2-jet}\}$ $\{\text{1-jet, 2-jet}\}$	13 13 13	[7]
$VH \rightarrow Vbb$	$Z \rightarrow \nu\nu$ $W \rightarrow \ell\nu$ $Z \rightarrow \ell\ell$	$E_T^{\text{miss}} \in \{120 - 160, 160 - 200, \geq 200 \text{ GeV}\} \otimes \{\text{2-jet, 3-jet}\}$ $p_T^W \in \{< 50, 50 - 100, 100 - 150, 150 - 200, \geq 200 \text{ GeV}\}$ $p_T^Z \in \{< 50, 50 - 100, 100 - 150, 150 - 200, \geq 200 \text{ GeV}\}$	13 13 13	[8]

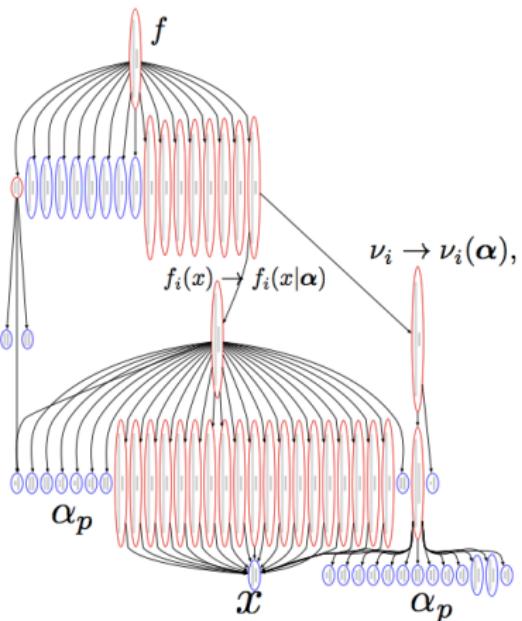
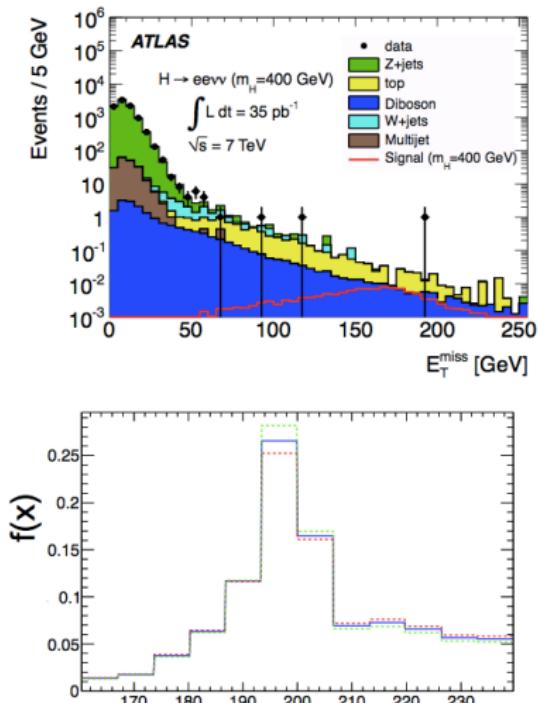


Individual channels

Higgs Boson Decay	Subsequent Decay	Modeling tools	$\int L dt$ [fb $^{-1}$]	Ref.
2011 $\sqrt{s} = 7$ TeV				
$H \rightarrow ZZ^{(*)}$	4ℓ	histfactory + parametrized pdf	4.6	[1]
$H \rightarrow \gamma\gamma$	–	parametrized pdf	4.8	[5]
$H \rightarrow \tau\tau$	$\tau_{\text{lep}}\tau_{\text{lep}}$ $\tau_{\text{lep}}\tau_{\text{had}}$ $\tau_{\text{had}}\tau_{\text{had}}$	histfactory	4.6 4.6 4.6	[7]
$VH \rightarrow Vbb$	$Z \rightarrow \nu\nu$ $W \rightarrow \ell\nu$ $Z \rightarrow \ell\ell$	histfactory	4.6 4.7 4.7	[8]
2012 $\sqrt{s} = 8$ TeV				
$H \rightarrow ZZ^{(*)}$	4ℓ	histfactory + parametrized pdf	13	[6]
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$H \rightarrow WW^{(*)}$	$e\nu\mu\nu$	histfactory	13	[9]
$H \rightarrow \tau\tau$	$\tau_{\text{lep}}\tau_{\text{lep}}$ $\tau_{\text{lep}}\tau_{\text{had}}$ $\tau_{\text{had}}\tau_{\text{had}}$	histfactory	13 13 13	[7]
$VH \rightarrow Vbb$	$Z \rightarrow \nu\nu$ $W \rightarrow \ell\nu$ $Z \rightarrow \ell\ell$	histfactory	13 13 13	[8]



