Latest results on pair production in strong inhomogeneous external fields

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Zimányi School, Budapest, 2012. 12. 07.

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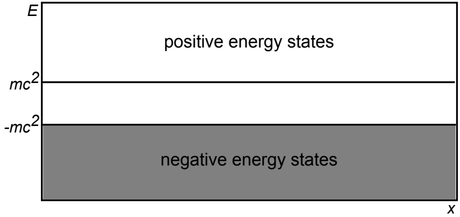
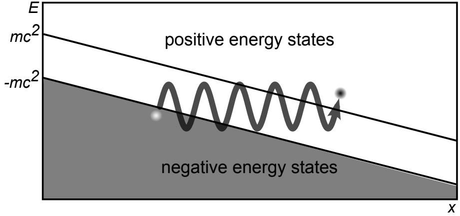
Phenomenology of pair production from vacuum:

Without external field:



With external field:



Introduction

Approximate rate of tunnelling from QM:

$$P \propto \exp\left(-\mathrm{constant} \times \frac{\mathrm{measure-of-the-barrier}}{\mathrm{measure-of-barrier-reduction}}\right)$$

Schwinger-formula: pair creation rate in constant electric field:

$$P = \frac{e^2 E^2}{4\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\pi \frac{n m^2 c^3}{\hbar e E}\right)$$

Introduction

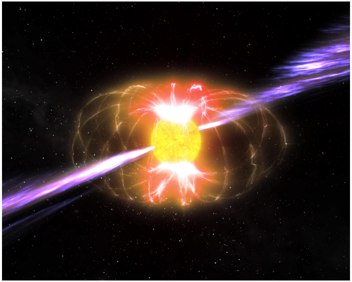
Relevant scales:

- Field strength: $E_c = \frac{m^2 c^3}{e\hbar} \approx 1.3 \cdot 10^{18} \frac{\mathrm{V}}{\mathrm{m}}$
- Time: $t_c = \frac{\hbar}{mc^2} \approx 1 \cdot 10^{-21} \mathrm{s}$
- Frequency: $\omega_c = \frac{e\mathcal{E}}{mc} \approx 8 \cdot 10^{20} \mathrm{Hz}\ (E = E_c)$
- Spatial gradient: $\partial_r = \frac{mc}{\hbar} \approx 6.6 \cdot 10^{10} \mathrm{m}^{-1}$

Motivation

Pair Production in nature:

QED pair production near massive astrophysical objects (black holes, magnetars, possible source of gamma ray bursts?)



John Rowe Animations

Pair Production in theory and experiment:

- QED pair production from vacuum was predicted half a century ago, but was not yet observed.
- Rapid development of laser technology may enable the observation in the near future.
- Pair production in strong QCD fields (Heavy Ion collisions).

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History

Some results from the early history of pair production description:

- Homogeneous static electric field. (J. Schwinger)
- Some special analytic time dependent, homogeneous electric fields.

  (V. S. Popov, M. S. Marinov, N. B. Narozhnyi, A. I. Nikishov, ...)

Recently a different approach is gaining attention: kinetic formulation

- Transport equation for QCD Wigner operator, later for Abelian plasmas. (D. Vasak, M. Gyulassy, H.-T. Elze)
- Equal time formulation of QED transport equations for the Wigner function (named the Dirac-Heisenberg-Wigner, DHW equations).

  (I. Bialynicki-Birula, P. Górnicki, J. Rafelski)
- Study of inhomogeneous and time dependent QED particle production (F. Hebenstreit, R. Alkofer, H. Gies)

Dirac-Heisenberg-Wigner formalism

What is the Wigner function?

- Quantum analogue of the classical phase space distribution.

How it is defined?

- Take the equal time density matrix in terms of 'center of mass' coordinates:

$$\hat{C}(\vec{x},\vec{s},t) = e^{-ie\int_{-1/2}^{1/2}\vec{\mathcal{A}}(\vec{x}+\lambda\vec{s},t)\vec{s}\mathrm{d}\lambda}\left[\Psi(\vec{x}+\frac{\vec{s}}{2},t),\bar{\Psi}(\vec{x}-\frac{\vec{s}}{2},t)\right] \quad (1)$$

- Take the expectation value.
- Fourier transform it w.r.t the coordinate difference:

$$W(\vec{x},\vec{p},t) = -\frac{1}{2}\int e^{-i\vec{p}\vec{s}}\langle 0|\hat{C}(\vec{x},\vec{s},t)|0\rangle \mathrm{d}^3 s \quad (2)$$

Dirac-Heisenberg-Wigner formalism

- The time derivative of the Wigner function gives us the evolution of the system:

$$D_t W = -\frac{1}{2}\vec{D}_{\vec{x}}[\gamma^0\vec{\gamma}, W] - im[\gamma^0, W] - i\vec{P}\{\gamma^0\vec{\gamma}, W\} \qquad (3)$$

- Theoretical approximation: external field is classical (quantum fluctuations are neglected).

