Event-by-event Fluctuations and Hadron Production in e^+e^- and pp collision

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with

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Content

1, *Tsallis-distribution* in highenergy reactions

- Hadron spectra in *AA, pp* and e^+e^- collisions
- Cross-section in *elastic pp* collisions

2, *Jet-fragmentation* in *pp* and e^+e^- collisions

- *Microcanonical jets* + *multiplicity fluctuations*
- Scale evolution and the *DGLAP* equations

(Gergely Kalmár's Talk today)

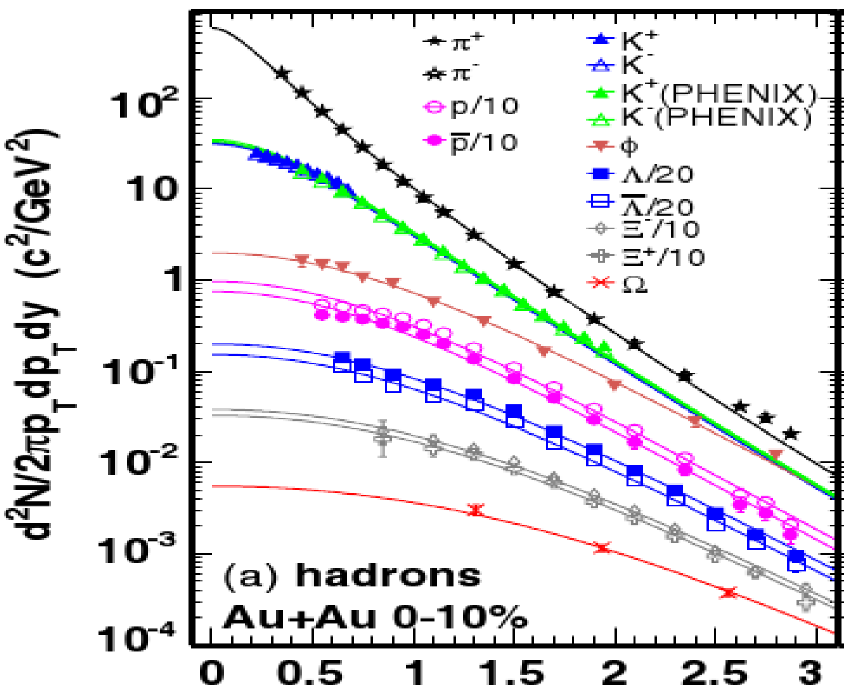
3, Transverse Spectra of *π*, *K*, *p* in *pp* collisions

(Ferenc Siklér's Talk on wednesday)

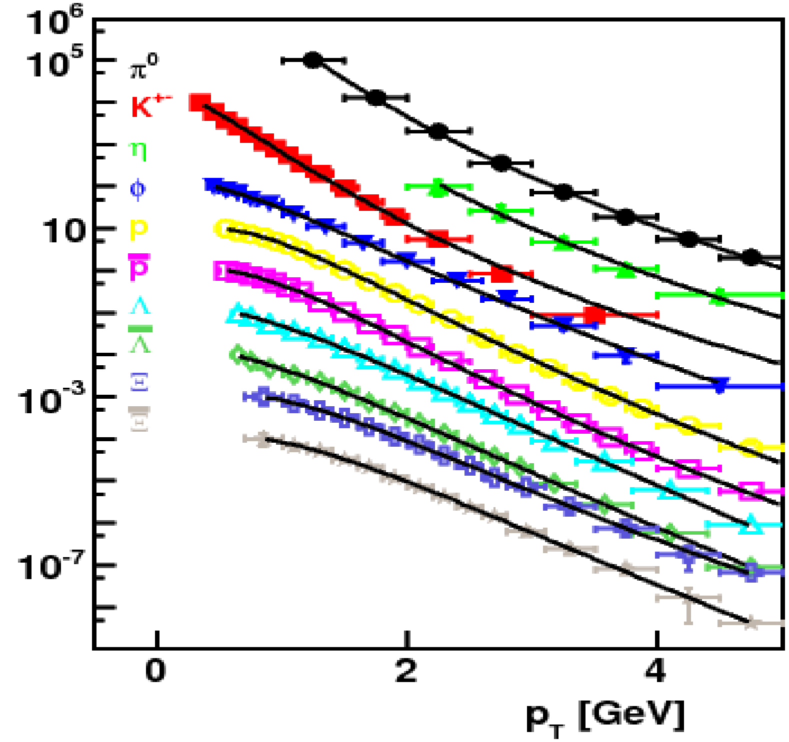
- *Canonical* distribution inside *single events*
- *+ Fluctuations* of the *total transverse energy* and *multiplicity*

1) Hadron Spectra in Heavy-ion Collisions

J. Phys. G: Nucl. Part. Phys. **37** 085104 (2010)



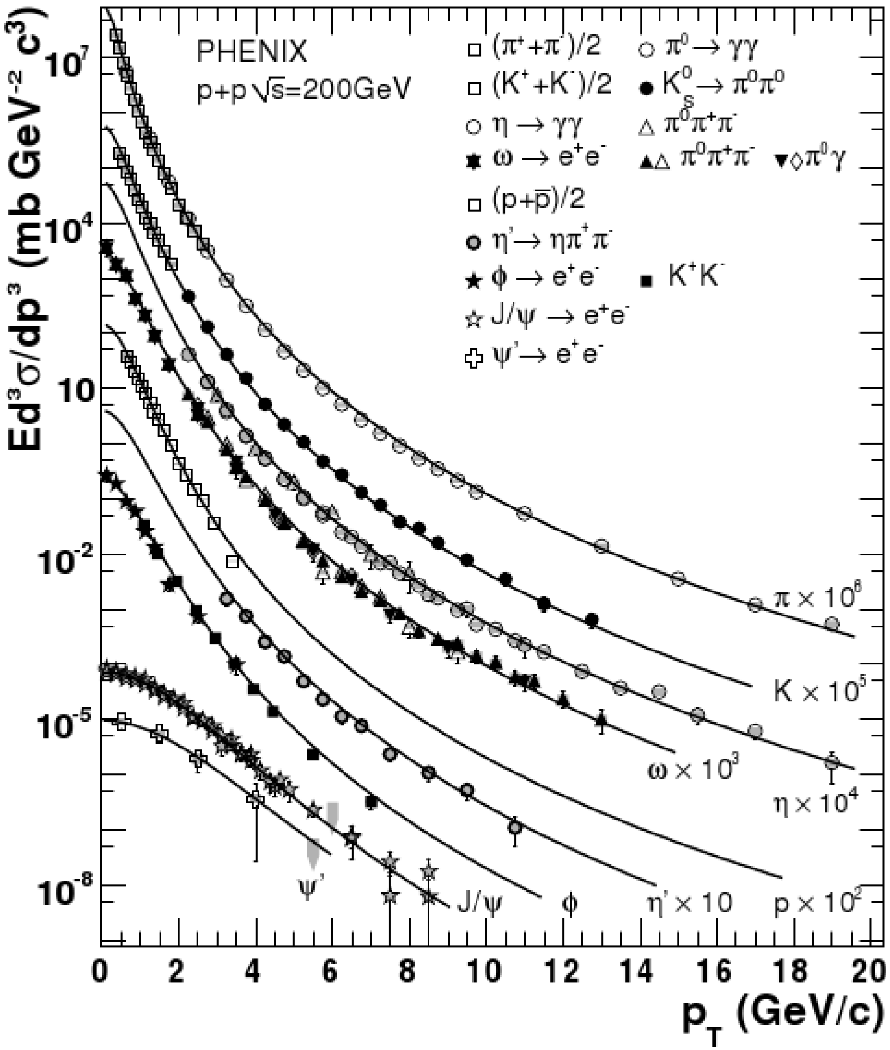
Phys. Lett. B, **689,** 14-17 (2010)



Theoretical model: Recombination of Tsallis distributed thermal quarks + „*Blast Wave*" flow profile for the expanding QGP