Individual channels





Individual channels

- Individual channels provide their own workspaces in the form of workspaces.
 - ① $H \rightarrow \gamma\gamma$ and $H \rightarrow ZZ^{(*)} \rightarrow 4\ell$ provide mass parametrized models to facilitate mass combination
 - ② Other channels provide fixed-mass models
- Embed handles for coupling study.
 - ① `mu_XS_*` will scale the production XS
 - ② `mu_BR_*` will scale the decay BR
 - ③ They will be re-parametrized for coupling study



Outline

① Combination inputs

② Combination model

Combination model
Systematics

③ Model Construction

Combination tool
 μ /rate combination
Mass combination
Coupling measurements



Modeling

- A poisson model, where $f(x)$ describes the probability density for the observable x for a single event

$$\mathbf{f}(\mathcal{D}|\nu, \boldsymbol{\alpha}) = \text{Pois}(n|\nu) \prod_{e=1}^n f(x_e|\boldsymbol{\alpha}), \quad (1)$$

- A combined model:

$$\mathbf{f}_{\text{sim}}(\mathcal{D}_{\text{sim}}|\boldsymbol{\alpha}) = \prod_{c \in \text{cats}} \left[\text{Pois}(n_c|\nu_c(\boldsymbol{\alpha})) \prod_{e=1}^{n_c} f_c(x_{ce}|\boldsymbol{\alpha}) \right], \quad (2)$$

- Including the constraint terms:

$$\mathbf{f}_{\text{tot}}(\mathcal{D}_{\text{sim}}, \mathcal{G}|\boldsymbol{\alpha}) = \prod_{c \in \text{cats}} \left[\text{Pois}(n_c|\nu_c(\boldsymbol{\alpha})) \prod_{e=1}^{n_c} f_c(x_{ce}|\boldsymbol{\alpha}) \right] \cdot \prod_{p \in \mathbb{S}} f_p(a_p|\alpha_p) \quad (3)$$



Modeling

-

$$f_{\text{tot}}(\mathcal{D}_{\text{sim}}, \mathcal{G} | \boldsymbol{\alpha}) = \prod_{c \in \text{cats}} \left[\text{Pois}(n_c | \nu_c(\boldsymbol{\alpha})) \prod_{e=1}^{n_c} f_c(x_{ce} | \boldsymbol{\alpha}) \right] \prod_{p \in S} f_p(a_p | \alpha_p)$$

**Auxiliary
pdf of Observables “measurement”**

- The *likelihood function* $L(\boldsymbol{\alpha})$ is numerically equivalent to $f(x|\boldsymbol{\alpha})$ with x fixed – or $\mathbf{f}(\mathcal{D}|\boldsymbol{\alpha})$ with \mathcal{D} fixed. It is common to work with the log-likelihood (or negative log-likelihood) function.



Profile Likelihood Ratio

- Measure μ :

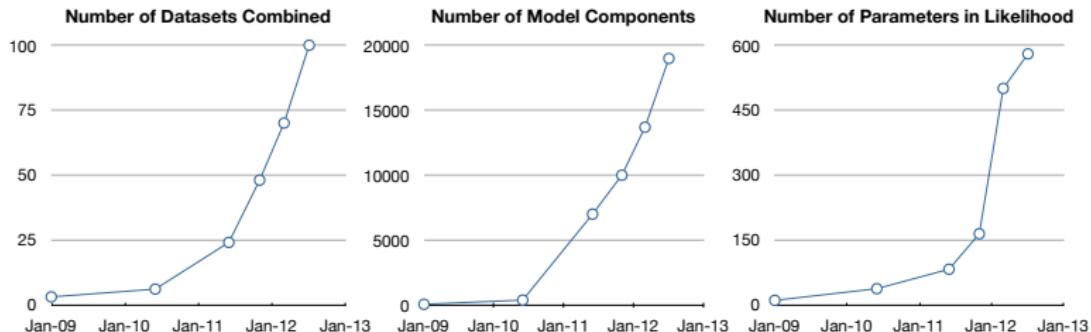
$$\Lambda(\boldsymbol{\mu}) = \frac{L(\boldsymbol{\mu}, \hat{\boldsymbol{\theta}}(\boldsymbol{\mu}))}{L(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\theta}})}, \quad (1)$$