Dirac-Heisenberg-Wigner formalism

The evolution equation:

$$D_t W = -\frac{1}{2}\vec{D}_{\vec{x}}[\gamma^0\vec{\gamma}, W] - im[\gamma^0, W] - i\vec{P}\{\gamma^0\vec{\gamma}, W\} \tag{4}$$

The equation has the following non-local differential operators:

$$D_t = \partial_t + e\vec{\mathcal{E}}(\vec{x}, t)\vec{\nabla}_{\vec{x}} - \frac{e\hbar^2}{12}(\vec{\nabla}_{\vec{x}}\vec{\nabla}_{\vec{p}})^2\vec{\mathcal{E}}(\vec{x}, t)\vec{\nabla}_{\vec{p}} + ... \tag{5}$$

$$\vec{D}_{\vec{x}} = \vec{\nabla}_{\vec{x}} + e\vec{\mathcal{B}}(\vec{x}, t) \times \vec{\nabla}_{\vec{x}} - \frac{e\hbar^2}{12}(\vec{\nabla}_{\vec{x}}\vec{\nabla}_{\vec{p}})^2\vec{\mathcal{B}}(\vec{x}, t) \times \vec{\nabla}_{\vec{p}} + ... \tag{6}$$

$$\vec{P} = \vec{p} + \frac{e\hbar}{12}(\vec{\nabla}_{\vec{x}}\vec{\nabla}_{\vec{p}})\vec{\mathcal{B}}(\vec{x}, t) \times \vec{\nabla}_{\vec{p}} + ... \tag{7}$$

Solution is expanded on irreducible 4x4 matrices:

$$W(x, p, t) = \frac{1}{4}\left[\mathbb{1}\mathbb{s} + i\gamma_5\mathbb{p} + \gamma^\mu\mathbb{v}_\mu + \gamma^\mu\gamma_5\mathbb{a}_\mu + \sigma^{\mu\nu}\mathbb{t}_{\mu\nu}\right] \tag{8}$$

Dirac-Heisenberg-Wigner formalism

We arrive to a system for 16 unknown real functions, the DHW functions:

$$D_t\mathbb{s} - 2\vec{P}\cdot\vec{\mathbb{t}}_1 = 0 \quad (9)$$

$$D_t\mathbb{p} + 2\vec{P}\cdot\vec{\mathbb{t}}_2 = 2m\mathbb{a}_0 \quad (10)$$

$$D_t\mathbb{v}_0 + \vec{D}_{\vec{x}}\cdot\vec{\mathbb{v}} = 0 \quad (11)$$

$$D_t\mathbb{a}_0 + \vec{D}_{\vec{x}}\cdot\vec{\mathbb{a}} = 2m\mathbb{p} \quad (12)$$

$$D_t\vec{\mathbb{v}} + \vec{D}_{\vec{x}}\mathbb{v}_0 + 2\vec{P}\times\vec{\mathbb{a}} = -2m\vec{\mathbb{t}}_1 \quad (13)$$

$$D_t\vec{\mathbb{a}} + \vec{D}_{\vec{x}}\mathbb{a}_0 + 2\vec{P}\times\vec{\mathbb{v}} = 0 \quad (14)$$

$$D_t\vec{\mathbb{t}}_1 + \vec{D}_{\vec{x}}\times\vec{\mathbb{t}}_2 + 2\vec{P}\mathbb{s} = 2m\vec{\mathbb{v}} \quad (15)$$

$$D_t\vec{\mathbb{t}}_2 - \vec{D}_{\vec{x}}\times\vec{\mathbb{t}}_1 - 2\vec{P}\mathbb{p} = 0 \quad (16)$$

The Quantum Kinetic limit

- A special case is when $\mathcal{B} = 0$, and $\mathcal{E}(x, y, z, t) = \mathcal{E}(t)$
- This leads to the Quantum Kinetic equation on f, u, v:

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{e\mathcal{E}\varepsilon_\perp}{\omega^2} v \tag{17}$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{1}{2}\frac{e\mathcal{E}\varepsilon_\perp}{\omega^2}(1 - 2f) - 2\omega u \tag{18}$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = 2\omega v \tag{19}$$

where:

$$\omega^2(\vec{p}, t) = \varepsilon_\perp^2 + \vec{p}_\parallel^2 \tag{20}$$

$$\varepsilon_\perp^2 = m^2 + \vec{p}_\perp^2 \tag{21}$$

$$\vec{p} = (\vec{q}_\perp, q_\parallel - e\mathcal{A}(t)) \tag{22}$$

Illustration of similarity with ionisation

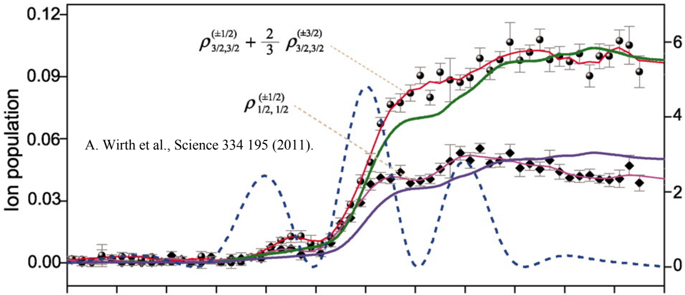
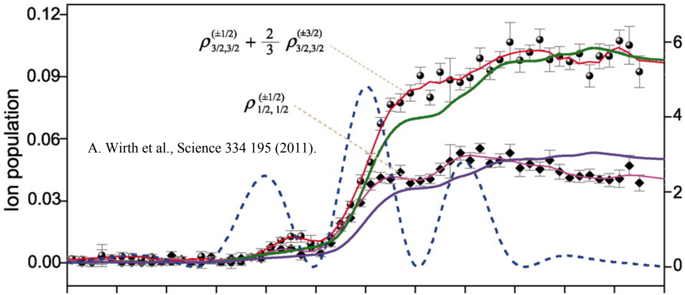
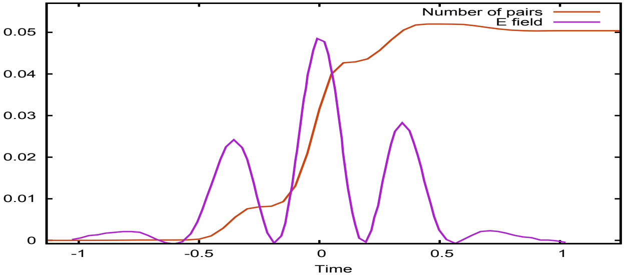


Illustration of similarity with ionisation





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Influence of external field parameters

What is needed for experimentalists?
$\Rightarrow$ Realistic laser fields!
But they are complicated, even an idealised one may look like:

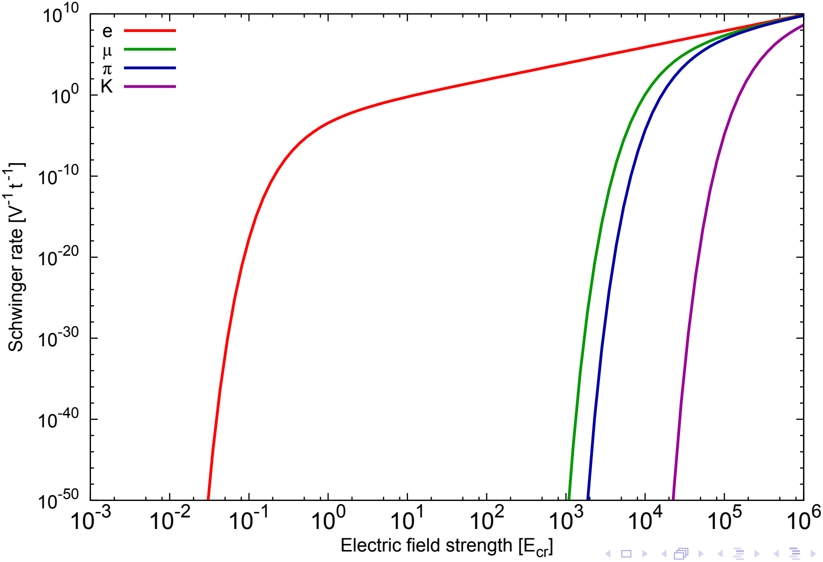
$$\mathcal{E}(t,x,y,z) = E_0 e^{\left(-\frac{x^2}{\Delta x^2} - \frac{y^2}{\Delta y^2} - \frac{(t-z)^2}{\Delta z^2}\right)} \times \cos\left(\phi + \omega(t-z) + \frac{c}{2}(t-z)^2\right). \tag{23}$$

Many-many parameters... we need to understand their influence on particle creation...

