Meson vacuum phenomenology: which scalar mesons are members of the meson nonet?

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- Motivation, scalar mesons
- QCDs chiral symmetry, effective models
- Axial(vector) meson extended linear σ-model
- Technical difficulty: particle mixing
- Tree-level masses, Decay widths
- Parametrization
- Particle identification
- Conclusions

Scalar mesons

	Mass (MeV)	width (MeV)	decays
$a_0(980)$	(980 ± 20)	$50 - 100$	$\pi\pi$ dominant
$a_0(1450)$	(1474 ± 19)	(265 ± 13)	$\pi\eta$, $\pi\eta'$, $K\bar{K}$
$K_0^\star(800) = \kappa$	(676 ± 40)	(548 ± 24)	$K\pi$
$K_0^\star(1430)$	(1425 ± 50)	(270 ± 80)	$K\pi$ dominant
$f_0(600) = \sigma$	$400 - 1200$	$600 - 1000$	$\pi\pi$ dominant
$f_0(980)$	(980 ± 10)	$40 - 100$	$\pi\pi$ dominant
$f_0(1370)$	$1200 - 1500$	$200 - 500$	$\pi\pi \approx 250$, $K\bar{K} \approx 150$
$f_0(1500)$	(1505 ± 6)	(109 ± 7)	$\pi\pi \approx 38$, $K\bar{K} \approx 9.4$
$f_0(1710)$	(1720 ± 6)	(135 ± 8)	$\pi\pi \approx 30$, $K\bar{K} \approx 71$

scalar nonet: a_0, K_0, $2\ f_0$ $\rightarrow$ pseudoscalar nonet: π, K, η, η'
Possible scalar states: $\bar{q}q$, $\bar{q}q\bar{q}q$, meson-meson molecules, glueballs
multiquark states: $f_0(980)$, $a_0(980)$, $f_0(600)$, $K_0^\star(800)$???
meson-meson bound state $(K\bar{K})$: $f_0(980)$???
glueballs: $f_0(1500)$, $f_0(1710)$???

Chiral symmetry

If the quark masses are zero (chiral limit) $\Longrightarrow$ QCD invariant under the following global transformation (chiral symmetry):

$$U(3)_L \times U(3)_R \simeq U(3)_V \times U(3)_A = SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$$

$U(1)_V$ term $\longrightarrow$ barion number conservation

$U(1)_A$ term $\longrightarrow$ broken through axial anomaly

$SU(3)_A$ term $\longrightarrow$ broken down by any quark mass

$SU(3)_V$ term $\longrightarrow$ broken down to $SU(2)_V$ if $m_u = m_d \neq m_s$ (isospin symmetry)
$\longrightarrow$ totally broken if $m_u \neq m_d \neq m_s$ (realized in nature)

Since QCD is very hard to solve $\longrightarrow$ low energy effective models can be set up $\longrightarrow$ reflecting the global symmetries of QCD $\longrightarrow$ degrees of freedom: observable particles instead of quarks and gluons

Linear realization of the symmetry $\longrightarrow$ linear sigma model
(nonlinear representation $\longrightarrow$ chiral perturbation theory (ChPT))

Vector meson extended linear sigma model

(based on: chiral symmetry + dilatation symmetry)

$$
\begin{aligned}
\mathcal{L}_{\text{vec}} &= \text{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2\text{Tr}(\Phi^\dagger\Phi) - \lambda_1[\text{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\text{Tr}(\Phi^\dagger\Phi)^2 \\
&- \frac{1}{4}\text{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \text{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + \text{Tr}[\hat{\epsilon}(\Phi + \Phi^\dagger)] \\
&+ c_1(\det\Phi - \det\Phi^\dagger)^2 + i\frac{g_2}{2}(\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
&+ \frac{h_1}{2}\text{Tr}(\Phi^\dagger\Phi)\text{Tr}(L_\mu^2 + R_\mu^2) + h_2\text{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\text{Tr}(L_\mu\Phi R^\mu\Phi^\dagger). \\
&+ g_3[\text{Tr}(L_\mu L_\nu L^\mu L^\nu) + \text{Tr}(R_\mu R_\nu R^\mu R^\nu)] + g_4[\text{Tr}\,(L_\mu L^\mu L_\nu L^\nu) + \text{Tr}\,(R_\mu R^\mu R_\nu R^\nu)] \\
&+ g_5\text{Tr}\,(L_\mu L^\mu)\,\text{Tr}\,(R_\nu R^\nu) + g_6[\text{Tr}(L_\mu L^\mu)\,\text{Tr}(L_\nu L^\nu) + \text{Tr}(R_\mu R^\mu)\,\text{Tr}(R_\nu R^\nu)],
\end{aligned}
$$