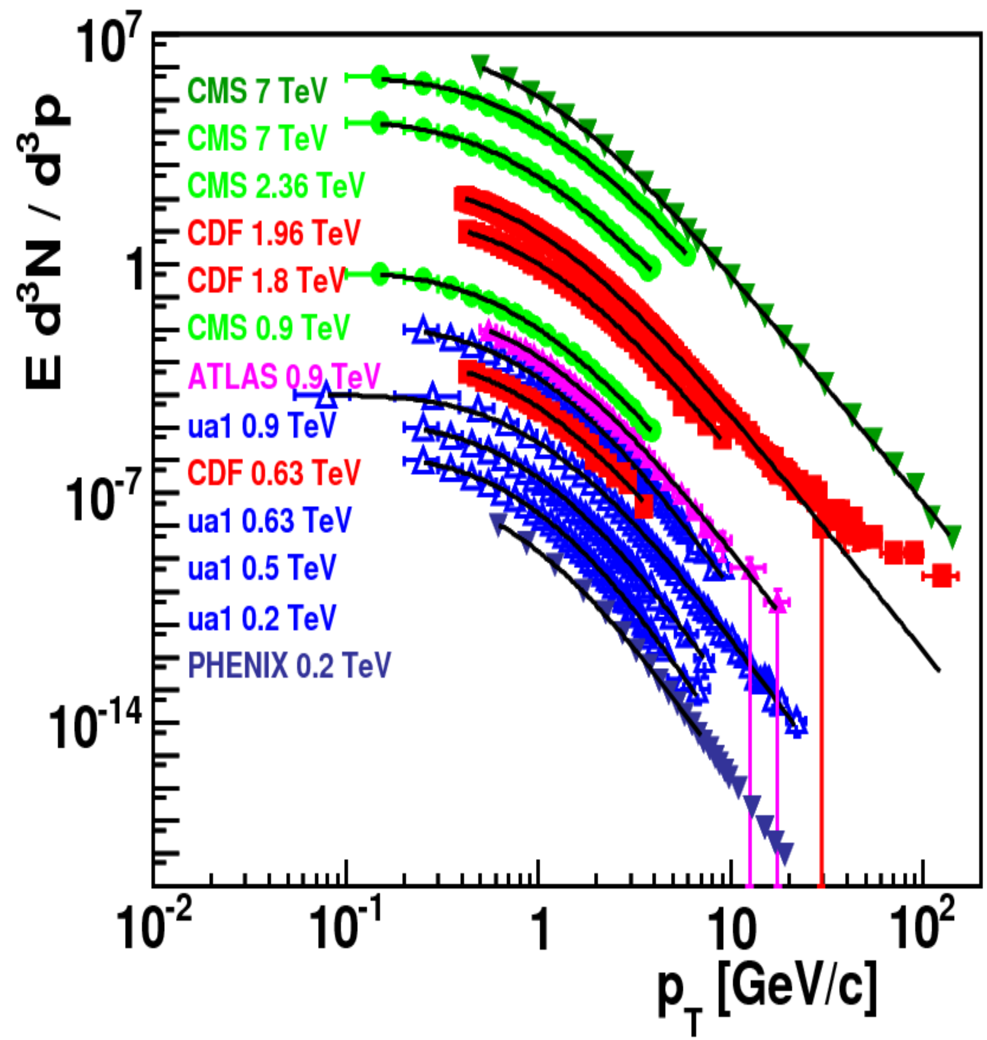
1) Hadron Spectra in Proton-proton Collisions

Phys. Rev. D, **83,** 052004 (2011)



$$E\frac{d\sigma}{d^3p} \propto \left(1+m_T/nT\right)^{-n}$$

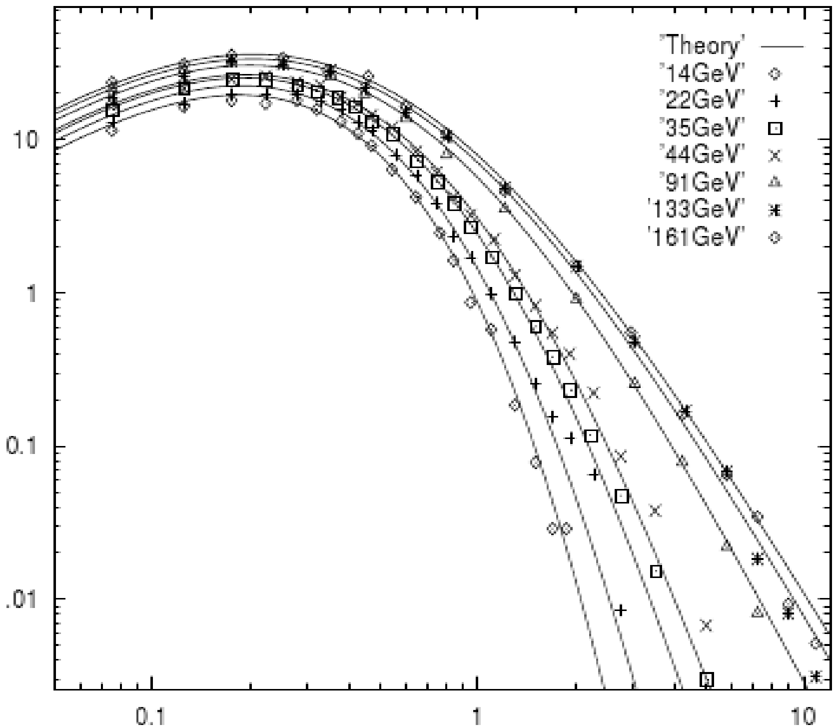
J. Phys.: Conf. Ser., **270** 012008 (2011)



$$E\frac{d\sigma}{d^3p} \propto m_T\left(1+m_T/nT\right)^{-n}$$

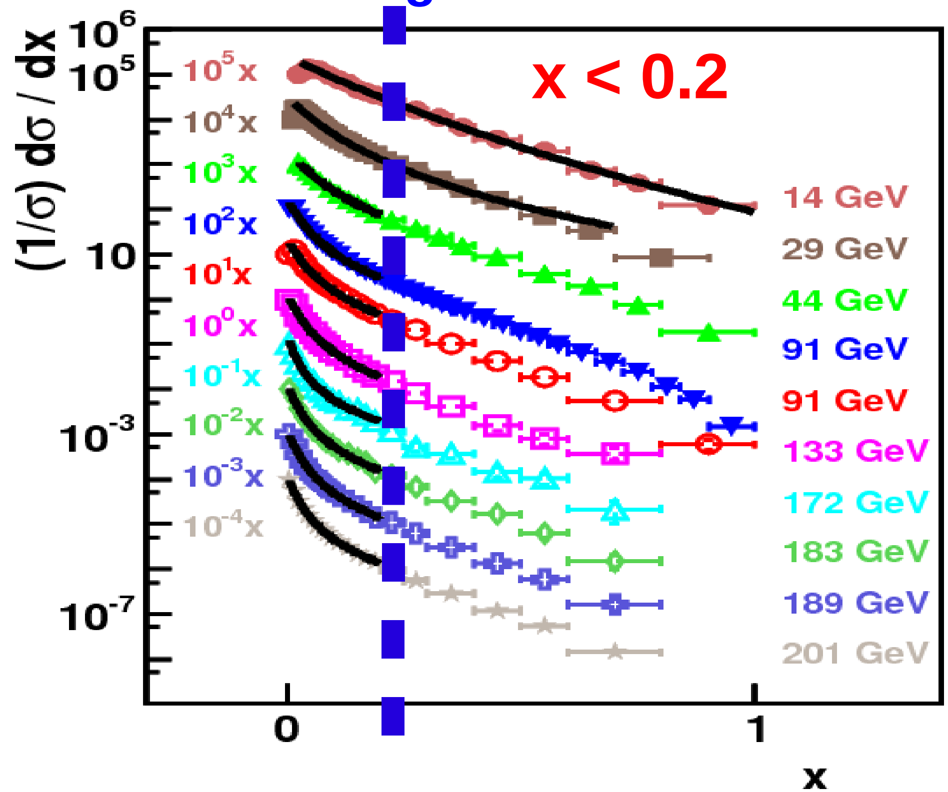
1) Hadron Spectra in Electron-positron Collisions

