

Zimányi Winter School, Budapest, 3-7 December, 2012

Integrable aspects of AdS/CFT

Z. Bajnok

MTA-Lendület Holographic QFT Group

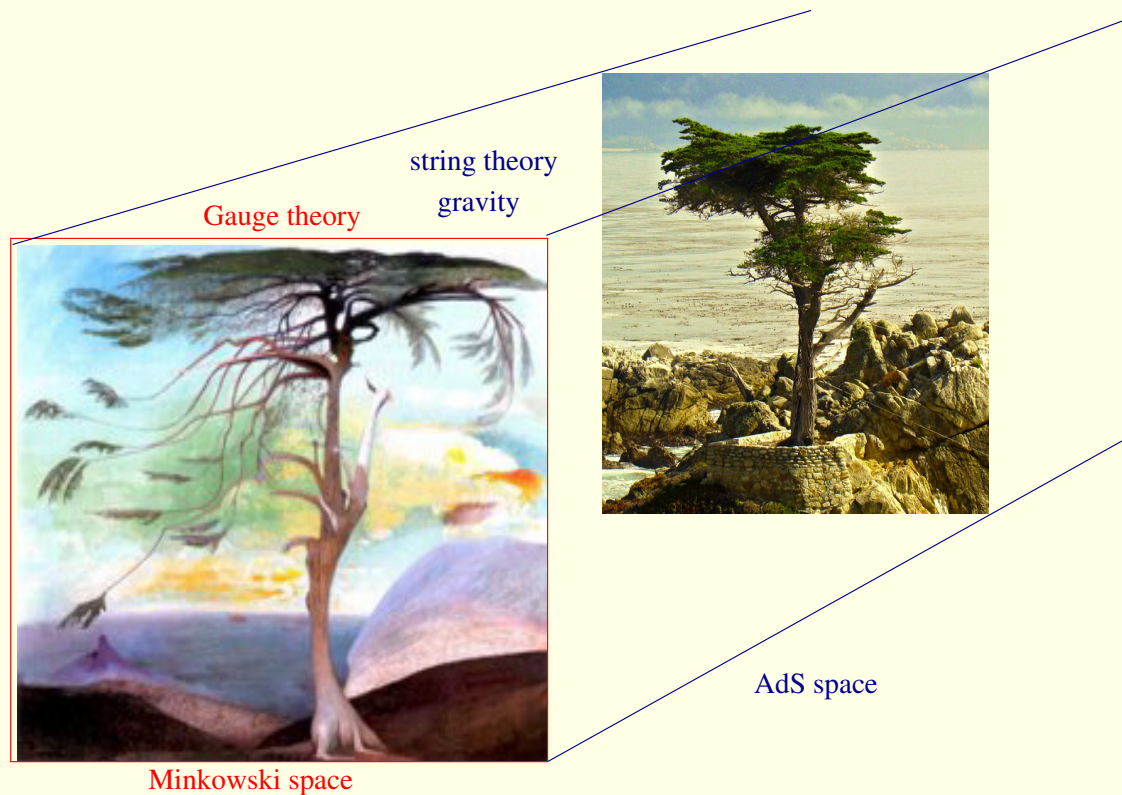
Wigner Research Centre for Physics, Budapest

Integrable aspects of AdS/CFT

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AdS/CFT correspondence \subset gauge/gravity duality

Motivation: Organizing matter

Periodic Table of the Elements © www.elementsdatabase.com

- hydrogen
- alkali metals
- alkali earth metals
- transition metals
- poor metals
- nonmetals
- noble gases
- rare earth metals

1																	2
3	4											5	6	7	8	9	10
11	12											13	14	15	16	17	18
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
55	56	57	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86
87	88	89	104	105	106	107	108	109	110								
		58	59	60	61	62	63	64	65	66	67	68	69	70	71		
		90	91	92	93	94	95	96	97	98	99	100	101	102	103		

Leptons		Quarks		Bosons (Forces)	
electron e 0.511 MeV -1 1/2	electron neutrino ν_e <2.2 eV 0 1/2	down d 4.8 MeV -1/3 1/2	up u 2.4 MeV 2/3 1/2	weak force W 80.4 GeV ±1 1	weak force Z 91.2 GeV 0 1
muon μ 105.7 MeV -1 1/2	muon neutrino ν_μ <0.17 MeV 0 1/2	strange s 104 MeV -1/3 1/2	charm c 1.27 GeV 2/3 1/2		
tau τ 1.777 GeV -1 1/2	tau neutrino ν_τ <15.5 MeV 0 1/2	bottom b 4.2 GeV -1/3 1/2	top t 171.2 GeV 2/3 1/2		
		gluon g 0 0 1	photon γ 0 0 1		
				I II III	

↑ mass
↑ charge
↑ spin
↑ name

Three Generations of Matter (Fermions)

Motivation: Organizing matter

Periodic Table of the Elements © www.elementsdatabase.com

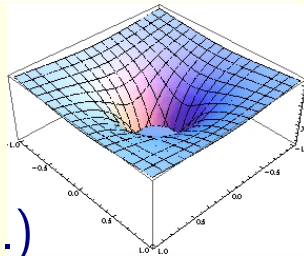
<ul style="list-style-type: none"> ■ hydrogen ■ alkali metals ■ alkali earth metals ■ transition metals ■ poor metals nonmetals ■ noble gases ■ rare earth metals 										<table border="1" style="width: 100%; text-align: center;"> <tr><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>B</td><td>C</td><td>N</td><td>O</td><td>F</td><td>Ne</td></tr> <tr><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td></tr> <tr><td>Al</td><td>Si</td><td>P</td><td>S</td><td>Cl</td><td>Ar</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td></tr> <tr><td>Ga</td><td>Ge</td><td>As</td><td>Se</td><td>Br</td><td>Kr</td></tr> <tr><td>49</td><td>50</td><td>51</td><td>52</td><td>53</td><td>54</td></tr> <tr><td>In</td><td>Sn</td><td>Sb</td><td>Te</td><td>I</td><td>Xe</td></tr> <tr><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td><td>86</td></tr> <tr><td>Tl</td><td>Pb</td><td>Bi</td><td>Po</td><td>At</td><td>Rn</td></tr> <tr><td>87</td><td>88</td><td>89</td><td>104</td><td>105</td><td>106</td><td>107</td><td>108</td><td>109</td><td>110</td></tr> <tr><td>Fr</td><td>Ra</td><td>Ac</td><td>Unq</td><td>Unp</td><td>Unh</td><td>Uns</td><td>Uno</td><td>Une</td><td>Unn</td></tr> </table>										5	6	7	8	9	10	B	C	N	O	F	Ne	13	14	15	16	17	18	Al	Si	P	S	Cl	Ar	31	32	33	34	35	36	Ga	Ge	As	Se	Br	Kr	49	50	51	52	53	54	In	Sn	Sb	Te	I	Xe	81	82	83	84	85	86	Tl	Pb	Bi	Po	At	Rn	87	88	89	104	105	106	107	108	109	110	Fr	Ra	Ac	Unq	Unp	Unh	Uns	Uno	Une	Unn
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Leptons		Quarks		Bosons (Forces)	
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tau τ 1.777 GeV $-\frac{1}{2}$	tau neutrino ν_τ <15.5 MeV 0	bottom b 4.2 GeV $-\frac{1}{3}$	top t 171.2 GeV $\frac{2}{3}$		

Three Generations of Matter (Fermions)

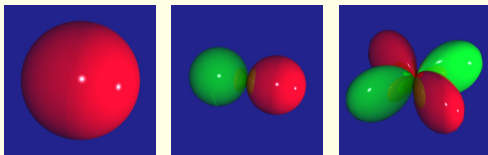
Electric interaction

(potential $\Phi(r) = k \frac{Zq}{r}$)