where

$$D^\mu\Phi = \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ieA^\mu[T_3, \Phi]$$

$$\Phi = \sum_{i=0}^{8} (\sigma_i + i\pi_i) T_i$$

$$R^\mu = \sum_{i=0}^{8} (\rho_i^\mu - b_i^\mu) T_i$$

$$L^\mu = \sum_{i=0}^{8} (\rho_i^\mu + b_i^\mu) T_i$$

$$L^{\mu\nu} = \partial^\mu L^\nu - ieA^\mu [T_3, L^\nu] - \{\partial^\nu L^\mu - ieA^\nu [T_3, L^\mu]\}$$

$$R^{\mu\nu} = \partial^\mu R^\nu - ieA^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - ieA^\nu [T_3, R^\mu]\}$$

$$\hat{\epsilon} = \sum_{i=0}^{8} \varepsilon_i T_i$$

$U(3)$ generators: $T_0 := \frac{1}{\sqrt{6}} \mathbf{1}$, $T_i = \frac{\lambda_i}{2}$ $i = 1 \dots 8$

determinant breaks $U_A(1)$ symmetry

explicit symmetry breaking: external fields $\varepsilon_0, \varepsilon_8 \neq 0 \Longleftrightarrow m_u = m_d \neq 0, m_s \neq 0$ or $\varepsilon_0, \varepsilon_3, \varepsilon_8 \neq 0 \Longleftrightarrow m_u \neq m_d \neq 0, m_s \neq 0$

non strange – strange base:

$$\varphi_N = \sqrt{2/3}\varphi_0 + \sqrt{1/3}\varphi_8,$$

$$\varphi_S = \sqrt{1/3}\varphi_0 - \sqrt{2/3}\varphi_8, \qquad \varphi \in (\sigma, \pi, \varepsilon)$$

broken symmetry: non-zero condensates $\langle \sigma_N \rangle, \langle \sigma_S \rangle \longleftrightarrow \phi_N, \phi_S$

Pseudoscalar and Scalar Meson nonets

$$\Phi_{PS} = \sum_{i=0}^{8} \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} \quad (\sim \bar{q}_i \gamma_5 q_j)$$

$$\Phi_{S} = \sum_{i=0}^{8} \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} \quad (\sim \bar{q}_i q_j)$$

Particle contents:

Pseudoscalars: $\pi(138), K(495), \eta(548), \eta'(958)$

Scalars: $a_0(980 \text{ or } 1450), K_0^\star(800 \text{ or } 1430)$,

(σ_N, σ_S) : 2 of $f_0(600, 980, 1370, 1500, 1710)$

Vector Meson nonets

$$V^\mu = \sum_{i=0}^{8} \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star+} \\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star0} \\ K^{\star-} & \bar{K}^{\star0} & \omega_S \end{pmatrix}^\mu$$

$$A_V^\mu = \sum_{i=0}^{8} b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

Particle contents:
Vector mesons: $\rho(770), K^\star(894), \omega_N = \omega(782), \omega_S = \phi(1020)$
Axial vectors: $a_1(1230), K_1(1270), f_{1N}(1280), f_{1S}(1426)$

Characteristics of the model

Parameters of the Lagrangian at $T = 0$:

$$m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_N, \delta_S, \Phi_N, \Phi_S$$

$\longrightarrow$ choose $\delta_N = 0 \longrightarrow 13$ unknown parameters

particles (mesons up to ~ 2 GeV):

