Zimányi Winter School, Budapest, 3-7 December, 2012

# Integrable aspects of AdS/CFT


## Z. Bajnok

*MTA-Lendület Holographic QFT Group*

*Wigner Research Centre for Physics, Budapest*

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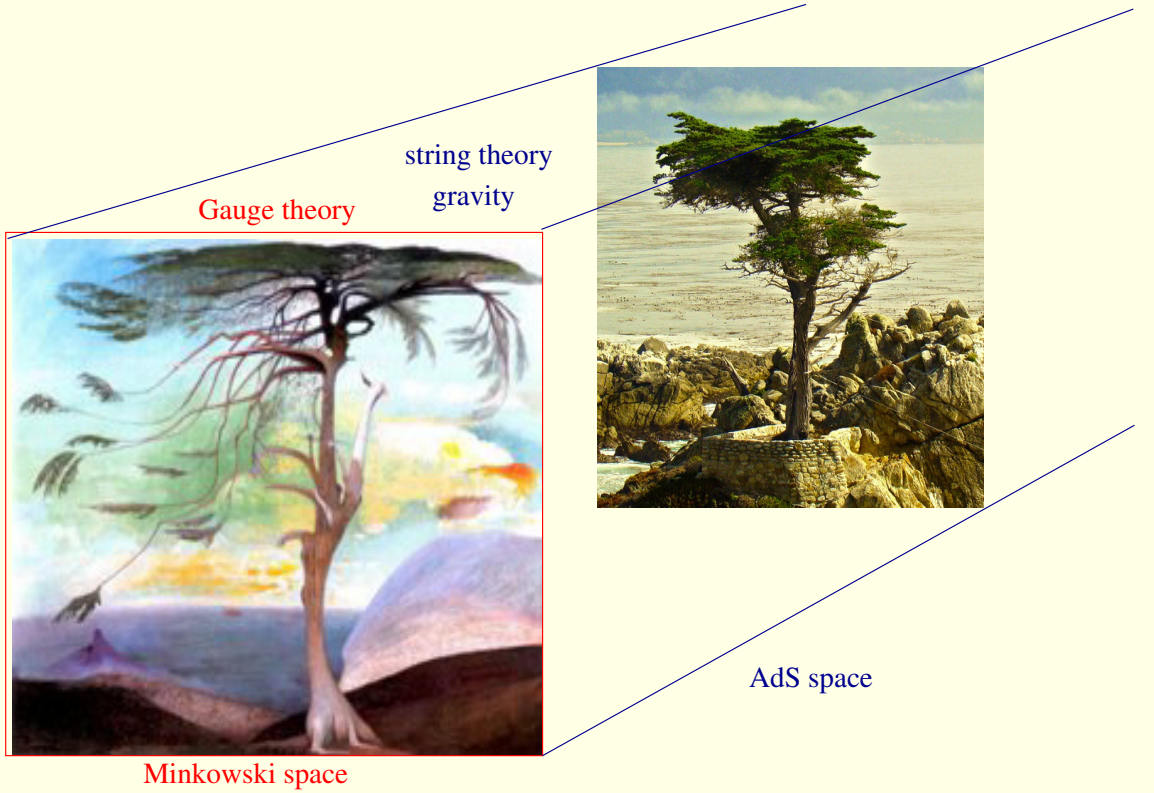
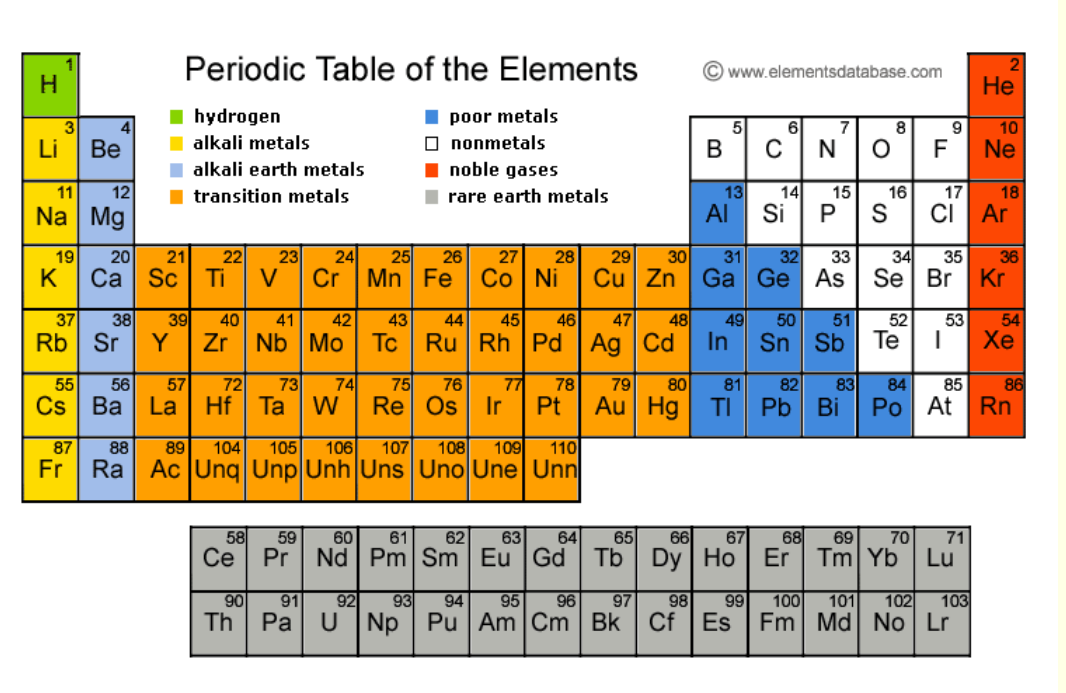
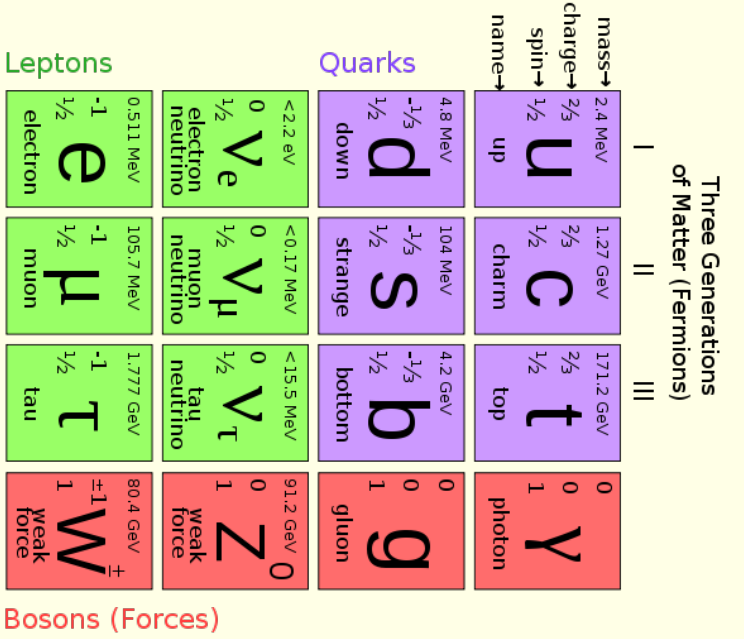