Quantum mechanics (Schrödinger eq.)

$$H\Psi = \left(-\frac{(\hbar\nabla)^2}{2m} + \Phi(r)\right)\Psi = E\Psi$$



Motivation: Organizing matter

Periodic Table of the Elements © www.elementsdatabase.com

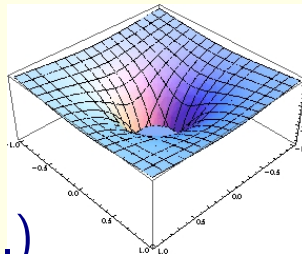
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K	Ca	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe								
Rb	Sr	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn								
Cs	Ba	Ac	Unq	Unp	Unh	Uns	Uno	Une	Unn																
Fr	Ra											Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu
										Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr		

Leptons		Quarks		Bosons (Forces)	
name	spin	charge	mass	name	spin
electron e	$1/2$	-1	0.511 MeV	down d	$1/2$
electron neutrino ν_e	$1/2$	0	< 2.2 eV	up u	$1/2$
muon μ	$1/2$	-1	105.7 MeV	strange s	$1/2$
muon neutrino ν_μ	$1/2$	0	< 0.17 MeV	charm c	$1/2$
tau τ	$1/2$	-1	1.777 GeV	bottom b	$1/2$
tau neutrino ν_τ	$1/2$	0	< 15.5 MeV	top t	$1/2$
weak force W	1	± 1	80.4 GeV	gluon g	1
weak force Z	1	0	91.2 GeV	photon γ	1

Three Generations of Matter (Fermions)

Electric interaction

$$\text{(potential } \Phi(r) = k \frac{Zq}{r} \text{)}$$



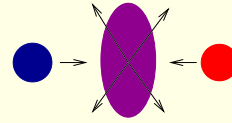
Quantum mechanics (Schrödinger eq.)

$$H\Psi = \left(-\frac{(\hbar\nabla)^2}{2m} + \Phi(r) \right) \Psi = E\Psi$$



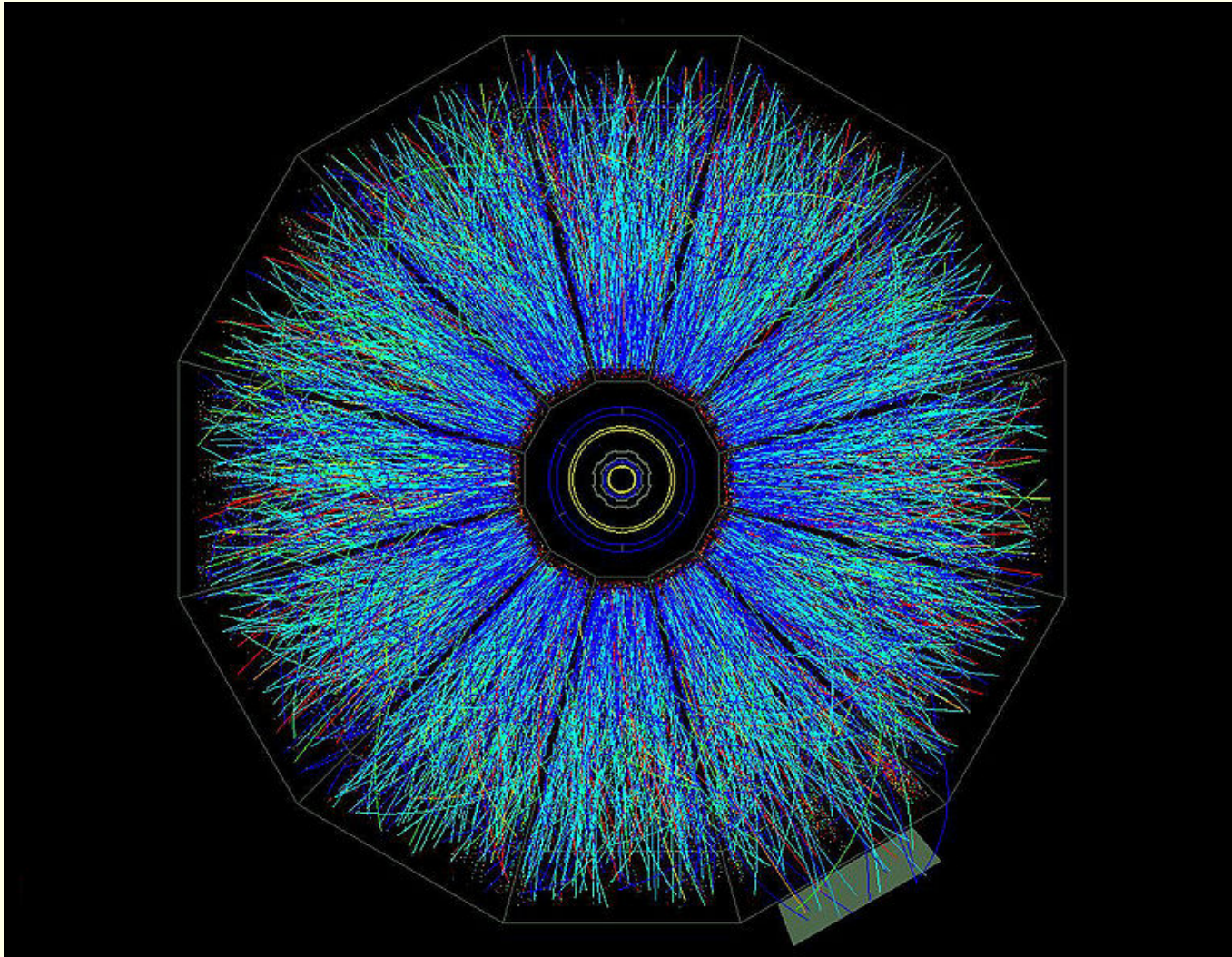
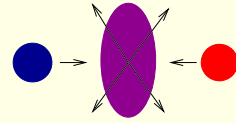
Heavy ion collision

Brookhaven: Relativistic heavy ion collider (gold ion)



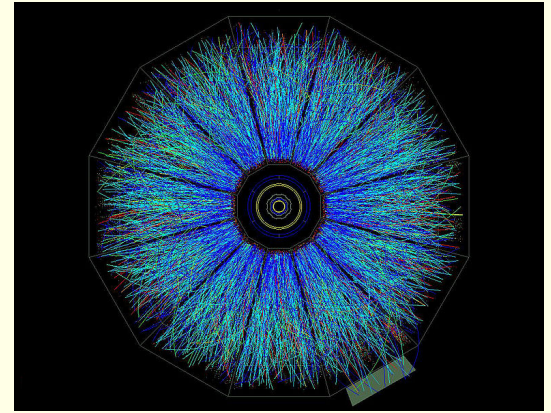
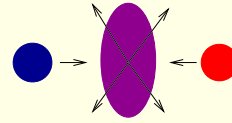
Heavy ion collision

Brookhaven: Relativistic heavy ion collider (gold ion)



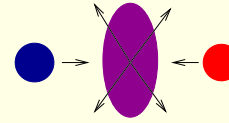
Heavy ion collision

Brookhaven: Relativistic heavy ion collider (gold ion)



Heavy ion collision

Brookhaven: Relativistic heavy ion collider (gold ion)



