NEW NON-PERTURBATIVE ANALYTICAL EQUATION OF STATE FOR $SU(3)$ GLUON PLASMA

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PT QGP EoS

$$P = \frac{1}{3} f(\alpha_s, N_f, \mu_f/T) T^4 + \frac{1}{3}\phi_2(\alpha_s, \mu_f) T^2 + \frac{1}{3}\phi_4(\alpha_s, \mu_f) - B$$

$$f(\alpha_s, N_f, \mu_f/T) = f_0(N_f) - f_2(N_f)\alpha_s + f_3(N_f, \mu_f/T)\alpha_s^{3/2}$$

$$+ f_4(N_f, \mu_f/T, \ln\alpha_s)\alpha_s^2 + f_5(N_f)\alpha_s^{5/2} + O(\alpha_s^3 \ln\alpha_s)$$

$$\alpha_s = g^2/4\pi, \qquad m_f^{(0)} = 0, \qquad N_f = 0, 1, 2.$$

The 3d reduction exhibits serious problems

$$\int \frac{dk_0}{(2\pi)} \to \frac{1}{\beta} \sum_{n=-\infty}^{\infty}, \qquad k_0 \to (2n+1)\frac{\pi}{\beta}, \quad \beta = T^{-1}$$

I. Much more severe IR structure in 3d QCD
II. Coupling constant becomes dimensional

$$g = [M]^{\frac{4-d}{2}} g', \quad g_{3d}^2 = [M], \quad T, \quad gT, \quad g^2 T$$

All this results in the non-analytical dependence on α_s, so thermal PT QCD is ???

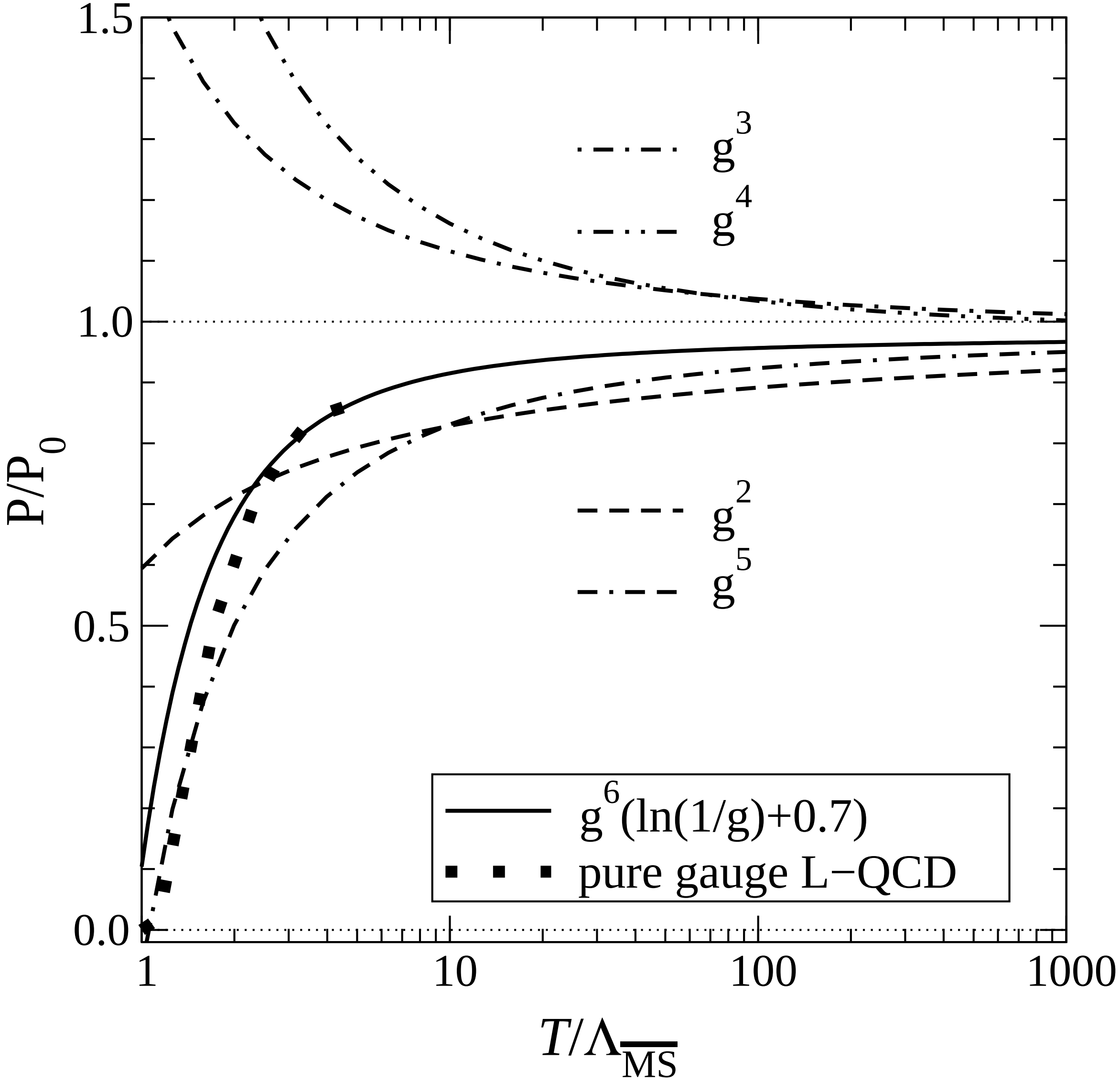


Figure 1: K. Kajantie et al, Phys. Rev. D 67, 105008 (2003)

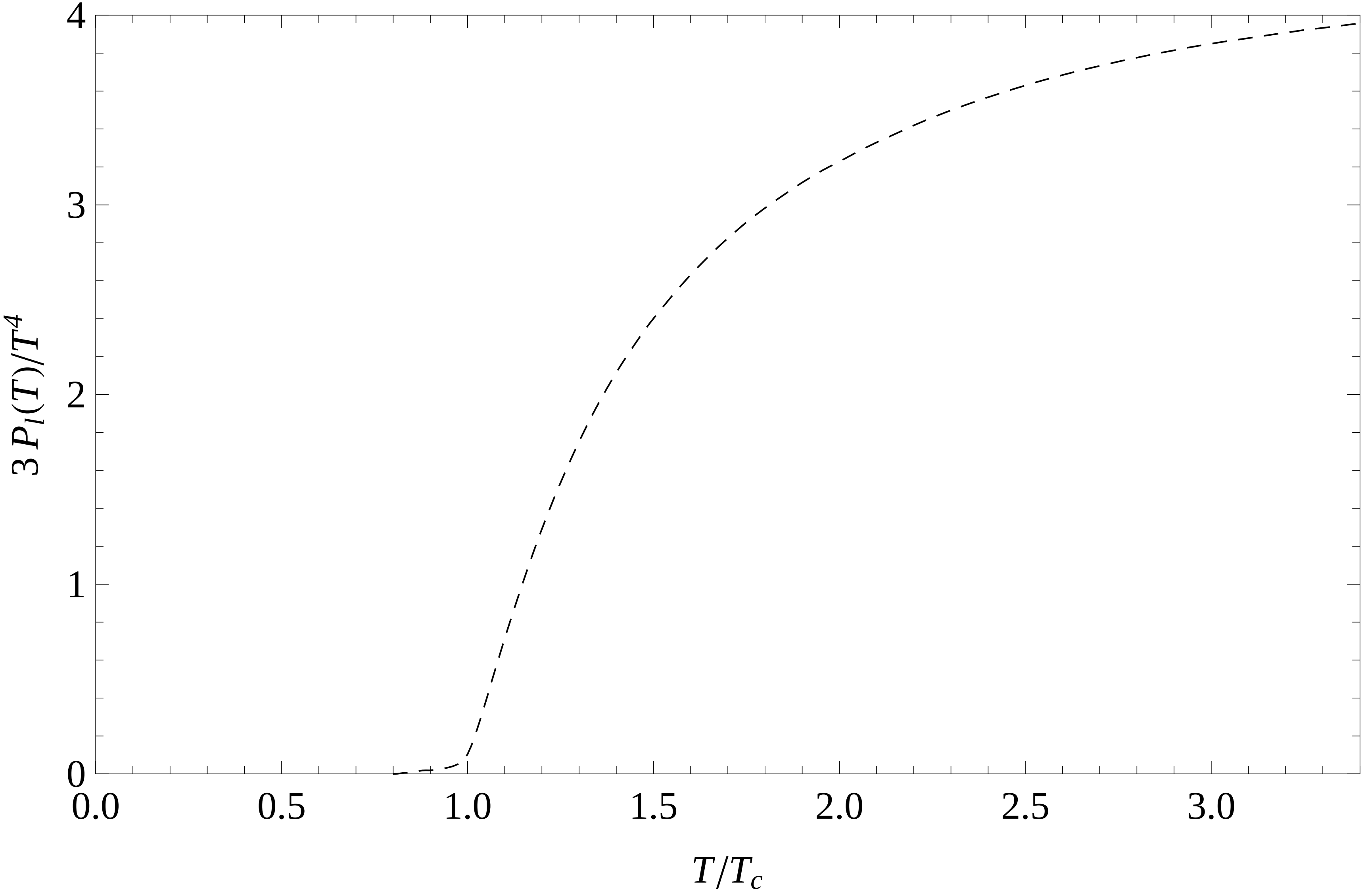


Figure 2: Lattice QCD, M. Panero, Phys. Rev. Lett., 103, 23001 (2009)

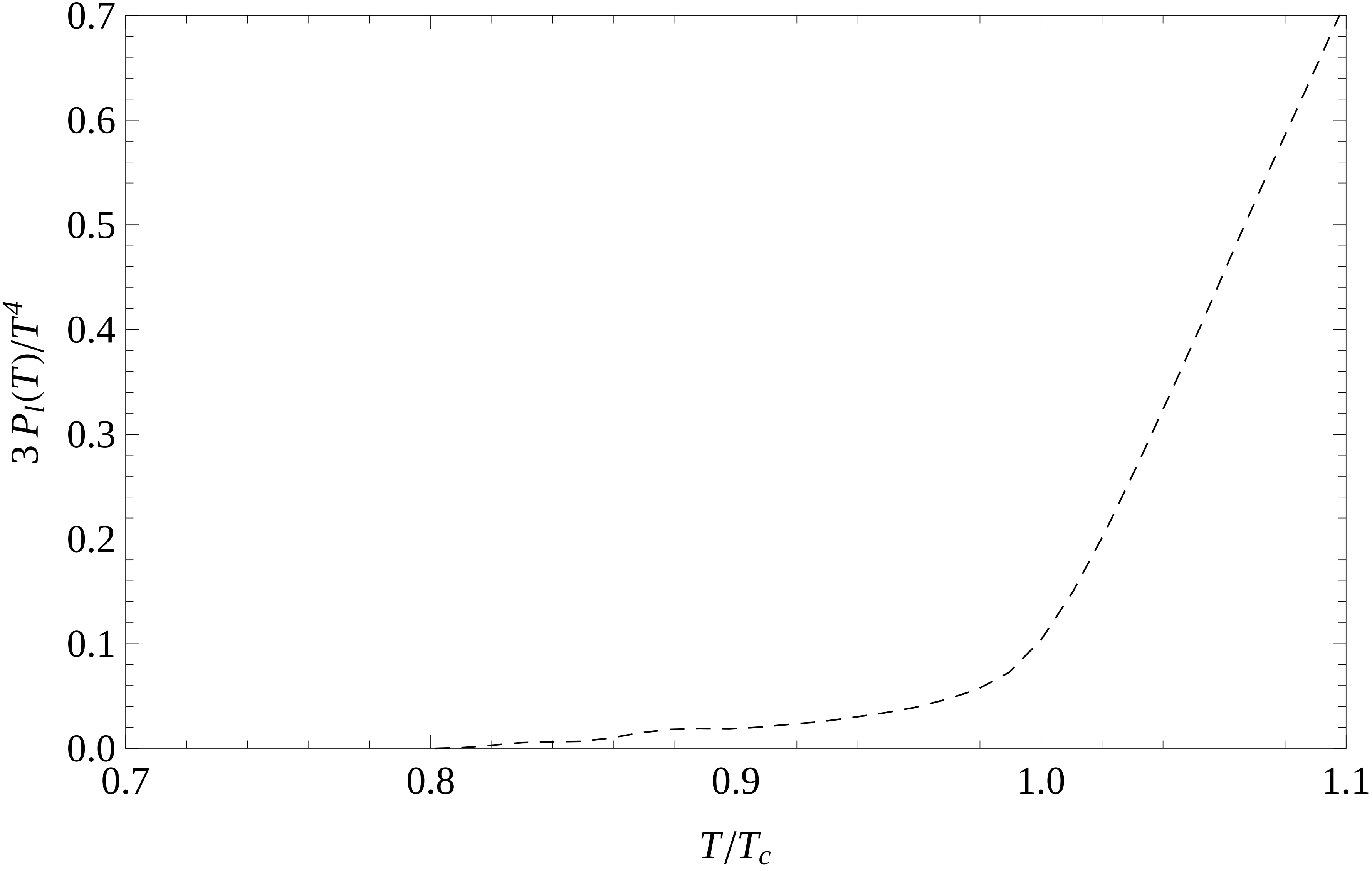
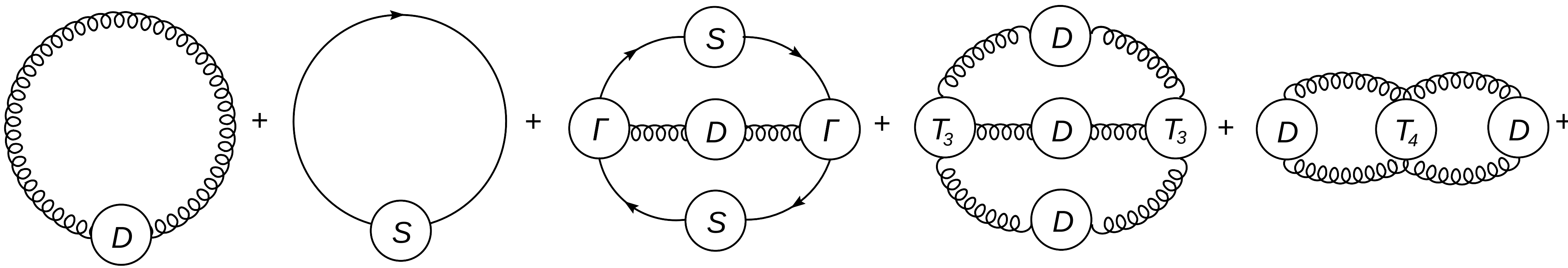