- Coupling determination:

$$\Lambda(\boldsymbol{\kappa}) = \frac{L(\boldsymbol{\kappa}, \hat{\boldsymbol{\theta}}(\boldsymbol{\kappa}))}{L(\hat{\boldsymbol{\kappa}}, \hat{\boldsymbol{\theta}})}, \quad (2)$$



Model



- The model has become much bigger along with higgs searches
- more than 100 categories, more than 1000 nuisance parameters, around 700 of them are mc-stat ones



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Treatment of systematics

- Systematics are becoming increasingly important, as they dominate sensitivity as the dataset grows.
 - Being conservative is not acceptable, as the main goal is to reduce the total uncertainty
 - Different treatment of systematics can potentially change the results
- All sources of uncertainties are taken to be either 100%-correlated (positively or negatively) or uncorrelated (independent)
 - Likelihood can be written in a clean factorized form
 - May need break-down of the systematics into different sources

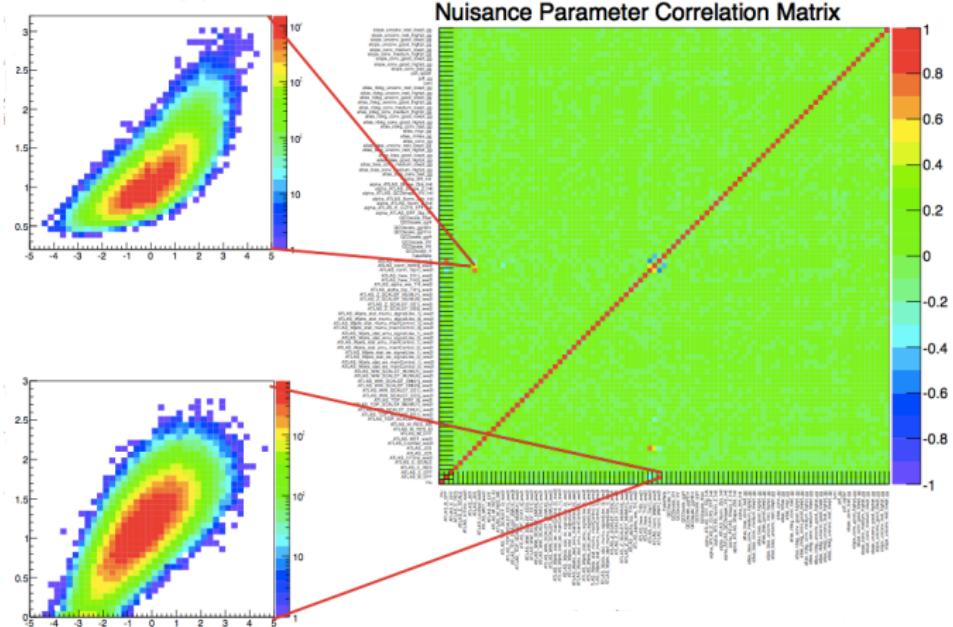


Treatment of systematics

- Decision to correlate or not those parameters are taken based on physics judgement.
 - Jet Energy scale uncertainty is splitted into different sub-ones, each originating from a different source:
ATLAS_JES_BASE, ATLAS_JES_BJET,
ATLAS_JES_CLOSEBY, ATLAS_JES_FLAV,
ATLAS_JES_FWD, ATLAS_JES_MU, ATLAS_JES_NPV
 - In some cases, the source of several uncertainties could be the same, but the event kinematics, flavor composition, etc could be different, thus need uncorrelate them