Meson spectrum

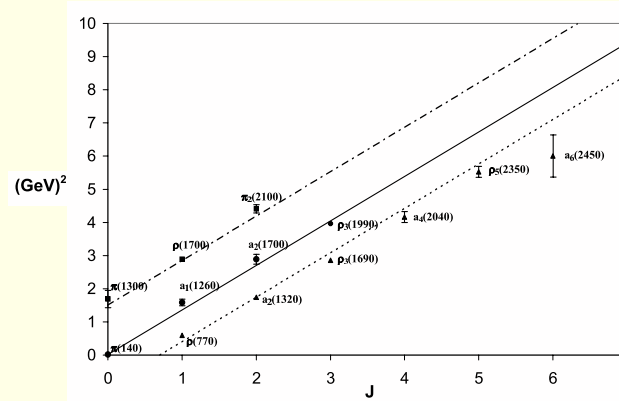
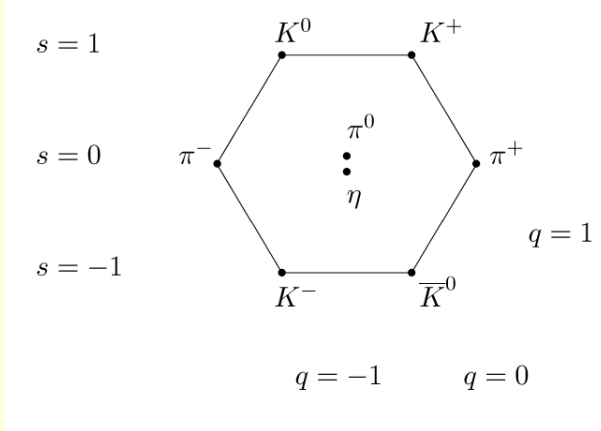
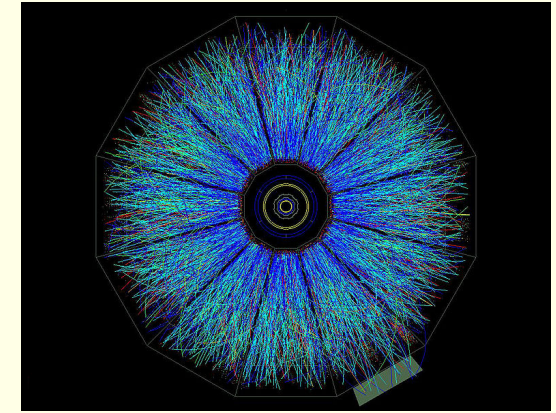


Fig. 1.



Heavy ion collision

Brookhaven: Relativistic heavy ion collider (gold ion)

Meson spectrum

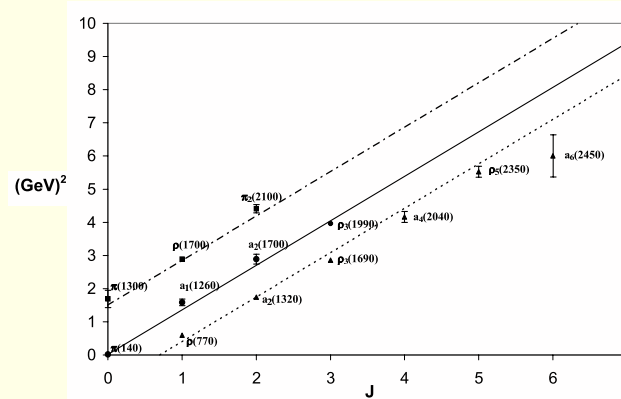
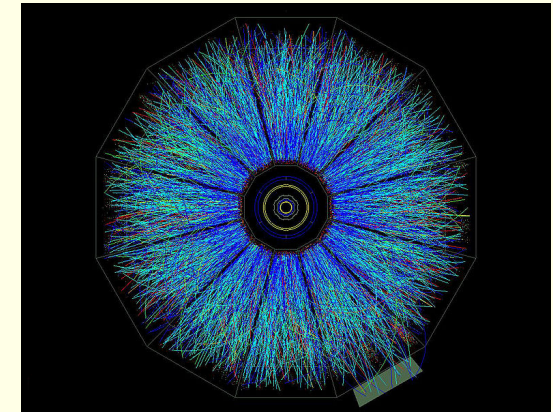
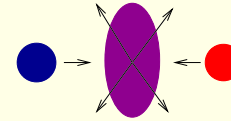
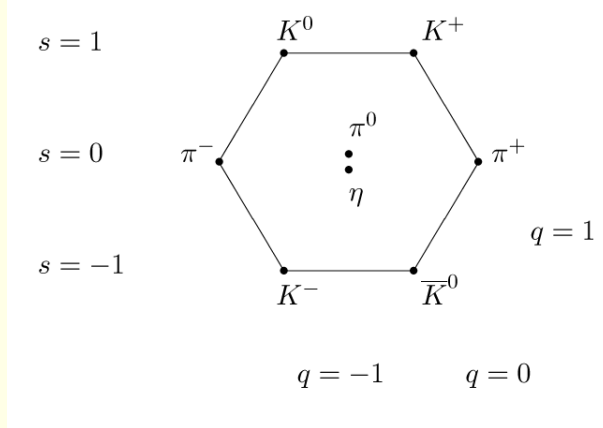


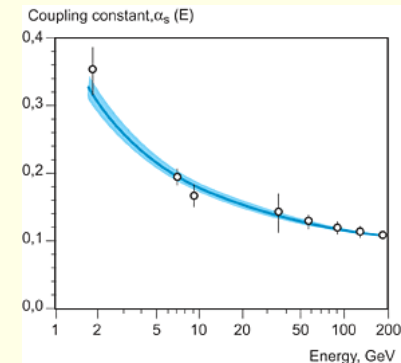
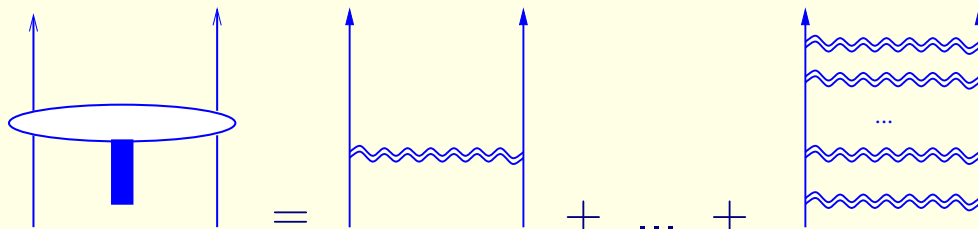
Fig. 1.

QCD = $SU(3)$ gauge theory: $G_{\mu}^{1..8}$ gluon $\rightarrow F_{\mu\nu}^{1..8}$, Ψ_{kvarik}^{123}

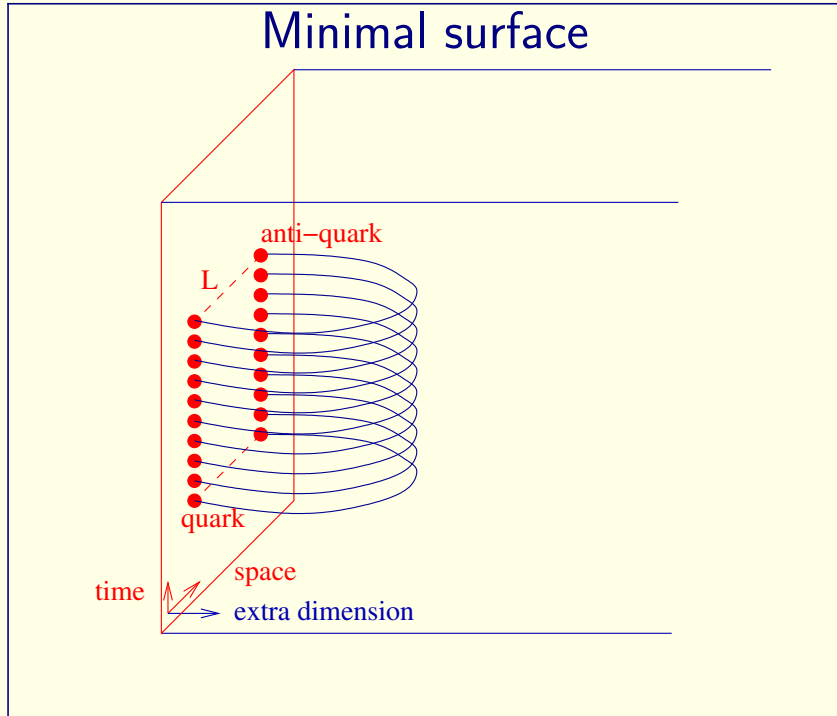
$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\Psi}(i\partial - m)\Psi - g\bar{\Psi}G\Psi$$