Try to learn from even simpler models!

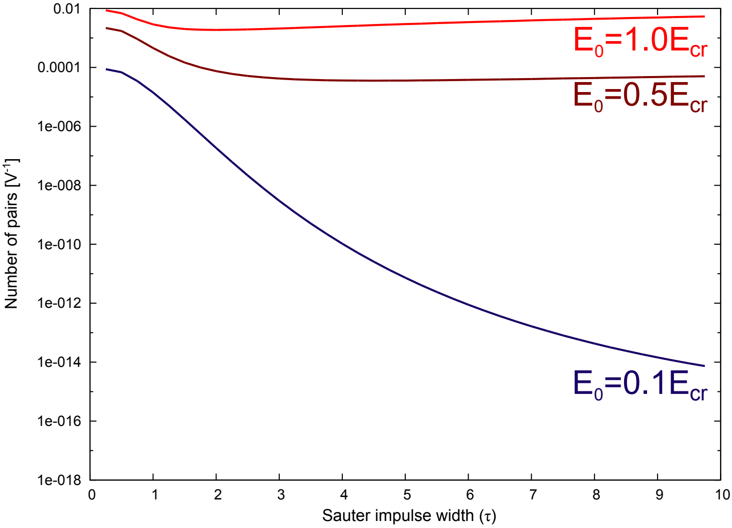
Field strength dependence

Schwinger result (constant electric field): $n \simeq \frac{e^2 E^2}{4\pi^3} \exp\left(-\frac{m^2 \pi}{eE}\right)$



Pulse width dependence

Sauter field: $\mathcal{E}(t) = E_0 \mathrm{sech}^2\left(\frac{t}{\tau}\right)$



Inhomogeneity

- So far only homogeneous, time dependent fields...
  They can be calculated at most by the Quantum Kinetic equations.

- Inhomogeneity requires the solution of all the 16 DHW equations, in at least 3 dimensions!
  An analytically unmanageable and numerically demanding problem!

Dirac-Heisenberg-Wigner equations

$$
\begin{aligned}
D_t \mathbb{s} & & & - & 2\vec{P}\cdot\vec{\mathbb{t}}_1 & = 0 \\
D_t \mathbb{p} & & & + & 2\vec{P}\cdot\vec{\mathbb{t}}_2 & = 2m\mathbb{a}_0 \\
D_t \mathbb{v}_0 & + & \vec{D}_{\vec{x}}\cdot\vec{\mathbb{v}} & & & = 0 \\
D_t \mathbb{a}_0 & + & \vec{D}_{\vec{x}}\cdot\vec{\mathbb{a}} & & & = 2m\mathbb{p} \\
D_t \vec{\mathbb{v}} & + & \vec{D}_{\vec{x}}\mathbb{v}_0 & + & 2\vec{P}\times\vec{\mathbb{a}} & = -2m\vec{\mathbb{t}}_1 \\
D_t \vec{\mathbb{a}} & + & \vec{D}_{\vec{x}}\mathbb{a}_0 & + & 2\vec{P}\times\vec{\mathbb{v}} & = 0 \\
D_t \vec{\mathbb{t}}_1 & + & \vec{D}_{\vec{x}}\times\vec{\mathbb{t}}_2 & + & 2\vec{P}\mathbb{s} & = 2m\vec{\mathbb{v}} \\
D_t \vec{\mathbb{t}}_2 & - & \vec{D}_{\vec{x}}\times\vec{\mathbb{t}}_1 & - & 2\vec{P}\mathbb{p} & = 0
\end{aligned}
$$

Dirac-Heisenberg-Wigner equations

- The DHW equations are known since 20 years, but despite their potential in the description of pair production, they are difficult to solve.
- Problem arises from the high dimensionality, and high derivatives in momentum and coordinate space etc.