Treatment of systematics

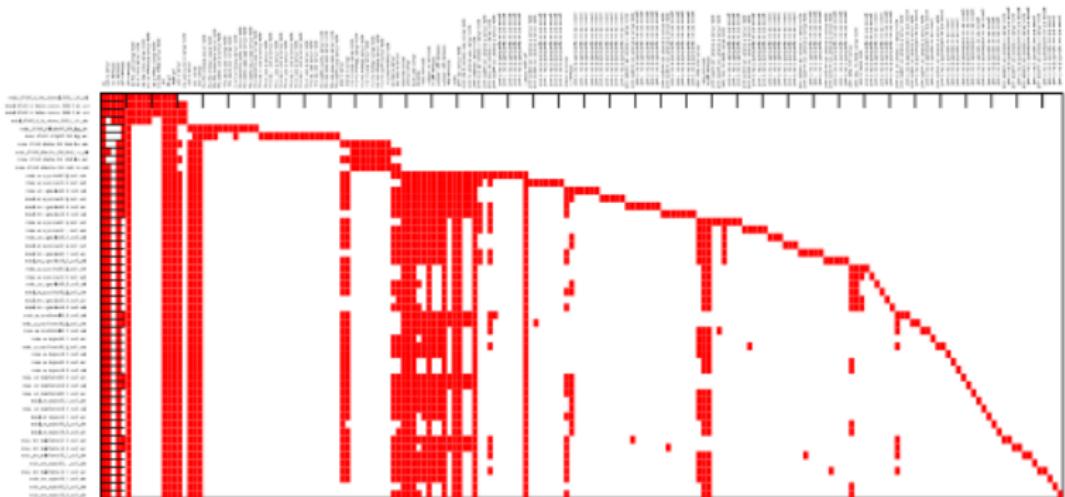


- A suite of standard RooStats tool performing these checks.



Treatment of systematics

- Systematics in each category





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Combination tool

- Our model has become more complicated along with the higgs search
- Better care must be taken to correlate parameters and perform other operations, with all the information documented. Using xml file is one good choice.
 - ① Easy to add/remove individual channels
 - ② Clear to see the structure of each input
 - ③ Remapping of the nuisance parameters can be done easily and transparently
 - ④ Most of the common operations can be done without touching the source code



Combination tool



Connecting the variables

- Renaming of nuisance parameters

```
<Syst OldName="atlas_nui_EM_ES_Z_unconv_good_lowpt
(atlas_EM_ES_Z, RNDM_atlas_EM_ES_Z)"
NewName="ATLAS_EM_ES_Z"/>
```

- 1 Nuisance Parameter: atlas_EM_ES_Z → ATLAS_EM_ES_Z
- 2 Global Observable: RNDM_atlas_EM_ES_Z → ATLAS_EM_ES_Z_In
- 3 Pdf: atlas_nui_EM_ES_Z_unconv_good_lowpt → ATLAS_EM_ES_Z_Pdf

- Assume default naming convention in input workspaces:

```
<Syst OldName="alpha_ATLAS_BR_VV" NewName="ATLAS_BR_VV"/>
```



Combination Tools

- Connecting the parameters of interest

combined model:

```
<ModelPOI Name="mu(1~0~5),  
mu_XS7_ggF[1-1], mu_XS7_VBF[1-1],  
mu_XS7_WH[1-1], mu_XS7_ZH[1-1],  
mu_XS7_ttH[1-1], mu_BR_gamgam[1-1],  
mu_BR_WW[1-1], mu_BR_ZZ[1-1],  
mu_BR_bb[1-1], mu_BR_tautau[1-1]" />
```

individual model:

```
<ModelPOI Name="mu,  
mu_ggF,mu_VBF,mu_WH,mu_ZH,mu_ttH,  
mu_BR_gg,dummy,dummy,dummy,dummy" />
```



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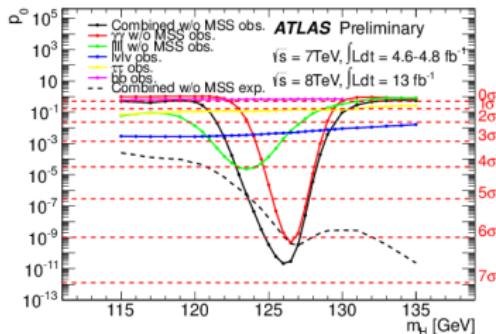
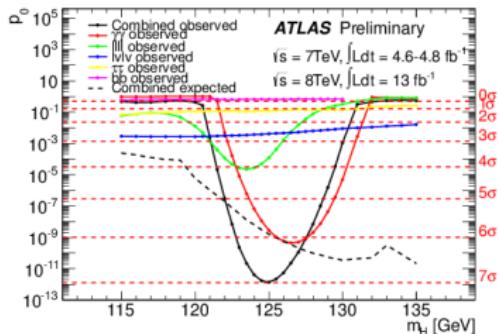
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Significance



- The local probability p_0 for a background-only experiment to be more signal-like than the observation.
- Mass scale systematics(MSS) are included in the left plot, but NOT in the right plot
- MSS can be pulled so that the excess can be described by the model in a wilder range, thus forming a broader curve.



