Investigating m_T dependence of HBT radii in hydrodynamic models

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Outline

- Correlation function - theory
- Transverse mass scaling in identical particles correlations
- Results
- Summary

Motivation of studies

- Study of quark-gluon plasma produced in Pb-Pb collisions
- Confirmation of hydrodynamics predictions for quark-gluon plasma
- Verification of HBT radii scaling not only for pions, but also for heavier particles like kaons and protons.

Correlation function

Size of emitting source can be investigated using correlation of two particles:

$$C(\vec{p}_a, \vec{p}_b) = \frac{P_2(\vec{p}_a, \vec{p}_b)}{P_1(\vec{p}_a) P_2(\vec{p}_b)}$$

Probability to observe a particle with momentum $\vec{p}_a$ if a particle with momentum $\vec{p}_a$ was observed

Probability to observe a particle with given momentum independently

Relation between correlation function and emission function $S_{ab}(\cdot)$ is given by:

$$C(\vec{p}_a, \vec{p}_b) = \int S_{ab}(\vec{p}_a, \vec{x}_a, \vec{p}_b, \vec{x}_b) |\Psi_{ab}|^2 d^4 \vec{x}_a d^4 \vec{x}_b$$

$S_{ab}(\vec{p}_a, \vec{x}_a, \vec{p}_b, \vec{x}_b)$ – emission function – generalized density function of the source or a probability density of emission of the particle pair with momenta $\vec{p}_a$, $\vec{p}_b$ from space-time points $\vec{x}_a$, $\vec{x}_b$.

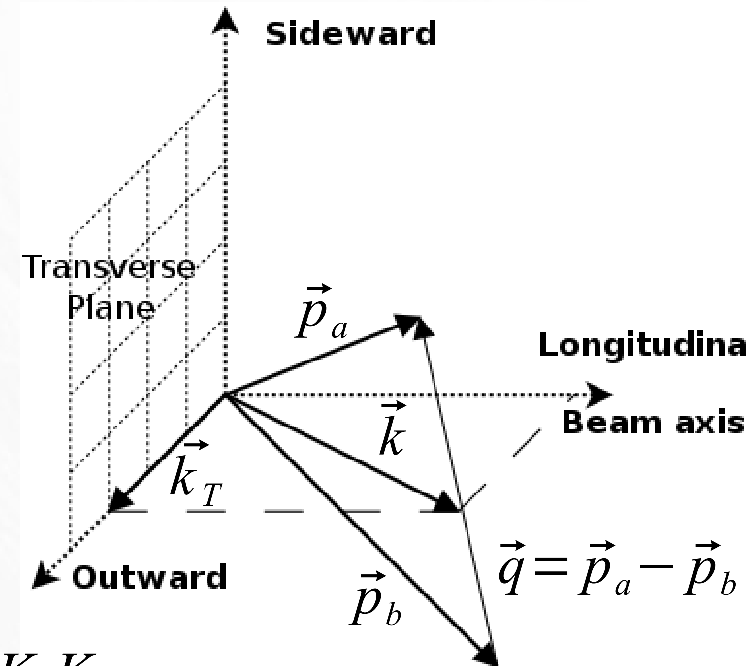
Pair wave function

- In general $|\Psi_{ab}|$ includes all types of interactions between two particles: Coulomb, strong and symmetrization or antisymmetrization of wavefunction
- Effects coming from Coulomb and strong interaction are difficult to calculate and analyse, especially in 3D. That is why currently experimental analysis for heavy particles is limited to 1D
- But in simulation we can simplify the analysis and take into account only effects of quantum statistics. This makes the analysis easier, less time consuming and enables 3D analysis

Correlation function

Source emission function is assumed to be a 3D gaussian distribution:

$$S(\vec{r}) \sim \exp\left(-\frac{r_{out}^2}{R_{out}^2} - \frac{r_{side}^2}{R_{side}^2} - \frac{r_{long}^2}{R_{long}^2}\right)$$ LCMS



$$C(\vec{p}_a, \vec{p}_b) = \int S_{ab}(\vec{p}_a, \vec{x}_a, \vec{p}_b, \vec{x}_b) |\Psi_{ab}|^2 d^4\vec{x}_a d^4\vec{x}_b$$

Solution for provided emission function:

$$C(q_O, q_S, q_L) = 1 + \lambda \exp(-R_{out}^2 q_O - R_{side}^2 q_S^2 - R_{long}^2 q_L^2)$$

symmetrization → $\lambda \to 1$ for π-π and *K*-*K*

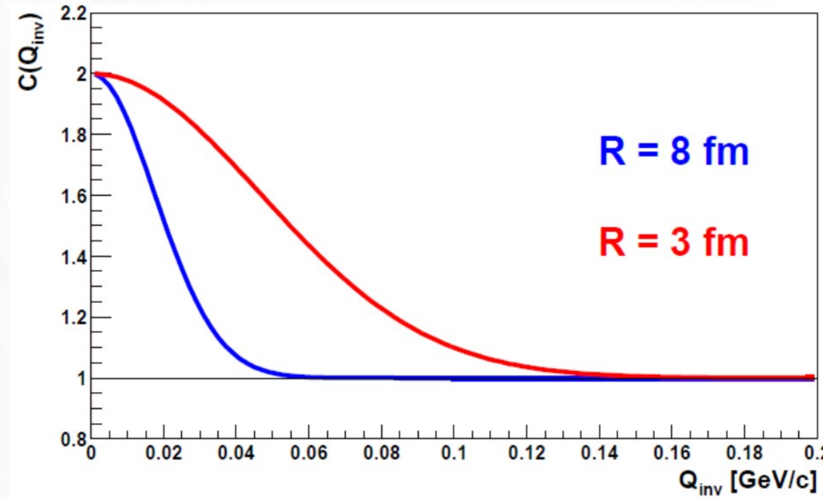
¼ symmetrization + ¾ antisymmetrization → $\lambda \to -0{,}5$ for *p*-*p*

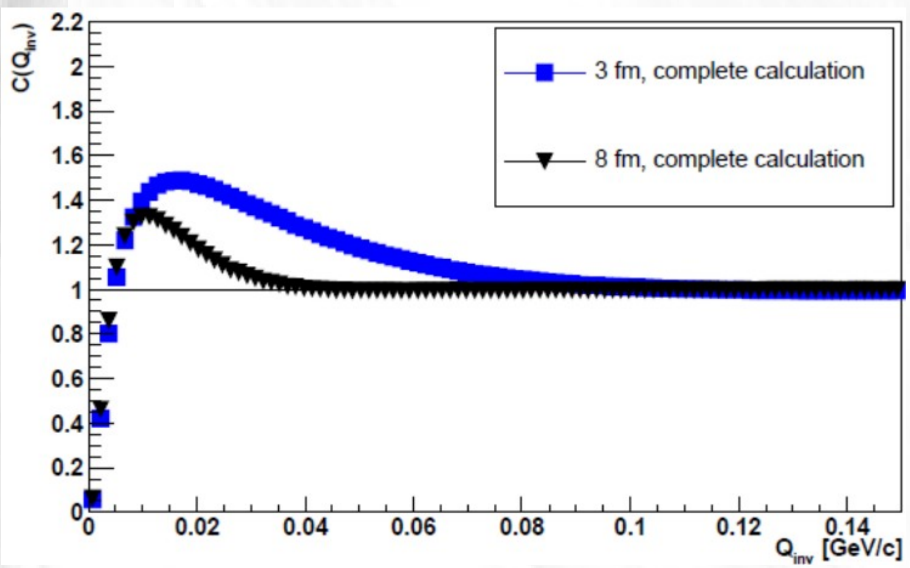
1D correlation function:

$$C(q) = 1 + \lambda \exp(-R_{inv}^2 |q|^2)$$ PRF

Correlation function

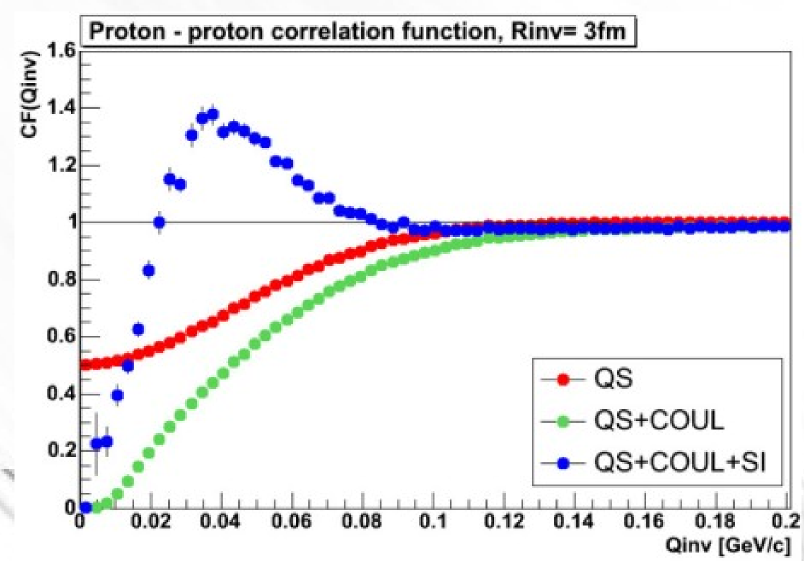
theoretical π-π correlation function – including only wavefunction symmetrization





theoretical π-π correlation function – including wavefunction symmetrization and coulomb interaction

theoretical p-p correlation function – including wavefunction anti- and symmetrization (QS), coulomb(COUL) and strong interaction (SI)



m_T scaling for identical particles

Hydrodynamics predicts femtoscopic radius scaling: $R_{LCMS} \sim m_T^{-1/2}$

Relation between radii in PRF and LCMS:

$$R^*_{out} = R_{out}\langle\gamma_t\rangle,$$
$$R^*_{side} = R_{side},$$
$$R^*_{long} = R_{long},$$

$$R_{LCMS} = \sqrt{(R^2_{out} + R^2_{side} + R^2_{long})/3}$$

Conversion from LCMS to PRF causes the increase of R_{out} which has two effects: the overall radius of the system increases and the particle source becomes non-gaussian. The interplay of the two effects can be accounted for with an approximate formula: $R_{inv} = \sqrt{(R^2_{out}\sqrt{\gamma} + R^2_{side} + R^2_{long})/3}$

Assuming $R_{out} = R_{side} = R_{long}$, an approximate formula to recover a scaling behaviour from R_{inv} is proposed: $R_{LCMS} \sim R_{inv}/[(\sqrt{\gamma}+2)/3]^{1/2}$

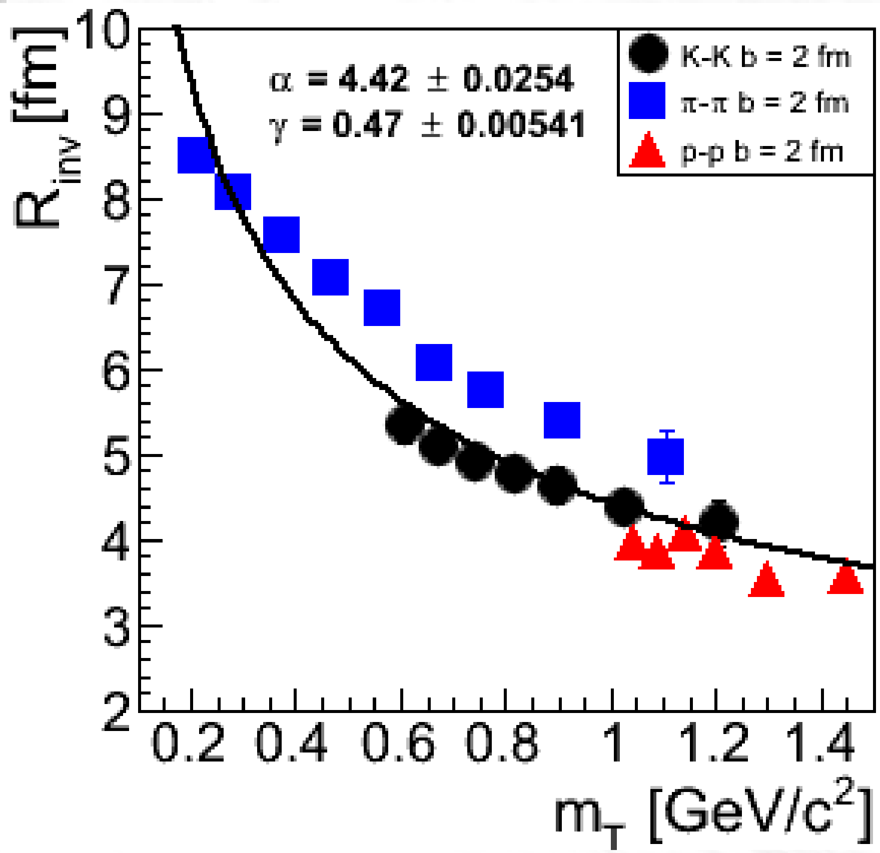
$$\gamma = \left(1 - \frac{k_T^2}{m_T^2}\right)^{-1/2} \qquad m_T = \sqrt{k_T^2 + m^2}$$