- pseudoscalars: $\pi(138)$, $K(495)$, $\eta(548)$, $\eta'(958)$
- vectormesons: $\rho(770)$, $K^\star(894)$, $\omega(782)$, $\Phi(1020)$
- axialvector-mesons: $a_1(1230)$, $K_1(1270)$, $f_1(1280)$, $f_1(1426)$
- scalars: more physical states than we can describe:
  $2\ a_0$'s $(a_0(980), a_0(1450))$, $2\ K_S$'s $(K_0^\star(800), K_0^\star(1430))$,
  $5\ f_0$'s $(f_0(600), f_0(980), f_0(1370), f_0(1500), f_0(1710))$

technical difficulty: mixing in the $N - S$ sector

Spontaneous symmetry breaking and particle mixing

SSB $\longrightarrow$ through Higgs mechanism generates particle masses $\longrightarrow$ since vacuum has zero quantum numbers $\longrightarrow$ only $\sigma_0, \sigma_8, \sigma_3$ (equivalently $\sigma_N, \sigma_S, \sigma_3$) can have non-zero vev ($\sigma_3 \longrightarrow$ isospin violation $\longrightarrow$ neglected)

note: pion/kaon condensates $\longrightarrow$ even other σ's have non-zero expectation values ($\longrightarrow$ parity, charge violation)

shifting with vev in the Lagrangian: $\sigma_i \to \sigma_i + \phi_i$ ($\longrightarrow$ mass generation)

- For (pseudo)scalars this shifting results in particle mixing in the $N - S$ sector $\longrightarrow \sigma_N/\pi_N, \sigma_S/\pi_S$ fields are not mass eigenstates $\longrightarrow$ orthogonal transformations needed to resolve

- For (axial)vectors $\longrightarrow$ mixing between different nonets $\longrightarrow$ resolved by certain field shiftings

Mixing in the extended model

Making the $\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S}$ transformation in $\mathcal{L}_{\text{vec}}$

Quadratic terms after shifting:

$$\begin{aligned}
\mathcal{L}^{quad} = & -\frac{1}{2}\sigma_a(\partial^2\delta_{ab} + (m_\sigma^2)_{ab})\sigma_b - \frac{1}{2}\pi_a(\partial^2\delta_{ab} + (m_\pi^2)_{ab})\pi_b \\
& - \frac{1}{2}\rho_a^\mu \left[(-g_{\mu\nu}\partial^2 + \partial_\mu\partial_\nu)\delta_{ab} - g_{\mu\nu}(m_\rho^2)_{ab}\right]\rho_b^\nu \\
& - \frac{1}{2}b_a^\mu \left[(-g_{\mu\nu}\partial^2 + \partial_\mu\partial_\nu)\delta_{ab} - g_{\mu\nu}(m_b^2)_{ab}\right]b_b^\nu \\
& -\frac{1}{2}\rho_a^\mu(g_1 f_{abc} v_c \partial_\mu)\sigma_b - \frac{1}{2}\sigma_a(g_1 f_{abc} v_c \partial_\mu)\rho_b^\mu \\
& -\frac{1}{2}b_a^\mu(g_1 d_{abc} v_c \partial_\mu)\pi_b + \frac{1}{2}\pi_a(g_1 d_{abc} v_c \partial_\mu)b_b^\mu
\end{aligned}$$

Mixing in the $N-S$ sector for σ and $\pi \longrightarrow (m_\sigma^2)_{NS} \neq 0$, $(m_\pi^2)_{NS} \neq 0$

resolved by simple 2 dim. orthogonal transformations

Mixing between nonets $\longrightarrow \rho_a^\mu \leftrightarrow \sigma$ and $b_a^\mu \leftrightarrow \pi$

take a closer look $\longrightarrow$

Explicit form of nonet mixing crossterms:

$$-g_1\phi_N(f_{1N}^{\mu}\partial_\mu\eta_N+\vec{a}_1^{\mu}\cdot\partial_\mu\vec{\pi})-\sqrt{2}g_1\phi_S f_{1S}^{\mu}\partial_\mu\eta_S-\left(\frac{g_1}{\sqrt{2}}\phi_S+\frac{g_1}{2}\phi_N\right)\Big(K_1^{\mu 0}\partial_\mu\bar{K}^0$$
$$+K_1^{\mu+}\partial_\mu K^-+\text{h.c.}\Big)+\left(i\frac{g_1}{\sqrt{2}}\phi_S-i\frac{g_1}{2}\phi_N\right)\left(\bar{K}^{\star\mu 0}\partial_\mu K_S^0+K^{\star\mu -}\partial_\mu K_S^+\right)$$
$$+\left(-i\frac{g_1}{\sqrt{2}}\phi_S+i\frac{g_1}{2}\phi_N\right)\left(K^{\star\mu 0}\partial_\mu\bar{K}_S^0+K^{\star\mu+}\partial_\mu K_S^-\right)$$