AdS/CFT correspondencce $\subset$ gauge/gravity duality

# Motivation: Organizing matter



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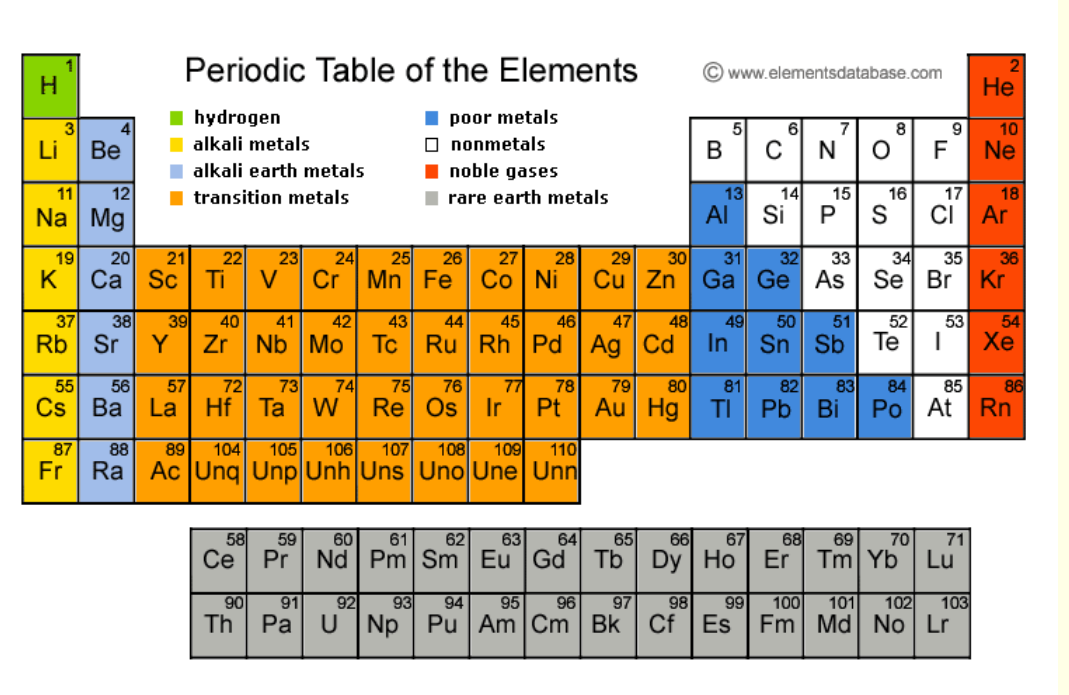
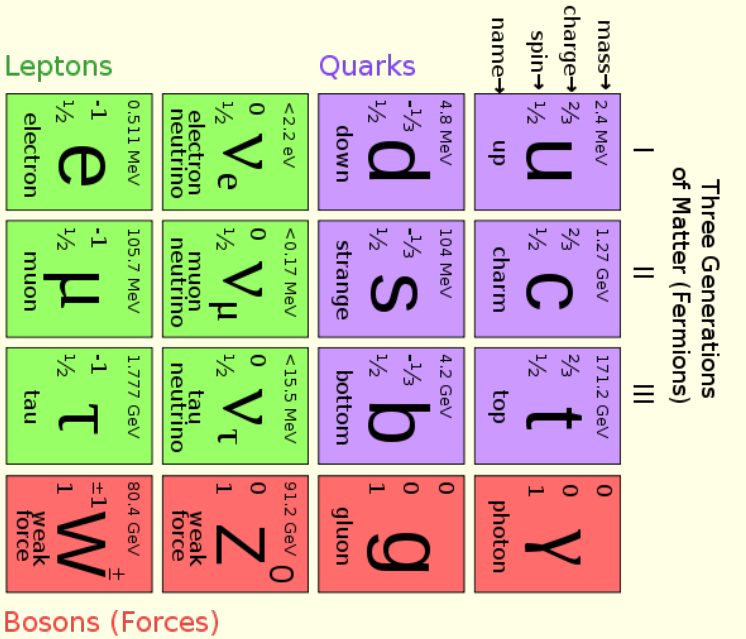
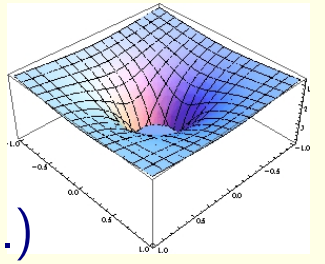


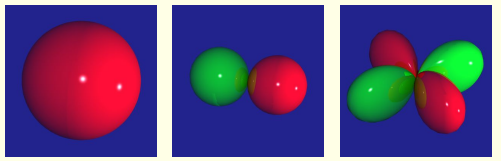




Electric interaction
(potential $\Phi(r) = k\frac{Zq}{r}$)



Quantum mechanics (Schrödinger eq.)

$$H\Psi = (-\frac{(\hbar\nabla)^2}{2m} + \Phi(r))\Psi = E\,\Psi$$



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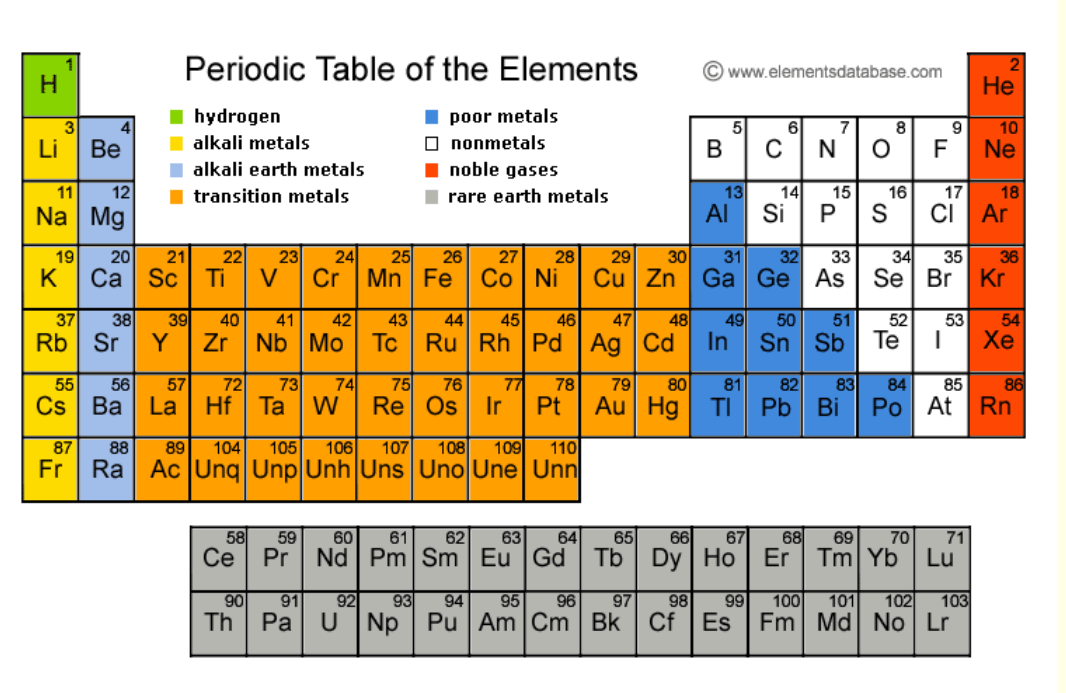


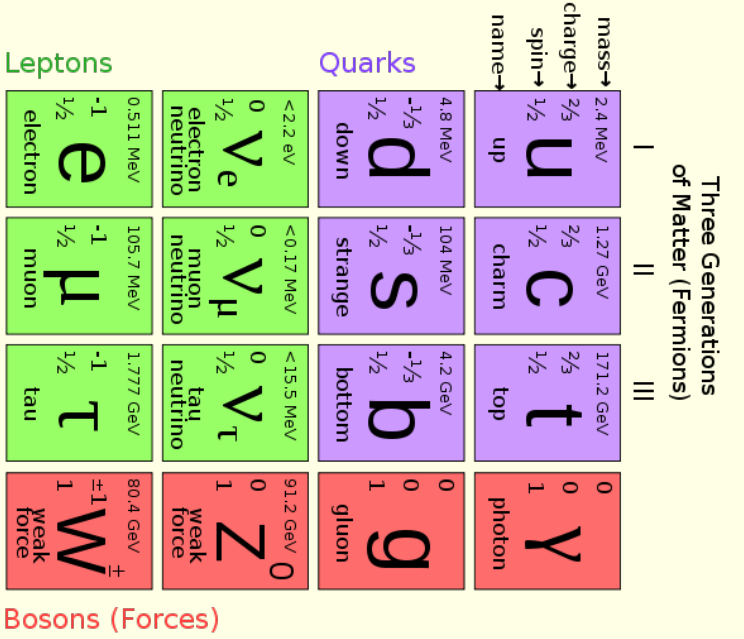


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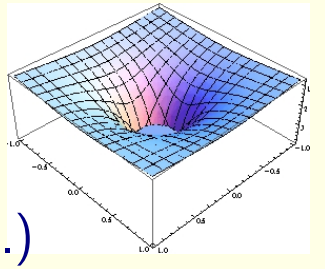