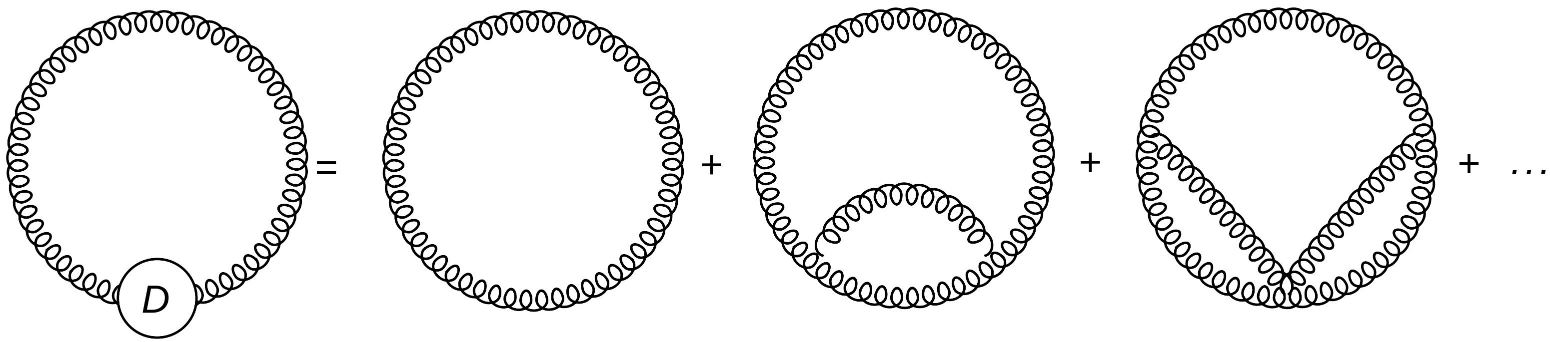
Figure 3: Lattice QCD, M. Panero, Phys. Rev. Lett., 103, 23001 (2009)



The CJT effective potential approach for composite operators

J.M. Cornwall, R. Jackiw, E. Tomboulis, Phys. Rev. D 10, (1974) 2428

The vacuum energy density (VED)



The gluon effective potential to the leading order (log-loop level $\sim \hbar$) is

$$V(D) = \frac{i}{2} \int \frac{d^4 q}{(2\pi)^4} \mathrm{Tr} \left\{ \ln(D_0^{-1} D) - (D_0^{-1} D) + 1 \right\}$$

where the normalization is $V(D_0) = 0$.

$$iD_{\mu\nu}(q) = \left\{T_{\mu\nu}(q)\alpha(-q^2,\xi) + \xi L_{\mu\nu}(q)\right\}\frac{1}{q^2}$$

$\alpha_s(-q^2)$ – being the full effective ("running") charge

$$T_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} = g_{\mu\nu} - L_{\mu\nu}(q)$$

$iD_{\mu\nu}(q) \to iD^0_{\mu\nu}(q)$ when $\alpha(-q^2,\xi) = 1.$

$$Tr\,\ln(D_0^{-1}D) = 8\times 4\ln det(D_0^{-1}D) = 32\ln\left\{\frac{3}{4}\alpha(-q^2,\xi) + \frac{1}{4}\right\}$$

and going over to Euclidean space, one obtains $[\epsilon_g = V(D), \;\; a = (3/4) - 2\ln 2]$.

$$\epsilon_g = -16 \int \frac{d^4q}{(2\pi)^4} \left\{ \ln[1 + 3\alpha(q^2)] - \frac{3}{4}\alpha(q^2) + a \right\}$$

Colorless, Transversal, $16 = 8 \times 2$

$$\alpha(q^2) \equiv \alpha(q^2; \Delta^2)$$

Millenium Prize Problem Theorem

If QCD exists then it has a mass gap $\Delta > 0$

The intrinsically NP (INP) gluon effective charge

$$\alpha^{INP}(q^2;\Delta^2) = \alpha(q^2;\Delta^2) - \alpha(q^2;\Delta^2 = 0)$$

$$= \alpha(q^2;\Delta^2) - \alpha^{PT}(q^2)$$

Δ^2 is the mass gap responsible for the NP dynamics in the QCD ground state (the so-called Jaffe-Witten (JW) mass gap).

$$\alpha(q^2;\Delta^2) = \alpha^{INP}(q^2;\Delta^2) + \alpha^{PT}(q^2)$$

$$\alpha^{INP}(q^2;\Delta^2) = \frac{\Delta^2}{q^2}$$

$$q^2 \frac{d\alpha^{INP}(q^2,\Delta^2)}{dq^2} = \beta(\alpha^{INP}(q^2;\Delta^2))$$

$$\beta(\alpha^{INP}(q^2;\Delta^2)) = -\alpha^{INP}(q^2;\Delta^2) = -\frac{\Delta^2}{q^2}$$

V. Gogokhia, G.G. Barnaföldi,
The Mass Gap and its Applications (Word Scientific, 2012)

$$\alpha^{PT}(q^2) = \frac{\alpha_s}{1+\alpha_s b \ln(q^2/\Lambda_{YM}^2)},\ b = \frac{11}{4\pi},\ \alpha_s \equiv \alpha_s(M_z) = 0.1187.$$

THE BAG CONSTANT

$$B = VED^{PT} - VED = VED^{PT} - [VED - VED^{PT} + VED^{PT}] =$$

$$VED^{PT} - [VED^{INP} + VED^{PT}] = -VED^{INP} > 0$$

$$B_{YM} = 16 \int^{q^2_{eff}} \frac{d^4q}{(2\pi)^4} \left\{ \ln[1 + 3\alpha^{INP}(q^2)] - \frac{3}{4}\alpha^{INP}(q^2) \right\}$$

q^2_{eff} separates the NP from the PT regions

G.G. Barnaföldi, V. Gogokhia,
J. Phys. G: Nucl. Part. Phys. 37, 025003 (2010)

$$P_g = \epsilon_g + B_{YM} = P_{NP} + P_{PT} = B_{YM} + P_{YM} + P_{PT}$$

$$B_{YM} = 16 \int^{q^2_{eff}} \frac{d^4q}{(2\pi)^4} \left\{ \ln[1 + 3\alpha^{INP}(q^2)] - \frac{3}{4}\alpha^{INP}(q^2) \right\}$$

$$P_{YM} = -16 \int \frac{d^4q}{(2\pi)^4} \left\{ \ln[1 + \frac{3}{4}\alpha^{INP}(q^2)] - \frac{3}{4}\alpha^{INP}(q^2) \right\}$$

$$P_{PT} = -16 \int_{\Lambda^2_{YM}} \frac{d^4q}{(2\pi)^4} \left\{ \ln\left[1 + \frac{3\alpha^{PT}(q^2)}{4 + 3\alpha^{INP}(q^2)}\right] - \frac{3}{4}\alpha^{PT}(q^2) \right\}$$

Generalization to non-zero temperatures

In the imaginary time formalism these expressions can be easily generalized to non-zero temperatures T according to the prescription (there is already Euclidean signature)

$$\int \frac{dq_0}{(2\pi)} \to T \sum_{n=-\infty}^{+\infty}, \quad q^2 = \mathbf{q}^2 + q_0^2 = \mathbf{q}^2 + \omega_n^2 = \omega^2 + \omega_n^2 = \omega^2 + (2n\pi T)^2$$

In frequency-momentum space ($T^{-1} = \beta, \quad \omega = \sqrt{\mathbf{q}^2}$),

$$\alpha^{INP}(q^2) = \alpha^{TNP}(\omega^2, \omega_n^2) = \frac{\Delta^2}{\omega^2 + \omega_n^2},$$

$$\alpha^{PT}(q^2) = \alpha^{PT}(\omega^2, \omega_n^2) = \frac{\alpha_s}{1 + \alpha_s b \ln[(\omega^2 + \omega_n^2)/\Lambda_{YM}^2]}.$$

$$P_g(T) = P_{NP}(T) + P_{PT}(T) = B_{YM}(T) + P_{YM}(T) + P_{PT}(T),$$

$$B_{YM}(T) = \frac{8}{\pi^2} \int_0^{\omega_{eff}} d\omega \, \omega^2 \, T \sum_{n=-\infty}^{+\infty} \left[\ln \left(1 + 3\alpha^{INP}(\omega_n^2) \right) - \frac{3}{4} \alpha^{INP}(\omega_n^2) \right],$$

$$P_{YM}(T) = -\frac{8}{\pi^2} \int_0^{\infty} d\omega \, \omega^2 \, T \sum_{n=-\infty}^{+\infty} \left[\ln \left(1 + \frac{3}{4} \alpha^{INP}(\omega_n^2) \right) - \frac{3}{4} \alpha^{INP}(\omega_n^2) \right],$$

$$P_{PT}(T) = -\frac{8}{\pi^2} \int_{\Lambda_{YM}}^{\infty} d\omega \, \omega^2 \, T \sum_{n=-\infty}^{+\infty} \left[\ln \left(1 + \frac{3\alpha^{PT}(\omega_n^2)}{4 + 3\alpha^{INP}(\omega_n^2)} \right) - \frac{3}{4} \alpha^{PT}(\omega_n^2 \right.$$

$$\sum_{n=-\infty}^{+\infty} \frac{1}{\mathbf{q}^2 + \omega_n^2} = \sum_{n=-\infty}^{\infty} \frac{1}{\omega^2 + (2\pi T)^2 n^2} = (2\pi/\beta)^{-2} \sum_{n=-\infty}^{+\infty} \frac{1}{n^2 + (\beta\omega/2\pi)^2}$$

$$= (2\pi/\beta)^{-2}(2\pi^2/\beta\omega)\left(1 + \frac{2}{e^{\beta\omega} - 1}\right) = \frac{\beta}{2\omega}\left(1 + \frac{2}{e^{\beta\omega} - 1}\right).$$