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Mass combination

- Use high mass-resolution channels, $H \rightarrow \gamma\gamma$ and $H \rightarrow \ell\ell\ell\ell$
- Model is parametrized in terms of mass
- Estimate the mass:

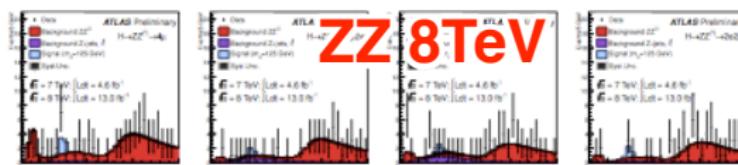
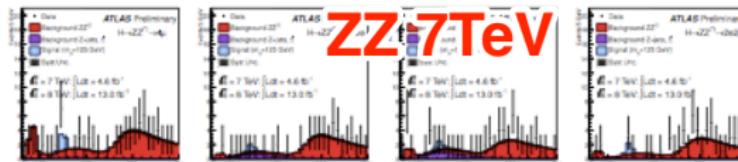
$$\Lambda(m_H) = \frac{L(m_H, \hat{\mu}_{\gamma\gamma}(m_H), \hat{\mu}_{4\ell}(m_H), \hat{\theta}(m_H))}{L(\hat{m}_H, \hat{\mu}_{\gamma\gamma}, \hat{\mu}_{4\ell}, \hat{\theta})} . \quad (3)$$

- Directly quantify the consistency between the measurements of $m_H^{\gamma\gamma}$ and $m_H^{4\ell}$:

$$\Lambda(\Delta m_H) = \frac{L(\Delta m_H, \hat{\mu}_{\gamma\gamma}(\Delta m_H), \hat{\mu}_{4\ell}(\Delta m_H), \hat{m}_H(\Delta m_H), \hat{\theta}(\Delta m_H))}{L(\hat{\Delta m}_H, \hat{\mu}_{\gamma\gamma}, \hat{\mu}_{4\ell}, \hat{m}_H, \hat{\theta})} . \quad (4)$$

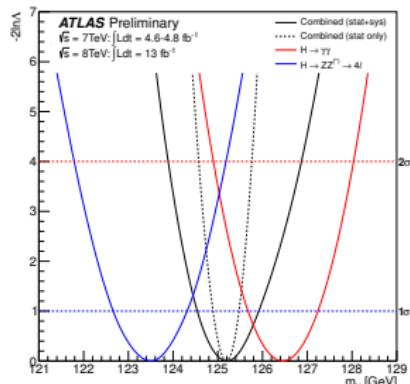
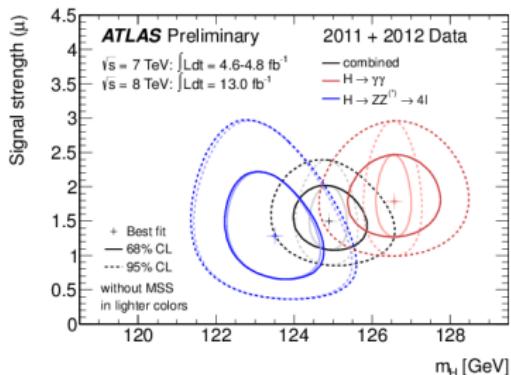


Mass combination





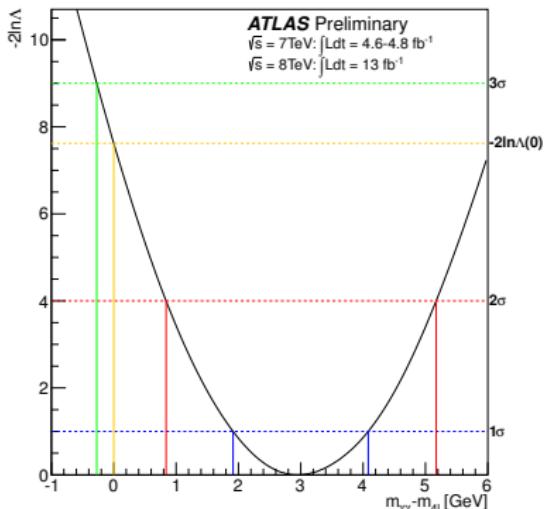
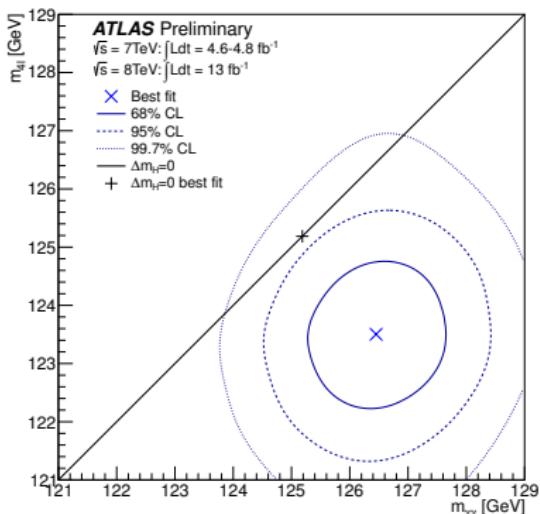
Mass combination