Transverse momentum distribution of charged hadrons in jets



$$\frac{d\sigma}{dp_T} \propto p_T \int_0^{\infty} dp_L f_{TS}(E)$$

Energy-fraction distribution of charged hadrons

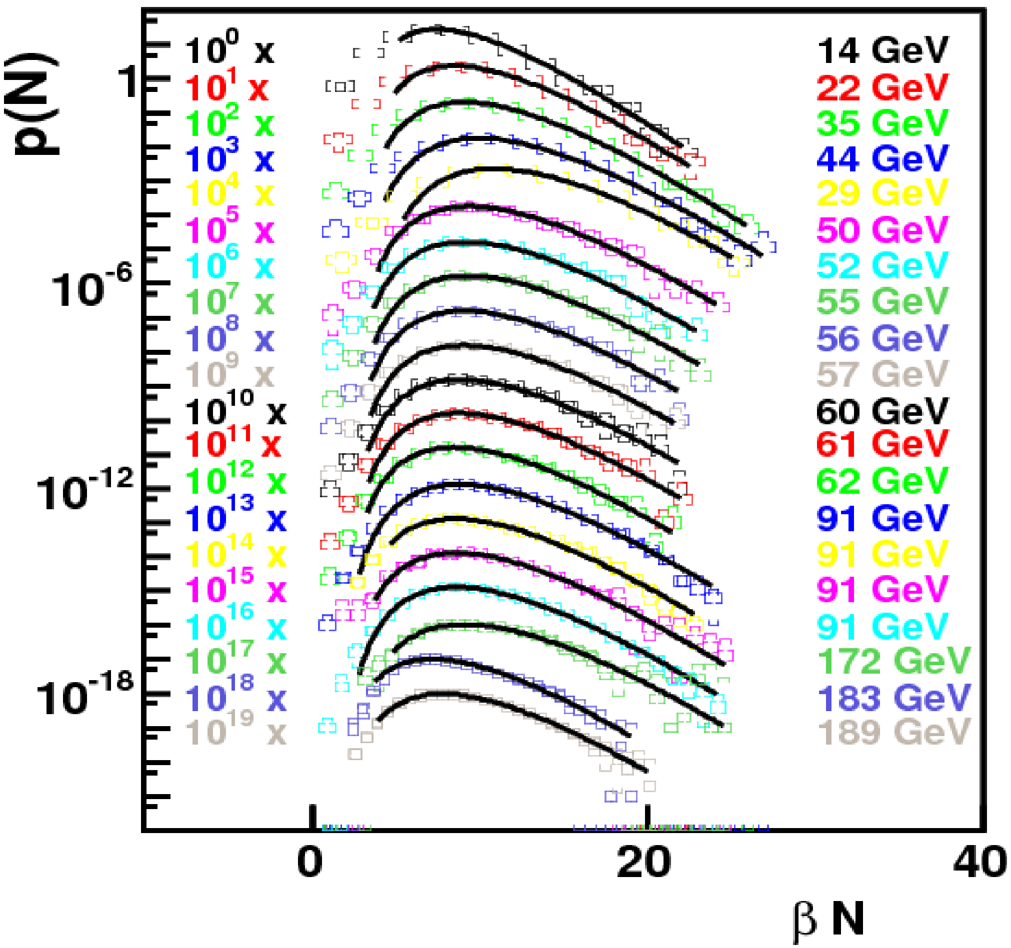


$$\frac{d\sigma}{dx} \propto \left(1 + \frac{q-1}{T/(\sqrt{s}/2)} x\right)^{-1/(q-1)}$$

- C. Beck, *Physica A*, **286**, 164-180 (2000)
- I. Bediaga, et.al., *Physica A,* **286**, 156-163 (2000)
- Urmossy et.al., *Phys.Lett.B*, **701,** 111-116 (2011), arXiv:1101.3023

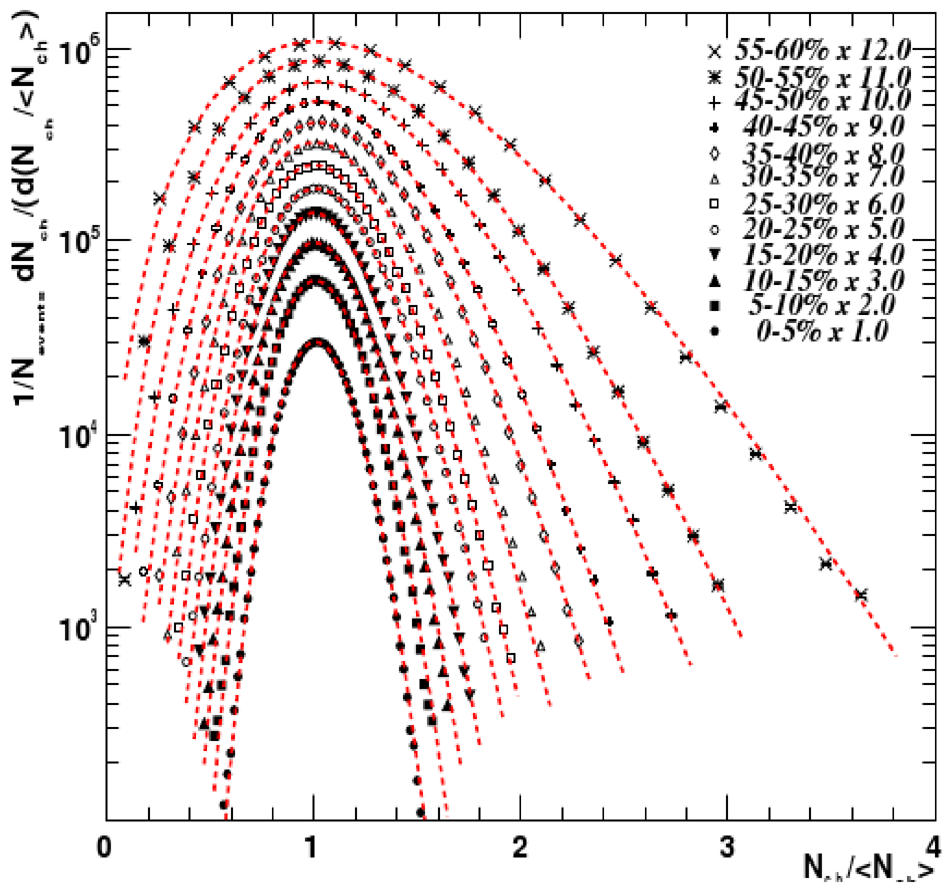
1) Multiplicity Fluctuations

Multiplicity distributions in *electron-positron* collisions



Urmossy et.al., *Phys. Lett. B*, **701**: 111-116 (2011), arXiv:1101.3023

Multiplicity distributions in *AuAu* collisions @200 AGeV



PHENIX Collab., arXiv:0805.1521v1

2) *Jet-fragmentation in e^+e^- and pp Collisions*

- A ***jet*** is a bunch of hadrons flying ***almost colinearly*** ***(quasi – 1 dimension!)***.
  If the ***cross-section*** of the production of these hadrons is proportional to their phasespace ***restricted only by energy conservation***, these hadrons form a ***microcanonical ensemble***.
  Thisway, in a jet of ***N*** (massless) hadrons, hadrons have the ***energy distribution***:

$$f_N(z) = A_N (1-z)^{N-2}, \qquad z = \frac{\epsilon_h}{E_{jet}}$$

- The ***number of hadrons*** in a ***jet fluctuates*** as ***(experimental observation)***

$$p(N) \propto (N-N_0)^{\alpha-1} e^{-\beta(N-N_0)}$$

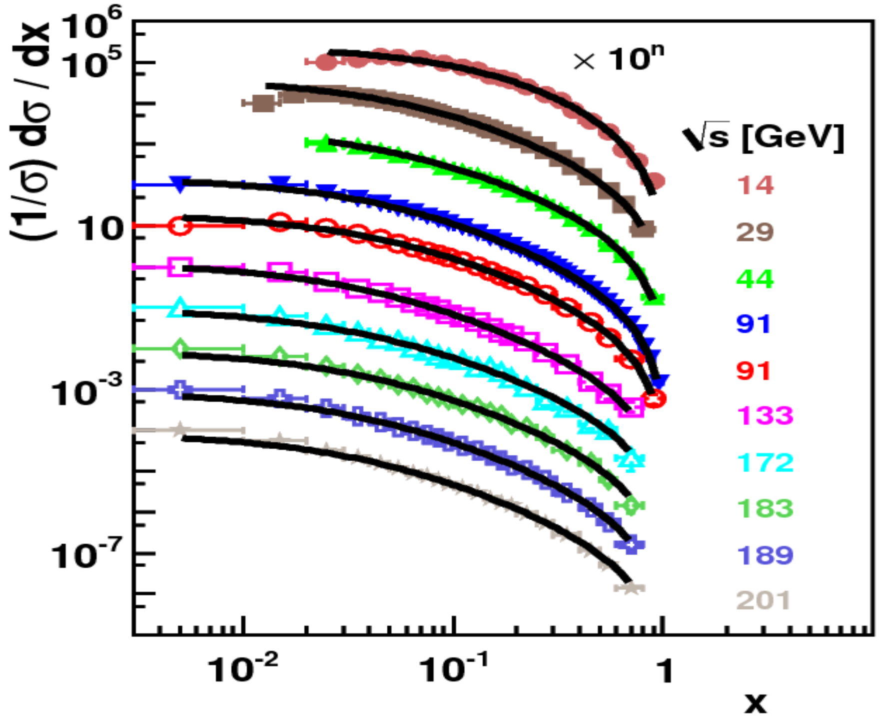
or negative-binomial distribution

- Thus, the ***multiplicity averaged hadron distribution*** (***fragmentation function***) becomes

$$\frac{d\sigma}{dz} = \sum_{N=N_0}^{\infty} f_N(z) N p(N) \propto \frac{(1-z)^{\nu(N_0)}}{\left(1 - \frac{(q-1)}{T/E_{jet}} \ln(1-z)\right)^{1/(q-1)}}$$