The summation of the thermal logarithms

$$\sum_{n=-\infty}^{+\infty} \ln[3\Delta^2 + \mathbf{q}^2 + \omega_n^2] = \ln \omega'^2 + 2\sum_{n=1}^{\infty} \ln(2\pi/\beta)^2[n^2 + (\beta\omega'/2\pi)^2]$$

$$\sum_{n=-\infty}^{+\infty} \ln[\mathbf{q}^2 + \omega_n^2] = \ln \omega^2 + 2 \sum_{n=1}^{\infty} \ln(2\pi/\beta)^2 [n^2 + (\beta\omega/2\pi)^2].$$

$$L(\omega') = \sum_{n=1}^{\infty} \ln[n^2 + (\beta\omega'/2\pi)^2] = \sum_{n=1}^{\infty} \ln n^2 + \sum_{n=1}^{\infty} \ln \left[1 - \frac{x'^2}{n^2\pi^2}\right]$$

$$L(\omega) = \sum_{n=1}^{\infty} \ln[n^2 + (\beta\omega/2\pi)^2] = \sum_{n=1}^{\infty} \ln n^2 + \sum_{n=1}^{\infty} \ln \left[1 - \frac{x^2}{n^2\pi^2}\right].$$

$$x'^2 = -\left(\frac{\beta\omega'}{2}\right)^2, \quad x^2 = -\left(\frac{\beta\omega}{2}\right)^2.$$

$$L(\omega') - L(\omega) = \sum_{n=1}^{\infty} \ln\left[1 - \frac{x'^2}{n^2\pi^2}\right] - \sum_{n=1}^{\infty} \ln\left[1 - \frac{x^2}{n^2\pi^2}\right]$$

$$= \ln\sin x' - \frac{1}{2}\ln x'^2 - \ln\sin x + \frac{1}{2}\ln x^2,$$

$$L(\omega') - L(\omega) = -\frac{1}{2}\ln\left(\frac{x'^2}{x^2}\right) + \ln\left(\frac{\sin x'}{\sin x}\right).$$

$$x' = \pm i\left(\frac{\beta\omega'}{2}\right), \quad x = \pm i\left(\frac{\beta\omega}{2}\right),$$

$$L(\omega') - L(\omega) = -\frac{1}{2}\ln\left(\frac{\omega'^2}{\omega^2}\right) + \frac{1}{2}\beta(\omega' - \omega) + \ln\left(\frac{1 - e^{-\beta\omega'}}{1 - e^{-\beta\omega}}\right).$$

The temperature dependence of B_{YM}

$$B_{YM}(T) = -\frac{6}{\pi^2}\Delta^2 B^{(1)}_{YM}(T) - \frac{16}{\pi^2} T \left[B^{(2)}_{YM}(T) - B^{(3)}_{YM}(T) \right]$$

$$B^{(1)}_{YM}(T) = \int_0^{\omega_{eff}} d\omega \frac{\omega}{e^{\beta\omega} - 1}, \qquad \beta^{-1} = T$$

$$B^{(2)}_{YM}(T) = \int_0^{\omega_{eff}} d\omega \; \omega^2 \ln\left(1 - e^{-\beta\omega}\right)$$

$$B^{(3)}_{YM}(T) = \int_0^{\omega_{eff}} d\omega \; \omega^2 \ln\left(1 - e^{-\beta\omega'}\right), \qquad \omega' = \sqrt{\omega^2 + 3\Delta^2}$$

The temperature dependence of P_{YM}

$$P_{YM}(T) = \frac{6}{\pi^2}\Delta^2 P_{YM}^{(1)}(T) + \frac{16}{\pi^2}T\left[P_{YM}^{(2)}(T) - P_{YM}^{(3)}(T)\right]$$

$$P_{YM}^{(1)}(T) = \int_0^\infty d\omega \frac{\omega}{e^{\beta\omega}-1} = \frac{\pi^2}{6}T^2$$

$$P_{YM}^{(2)}(T) = \int_0^\infty d\omega\ \omega^2 \ln\left(1 - e^{-\beta\omega}\right)$$

$$P_{YM}^{(3)}(T) = \int_0^\infty d\omega\ \omega^2 \ln\left(1 - e^{-\beta\bar{\omega}}\right), \qquad \bar{\omega} = \sqrt{\omega^2 + (3/4)\Delta^2}$$

The NP part of the gluon pressure

$$P_{NP}(T) = B_{YM}(T) + P_{YM}(T))$$

$$P_{NP}(T) = \frac{6}{\pi^2}\Delta^2 P_1(T) + \frac{16}{\pi^2} T[P_2(T) + P_3(T) - P_4(T)]$$

$$P_1(T) = \int_{\omega_{eff}}^{\infty} d\omega \frac{\omega}{e^{\beta\omega} - 1}$$

$$P_2(T) = \int_{\omega_{eff}}^{\infty} d\omega \ \omega^2 \ln\left(1 - e^{-\beta\omega}\right)$$

$$P_3(T) = \int_{0}^{\omega_{eff}} d\omega \ \omega^2 \ln\left(1 - e^{-\beta\omega'}\right)$$

$$P_4(T) = \int_0^\infty d\omega\ \omega^2 \ln\left(1 - e^{-\beta\bar{\omega}}\right)$$

$$\omega' = \sqrt{\omega^2 + 3\Delta^2} = \sqrt{\omega^2 + m'^2_{eff}}$$

$$\bar{\omega} = \sqrt{\omega^2 + \frac{3}{4}\Delta^2} = \sqrt{\omega^2 + \bar{m}^2_{eff}}$$

In the formal PT limit ($\Delta^2 = 0$) from these relations it follows that $\bar{\omega} = \omega' = \omega$. And always $\beta = T^{-1}$.

V. Gogokhia, M. Vasuth
J. Phys. G: Nucl. Part. Phys. 37 075015 (2010).

The PT part of the gluon pressure

$$P_{PT}(T) = -\frac{8}{\pi^2} \int_{\Lambda_{YM}}^{\infty} d\omega \; \omega^2 \; T$$

$$\times \sum_{n=-\infty}^{+\infty} \left[\ln \left[1 + \frac{3\alpha_s^{PT}(\omega^2, \omega_n^2)}{4 + 3\alpha_s^{INP}(\omega^2, \omega_n^2)} \right] - \frac{3}{4} \alpha_s^{PT}(\omega^2, \omega_n^2) \right]$$

$$= P_{PT}^s(T) + O(\alpha_s^2),$$

$$P_{PT}^s(T) \equiv P_{PT}^s(\Delta^2; T) = \alpha_s \times \frac{9}{2\pi^2} \Delta^2 \int_{\Lambda_{YM}}^{\infty} d\omega \; \omega^2 \; \frac{1}{\bar{\omega}} \frac{1}{e^{\beta \bar{\omega}} - 1}.$$

The scale-setting scheme

$$q_{eff}^2 = \mathbf{q}_{eff}^2 + \omega_c^2 = \omega_{eff}^2 + \omega_c^2, \quad \omega_c = 2\pi n_c T_c$$

$$\omega_{eff} = \sqrt{q_{eff}^2 - \omega_c^2}, \quad \omega_{eff} \leq q_{eff}$$

$$\omega_{eff} = \sqrt{q_{eff}^2} = 1\ GeV$$

$$\Delta^2 = 0.4564\ GeV^2, \qquad \Delta = 0.6756\ GeV$$

$$\omega' = \sqrt{\omega^2 + m'^2_{eff}}, \quad m'_{eff} = \sqrt{3}\Delta = 1.17\ GeV.$$

$$\bar{\omega} = \sqrt{\omega^2 + \bar{m}^2_{eff}}, \quad \bar{m}_{eff} = (\sqrt{3}/2)\Delta = 0.585\ GeV,$$

$$< G^2 >_{T=0} = 0.1052\ GeV^4$$

The confinement dynamics is nontrivially taken into account directly through the mass gap, and through the Bag pressure itself.

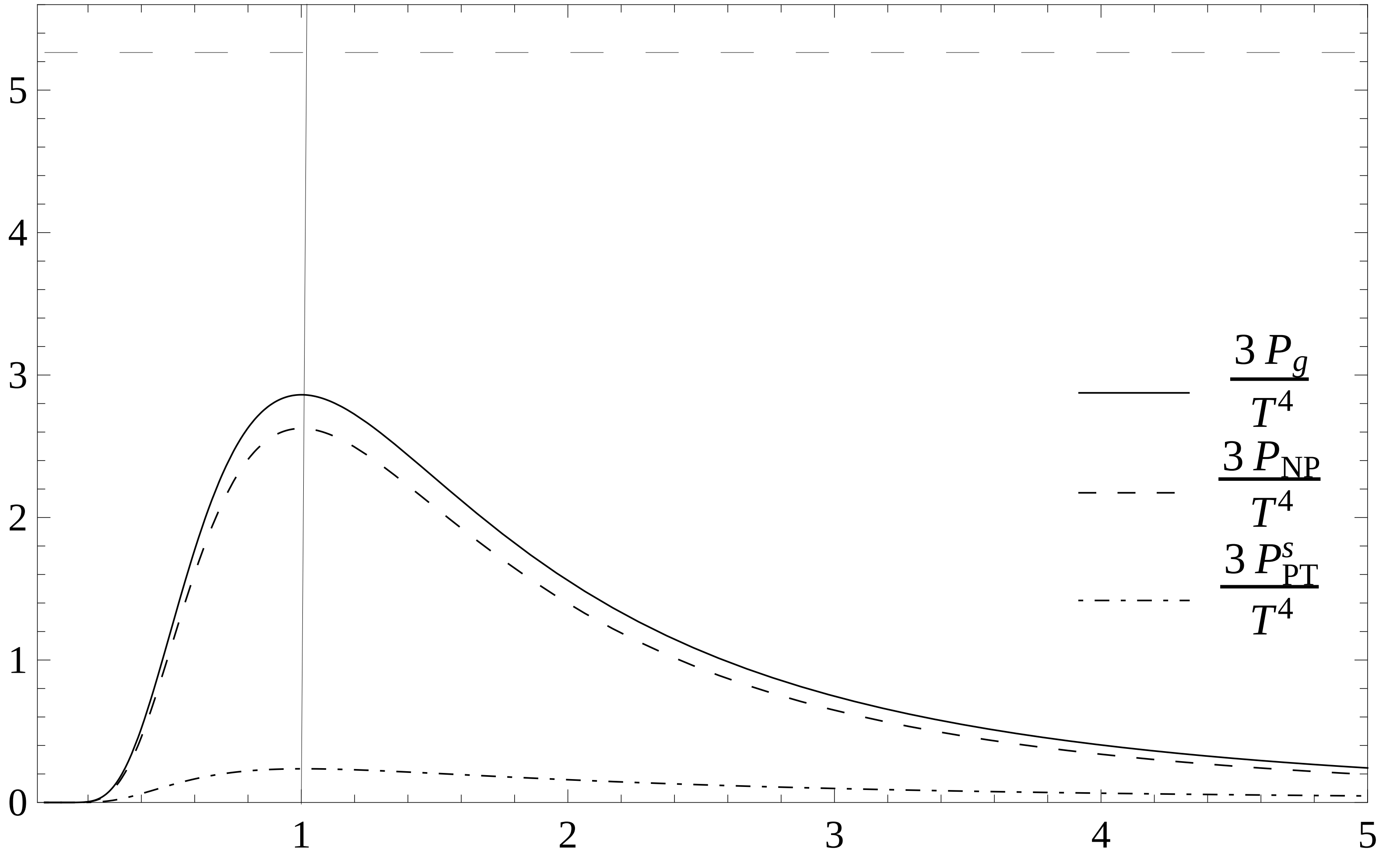


Figure 4: The gluon pressure $P_g(T) = P_{NP}(T) + P^s_{PT}(T)$ and its parts is shown as a function of T/T_c. All the pressures are properly scaled. The characteristic temperature is $T^* = 266.5\ MeV$.