- Left: Likelihood contours on the (μ, m_H) plane
- Right: The profile likelihood ratio $-2\ln\Lambda(m_H)$



Mass combination



- Left: Likelihood contours on the $(m_H^{\gamma\gamma}, m_H^{lll})$ plane
- Right: The profile likelihood ratio $-2\ln\Lambda(m_H^{\gamma\gamma} - m_H^{lll})$



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Coupling model

- For each production mode i , a signal strength factor μ_i defined by $\mu_i = \frac{\sigma_i}{\sigma_{i,SM}}$ is introduced
- For each decay final state f , a factor $\mu_f = \frac{B_f}{B_{f,SM}}$ is introduced
- For each analysis category k the number of signal events (n_{signal}^k) is parametrized as:
$$n_{signal}^k = (\sum \mu_i \sigma_{i,SM} \times A_i^k \times \epsilon_i^k) \times \mu_f \times B_{f,SM} \times \mathcal{L}^k$$



Coupling model

- Only modifications of couplings strengths, are taken into account: the observed state is assumed to be a CP-even scalar as in the SM.
- The signals observed in the different search channels originate from a single narrow resonance.
- The width of the Higgs boson with a mass of 126 GeV is assumed to be negligible. Hence:
$$\sigma \times BR(ii \rightarrow H \rightarrow ff) = \frac{\sigma_{ii} \times \Gamma_{ff}}{\Gamma_H}$$



Coupling model

- The leading order (LO) motivated scale factors κ_i are defined in such a way that the cross sections σ_{ii} and the partial decay widths Γ_{ff} associated with the SM particle i scale with the factor κ_i^2 when compared to the corresponding SM prediction.

Production modes

$$\frac{\sigma_{ggH}}{\sigma_{ggH}^{SM}} = \begin{cases} \kappa_g^2(\kappa_b, \kappa_t, m_H) \\ \kappa_g^2 \end{cases}$$

$$\frac{\sigma_{VBF}}{\sigma_{VBF}^{SM}} = \kappa_{VBF}^2(\kappa_W, \kappa_Z, m_H)$$

$$\frac{\sigma_{WH}}{\sigma_{WH}^{SM}} = \kappa_W^2$$

$$\frac{\sigma_{ZH}}{\sigma_{ZH}^{SM}} = \kappa_Z^2$$

$$\frac{\sigma_{t\bar{t}H}}{\sigma_{t\bar{t}H}^{SM}} = \kappa_t^2$$

Detectable decay modes

$$\frac{\Gamma_{WW^{(*)}}}{\Gamma_{WW^{(*)}}^{SM}} = \kappa_W^2$$

$$\frac{\Gamma_{ZZ^{(*)}}}{\Gamma_{ZZ^{(*)}}^{SM}} = \kappa_Z^2$$

$$\frac{\Gamma_{b\bar{b}}}{\Gamma_{b\bar{b}}^{SM}} = \kappa_b^2$$

$$\frac{\Gamma_{\tau^-\tau^+}}{\Gamma_{\tau^-\tau^+}^{SM}} = \kappa_\tau^2$$

$$\frac{\Gamma_{\gamma\gamma}}{\Gamma_{\gamma\gamma}^{SM}} = \begin{cases} \kappa_\gamma^2(\kappa_b, \kappa_t, \kappa_\tau, \kappa_W, m_H) \\ \kappa_\gamma^2 \end{cases}$$

$$\frac{\Gamma_{Z\gamma}}{\Gamma_{Z\gamma}^{SM}} = \begin{cases} \kappa_{(Z\gamma)}^2(\kappa_b, \kappa_t, \kappa_\tau, \kappa_W, m_H) \\ \kappa_{(Z\gamma)}^2 \end{cases}$$



Coupling model



Parametrization

- gluon fusion

mH_GeV	sigma_ggH_pb	sigma_tt/sigma_ggH	sigma_bb/sigma_ggH	sigma_tb/sigma_ggH
126.0	1.332933e+01	1.058802e+00	6.860053e-03	-6.566210e-02

```
<Item Name="expr::C2_G1G1H_7TeV('(@0*@0)*@2 + (@1*@1)*@3 + (@0*@1)*@4',
CF, CF, sigma_tt_o_sigma_ggH_G1G1H_7TeV[1.0592],sigma_bb_o_sigma_ggH_G1G1H_
sigma_tb_o_sigma_ggH_G1G1H_7TeV[-0.0662])"/>
```