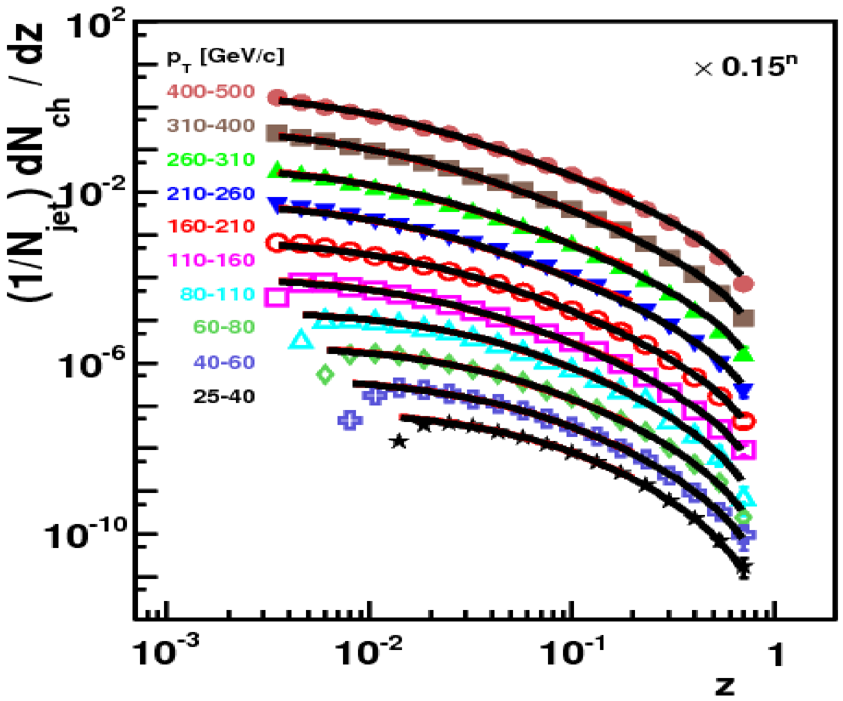
2) Confrontation with Measurements

e^+e^- annihilations @LEP ($\sqrt{s}$ = 14–200 GeV)



Urmossy et. al.,
Phys. Lett. B, 701, 111-116 (2011),
arXiv:1101.3023

proton-proton collisions @LHC (p_T = 25–500 GeV/c)

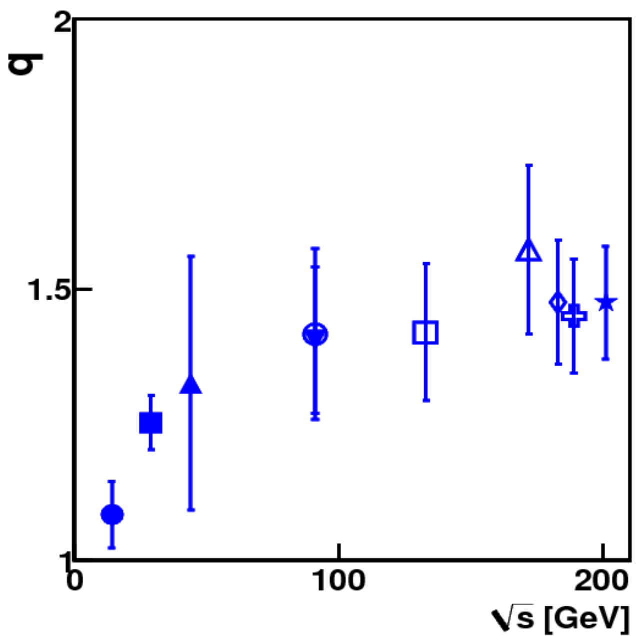


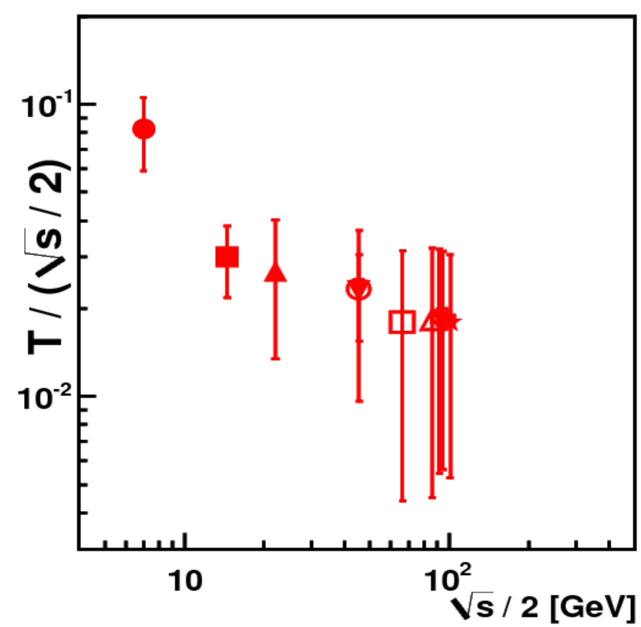
Urmossy et. al.,
arXiv:1204.1508v1

2) *Scale-evollution of the Parameters*

e^+e^- annihilations @LEP (√s = 14–200 GeV)

$$\frac{d\sigma}{dz} \propto (1-z)^{\nu} \left(1 - \frac{(q-1)}{T/E_{jet}} \ln(1-z)\right)^{-1/(q-1)}$$