The full gluon plasma pressure

$$P_{GP}(T) = P_g(T) + \Theta\left(\frac{T_c}{T} - 1\right) L(T) + \Theta\left(\frac{T}{T_c} - 1\right) H(T),$$

The GP pressure is continuous at T_c if and only if

$$L(T_c) = H(T_c)$$

$$L(T) = f_l(T) P_{SB}(T) - \phi_l(T) P_g(T),$$

$$H(T) = f_h(T) P_{SB}(T) - \phi_h(T) P_g(T).$$

Analytical simulation

$$N_g \equiv N_g(\beta, \omega) = \frac{1}{e^{\beta\omega} - 1}, \quad \beta = T^{-1},$$

$$f_l(T) = \sum_{i=1}^{n} A_i e^{-\alpha_i (T_c/T)}, \quad \alpha_i > 0, \quad \phi_l(T) = e^{-\alpha (T_c/T)}, \quad \alpha > 0,$$

$$f_h(T) = 1 - \alpha_s(T) = 1 - \frac{0.36}{1 + 0.55 \ln(T/T_c)}, \quad T \geq T_c,$$

$$\phi_h(a) = \sum_{k=0}^{\infty} c_k a^{-k} = c_0 + [\phi_h(1) - c_0] a^{-n}, \quad a = T/T_c.$$

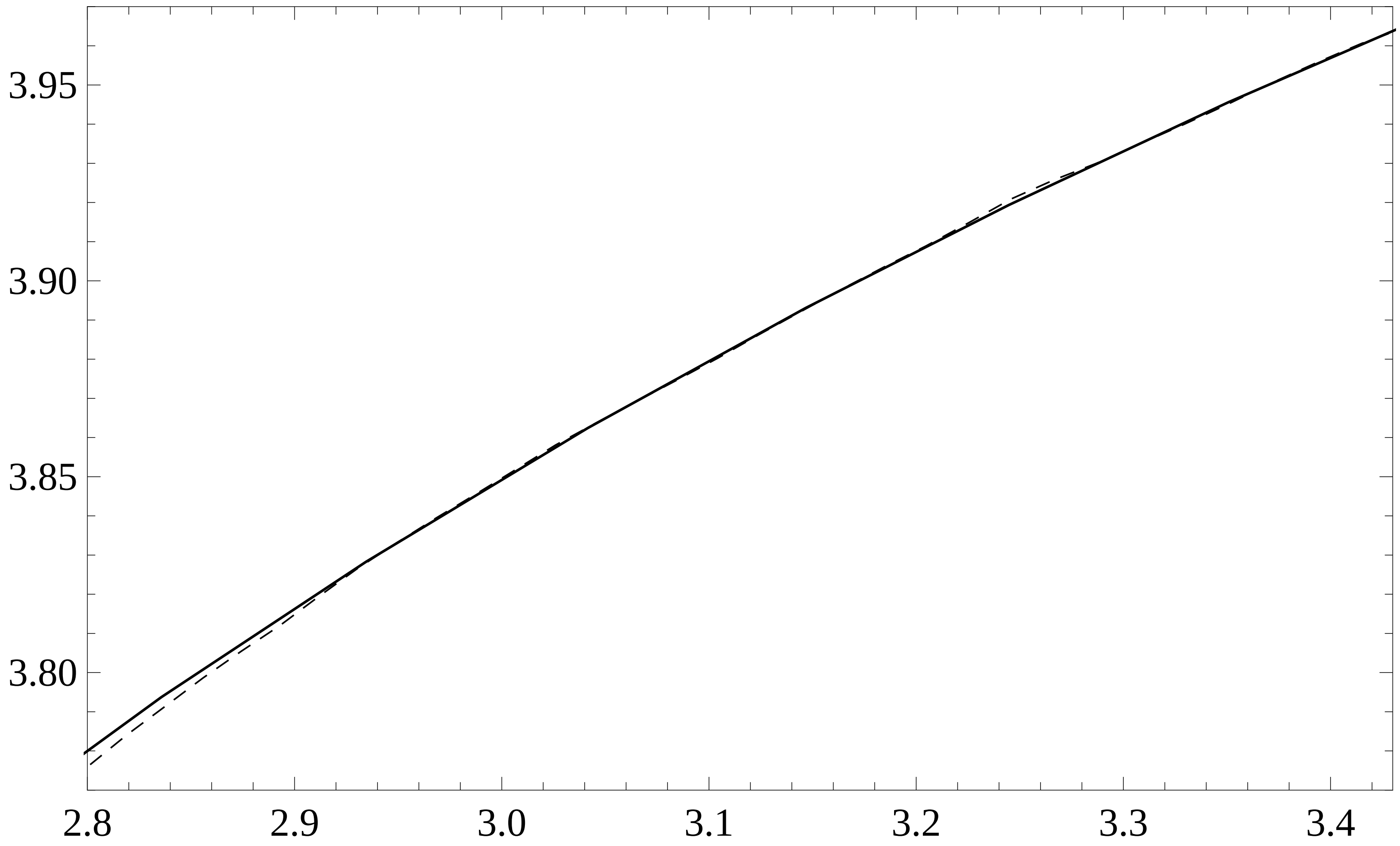


Figure 5: The gluon plasma pressure $P_{GM}(T)$ is shown as a function of $a = T/T_c$. Reproducing the lattice data has been done with the Least Mean Squares (LMS) method. The best values for its parameters are: $c_0 = 1.2815,\ \phi(1) = 2.1164,\ n = 3.$

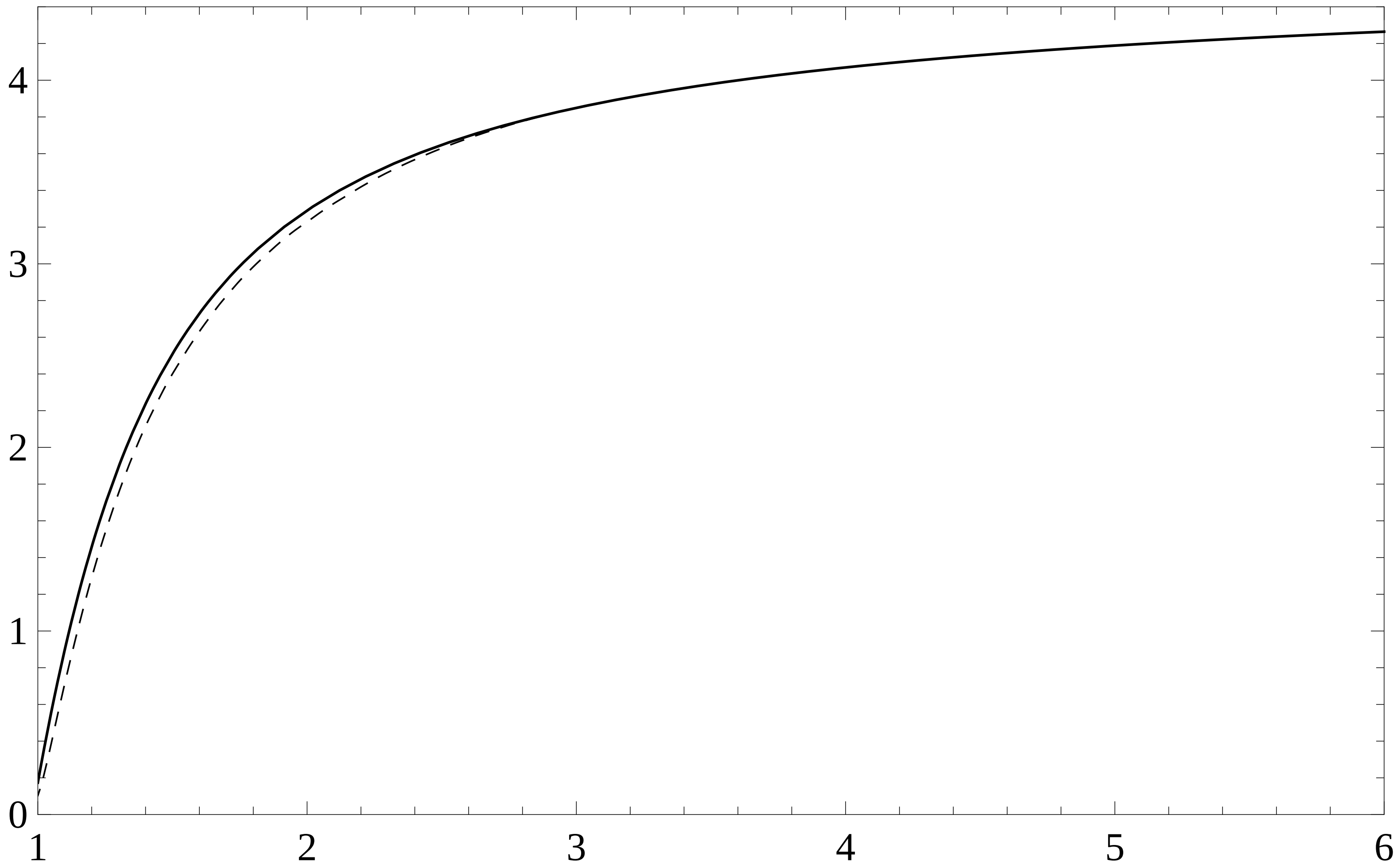


Figure 6: The gluon plasma pressure $P_{GM}(T)$ is shown as a function of $a = T/T_c$ up to $a = 1$.

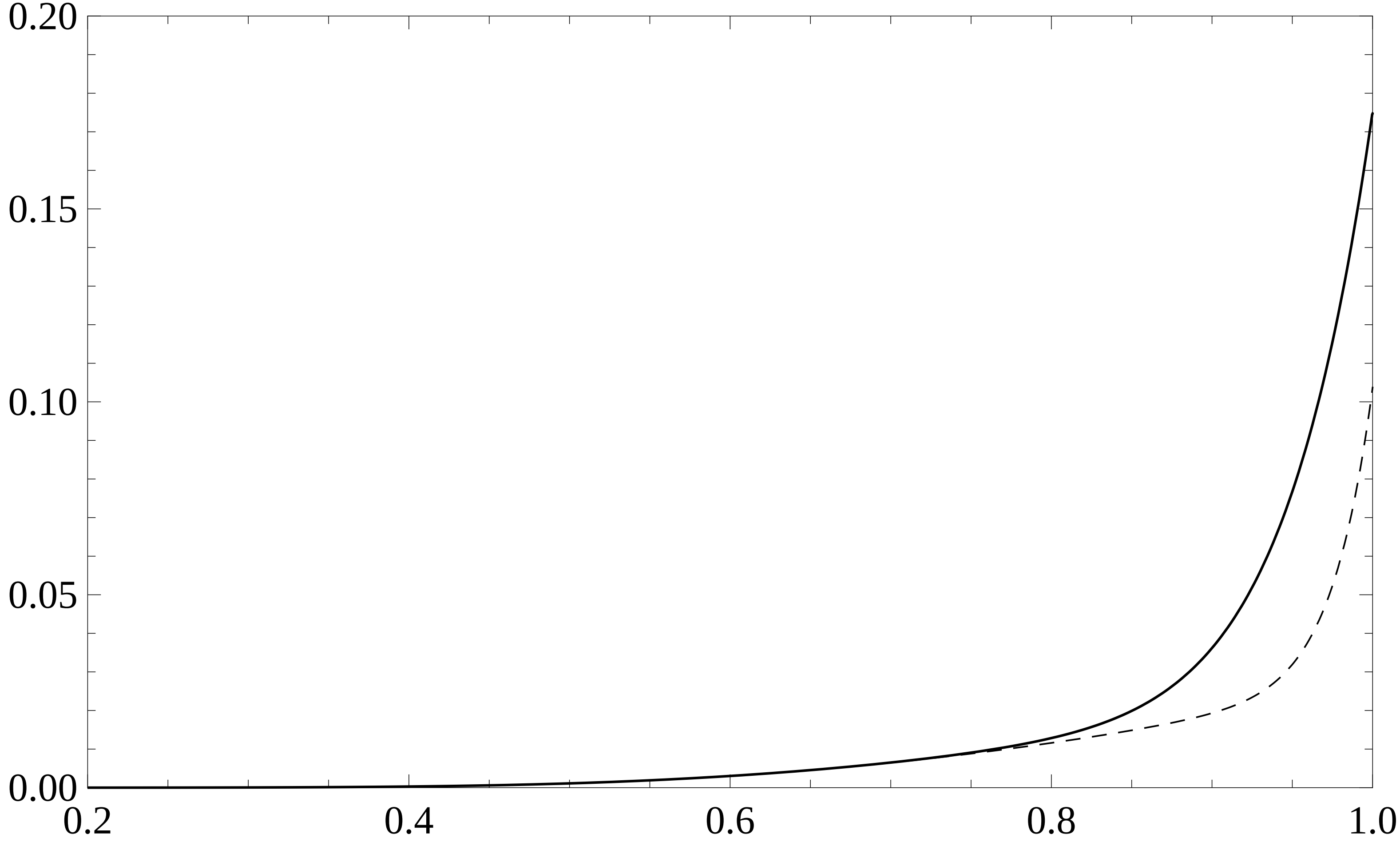


Figure 7: $3P_{GM}(T_c)/T_c^4 = 0.1738$ and $P_l(T_c) \times 5.2334/T_c^4 = 0.1036$.

