Anisotropic hydrodynamics and the early thermalization puzzle

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1. Motivation

1.1 Early thermalization puzzle at RHIC

- experimental data from RHIC (and the LHC) are commonly interpreted as the evidence for very fast local equilibration of matter, success of perfect fluid hydrodynamics ($\tau_{\rm th} \leq 1$ fm/c)
- fast equilibration contradicts the results of microscopic models of heavy-ion collisions, string models, color glass condensate, pQCD kinetic calculations, ...
- for $\tau < 1$ fm/c, the system exhibits high pressure anisotropy, typically $P_\perp \gg P_\parallel$
- only at such early times the transverse distribution of matter in nuclei is known and may be used to model the initial energy/entropy density for the hydrodynamic calculations
- the use of viscous hydrodynamics too early after the impact may be questioned – the corrections to the perfect-fluid energy-momentum tensor are very large
- AdS/CFT correspondence (Janik et al.) predicts also a large difference between $P_\perp$ and $P_\parallel$, which slowly decays with time
- this gives hints for the REORGANIZATION of the HYDRODYNAMIC EXPANSION, with the ANISOTROPY INCLUDED in the LEADING ORDER

1. Motivation

1.2 Forms of the energy-momentum tensor of matter produced in HIC

- (early stages) color glass condensate:

$$T^{\mu\nu}\Big|_{\tau\ll 1/Q_s} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & \varepsilon & 0 & 0 \\ 0 & 0 & \varepsilon & 0 \\ 0 & 0 & 0 & -\varepsilon \end{pmatrix}$$

T. Lappi, K. Fukushima

$$T^{\mu\nu}\Big|_{\tau\gg 1/Q_s\sim 0.2fm} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & \varepsilon/2 & 0 & 0 \\ 0 & 0 & \varepsilon/2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Y. V. Kovchegov, A. Krasnitz
Y. Nara, R. Venugopalan

- (late stages) perfect-fluid behavior:

$$T^{\mu\nu}_{\rm perfect\ hydro} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

- in the leading order, the energy momentum-tensor is diagonal but the pressure may be highly anisotropic

2. Concept of anisotropic hydrodynamics

2.1 Covariant form of the energy-momentum tensor

- $P_\perp \neq P_\parallel$

$$T^{\mu\nu} = (\varepsilon + P_\perp) U^\mu U^\nu - P_\perp g^{\mu\nu} - (P_\perp - P_\parallel) V^\mu V^\nu$$

- $P_\perp = P_\parallel \rightarrow$ isotropic fluid, $\quad T^{\mu\nu} \rightarrow T^{\mu\nu}_{\rm perfect\ hydro}$

$$U^\mu = \gamma(1, v_x, v_y, v_z), \quad \gamma = (1 - v^2)^{-1/2} \quad \text{hydrodynamic flow}$$

$$V^\mu = \gamma_z(v_z, 0, 0, 1), \quad \gamma_z = (1 - v_z^2)^{-1/2} \quad \text{longitudinal axis}$$

$$U^2 = 1, \quad V^2 = -1, \quad U \cdot V = 0$$

- local rest frame: $U^\mu = (1,0,0,0)$ and $V^\mu = (0,0,0,1)$

$$T^{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P_\perp & 0 & 0 \\ 0 & 0 & P_\perp & 0 \\ 0 & 0 & 0 & P_\parallel \end{pmatrix}$$

M.Martinez, M.Strickland, NPA848 (2010) 183, NPA856 (2011) 68

R.Ryblewski, WF, PRC83 (2011) 034907, PRC85 (2012) 064901

2. Concept of anisotropic hydrodynamics

2.2 Energy-momentum conservation and entropy production

- energy and momentum are conserved but the entropy may grow

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= 0 \\ \partial_\mu S^\mu &= \Sigma \end{aligned}$$

$S^\mu = \sigma U^\mu$ – entropy flow
Σ – entropy source, its specific form defines the model

- one has to specify:

generalized EOS	$\epsilon = \epsilon(P_\perp, P_\parallel)$
entropy production term	$\Sigma = \Sigma(P_\perp, P_\parallel)$

- system of 5 equations for 5 unknown functions: $\vec{v}$, $P_\perp$, $P_\parallel$
- in particular, for massless partons the condition $T^\mu_{\ \mu} = 0$ gives

$$\varepsilon(P_\perp, P_\parallel) = 2P_\perp + P_\parallel$$

3. Microscopic interpretation
3.1 Parton distribution function

- locally anisotropic systems of particles $\rightarrow$ two different scales $\lambda_\perp$ and $\lambda_\parallel$, may be interpreted as the transverse and longitudinal temperature

$$f_{LRF} = f\left(\frac{p_\perp}{\lambda_\perp}, \frac{|p_\parallel|}{\lambda_\parallel}\right)$$

