Analytic solutions of relativistic hydrodynamics for a lattice QCD inspired equation of state

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Outline

1. Hydrodynamics in relativistic heavy ion collisions
   - Basic equations and Equation of State
   - An already known solution
   - Observables with $\kappa =$const.

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   - Temperature dependent EoS with conserved charge
   - Temperature dependent EoS without conserved charge
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2. New solution for more general EoS
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   - Temperature dependent EoS without conserved charge
   - Pressure dependent EoS without conserved charge
3. Investigation with lattice QCD parametrization
   - Temperature dependence from the new solution
   - Connection between initial and final state

Basic equations

- Continuity equation and energy-momentum equation:

$$\partial_\nu(nu^\nu) = 0 \ , \quad \partial_\nu T^{\mu\nu} = 0.$$

- In a perfect fluid, $T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - g^{\mu\nu} p$.
- From the condition $\partial_\nu T^{\mu\nu} = 0$, the Euler equation and the energy conservation equation can be deduced:

$$(\epsilon + p)\partial_\nu u^\nu + u^\nu \partial_\nu \epsilon = 0,$$
$$(\epsilon + p)u^\nu \partial_\nu u^\mu = (g^{\mu\nu} - u^\mu u^\nu)\partial_\nu p.$$

- EoS is $\epsilon = \kappa p$, where κ is not necessarily constant.

A known solution

(Csörgő, Csernai, Hama, Kodama, Heavy.Ion.Phys. A21:73-84, 2004)

- Ellipsoidal symmetry in space-time: $s = \frac{x^2}{X^2(t)} + \frac{y^2}{Y^2(t)} + \frac{z^2}{Z^2(t)}$

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- The solution:
  - $n = n_0 \left(\frac{\tau_0}{\tau}\right)^3 \nu(s)$
  - $T = T_0 \left(\frac{\tau_0}{\tau}\right)^{3/\kappa} \frac{1}{\nu(s)}$
  - $p = nT$
- $\nu(s)$: arbitrary function of the s scale parameter (e.g. $e^{-bs/2}$)

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- In this solution κ is <u>constant</u>
- Entropy density σ can be calculated as well

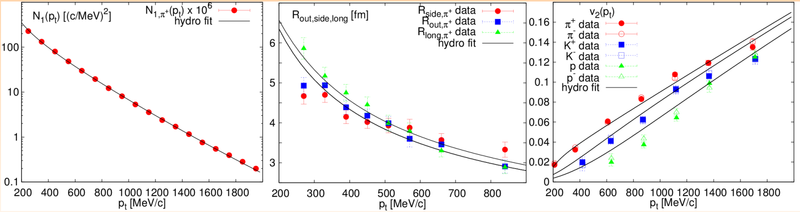
Observables with κ =constant

- Source function can be calculated from this solution

$$S(x,p)d^4x = \mathcal{N} n(x) p^\mu d^3\Sigma_\mu(x) H(\tau) d\tau \exp\left(-\frac{p_\mu u^\mu}{T}\right)$$

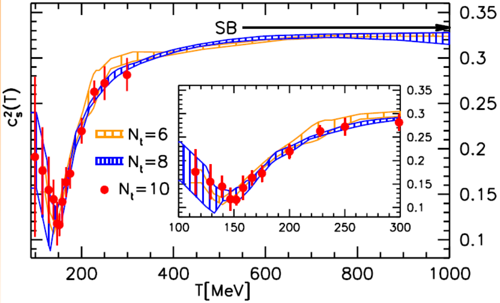
- Calculated observables are fitted to data succesfully

Csanád, Vargyas, Eur. Phys. J. A 44, 473 (2010), arXiv:0909.4842



More general EoS

- Constant EoS may not be realistic (κ may depend on temperature or pressure)



Borsányi, Fodor, Katz et al., JHEP 1011, 077 (2010), arXiv:1007.2580a

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- From lQCD trace anomaly ($I = \epsilon - 3p$) can be calculated
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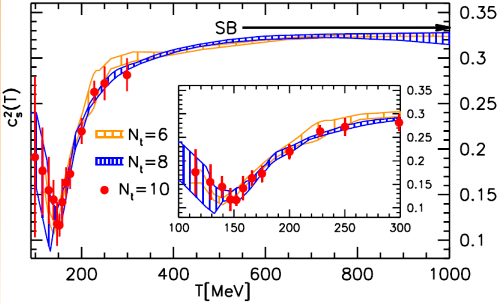

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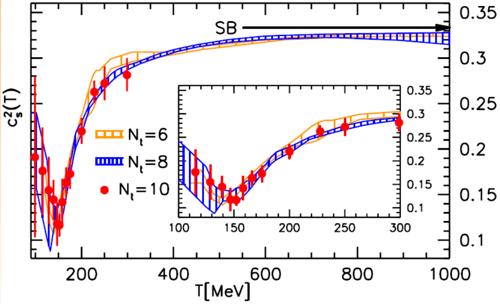
- From $I \to \kappa(T) = I(T)/p(T) + 3$, speed of sound: $\kappa = \frac{1}{c_s^2}$



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Temperature dependent EoS with conserved charge

- If there is a conserved charge (n), the energy equation yields

$$T\partial_\nu u^\nu + \frac{\mathrm{d}}{\mathrm{d}T}\left(T\kappa(T)\right) u^\nu \partial_\nu T = 0.$$

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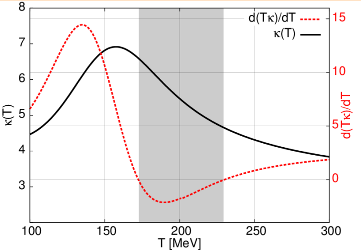