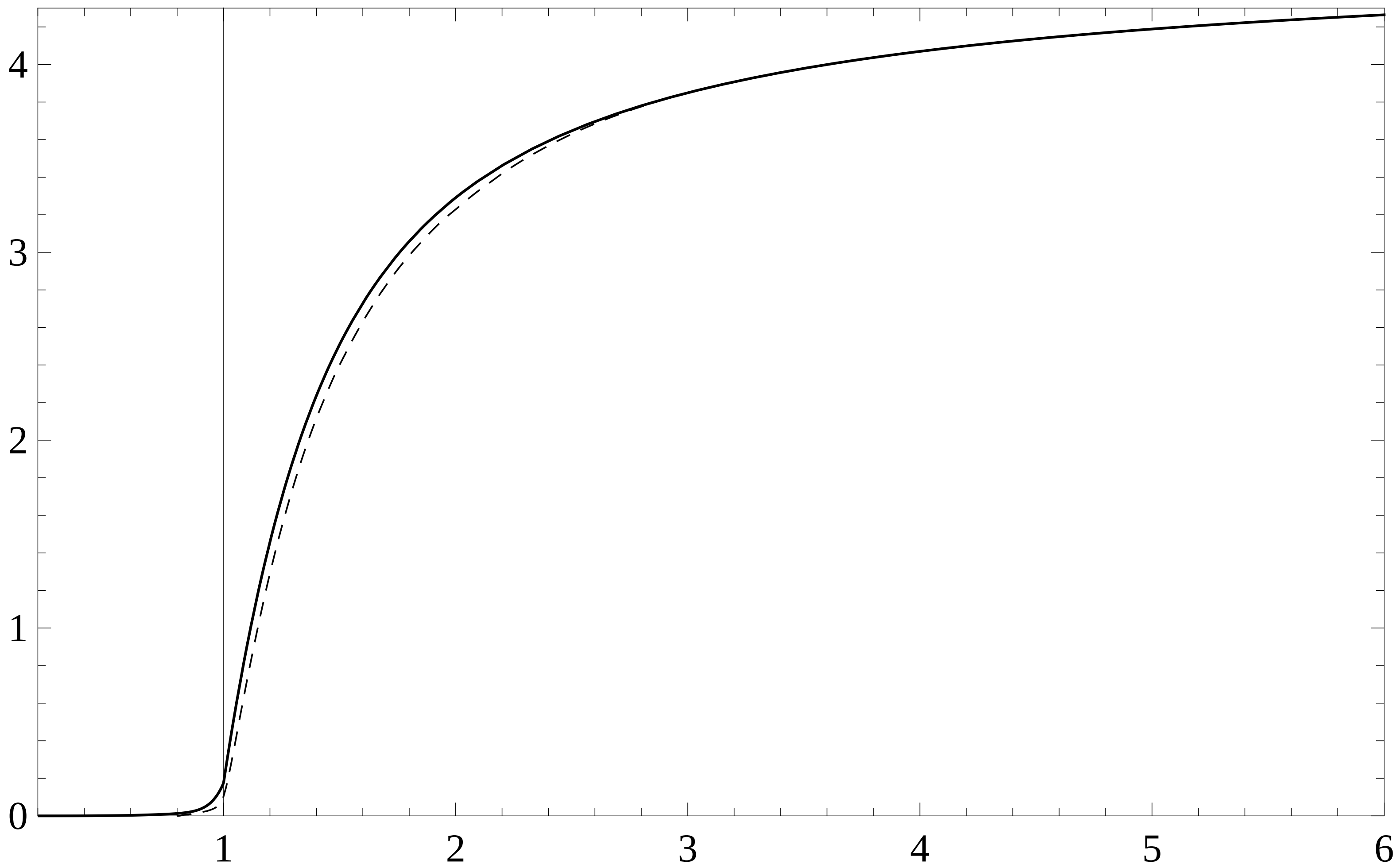


Figure 8: $3P_{GM}(T)/T^4$ and $P_l(T) \times 5.2334/T^4$ are shown in the whole temperature range.

Analytical expression for the full gluon plasma pressure

$$P_{GP}(T) = P_g(T) + \Theta\left(\frac{T_c}{T} - 1\right) \times$$
$$\left[[0.0282 e^{-19.1((T_c/T)-1)} + 0.005 e^{-3.4((T_c/T)-1)}] P_{SB}(T) - e^{-0.001(T_c/T)} P_g(T)\right]$$
$$+\Theta\left(\frac{T}{T_c} - 1\right)[(1 - \alpha_s(T))P_{SB}(T) - \phi_h(T)P_g(T)],$$

$$\alpha_s(T) = \frac{0.36}{1 + 0.55\ln(T/T_c)}, \quad T \geq T_c,$$

$$\phi_h(T) = 1.2815 + 0.8349\left(\frac{T_c}{T}\right)^3, \quad \phi_h(T_c) = 2.1164.$$

SB limits $(T \to \infty)$ and relations

$$\frac{3P_{SB}(T)}{T^4} = \frac{\epsilon_{SB}(T)}{T^4} = \frac{3s_{SB}(T)}{4T^3} = \frac{c_{V(SB)}(T)}{4T^3} = \frac{24}{45}\pi^2 \approx 5.2634,$$

$$C_{SB}(T) = c^2_{s(SB)}(T) = \frac{1}{3},$$

$$\epsilon_{SB}(T) - 3P_{SB}(T) = 0.$$

$$TP'_{SB}(T) = 4P_{SB}(T) = s_{SB}(T)T = \frac{4}{3}\epsilon_{SB}(T) = \frac{1}{3}Tc_{V(SB)}(T).$$

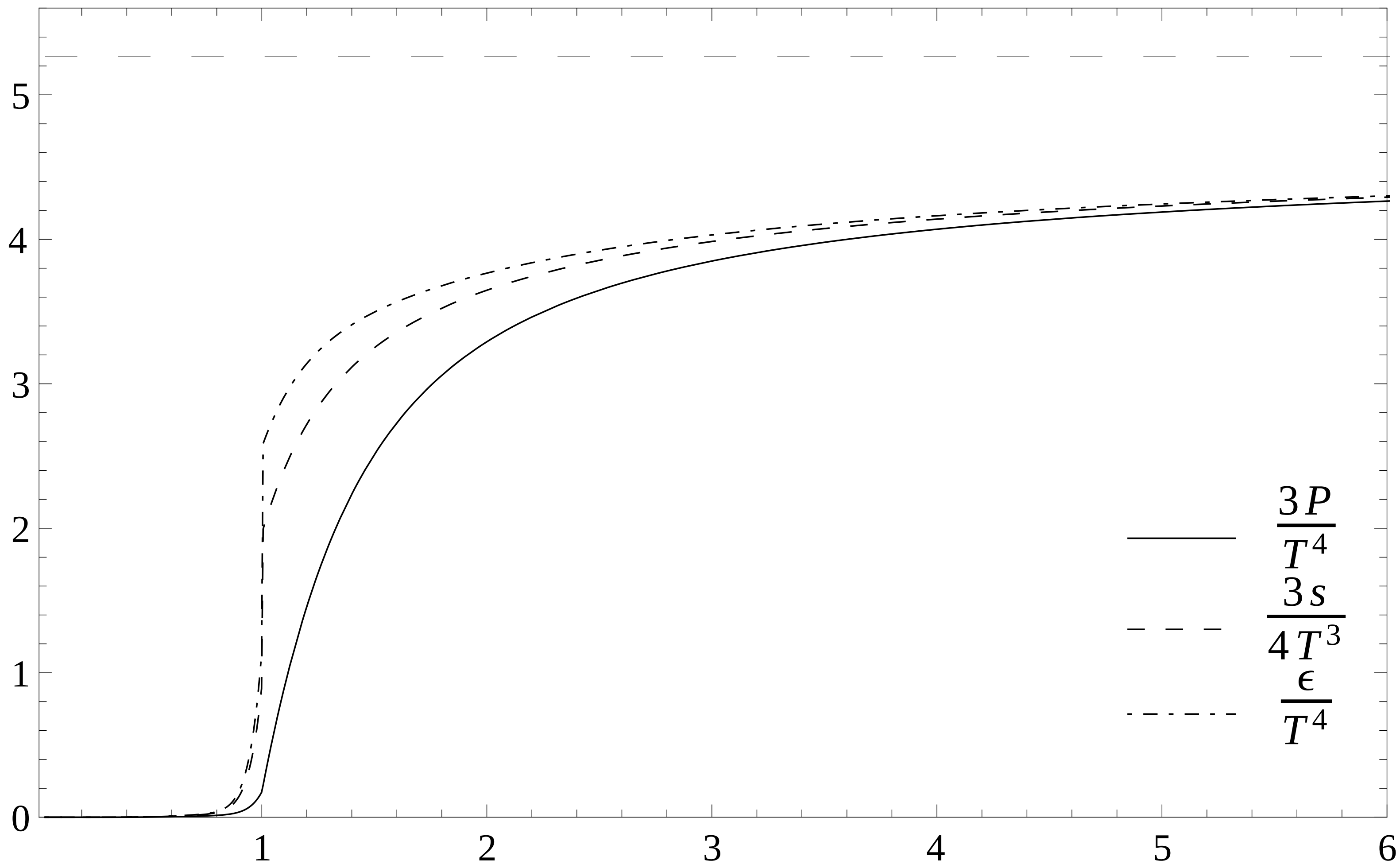


Figure 9: The gluon plasma pressure, $s(T) = (\partial P(T)/\partial T)$ and $\epsilon(T) = Ts(T) - P(T)$. The latent heat (LH) is $\epsilon_{LH} = 1.414$.