non-perturbative: $\frac{\alpha_s}{4\pi} = O(1)$

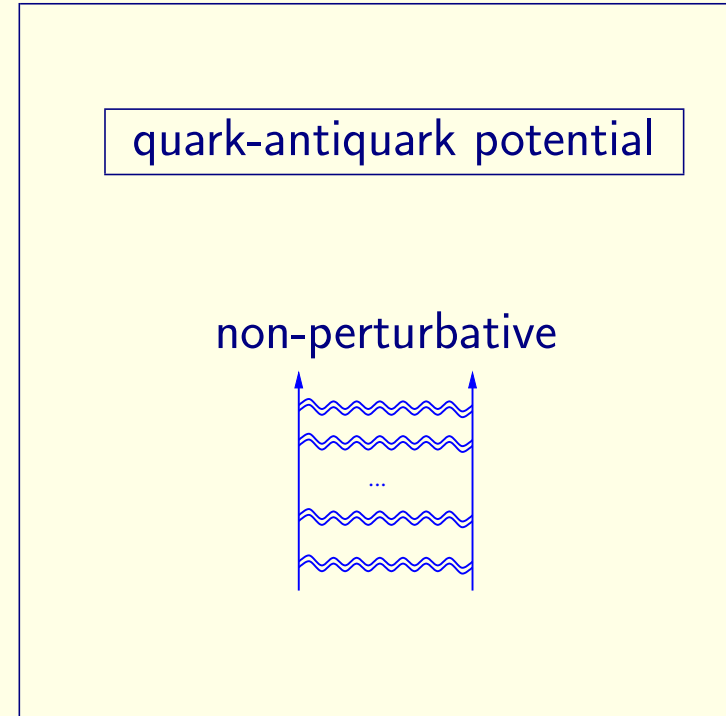
running coupl.: $\beta(\alpha_s) = \mu \frac{\partial \alpha_s}{\partial \mu} < 0$
asymptotic freedom, confinement



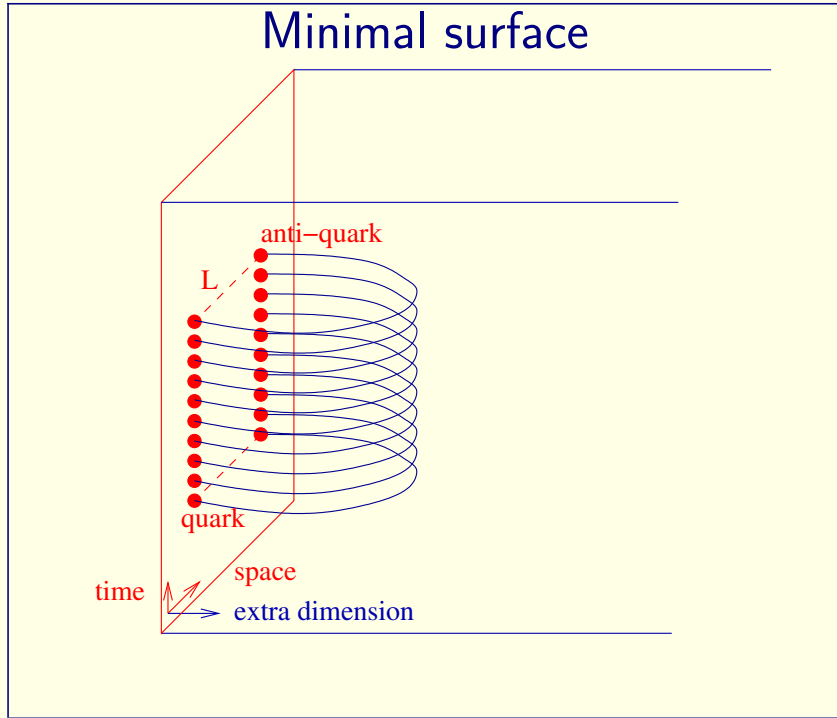
AdS/CFT correspondence: other explanation



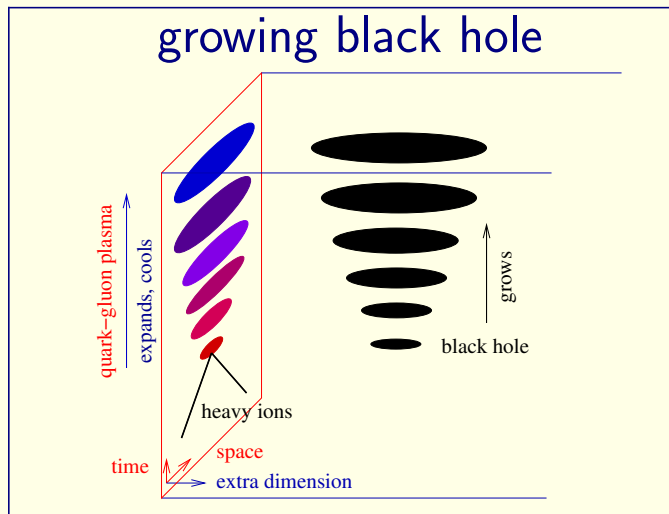
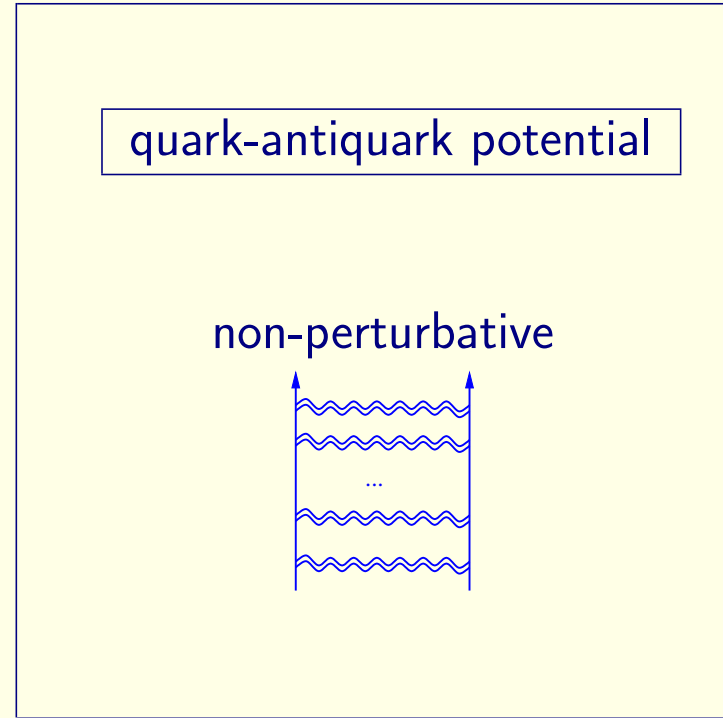
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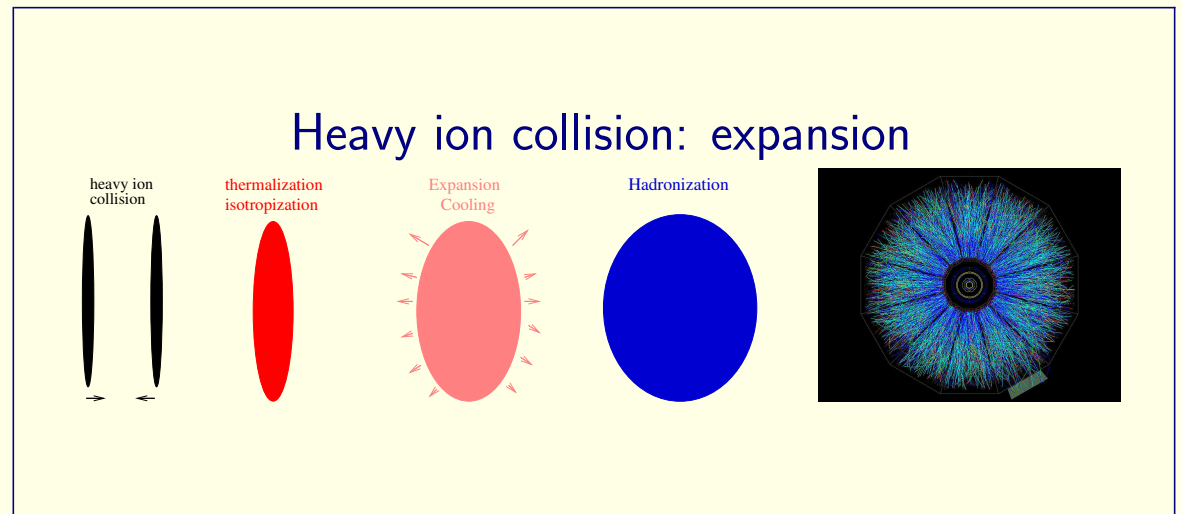
AdS/CFT correspondence: other explanation