- $H \rightarrow \gamma\gamma$

M_H	G_gaga	G_tt/G_gaga	G_bb/G_gaga	G_WW/G_gaga	G_tb/G_gaga	G_tW/G_gaga
126.	0.95959E-05	0.70904E-01	0.18760E-04	1.5863	-0.17319E-02	-0.67074

```
<!-- H->gamgam -->
<Item Name="expr::C2_HGaGa('(@0*@0)*@4 + (@1*@1)*@5 + (@2*@2)*@6 + (@0*@1)*
(@0*@2)*@8 + (@1*@2)*@9 + (@3*@3)*@10 + (@0*@3)*@11 + (@1*@3)*@12 + (@2*@3)*
CF,CF,CV,CF,G_tt_o_G_HGaGa[0.0715],G_bb_o_G_HGaGa[0.0000],G_WW_o_G_HGaGa[1.0592],
G_tb_o_G_HGaGa[-0.0018],G_tW_o_G_HGaGa[-0.6740],G_bW_o_G_HGaGa[0.0083],G_ll_o_G_HGaGa[0.0000],
G_t1_o_G_HGaGa[-0.0019],G_b1_o_G_HGaGa[0.0000],G_lW_o_G_HGaGa[0.0090])"/>
```



Parametrization

- total width

mH_GeV	H_gg	H_gamgam	H_Zgam	H_WW	H_ZZ	Total_Width_GeV
126.0	8.45E-02	2.29E-03	1.64E-03	2.33E-01	2.91E-02	4.18E-03

```
<!-- total width -->
<Item Name="expr::invC2H('1/(@0*@11 + @1*@12 + @2*@13 + @3*@14 + @4*@15 +
@5*@16 + @6*@17 + @7*@17 + @8*@16 + @9*@18 + @10*@18)',

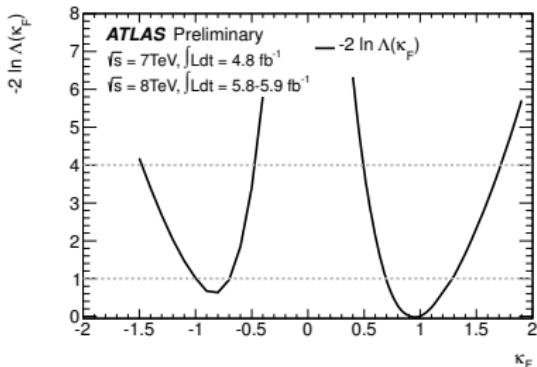
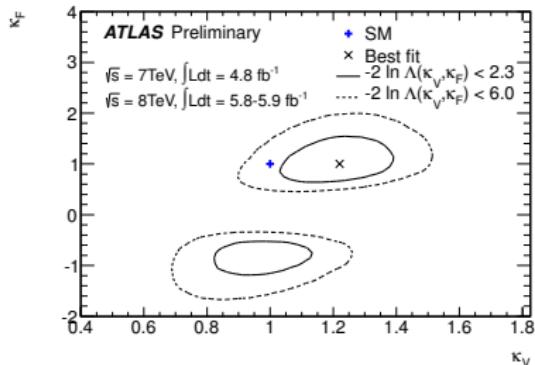
BR_H_gluglu_SM[0.0855],BR_H_gamgam_SM[0.0023],BR_H_Zgam_SM[0.0015],
BR_H_WW_SM[0.2160],BR_H_ZZ_SM[0.0266],BR_H_bb_SM[0.5770],
BR_H_tautau_SM[0.0637],BR_H_mumu_SM[0.0002],BR_H_ssbar_SM[0.0004],
BR_H_ccbar_SM[0.0267],BR_H_ttbar_SM[0.0000], C2_HG1G1, C2_HGaGa, C2_HZGa, C
```

- Replace the handles

```
mu_XS7_ggF=C2_G1G1H_7TeV
mu_BR_gamgam=C2_HGaGa
mu=invC2H)
```