Resolved by the following field shifts:

$$f_{1N/S}^{\mu}\longrightarrow f_{1N/S}^{\mu}+w_{f_{1N/S}}\partial^\mu\eta_{N/S},$$
$$a_1^{\mu+,0}\longrightarrow a_1^{\mu+,0}+w_{a_1}\partial^\mu\pi^{+,0},\ (+\text{h.c.})$$
$$K_1^{\mu+,0}\longrightarrow K_1^{\mu+,0}+w_{K_1}\partial^\mu K^{+,0},\ (+\text{h.c.})$$
$$K^{\star\mu+,0}\longrightarrow K^{\star\mu+,0}+w_{K^\star}\partial^\mu K_S^{+,0}(+\text{h.c.})$$

Vanishing of the crossterms $\longrightarrow$ determination of the w_i's

After these shifts, π, η_N, η_S, K, and K_S are not canonically normalized $\longrightarrow$ field renormalization $\longrightarrow$ renormalization factors: Z_π, Z_{η_N}, Z_{η_S}, Z_K, Z_{K_S}

Tree-level masses

Pseudoscalar mass squares:

$$m_\pi^2 = Z_\pi^2 \left[m_0^2 + \Lambda_N \Phi_N^2 + \lambda_1 \Phi_S^2\right]$$

$$m_K^2 = Z_K^2 \left[m_0^2 + \Lambda_N \Phi_N^2 - \frac{\lambda_2}{\sqrt{2}} \Phi_N \Phi_S + \Lambda_S \Phi_S^2\right]$$

$$m_{\eta_N}^2 = Z_\pi^2 \left[m_0^2 + \Lambda_N \Phi_N^2 + \lambda_1 \Phi_S^2 + c_1 \, \Phi_N^2 \Phi_S^2\right]$$

$$m_{\eta_S}^2 = Z_{\eta_S}^2 \left[m_0^2 + \lambda_1 \Phi_N^2 + \Lambda_s \Phi_S^2 + \frac{c_1}{4} \Phi_N^4\right]$$

$$m_{\eta_{NS}}^2 = Z_\pi Z_{\pi_S} \frac{c_1}{2} \Phi_N^3 \Phi_S$$

Scalar mass squares:

$$m_{a_0}^2 = m_0^2 + \Lambda_N' \Phi_N^2 + \lambda_1 \Phi_S^2$$

$$m_{K_S}^2 = Z_{K_S}^2 \left[m_0^2 + \Lambda_N \Phi_N^2 + \frac{\lambda_2}{\sqrt{2}} \Phi_N \Phi_S + \Lambda_S \Phi_S^2\right]$$

$$m_{\sigma_N}^2 = m_0^2 + 3\Lambda_N \Phi_N^2 + \lambda_1 \Phi_S^2$$

$$m_{\sigma_S}^2 = m_0^2 + \lambda_1 \Phi_N^2 + 3\Lambda_s \Phi_S^2$$

$$m_{\sigma_{NS}}^2 = 2\lambda_1 \Phi_N \Phi_S$$

Mass square eigenvalues for σ and π in the $N-S$ sector

$$m^2_{f_0^H/f_0^L} = \frac{1}{2}\left[m^2_{\sigma_N} + m^2_{\sigma_S} \pm \sqrt{(m^2_{\sigma_N} - m^2_{\sigma_S})^2 + 4m^2_{\sigma_{NS}}}\right]$$

$$m^2_{\eta'/\eta} = \frac{1}{2}\left[m^2_{\eta_N} + m^2_{\eta_S} \pm \sqrt{(m^2_{\eta_N} - m^2_{\eta_S})^2 + 4m^2_{\eta_{NS}}}\right]$$