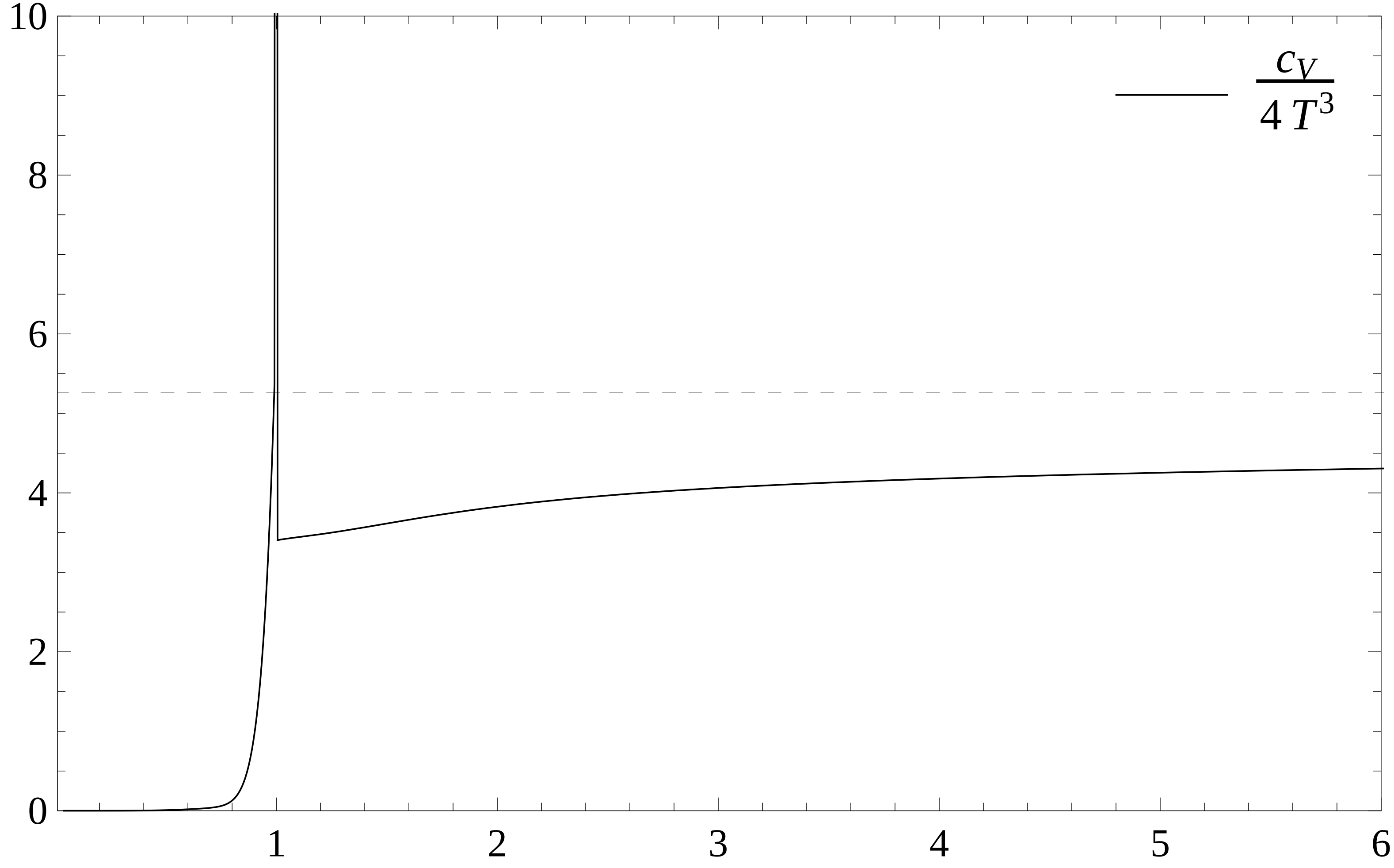


Figure 10: The heat capacity $c_V(T) = (\partial\epsilon(T)/\partial T)$. It has a δ-type singularity (an essential discontinuity) at T_c.

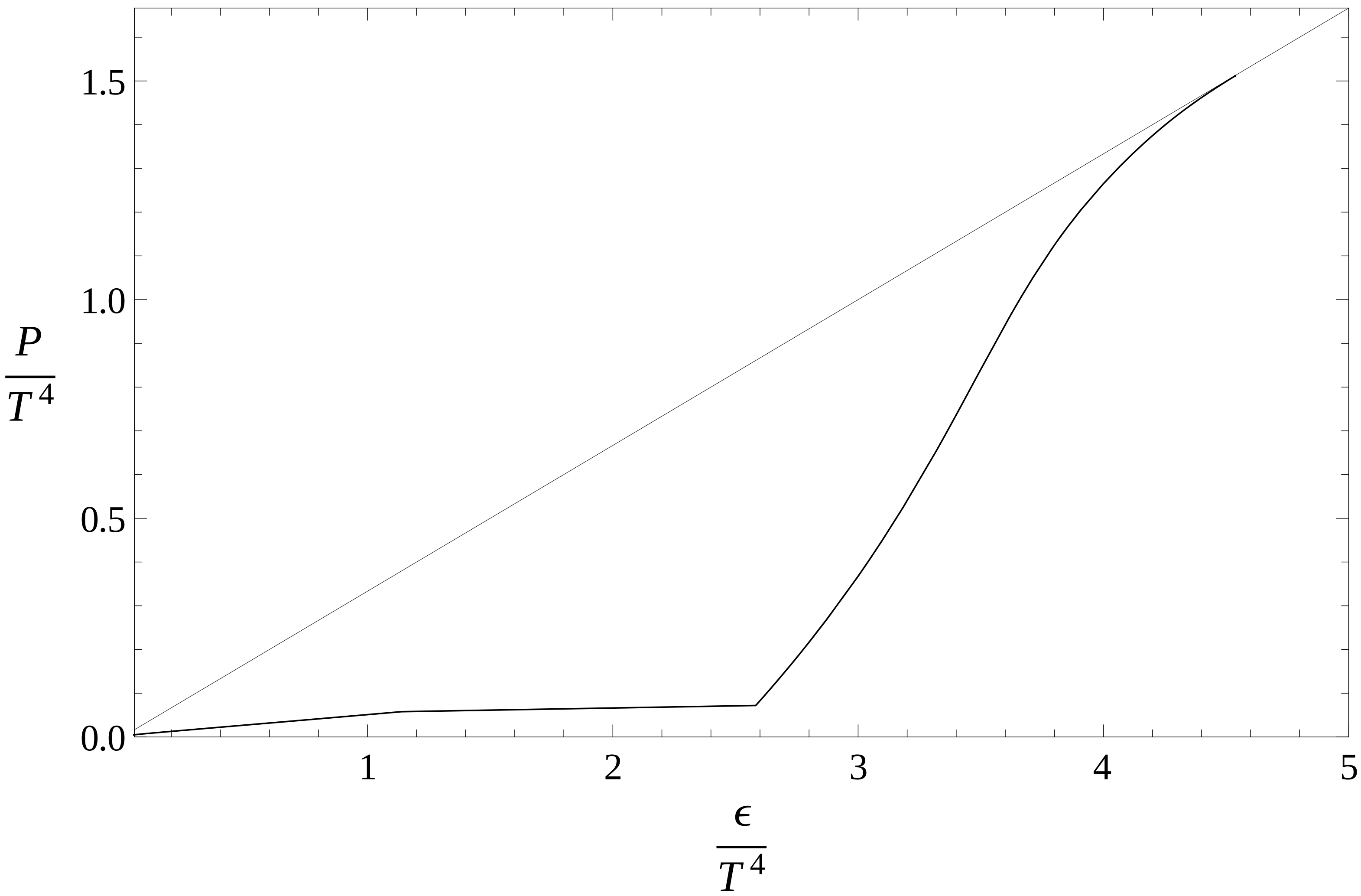


Figure 11: $P(\epsilon)$ EoS (solid line) which rather rapidly approaches $conformality = 1/3$ (diagonal line).

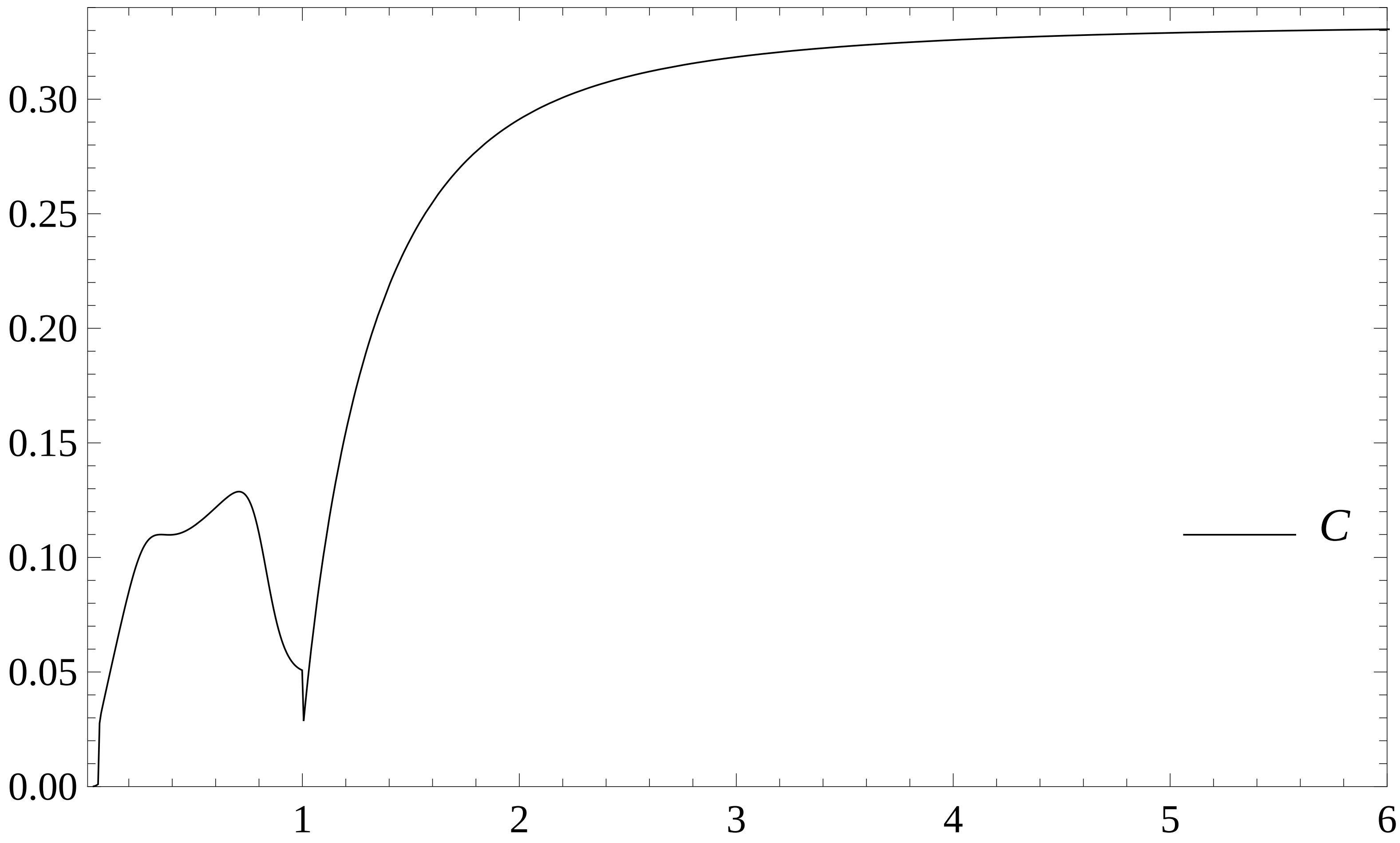


Figure 12: Conformity $C = P(T)/\epsilon(T)$.