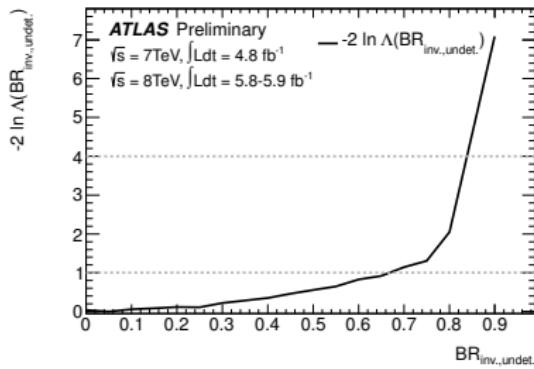
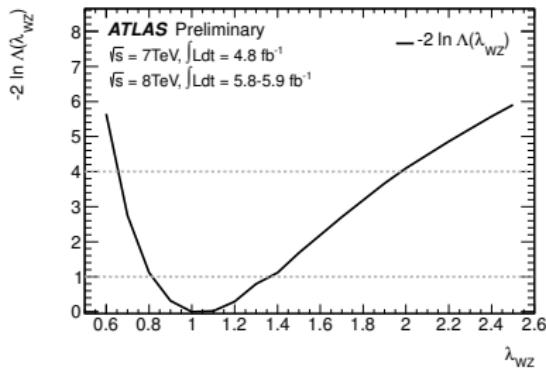
Coupling results



- κ_F and κ_V model, with assumption on total width



Coupling results

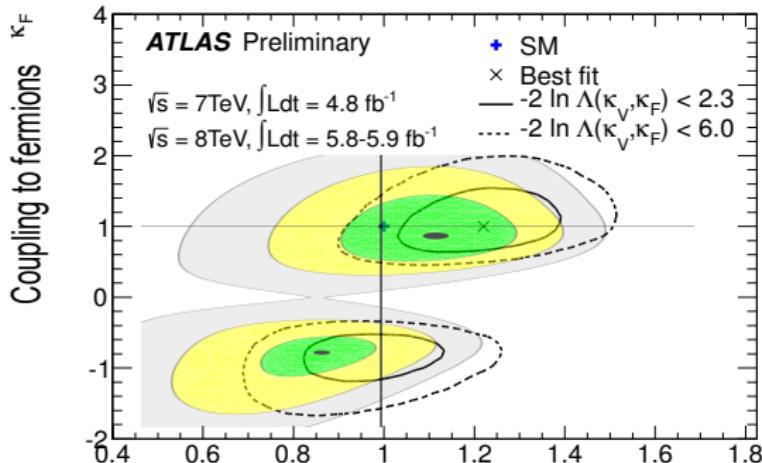


Left: model to probe custodial symmetry.

Right: model to probe invisible/undetected branching ratio



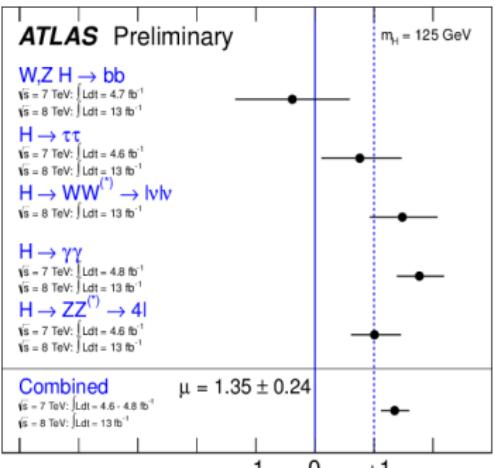
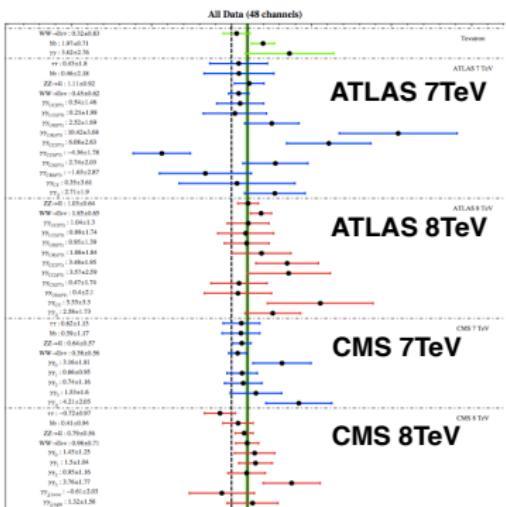
Coupling results



- ATLAS results (126GeV) overlayed with Figure^{k_V} 2 of arXiv:1207.1717, a back-of-envelope calculation
- We'll make public the results for each channel, in which case comparison would be easier



Coupling results



- Simple chi-square combination will always give lower strengths: neglecting the fact that the $\sigma_{\mu}^+ > \sigma_{\mu}^-$
- Provided a table of μ and it's SYMMETRIZED errors in ATLAS recently