- Romatschke-Strickland (RS) form, generalization of equilibrium/isotropic distributions, frequently used in the studies of anisotropic quark-gluon plasma (here as a modified Boltzmann distribution)

$$f_{LRF} = g_0 \exp\left(-\sqrt{\frac{p_\perp^2}{\lambda_\perp^2} + \frac{p_\parallel^2}{\lambda_\parallel^2}}\right) = g_0 \exp\left(-\frac{1}{\lambda_\perp}\sqrt{p_\perp^2 + x\, p_\parallel^2}\right)$$

where $x = 1 + \xi = \left(\frac{\lambda_\perp}{\lambda_\parallel}\right)^2$ is the anisotropy parameter

- covariant version

$$f = g_0 \exp\left(-\frac{1}{\lambda_\perp}\sqrt{(p \cdot U)^2 + \xi\, (p \cdot V)^2}\right)$$

3. Microscopic interpretation

3.2 Energy-momentum tensor and entropy flux

- moments of anisotropic distributions

$$T^{\mu\nu} = \int \frac{d^3p\, p^\mu p^\nu}{(2\pi)^3 E_p} f(p\cdot U, p\cdot V) = (\varepsilon + P_\perp)\, U^\mu U^\nu - P_\perp\, g^{\mu\nu} - (P_\perp - P_\parallel) V^\mu V^\nu$$

$$S^\mu = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{E_p} f(p\cdot U, p\cdot V) \left[1 - \ln\left(\frac{f(p\cdot U, p\cdot V)}{g_0}\right)\right] = \sigma U^\mu$$

- for further analysis most convenient two independent parameters are x (anisotropy parameter) and σ (non-equilibrium entropy density)

$$(P_\perp, P_\parallel) \text{ or } (\lambda_\perp, \lambda_\parallel) \longrightarrow (\sigma, x)$$

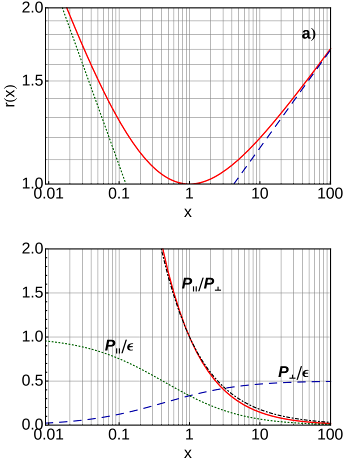
3. Microscopic interpretation

3.3 Energy, pressure, entropy for RS form

generalized equation of state

$$\begin{aligned}
\varepsilon(\sigma, x) &= \varepsilon_{\mathrm{id}}(\sigma) r(x) \\
P_{\perp}(\sigma, x) &= P_{\mathrm{id}}(\sigma)\left[r(x) + 3x r'(x)\right] \\
P_{\parallel}(\sigma, x) &= P_{\mathrm{id}}(\sigma)\left[r(x) - 6x r'(x)\right]
\end{aligned}$$

$$r(x) = \frac{x^{-\frac{1}{3}}}{2}\left[1 + \frac{x \arctan\sqrt{x-1}}{\sqrt{x-1}}\right]$$



3. Microscopic interpretation

3.4 Connection to kinetic theory

M.Martinez, M.Strickland, NPA848 (2010) 183, NPA856 (2011) 68
M.Martinez, R. Ryblewski, M.Strickland, PRC85 (2012) 064901
zeroth moment of the Boltzmann equation = entropy production = gluon emission

$$p^\mu \partial_\mu f = C \approx -p \cdot U\,\Gamma\,(f - f_{\rm eq}), \quad \partial_\mu \int dP\, p^\mu f = \int dP\, C$$

Γ is the inverse relaxation time, for the covariant RS form, $\sigma = 4n$, one gets

$$\partial_\mu\,(\sigma U^\mu) = \frac{1}{4}\int dP\ C = \Sigma \approx \frac{\Gamma}{4}(n_{\rm eq} - n) \quad \text{(one equation)}$$

first moment of the Boltzmann equation, energy-momentum conservation

$$\partial_\mu \int dP\, p^\nu p^\mu f = \int dP\, p^\nu C = \partial_\mu T^{\nu\mu} = 0 \quad \text{(four equations)}$$

$$\int dP\, p^\nu C = -\int dP\, p\cdot U\, p^\nu\,\Gamma\,(f - f_{\rm eq}) = 0 \quad \text{(Landau matching for } T(P_\perp, P_\parallel) \text{ in } f_{\rm eq})$$