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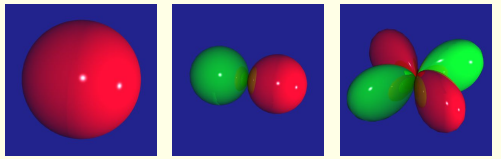
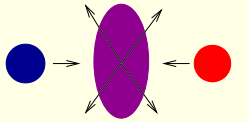


# Heavy ion collision

Brookhaven: Relativistic heavy ion collider (gold ion)



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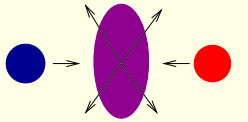

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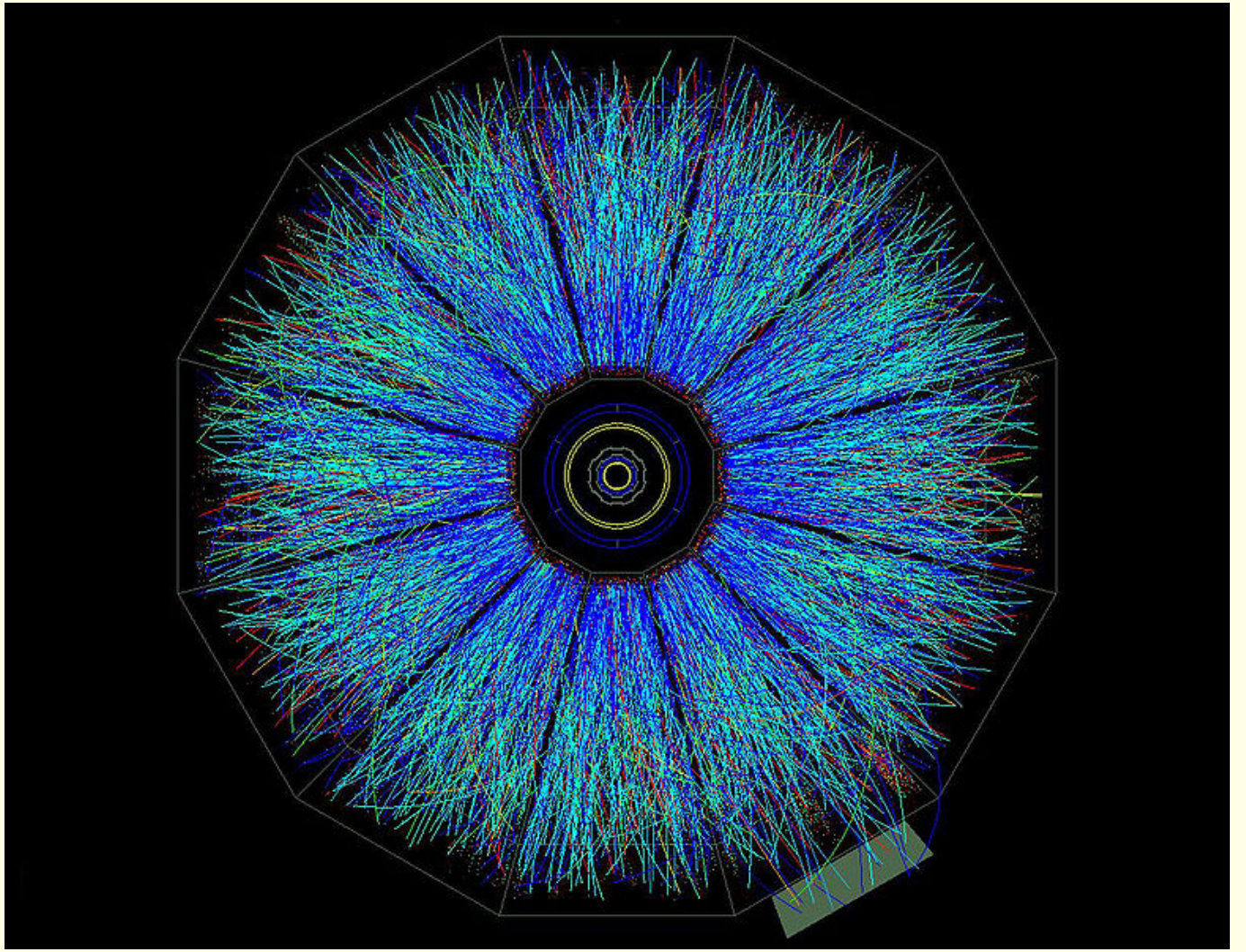

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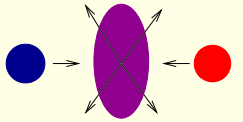

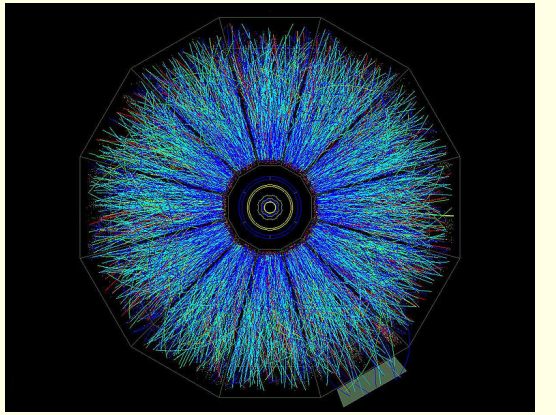

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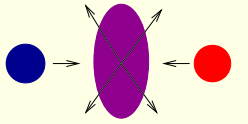
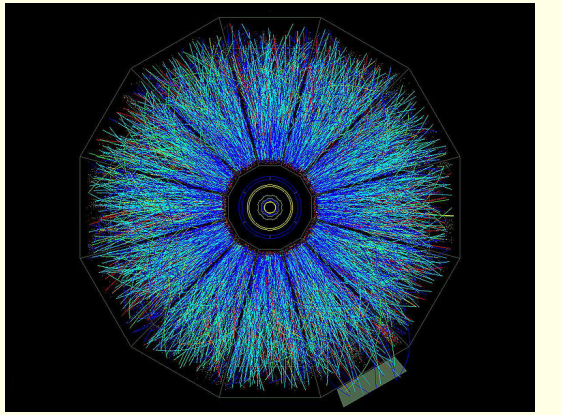
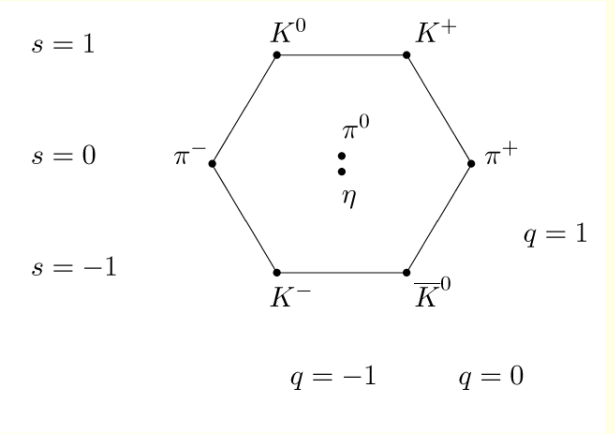