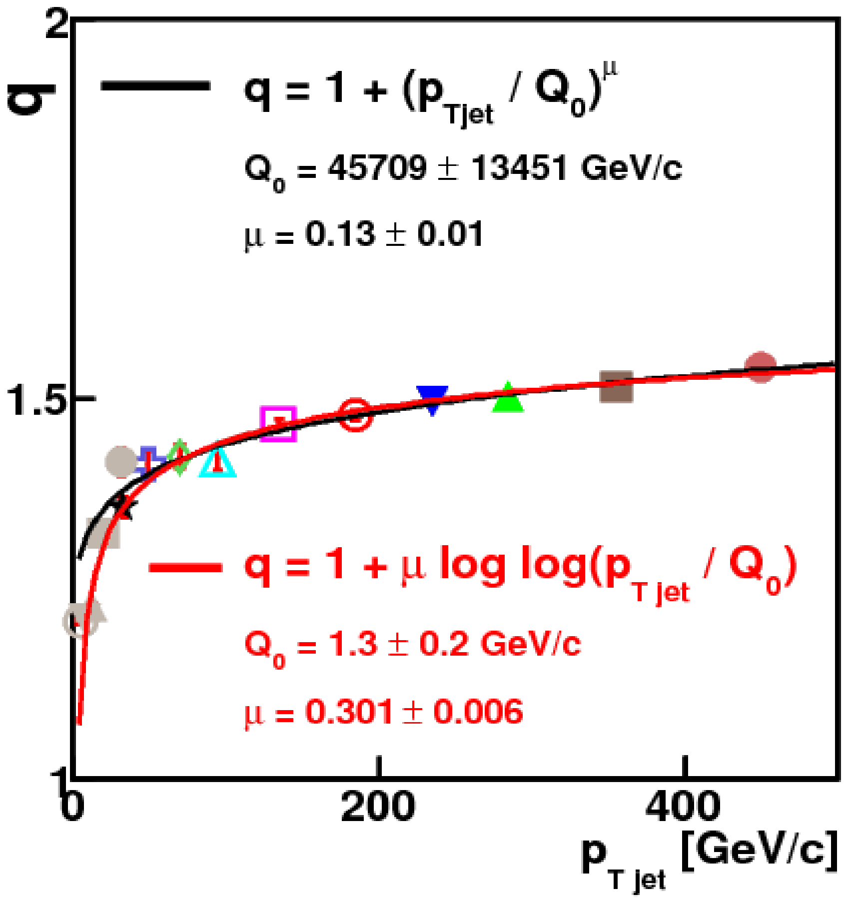


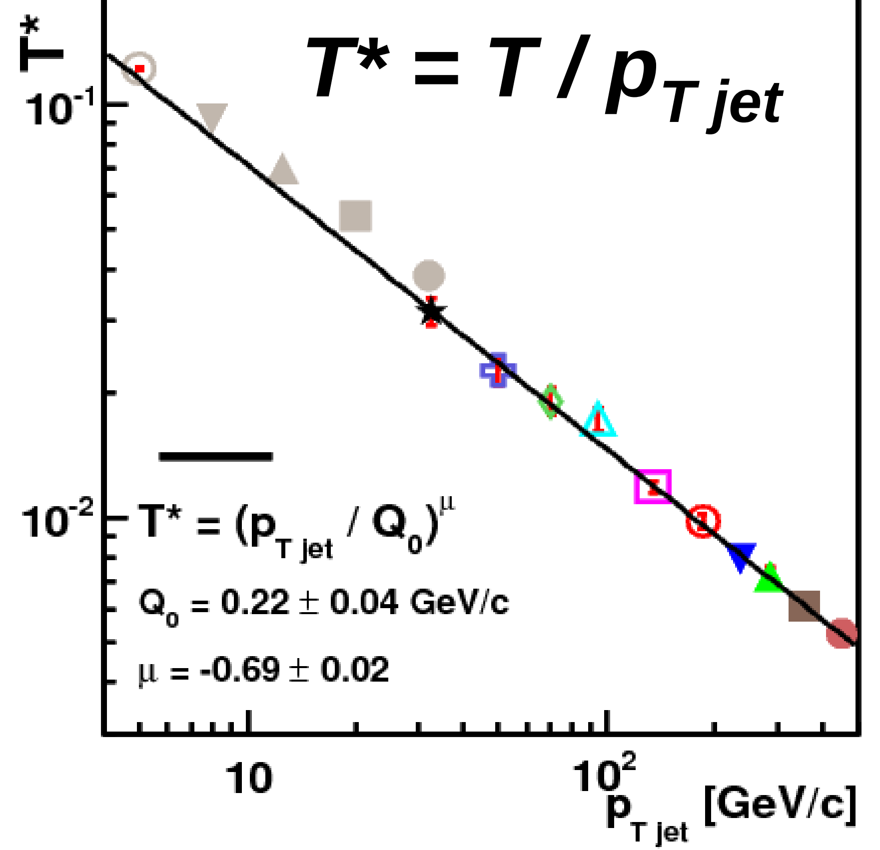
q rises, ***T/√s falls as*** **the collision energy (√s) grows**

Urmossy et. al., *Phys. Lett. B*, 701, 111-116 (2011), arXiv:1101.3023

2) *Similar Scale-depencence in*

proton-proton collisions @LHC (pT = 25–500 GeV/c)





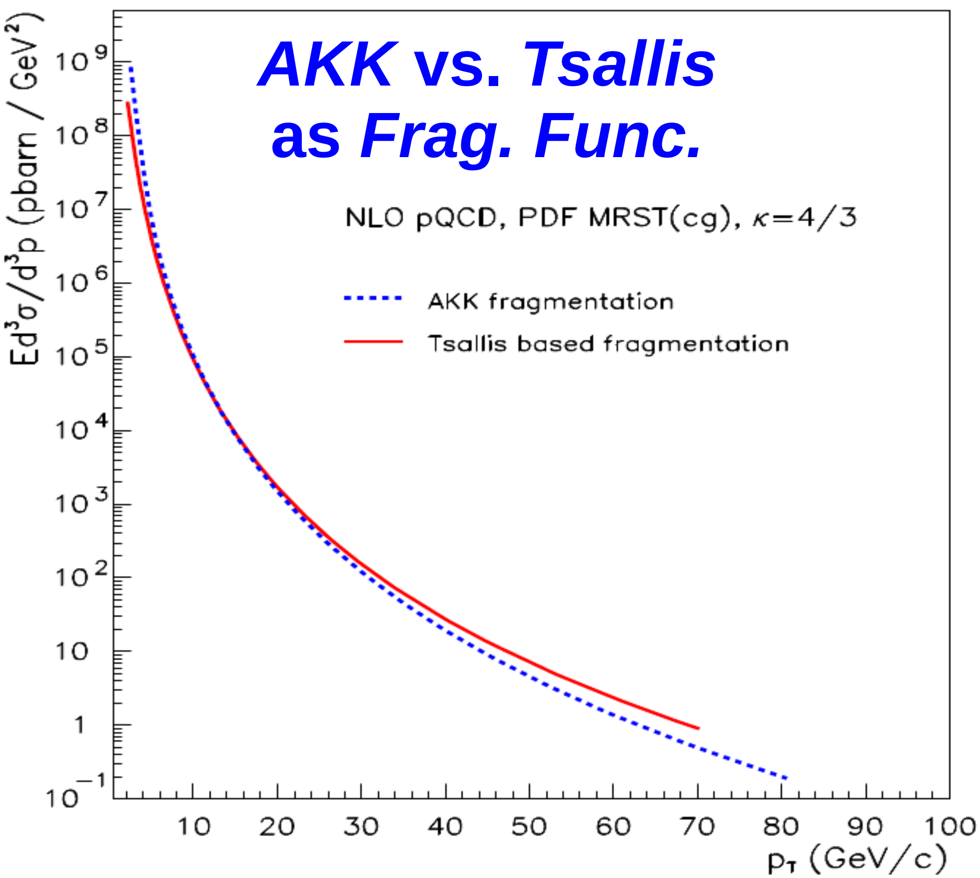
Urmossy et. al., arXiv:1204.1508v1

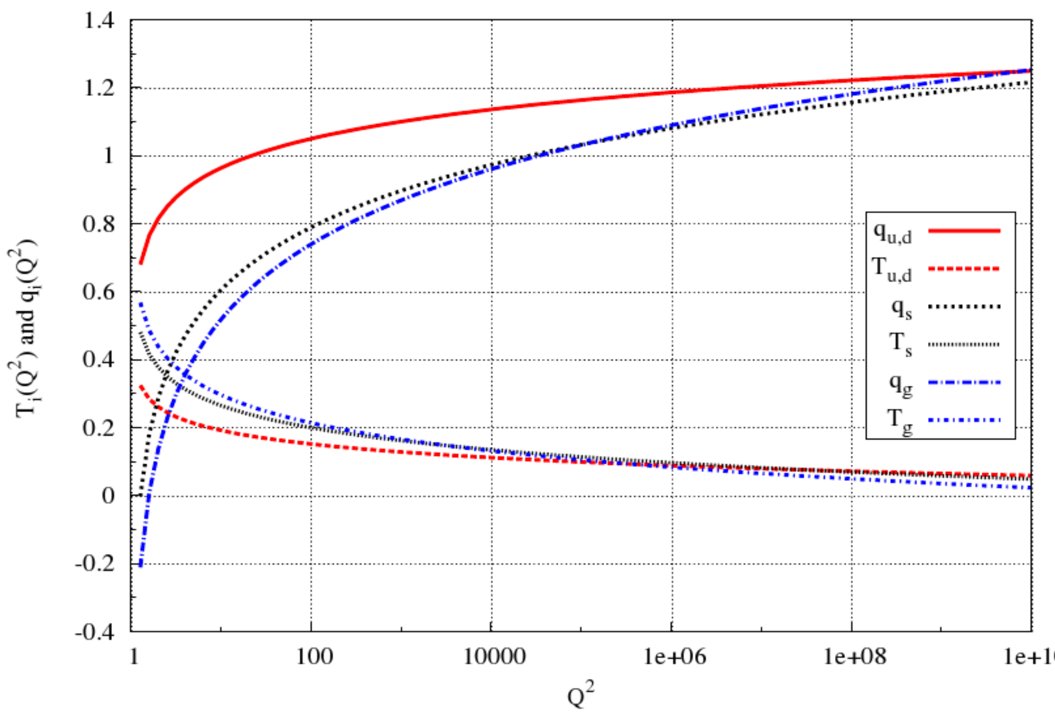
2) Why do we see such scale depencence?