Vector mass squares:

$$m^2_\rho = m^2_1 + \frac{1}{2}(h_1 + h_2 + h_3)\Phi^2_N + \frac{h_1}{2}\Phi^2_S + 2\delta_N$$

$$m^2_{K^\star} = m^2_1 + H_N\Phi^2_N + \frac{1}{\sqrt{2}}\Phi_N\Phi_S(h_3 - g^2_1) + H_S\Phi^2_S + \delta_N + \delta_S$$

$$m^2_{\omega_N} = m^2_\rho$$

$$m^2_{\omega_S} = m^2_1 + \frac{h_1}{2}\Phi^2_N + \left(\frac{h_1}{2} + h_2 + h_3\right)\Phi^2_S + 2\delta_S$$

Axialvector meson mass squares:

$$m^2_{a_1} = m^2_1 + \frac{1}{2}(2g^2_1 + h_1 + h_2 - h_3)\Phi^2_N + \frac{h_1}{2}\Phi^2_S + 2\delta_N$$

$$m^2_{K_1} = m^2_1 + H_N\Phi^2_N - \frac{1}{\sqrt{2}}\Phi_N\Phi_S(h_3 - g^2_1) + H_S\Phi^2_S + \delta_N + \delta_S$$

$$m^2_{f_{1N}} = m^2_{a_1}$$

$$m^2_{f_{1S}} = m^2_1 + \frac{h_1}{2}\Phi^2_N + \left(2g^2_1 + \frac{h_1}{2} + h_2 - h_3\right)\Phi^2_S + 2\delta_S$$

Decay widths

For a $A \to BC$ decay process the decay with is:

$$\Gamma_{A\to BC} = \frac{k}{8\pi m_A^2} \left|\mathcal{M}_{A\to BC}\right|^2$$

$k \longrightarrow$ three momentum of the produced particles in the restframe of A
$\mathcal{M}_{A\to BC} \longrightarrow$ transition matrix element

If A is a vector particle and $C = B^\dagger \Longrightarrow$

$$\left|\mathcal{M}_{A\to BB^\dagger}\right|^2 = \frac{4}{3} k^2 V_\mu V^{\mu\star}$$

$V_\mu \longrightarrow$ vertex function directly followed from the three-coupling terms of $\mathcal{L}$

If A is a vector particle, B scalar and $C = \gamma$ a photon $\Longrightarrow$

$$\left|\mathcal{M}_{A\to B\gamma}\right|^2 = \frac{1}{3}\left(g^{\alpha\beta} - \frac{k_A^\alpha k_A^\beta}{m_A^2}\right) V_{\alpha\alpha'} V_\beta^{\star\alpha'}$$

Some decay widths in the extended model

The $\rho \to \pi\pi$ decay width:

$$\Gamma_{\rho\to\pi\pi} = \frac{m_\rho^5}{48\pi m_{a_1}^4}\left[1-\left(\frac{2m_\pi}{m_\rho}\right)^2\right]^{3/2}\left[g_1 Z_\pi^2 - \frac{g_2}{2}\left(Z_\pi^2-1\right)\right]^2$$

The experimental value from the PDG: $\Gamma^{(\text{exp})}_{\rho\to\pi\pi} = (149.1 \pm 0.8)$ MeV

The $a_1 \to \pi\gamma$ decay width:

$$\Gamma_{a_1\to\pi\gamma} = \frac{e^2 g_1^2 \Phi_N^2}{96\pi m_{a_1}} Z_\pi^2 \left[1-\left(\frac{m_\pi}{m_{a_1}}\right)^2\right]^3$$

The experimental value: $\Gamma^{(\text{exp})}_{a_1\to\pi\gamma} = (0.640 \pm 0.246)$ MeV

Parametrization: general considerations

In order to make predictions $\longrightarrow$ unknown constants of the model must be determined

$\Longrightarrow$ choose a set of (well known) physical quantities/conditions for fitting procedure

For instance:

- PartiallyConservedAxialCurrent $\longrightarrow$ fix the condensates (2 parameter)
- Particle masses (which can be compared with PDG (K. Nakamura et al., J. Phys. G **37**, 075021 (2010)))
- Decay widths (which can be compared with PDG)