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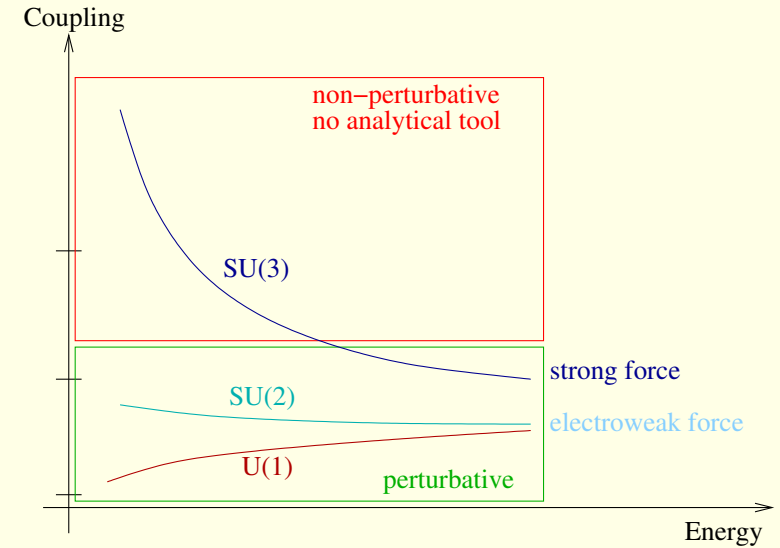


QCD \sim maximally supersymmetric gauge theory = CFT

Fundamental interactions

interaction	particles	gauge theory
electromagnetic	photon+electron	$U(1)$
electroweak	W^\pm, Z, μ, ν +Higgs	$SU(2) \times U(1)$
strong	gluon+quarks	$SU(3)$

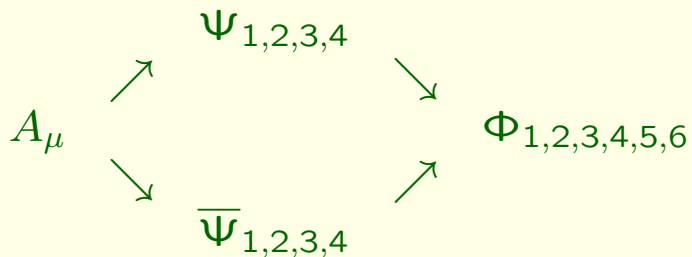
only analytical tool: perturbation theory



maximally supersymmetric ($\mathcal{N}=4$) gauge theory

interaction	particles	gauge theory
$\mathcal{N} = 4$ supersymm.	gluon+quarks+scalars	$SU(N)$

all fields $N^2 - 1$ component matrix



$$\mathcal{L} = \frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i \bar{\Psi} \not{D} \Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \bar{\Psi} [\Phi, \Psi]$$

no running $\beta = 0 \rightarrow$ CFT

no confinement

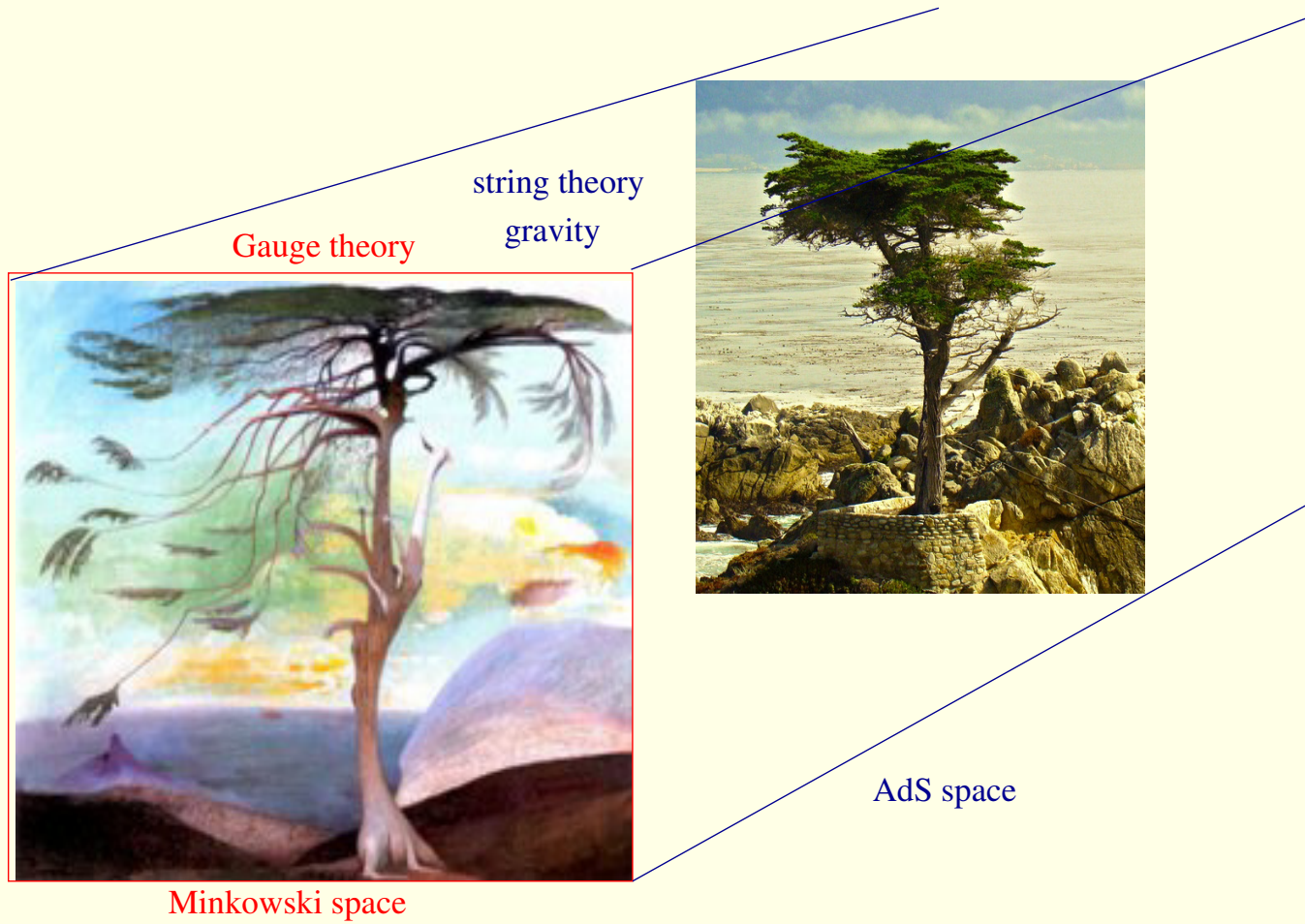
supersymmetric

heavy ion collision:

finite T \rightarrow SUSY is broken

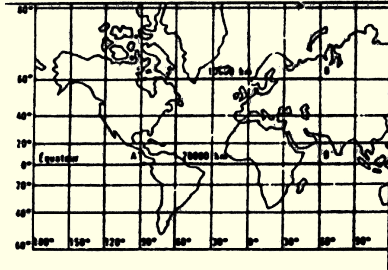
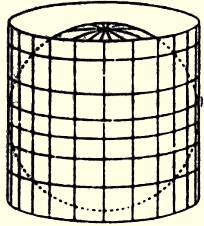
quark-gluon plasma is not confined

AdS/CFT correspondence (Maldacena 1998)



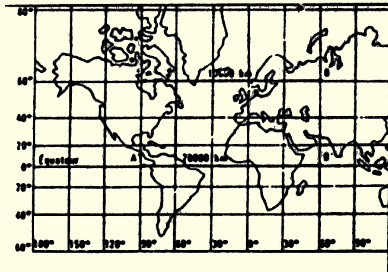
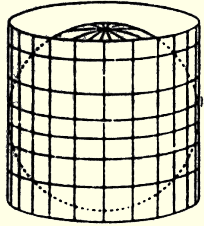
AdS: string theory on Anti de Sitter \supset gravitation

positively curved space

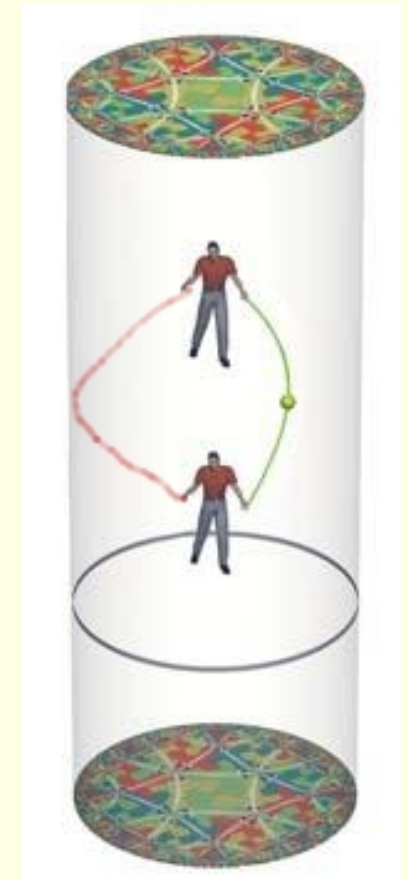


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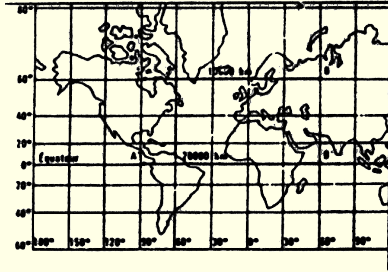
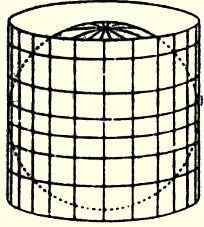


Anti de Sitter: negatively curved space

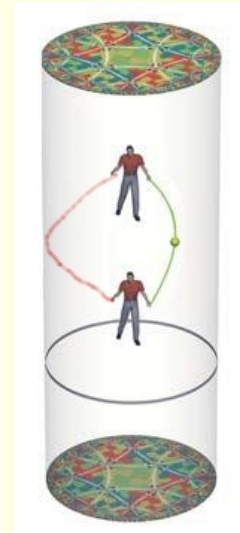


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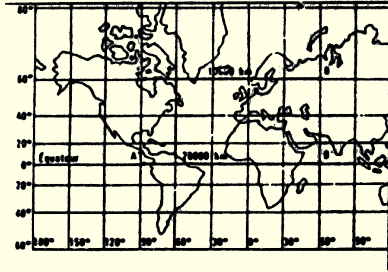
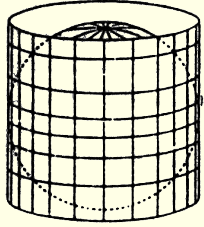


Anti de Sitter: negatively curved space



AdS: string theory on Anti de Sitter \supset gravitation

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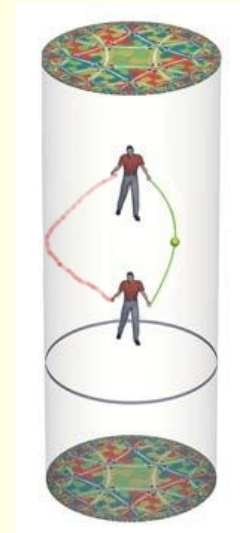
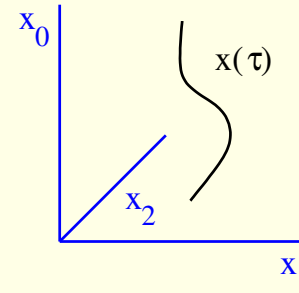


Anti de Sitter: negatively curved space



relativistic point particle: $ds^2 = -dx_0^2 + dx_1^2 + \dots$

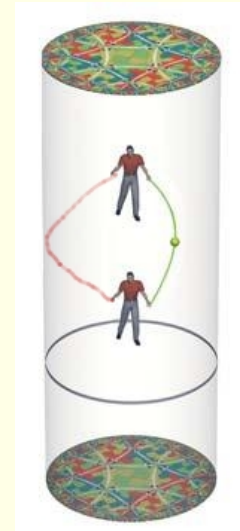
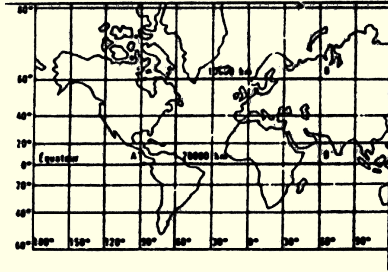
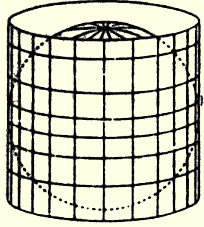
$S \propto \text{worldline} \propto \int ds = \int \sqrt{\dot{x} \cdot \dot{x}} d\tau$



AdS: string theory on Anti de Sitter \supset gravitation

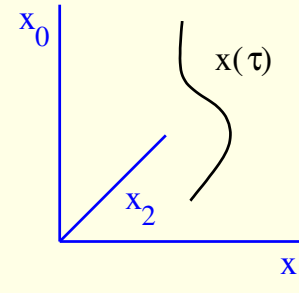
positively curved space

Anti de Sitter: negatively curved space



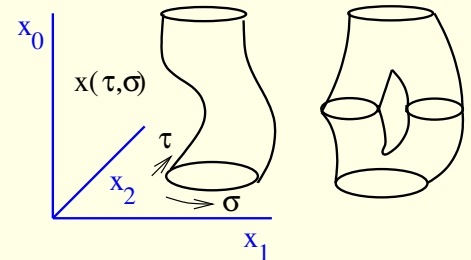
relativistic point particle: $ds^2 = -dx_0^2 + dx_1^2 + \dots$