(Gergely Kalmár's Talk today)

π^+ spectrum in pp --> π^+X @ √s=7 TeV (NLO pQCD)

Scale dependence of *q* and *T*





$$q_i = q_{0i} + q_{1i} \ln(\ln(Q^2))$$

$$T_i = T_{0i} + T_{1i} \ln(\ln(Q^2))$$

$$D_{p_i}^{\pi^+}(z) \sim (1 + (q_i - 1) z / T_i)^{-1/(q_i - 1)}$$

Barnaföldi et. al., *Proceedings of the Workshop Gribov '80* (2010)

3) Transverse Spectra of π, K, p in pp Collisions

(Ferenc Siklér's Talk on wednesday)

Hadron distribution in a single event of multiplicity *N* and total transverse energy

$E_T = \Sigma m_T$

$$f_{E_T,N}(\epsilon) \quad , \quad \epsilon = \sqrt{m^2 + p_T^2} - m$$

From experiments we know only the *averages*:

- **Hadron spectra measured at fix multiplicity *N* (*p_T < 2 GeV/c* new CMS data):**

$$f_N(\epsilon) \propto \left[1 + (q-1)\epsilon/T\right]^{-1/(q-1)} = \int dE_T \; f_{E_T,N}(\epsilon)$$

- **Multiplicity averaged hadron spectra:**

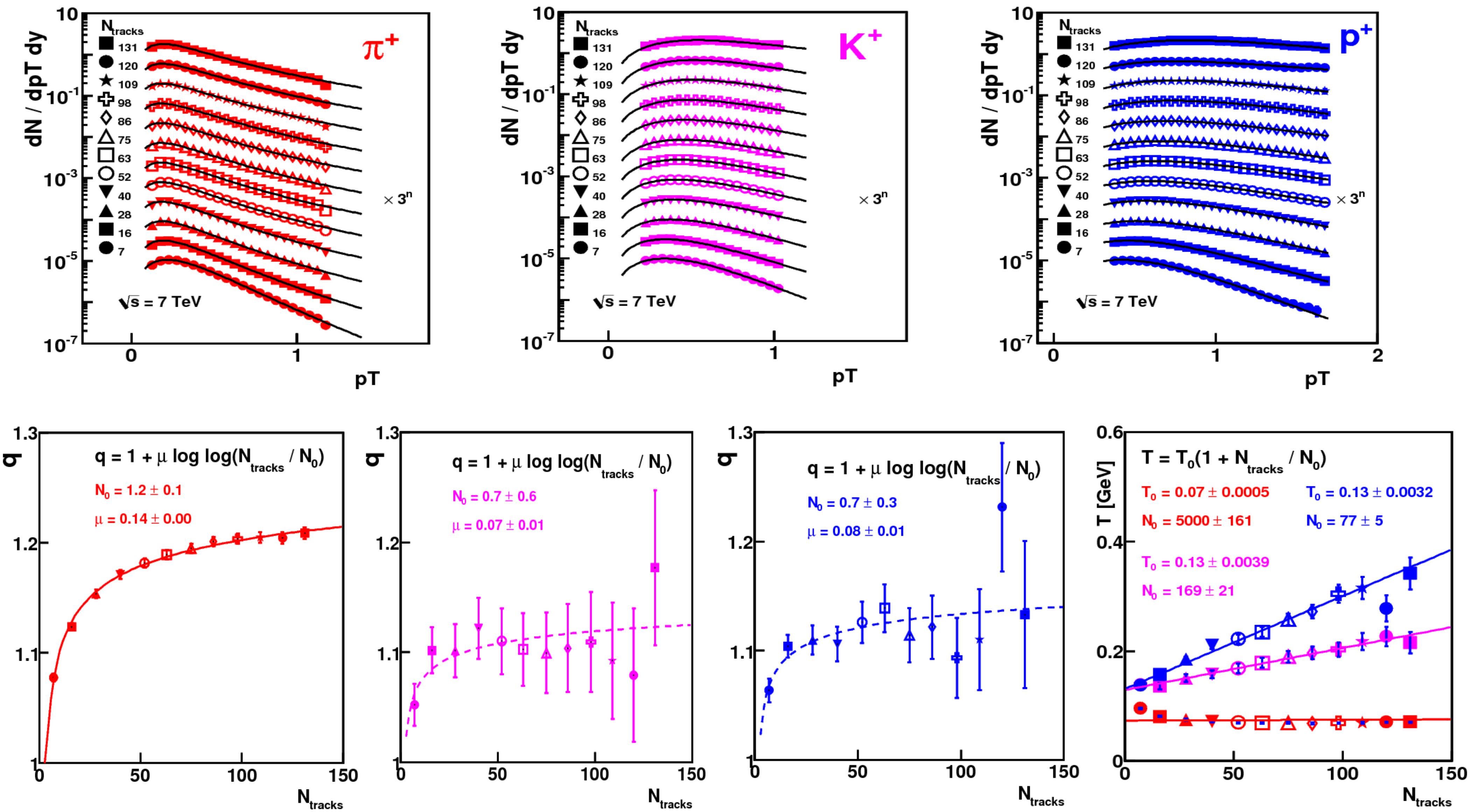
$$f(\epsilon) \propto \left[1 + (\tilde{q}-1)\epsilon/\tilde{T}\right]^{-1/(\tilde{q}-1)} = \sum_N \int dE_T \; f_{E_T,N}(\epsilon)$$

- **Multiplicity distributions of charged particles:**

$$p(N) \propto N^{a-1} e^{-aN/N_0} = \int dE_T \int d^3p \; f_{E_T,N}(\epsilon)$$

***Euler-gamma / negative-binomial* distribution**

3) Multiplicity Dependence of π, K, p Spectra in pp@7 TeV



$$q = 1+\mu \log\log(N/N_q) \quad , \quad T = T_0(1+N/N_T)$$

3) Transverse Spectra of π, K, p in pp Collisions

Let us suppose that

$$f_{E_T,N}(\epsilon) = p(N,E_T)\ A\ \exp\{-\beta\epsilon\}\ ,\quad \beta = 3\mathrm{N}/E_T$$

$p(N,E_T)$ containes all the information on N and E_T fluctuations. Let us choose

$$p(N,E_T) = h(N)\ g_N(E_T)$$

with independent multiplicity fluctuations

$$h(N) \sim N^{a-1} e^{-aN/N_0}$$

but multiplicity dependent energy distribution

$$g_N(E_T) \sim E_T^{-(\alpha+2)} e^{-\alpha E_0/E_T}$$

… and we will recover the measured marginal distributions:

$$p(N)\ ,\quad f_N(\epsilon)\ ,\quad f(\epsilon)$$

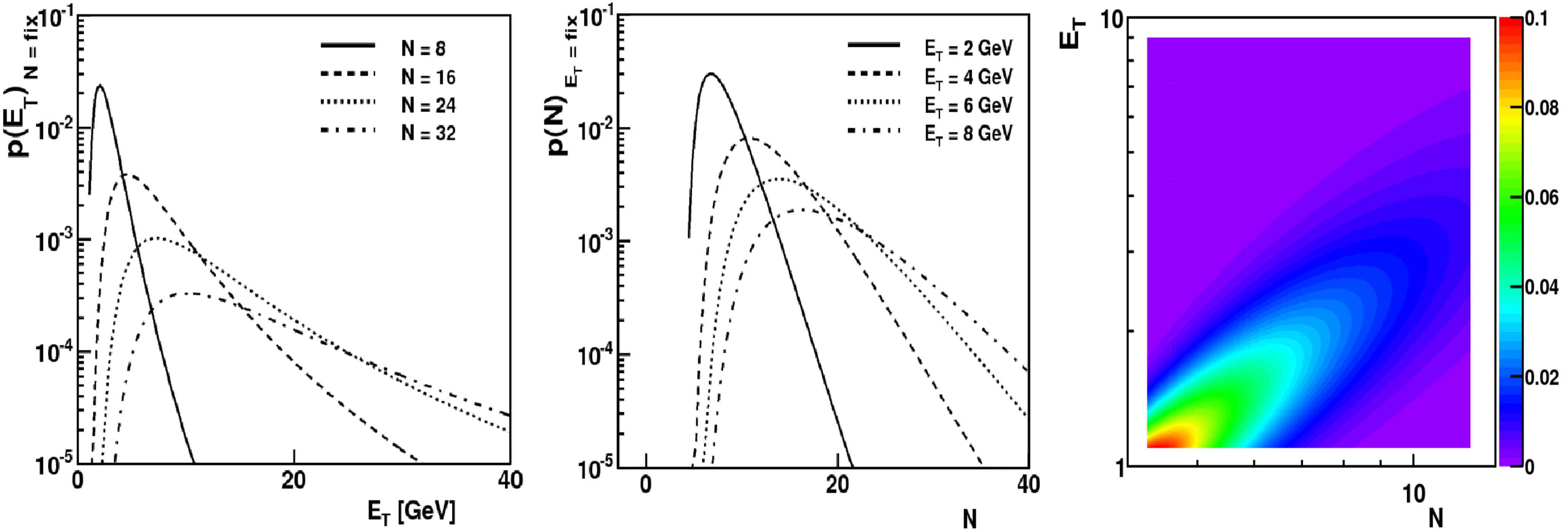
3) Predictions of the Model

N **and** E_T **are *correlated*! Their joint distribution:**

$$p(N, E_T) \sim E_T^{-(\alpha+2)} e^{-\alpha E_0/E_T} \times N^{a-1} e^{-aN/N_0}$$

with

$$\alpha = [\mu \ln\ln(N/N_q)]^{-1} - 4, \quad E_0 = \frac{3N T_0 (1 + N/N_T)}{1 - 4\mu \ln\ln(N/N_q)}$$



Take Home Message:

Multiplicity fluctuations hide event-by-event physics from our eyes.

We should measure the spectra in each multiplicity bin.

Back-up Slides……………………..

How to Obtain the Tsallis Distribution (a)

Generalise the free-energy functional in the Maximum Entropy variational ansatz

$$S\left[f(\epsilon)\right]-\beta\int \epsilon f(\epsilon) \;=\; max$$

Deform the entropy*, *S[f] **(F. Caruso and C. Tsallis., Phys Rev E 78, 021101 (2008); C. Tsallis, Eur. Phys. J. A, 40, 257-266 (2009))**

In a *linear spin-chain*, because of *entenglement*, the entropy of N adjacent spins becomes proportional to N (in the limit of N-->∞) if a new entropy functional is introduced:

$$S_{BG}=-\int f\ln f \quad \rightarrow \quad S_{TS}=-\int f\ln_q f$$

$$\ln_q f=\frac{f^{1-q}-1}{q-1}$$

The sollution of the variational problem is the Tsallis distribution:

$$f(\epsilon)=A\left(1+(q-1)\epsilon/T\right)^{-1/(q-1)}$$

How to Obtain the Tsallis Distribution (b)

Introduce ***special N-body interactions*** **of the type**

$$E = E_1 + E_2 + \ldots\ a\,(E_1 E_2 + E_1 E_3 + \ldots) + \ldots + a^{N-1} E_1 * \ldots * E_N$$

which is equivalent to

$$L(E) = L(E_1) + L(E_2) + \ldots + L(E_N) \quad \text{with} \quad L(E) = (1/a)\ln(1 + a\,E)$$

Now the variational problem becomes

$$-\int f \ln f - \beta \int L[f] = max$$

with the Tsallis distribution as the sollution:

$$f(\epsilon) = A\left(1 + (q-1)\epsilon/T\right)^{-1/(q-1)},$$

- K. Urmossy et. al., *EPJ Web of Conferences,* **13**, 05003 (2011)
- T. S. Biro et. al., *J. Phys. G,* **36** 064044 (2009)
- T. S. Biro et. al., *Eur. Phys. J. A* **40** 325-340 (2009)

What is T? *Equilibration of 2 Systems*

The sum of the energies *E_1* and *E_2* of 2 non-interacting sub-systems is the total energy *E* of the whole system

$$E = E_1 + E_2$$

In *equilibrium* the total energy is fixed, while the total entropy is maximal:

$$S(E) = S_1(E_1) + S_2(E_2) = max$$

$$E = E_1 + E_2 = fix$$

Separating the variables, both sides are equal to the inverse temperature:

$$\frac{\partial S_1(E_1)}{\partial E_1} = \frac{1}{T} = \frac{\partial S_2(E_2)}{\partial E_2}$$

What is T? Equilibration of 2 Systems

Suppose, that because of special interactions, the energy of 2 sub-systems, E_1 and E_2 and the total energy E are related as

$$L(E) = L(E_1) + L(E_2), \qquad L(E) = \frac{1}{a}\ln(1 + aE)$$

In *equilibrium* the total energy is fixed, while the total entropy is maximal:

$$S(E) = S_1(E_1) + S_2(E_2) \;=\; max$$

$$L(E) = L(E_1) + L(E_2) \;=\; fix$$

Separating the variables, both sides are equal to the inverse temperature:

$$\frac{\partial S_1(E_1)}{\partial L(E_1)} \;=\; \frac{1}{T} \;=\; \frac{\partial S_2(E_2)}{\partial L(E_2)}$$

T.S. Biró, P. Ván, *Physical Review E*, **83**, 061187 (2011)

Entropy and Energy Functionals with the Corresponding Equilibrium Distributions

$$S[f(\epsilon)] - \beta C[f(\epsilon)] = max$$

$S[f]$	$C[f]$	$f_{eq}(\epsilon)$	
$-\int f \ln f$	$\int \epsilon f$	$A \exp\{-\beta \epsilon\}$	
$-\int f \ln f$	$\int L(\epsilon) f$	$A \exp\{-\beta L(\epsilon)\}$	T. S. Biro et. al., *Eur. Phys. J. A* **40** 325-340 (2009)
$\int f \ln_q f$	$\int \epsilon f$	$A \left[1 + (q-1)\beta \epsilon\right]^{-1/(q-1)}$	C. Tsallis
$\int f \ln_q f$	$\int L(\epsilon) f$	$A \left[1 + (q-1)\beta L(\epsilon)\right]^{-1/(q-1)}$	T.S. Biró et. al., Phys. Rev. E, 83, 061187, (2011)

Deformed logarithm (C. Tsallis, Eur. Phys. J. A, 40, 257-266 (2009) ***):***

$$\ln_q(z) = \frac{z^{1-q} - 1}{q - 1}$$

How to Obtain the Tsallis Distribution (c)

SuperStatistics:

If the hadron distribution is *Boltzmann-Gibbs*,

$$f(\epsilon) = A \exp(-\beta \epsilon)$$

but the *temperature fluctuates* *event-by-event* or *position-to-position* as

$$p(\beta) \;\propto\; \beta^{\alpha-1} \exp(-\alpha \beta / \langle \beta \rangle)$$

the *average distribution* becomes the *Tsallis* distribution:

$$\frac{dN}{d^{3}p} = \int d\beta \, p(\beta) f_{\beta}(\epsilon) \;\propto\; \left(1 + \frac{\langle \beta \rangle \epsilon}{\alpha}\right)^{-(\alpha + D + 1)}$$

How to Obtain the Tsallis Distribution (c)

Or similarly,

If the hadron distribution is *Boltzmann-Gibbs*,

$$f(\epsilon) = A\exp(-\beta\epsilon), \qquad E/n = DT$$

but the *multiplicity fluctuates event-by-event while E = constant*

$$p(n) \propto n^{\alpha-1}\exp(-\alpha n/\langle n\rangle)$$

the *average distribution* becomes the *Tsallis* distribution:

$$\frac{dN}{d^3p} = \int dn\, p(n) f_n(\epsilon) \propto \left(1+\frac{D\langle n\rangle}{\alpha E}\epsilon\right)^{-(\alpha+D+1)}$$

How to Obtain the Tsallis Distribution (*c*)

Moreover,

If the hadron distribution is *Boltzmann-Gibbs*,

$$f(\epsilon) = A\exp(-\beta\epsilon), \qquad E/n = DT$$

but the *total transverse energy fluctuates event-by-event while n = fix*

$$p(E) \propto E^{-\alpha-2}\exp(-\alpha\langle E\rangle/E)$$

the *average distribution* becomes the *Tsallis* distribution:

$$\frac{dN}{d^{٣}p} = \int dE\, p(E) f_E(\epsilon) \propto \left(١ + \frac{Dn}{\alpha\langle E\rangle}\epsilon\right)^{-(\alpha+D+1)}$$

Now What Is the T Parameter?