5 equations for 5 unknown functions: $\vec{v}$, $P_\perp$, $P_\parallel$ similarly as in the phenomenological approach introduced earlier

3. Microscopic interpretation

3.5 Phenomenological ansatz for Σ

- the simplest ansatz for Σ has the form

$$\Sigma = \frac{(\lambda_\perp - \lambda_\parallel)^2}{\lambda_\perp \, \lambda_\parallel} \frac{\sigma}{\tau_{\rm eq}} = \frac{(1-\sqrt{x})^2}{\sqrt{x}} \frac{\sigma}{\tau_{\rm eq}}$$

  where $\tau_{\rm eq}$ is a timescale parameter
- consistent with Israel-Stewart (IS) theory for small $\xi = x - 1$ and for purely longitudinal boost-invariant motion (more details given later)

$$\Sigma \approx \frac{\xi^2}{4\tau_{\rm eq}} \sigma$$

- for large $\xi = x - 1$ various forms of Σ are conceivable, results of microscopic models may be used to introduce time dependence of x, in particular one may use the AdS/CFT correspondence

4. Purely-longitudinal boost-invariant motion

4.1 Implementation of boost-invariance

- boost-invariant ansatz for U and V

$$U^\mu = (\cosh\eta, 0, 0, \sinh\eta), \qquad V^\mu = (\sinh\eta, 0, 0, \cosh\eta)$$

$$\tau = \sqrt{t^2 - z^2}, \qquad \eta = \frac{1}{2}\ln\frac{t+z}{t-z}$$

- leads to the two equations of motion

$$\frac{d\varepsilon}{d\tau} = -\frac{\varepsilon + P_\parallel}{\tau}, \qquad \frac{d\sigma}{\sigma d\tau} + \frac{1}{\tau} = \frac{\Sigma}{\sigma}$$

- the first equation is equivalent to:

$$r'(x)\left(\frac{dx}{d\tau} - \frac{2x}{\tau}\right) = -\frac{4}{3} r(x) \left(\frac{d\sigma}{\sigma d\tau} + \frac{1}{\tau}\right)$$

$\Sigma = 0 \longrightarrow \quad x = 1$ or $dx/d\tau = 2x/\tau$ (local equilibrium or free streaming)

4. Purely-longitudinal boost-invariant motion

4.2 Connection with the Israel-Stewart theory

- close to equilibrium, $|\xi| \ll 1$, $P_\parallel(x) = P_{\rm eq} - \bar{\pi}$, $P_\perp(x) = P_{\rm eq} + \frac{\bar{\pi}}{2}$

$$\frac{\bar{\pi}}{\varepsilon_{\rm eq}} = \frac{8}{45}(x-1) = \frac{8}{45}\xi$$

- our equations agree with the evolution equation for $\bar{\pi}$ in 0+1 I-S theory:

$$\frac{d\overline{\pi}}{d\tau} + \frac{4\overline{\pi}}{3\tau} - \frac{16}{45}\frac{\varepsilon}{\tau} = -\frac{15\overline{\pi}}{4\tau_{\rm eq}} \quad \rightarrow \quad \frac{d\overline{\pi}}{d\tau} = -\frac{4\overline{\pi}}{3\tau} + \frac{4\eta_\pi}{3\tau_\pi\tau} - \frac{\overline{\pi}}{\tau_\pi}$$

with the identification

$$\frac{1}{\tau_{\rm eq}} = \frac{4}{15\tau_\pi}, \quad \tau_\pi = \frac{5\eta_\pi}{T\sigma_{\rm eq}}$$

- similar agreement for the entropy production with IS:

$$\partial_\mu S^\mu = \sigma_{\rm eq}\frac{\xi^2}{4\tau_{\rm eq}} \quad \longrightarrow \quad \partial_\mu S^\mu = \frac{3\bar{\pi}^2}{4\eta_\pi T}$$

- the results shown in this presentation are for FIXED $\tau_{\rm eq}$,
in this way we REPRODUCE the perfect-fluid behavior for $\tau \geq 2\tau_{\rm eq}$