## Brookhaven: Relativistic heavy ion collider (gold ion)

## Meson spectrum



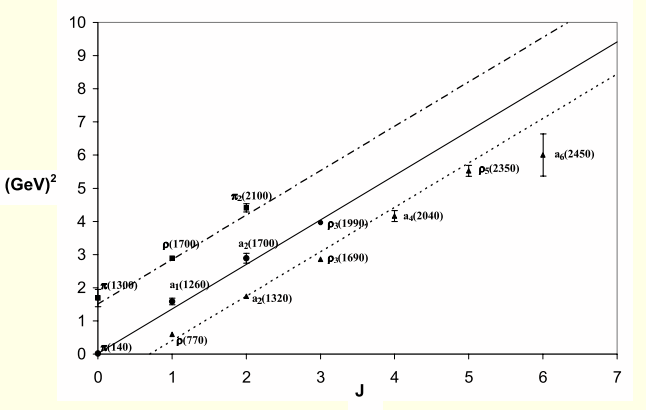
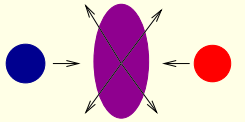
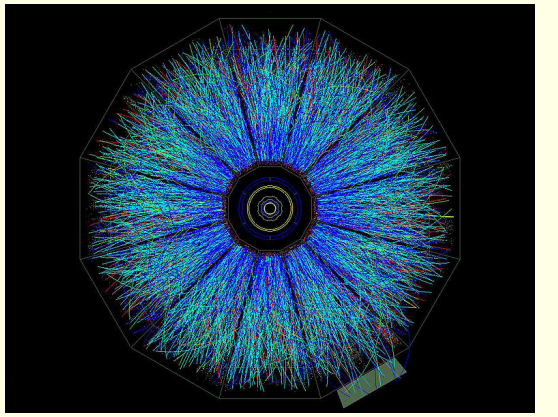




Fig. 1.

## Heavy ion collision





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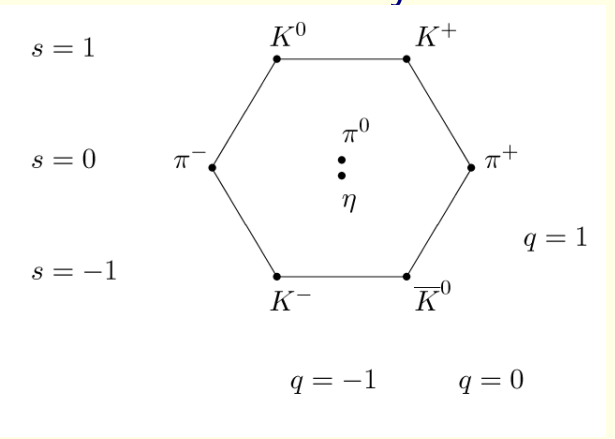


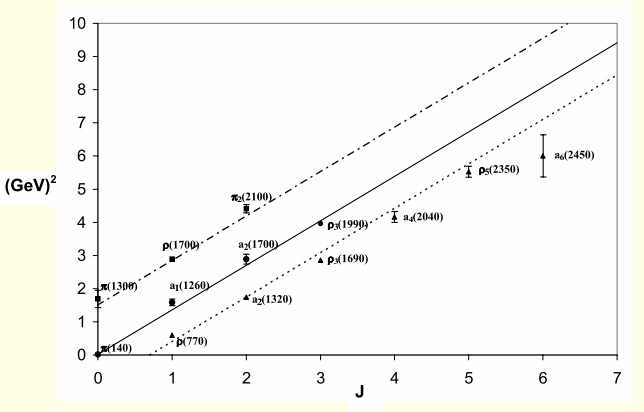




Fig. 1.

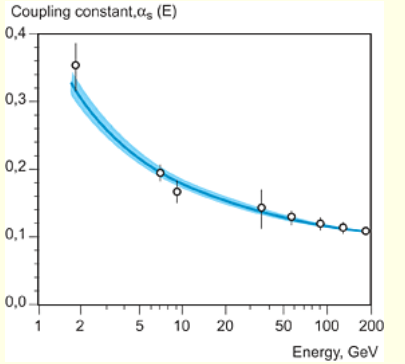
QCD= $SU(3)$ gauge theory: $G_\mu^{1..8}$ gluon $\rightarrow F_{\mu\nu}^{1..8}$, $\Psi_{kvark}^{123}$

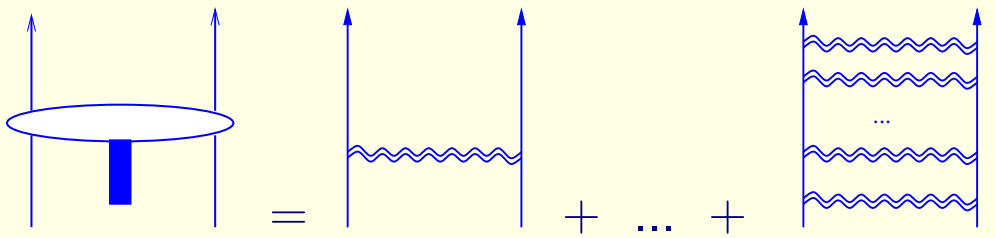
$$\mathcal{L} = -\tfrac{1}{4}F^2 + \bar{\Psi}(i\partial\!\!\!/ - m)\Psi - g\bar{\Psi}G\!\!\!/\Psi$$

non-perturbative: $\frac{\alpha_s}{4\pi} = O(1)$

running coupl.: $\beta(\alpha_s) = \mu\frac{\partial\alpha_s}{\partial\mu} < 0$

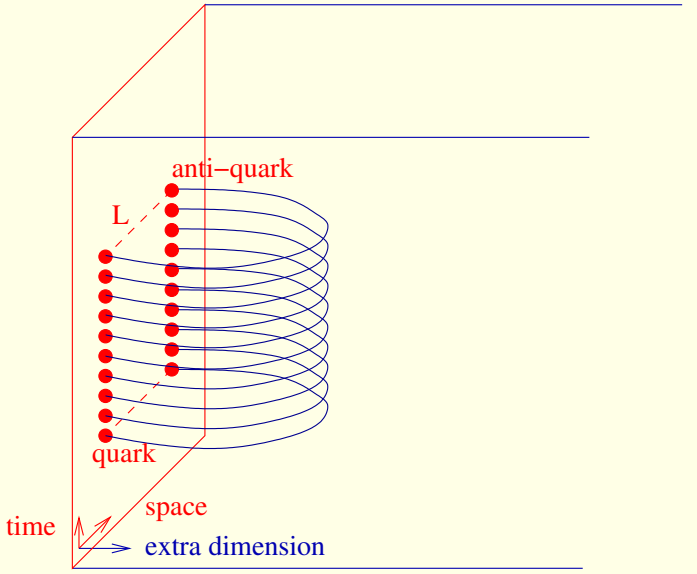
asymptotic freedom, confinement



# AdS/CFT correspondence: other explanation

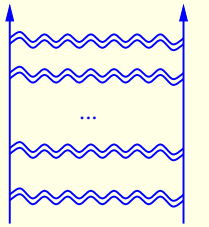
Minimal surface




≡

quark-antiquark potential

non-perturbative



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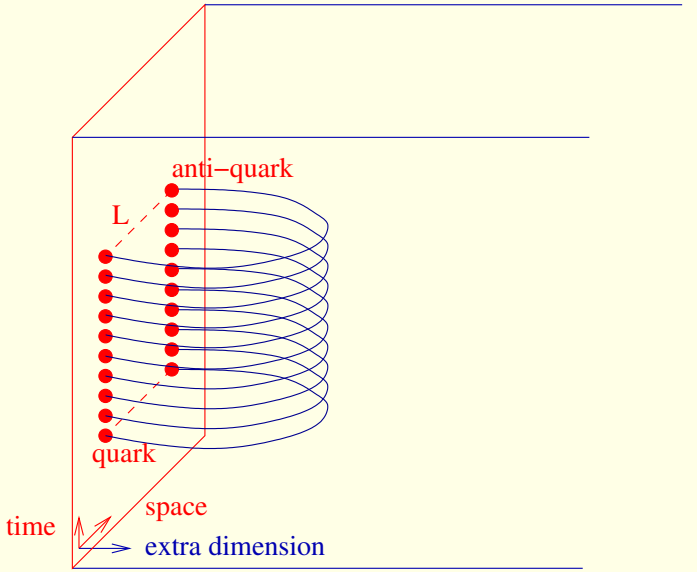