$S \propto \text{worldline} \propto \int ds = \int \sqrt{\dot{x} \cdot \dot{x}} d\tau$



relativistic string: $ds^2 = -dx_0^2 + dx_1^2 + \dots$

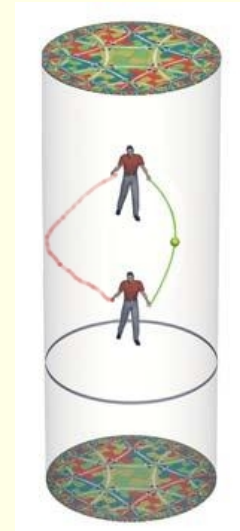
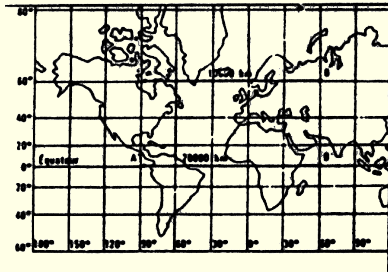
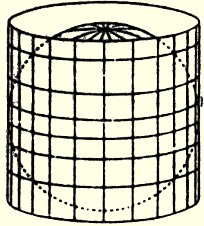
$S \propto \text{worldsheet} \propto \int dA = \int \sqrt{(\dot{x} \cdot x')^2 - \dot{x}^2 x'^2} d\tau d\sigma$



AdS: string theory on Anti de Sitter \supset gravitation

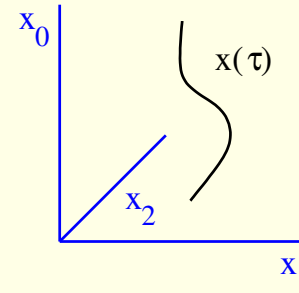
positively curved space

Anti de Sitter: negatively curved space



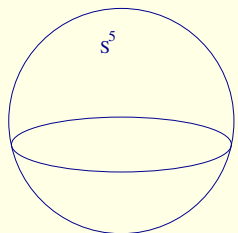
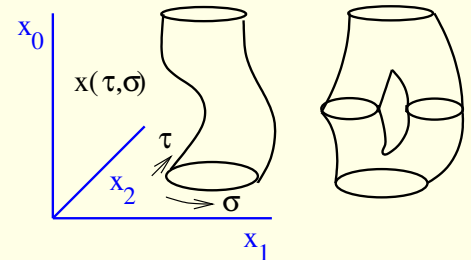
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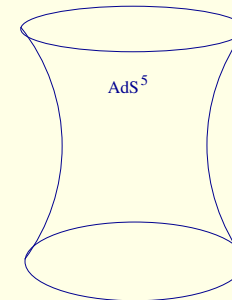
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$$S^5 : Y_0^2 + Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2 + Y_5^2 = R^2$$

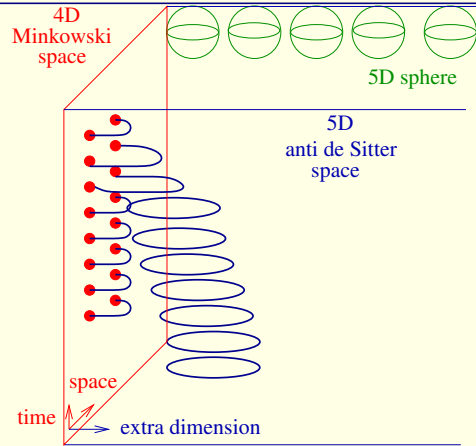
$$AdS_5 : -X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 - X_5^2 = -R^2$$



$$S = \frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} \left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M \right) + \text{fermionok} \quad \text{supercoset} \quad \frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$$

AdS/CFT correspondence (Maldacena 1998)

II_B superstring on AdS₅ × S⁵



$$\sum_1^6 Y_i^2 = R^2 \quad - + + + + - = -R^2$$

$$\frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} (\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M) + \dots$$

≡

N = 4 D=4 SU(N) SYM

$$\frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i\bar{\Psi} \not{D}\Psi + V \right]$$

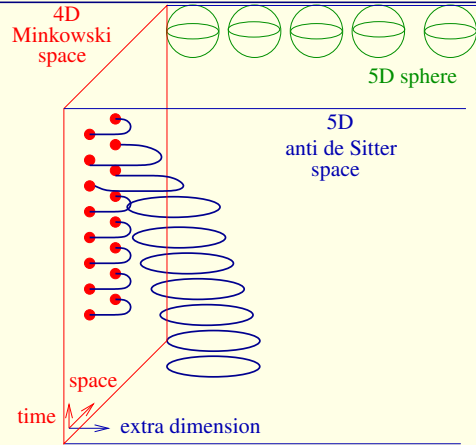
$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \bar{\Psi} [\Phi, \Psi]$$

β = 0 superconformal $\frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$

gaugeinvariants: $\mathcal{O} = \text{Tr}(\Phi^2), \det(\)$

AdS/CFT correspondence (Maldacena 1998)

II_B superstring on $AdS_5 \times S^5$



$$\sum_1^6 Y_i^2 = R^2 \quad - + + + + - = -R^2$$

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\equiv

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Couplings: $\sqrt{\lambda} = \frac{R^2}{\alpha'}$, $g_s = \frac{\lambda}{N} \rightarrow 0$

2D QFT

String energy levels: $E(\lambda)$

$$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \dots$$

Dictionary

strong \leftrightarrow weak

\Downarrow

$\lambda = g_{YM}^2 N$, $N \rightarrow \infty$ planar limit

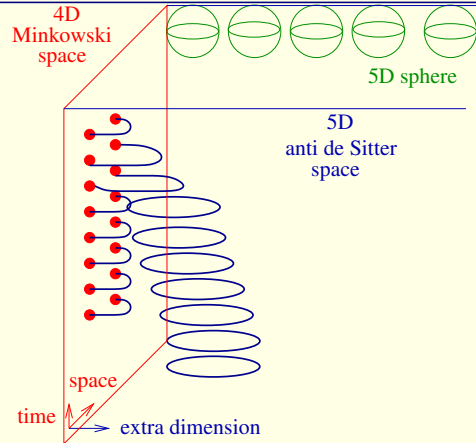
$$\langle \mathcal{O}_n(x) \mathcal{O}_m(0) \rangle = \frac{\delta_{nm}}{|x|^{2\Delta_n(\lambda)}}$$

Anomalous dim $\Delta(\lambda)$

$$\Delta(\lambda) = \Delta(0) + \lambda \Delta_1 + \lambda^2 \Delta_2 + \dots$$

AdS/CFT correspondence (Maldacena 1998)

II_B superstring on $AdS_5 \times S^5$



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Anomalous dim $\Delta(\lambda)$

$$\Delta(\lambda) = \Delta(0) + \lambda \Delta_1 + \lambda^2 \Delta_2 + \dots$$

2D integrable QFT

spectrum: $Q = 1, 2, \dots, \infty$ dispersion: $\epsilon_Q(p) = \sqrt{Q^2 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}}$
Exact scattering matrix: $S_{Q_1 Q_2}(p_1, p_2, \lambda)$