5. Non-boost-invariant (3+1)D case
5.1 Initial conditions

- Initial evolution time $\tau_0 = 0.25$ fm, equilibration time $\tau_{\rm eq} = 0.25$ fm and 1 fm
- initial anisotropy choices: $x_0 = 100$ (transverse thermalization), $x_0 = 1$ (perfect fluid), and $x_0 = 0.032$ (longitudinal thermalization)
- initial energy density profile (tilted source by P.Bozek)

$$\varepsilon_0(\tau_0, \eta, \mathbf{x}_\perp) = \varepsilon_{\rm i}\, \tilde{\rho}(b, \eta, \mathbf{x}_\perp) \qquad \tilde{\rho}(b, \eta, \mathbf{x}_\perp) = \frac{\rho(b, \eta, \mathbf{x}_\perp)}{\rho(0, 0)}$$

$$\rho(b, \eta, \mathbf{x}_\perp) = (1 - \kappa)\left[\rho_W^+(b, \mathbf{x}_\perp) f^+(\eta) + \rho_W^-(b, \mathbf{x}_\perp) f^-(\eta)\right] + \kappa \rho_B(b, \mathbf{x}_\perp) f(\eta)$$

- initial longitudinal profile

$$f(\eta) = \exp\left[-\frac{(\eta - \Delta\eta)^2}{2\sigma_\eta^2}\theta(|\eta| - \Delta\eta)\right] \qquad \Delta\eta = 1, \sigma_\eta^2 = 1.3$$

- mixing factor $\kappa = 0.14$, initial energy density in the center $\varepsilon_{\rm i}$ chosen separately for each pair of x and $\tau_{\rm eq}$

5. Non-boost-invariant (3+1)D case

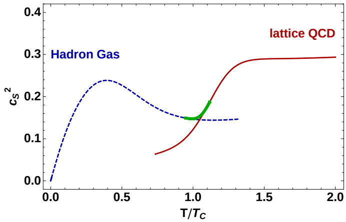
5.2 Generalized EOS – inclusion of the phase transition

to connect the isotropization with the process of formation of the equilibrated quark-gluon plasma we may consider the following ansatz

$$
\begin{aligned}
\varepsilon(\sigma, x) &= \varepsilon_{\rm qgp}(\sigma) r(x) \\
P_\perp(\sigma, x) &= P_{\rm qgp}(\sigma)\left[r(x) + 3x r'(x)\right] \\
P_\parallel(\sigma, x) &= P_{\rm qgp}(\sigma)\left[r(x) - 6x r'(x)\right]
\end{aligned}
$$

Here, the functions $\varepsilon_{\rm qgp}(\sigma)$ and $P_{\rm qgp}(\sigma)$ describe the realistic equation of state :

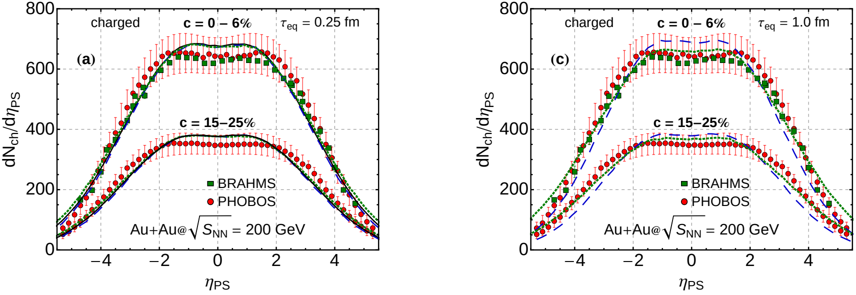
M. Chojnacki and WF, Acta Phys. Pol. B38 (2007) 3249.



5. Non-boost-invariant 3+1D case

5.3 $dN/d\eta$ of charged particles

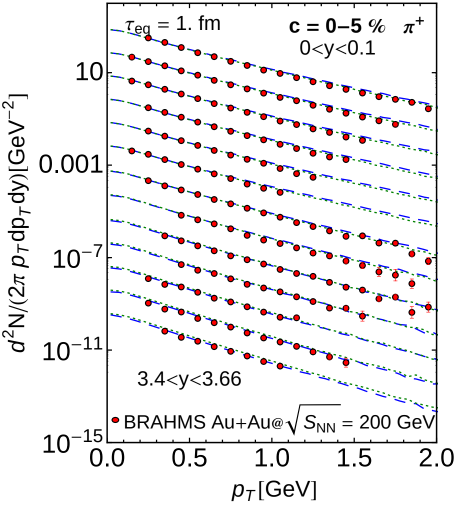
Initial anisotropy : x_0 = 1 (black), x_0 = 100 (blue), and x_0 = 0.032 (green),
freeze-out at constant entropy density corresponding to $T = 150$ MeV,
first 1 fm/c of the freeze-out hypersurface excluded