Results

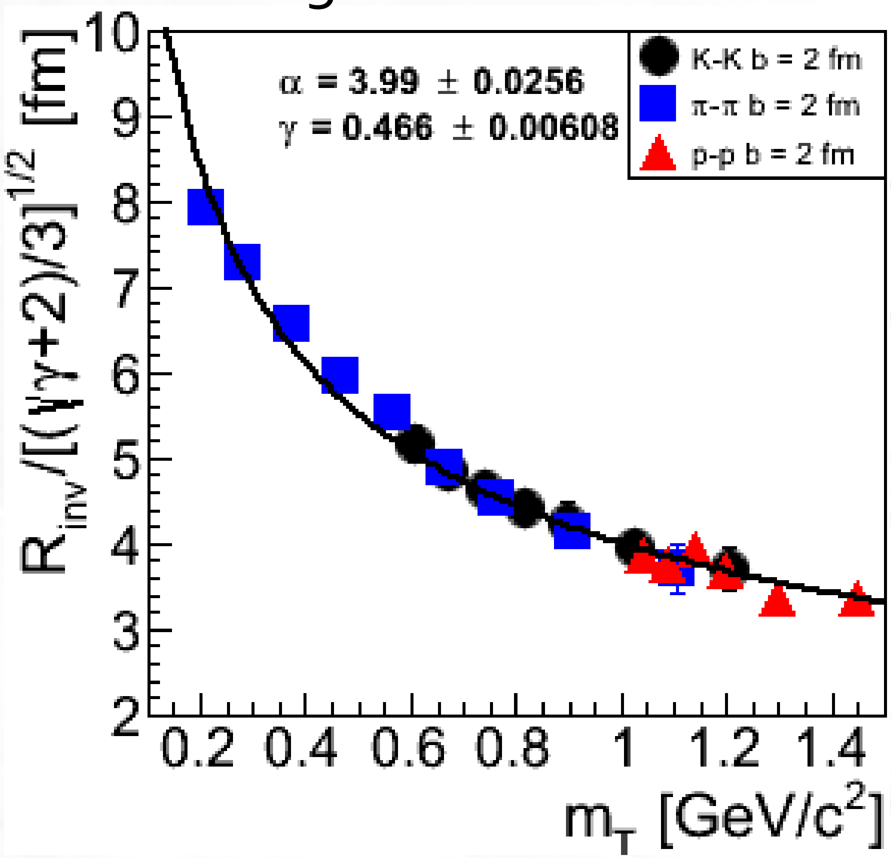
- Analysis was performed for data generated using three different models for Pb-Pb collisions at $\sqrt{s_{NN}}=2.76\,TeV$:
  - **Therminator**: hydrodynamics, resonances, statistical hadronisation, no interaction between final state hadrons
  - **hHKM**: hydrodynamics, UrQMD hadronic cascade
  - **EPOS**: multiple rescattering with hydrodynamics, hadronic cascade
- To the R_{inv} proposed scaling factor was applied to test scaling hypothesis and recover transverse mass scaling. For all of the results following equation was fitted: $R=\alpha m_T^{-\gamma}$

Scaling factor influence

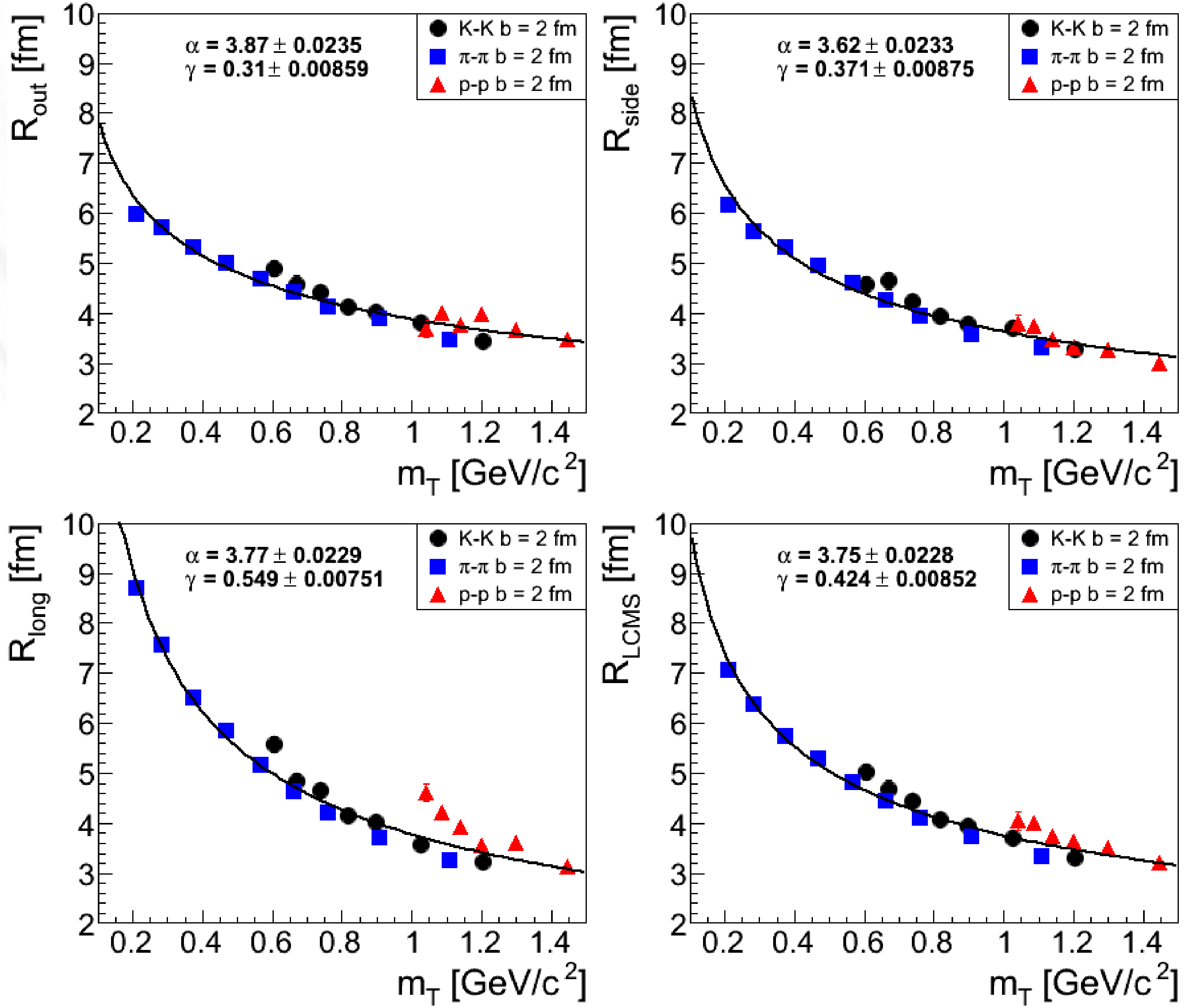
Without scaling factor:



With scaling factor:

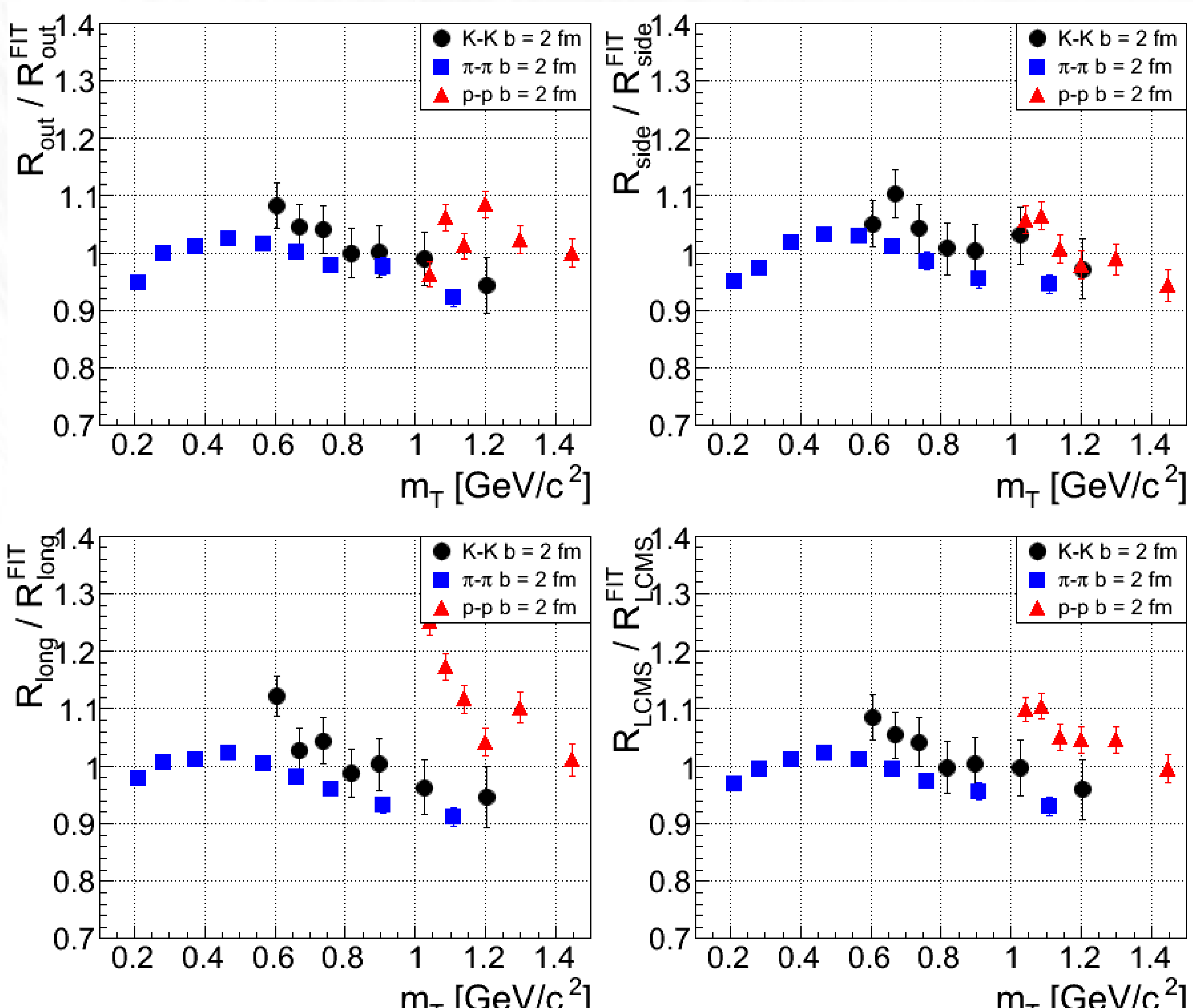


Therminator b = 2 fm - LCMS

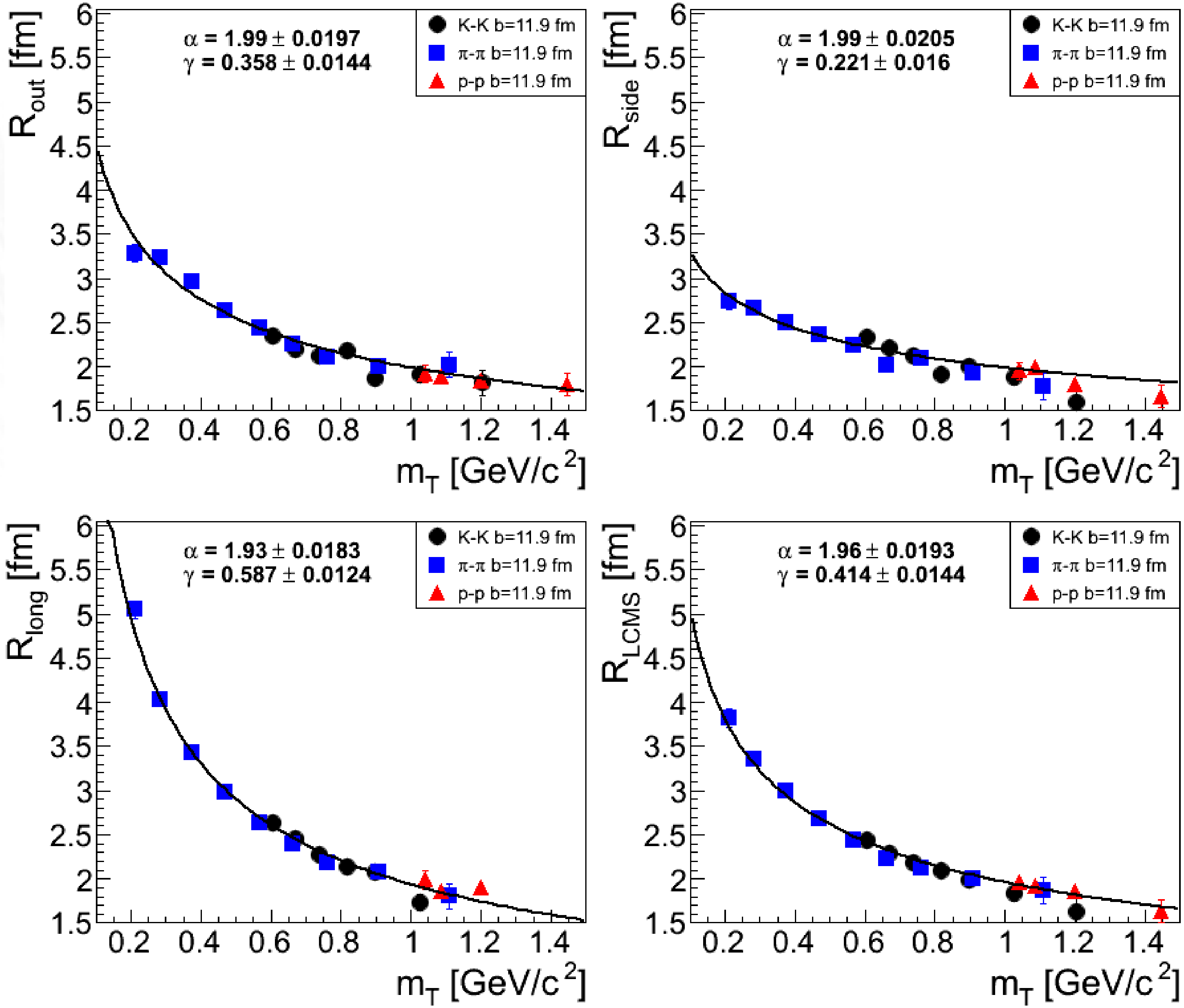


Therminator b = 2 fm - LCMS

Scaling quality:

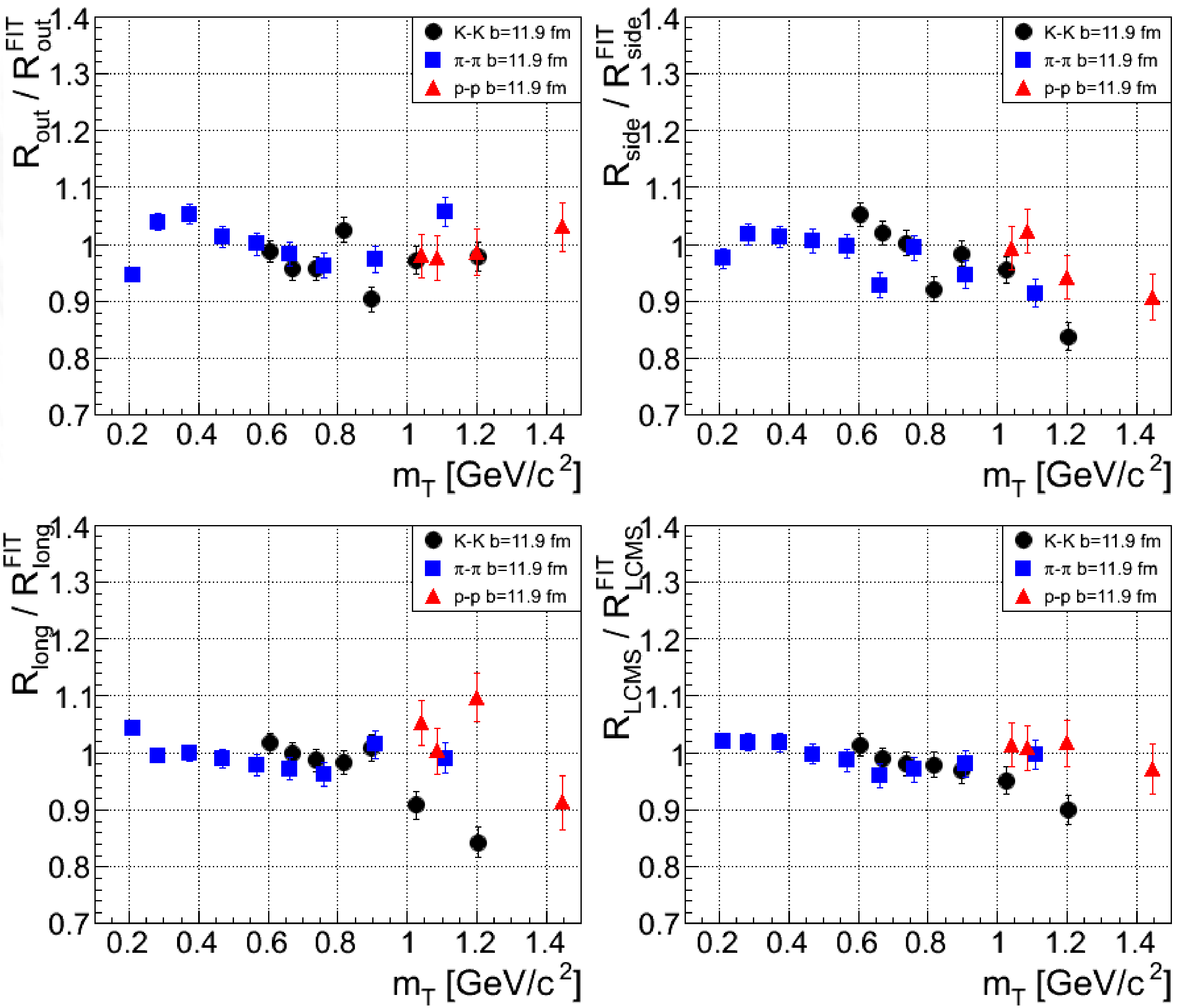


Therminator b = 11.9 fm - LCMS



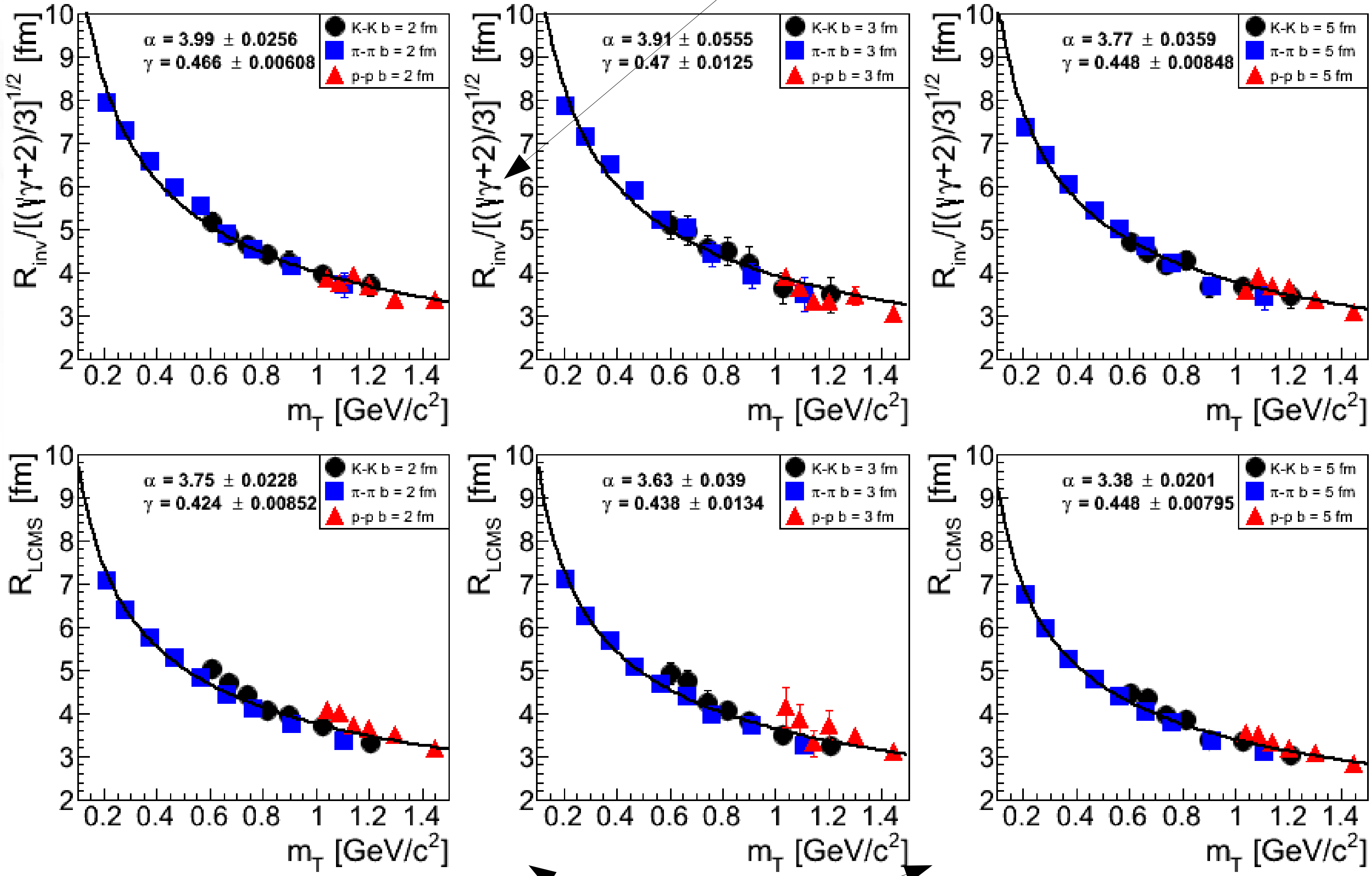
Therminator b = 11.9 fm - LCMS

Scaling quality:



Therminator - all centralities – LCMS & PRF

PRF radii divided by scaling factor

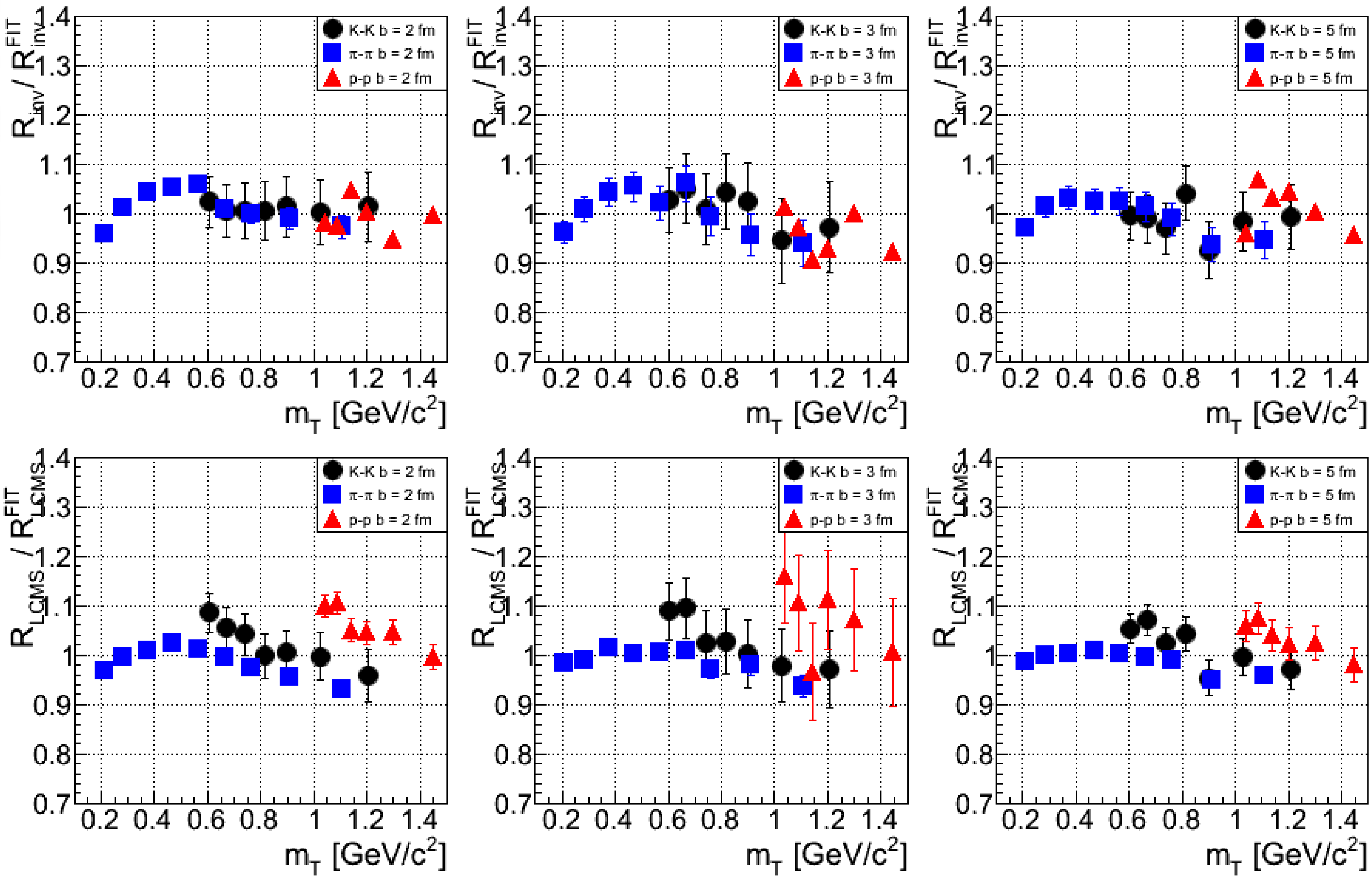


LCMS

Therminator - all centralities – LCMS & PRF

Scaling quality:

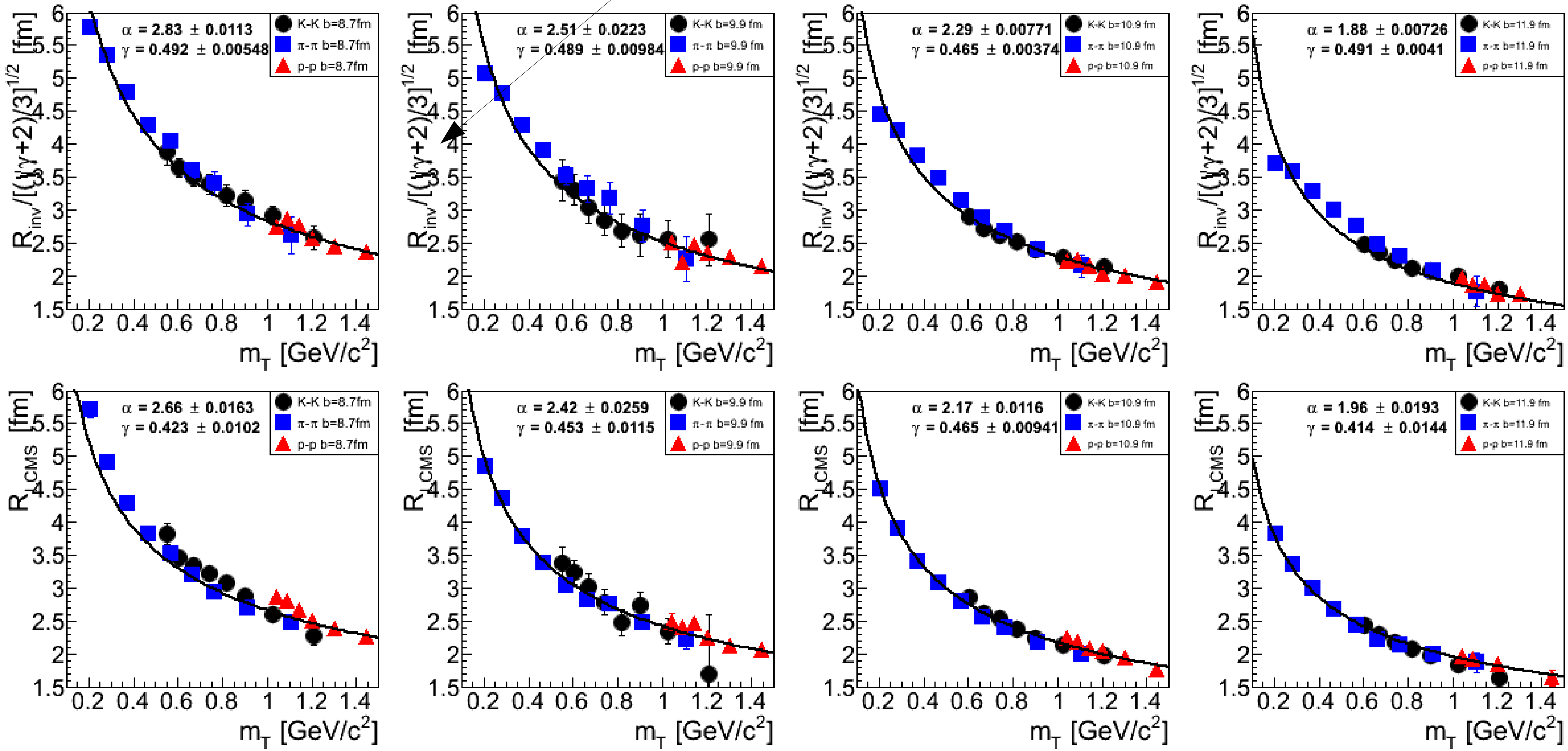
PRF



LCMS

Thermінator - all centralities – LCMS & PRF

PRF radii divided by scaling factor

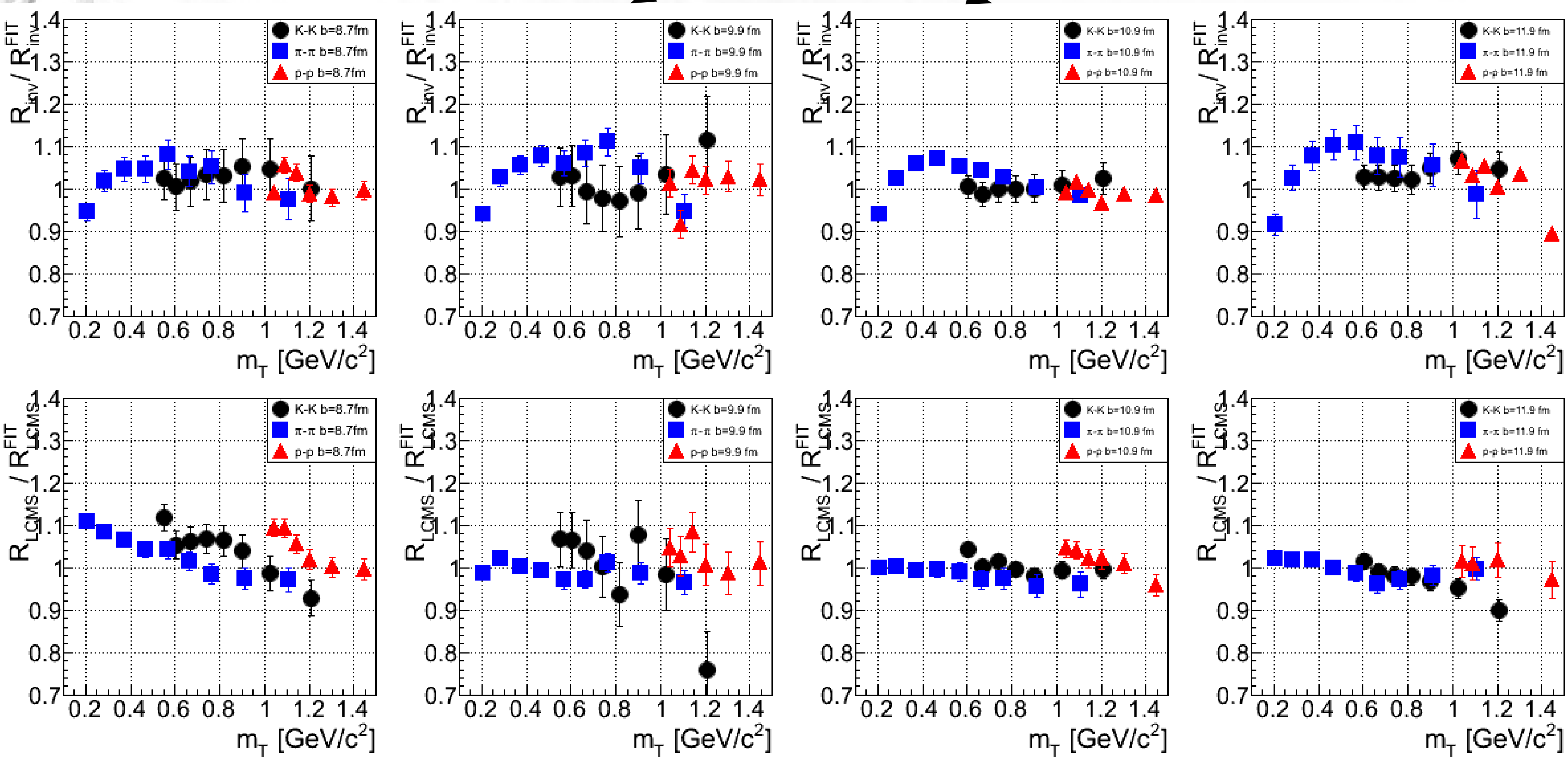


LCMS

Therminator - all centralities – LCMS & PRF

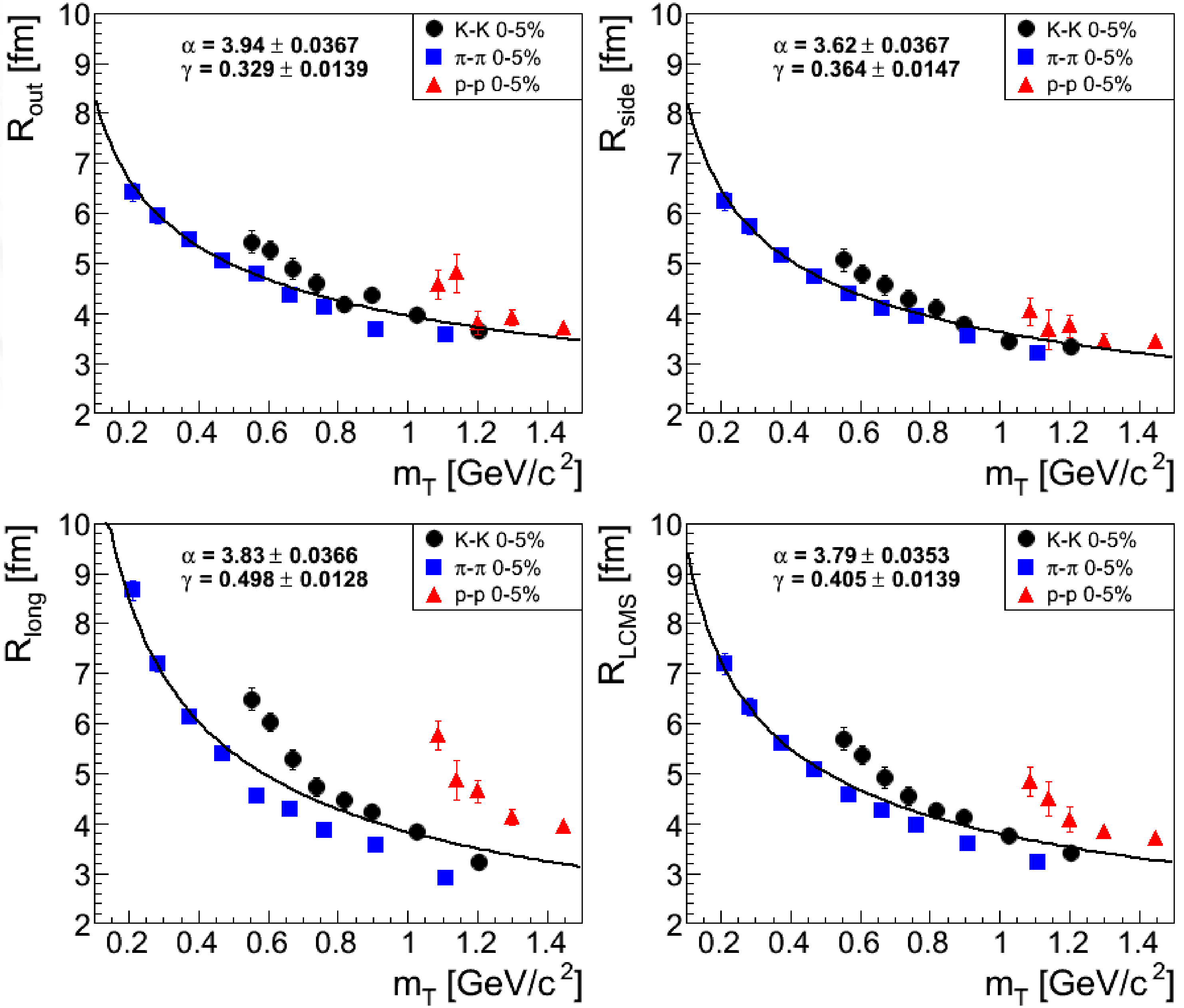
Scaling quality:

PRF



LCMS

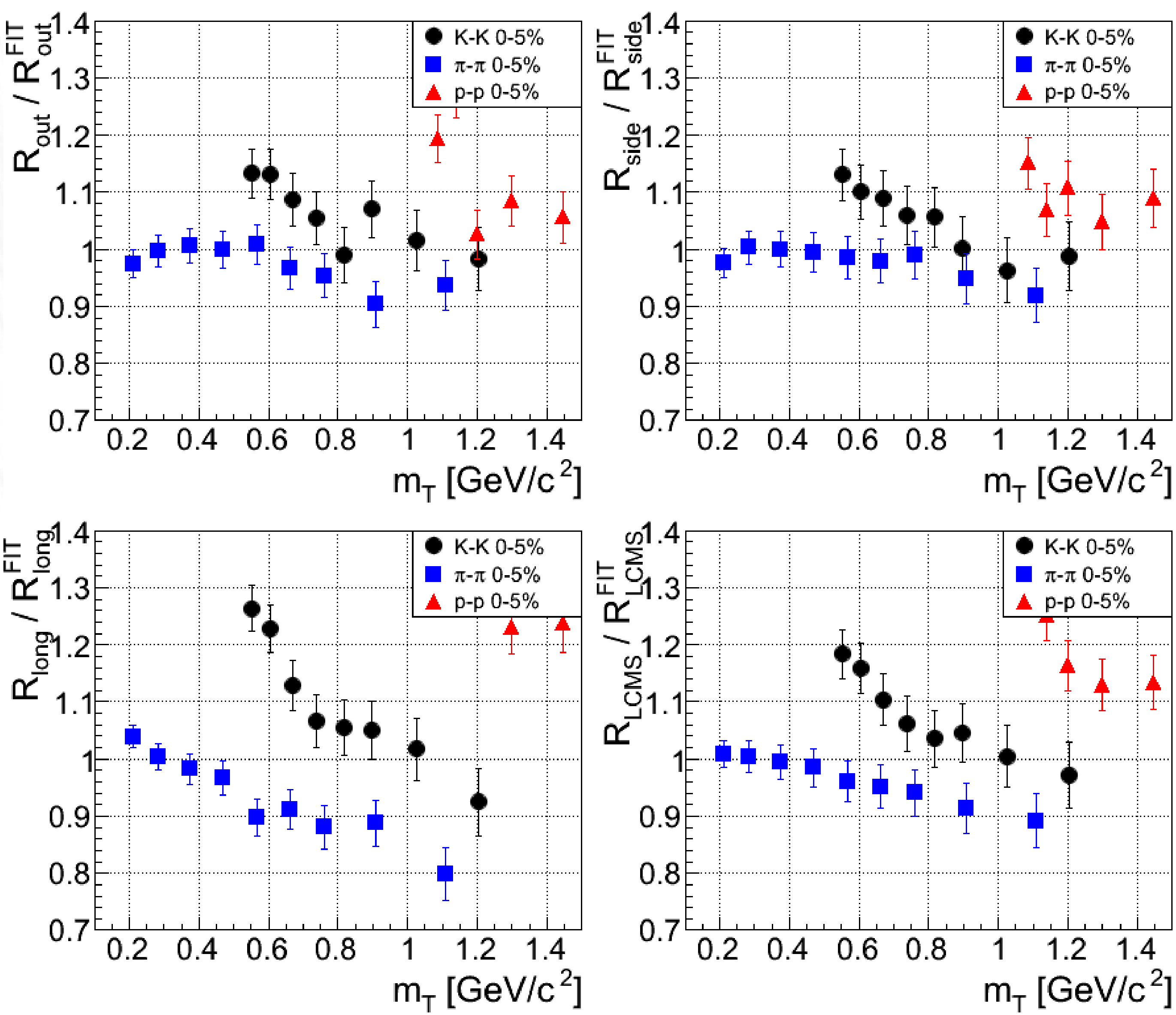
hHKM 0-5% - LCMS



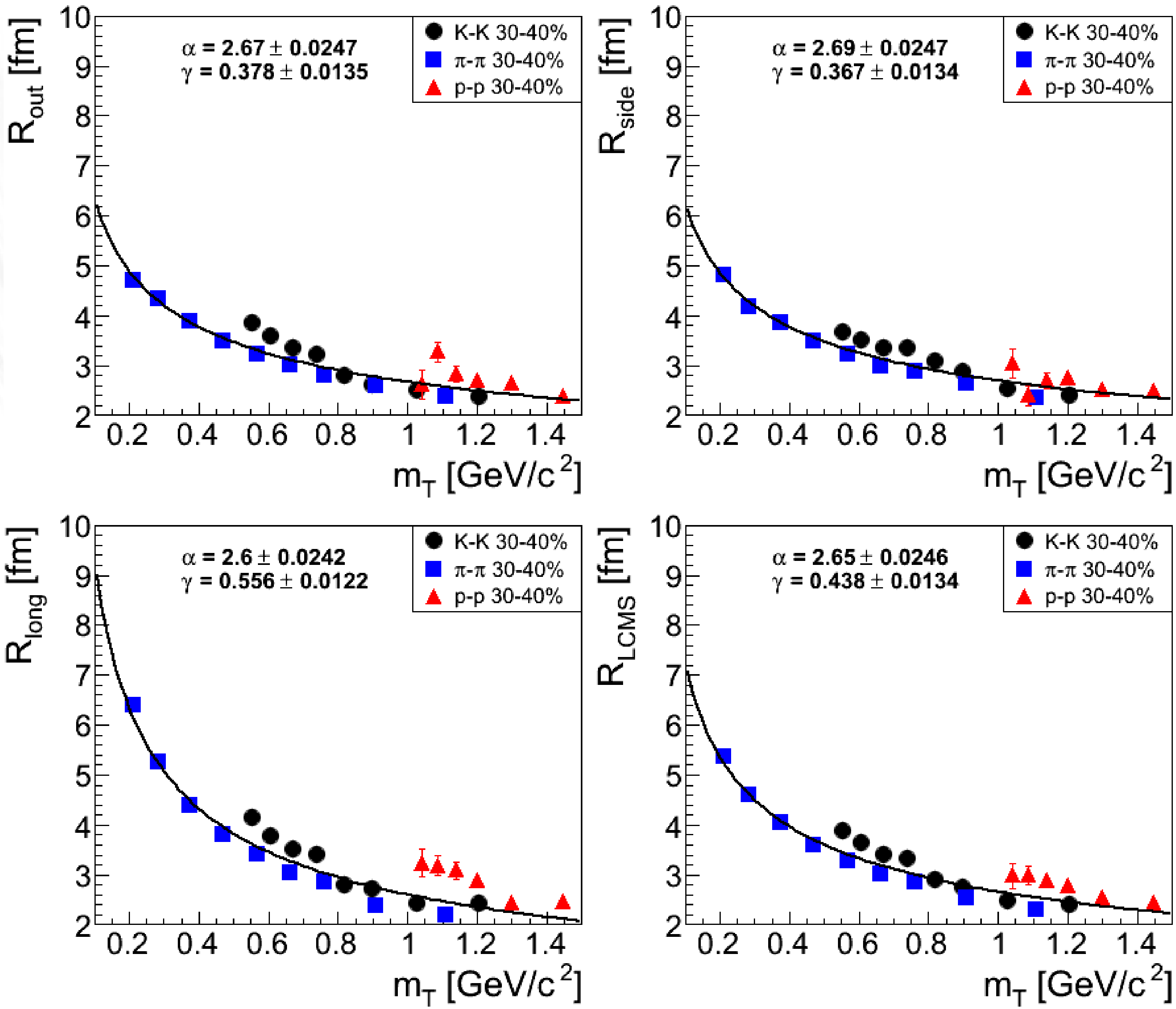
R_{out} [fm]
α = 3.94 ± 0.0367
γ = 0.329 ± 0.0139
K-K 0-5%
π-π 0-5%
p-p 0-5%
m_T [GeV/c^2]
R_{side} [fm]
α = 3.62 ± 0.0367
γ = 0.364 ± 0.0147
K-K 0-5%
π-π 0-5%
p-p 0-5%
m_T [GeV/c^2]
R_{long} [fm]
α = 3.83 ± 0.0366
γ = 0.498 ± 0.0128
K-K 0-5%
π-π 0-5%
p-p 0-5%
m_T [GeV/c^2]
R_{LCMS} [fm]
α = 3.79 ± 0.0353
γ = 0.405 ± 0.0139
K-K 0-5%
π-π 0-5%
p-p 0-5%
m_T [GeV/c^2]

hHKM 0-5% - LCMS

Scaling quality:

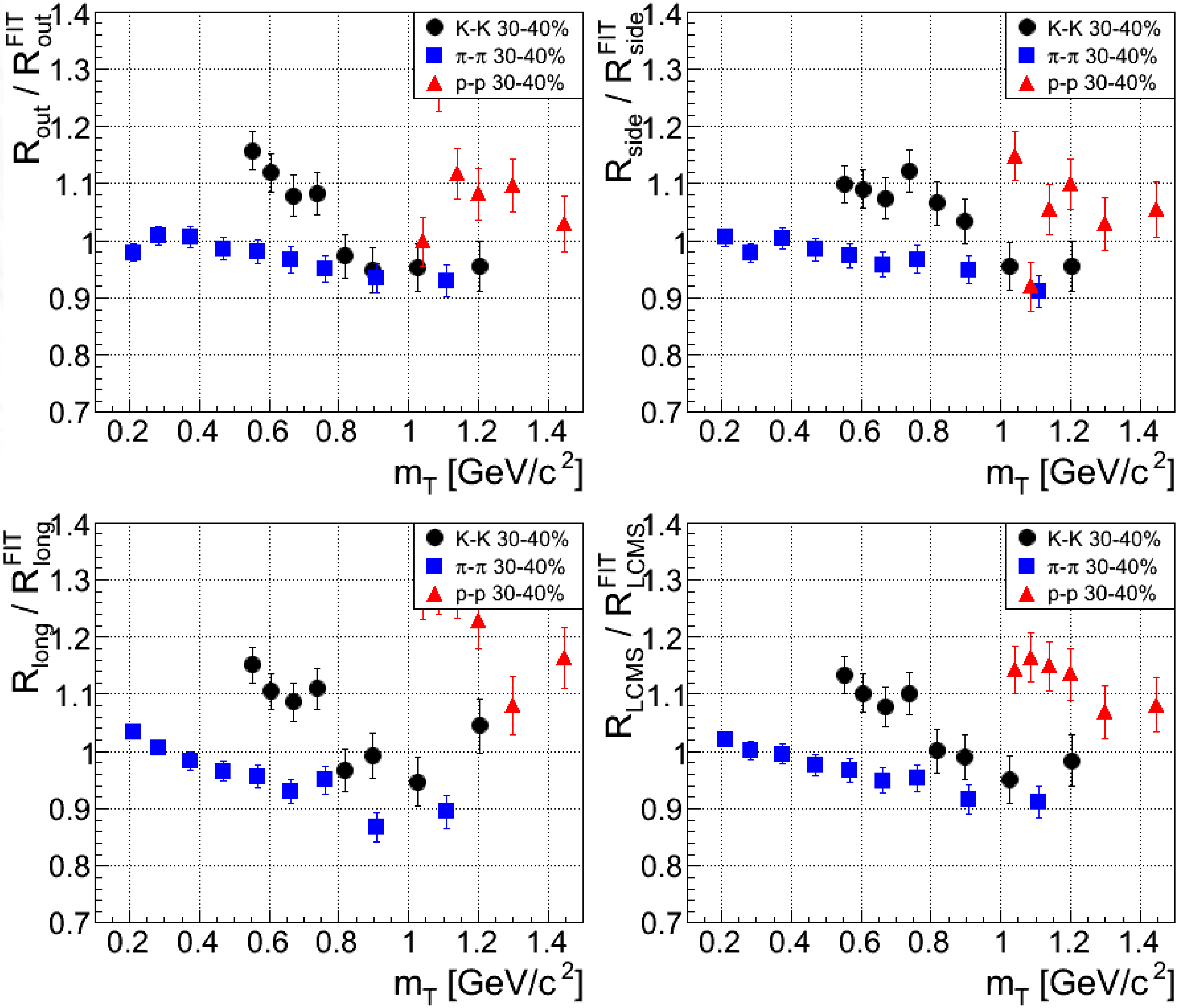


hHKM 30-40% - LCMS



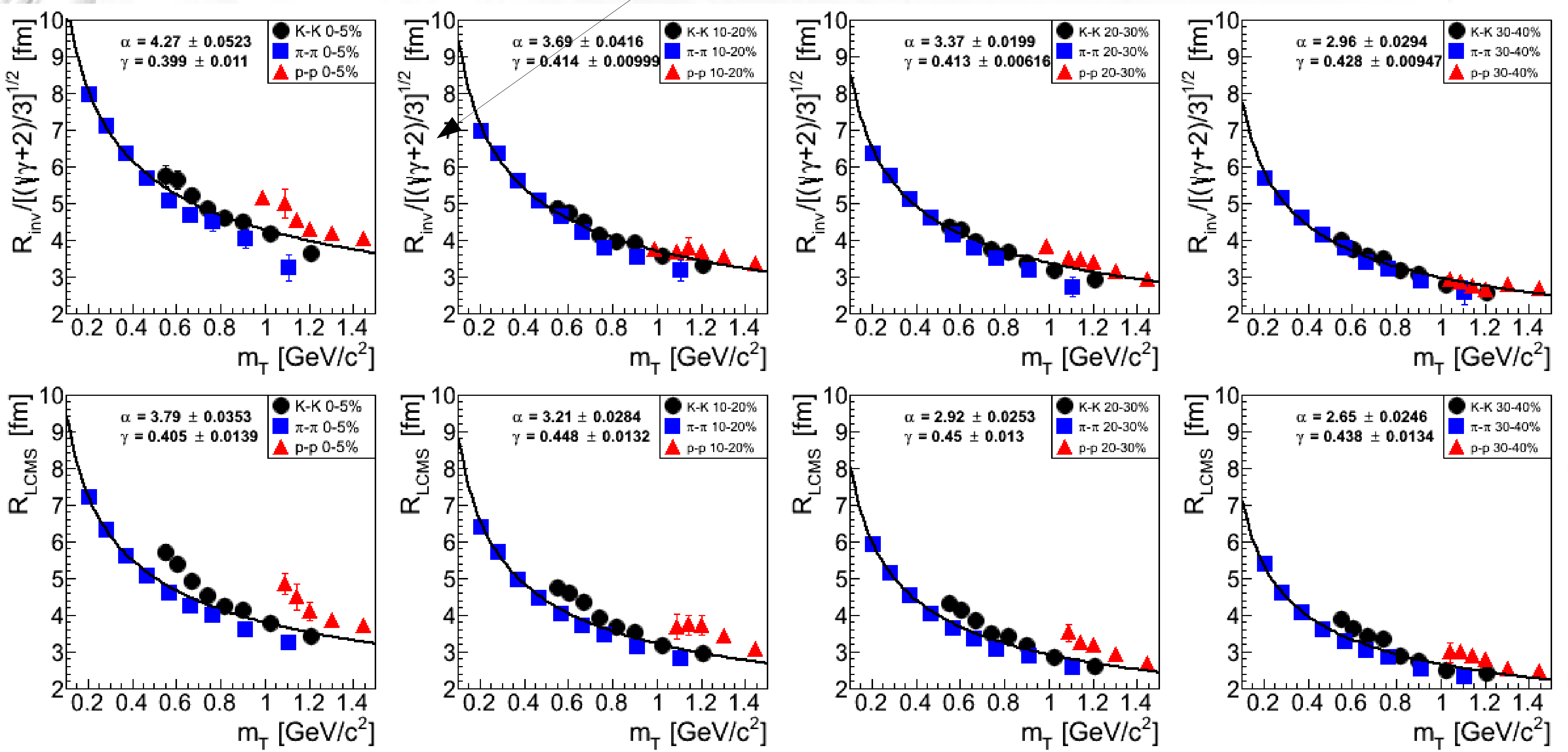
hHKM 30-40% - LCMS

Scaling quality:



hHKM - all centralities – LCMS & PRF

PRF radii divided by scaling factor

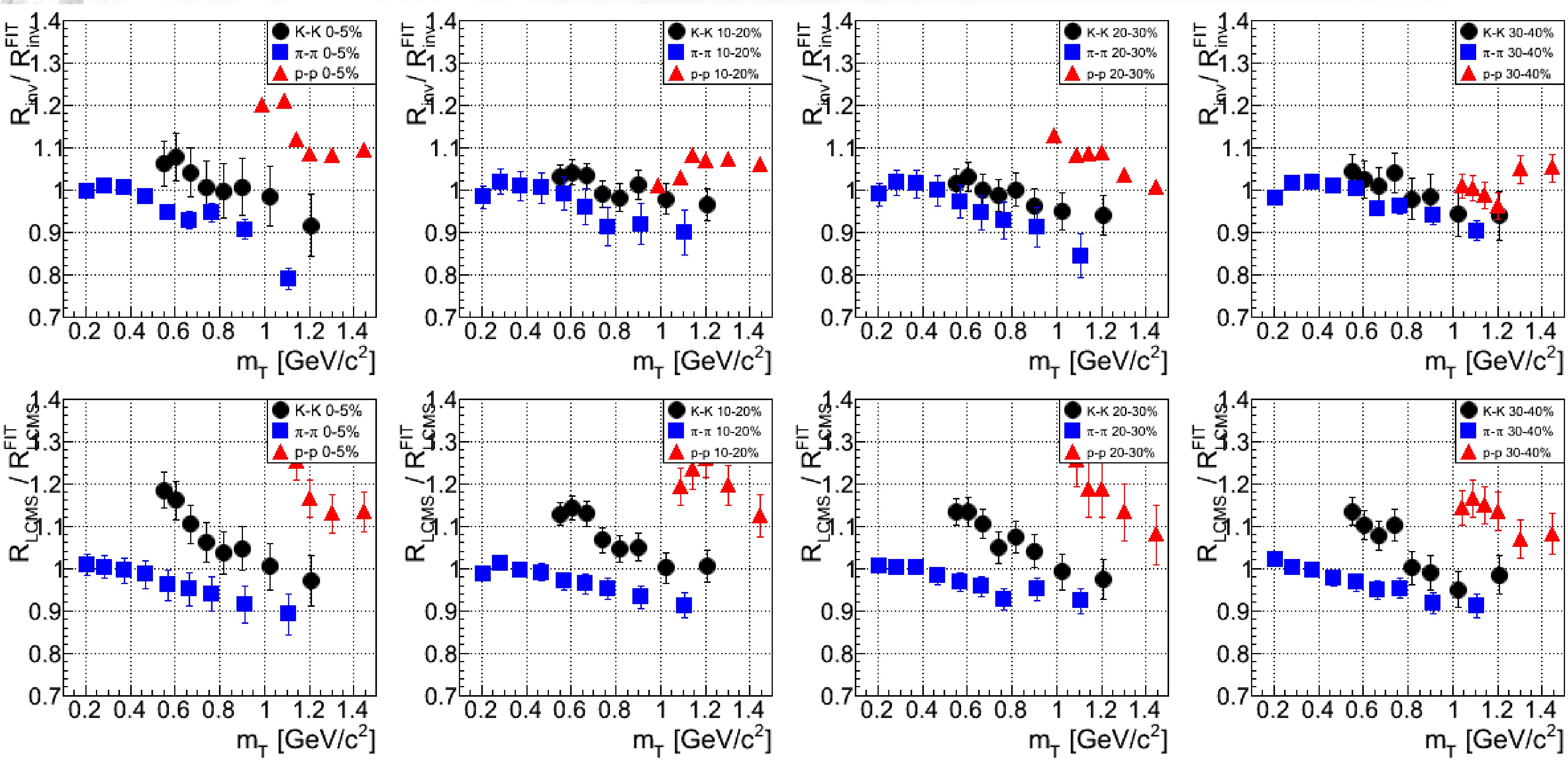


LCMS

hHKM - all centralities – LCMS & PRF

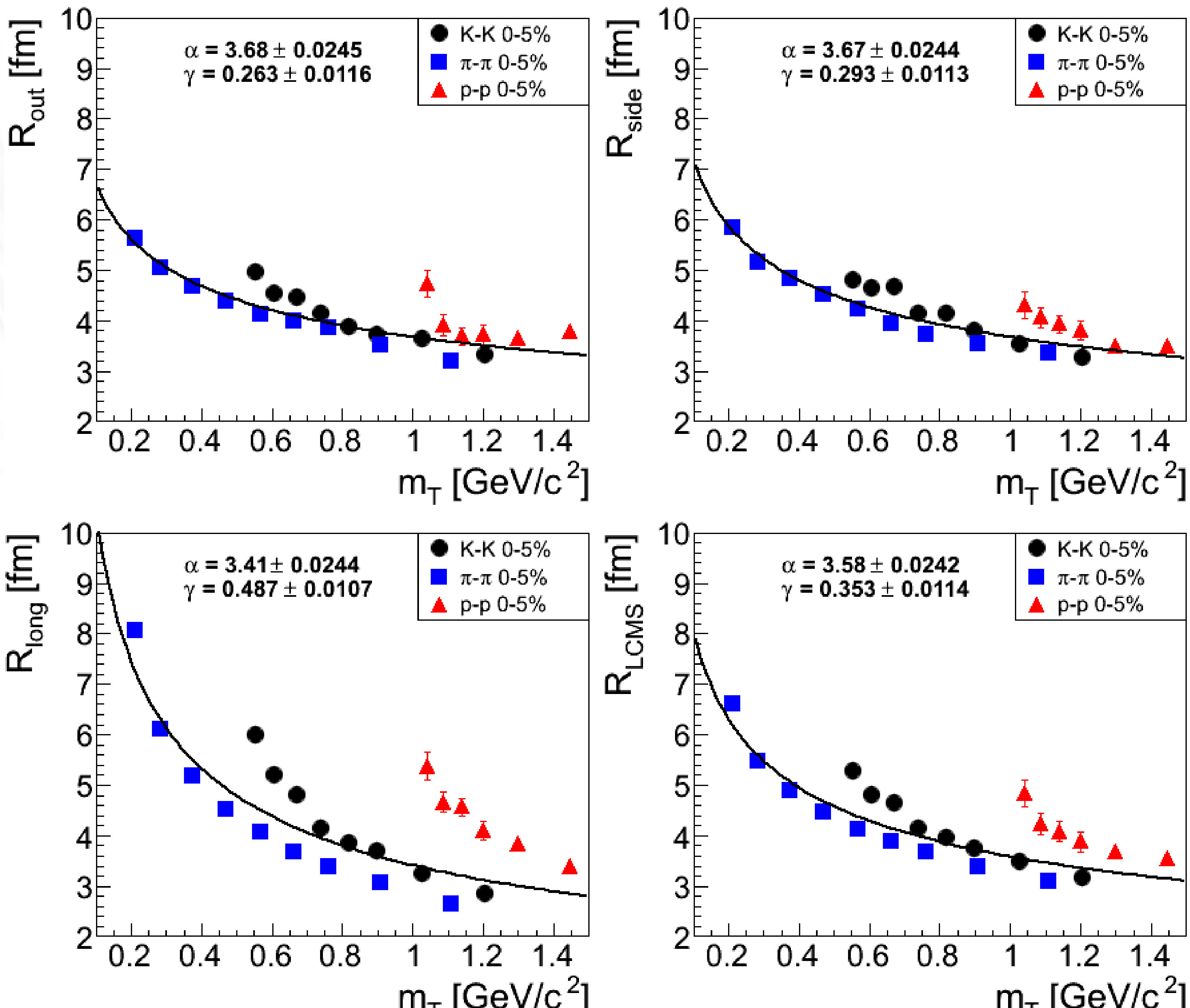
Scaling quality:

PRF



LCMS

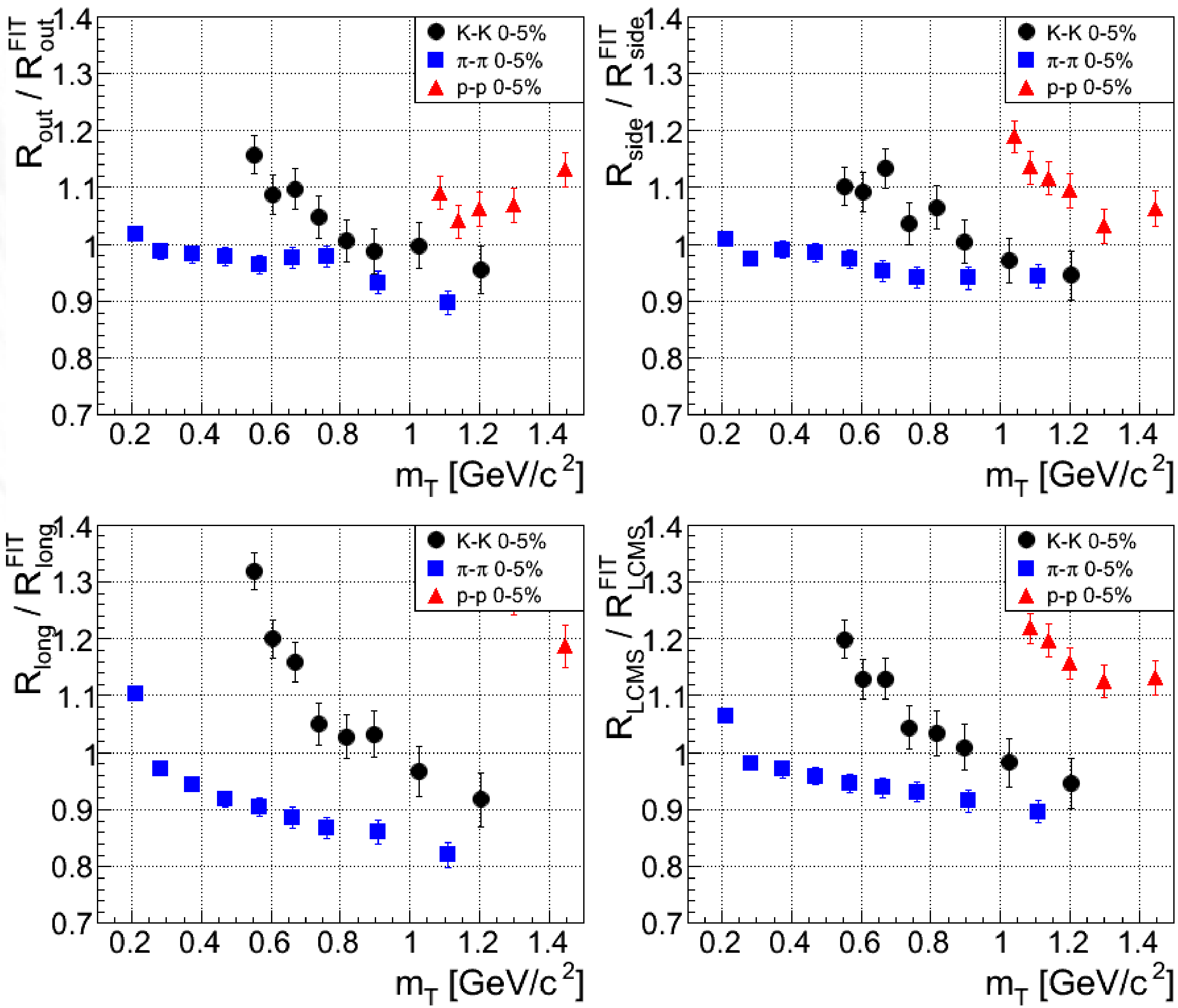
EPOS 0-5% - LCMS



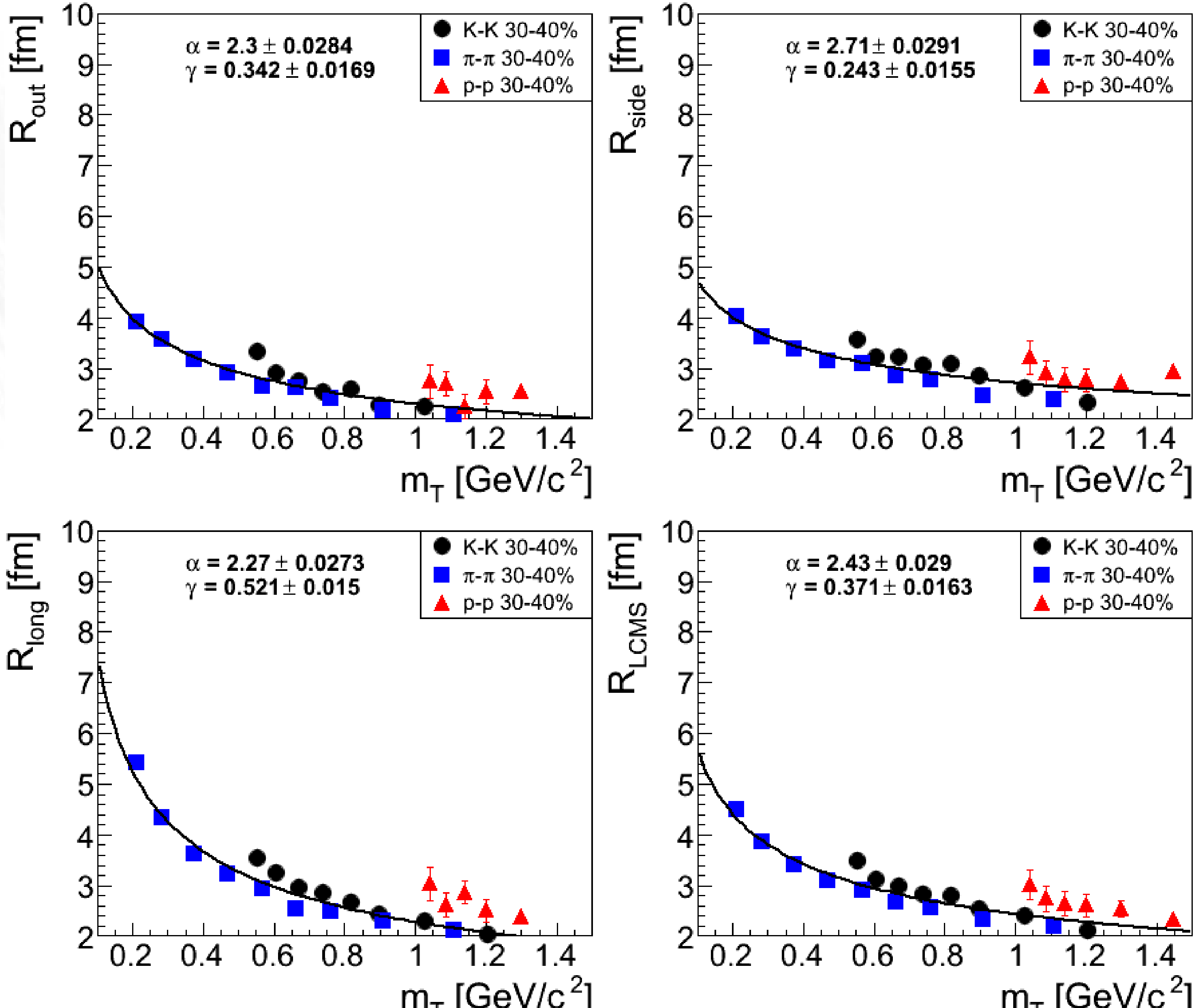
R_{out} [fm]
α = 3.68 ± 0.0245
γ = 0.263 ± 0.0116
K-K 0-5%
π-π 0-5%
p-p 0-5%
m_T [GeV/c^2]
R_{side} [fm]
α = 3.67 ± 0.0244
γ = 0.293 ± 0.0113
K-K 0-5%
π-π 0-5%
p-p 0-5%
m_T [GeV/c^2]
R_{long} [fm]
α = 3.41± 0.0244
γ = 0.487 ± 0.0107
K-K 0-5%
π-π 0-5%
p-p 0-5%
m_T [GeV/c^2]
R_{LCMS} [fm]
α = 3.58 ± 0.0242
γ = 0.353 ± 0.0114
K-K 0-5%
π-π 0-5%
p-p 0-5%
m_T [GeV/c^2]

EPOS 0-5% - LCMS

Scaling quality:

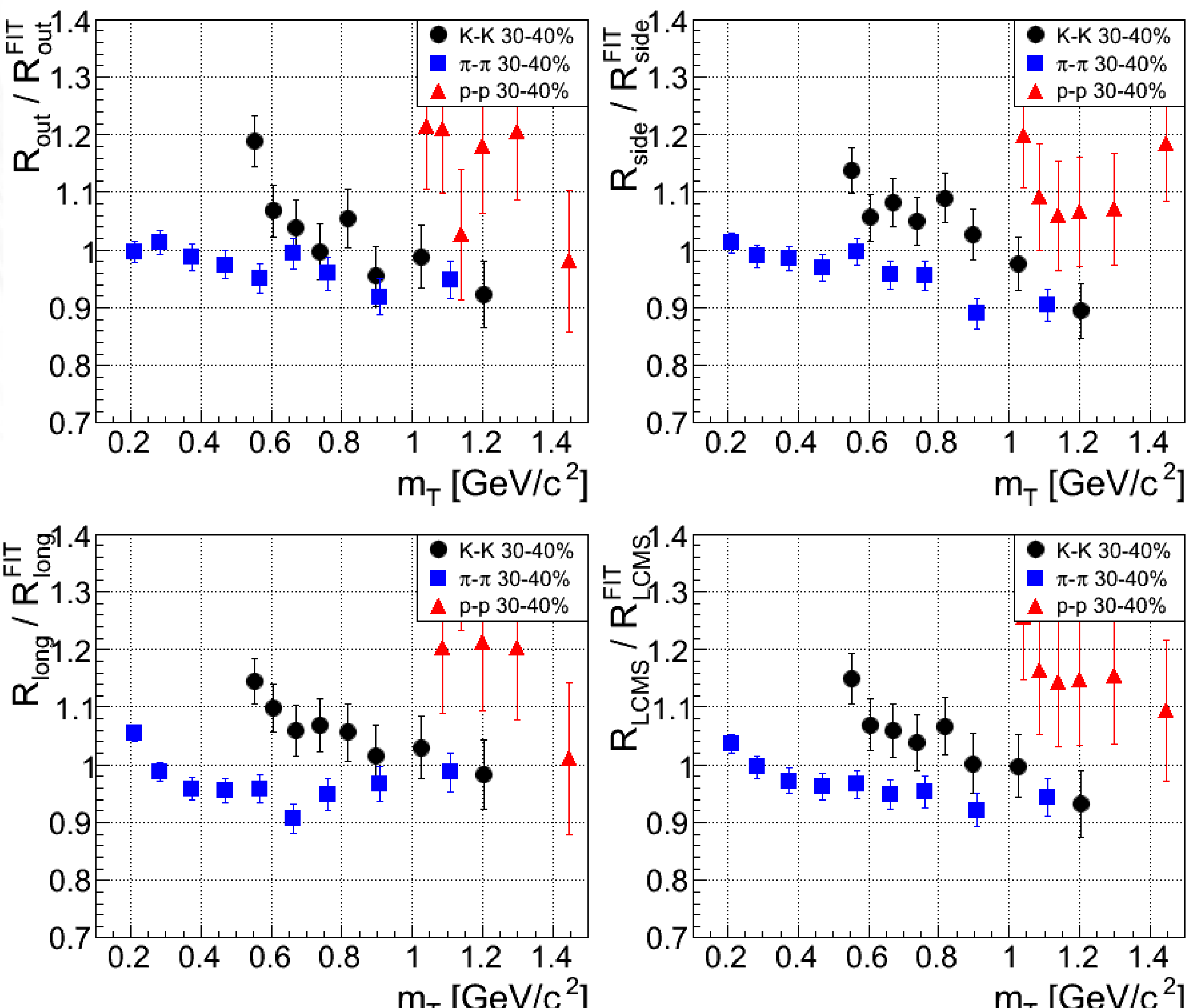


EPOS 30-40% - LCMS



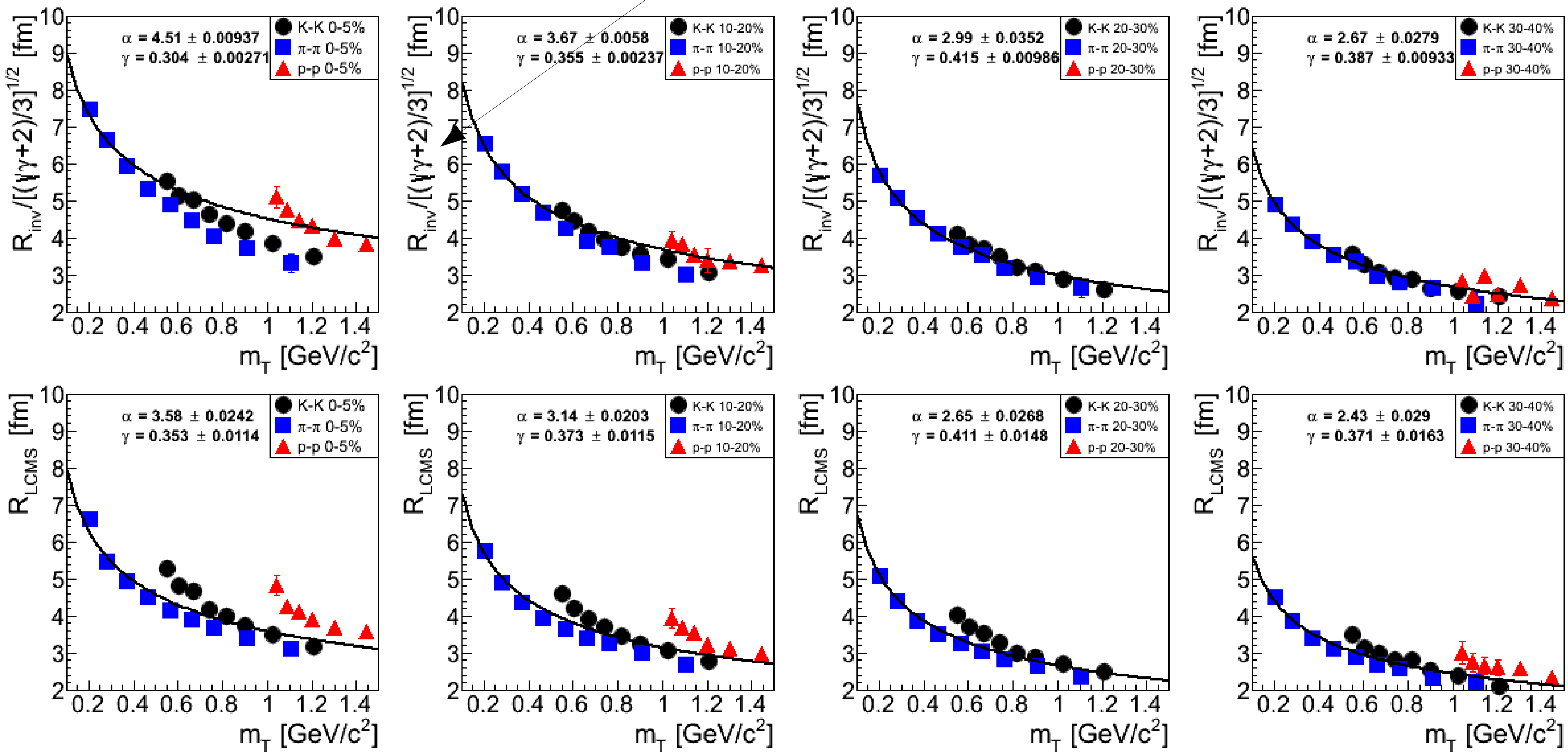
EPOS 30-40% - LCMS

Scaling quality:



EPOS - all centralities – LCMS & PRF

PRF radii divided by scaling factor

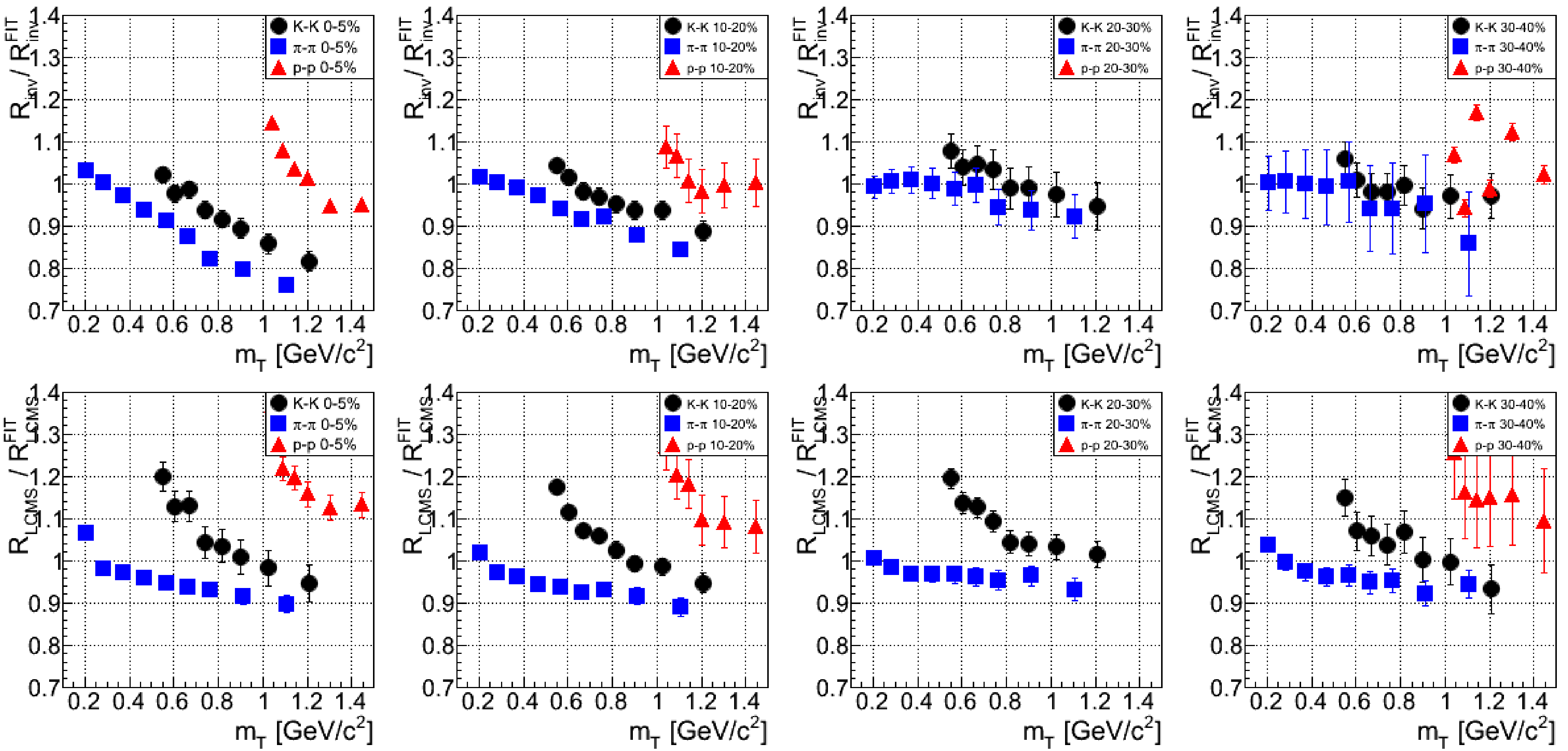


LCMS

EPOS - all centralities – LCMS & PRF

Scaling quality:

PRF



LCMS

Summary

- Simulation agrees with experimental data: π obey m_T scaling predicted by hydrodynamics, this also applies to K and p .
- Scaling of R_{inv} can be recovered by dividing by proposed factor: $[(\sqrt{\gamma}+2)/3]^{1/2}$
- Scaling works well in Therminator model within 10% range.
- For the hHKM and EPOS models scaling works within 20% range.

Backup slides

EPOS model

Energy conserving quantum mechanical multiple scattering approach based on:

- binary parton-parton interactions
- off-shell remnants
- splitting of parton ladders

It includes:

- 3+1 dimensional hydrodynamic evolution
- hadronic cascade procedure after hadronization

hHKM

Hybrid hydrokinetic model is HKM with UrQMD hadronic cascade.
hHKM is based on:

- hydrodynamical expansion of the system
- dynamical decoupling described by escape probabilities

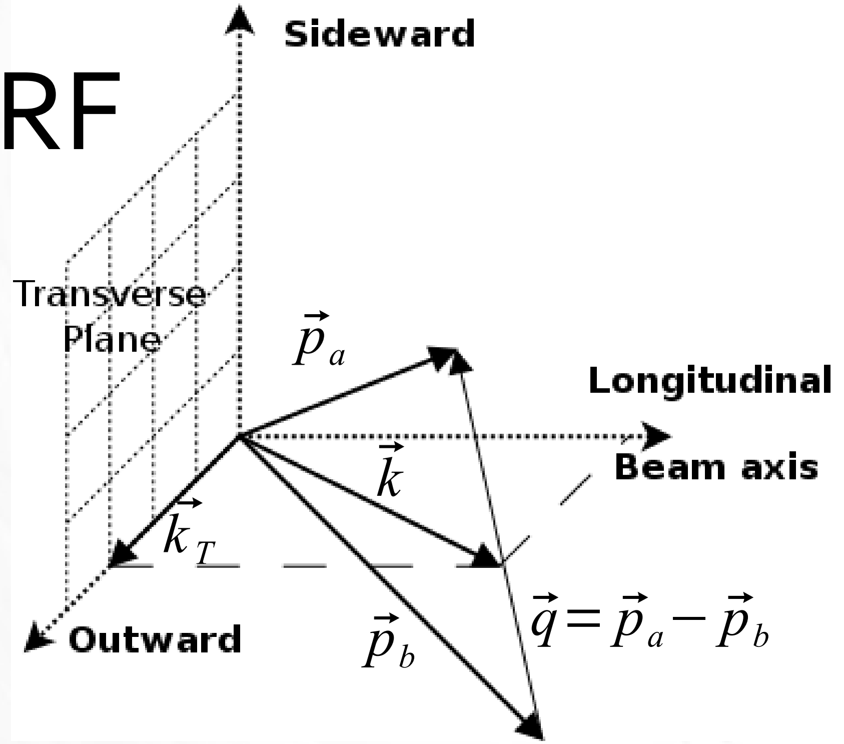
This method corresponds to a generalized relaxation time (τ_{rel}) approximation for the Boltzmann equation applied to inhomogeneous expanding systems; at small τ_{rel} allows one to catch the viscous effects in hadronic component – hadron-resonance gas.

Therminator model

Therminator:

- Performs generation of stable particles and unstable resonances at chosen freeze-out hypersurface based on statistical distribution factors
- Provides subsequent space-time evolution and decays of hadronic resonances in cascades
- 3+1 D viscous hydrodynamic freeze-out hypersurface was used
- No interaction between final state hadrons

LCMS and PRF



Longitudinal Co-Moving System – a system where the pair longitudinal momentum vanishes:

$$p_{a,\,long}=-p_{b,\,long}$$

Pair Rest Frame – a system where the center of mass rests:

$$\vec{p}_a=-\vec{p}_b=\vec{k}^*$$

Pair wave function is most easily expressed and calculated in PRF. The 1D analysis is also performed in PRF.

The 3D analysis gives more information about the system and is usually made in LCMS. The scaling predicted by hydrodynamics is also in LCMS.

Spherical harmonics representation

The 3D space is converted into infinite set of 1-dimensional functions: $Y_l^m(\theta, \varphi)$

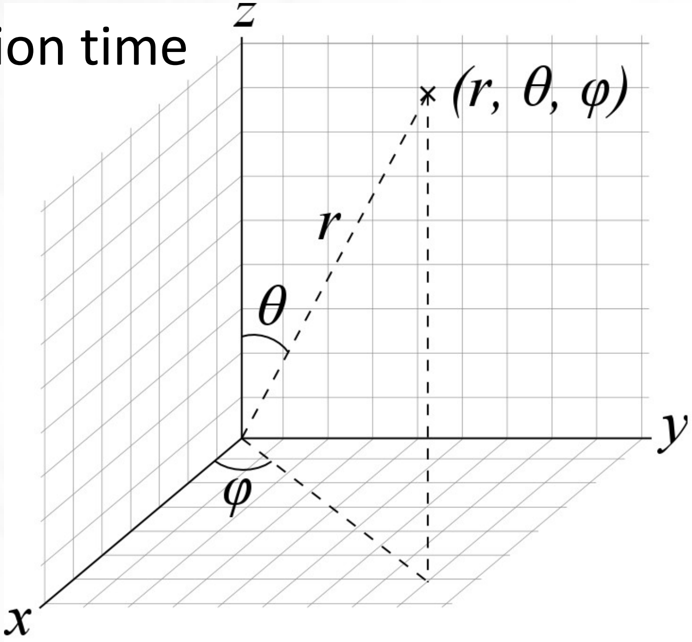
Advantages:

- Symmetries in correlation function reduces necessary amount of data to be stored
- Saves memory at the cost of computation time

$$C(\mathbf{q}) = \sum_{l,m} C_l^m(q) Y_l^m(\theta, \phi),$$

$$C_l^m(q) = \int_\Omega C(q, \theta, \phi) Y_l^m(\theta, \phi) d\Omega,$$

full solid angle



Spherical harmonics representation

A full femtoscopic information is stored only in four l,m components:

$$\Re C_0^0 = N \int C(q, \cos(\theta_q), \varphi_q)\, d\theta_q\, d\varphi_q \quad \rightarrow R_{LCMS}$$

$$\Re C_1^1 \quad \rightarrow space-time\ emission\ asymmetry$$

$$\Re C_2^0 = N \int C(q, \cos(\theta_q), \varphi_q)(3\cos^2(\theta_q)-1)\, d\theta_q\, d\varphi_q \quad \rightarrow \frac{R_{long}}{R_T}$$

$$\Re C_2^2 = N \int C(q, \cos(\theta_q), \varphi_q) \sin^2(\theta_q) \cos(2\varphi_q)\, d\theta_q\, d\varphi_q \quad \rightarrow \frac{R_{out}}{R_{side}}$$

$$R_T = \sqrt{(R_{out}^2 + R_{side}^2)/2} \qquad R_{LCMS} = \sqrt{(R_{out}^2 + R_{side}^2 + R_{long}^2)/3}$$

Symmetries in 3D correlation function of identical particles causes spherical harmonics with odd values of l and m disappear.

For non-identical particles information about emission asymmetry between particle of different mass is stored in $\Re C_1^1$ component.