We combined pseudo-spectral methods with finite difference and characteristics and were able to create a reliable numeric solver to evolve the DHW equations.

Influence of inhomogeneity

Consider the modified Sauter field:

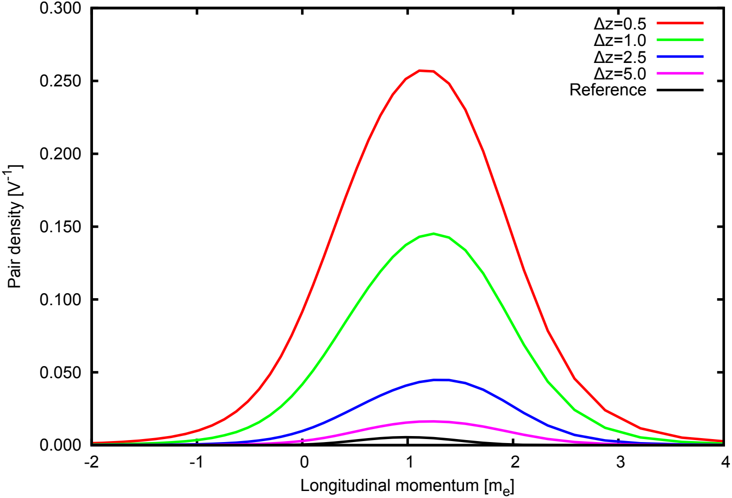
$$\vec{\mathcal{E}}(t,z) = \vec{e}_x E_0 \exp\left(-\frac{z^2}{\Delta z^2}\right) \mathrm{sech}^2\left(\frac{t}{\tau}\right).$$

Clearly $\Delta z \longrightarrow \infty$ recovers the known analytic result.

Let's fix $E_0 = 0.5 E_{cr}$ and investigate the interplay of τ andΔz.

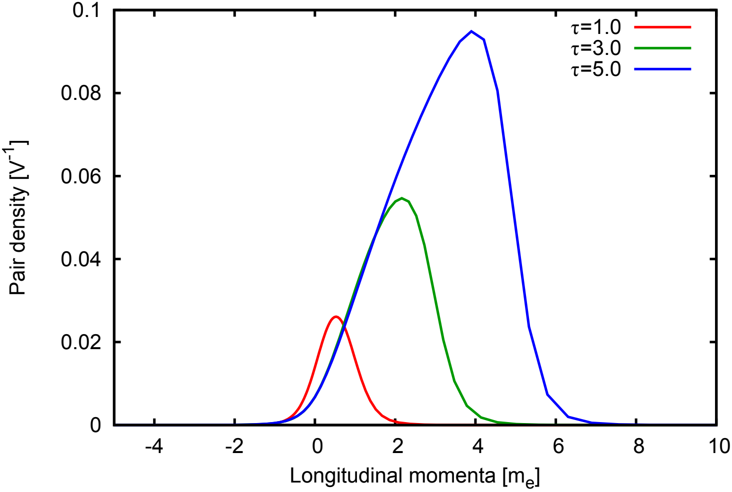
Influence of inhomogeneity

Pulse duration: $\tau = 2.0$



Influence of inhomogeneity

Pulse duration dependence at $\Delta z = 3.0$



Influence of inhomogeneity

Summary of τ, Δz interplay:

- Larger gradients increase particle production.
- As the system evolves for longer times, even smaller gradients can increase density considerably.

- Spatial gradients can counter balance or even overcome the decreasing effect of pulse widening.
  Important factor in planning future laser experiment parameters (effect of focal area)

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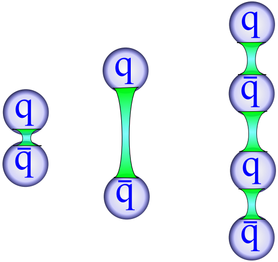
QCD pair production



Quark potential is linear with separation: if a $q - \bar{q}$ pair is separating, the interaction creates more and more quark pairs until energy is depleted.

This process is modelled by color ropes/strings that fragment into final particles.

Success in describing particle spectra in ultra-relativistic nucleus-nucleus collisions!