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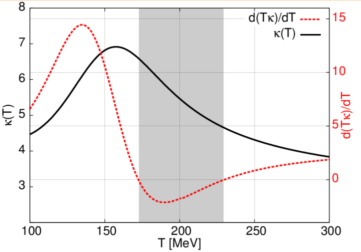

- Solution cannot be applied if 173 MeV $< T <$ 225 MeV
- This problem is absent, if there is no conserved charge

Temperature dependent EoS without conserved charge

- If there is no conserved charge, we have

$$\epsilon = Ts - p \ \rightarrow d\epsilon = Tds. \tag{1}$$

(because of Gibbs–Duhem relation: $dp = sdT$)

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- Put (1) and $\epsilon + p = Ts$ to the energy equation:

$$T\sigma\partial_\nu u^\nu + u^\nu T\partial_\nu\sigma \rightarrow \partial_\nu(\sigma u^\nu) = 0$$

This is a continuity equation to entropy-density.

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- Put $\epsilon + p = Ts$ and $\epsilon = \kappa(T)p$ to the energy equation

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- It's not the same equation (only if κ=const.)

New solution with more general EoS

Csanád, Nagy, Lökös, Eur. Phys. J. A (2012) 48: 173

If there is a conserved charge

$$
\begin{aligned}
n &= n_0 \frac{\tau_0^3}{\tau^3}, \\
u^\nu &= \frac{x^\nu}{\tau}, \\
\frac{\tau_0^3}{\tau^3} &= \exp\left[\int_{T_0}^{T} \frac{d\kappa(\zeta)\zeta}{d\zeta}\frac{1}{\zeta} d\zeta\right]
\end{aligned}
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- Arbitrary $\kappa(T)$ function may be used
- If $d\kappa(T)T/dT < 0 \rightarrow \partial_\nu u^\nu < 0$! It's not realistic!

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If there is no conserved charge

$$\sigma = \sigma_0 \frac{\tau_0^3}{\tau^3},$$

$$u^\nu = \frac{x^\nu}{\tau},$$

$$\frac{\tau_0^3}{\tau^3} = \exp\left[\int_{T_0}^{T}\left(\frac{\kappa(\zeta)}{\zeta} + \frac{1}{\kappa+1}\frac{d\kappa(\zeta)}{d\zeta}d\zeta\right)\right]$$

- It can be used if κ is given as a function of temperature

Pressure dependent κ without conserved charge

Csanád, Nagy, Lökös, Eur. Phys. J. A (2012) 48: 173

- If EoS is given as a function of pressure: $\epsilon = \kappa(p)p$

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- If EoS is given as a function of pressure: $\epsilon = \kappa(p)p$
- The energy equation can written with this EoS as

$$\partial_\nu(\epsilon u^\nu) + p\partial_\nu u^\nu = 0$$

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$$\left(\frac{\epsilon}{p} + 1\right)\partial_\nu \ln\left(\frac{V_0}{V}\right) = \frac{\partial_\nu \epsilon}{p}$$

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$$\left(\frac{\epsilon}{p}+1\right)\partial_\nu \ln\left(\frac{V_0}{V}\right) = \frac{\partial_\nu \epsilon}{p}$$

- With an integral trasformation it can be „solved" by

$$\frac{V_0}{V} = \exp\left[\int_{p_0}^{p}\left(\frac{\kappa(\zeta)}{\zeta}+\frac{\mathrm{d}\kappa}{\mathrm{d}\zeta}\right)\frac{\mathrm{d}\zeta}{\kappa(\zeta)+1}\right]$$

A new solution if $\kappa = \kappa(p)$

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This new solution can be written as

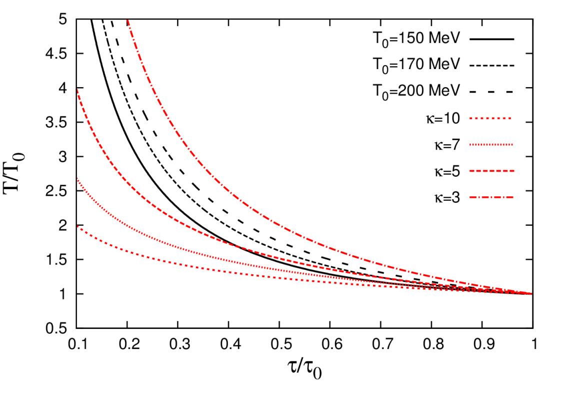
$$\sigma = \sigma_0 \frac{\tau_0^3}{\tau^3},$$
$$u^\nu = \frac{x^\nu}{\tau},$$
$$\frac{\tau_0^3}{\tau^3} = \exp\left[\int_{p_0}^{p} \left(\frac{\kappa(\zeta)}{\zeta} + \frac{\mathrm{d}\kappa}{\mathrm{d}\zeta}\right) \frac{\mathrm{d}\zeta}{\kappa(\zeta)+1}\right]$$

- It can be used if a parametrization to $\epsilon(p)$ is given
- If $p = p(T)$, we get the previous solution

Investigation without conserved charge

- Let assume lQCD EoS
- $T(\tau)$ can be calculated

$$\frac{V_0}{V} = \exp\left[\int_{T_0}^{T}\left(\frac{\kappa(\zeta)}{\zeta} + \frac{1}{\kappa+1}\frac{d\kappa(\zeta)}{d\zeta}d\zeta\right)\right]$$



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- Found new solutions with temperature, pressure dependent EoS
- $T(\tau)$ can be calculated $\rightarrow$ lQCD EoS applicable
- If assuming τ_f/τ_{init}, T_f/T_{init} can be calculated

Thank you for your attention!