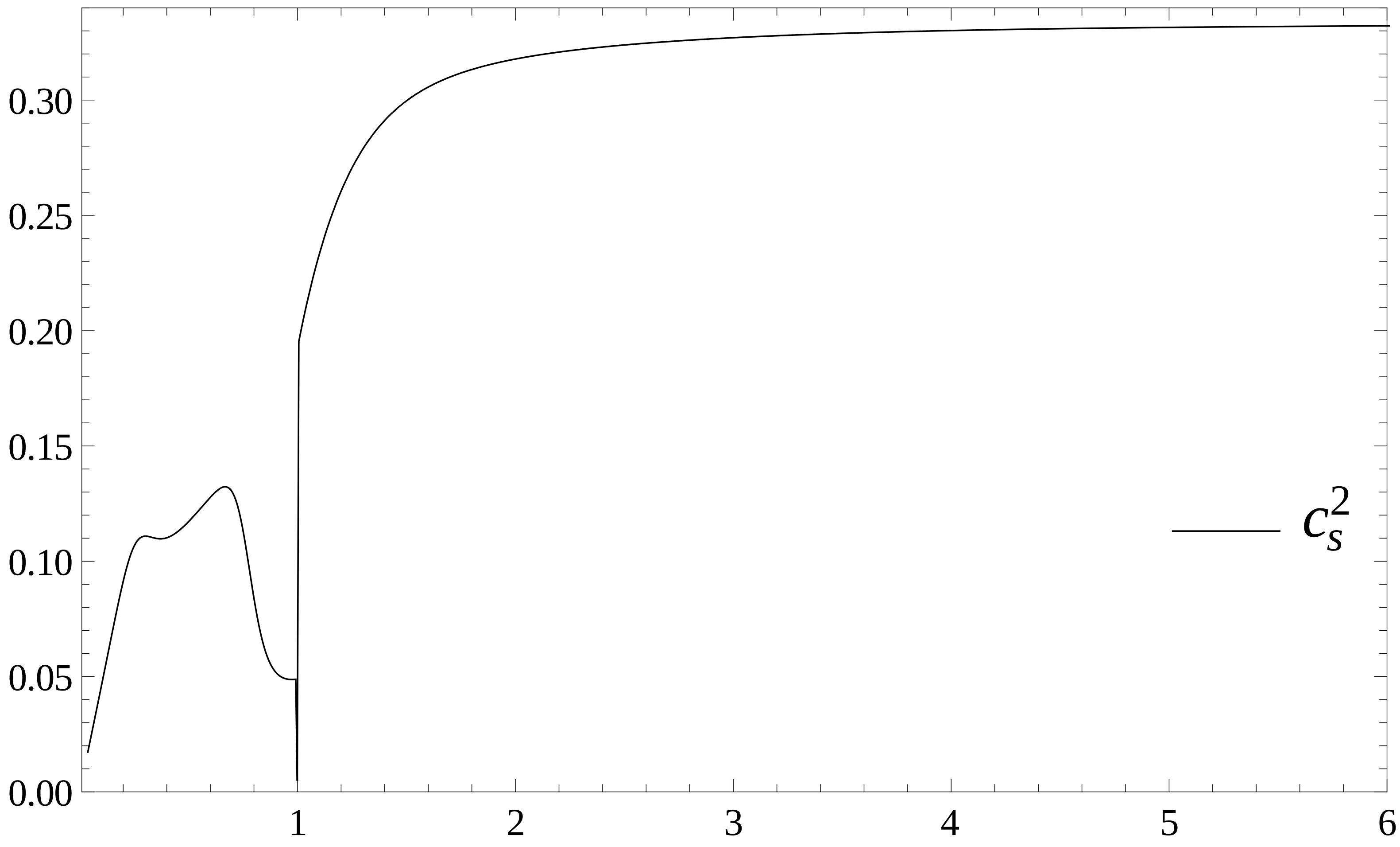


Figure 13: The velocity of sound squared $c_s^2(T) = \partial P(T)/\partial \epsilon(T) = s(T)/c_V(T)$.

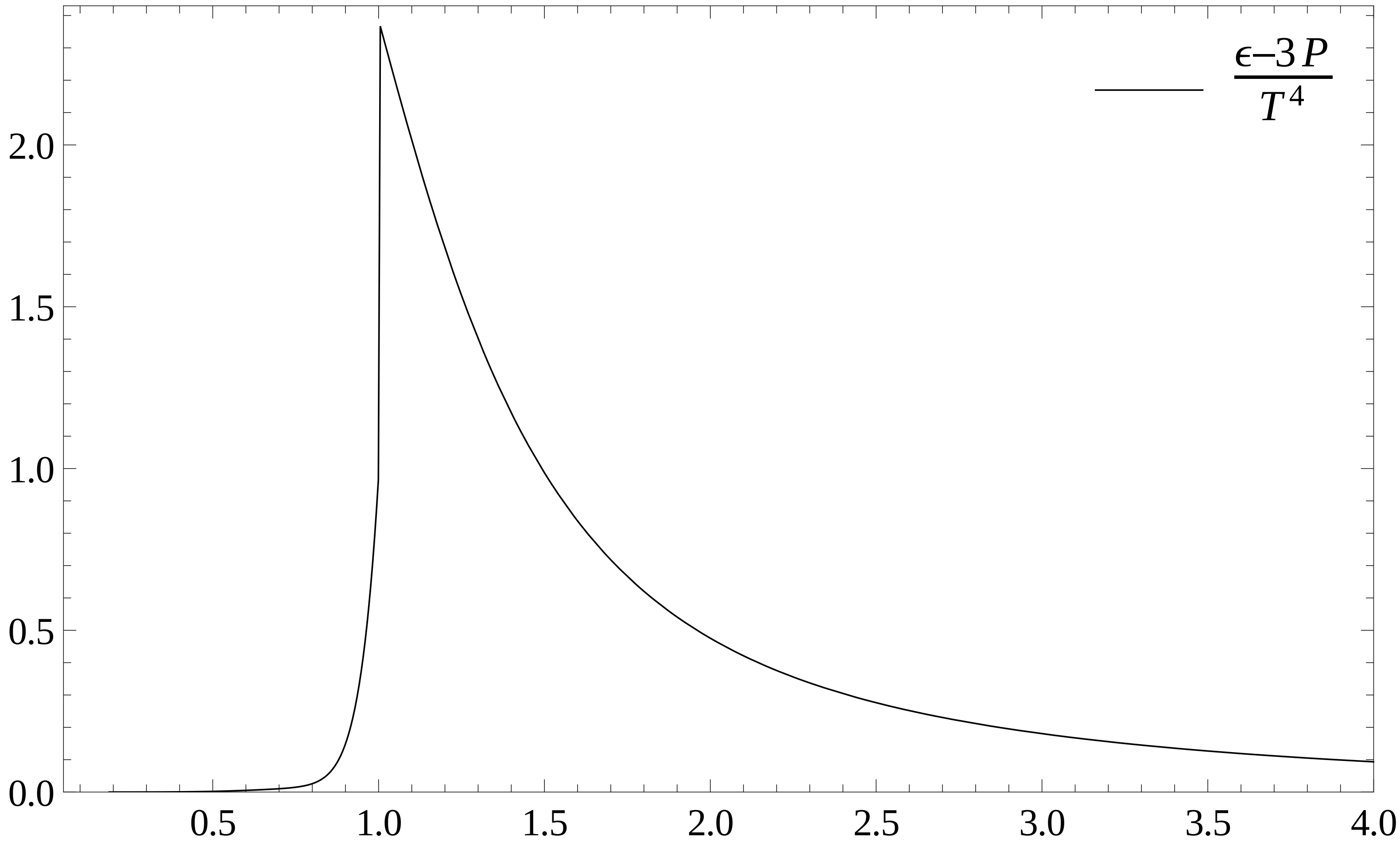


Figure 14: The trace anomaly $\epsilon(T) - 3P(T)$.

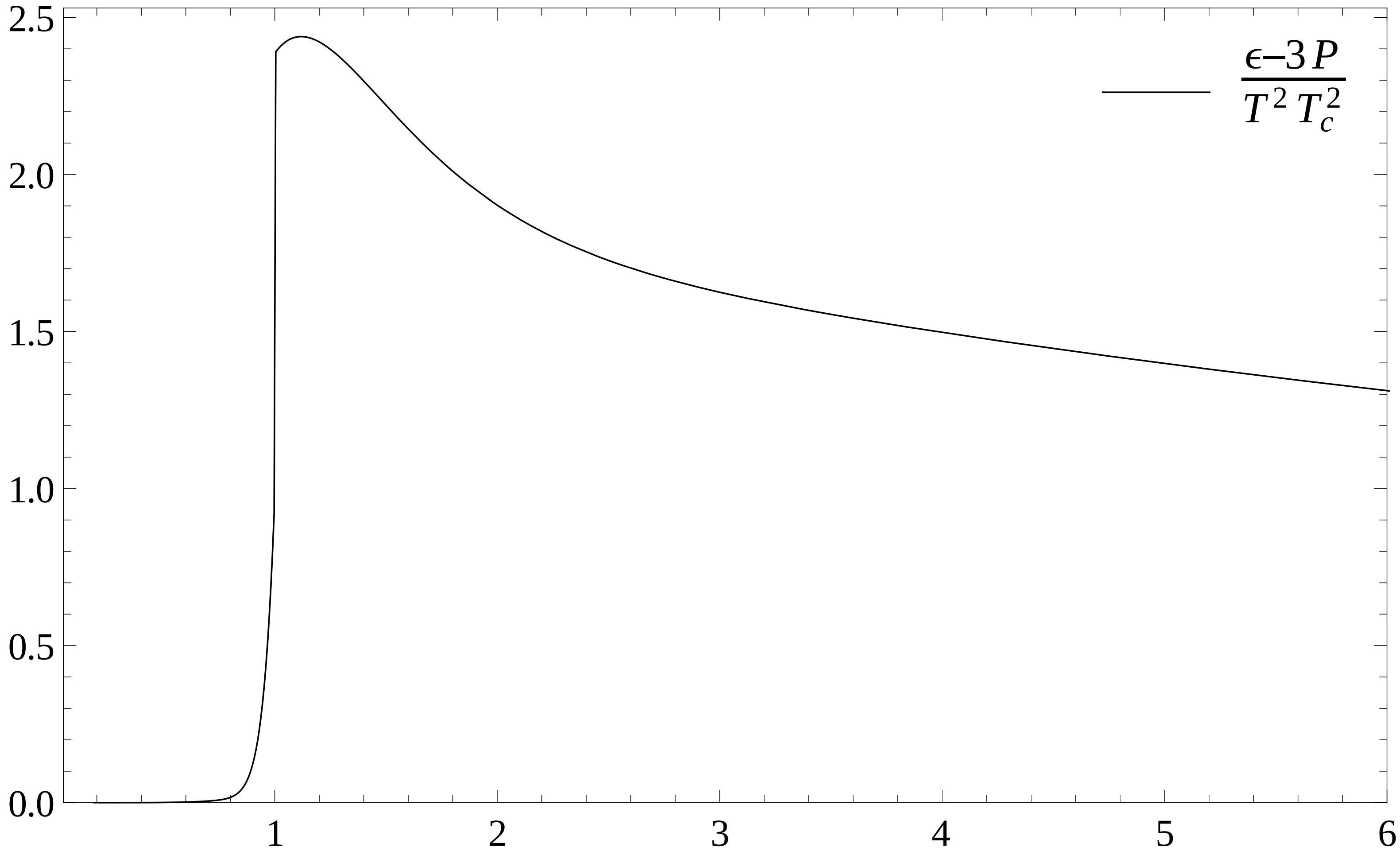


Figure 15: The trace anomaly as a function of T/T_c. And at high temperature $(\epsilon(T) - 3P(T))/T^2T_c^2 \sim 0.6\bar{B}_2\alpha_s + 0.9\bar{B}_3(T_c/T)$.