Outlook to the non-Abelian case

The particle production from these extreme strong color fields can be calculated by the evolution of the non-Abelian Wigner function: ($SU(2)$, homogeneous external field)

$$D_t W = -\frac{g}{8}\frac{\partial}{\partial p_i}\left(4\left\{W, F_{0i}\right\} + 2\left\{F_{i\nu}, \left[W, \gamma^0\gamma^\nu\right]\right\} - \left[F_{i\nu}, \left\{W, \gamma^0\gamma^\nu\right\}\right]\right) + ip_i\left\{\gamma^0\gamma^i, W\right\} - im\left[\gamma^0, W\right] + ig\left[A_i, \left[\gamma^0\gamma^i, W\right]\right] \tag{24}$$

A_μ: color four-potential,

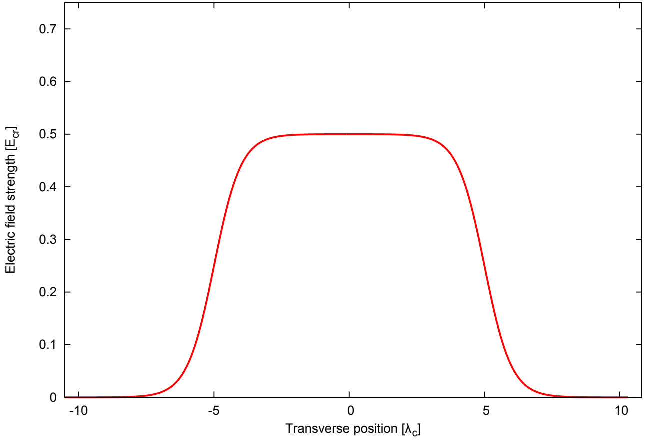
$F_{\mu\nu}$: color field tensor

Unitary generators recover the Abeilan formulas, moreover the particle spectras are very similar to the Abelian case!

V.V. Skokov, P. Lévai, Phys. Rev. D71 054004 (2008).

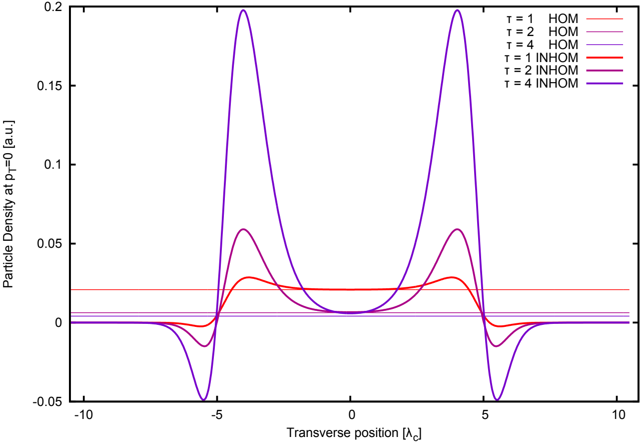
Abelian toy model of a color string

Let's consider an inhomogeneous plateu field in the transverse spatial direction and Sauter-like time dependence. ($E_0 = 0.5$).



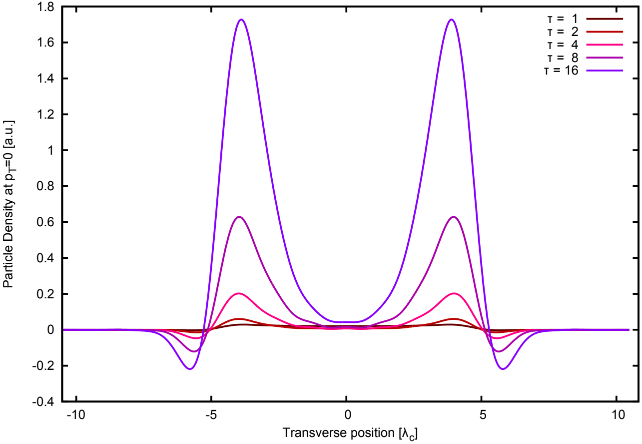
Abelian toy model of a color string

The inhomogeneity on the edges significantly increase the particle density!



Abelian toy model of a color string

Conclusion: the homogeneous models may significantly under estimate the particle yields!



Summary

- So far pair production was calculated in spatially homogeneous external fields only.
- The solution of the DHW system in time dependent and inhomogeneous external fields is now possible.
- We have shown some effects of inhomogeneity that deserve further studies, because they may significantly change earlier estimates of pair production yields.
- The Wigner function formalism is a versatile tool for describing and connecting different areas of high energy physics, from lasers to heavy ion collisions.

Supporters: OTKA Grants No. 77816, No. 104260, No. 106119 and US DoE contract DE-AC02-98CH10886.