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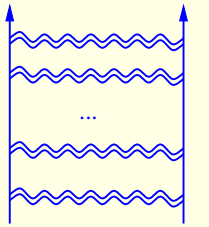
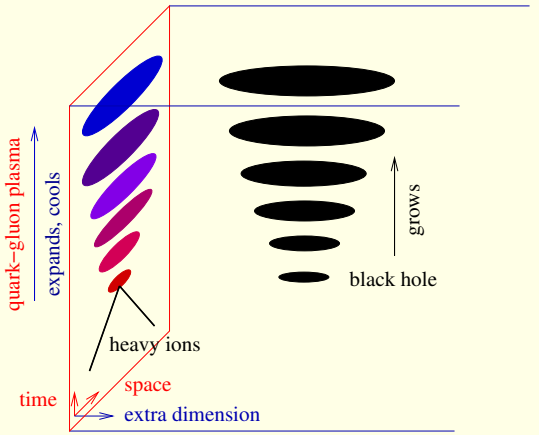
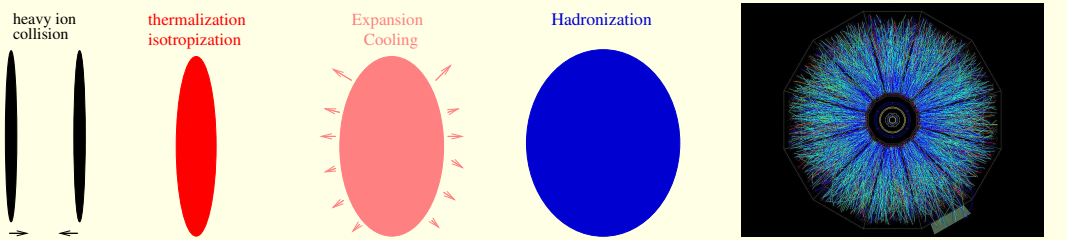



## growing black hole




≡

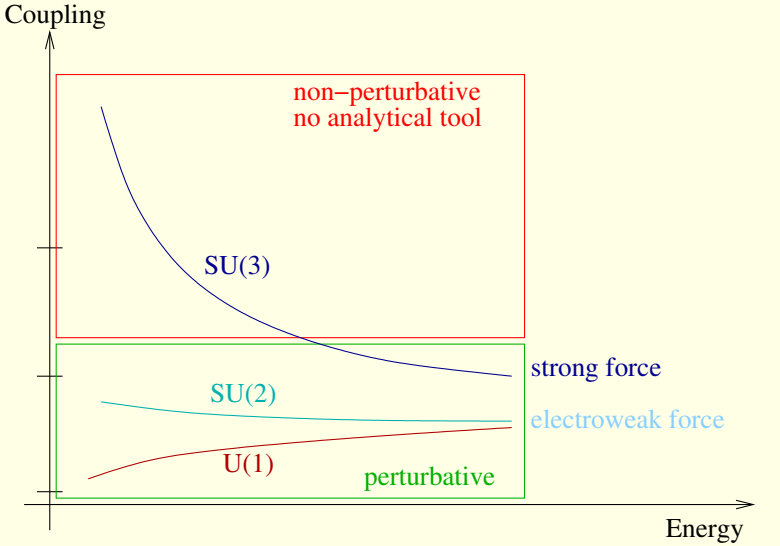
## Heavy ion collision: expansion



# QCD $\sim$ maximally supersymmetric gauge theory = CFT

Fundamental interactions

| interaction | particles | gauge theory |
|---|---|---|
| electromagnetic | photon+electron | $U(1)$ |
| electroweak | $W^{\pm}, Z$ $\mu$,$\nu$+Higgs | $SU(2) \times U(1)$ |
| strong | gluon+quarks | $SU(3)$ |

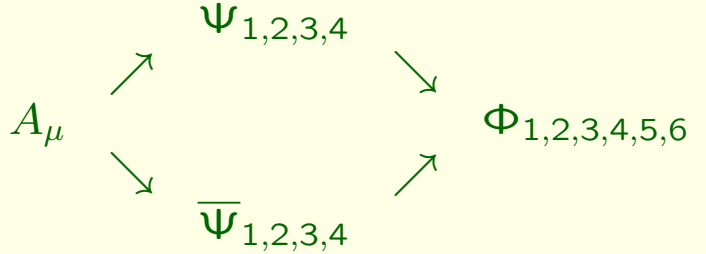



only analytical tool: perturbation theory

maximally supersymmetric ($\mathcal{N}$=4) gauge theory

| interaction | particles | gauge theory |
|---|---|---|
| $\mathcal{N} = 4$ supersymm. | gluon+quarks+scalars | $SU(N)$ |

all fields $N^2 - 1$ component matrix



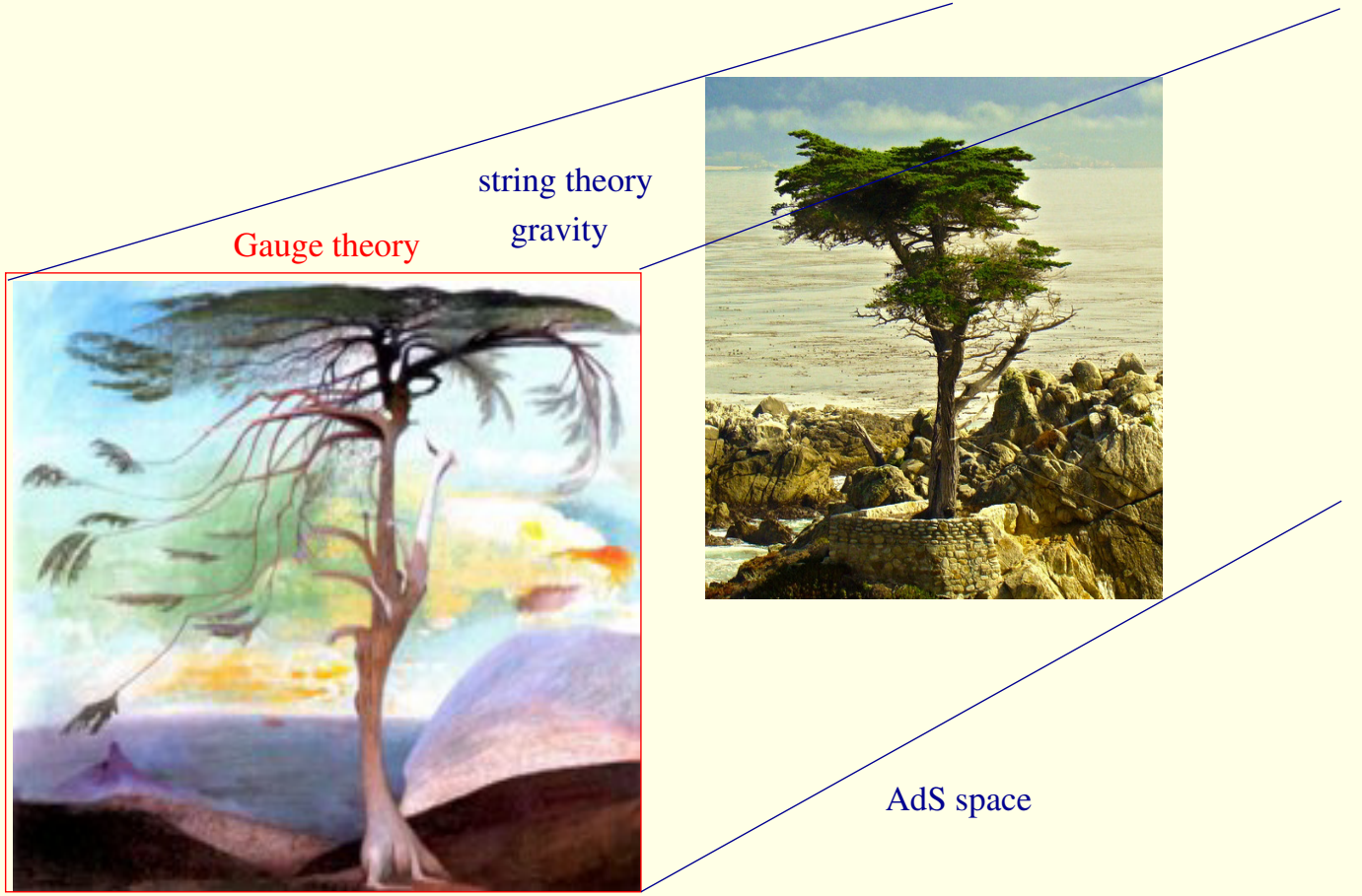

$$\mathcal{L} = \frac{2}{g_{YM}^2} \int d^4x \mathrm{Tr}\left[-\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\overline{\Psi}\not{D}\Psi + V\right]$$