Finding a good parameter set $\longrightarrow$ non-trivial task (usually there are lots of solutions, but non of them is perfect)

Parametrization in the extended model

13 unknown parameters $\longrightarrow$ Determined by the minimalization of the χ^2:

$$\chi^2(x_1,\ldots,x_N) = \sum_{i=1}^{M} \left[\frac{Q_i(x_1,\ldots,x_N) - Q_i^{\text{exp}}}{\delta Q_i} \right]^2 ,$$

where $(x_1,\ldots,x_N) = (m_0,\lambda_1,\lambda_2,\ldots)$, $Q_i(x_1,\ldots,x_N)$ calculated from the model, while $Q_i^{\text{exp}} \pm \delta Q_i$ taken from the PDG

multiparametric minimalization $\longrightarrow$ MINUIT

- PCAC $\rightarrow 2$ physical quantities: f_π, f_K
- Tree-level masses $\rightarrow 14$ physical quantities: $m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{K_1}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\rightarrow 12$ physical quantities: $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi}, \Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$

The question: which a_0, $K_0^\star$ and f_0s belong to the scalar nonet?

Particle identification, results

In the first step f_0 mesons were left out $\rightarrow$ their properties are very uncertain (Different analyses give different results)

First run $\rightarrow$ which pairs of $a_0, K_0^\star$ give acceptable fits

Then we continue by studying which pair of f_0's can be described better

13 parameters to fit 28 measured quantities

For η we have never found a very good fit ($\rightarrow$ due to: very small error of m_η + functional connection to m_K)

The best solution is:
$a_0(1450), K_0^\star(1430)$ and $f_0(1370), f_0(1710)$

$f_0(600)$ and $f_0(980)$ might be tetraquarks
$(f_0(1370), f_0(1500), f_0(1710))$: mixing of glueball and the 2 states above

$\chi^2 = 49.09 \rightarrow$ best solution

$K_S \rightarrow K_0^\star(1430)$
$a_0 \rightarrow$ just between the two a_0's
$f_0^L \rightarrow f_0(600)$
$f_0^H \rightarrow f_0(1370)$

Qty	PDG [GeV]	Fit [GeV]	χ^2
f_π	0.0922 ± 0.0009	0.0918	0.1910
f_K	0.1100 ± 0.0011	0.1106	0.2920
m_π	0.1380 ± 0.0026	0.1385	0.0357
m_η	0.5479 ± 0.0055	0.5298	10.9518
$m_{\eta'}$	0.9578 ± 0.0096	0.9674	1.0175
m_K	0.4956 ± 0.0050	0.5056	4.0506
m_ρ	0.7755 ± 0.0078	0.7708	0.3690
m_Φ	1.0195 ± 0.0102	1.0134	0.3525
$m_{K^\star}$	0.8938 ± 0.0089	0.9019	0.8195
m_{a_1}	1.2300 ± 0.0400	1.1636	2.7543
$m_{f_1^H}$	1.4264 ± 0.0143	1.4088	1.5242
m_{K_1}	1.2720 ± 0.0127	1.2909	2.2136
m_{a_0}	1.4740 ± 0.0737	1.2007	13.7494
m_{K_s}	1.4250 ± 0.0713	1.3128	2.4806
$m_{f_0^L}$	0.6000 ± 0.2000	0.8940	2.1612
$m_{f_0^H}$	1.3700 ± 0.1500	1.3642	0.0015
$\Gamma_{\rho\rightarrow\pi\pi}$	0.149100 ± 0.007455	0.156776	1.060158
$\Gamma_{\Phi\rightarrow KK}$	0.001770 ± 0.000089	0.001684	0.944766
$\Gamma_{K^\star\rightarrow K\pi}$	0.046200 ± 0.002310	0.045459	0.103017
$\Gamma_{a_1\rightarrow\pi\gamma}$	0.000640 ± 0.000250	0.000605	0.019423
$\Gamma_{a_1\rightarrow\rho\pi}$	0.425000 ± 0.175000	0.551275	0.520668
$\Gamma_{f_1\rightarrow KK^\star}$	0.043900 ± 0.002195	0.043896	0.000003
Γ_{a_0}	0.265000 ± 0.013250	0.267903	0.047995
$\Gamma_{K_S\rightarrow K\pi}$	0.270000 ± 0.080000	0.348810	0.970466
$\Gamma_{f_0^L\rightarrow\pi\pi}$	0.800000 ± 0.200000	0.451370	3.038596
$\Gamma_{f_0^L\rightarrow KK}$	0.000000 ± 0.100000	0.000000	0.000000
$\Gamma_{f_0^H\rightarrow\pi\pi}$	0.250000 ± 0.100000	0.278108	0.079004
$\Gamma_{f_0^H\rightarrow KK}$	0.150000 ± 0.100000	0.258269	1.172223