Alas, from measurements, we do not see the *interactions* and *internal fluctuations* inside the quark-matter, however, we measure the ***mean energy per particle*** **(**for $\epsilon(p) = p$ disp. rel.**):**

$$\frac{E}{N} = \frac{\int \epsilon f_{TS}(\epsilon)}{\int f_{TS}(\epsilon)} = \frac{DT}{1-(q-1)(D+1)}$$

Thus, the fitted *Tsallis T is much smaller then the* fitted *Boltzmann T*!

Fluctuations of the total transverse energy can describe pp data

If the distribution of the total transverse energy is

$$p(E) \propto E^{-\alpha-2} \exp\left(-\alpha \langle E \rangle / E\right)$$

where the *mean energy* and the *width* of the distribution *varies with n* as

$$\alpha = \frac{1}{\mu \ln\ln(N/N_q)} - (D+1)$$

$$\langle E \rangle = \frac{D T_0 (1 + N/N_T)}{1 - (D+1)\mu \ln\ln(N/N_q)}$$

This prediction could be tested experimentally ...

Experimentally multiplicity averaged spectra are measured

Miltiplicity distributions show KNO-scaling (Koba-Nielsen-Olesen)

$$p(N) = \frac{1}{\langle N(s) \rangle} \Psi\left(\frac{N - N_0}{\langle N(s) \rangle} \right)$$

- A. Rényi, Foundations of Probability, Holden-Day (1970).
- A. M. Polyakov, Zh. Eksp. Teor. Fiz. 59, 542 (1970).
- Z. Koba, H. B. Nielsen, P. Olesen, Nucl. Phys. B 40, 317 (1972).
- S. Hegyi, Phys. Lett. B: 467, 126-131, 1999.
- S. Hegyi, Proc. ISMD 2000, Tihany, Lake Balaton, Hungary, 2000
- Yu.L. Dokshitzer, Phys. Lett. B, 305, 295 (1993); LU-TP/93-3 (1993).

A kírérletekkel konzisztens konkrét függvényalak:

$$p(N) \propto (N - N_0)^{\alpha - 1} e^{-\beta (N - N_0)}$$

Amiből az átlag hadron eloszlás:

$$\frac{d\sigma}{d^D x} = \sum f_N(x) N p(N) \propto \frac{(1-x)^{D(N_0 - 1) - 1}}{\left(1 - a \ln(1-x)\right)^b}$$

Urmossy et. al., Phys. Lett. B, 701: 111-116 (2011), arXiv:1101.3023

We see Statistical Physical distributions if

- ***Matter created in the collisions reaches equilibrium***
- ***or the cross section of the creation of particles $h_1, \ldots, h_N$***

$$d\sigma^{h_1,\ldots,h_N} = |M|^2 \delta^{(4)}\left(\sum_i p^{\mu}_{h_i} - P^{\mu}_{tot}\right) d\Omega_{h_1,\ldots,h_N}$$

is such that

$$d\sigma^{h_1,\ldots,h_N} \propto \delta\left(\sum_i \epsilon_{h_i} - E_{tot}\right) d\Omega_{h_1,\ldots,h_N}$$

Entropy is maximal in both cases, so the created particles form a microkanonikus ensembles and thus the single-particle distribution is (m = 0)

$$f_N(x) \propto \frac{\Omega_{N-1}(E-\epsilon)}{\Omega_N(E)} \propto (1-x)^{D(N-1)-1}, \qquad x = \frac{\epsilon}{E} = \frac{p}{\sqrt{s}/2}$$

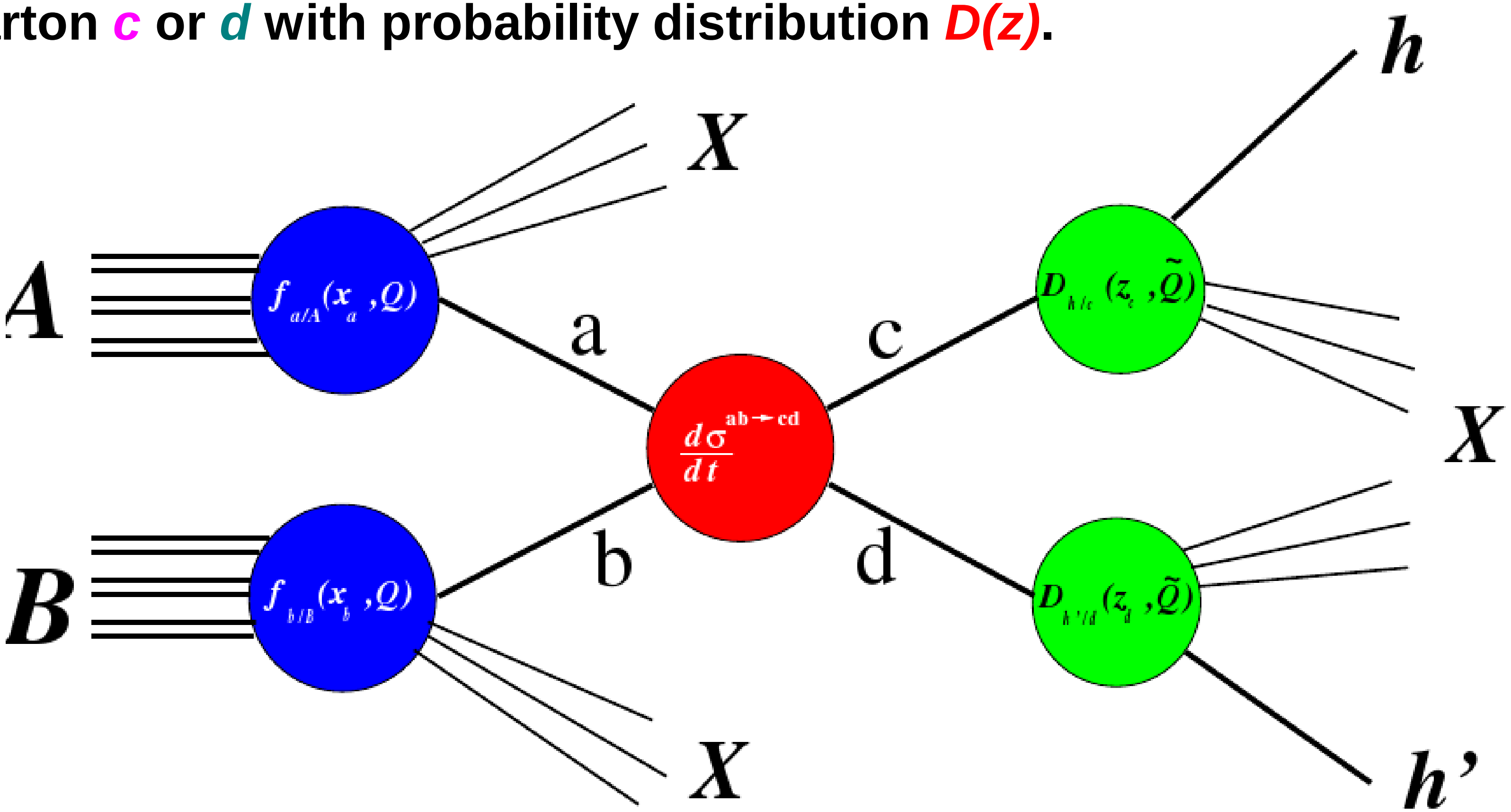
for the N-particle phasespace is

$$\Omega_N(E) = \int \prod d^D p_i \;\; \delta\left(E - \sum \epsilon_j\right) \propto E^{DN-1}$$

Parton-model calculation in pp collisions

Idea: **partons *a, b* inside protons *A, B* scatter off of each other.**

- **A parton carries some momentum fraction *x* of the momentum of its proton with probability-distribution *f(x)*.**
  Throughout the scattering of *a* and *b*, partons *c* and *d* are produced.
- ***c* and *d* induce *jets* (showers of hadron, whose distribution is measured).**
  Hadrons inside the jets, carry momentum fraction *z* of the momentum of the leading parton *c* or *d* with probability distribution *D(z)*.



Goal: ***find a satisfactory model for*** $D(z)$