$$V(\Phi,\Psi) = \frac{1}{4}[\Phi,\Phi]^2 + \overline{\Psi}[\Phi,\Psi]$$

| no running $\beta = 0 \rightarrow$ CFT |
|---|
| no confinement |
| supersymmetric |

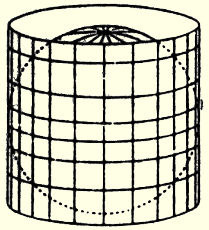
| heavy ion collision: |
|---|
| finite T $\rightarrow$ SUSY is broken |
| quark-gluon plasma is not confined |

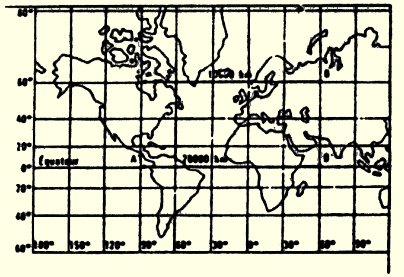
# AdS/CFT correspondence (Maldacena 1998)



# AdS: string theory on Anti de Sitter $\supset$ gravitation

positively curved space





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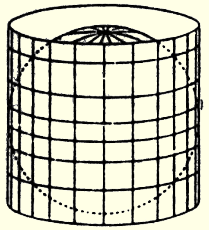

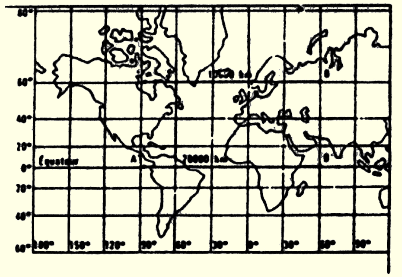

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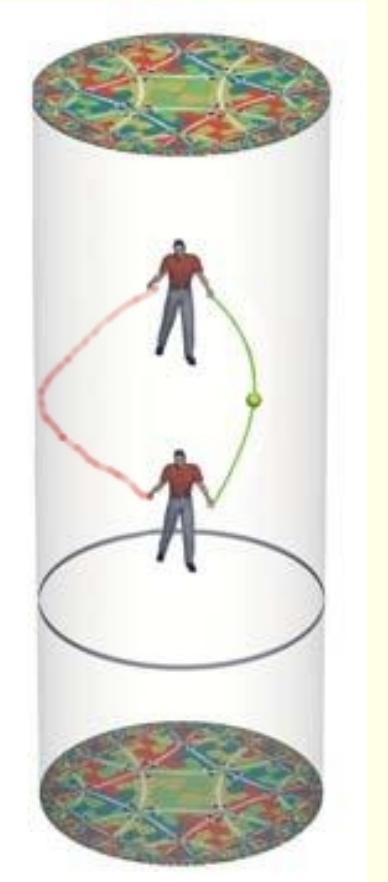

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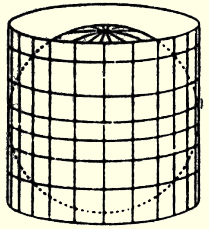

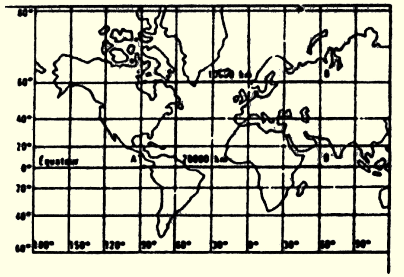

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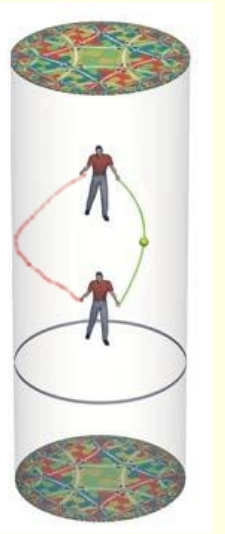

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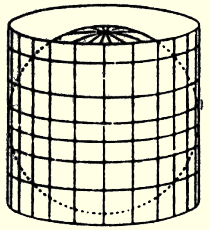

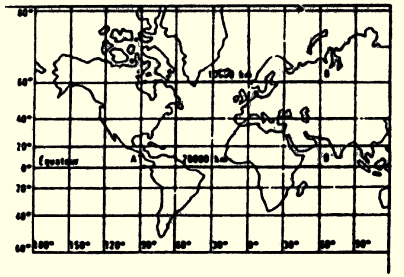

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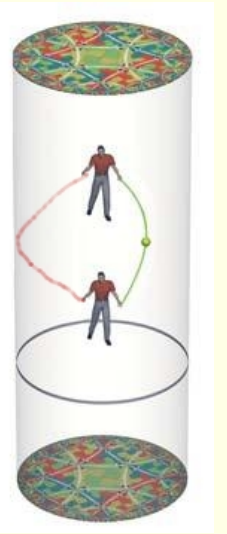
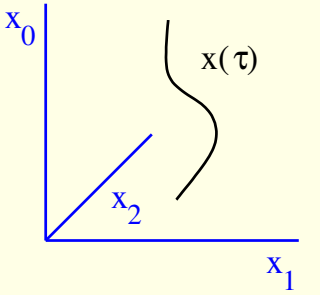


relativistic point particle: $ds^2 = -dx_0^2 + dx_1^2 + \ldots$

$S \propto$ worldline $\propto \int ds = \int \sqrt{\dot{x} \cdot \dot{x}} d\tau$



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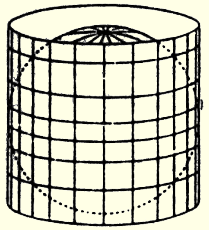

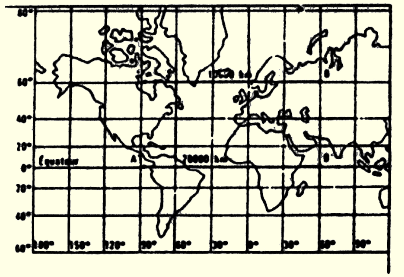

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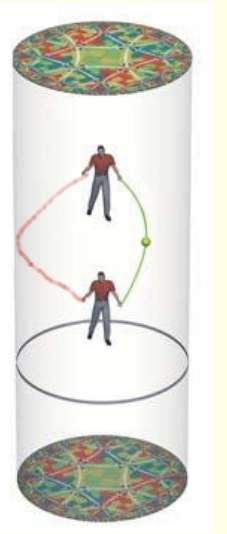

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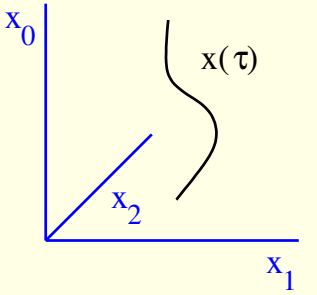
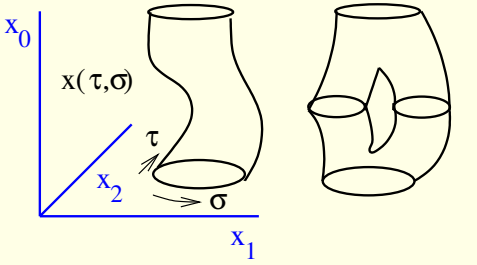