$\chi^2 = 59.66 \rightarrow$ second best solution

$$K_S \rightarrow K_0^\star(1430)$$
$$a_0 \rightarrow a_0(1450)$$
$$f_0^L \rightarrow f_0(1370)$$
$$f_0^H \rightarrow f_0(1710)$$

In this solution there are no identification problems $\rightarrow$ physically the best solution

Qty	PDG [GeV]	Fit [GeV]	χ^2
f_π	0.0922 ± 0.0009	0.0925	0.1173
f_K	0.1100 ± 0.0011	0.1096	0.1103
m_π	0.1380 ± 0.0026	0.1390	0.1432
m_η	0.5479 ± 0.0055	0.5265	15.2159
$m_{\eta'}$	0.9578 ± 0.0096	0.9677	1.0668
m_K	0.4956 ± 0.0050	0.5039	2.8260
m_ρ	0.7755 ± 0.0078	0.7672	1.1528
m_Φ	1.0195 ± 0.0102	1.0140	0.2880
$m_{K^\star}$	0.8938 ± 0.0089	0.8999	0.4698
m_{a_1}	1.2300 ± 0.0400	1.1789	1.6338
$m_{f_1^H}$	1.4264 ± 0.0143	1.4051	2.2211
m_{K_1}	1.2720 ± 0.0127	1.2964	3.6703
m_{a_0}	1.4740 ± 0.0737	1.4417	0.1920
m_{K_s}	1.4250 ± 0.0713	1.5365	2.4511
$m_{f_0^L}$	1.3700 ± 0.1500	1.2141	1.0802
$m_{f_0^H}$	1.7200 ± 0.0860	1.5841	2.4960
$\Gamma_{\rho\to\pi\pi}$	0.149100 ± 0.007455	0.166519	5.459288
$\Gamma_{\Phi\to KK}$	0.001770 ± 0.000089	0.001544	6.518627
$\Gamma_{K^\star\to K\pi}$	0.046200 ± 0.002310	0.044303	0.674290
$\Gamma_{a_1\to\pi\gamma}$	0.000640 ± 0.000250	0.000650	0.001727
$\Gamma_{a_1\to\rho\pi}$	0.425000 ± 0.175000	0.736729	3.173052
$\Gamma_{f_1\to KK^\star}$	0.043900 ± 0.002195	0.043789	0.002548
Γ_{a_0}	0.265000 ± 0.013250	0.253140	0.801181
$\Gamma_{K_S\to K\pi}$	0.270000 ± 0.080000	0.350839	1.021096
$\Gamma_{f_0^L\to\pi\pi}$	0.250000 ± 0.100000	0.122365	1.629078
$\Gamma_{f_0^L\to KK}$	0.150000 ± 0.100000	0.125730	0.058903
$\Gamma_{f_0^H\to\pi\pi}$	0.029700 ± 0.006500	0.031280	0.059121
$\Gamma_{f_0^H\to KK}$	0.071400 ± 0.029100	0.141566	5.813976

Conclusions and outlook

- With multiparametric χ^2 minimalization, the meson assignment to a $q\bar{q}$ state can be constrained
- According to the model the $a_0(q\bar{q})$ must be assigned to $a_0(1450)$, while the $K_S(q\bar{q})$ to $K_0^\star(1430)$
- It seems that most probably the two f_0's are both above 1 GeV, namely they should be assigned to $f_0(1370)$ and $f_0(1710)$
- In the case when one of the f_0 is below 1 GeV, the only possibility is $f_0^L = f_0(600)$. However in this case the assignment of a_0 becomes problematic.

details in: arXiv:1208.0585