AdS/CFT correspondence: confirmation

supersymmetric **BPS** operators

$$V(\Phi, \Psi) = \frac{1}{4}[\Phi, \Phi]^2 + \bar{\Psi}[\Phi, \Psi]$$

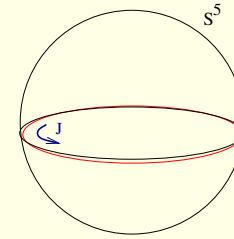
$$Z = \Phi_1 + i\Phi_2, X = \Phi_3 + i\Phi_4$$

$$\mathcal{O}_{BPS} = \text{Tr}(Z^J) \leftrightarrow |\uparrow\uparrow \dots \uparrow\rangle$$

$$\Delta_{BPS} = J$$

weak \leftrightarrow strong

BPS string configuration



$$E_{BPS}(\lambda) = J$$

AdS/CFT correspondence: confirmation

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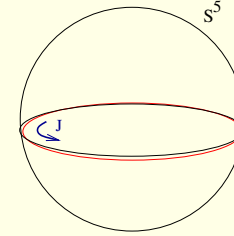
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2D integrable QFT

$$\text{supersymmetric groundstate } E_0(J) = \Delta(\lambda) - J = 0$$

AdS/CFT correspondence: confirmation

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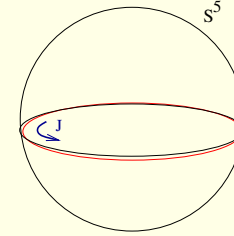
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BPS string configuration



$$E_{BPS}(\lambda) = J$$

2D integrable QFT

$$\text{supersymmetric groundstate } E_0(J) = \Delta(\lambda) - J = 0$$

Nontrivial anomalous dimension

supersymmetric theory: Excited state

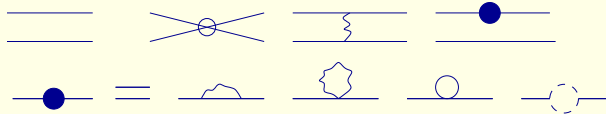
$$\mathcal{O}_K = \text{Tr}(ZXZX + \dots) \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle + \dots$$

Confirmation: excited state - Konishi operator

nonsupersymmetric operator: Konishi

$$\mathcal{O}_K = \text{Tr}(ZXZX + \dots) \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle + \dots$$

operator mixing



$$\Delta(\lambda) = \Delta(0) + \lambda\Delta_1 + \dots + \lambda^4\Delta_4 + \dots$$

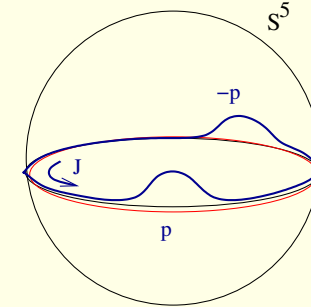


[Fiamberti ..'08]

$$\Delta_4 = -2496 + 576\zeta_3 - 1440\zeta_5$$

≡

string configuration

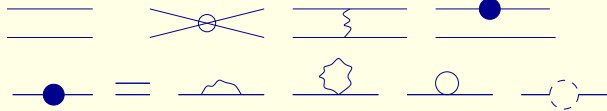


moving bumps (sine-Gordon) [Hofman .. '07]
string action=saddle point+loop corr.


$$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \dots$$

Confirmation: excited state - Konishi operator

nonsupersymmetric operator: Konishi
 $\mathcal{O}_K = \text{Tr}(ZXZX + \dots) \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle + \dots$
 operator mixing



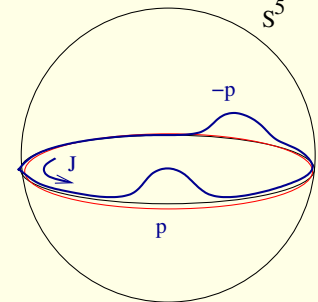
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[Fiamberti ..'08]
 $\Delta_4 = -2496 + 576\zeta_3 - 1440\zeta_5$

\equiv

string configuration



moving bumps (sine-Gordon) [Hofman .. '07]
 string action = saddle point + loop corr.
 $E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \dots$

two particle state

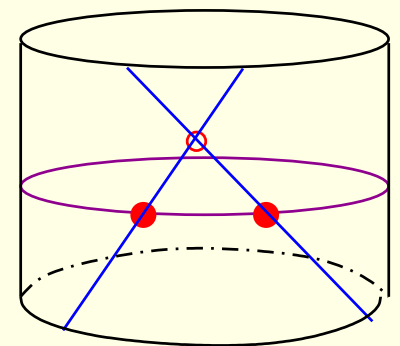
$$E = E_{BA} + E_{FSC}$$

Bethe Ansatz: $e^{ipJ} S(p, -p) = 1$

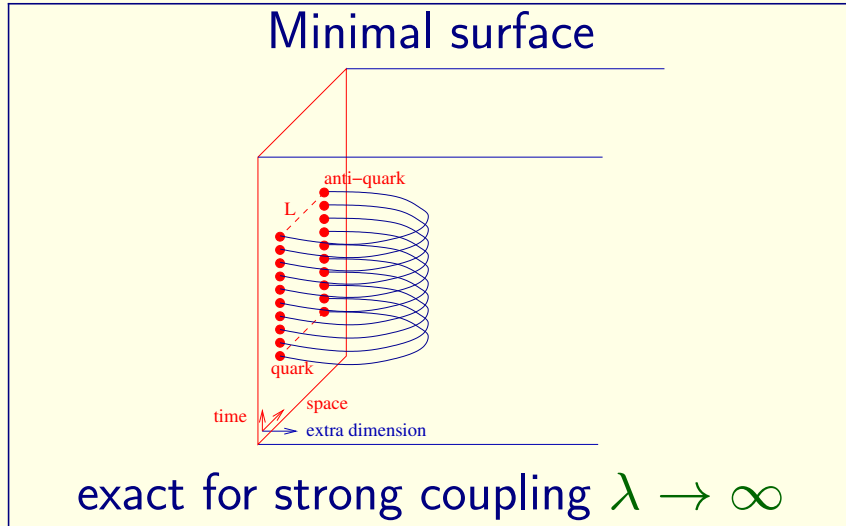
$$E_{BA} = 2E(p, \lambda) = 2\sqrt{1 + \frac{\lambda}{\pi^2} (\sin \frac{p}{2})^2}$$

$$E_{FSC} = \sum_Q \int \frac{dq}{2\pi} S_{Q1}(q, p) S_{Q1}(q, -p) e^{-\epsilon_Q L}$$

$E_4 = \Delta_4 = -2496 + 576\zeta_3 - 1440\zeta_5$ [Z.B., R. Janik '09]



AdS/CFT correspondence: applications



\equiv

quark-antiquark potential

Wilson loop: $\langle \oint_C A_\mu dx^\mu \rangle$
non-perturbative

$$V(r) = -\frac{4\pi^2 \sqrt{2\lambda} 1}{\Gamma(\frac{1}{4})^4 r}$$

AdS/CFT correspondence: applications

Minimal surface

exact for strong coupling $\lambda \rightarrow \infty$

quark-antiquark potential

≡

Wilson loop: $\langle \oint_C A_\mu dx^\mu \rangle$
non-perturbative

$$V(r) = -\frac{4\pi^2 \sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4} \frac{1}{r}$$

growing black hole

metric $\delta g(x, 0) \propto \langle T_{\mu\nu} \rangle$

$$ds^2 = \frac{1}{z^2} (g(x, z)_{\mu\nu} dx^\mu dx^\nu + dz^2)$$

Einstein equation

$$R_{ab} - \frac{1}{2} g_{ab} R - 6g_{ab} = 0$$

growing black hole

$$g_{tt} = -\frac{(1-z^4/z_0^4)^2}{(1+z^4/z_0^4)^2}; \quad g_{xx} = 1 + \frac{z^4}{z_0^4}$$

Heavy ion collision: expansion

≡

$\langle T_{\mu\nu} \rangle$ matter distribution
relativistic hydrodynamics
 $\partial_\mu T^{\mu\nu} = 0$ and $T^\mu_\mu = 0$
viscous quark-gluon plasma

expansion in time: perfect fluid + $\frac{\eta}{s} = \frac{1}{4\pi} + \dots$

Conclusion= Trust the AdS/CFT correspondence!