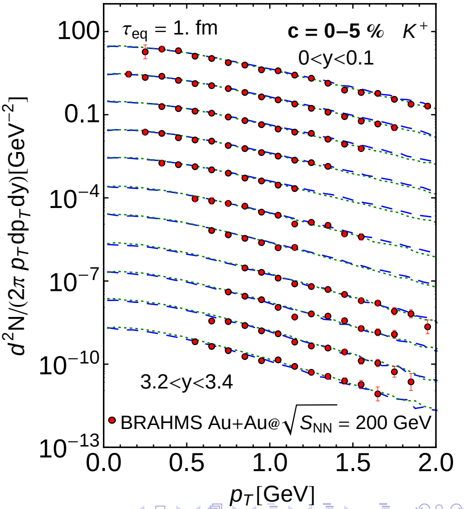


5. Non-boost-invariant (3+1)D case

5.4 $p_\perp$ spectra in different y windows

Initial anisotropy : x_0 = 1 (black), x_0 = 100 (blue), and x_0 = 0.032 (green)

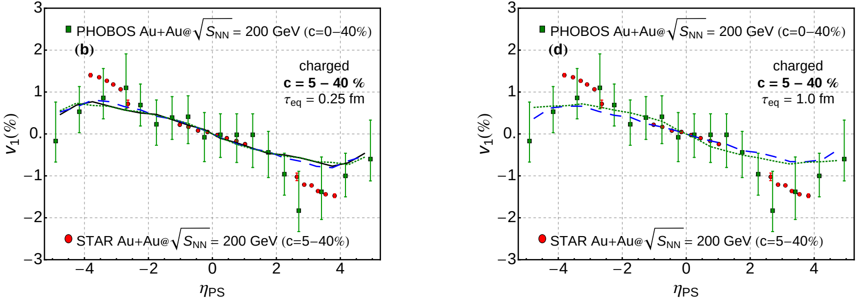




5. Non-boost-invariant (3+1)D case

5.5 pseudorapidity dependence of v_1 for charged particles

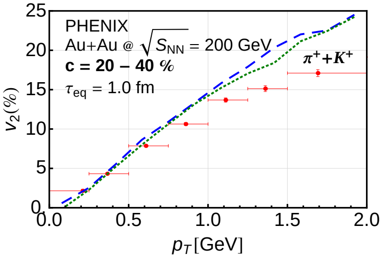
Initial anisotropy : $x_0 = 1$ (black), $x_0 = 100$ (blue), and $x_0 = 0.032$ (green)



5. Non-boost-invariant (3+1)D case

5.6 $v_2(p_T)$ in midrapidity

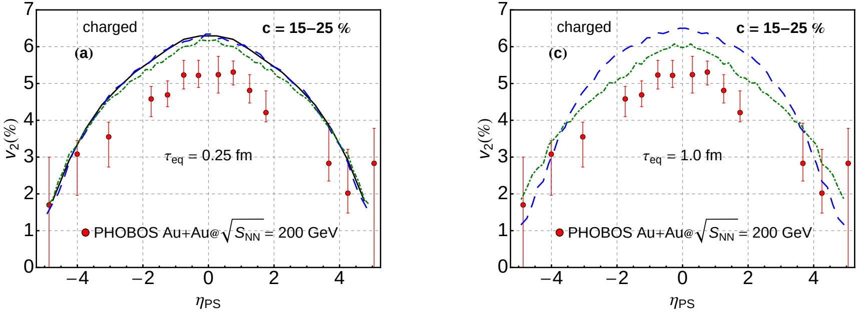
Initial anisotropy : x_0 = 1 (black), x_0 = 100 (blue), and x_0 = 0.032 (green)



5. Non-boost-invariant (3+1)D case

5.7 pseudorapidity dependence of v_2 for charged particles

Initial anisotropy : $x_0 = 1$ (black), $x_0 = 100$ (blue), and $x_0 = 0.032$ (green)



6. Conclusions

- A new framework of ANISOTROPIC HYDRODYNAMICS has been introduced. The effects of dissipation are defined by the form of the entropy source. The entropy production is directly connected with the soft particle (gluon) production. This process is responsible for isotropization/thermalization.
- The RHIC soft hadronic data described with (3+1)D code. Initial conditions with extremely different anisotropies lead to similar results, provided the initial energy density and rapidity profiles are properly readjusted.
- Complete THERMALIZATION of the system MAY BE DELAYED to easily acceptable times of about 1–2 fm/c. The early-thermalization puzzle may be circumvented.