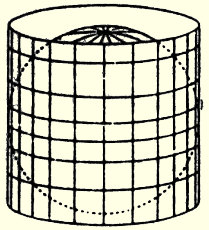

relativistic string: $ds^2 = -dx_0^2 + dx_1^2 + \ldots$

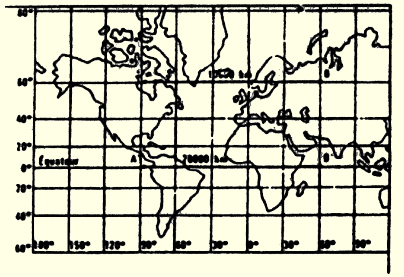
$S \propto$ worldsheet $\propto \int dA = \int \sqrt{(\dot{x} \cdot x')^2 - \dot{x}^2 x'^2} d\tau d\sigma$



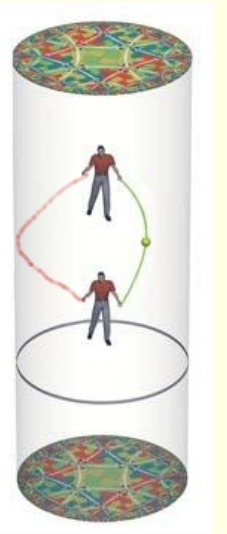
# AdS: string theory on Anti de Sitter $\supset$ gravitation

positively curved space



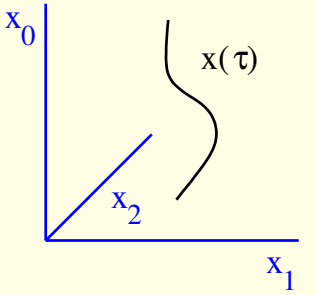


Anti de Sitter: negatively curved space



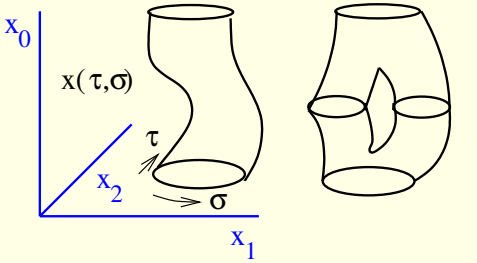
relativistic point particle: $ds^2 = -dx_0^2 + dx_1^2 + \ldots$

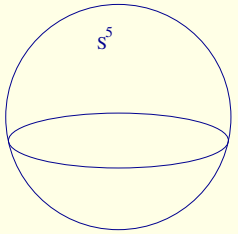
$S \propto$ worldline $\propto \int ds = \int \sqrt{\dot{x} \cdot \dot{x}} d\tau$




relativistic string: $ds^2 = -dx_0^2 + dx_1^2 + \ldots$

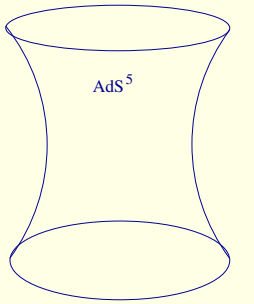
$S \propto$ worldsheet $\propto \int dA = \int \sqrt{(\dot{x} \cdot x')^2 - \dot{x}^2 x'^2} d\tau d\sigma$




$$S^5 : Y_0^2 + Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2 + Y_5^2 = R^2$$

$$AdS_5 : -X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 - X_5^2 = -R^2$$

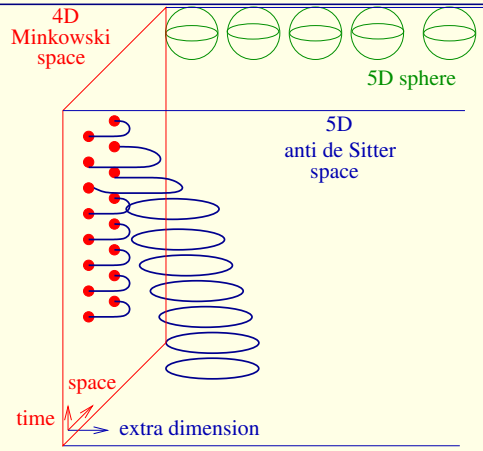



$$\boxed{S = \frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} \left( \partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M \right) + fermionok}$$

supercoset $\frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$

# AdS/CFT correspondence (Maldacena 1998)

$\mathrm{II}_B$ superstring on $AdS_5 \times S^5$




$\sum_1^6 Y_i^2 = R^2 \qquad -++++- = -R^2$

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$\equiv$

$\mathcal{N} = 4$ D=4 $SU(N)$ SYM

$\frac{2}{g_{YM}^2} \int d^4x \,\mathrm{Tr}\left[-\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\overline{\Psi}\not{D}\Psi + V\right]$

$V(\Phi,\Psi) = \frac{1}{4}[\Phi,\Phi]^2 + \overline{\Psi}[\Phi,\Psi]$

$\beta = 0$ superconformal $\frac{PSU(2,2|4)}{SO(5)\times SO(1,4)}$

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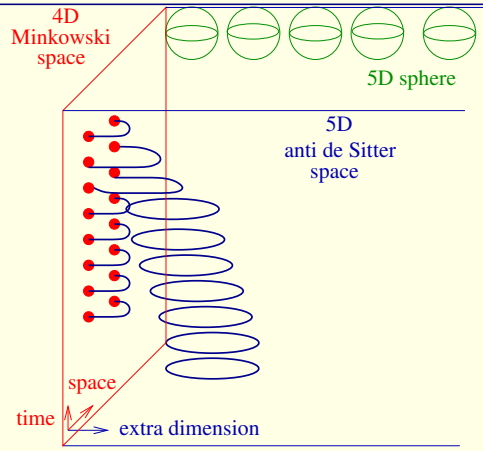


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## Dictionary

Couplings: $\sqrt{\lambda} = \frac{R^2}{\alpha'}$, $g_s = \frac{\lambda}{N} \to 0$

2D QFT

String energy levels: $E(\lambda)$

$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \ldots$

strong$\leftrightarrow$weak

$\Downarrow$

$\lambda = g_{YM}^2 N$ , $N \to \infty$ planar limit

$\langle \mathcal{O}_n(x)\mathcal{O}_m(0)\rangle = \frac{\delta_{nm}}{|x|^{2\Delta_n(\lambda)}}$

Anomalous dim $\Delta(\lambda)$

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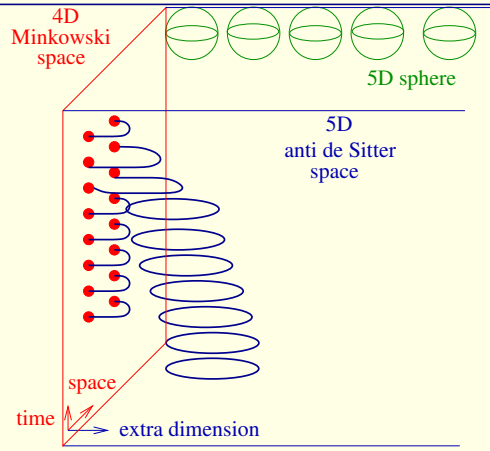


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## 2D integrable QFT

spectrum: $Q = 1, 2, \ldots, \infty$ dispersion: $\epsilon_Q(p) = \sqrt{Q^2 + \frac{\lambda}{\pi^2}\sin^2\frac{p}{2}}$