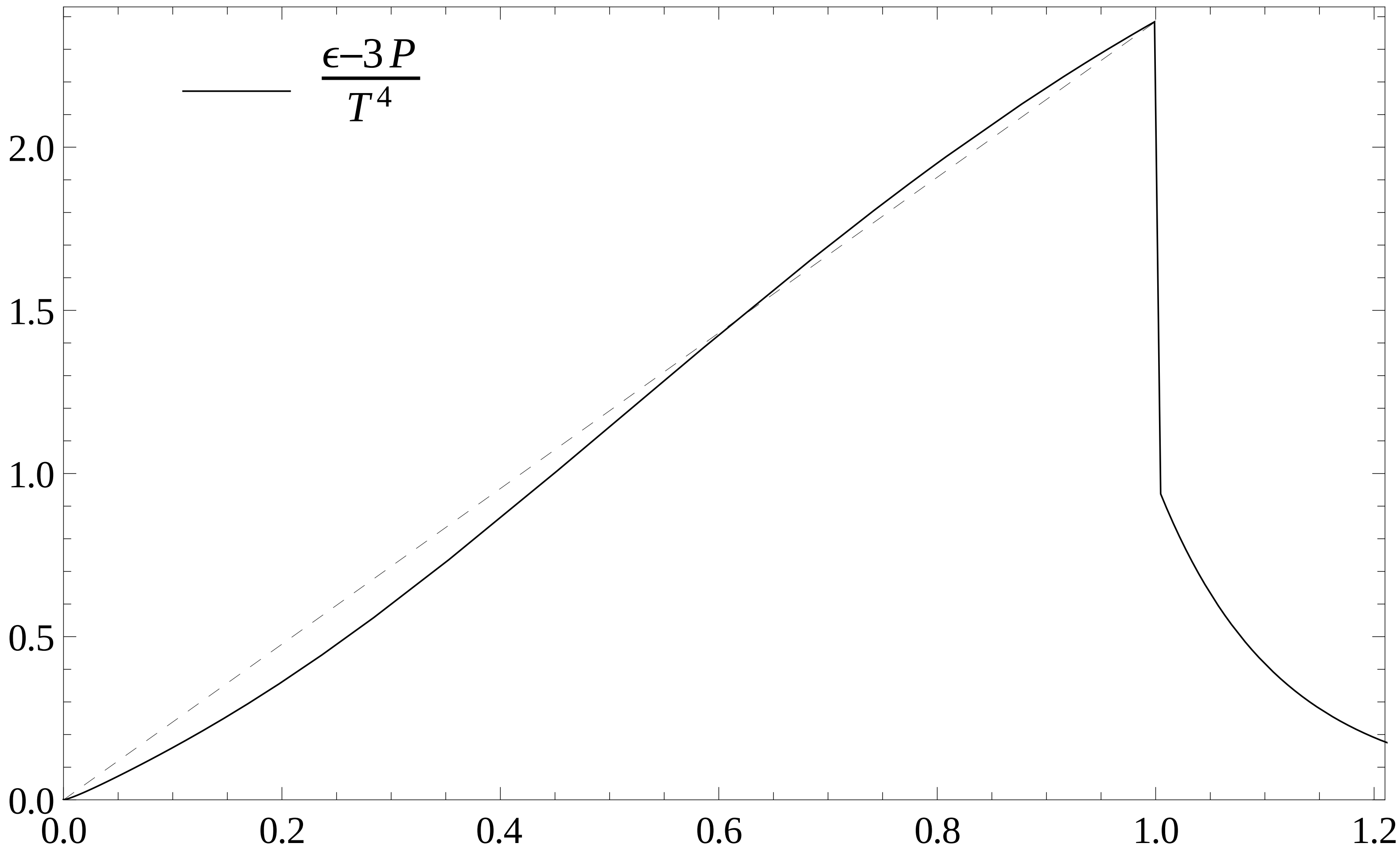


Figure 16: The trace anomaly $(\epsilon(T) - 3P(T))$ as a function of (T_c^2/T^2).

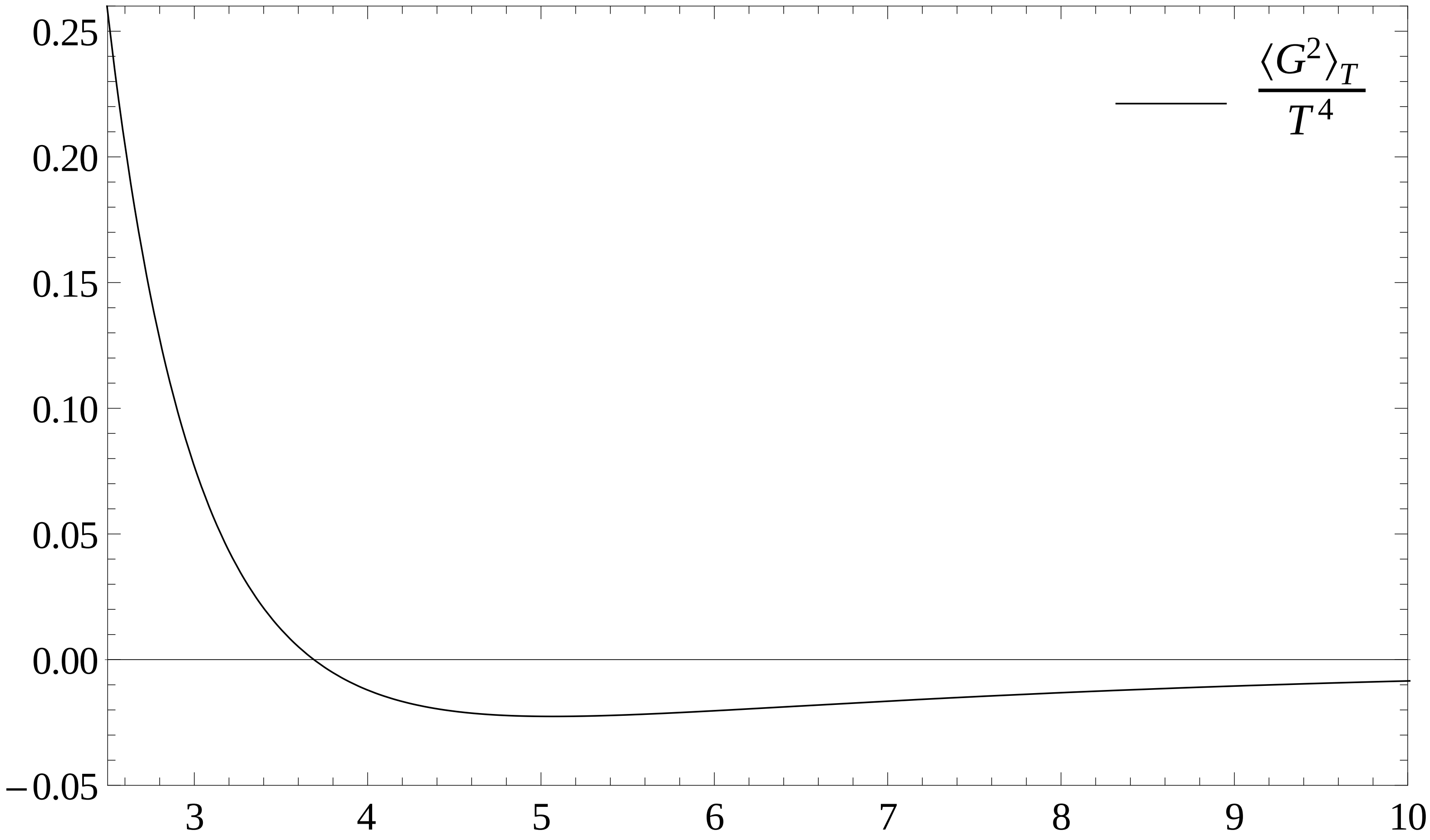


Figure 17: The gluon condensate $< G^2 >_T = < G^2 >_0 - [\epsilon(T) - 3P(T)]$ as a function of T/T_c.

The dynamical structure of the GP

1. $\omega' = \sqrt{\omega^2 + 3\Delta^2} = \sqrt{\omega^2 + m'^2_{eff}}, \quad m'_{eff} = 1.17\ GeV$

2. $\bar{\omega} = \sqrt{\omega^2 + \frac{3}{4}\Delta^2} = \sqrt{\omega^2 + \bar{m}^2_{eff}}, \quad \bar{m}_{eff} = 0.585\ GeV$

3. The NP massive excitation $\alpha_s \bar{\omega}$

4. The two NP massless excitations due to Δ^2

5. The two PT contributions, $P_{SB}(T)$ and $\alpha_s(T) \cdot P_{SB}(T)$

6. Very slow approach of all the independent thermodynamic quantities to their respective SB limits

7. While their ratios approach SB limits rather rapidly

8. Substantial deviation from lattice data for the trace anomaly due its special sensivity to the NP effects: the mass gap, confining β-function, etc.

$$P(T) = [1 - \alpha_s(T)]\, P_{SB}(T) - 0.3 P_g(T), \quad T > T_c,$$

$$P_g(T) \sim B_2 \alpha_s \Delta^2 T^2 + [B_3 \Delta^3 + M^3] T, \quad T \gg T_c,$$

V. Gogokhia, M. Vasuth, arXiv:1007.1573v2

V. Gogokhia, M. Vasuth, arXiv:1012.4157v6

Quark Confinement

I. Necessary condition $S(p) \neq Z_2/(\hat{p} - m_{phys})$,

i.e., quarks are always off mass-shell objects

II. Sufficient condition
Discrete spectrum (no continuum) in bound states

Quark Confinement is absolute and permanent
Deconfinement phase transition does not exist
What is known as
the Deconfinement phase transition is, in fact,
the Dehadronization phase transition

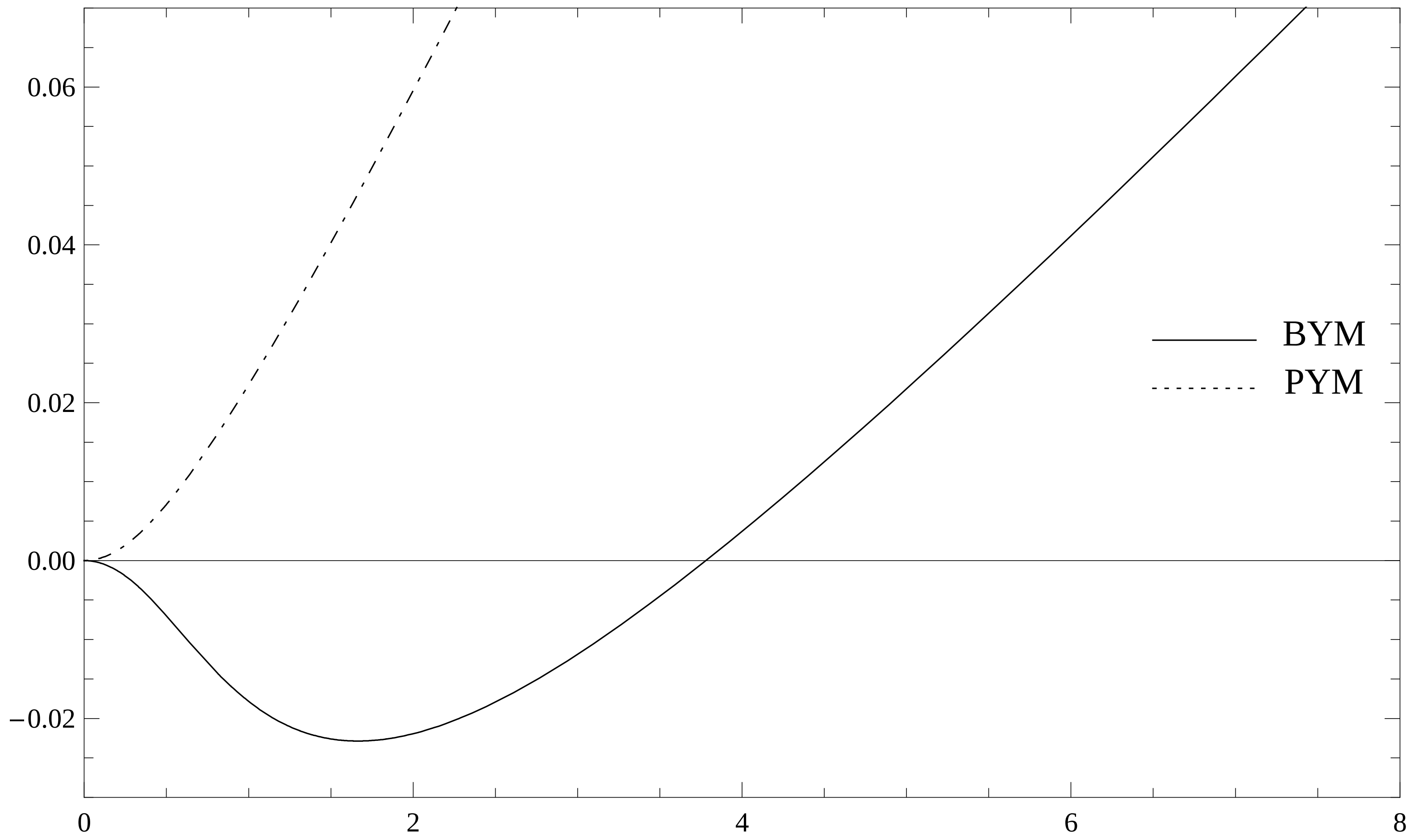


Figure 18: The Bag constant and the NP YM part as a functions of T/T_c. The Bag constant at $T/T_c = 3.75$ is zero, so that up to this value the Bag constant is responsible for the NP vacuum contributions to the pressure.