Exact scattering matrix: $S_{Q_1Q_2}(p_1, p_2, \lambda)$

# AdS/CFT correspondence: confirmation

supersymmetric BPS operators

$$V(\Phi,\Psi)=\tfrac{1}{4}[\Phi,\Phi]^2+\overline{\Psi}[\Phi,\Psi]$$

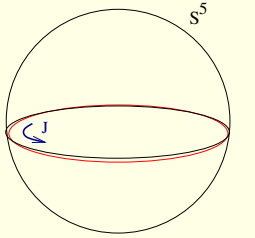
$$Z=\Phi_1+i\Phi_2, X=\Phi_3+i\Phi_4$$

$$\mathcal{O}_{BPS}=\mathrm{Tr}(Z^J)\leftrightarrow|\uparrow\uparrow\ldots\uparrow\rangle$$

$$\Delta_{BPS}=J$$

weak↔strong

BPS string configuration

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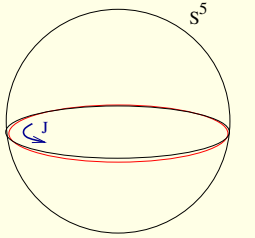


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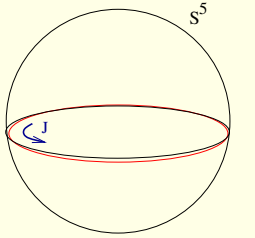


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Nontrivial anomalous dimension

supersymmetric theory: Excited state

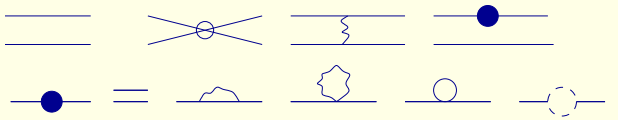
$\mathcal{O}_K=\mathrm{Tr}(ZXZX+...)\leftrightarrow|\uparrow\downarrow\uparrow\downarrow\rangle+.$

# Confirmation: excited state - Konishi operator

nonsupersymmetric operator: Konishi

$\mathcal{O}_K = \mathrm{Tr}(ZXZX + ...) \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle + .$

operator mixing



$\Delta(\lambda) = \Delta(0) + \lambda\Delta_1 + \ldots + \lambda^4\Delta_4 +$

[Fiamberti ..'08]

$\Delta_4 = -2496 + 576\zeta_3 - 1440\zeta_5$

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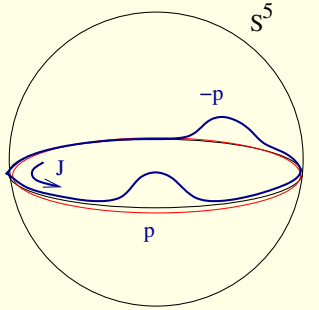


moving bumps (sine-Gordon) [Hofman .. '07]

string action=saddle point+loop corr.

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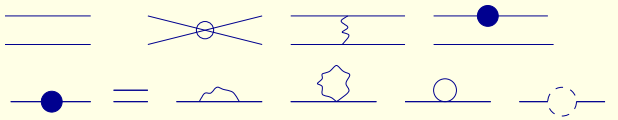

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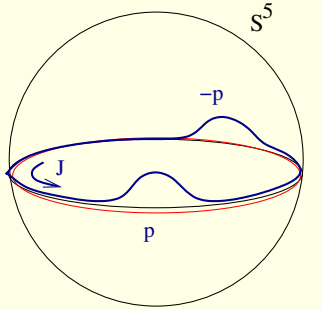


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two particle state

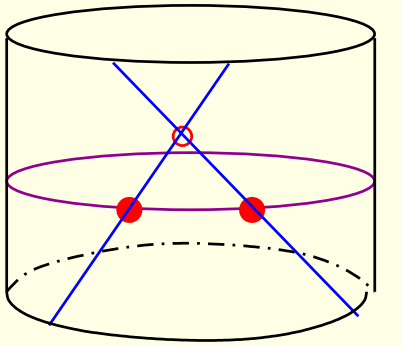
$E = E_{BA} + E_{FSC}$

Bethe Ansatz: $e^{ipJ}S(p,-p) = 1$

$E_{BA} = 2E(p,\lambda) = 2\sqrt{1 + \frac{\lambda}{\pi^2}(\sin\frac{p}{2})^2}$

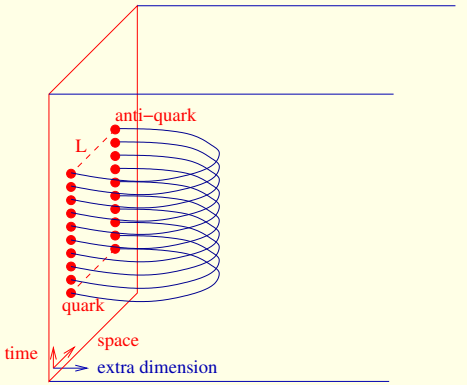
$E_{FSC} = \sum_Q \int \frac{dq}{2\pi} S_{Q1}(q,p) S_{Q1}(q,-p) e^{-\epsilon_Q L}$

$E_4 = \Delta_4 = -2496 + 576\zeta_3 - 1440\zeta_5$ [Z.B., R. Janik '09]



# AdS/CFT correspondence: applications

Minimal surface




exact for strong coupling $\lambda \to \infty$

$\equiv$

quark-antiquark potential

Wilson loop: $\langle \oint_C A_\mu dx^\mu \rangle$

non-perturbative

$$V(r) = -\frac{4\pi^2\sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4}\frac{1}{r}$$

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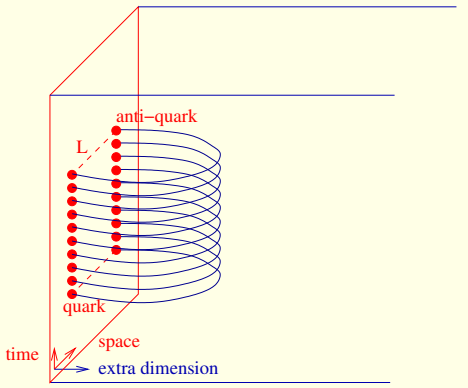


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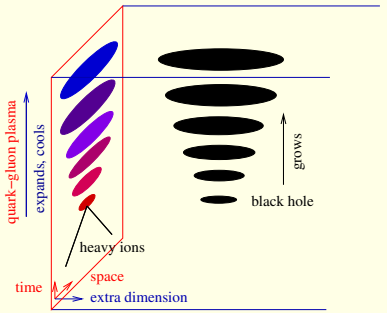
## growing black hole




metric $\delta g(x,0) \propto \langle T_{\mu\nu} \rangle$

$$ds^2 = \frac{1}{z^2}(g(x,z)_{\mu\nu}dx^\mu dx^\nu + dz^2)$$

Einstein equation

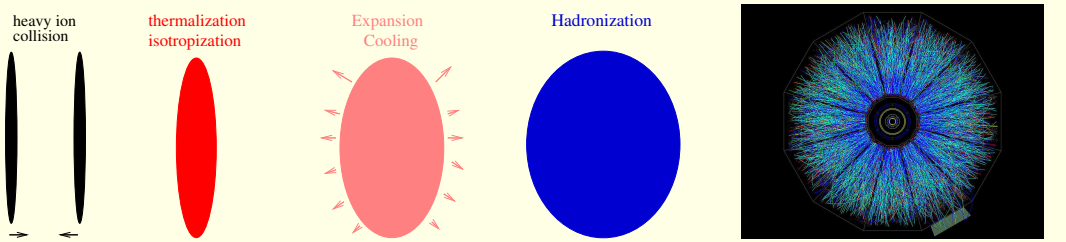
$$R_{ab} - \frac{1}{2}g_{ab}R - 6g_{ab} = 0$$

growing black hole

$$g_{tt} = -\frac{(1-z^4/z_0^4)^2}{(1+z^4/z_0^4)^2}\ ;\ g_{xx} = 1 + \frac{z^4}{z_0^4}$$

$\equiv$

## Heavy ion collision: expansion




$\langle T_{\mu\nu} \rangle$ matter distribution

relativistic hydrodynamics

$\partial_\mu T^{\mu\nu} = 0$ and $T^\mu_\mu = 0$

viscous quark-gluon plasma

expansion in time: perfect fluid + $\frac{\eta}{s} = \frac{1}{4\pi}$ +...

# Conclusion= Trust the AdS/CFT correspondence!